A method of computing many functions simultaneously by using many parallel quantum systems

K. Nagata,[1] G. Resconi,[2] T. Nakamura,[3] J. Batle,[4] S. Abdalla,[5] A. Farouk,[6,7,8] and H. Geurdes[9]

[1] *Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 34141, Korea*
[2] *Department of Mathematics and Physics, Catholic University, Brescia, Italy, I-25121*
[3] *Department of Information and Computer Science, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan*
[4] *Departament de Física, Universitat de les Illes Balears, 07122 Palma de Mallorca, Balearic Islands, Europe*
[5] *Department of Physics, Faculty of Science, King Abdulaziz University Jeddah, P.O. Box 80203, Jeddah 21589, Saudi Arabia*
[6] *Computer Sciences Department, Faculty of Computers and Information, Mansoura University*
[7] *University of Science and Technology, Zewail City of Science and Technology, Giza, Egypt*
[8] *Scientific Research Group, Egypt*
[9] *Geurdes datascience, 2593 NN, 164, Den Haag, Netherlands*

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We suggest a method of computing many functions in the same time by using many parallel quantum systems. We use the Bernstein-Vazirani algorithm. Given the set of real values $\{a_1, a_2, a_3, \ldots, a_N\}$, and the function $g : \mathbf{R} \to \{0, 1\}$, we shall determine the following values $\{g(a_1), g(a_2), g(a_3), \ldots, g(a_N)\}$ simultaneously. By using M parallel quantum systems, we can compute M functions $g^1, g^2, ..., g^M$ simultaneously. The speed of determining the $N \times M$ values will be shown to outperform the classical case by a factor of N.

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Quantum mechanics [1–6] provides exact and frequently remarkably accurate numerical predictions, as reported for over a century. There has been a remarkable link in recent decades between the quantum theory and information theory, givin rise to the rich field of quantum information theory (QIT), which novel proposals that outperform classical tasks or simply have no classical counterpart [6].

One case that involves both quantum theory and information theory that can be found in the foundations of the quantum theory is the Leggett-type non-local variables theory [7], which has been experimentally explored [8–10]. These experiments report that quantum theory does not accept a Leggett-type non-local variables interpretation, although some controversy remains around the conclusions and interpretations of the experimental outcomes [11–13].

Applications of QIT also include the implementation of quantum algorithms. One such case is provided for instance by the Deutsch's problem [14], first experimetally realized on a nuclear magnetic resonance proof-of-principle quantum computer [15]. Implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer has also been achieved [16]. There have been, as well, several other attempts to use single-photon two-qubit states for quantum computing. Oliveira *et al* implemented the Deutsch's algorithm with polarization and transverse electromagnetic spatial modes as qubits [17]. Other achievements also include single-photon Bell states preparation and measurement [18], a decoherence-free implementation of Deutsch's algorithm using single-photon and using two logical qubits [19] and, more recently, a one-way quantum computing implementation of the algorithm[20].

These achievements involving the Deutsch-Jozsa algorithm are very well related to the so called Bernstein-Vazirani algorithm [21, 22], which can be considered as an extended version of the previous one. After these two algorithms, Simon's algorithm [23] was discovered, among others. There has been an experimental implementation of a quantum algorithm that solves the Bernstein-Vazirani parity problem without entanglement [24]. Additionally, fiber-optics implementations of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits have been realized [25]. Also, a variant of the algorithm for quantum learning being robust against noise has been introduced [26], as well as a quantum algorithm for approximating the influences of Boolean functions and its applications [27]. The Bernstein-Vazirani algorithm has been also versatile in quantum key distribution [28] and transport implementation with ion qubits [29].

In the present work we suggest a method of computing many functions in the same time by using many parallel quantum systems. We use the Bernstein-Vazirani algorithm. Given the set of real values $\{a_1, a_2, a_3, \ldots, a_N\}$, and the function $g : \mathbf{R} \to \{0, 1\}$, we shall determine the following values $\{g(a_1), g(a_2), g(a_3), \ldots, g(a_N)\}$ simultaneously. By using M parallel quantum systems, we can compute M functions $g^1, g^2, ..., g^M$ simultaneously. The speed of determining the $N \times M$ values will be shown to outperform the classical case by a factor of N.

Let us suppose that we are given the following sequence of real values

$$a_1, a_2, a_3, \dots, a_N. \tag{1}$$

Let us now introduce the function

$$g : \mathbf{R} \to \{0, 1\}. \tag{2}$$

One step is to determine the following values:

$$g(a_1), g(a_2), g(a_3), \dots, g(a_N). \tag{3}$$

Recall that in the classical case, we need N queries, that is, N separate evaluations of the function (2). In our quantum algorithm, we shall require a single query. Suppose now that we introduce another function

$$f : \{0, 1\}^N \to \{0, 1\}, \tag{4}$$

which is a function with a N-bit domain and a 1-bit range. We construct the following function:

$$\begin{aligned} f(x) &= g(a) \cdot x = \sum_{i=1}^{N} g(a_i) x_i (\mathrm{mod} 2) \\ &= g(a_1)x_1 \oplus g(a_2)x_2 \oplus g(a_3)x_3 \oplus \cdots \oplus g(a_N)x_N, \\ & x_i \in \{0, 1\}^N, g(a_i) \in \{0, 1\}, a_i \in \mathbf{R}, \end{aligned} \tag{5}$$

where a_i is a real value. Here $g(a)$ symbolizes

$$g(a_1)g(a_2)\cdots g(a_N). \tag{6}$$

Let us follow the quantum states through the algorithm. The input state is

$$|\psi_0\rangle = |0\rangle^{\otimes N} |1\rangle. \tag{7}$$

After the Hadamard transform on the state, we have

$$|\psi_1\rangle = \sum_{x \in \{0,1\}^N} \frac{|x\rangle}{\sqrt{2^N}} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]. \tag{8}$$

Next, the function f is evaluated using

$$U_f : |x, y\rangle \to |x, y \oplus f(x)\rangle, \tag{9}$$

giving

$$|\psi_2\rangle = \pm \sum_x \frac{(-1)^{f(x)}|x\rangle}{\sqrt{2^N}} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]. \tag{10}$$

After the Hadamard transform, using the previous equation and (10) we can now evaluate $|\psi_3\rangle$,

$$|\psi_3\rangle = \pm \sum_z \sum_x \frac{(-1)^{x \cdot z + f(x)}|z\rangle}{2^N} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]. \tag{11}$$

Thus,

$$|\psi_3\rangle = \pm \sum_z \sum_x \frac{(-1)^{x \cdot z + g(a) \cdot x}|z\rangle}{2^N} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right]. \tag{12}$$

We notice

$$\sum_x (-1)^{x \cdot z + g(a) \cdot x} = 2^N \delta_{g(a), z}. \tag{13}$$

Thus,

$$\begin{aligned} |\psi_3\rangle &= \pm \sum_z \sum_x \frac{(-1)^{x \cdot z + g(a) \cdot x}|z\rangle}{2^N} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] \\ &= \pm \sum_z \frac{2^N \delta_{g(a), z}|z\rangle}{2^N} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] \\ &= \pm |g(a)\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] \\ &= \pm |g(a_1)g(a_2)\cdots g(a_N)\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right], \end{aligned} \tag{14}$$

from which

$$|g(a_1)g(a_2)\cdots g(a_N)\rangle. \tag{15}$$

can be obtained. That is to say, if we measure $|g(a_1)g(a_2)\cdots g(a_N)\rangle$ then we can retrieve the following values

$$g(a_1), g(a_2), g(a_3), \dots, g(a_N) \tag{16}$$

using a single query. All we have to do is to perform one quantum measurement.

The speed to determine N values improves by a factor of N as compared to the classical counterpart. Notice that we recoiver the Bernstein-Vazirani algorithm when $g : a_i \to a_i$.

We suggest a method to compute M functions in the same time as many parallel quantum systems. By using M parallel quantum systems, we can compute M functions $g^1, g^2, ..., g^M$ simultaneously. That is, we can retrieve the following values

$$g^1(a_1), g^1(a_2), g^1(a_3), \dots, g^1(a_N), \tag{17}$$

$$g^2(a_1), g^2(a_2), g^2(a_3), \dots, g^2(a_N), \tag{18}$$

$\cdots$

$$g^M(a_1), g^M(a_2), g^M(a_3), \dots, g^M(a_N) \tag{19}$$

In the case, we measure the following quantum state:

$$\begin{aligned} &|g^1(a_1)g^1(a_2)\cdots g^1(a_N)\rangle \otimes \\ &|g^2(a_1)g^2(a_2)\cdots g^2(a_N)\rangle \otimes \\ &\cdots \otimes |g^M(a_1)g^M(a_2)\cdots g^M(a_N)\rangle. \end{aligned} \tag{20}$$

All we have to do is to perform one quantum measurement. [30].

In conclusions, we have presented a generalization of the Bernstein-Vazirani algorithm. And by using the new quantum algorithm, we have suggested a new method to compute M functions in the same time as M parallel quantum systems. The speed of computing the $N \times M$ values will be shown to outperform the classical case by a factor of N.

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