Why

$\rho+3p = 0$

is the equation of state of the universe?

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Abstract

Some thirty years ago, the model of a closed cosmos expanding at the speed of light was developed deriving from basic quantum-theoretical arguments. In this model, the equation of state reads $\rho+3p=0$ (ρ and p denoting cosmological energy density and pressure). At the core of the model, there are the quantum bits of absolute quantum information (AQI), the simplest of all possible quantum structures. Taken as the ultimate substance, referred to as protyposis, the AQI concept affords a unifying explanation of the structure of space and time and the evolution of the cosmos and its inventory.

Recent astrophysical investigations show that the protyposis model is in better agreement with the observation data than the present standard flat - Λ CDM model. *Ad hoc* assumptions such as Dark Energy and Inflation are dispensable.

The protyposis theory has been advanced to account for the formation of relativistic quantum particles from AQI bits. Moreover, a rationalization has been given of the General Theory of Relativity and of the three non-gravitational forces.

Ultimately, the protyposis concept will allow us to understand the emergence of both matter and consciousness.

Keywords: State Equation, Cosmology, Quantum Information, Protyposis, Dark Energy, Dark Matter, Cosmological Term, Non-Accelerated Cosmic Expansion, Fundamental Interactions

1. Introduction

Recent examinations of extensive astronomic data sets[1, 2, 3, 4, 5, 6] have shown that the best agreement with the data is afforded by a model referred to as $R_h=ct$ universe by the authors of these studies. It is thus a universe expanding at constant speed, the speed of light.

In the $R_h=ct$ universe, problems arising in the previous flat-ΛCDM standard model are absent. As the authors of Ref.[7] put it:

> „Cosmological models with a geometry different from that in the current standard model have fallen out of favour and are rarely considered in ongoing tests using the latest high-precision measurements. However, even within the framework of the standard model, not all the data fit together tension free. At least some controversy still surrounds the interpretation of various measurements, and other competing models often fit at least some of these observations better than the concordance model does. It is therefore useful to re-examine how these alternative scenarios fare compared to ΛCDM when new, improved data become available."

> „But whereas this optimization of parameters in ΛCDM/*w*CDM creates some tension with their concordance values, the $R_\mathrm{h}=ct$ universe has the advantage of fitting the QSO and AP data without any free parameters."

This is corroborated by other authors:

> "In contrast to the perception based on Type Ia SNe that ΛCDM can best account for the observed expansion of the Universe, the conclusion from these other studies is that the cosmic dynamics is better described by a cosmology we refer to as the $\mathrm{R_h}$ = ct Universe." [8]

Another quote reads:

> "As we shall show here, the use of this diagnostic, […], disfavors the current concordance (ΛCDM) model at 2.3σ. Within the context of expanding Friedmann-Robertson-Walker (FRW) cosmologies, these data instead favor the zero active mass equation-of-state, $\rho + 3p = 0$, where ρ and p are, respectively, the total density and pressure of the cosmic fluid, the basis for the $\mathrm{R_h}$ = ct universe." [9]

See also the conclusions in other work[10], confirmed by Ref.[11]:

> "The 'standard' model of cosmology is founded on the basis that the expansion rate of the universe is accelerating at present — as was inferred originally from the Hubble diagram of Type Ia supernovae. There exists now a much bigger database of supernovae so we can perform rigorous statistical tests to check whether these 'standardisable candles' indeed indicate cosmic acceleration. Taking account of the empirical procedure by which corrections are made to their absolute magnitudes to allow for the varying shape of the light curve and extinction by dust, we find, rather surprisingly, that the data are still quite consistent with a constant rate of expansion."

However, a constant expansion is not sufficient yet. Essential for the interpretation of the data is the equation of state $\rho+3p=0$. The Authors call this as „the zero-active mass condition". This condition is critical to the whole picture. There are various ways of attaining a constant expansion rate in the Universe including the famous Milne cosmology (i.e., an empty Universe). However, only this one produces measurable quantities, such as the Hubble expansion rate and luminosity distance, that are consistent with the observations. All the other constant-expansion rate scenarios have been ruled out at a very high statistical significance.[12]

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[1] López-Corredoira, M, Melia, F, E. Lusso, F E, G. Risaliti, G (2016)
[2] Melia, F, López-Corredoira, M (2015)
[3] Wei, J.-J., Wu, X.-F., Melia, F. & Maier, R. S. (2015)
[4] Melia, F (2012)
[5] Melia, F, Maier, R S (2013)
[6] Nielsen, J T, Guffanti, A, Sarkar, S (2016)
[7] López-Corredoira, M, Melia, F, E. Lusso, F E, G. Risaliti, G (2016)
[8] Wei, J-J, Wu, X-F, Melia, F & Maier, R S (2015);
[9] Melia, F, López-Corredoira, M (2015)
[10] Melia, F (2012); Melia, F, Maier, R S (2013)
[11] Nielsen, J T, Guffanti, A, Sarkar, S (2016)
[12] Melia, F. (Jan. 2017), private Communication

The better model than the present flat-ΛCDM model was derived using fundamental quantum-theoretical arguments 30 years ago.[13] Today, it is referred to as the „protyposis model". Its basic entities are the mathematically simplest quantum structures, the abstract quantum bits of absolute (free-of-meaning) information – AQI bits.

This model was opposing the mainstream then, because at that time almost all of the experts believed that the cosmos would recollapse – at least if its volume is not infinitely large. Presently the model is at odds with the view of an accelerated cosmic expansion, a view still held by many experts.

The quantum-theoretically founded protyposis model has the great epistemological advantage that it dispenses with the concepts needed and conceived in the standard model, such as „dark energy" and „inflation" together with required, freely adjustable parameters.

The protyposis model eliminates a large part of the all but intractable „cosmological problems". The hitherto mysterious „dark energy" can readily be explained. In the case of „dark matter", various fictitious particles were postulated to explain the apparent gravitational effect, which have yet to be found. In the protyposis model, there is no need for such particles. The gravitation effect due to non-luminescent entities derives from the model. The AQI bits entail dark matter-type local gravitational effects, without the necessity of appearing as massive particles or massless photons.

2. The quantum theoretical foundation of cosmology

Regarding the connections between quantum theory and cosmology, there have been misconceptions in the historical development of physics, which, however, can be cleared in light of the present knowledge.

Historically, quantum theory was the very theory that, for the first time, allowed one to really understand micro-physical phenomena, that is, atomic and intra-atomic processes. Also for historical reasons, quantum theory was seen as carrying forward the thousend of years old concept of „atoms". It was supposed that the path would lead to ever smaller „elementary structures" , e.g., preons and strings. Here „smallness" was always seen as a spatial aspect. That this is still the predominant view is exemplified in the „holographic universe". There the „smallest structures" are supposedly areas of a Planck-length diameter.

Einstein's General Theory of Relativity (GRT) allowed physicists to understand, for the first time, space and time as dynamic entities and describe adequately the interaction of space and time with the material and energetical content of space.

The applications of GRT, from the perihel rotation of the Mercury to the pairs of neutron stars and the detection of gravity waves, shows that the theory describes the gravitational effects extremely well. These successful applications are normally based on approximations to the GRT, and as approximations they can be quantized, such as in the spin-2 quantization of gravitational waves. However, it is seen a major theoretical problem that one has not yet succeeded in the quantization of the full GRT.

Unlimited exact solutions of the GRT always describe a whole universe. Of these infinitely many solutions at most a single one can apply to the actual cosmos. Thus, the GRT describes much more than exists in the reality.

2.1. A new look at the foundations

From Planck's and Einstein's fundamental formulas

(2.1) $$E = mc^2 = h\nu = hc/\lambda$$

it is apparent that a smaller wavelength or Compton wavelength comes with higher energy or mass, respectively.

[13] Görnitz (1986, 1988[1], 1988[2], 2009, 2010)

Taking off the blinders of the antique atom conceptions, would not these formulas make it implausible that the path into the small will lead to simpler structures?

Yet the answer is not entirely simple, as the great historical success of the concept of atoms shows. In fact, down to the atoms of chemistry the disassembling into smaller parts, e.g., of a molecule into its atoms, affords a gain in simplicity .

„The higher the energy, the simpler the system“: this becomes more and more an entirely incomprehensible thesis the deeper one proceeds into the interior of the atoms. The required increase of energy causes a concomitant increase of the probability for the occurrence of ever more virtual and also real quanta.

By contrast, what is plausible is that the simplest quantum structures possess the smallest energies and thereby the largest extensions. This insight makes it possible to release quantum theory out of the „ghetto of the spatially small“. As ever more sophisticated experiments show, quantum systems extending over large distances can nowadays be handled. [14]

With regard to cosmology, a relevant aspect of quantum theory is the composition of subsystems via tensor products of their state spaces. From a mathematical point of view, this structure allows for holistic entities lacking parts while being extended.

What are the simplest structures?

For good reasons, quantum field theories are at the core of modern theoretical studies. They involve the most complex mathematical structures with which one can operate in physics. The simplest way to understand quantum fields is to see them as representing an infinite number of field quanta, that is, quantum particles. This of course implies that quantum particles are considerably simpler structures than quantum fields.

In analogy to the field-particle concept, it has been shown that relativistic quantum particles can be understood as an infinite number of quantum bits. [15]

While quantum fields have a state space of uncountably infinite dimensions and the state space of quantum particles is countably infinite, that of the quantum bits is just two-dimensional.

By mathematical reasons the quantum bits are the simplest possible quantum structures.

One bit is the smallest possible amount of information. Obviously, a quantum bit must not be conceived as something „spatially small“. The more accurately a position in space is to be determined, the more information is required to this end. This fact is however often hidden as a bit can be „attached“ to a „carrier“, like the spin of an electron; by constructing ever smaller carriers, ever more bits of the special intended information can be stored in a given volume.

In accepting the quantum bits as the basic constituents of physics, one has to dispense with the familiar connotation „information = meaning“. „Meaning“ always has a large subjective contribution and is therefore not suitable for defining objectifiable physical quantities.

In descending the stages from the complex to the simple, „quantum fields -> quantum particles -> quantum bits“, the physical structure at the lowest stage is formed by bits of abstract and **a**bsolute (free-of-meaning) **q**uantum **i**nformation (AQI) referred to as „protyposis“ (Greek: preformation). [16] The introduction of a new designation was motivated by the need to distinguish from the commonplace understanding of „information“. The abstractness and absolutness of the AQI bits also implies the absence of any reference to the notions of a transmitter and a receiver.

2.2. The construction of the physical space from the elementary structures of quantum theory

A basic fact of human experience is that the reality is three-dimensional. An actual foundation of this finding can be derived from the simplest quantum structures, the AQI bits.

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[14] Zeiliger Group: Experimental Quantum Foundations and Applications. http://www.iqoqi-vienna.at/research/zeilinger-group/satellite-based-quantum-communication/

[15] Görnitz, T, Graudenz, D, Weizsäcker, C F v (1992), Görnitz, T, Schomäcker, U (2012)

[16] siehe Görnitz & Görnitz (2002, 2008, 2016)

More than half a century ago, Carl Friedrich v. Weizsäcker had shown why the physical space is three-dimensional[17] as a consequence of the simplest quantum-theoretical entities which we now call AQI bits. Weizsäcker had termed these quantum objects „Ure“ (singular: Ur) or „Ur-Alternativen“.

To relate to the familiar physics, Weizsäcker’s concept of information, still committed to „meaning“ and „knowledge“, had to be carried further and made radically abstract, which is reflected in the notational change from the „Ur“ to the AQI bit.

The AQI bits are the simplest possible structures – already for mathematical reasons. Therefrom space and time and their dynamics originate, as well as the material and energetic structures encountered.

Weizsäcker’s summarizing considerations are to be found in his book „Aufbau der Physik“.[18] A brief sketch is as follows: The symmetry groups of the two-dimensional complex state space of the quantum bit are the SU(2) and U(1) groups. These groups span a real 4-dimensional manifold. The U(1) group was linked to the empirical time. From the group-theoretical point of view, the physical space was supposed to be a homogeneous space of the SU(2), that is, the group itself, being a S^3, the treedimensional boundary of a fourdimensional sphere.

Missing in Weizsäcker’s design was a metric.[19] Using group-theoretical arguments a metric could be established.[20]

2.3. Establishing the metric of space-time via the elementary structures of quantum theory

The regular representation of a semi-simple compact topological group such as the SU(2) operates in the Hilbert space of L^2-functions on the maximal homogeneous space of the group, that is the group as S^3. The regular representation comprises all irreducible repesentations of the group. An irreducible representation stands for an indivisible quantum system, a whole without parts.

The states of an AQI bit generate a two-dimensional representation of the SU(2). In physics, for obvious reasons, this is often referred to as „spin-½ representation“. The representation is formed by functions on the S^3, which divide the S^3 into two halves. For an analogy, consider the sine

Let ${}^{2k+1}\boldsymbol{D}_k$ denote an irreducible representation of the *SU(2)*. Here, the subscript indicates a „spin“value , *k=0, ½, 1, 3/2, ...,* while the superscript *2k+1* specifies the respective dimension of the representation.

A tensor product of two-dimensional representations can be decomposed into a direct sum of irreducible representations. From a physical point of view, this means that a manifold of qubits will appear, with a certain probability, as an indivisible quantum system.

Let *N* be the number of AQI bits in the cosmos. The state space of an AQI bit is the two-dimensional space of a spin-½ representation. Thus, the state space of the tensor product of *N* AQI bits has the dimension 2^N.[21]

The decomposition of this tensor product into irreducible representations can be written as

(2.2) $$\left({}^{2}\mathrm{D}_{1/2}\right)^{\otimes N} = \bigoplus_{j=0}^{\{N/2\}} f(N,j)^{2\{N/2\}-2j+1}\mathrm{D}_{\{N/2\}-j}$$

where *{N/2}* denotes the integer *N/2* or *(N-1)/2* for even or odd *N*, respectively. *f(N, j)* is an integer factor given by

[17] Weizsäcker, C F v (1955, 2006)
[18] Weizsäcker 1985, 2006
[19] Weizsäcker 1985, p. 399
[20] Görnitz 1986, 1988[1], 1988[2]
[21] Görnitz, T, (1988[1])

(2.3) $$f(N,j)=\frac{N!(N+1-2j)}{(N+1-j)!\,j!}$$

The quantum-physical meaning of this decomposition is that, with a specific probability, the system will be in a state belonging a representation ${}^{2k+1}\boldsymbol{D}_k$. That probability is proportional to the factor *f(N, j)*, being the frequency with which the respective representation occurs in the decomposition of the tensor product.

As to be expected, the weighted sum of the dimensions becomes

(2.4) $$\bigoplus_{j=0}^{\{N/2\}} f(N,j)(2\{N/2\}-2j+1)=2^N$$

(2.5) $$Z=\sum_{j=0}^{2k}\frac{(2k)!(2k+1-2j)}{(2k+1-j)!\,j!}=\frac{(2k)!}{(k!)^2}\qquad \text{for}\quad N=2k$$

(2.6) $$Z=\sum_{j=0}^{2k}\frac{(2k+1)!(2k+2-2j)}{(2k+2-j)!\,j!}=\frac{(2k+1)!}{k!(k+1)!}\approx 2\frac{(2k)!}{(k!)^2}\qquad \text{for}\quad N=2k+1$$

To obtain probabilities for the occurence of a representation, the frequency factors have to divided by the total number Z of all representations:
The maximum of the frequency factor *f(N, j)* is assumed for

(2.7) $$j_{max}= (1/2)[N-\sqrt{(N+2)}]$$

which for $N>>1$ can be approximated as

(2.8) $$j_{max} \approx (1/2)[N-\sqrt{(N)}]$$

Using the approximation formula for the factorial,

(2.9) $$ln(n!) \approx [n+(1/2)]\ ln\ n - n + (1/2)\ ln\ (2\pi)$$

one obtains the following orders of magnitudes

(2.10) $$f(N, \{N/2\}) \approx O(2^N N^{-3/2})$$

(2.11) $$f(N, j_{max}) \approx f(N, (1/2)[N-\sqrt{N}]) \approx O(2^N N^{-1})$$

for the frequencies of the one-(or two-)dimensional representation ($j=\{N/2\}$) and the $2^{\sqrt{N}}$ dimensional representation ($j=j_{max}$), respectively.

Below j_{max} the frequencies decrease exponentially towards the value 1 for j = 0. A numerical example can give an illustration, see Fig. 1.

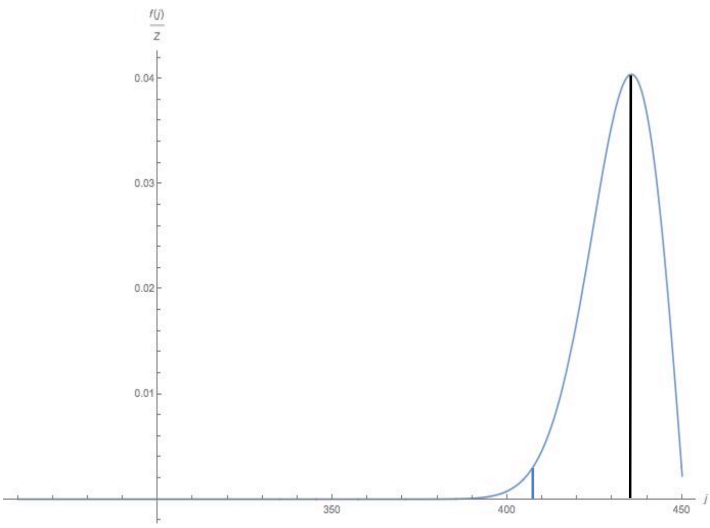

Fig. 1: Relative frequencies of irreducible representations in the decomposition of the N-fold tensor product of two-dimensional representations of the *SU(2)* group.

The chosen example is: $N=900$, $N/2 = 450$, $(N - \sqrt{N})/2 = 435$, $N/2-\sqrt{(2N)} = 407.5$. The largest wavelength (or smallest dimension) belongs to the representations $j=N/2$ (here 450), the most abundant ones to $j=(N-\sqrt{N})/2$ (here 435), and the smallest wavelengths (largest dimensions) with non-negligible frequency to $j=(N/2-\sqrt{2N})$ (here 407.5).

A numerical evaluation of the area below the curve gives $0.999284 \approx 1$. The integral from $j= 407.5$ to $j=450$ amounts to 0.984347. As this graphical example shows, the frequencies of representations with $j < N/2 -\sqrt{(2N)}$, that is, representations of the dimension between $\sqrt{(2N)}$ und N, can be safely neglected. In these representations, the corresponding wavefunctions are of a wavelength that is considerably shorter than that associated with the representation of maximal frequency at $j=(N-\sqrt{N})/2$.

Let R denote the radius of the S^3, of the cosmic space. An $SU(2)$ representation of dimension n implies wave functions that allow for a partitioning of the space down to a size of the order of R/n. (See appendix 1)

In the quantum-theoretical context of our argumentation „facts“ are not given *a priori*, but have to be defined in an appropriate way. It is reasonable to relate facts to large probabilities.

Accordingly, an S^3-space accomodating N AQI bits will have a smallest „factual“ length of an extension of the order $R/\sqrt{(2N)}$, which we shall identify with the Planck length.

Smaller lengths are to be seen as being merely „virtually“ existent. This insight is relevant for our perception of the reality.

As a consequence of Planck‘s formula $E=hc/\lambda$ together with Einstein's formula $E=mc^2$, the characteristic extension of a quantum system, that is, its wavelength or Compton wavelength for massless or massive quanta, respectively, decreases with increasing energy or mass. Corresponding to a smallest „factually possible“ length, there is a largest factually possible energy or mass, and thus also a largest number of AQI bits for which a system can be regarded a factually indivisible quantum particle.

For systems with energy or mass larger than the Planck mass to be considered as „indivisible quantum systems“, the „particle“ model is no longer adequate. Here the black hole model may apply.

Objects with a mass exceeding the Planck mass, other than black holes, cannot be treated as factually indivisible quantum objects.

This is of relevance to the issue of the transition from a quantum to the classical description. That transition does not relate to the spatial extension but rather to the energies or masses here involved.

The border is the Planck mass. More massive systems (other than black holes) are at most „virtually indivisible quantum systems“.

2.4. Plenty of extended becomes „small“

A single AQI bit was represented by a state function spreading out over S^3, the entire cosmic space. It seems to be at odds with common sense that it should be possible to construct from many of those AQI bits a strongly localized wave function. While our experience is „plenty“ implies „big“, it is postulated here that „plenty“ results in „small“. Seen from the information perspective, however, it is apparent that plenty of information affords a better localisation than little information.

What comes into effect with the AQI bits is an important quantum-physical feature. In quantum theory there is a multiplicative composition of a system from parts – as opposed by the additive composition in classical physics.

A simple visualization is the product of sine functions. In Fig. 2, the graph of a single sine function on the circle is compared with the graphs of powers of that sine function.

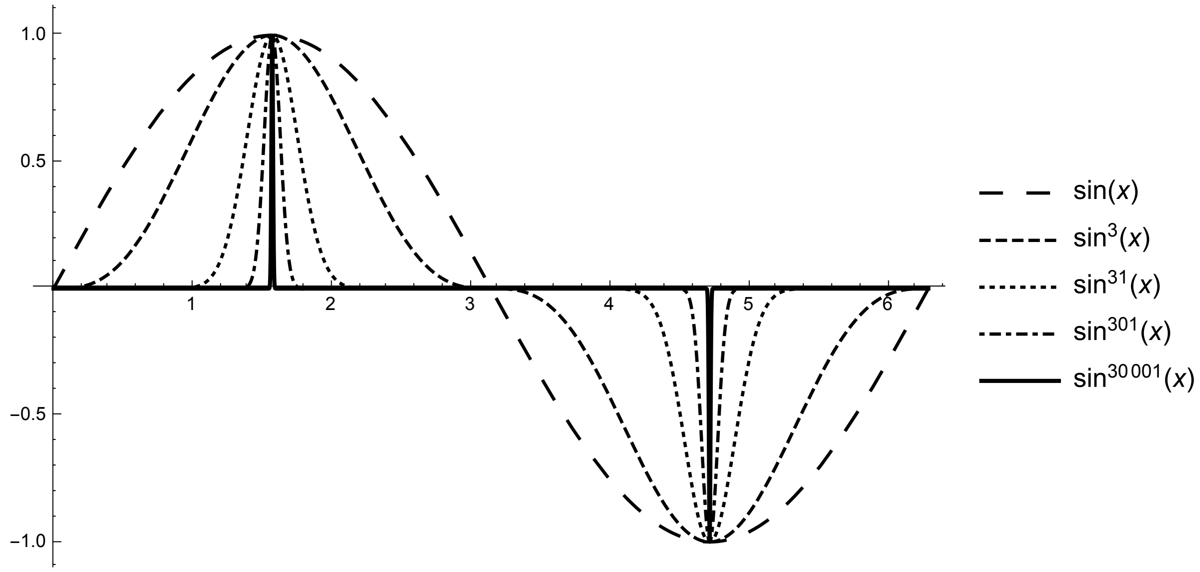

Fig. 2: The sine function and powers of the sine functions on the interval 0 - 2π.
The larger the number of sine factors the more localized is the graph of the product.

Using a linear combination of various powers of the sine function, a single peak graph can be generated.

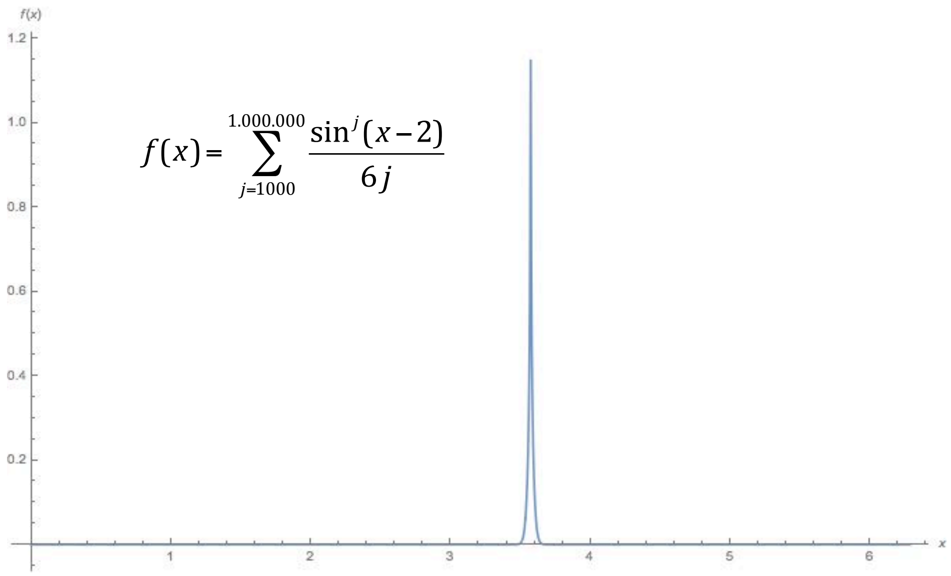

Fig. 3: Superimposing different powers of the sine function results in a single peak function.

3. Establishing a metric

As a science founding on empiricism, physics cannot be solely based on *a priori* arguments, but needs the connection to observations and experimental results.

As already recognized by Max Planck in 1899, the three fundamental natural constants – the velocity of light, the gravitational constant, and the action quantum or Planck's constant – allow one to derive units of length, time, and mass. These units are called Planck units nowadays. The Planck length and Planck time mark the smallest length and the shortest time that can be conceived as being factually realizable. The Planck mass characterizes the most massive quantum particle and also the border to the smallest conceivable black hole.

- Backed by this empirical knowledge we will identify the smallest realizable length following from our group-theoretical arguments with the Planck length l_{Pl}.Thus, the relation between the radius R of the S^3 space and the number of AQI bits reads [22]

(3.1) $$R = l_{Pl} \sqrt{2N}$$

This allows us to define distances in the S^3 space, that is, the physical space according to the derivation given here.

Assuming an invariable Planck length l_{Pl}, the radius R of the S^3 space will increase with a growing number N of AQI bits.

When we speak of a "growing number" of AQI bits, a definition of time is required in which such a process shall be measured.

Having a definition of a distance via the Planck length, time (or duration) follows if a velocity is given.

- There is a distinguished velocity in nature, namely the velocity c of light in vacuum.

For the present cosmological model, this suggests to postulate a definition of the cosmic time t such that the cosmic radius changes at the rate of the distinguished velocity c:

(3.2) $$R = ct$$

Using Planck units, c=1, this simply reads $R = t$.

A basic formula of quantum theory is Planck's relation between energy and the characteristic extension.

(3.3) $$E = h\nu = hc/\lambda$$

The „wavelength" λ of an AQI bit is of the order of the radius R of the S^3 space. Corresponding to that largest possible length, there is a smallest energy which can be understood as a factual quantity, that is, the energy E_{AQI} of a qubit, being inversely proportional to R

(3.4) $$E_{AQI} \sim 1/R$$

Since $R=\sqrt{(2N)}$ or $N=R^2/2$, the total energy of N AQI bits in the S^3 cosmos relates to R according to

(3.5) $$U \sim N/R \sim R/2$$

We consider a multitude of AQI bits in an expanding volume. Accordingly, it is necessary to resort also to statistical arguments and observe thermodynamic relations. The first law of thermodynamics addresses the behaviour of the energy U of a system with varying volume, that is, for $dV \neq 0$.

> Here a few explanatory remarks are in order, addressing also aspects of the GTR, even though that theory will not be used in deriving our arguments. Originally, the first law of thermodynamics allowed one to rationalize the performance of heat engines. An inflow of an amount of heat δQ can result in an increase of the internal energy or in an expansion of the volume dV towards an environment at pressure p: $\delta Q = dU + pdV$. In an thermally isolated system $\delta Q=0$. Obviously, the cosmos is thermally isolated so that $\delta Q=0$ is a reasonable assumption.
>
> It is, however, important to note that the AQI bits are no particles. Arguments deriving from the thermodynamics of gases cannot simply be transferred to the present model.
>
> For the „normal" materials considered in thermodynamics, almost always a positive pressure applies. It takes a positive pressure to increase the volume against external forces. However, such arguments apply only as long as the GTR is left out of consideration.
>
> In the framework of the GTR all components of the energy-momentum tensor and thus also a positive pressure contribute to the gravitational effect and thereby to a contraction. There are two different aspects of the pressure. For a system in a volume the pressure means a force against the

[22] While an expanding cosmos better conforms to common sense, one might equally well postulate $R= constant$ and assume a correspondingly shrinking Planck length l_{Pl}.

boundary. Beside this the pressure has the dimension of an energy density and any energy desity reacts on the structure of space and time. Therefore, a sufficiently large positive pressure will cause an appreciable contracting effect by its reaction on space. In a sufficiently large neutron star an increasing positive internal pressure contributes to the gravitational collapse rather than preventing it. In contrast to to our everyday conceptions, where a positive pressure inflates a tire, a negative pressure in the GTR counteracts a collapse.

Due to the quantum-theoretical relations for the AQI bits (3.5) the internal energy U of the cosmos and its volume grow in the same way. Accordingly, in the present model the inner pressure assumes a negative value as will be shown below.

The negative pressure associated with the fictitious dark energy is of such a magnitude that – if real – it would effect an accelerated cosmic expansion. Without resorting to the GTR, the value of the negative pressure deriving from the protyposis model entails a constant expansion.

For our cosmological model featuring a closed yet varying volume the first law of thermodynamics applies,(3.6) $dU + p\, dV = 0$

Since $dU \sim dR/2$ we obtain

(3.7) $dR/2 + 2\,\pi^2\, p\, 3R^2\, dR = 0$

As the cosmic radius is changing, $dR \neq 0$, yielding the relation

(3.8) $p = -\, 1/4\, \pi^2\, 3R^2$

Defining an energy density ρ according to $U = \rho\, V = \rho\, 2\, \pi^2\, R^3$, ρ can be written as

(3.9) $U = R/2 = \rho\, 2\, \pi^2\, R^3$ or $\rho = 1/\, 4\, \pi^2\, R^2$

The result is an equation of state for the AQI bits of the protyposis of the form

(3.10) $p = -\rho/3$ or $\rho + 3p = 0$

The cited authors[23] of the studies on the linear expansion of the cosmos refer to this equation of state as the „zero active mass condition“.

The equation of the quantum-cosmological protyposis model suggests to understand the cosmos as a splitting of the AQI vacuum into two equally large parts: a positive energy part comprising also matter and light, and a negative pressure part to be associated with a gravitational effect, invoking here an interpretation according to Newton’s theory. In that interpretation gravitation is assigned, so to say, to a „negative energy“.

Even though both parts become ever bigger due to the increase of the AQI bits and the thereby effected expansion of the cosmos, the sum of the parts stays always zero. This calls to mind the antique and medieval philosophers who pondered on a „creatio ex nihilo“.

The protyposis concept establishes a real and moreover understandable foundation of the „substance of the cosmos“ and its evolution.

The arbitrary fabrications such as „inflation“ or „dark energy“ become no longer necessary. The problems to be tackled by these hypotheses are solved in a natural way via the negative pressure entailed by the protyposis concept.

4. The cosmological term and dark matter

The cosmological constant Λ presents an interesting aspect in the history of scientific cosmology. Albert Einstein firmly believed that the universe should not have a temporal beginning. He had to realize, however, that his equations did not allow for a corresponding solution. Hence, he changed them by introducing an additional free constant Λ. This constant, having a vacuum-type energy-momentum tensor, enables a cosmos with matter but without a temporal onset. That „Einstein cosmos“ is an S^3 space with a constant radius. The negative pressure introduced by Λ

[23] see footnotes 1-5

can, suitably adapted, balance a collapse of the space to be expected otherwise as an effect of the gravitating matter.

Shortly after, the redshift of the galaxies was detected and thereby the expansion of the cosmic space – implying a beginning in time, and Λ was dropped.

4.1. The cosmic substrate as an „ideal fluid"

If both the energy density and the pressure are uniform in the entire volume, then the physical model of an „ideal fluid" applies. In this model the energy-momentum tensor T_{ik} assumes the form of a diagonal matrix with the elements ρ (energy density) and p (pressure) dependening only on time but not of the position in space or the orientation.

The vacuum is a special case. Here the energy-momentum tensor is the unit matrix multiplied by a factor.

$$(4.1) \qquad T^i_k = \begin{bmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{bmatrix} \qquad {}_{Vakuum}T^i_k = \Lambda \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Due to the indefinite metric of the Minkowski space, a positive pressure appears with a minus sign.

When in the 1980-ties the observation data improved, it was realized that the solutions of Einstein's equations seen as valid then, differed too much from the data. Therefore, the Λ concept was revived, allowing one to adapt a free parameter to the new data, without having to resort to new ideas.

The problem with Λ is that it has to be extremely small, though larger than zero.

The protyposis concept affords a solution to that problem. In cosmology, it is necessary to distinguish matter and light. Trying to partition the AQI substance accordingly into forms appearing as matter and light, one finds that another part is needed that has to be of the form of an energy-momentum tensor of a „vacuum". As noted, the energy-momentum tensor associated with Λ is of this form. Accordingly, we denote that part by Λ_{eff} . As will be seen, the theoretically derived value for Λ_{eff} leads to good agreement with the empirical findings. However, Λ_{eff} cannot be a time-independent constant. Hence, we refer to Λ_{eff} , being temporally variable, as an „effective cosmological constant" or, preferably, as a „cosmological term".

Moreover, as mentioned before, Λ_{eff} solves the issue of the „dark matter" sought for since long. Trying to associate those particular forms of the AQI bits with a known quantum field, such a field would have to display the quantum characteristics of a vacuum. The Higgs field with charge and spin zero might be a possible candidate. However, there are arguments against that. The quanta of the Higgs field are of extreme mass and therefore decay utmost rapidly. While this would not yet be a compelling case against that assumption, it is more likely that the dark matter will not appear in the form of any kind of particles.

The protyposis energy-momentum tensor ${}_{AQI}T_{ik}$, in which the negative pressure enters with a positive sign, can be partitioned into contributions traditionally considered in cosmology. There the astronomers distinguish pressureless dust, comprising stars, planets, and black holes, from light.

The AQI bits of the protyposis divide into matter, that is, ${}_{dust}T_{ik}$, light, ${}_{light}T_{ik}$ (positive pressure, implying a negative sign), and a further quantity having the structure of a vacuum, that is, of a cosmological term[24] ${}_{(vacuum)}T_{ik}$, displaying a negative pressure too.

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[24] Görnitz (1988[1])

$$(4.2)\quad {}_{AQIs}T^i_k = \begin{bmatrix} \rho & & & \\ & \frac{\rho}{3} & & \\ & & \frac{\rho}{3} & \\ & & & \frac{\rho}{3} \end{bmatrix} = \begin{bmatrix} {}_{dust}\rho & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix} + \begin{bmatrix} {}_{light}\rho & & & \\ & -\frac{{}_{light}\rho}{3} & & \\ & & -\frac{{}_{light}\rho}{3} & \\ & & & -\frac{{}_{light}\rho}{3} \end{bmatrix} + \begin{bmatrix} \Lambda_{eff} & & & \\ & \Lambda_{eff} & & \\ & & \Lambda_{eff} & \\ & & & \Lambda_{eff} \end{bmatrix}$$

According to the form of Eq. (4.2) for the tensor ${}_{AQI}T_{ik}$, there are two relations for the three parameters ${}_{dust}\rho$, ${}_{light}\rho$, and Λ_{eff}:

(4.3) $$\rho = {}_{dust}\rho + {}_{light}\rho + \Lambda_{eff}$$

(4.4) $$\rho/3 = -\, {}_{light}\rho/3 + \Lambda_{eff}$$

Obviously, these equations would not make sense without the Λ_{eff} term. The energy density of light would be negative, clearly an absurd consequence.

Let q denote the ratio of ${}_{dust}\rho + {}_{light}\rho$ and Λ_{eff}:

(4.5a) $$q = ({}_{dust}\rho + {}_{light}\rho)/\,\Lambda_{eff}$$

(4.5b) $$\Lambda_{eff} = \rho\,/(q+1)$$

According to

(4.6a) $$\rho = -\,{}_{light}\rho + 3\,\Lambda_{eff} = -\,{}_{light}\rho + 3\,\rho\,/(q+1)$$

we may write ${}_{light}\rho$ as

(4.6b) $${}_{light}\rho = \rho\,(-1 + 3/q+1)) = \rho\,(3-q-1)/(q+1) = \rho\,(2-q)/q+1))$$

In a similar way, we obtain

(4.7) $$\rho = {}_{dust}\rho + \rho\,(2-q)/q+1)) + \rho/(q+1)$$

and thus

(4.8) $${}_{dust}\rho = \rho - \rho\,(2-q)/q+1) - \rho\,/(q+1) = \rho\,[q + 1 - 2 + q - 1]\,/(q+1) = 2\,\rho\,[q - 1]\,/(q+1)$$

Since ρ, ${}_{light}\rho$, and ${}_{dust}\rho$ are positive, it follows from

(4.9a) $${}_{light}\rho = \rho\,(2-q)/(q+1)$$

(4.9b) $${}_{dust}\rho = 2\,\rho\,(q - 1)/(q+1)\,,$$

that q is bounded according to

(4.10) $$1 \le q \le 2$$

This means that Λ_{eff} has an energy density at least as large as that of matter and light combined, yet not exceeding twice that sum. This results in the correct order of magnitude of the cosmological term (or a time-dependent „effective cosmological constant").

4.2. The explanation of the „dark matter"

In a further approximation to the reality, the initial idealization of a „cosmic fluid", entailing the model of a uniformly expanding cosmos, can be refined as to assuming spatial variability. Then there may be non-uniform distributions of the three parts ${}_{dust}\rho$, ${}_{light}\rho$, and Λ_{eff} in the cosmos as fluctuations around a mean value, entailing local gravitational effects.[25] The protyposis part Λ_{eff} – as distinguished from dust (stars, planets, black holes) and light – does not appear as matter or light. However, as with matter and light one may assume local density variations of Λ_{eff} as well. Hence, the Λ_{eff} part of the cosmic content can be interpreted as part of the „dark matter" gravita-

[25] Görnitz (2010)

tion. The gravitational effect of the black holes, that is, a non-luminescent „dust“ part of the protyposis not appearing in the form of „quantum particles“ will be discussed in Sec. 6 below.

The protyposis concept dispenses with the two and a half thousend years old idea of atomism that ultimately the entire content of the cosmos should be reducible to „spatially smallest“ building blocks.

5. A rationalization of Einstein's equations

The truly simplest structures consistent with physical and mathematical arguments are the protyposis AQI bits. As a necessary consequence of their existence, they cause a gravitational effect, as gravitation is the reaction of the cosmic evolution, caused by the increasing number of the AQI bits, to the content of the cosmos and the processes in it .

A cosmos with only a few AQI bits has a small radius and a large energy density (see Eq. 3.9), one with many AQI bits has a large radius and a small energy density. If one supposes that these relations, which apply to the content and the radius of the cosmos as a whole, can be transferred to the case of local fluctuations within the cosmos, then Einstein's equations follow as a classical approximation from the quantum-theoretical protyposis cosmology.[26]

The derivation along that path is permitted in so far as our argumentation up to here has been completely independent of Einstein's GTR equations.

The introduction of a cosmic time and, thus, of a distinguished coordinate or reference system seems to be at odds with what often is written about the GTR. There one may read: „There is no distinguished reference system.“ With regard the cosmology, however, this is incorrect.[27]

> Already Dirac has referred to the fact that such a distinguished background metric remains unnoticeable as long as one is included into an "Einsteinian elevator" without any windows. The situation changes if a window to the cosmos is opened and the background radiation is included into the researches.[28] That all reference systems are of equal rights is therefore a local property in the description of localized systems with arbitrary accelerations, however, it is not necessary to claim this property for the whole cosmic space as well.

Since the cosmological model established here is a rigorous solution of Einstein's equations, one may invert the argumentation and, in an inductive manner, infer the structure of the equation from the solution:

The reasonable postulation that local density variations within the cosmos obey the same laws as those applying to the density and the radius of the entire cosmos justifies the general validity of the relations between the energy-momentum tensor and the Einstein tensor in their GTR form.

This explains why Einstein's equations allow for such an excellent description of local gravitational phenomena within the cosmos.

As to the linear approximations of the GTR, used for example to describe gravitational waves, their quantization is of course always possible, allowing for a quantum-theoretical treatment of gravitational waves in terms of gravitons.

6. A model of black holes

Already within the framework of Newtonian mechanics, the idea was conceived of an astronomical body having a gravity at its surface of such a magnitude that the theoretical escape velocity would surpass the velocity of light. According to the outcome of GTR calculations, there is a horizon enclosing a region in space from which nothing can escape. In its interior, as a consequence of the strict GTR description, all of the matter would vanish in a singular mathematical point.

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[26] Görnitz (2009, 2010, 2011)
[27] Görnitz (2011)
[28] Dirac (1980)

Invoking for the first time quantum-theoretical aspects, Bekenstein[29] and Hawkings[30] could show that entropy has to be assigned to a black hole. The value of this entropy surpassed by many orders of magnitude anything that hitherto could be inferred from thermodynamical considerations for particles.

Considering AQI bits rather than particles in the treatment of black holes, already a simple toy model will make it plausible that black holes have entropies of the encountered order of magnitude.

Let us see what follows from the assumption that there are irreducible volumes – that is black hole equivalents – in the cosmos possessing a larger mass than Planck's mass.[31]

Elementary particles are associated with irreducible representations of the Poincaré group, and this mathematical concept implies the absence of any interior structure. Except for mass and spin, other physical quantities cannot be accomodated by those representations. Postulating the absence of accessible inner structure for an object with a mass exceeding Planck's mass, one obtains the black hole model. Here the event horizon precludes, on principle, any exterior knowledge of the conditions inside the black hole.

The challenge was to construct a simple model which would allow one to illustrate the order of magnitude of the black-hole entropy.

Let n_{BH} denote the number of AQI bits within such an irreducible volume and N the total cosmic AQI bit number. The black-hole mass is larger than Planck's mass, so that $n_{BH} \geq \sqrt{(2N)}$.

Since each AQI bit has the energy $1/R$ the mass m_{BH} of the black hole is given by

$$m_{BH} = n_{BH}\,(1/R) \tag{6.1}$$

The horizon property can be modeled by requiring that there must not be any overlap of the black holes within the cosmos, even if all cosmic AQI bits not assigned to the cosmic term Λ_{eff}, that is N/2 according to Eq. (4.10), participate in forming altogether z_{BH} black holes. Assuming that each of these z_{BH} black holes are of the same mass (and size), the product $z_{BH}\,n_{BH}$ satisfies

$$z_{BH}\,n_{BH} = N/2 = R^2/4 \tag{6.2}$$

Denoting the maximal extension (radius) of such an entity by r_{BH}, the horizon property requires that

$$z_{BH}\,r_{BH} = R \tag{6.3}$$

This means that the black holes can be aligned next to each other, yet maintaining their mutual separation. If the sum of the black-hole radii is smaller than or at most equal to the radius R of the cosmos, then the z_{BH} black holes can always be arranged within the cosmos without necessitating any overlaps.

Combining the last two equations results in the relation

$$n_{BH} = R\,r_{BH}\,/4 \tag{6.4}$$

between n_{BH} and r_{BH}, which can be used in Eq. (6.1) to establish the linear relation

$$m_{BH} = r_{BH}/4 \tag{6.5}$$

for the mass of the black hole and its radius.

To address the black hole entropy, we will introduce two more quantities.

We consider black-hole inside-oscillations with an extension r_{BH}, of which each is formed by a subset of the n_{BH} black-hole AQI bits. Let k_{BH} denote the number of qubits in these subsets.

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[29] Bekenstein (1973, 1974)
[30] Hawking (1975)
[31] Görnitz (1988[1])

These oscillations, resulting from a coherent product of AQI oscillations, can be interpreted as the fundamental modes of the black hole.

According to Eq.(2.2) j qubits can form an j-dimensional irreduzible representation of the SU(2). In appendix 1 it is mentioned that such an irreducible representation of the SU(2) containes functions with wavelengths of order R/j. As the extension of these oscillations (its wavelengths) is given by r_{BH}, the following relation holds

(6.6) $$r_{BH} = R/k_{BH} \quad \text{or} \quad k_{BH} = R/r_{BH}$$

Now let us examine how many of those fundamental oscillation modes can arise in our constructs. Obviously, the fundamental modes within the black hole are analogous to the AQI bits in the cosmos as a whole. An AQI bit has a “wavelength” of the order of the cosmic radius R. To generate a much smaller “wavelength” of the order of the black-hole radius r_{BH}, it takes many, that is, k_{BH} coherent AQI oscillations.

Just as the total number of AQI bits correlates with the set of all possible states in the cosmos, the number of the fundamental modes of the black hole, to be denoted by s_{BH}, corresponds to all possible inner states of the black hole, that is, information inaccessible from outside the black hole. Insofar s_{BH} is a measure of the entropy of the black hole.

Obviously, s_{BH} satisfies the following relation

(6.7) $$n_{BH} = s_{BH} \times k_{BH} \quad [\text{or} \quad s_{BH} = n_{BH}/k_{BH}]$$

Using here Eqs. (6.4) and (6.6) for n_{BH} and k_{BH}, respectively, yields an explicit expression for s_{BH}:

(6.8) $$s_{BH} = r_{BH}^2/4 = 4\, m_{BH}^2$$

where the second equation follows from the mass-radius relation (6.5). This is essentially the the black-hole entropy result according to Bekenstein and Hawking[32].

As these rather simplistic arguments show, the “wavelength” r_{BH} of the fundamental black-hole oscillations, each formed of k_{BH} AQI bits, corresponds to the extension of the event horizon. Like the cosmic AQI bits of wavelength R partition the space into two halves, the “k_{BH}-modes“ act as internal AQI-type bits for the irreducible black-hole volume. Apart from the number of these quasi-qubits, their actual states are unrecognizable from outside the black hole. At least the order of magnitude (up to a factor of π) of the black-hole entropy s_{BH} can be rationalized by these considerations. Note that a factor of 2 is missing in the present mass-radius relation (6.5), the correct result being $2m_{BH} = r_{BH}$.

The total number n_{BH} of the AQI bits forming the black hole is larger than its entropy s_{BH}. Entropy is to be understood as a measure of basically inaccessible information. In our black-hole model, s_{BH} is the share of the n_{BH} qubits for which the specific state cannot be recognized from outside. By contrast, it is possible, at least in principle, to come to know the states of the remaining AQI bits,

(6.9) $$n_{BH} - s_{BH} = n_{BH}(1-1/k_{BH})$$

The information on the position of a relatively small black hole within the vast cosmos is, among others, encoded in those qubits. Any localisation requires much information, as already our everyday experience tells us. The more information is available, the easier a search will succeed. Much information or, likewise, a large number of AQI bits is required to determine the position of an object in the cosmos.

This shows that „entropy“ cannot simply be equated with a respective number of AQI bits. In the case of an particle, for example, all the constituting AQI bits are accessible. Thus, the entropy concept does not apply to a particle in a known quantum state. Black holes, by contrast, have inner states that, as a matter of principle, cannot be known. Accordingly, an entropy is to be at-

[32] Bekenstein (1973, 1974), Hawking (1975)

tributed to a black hole. The entropy of a black hole, being an object in the cosmos, is always smaller than the number of AQI bits forming the black hole.

7. The interior of Black Holes

Usually, Einstein's theory is also applied to describe black holes within their horizons. Quantum-theoretical aspects are considered only close to the Schwarzschild singularity.

> The classical phase of the hole's internal evolution presents us with a problem which is mathematically quite definite and, in principle, straightforward. It is a hyperbolic initial-value problem of Cauchy's type. The evolution equations are the classical Einstein field equations. The initial data are set on or near the event horizon. The task is to evolve these data forward in time up to the point where a singularity is imminent. *(At this stage the classical evolution equations fail and the quantum regime takes over.)* [33] (highlighting by TG)

Postulating that the entire interior matter vanishes in a singular mathematical point is, in terms of physics, an utterly senseless statement, which certainly has contributed to the fact that until today there is serious resistance against the concept of black holes.

The non-physical hypothesis of a Schwarzschild singularity has to be dropped if quantum-theoretical arguments are taken into account.[34] Here one has to abandon the idea that quantum theory applies only to "the small".

According to quantum theory, a limited volume, an impenetrable box, effects the ground state of the interior space.

Therefore - other than implied by the GTR – one cannot suppose that the vacuum inside and outside of the event horizon is the same.

The present simple black-hole protyposis model, yielding the correct order of magnitude of the entropy, was essentially based on the analogy was between the inner state of a black hole and the cosmos in the protyposis description.

The energy density ρ_{BH} in a black hole is given by

(7.1) $$\rho_{BH} \approx m_{BH}/\, r_{BH}{}^3 = 1/4\; r_{BH}{}^2$$

where the mass-radius relation (6.5) was used to obtain the second equation.

Analogous to Eq. (3.7) we find

(7.2) $$dm_{BH} + p_{BH}\, dV_{BH} \approx (1/4)\; dr_{BH} + p_{BH}\; 3\; r_{BH}{}^2\, dr_{BH} = 0$$

which results in the familiar equation of state

(7.3) $$p_{BH} = -\, 1/(4{\times}3) r_{BH}{}^2 = -\, \rho_{BH}/3$$

for the interior of the black hole.

As has been elaborated elsewhere[35], this equation of state for the interior of a black hole gives rise to the model of a Friedmann-Robertson-Walker cosmos, which corresponds to the model of our cosmos. In view of this finding, one might be tempted to think of the possibility that our cosmos could be interpreted as the interior of a vast black hole.

8. The connection to the observations

8.1. The astrophysical data

At present, the generally accepted age of the cosmos is about 13.8 billion years, which, converted to Planck units, amounts to $8{\times}10^{60}$ Planck times:

(8.1) $$t_{cosmos} \approx 4.3588{\times}10^{17} s = 8{\times}10^{60}\, t_{Pl}$$

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[33] Israel (1998)
[34] Görnitz, Ruhnau (1989), Görnitz (2013)
[35] Görnitz, Ruhnau (1989), Görnitz (2013)

Using that age as empirical input, we now may draw connections to the orders of other physical quantities.

According to our model, the present number of AQI bits is given by $N = R^2/2 = t_{cosmos}{}^2/2$, which amounts to

(8.2) $$N = 3.2\times10^{121}.$$

while the value of the cosmic radius becomes

(8.3) $$R = 8\times10^{60}\, l_{Pl} = 1.28\times10^{28}\ cm$$

where $l_{Pl} = 1.616\times10^{-33}\ cm$ was used in the conversion to cm.

In the $R=ct$ model the product of the Hubble parameter and the cosmic time t_{cosmos} is 1, which is in good agreement with the present findings. The Hubble parameter is defined by

(8.4a) $$H_0 = \frac{\dot{R}(t)}{R(t)}$$

which for $R(t) = ct$ becomes

(8.4b) $$H_0 = \frac{c}{ct} = \frac{1}{t}$$

According to WMAP5[36] the current value is

(8.5) $$H_0 = 70.5\ km\ s^{-1}\ Mpc^{-1} = 2.285\times10^{-18}\ s^{-1}$$

and the multiplication with t_{cosmos} gives

(8.6a) $$H_0 \times t_{cosmos} = 0.996.$$

A recent paper[37] concludes a somewhat larger value, $H_0 = 73.1\ km\ s^{-1}\ Mpc^{-1}$, for which the product is

(8.6b) $$H_0 \times t_{cosmos} = 1.033.$$

Both results support the thesis of an expansion of the cosmic space at light velocity.

8.2. Extension of Einstein's equivalence to an equivalence of matter, energy, and quantum information

The formation of material and energetic quantum objects from the protyposis AQI bits suggests an equivalence between matter, energy, and quantum information, extending Einstein's equivalence between matter and motion.

With Planck's units the gravitational constant G, the action quantum $\hbar$, and the velocity of light c have the value 1 and, thus, their actual role in the formulas is not manifest.

According to Eq.(3.4), the energy E_{AQI} of an AQI bit was seen to be equivalent to the inverse of the cosmic radius or, likewise, the inverse of the cosmic age. In a dimensionally correct form this relation reads

(8.7) $$E_{AQI} = \hbar/t_{cosmos}$$

With the values $t_{cosmos} = 4.358\times10^{17}\ s$ and $\hbar = 6.582\times10^{-16}\ eV\, s$, the AQI bit energy amounts to

(8.8) $$E_{AQI} = 1.51\times10^{-33}\ eV$$

or in cgs-units

(8.9) $$E_{AQI} = 0.242\times10^{-44}\ erg$$

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[36] Komatsu (2009)

[37] Wong, K C et al (2016)

A $10\ eV$ photon would be formed from about 6×10^{33} qubits, an electron, having a mass of $511\ keV$, from 3×10^{38} qubits, and a proton with a rest mass of about $1\ GeV$ from 6×10^{41} qubits. The proton number is in the order of the value 10^{40} obtained by Weizsäcker already in 1974 as the result of rather demanding arguments. [38]

For the total cosmic energy $U \approx R/2$ we obtain the value

(8.10) $$U = 4.94\times10^{88}\ eV = 7.91\times10^{76}\ erg$$

With the value $c= 29\ 979\ 245\ 800\ cm/s$ the total "cosmic mass" $M=U/c^2$ amounts to

(8.11) $$M = 5.5\times10^{56}\ g = 5.5\times10^{53}\ kg$$

For the energy density, $\rho = U/V = U/2\pi^2R^3$, we find

(8.12) $$\rho = 1.6\times10^{-8}\ erg/cm^3$$

and the corresponding matter density, $\rho_M = M/V = M/2\pi^2R^3$, is given by

(8.13) $$\rho_M = 1.34\times10^{-29}\ g/cm^3 = 1.34\times10^{-26}\ kg/m^3$$

The so-called critical density is defined by $\rho_c = 3H_0^2/(8\pi G)$, which amounts to $\rho_c = 0.934\times10^{-29}\ g/cm^3 = 0.934\times10^{-26}\ kg/m^3$, where the H_0 value according to Eq.(8.6a) and $G=6.674\ cm^3/g\ s^2$ have been assumed. [39]The corresponding energy density is $8.38\times10^{-9}\ erg/cm^3$.

The critical density corresponds to a flat universe. For larger values, such as those obtained in the present protyposis model, a closed cosmos with a finite volume results.

The value of the cosmological term Λ_{eff}, that is, the time-dependent „effective cosmological constant" is in the range

(8.14) $$\rho/2 \leq \Lambda_{eff} \leq 2\ \rho/3$$

where ρ is the total energy density of the cosmos. While there is abundant speculation on the ratio of the as yet completely unknown „dark energy" and „dark matter" ingredients of the cosmos, specific assessments (in erg/cm^3) of the actual total energy density are hard to come by. A vacuum energy of $10^{-10}\ erg/cm^3$ has been listed by A. Riess in the Encyclopedia Britannica[40]. For the matter density calculated values between[41] $4.7\times10^{-27}\ kg/m^3$ or[42] $8.47\times10^{-27}\ kg/m^3$ are given in Wikipedia. The uncertainties with regard to these quantities are apparent.

8.3. A test of consistency: The protyposis concept and the black hole entropy

A an interesting supplement, we perform a consistency check of the protyposis model with regard to cosmological and astrophysical data, referring here also to the GTR.

Since there is no principial theoretical upper bound for the mass of a black hole, one may suppose, as a thought experiment, that the total mass of the universe is contained within a single black hole.

According to the GTR the Schwarzschild radius is given by $r_S = 2GM/c^2$, or, in Planck units,

(8.15) $$r_S = 2\ M$$

This can be compared with the relation $R/2 = U$ of the protyposis model, to be written also as $R/2 = M$ according to Einstein's relation between U and M with $c = 1$. This shows that the cosmic radius and the Schwarzschild radius of the hypothetical black hole are of the same order of magnitude.

The value of Planck's mass is $m_{Pl} =2.176\times10^{-8}\ kg$. The proton mass is $m_p = 1.672621\times10^{-27}\ kg$ or $7.7\times10^{-20}\ m_{Pl}$. Now suppose a proton drops into the mass M black hole (of $8\times10^{60}\ m_{Pl}$)so that the

[38] Weizsäcker (1974)
[39] H_0 aus (8.6b) implies $\rho_c = 1.004\times10^{-29}\ g/cm^3$
[40] https://www.britannica.com/topic/dark-energy, dated 12/30/2016
[41] https://de.wikipedia.org/wiki/Universum dated 30.12.2016
[42] https://de.wikipedia.org/wiki/Lambda-CDM-Modell dated 30.12.2016

total mass becomes $M+m_p$.

The entropy of a black hole is given by[43] $S = k_B c^3 A/4\hbar G$ where $A=16\,\pi\,(GM/c^2)^2$ and k_B is Boltzmann's constant. [44]

In Planck units this formula reduces to $S = 4\pi M^2$. The difference of the entropies before and after addition of the proton is given by

(8.16) $$\Delta S/4\pi = (M + m_p)^2 - M^2 \approx 2\,M\,m_p$$

Inserting the numerical values we find

(8.17) $$\Delta S = 8\times 10^{60}\times 7.7\times 10^{-20}/4\pi = 9.8\times 10^{40}$$

Thereby we have captured the order of magnitude of all AQI bits that become non-accessible via a single proton. Stated differently: A proton „is" 10^{41} AQI bits.

The comparison with the findings according to Eq. (8.9), by which 6×10^{41} *AQI* bits were attributed to a proton, shows that the data based on estimates of the orders of magnitude are rather consistent and quite realistic.

9. The protyposis concept and the foundation of interactions

Aspiring to be the basis for a physical modelling of the reality, the protyposis concept should also allow one to establish the mathematical form of interaction. This has in fact been achieved[45], as will be briefly outlined in the following.

For a long time, the quest to unify the scientific description of nature followed the path towards ever greater symmetries underlying the forces.

In the protyposis concept, the issue of a unification of the forces is shifted to the level of the simplest and thus fundamental structures of nature. This allows one to derive the basic forms of interactions and thereby establish the distinct interactions.

9.1. Interaction and gauge groups

As is well known, the interactions, seen as relevant in the atomic realm, can be understood in terms of local gauge groups. The compact groups $U(1)$, $SU(2)$, and $SU(3)$ are the gauge groups for the electromagnetic, the weak, and the strong interaction. Aiming at an eventual unification, one has searched for larger groups comprising these three gauge groups as subgroups. As possible candidates for such a larger group, the groups $SU(5)$, SO(10), and $E(8)$ have been investigated. However, all these big groups have the undesirable feature of grossly inflating the manifold of the concomitant „fundamental quantum species", that is, of entities to be considered as simple and elementary. The corresponding hypothetical particles, which have not been found in the energy region known so far, must be expected to be extremely instable in view of their large mass, as the experimental experience suggests. This could already be seen in the case of the Higgs particle, which has a life time in the order of 10^{-22} *s*. Obviously, one here reaches the limit for the applicability of the notion of „existence".

Interaction is a concept deriving from classical physics. The tensor product in the quantum-theoretical composition generates „entireness without parts". Therefore interaction can only be a valid concept for systems that can be considered as consisting of separate parts.

Already in Newtonian mechanics an individual coordinate cosmos is assigned to each particle. The time-evolution of the particle state is monitored by the variation of the respective coordinates. In the quantum-theoretical description, change is associated with the momentum operators P_k, being essentially the derivatives with respect to the coordinates of the wave function,

(9.1) $$P_k = -i\,\partial/\partial x_k$$

[43] Bekenstein (1973, 1974), Hawking (1975)
[44] measuring the temperature in erg, then $k_B=1$
[45] Görnitz, T (2014), Görnitz, Schomäcker (2016)

As is empirically well confirmed, the „switching on" of an interaction can be effected by replacing the partial derivatives with the covariant derivatives

(9.2) $$\partial/\partial x_k \Rightarrow \partial/\partial x_k + g\, A_a^k\, \theta^a$$

Here Θ^a is a generator in the Lie algebra of the respective gauge group and g denotes a coupling constant.

9.2. Electromagnetic and weak interaction

The description of the electromagnetic and the weak interaction follows rather directly from the preceeding considerations.

For a particle in Minkowski space, the interaction concept requires to choose the coordinate cosmos of a possible interacting partner. According to the protyposis cosmology this is a homogeneous space of the *U(1)* and *SU(2)* groups.

The operator for the motion in Minkowski space has to be augmented by the operators for the motion in the *U(1)×SU(2)* manifold, where a linear approximation is enabled in form of the Lie algebras ***u(1)+su(2)*.** This means that the motions in the homogeneous spaces of the *U(1)* and *SU(2)* groups are approximated in the vicinity of the unit element of the respective group. An element γ of the *U(1)* group can be represented according to

(9.3) $$\gamma = exp\{i\, A\, \zeta\} \approx 1 + i\, A\, \zeta$$

and similarily for the *SU(2)* group,

(9.4) $$\gamma = exp\{i\Sigma\, B_a\, \sigma^a\} \approx 1 + i\, \Sigma\, B_a\, \sigma^a$$

Here, as usual, the expansions of the exponential functions have been truncated after the linear term. The generators σ^a of the Lie algebra ***su(2)*** are the three Pauli matrices.

In the electromagnetic and weak interaction the momentum operators P^k are extended to the well-known forms

(9.5) $$P^k \rightarrow -i\partial/\partial x_k + g_1 A^k\, \zeta + g_2\, B^k{}_a\, \sigma^a$$

The two coupling constants g_1 and g_2 serve a measure of the respective charge and, thus, of the strengths of the interactions as well as the mass generated by the charge.

9.3. The strong interaction

In the foundation of the strong interaction it has to be taken into account that quantum theory requires the use of complex numbers. Quantum theory accounts for real effects deriving from possibilities. In classical physics, possibilities merely reflect the lack of knowledge on part of the describer and, thus, do not at all affect the behaviour of the systems.

Being much more accurate, quantum theory captures the temporally non-local influence of future possibilities on the current process. This cannot be modelled using only real numbers.

Real and imaginary numbers satisfy the well-known relations

(9.6) *real×real = real, imaginary×imaginary = real, real×imaginary = imaginary*

In the field of groups, these relations are mirrored by the so-called Cartan decomposition[46].

Consider a group *G* with a subgroup *K,* and let *P* denote the corresponding coset. Supposing that the Lie algebra **G** can be decomposed according to

(9.7) $$\mathbf{G} = \mathbf{K} + \mathbf{P}$$

where **K** is the subalgebra associated with *K*, then this is referred to a Cartan decomposition of the Lie group **G** if the elements

(9.8) $$k_i \in \mathbf{K} \quad \text{and} \quad p_i \in \mathbf{P}$$

[46] Böhm (2011) S. 377 ff.

satisfy the relations

(9.9) $$\lfloor k_i, k_j \rfloor \in \mathbf{K}, \quad \lfloor p_i, p_j \rfloor \in \mathbf{K}, \quad \text{and} \quad \lfloor k_i, p_j \rfloor \in \mathbf{P}$$

Here $[a,b] = ab - ba$ denotes the commutator of a and b.

If the the manifolds **K** and **P** are of same dimension, this algebraic structure is an analogue to the relations (9.4), underlying the transition from real to complex numbers, representing here $\boldsymbol{C^1}$ by $\boldsymbol{R^2}$.

To incorporate the quantic possibilities this suggests to double the coordinate cosmos and establish a structure that is analogous to that of Eqs. (9.6) and (9.9).

This postulate leads from $U(1)\times SU(2)$ directly to the $SU(3)$ group.

Following a paper by Byrd[47], it can be shown that the $SU(3)$ group, having 8 parameters to be denoted by the string $(\alpha,\beta,\gamma,\theta,a,b,c,\varphi)$, features together with the $U(1)$ and $SU(2)$ groups exactly the structure according to Eqs. (9.6-9.9).

As Byrd has shown, an arbitrary element of the $SU(3)$ group can be represented as

(9.10) $$D(\alpha,\beta,\gamma,\theta,a,b,c,\varphi) = e^{(i\lambda_3\alpha)}e^{(i\lambda_2\beta)}\, e^{(i\lambda_3\gamma)}e^{(i\lambda_5\theta)}e^{(i\lambda_3 a)}e^{(i\lambda_2 b)}e^{(i\lambda_3 c)}e^{(i\lambda_8\varphi)}$$

Here λ_i denote 3×3 matrices of the set of the 8 Gell-Mann matrices (see Appendix 2). In particular, the matrices λ_1, λ_2, and λ_3 result from augmenting the three Pauli matrices with each one row and one column of zero elements.

The representation (9.10) is of the form

(9.11) $$D(\alpha,\beta,\gamma,\theta,a,b,c,\varphi) = D^{(2)}(\alpha,\beta,\gamma)e^{(i\lambda_5\theta)}\, D^{(2)}(a,b,c)e^{(i\lambda_8\varphi)}$$

Here $e^{(i\lambda_5\theta)}$ and $e^{(i\lambda 8\theta)}$ are elements of the $U(1)$ group, and $D^{(2)}(\alpha,\beta,\gamma)$, $D^{(2)}(a,b,c)$ elements of $SU(2)$.

Thus, when the quantum-theoretical possibilities are taken into account, one retrieves, in addition to the electromagnetic and weak interaction, the structure of the strong interaction.

However, the corresponding quanta can only be structural quanta. Accordingly, quarks and gluons cannot appear as actual quantum particles in vacuum. This, however, does not prevent that they can act like particles within strongly interacting matter.

According to these insights, Eq. (9.5) is to be extended as

(9.12) $$P^k \rightarrow -i\partial/\partial x_k + g_1 A^k \zeta + g_2 B^k{}_a\, \sigma^a + g_3 C^k{}_a\, \lambda^a$$

The fact that the structural quanta of the strong interaction, quarks and gluons, cannot appear as free particles in vacuum is consistent with the empirical findings.

10. The general importance of the protyposis concept for a foundation of the natural sciences.

Natural science has the aim to explain the phenomena in the world, which will allow us to derive guidances for acting in adapted and preferably rational ways. To explain means to reduce complex structures to simple ones, unknown structures to ones already known. Such a reduction can be seen as successful, when something not understood becomes comprehensible.

Since for mathematical and physical reasons, there are no simpler structures than the protyposis AQI bits, the latter are the logical basis for any scientific explanations. In the practical implementation it is of course necessary to interprete and render comprehensible the mathematical transitions from a lower structural level to a higher one.

[47] Byrd (1998)

Given the amount of knowledge available today, it has become possible to base the observation data upon sound theoretical foundations. Today, the evolution within the cosmos can be understood. The evolution has begun with very simple structures, from which very complex structures have evolved. This makes apparent that natural science must reduce. Here, a dualistic concept, supposing two basic substances such as mind and matter, must be ruled out.

In principle, the actual structures can be derived from the simplest structure, the protyposis. Here „in principle“ means that the mathematical limits are understood and interpreted. Then it is not necessary or reasonable to treat the more complex structures at the next higher level without using the approximation methods suitable there and apply the procedures of the preceeding level. A „Schrödinger equation for the cell“ would not be a sensible approach.

In the cosmic evolution, after a very early and very hot phase, black holes are formed. Around them, in first galaxies, stars are formed from the then available hydrogen and helium. After the first supernova explosions had generated heavy elements, celestial bodies such as planets and comets can be formed.

If the conditions on a planet are suitable, life will develop. And if these conditions prevail sufficiently long, that is, if the star is not too big and thus explodes to soon and if the planet is not thrown off its course, then life will eventually develop forms of life capable of consciousness.

As has been known for a long time, quantum particles with a rest mass can take up and give off energy. Usually this is effected by absorption and emission of real and virtual photons. Being manifestations of the protyposis AQI bits, it is comprehensible that the material quanta and the photons can absorb or emit single AQI bits to be seen as meaningful. Mostly, these will be referred to as „properties“ of the particle.

Living beings differ from stable objects in that they permanently face a plethora of instable situations at every organisational level – from the constituents of the cells, to cells and organs, up to the entire being. In instable situations already tiniest causes can effect an influence.

Therefore life can be characterized in that living beings are thermodynamically instable systems that stabilize themselves via the processing of information. [48]

During the biological evolution, in rapidly movable life forms, such as animals, organs will develop that are specialized on information processing. Ultimately, in animals with a highly developed brain the formation of consciousness becomes possible. [49]

The protyposis concept allows us to understand that an advanced system of information processing such as the brain is distinguished in that there is no strict separation between „hardware“ and „software“.

As bits of quantum information, the AQI bits of the protyposis are to be related – if an every-day metaphor is suitable here – to our thoughts rather than to our body. With the protyposis as fundamental substance, it has become possible to explain the genesis of matter and the evolution, from the cosmos to life, up to the human psyche and consciousness, in a unifying way.

With the humans, who not only can enjoy and admire the creations of nature but, moreover, are destined to comprehend all that, the arc beginning with the big bang of the R=ct universe comes to a conclusion.

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[48] Görnitz & Görnitz (2016), Kap. 12
[49] see the summary in Görnitz & Görnitz (2016), S. 738 ff.

Appendix 1

An element g of the group $SU(2)$ has the form

(A1-1) $$g = \begin{pmatrix} x+iy & u+iv \\ -u+iv & x-iy \end{pmatrix} \quad \text{with} \quad \det|g| = x^2+y^2+u^2+v^2 = 1$$

With the Euler angles the usual parametrisation of the S^3 with radius 1 results in

(A1-2) $$g(\varphi,\psi,\theta) = \begin{pmatrix} e^{i(\varphi+\psi)}\cos\theta & ie^{i(\varphi-\psi)}\sin\theta \\ ie^{-i(\varphi-\psi)}\sin\theta & e^{-i(\varphi+\psi)}\cos\theta \end{pmatrix}$$

The Hilbert-space $L^2(S^3)$ of the square integrable functions $f(\varphi, \psi, \theta)$ over S^3 is the representation space for the regular representation of $SU(2)$.
An irreduzible subrepresentation of „spin j“ has a (2j+1)-dimensional representation space. In this space a basis is given by the functions $D_j^{kl}(\varphi, \psi, \theta)$.[50]

(A1-3) $$D^j(\varphi,\psi,\theta)_l^k = e^{-i(k\varphi+l\psi)} d^j(\theta)_l^k$$

The functions $d^j(\theta)^k{}_l$ are the Jacobian polynoms

(A1-4) $$d^j(\theta)_l^k = \sqrt{\frac{(j+k)!(j-k)!}{(j+l)!(j-l)!}} (\cos\tfrac{\theta}{2})^{k+l} (\sin\tfrac{\theta}{2})^{l-k} P_{j-k}^{k-l,k+l}(\cos\theta)$$

In the special cases $k=l$ Jacobian Polynoms transform into Legendre polynoms $P_j^k(\cos\theta)$

(A1-5) $$P_{j-k}^{k,k}(z) = (-2)^k \frac{j!}{(j-k)!} (1-z^2)^{-k/2} P_j^k(z)$$

In present connections the shortest wavelengths are of interest. For the trigonometric functions of φ resp. ψ the shortest wavelengths result for l resp. $k = j$, for the Legendre polynoms $P_j^k(\cos\theta)$ for $k=0$.
The wavelengths are of order $1/j$ resp. $2\pi/j$. For the case $j=30$ an illustration is given for $\cos(30x)$ und $P_{30}(\cos x)$

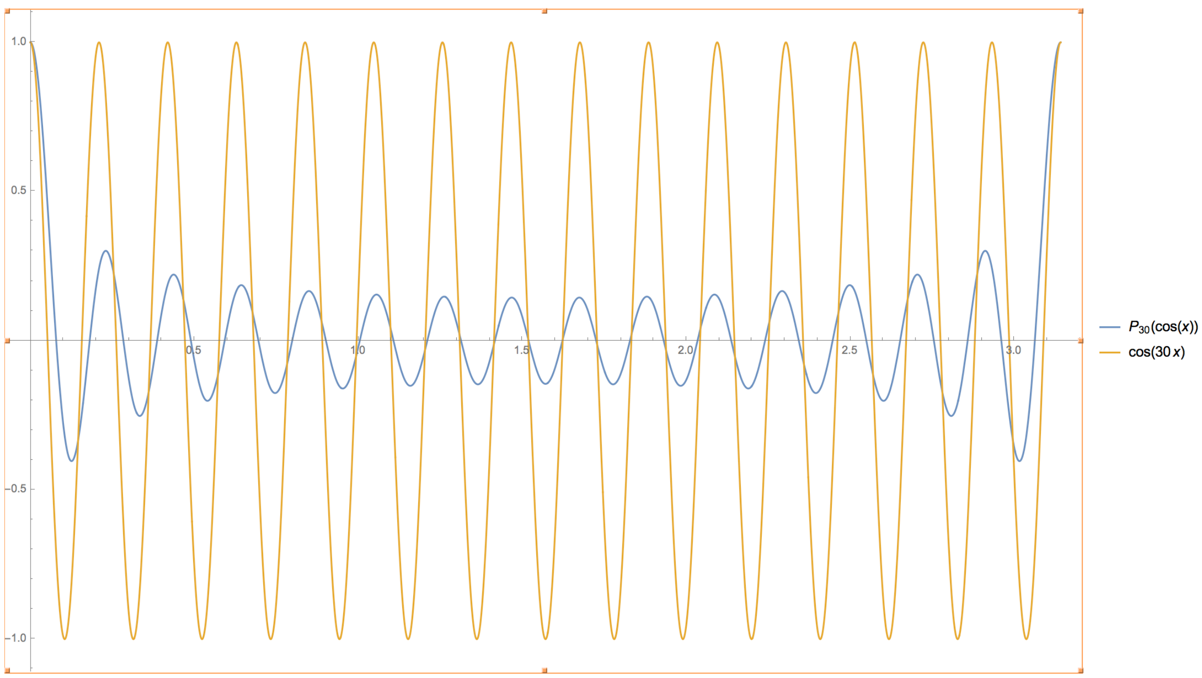

Figure A1-1: The Graphs of the funktions cos(30x) and P30(cosx) between 0 and 2π

[50] Siehe z.B. Tung, 1985, pp 141

Appendix 2

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Figure A2-1: The 8 generators of the Lie algebra of the SU(3) group in the so-called Gell-Mann representation. λ_1, λ_2 and λ_3 are extensions of the Pauli matrices; together with λ_8 they generate the U(2) group.

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
λ_1	0	$2i\lambda_3$	$-2i\lambda_2$	$i\lambda_7$	$-i\lambda_6$	$i\lambda_5$	$-i\lambda_4$	0
λ_2	$-2i\lambda_3$	0	$2i\lambda_1$	$i\lambda_6$	$i\lambda_7$	$-i\lambda_4$	$-i\lambda_5$	0
λ_3	$2i\lambda_2$	$-2i\lambda_1$	0	$i\lambda_5$	$-i\lambda_4$	$-i\lambda_7$	$i\lambda_6$	0
λ_4	$-i\lambda_7$	$-i\lambda_6$	$-i\lambda_5$	0	$i\lambda_3$ $+i\sqrt{3}\lambda_8$	$i\lambda_2$	$i\lambda_1$	$-i\sqrt{3}\lambda_5$
λ_5	$i\lambda_6$	$-i\lambda_7$	$i\lambda_4$	$-i\lambda_3$ $-i\sqrt{3}\lambda_8$	0	$-i\lambda_1$	$-i\lambda_2$	$i\sqrt{3}\lambda_4$
λ_6	$-i\lambda_5$	$i\lambda_4$	$i\lambda_7$	$-i\lambda_2$	$i\lambda_1$	0	$-i\lambda_3$ $+i\sqrt{3}\lambda_8$	$-i\sqrt{3}\lambda_7$
λ_7	$i\lambda_4$	$i\lambda_5$	$-i\lambda_6$	$-i\lambda_1$	$i\lambda_2$	$i\lambda_3$ $-i\sqrt{3}\lambda_8$	0	$i\sqrt{3}\lambda_6$
λ_8	0	0	0	$i\sqrt{3}\lambda_5$	$-i\sqrt{3}\lambda_4$	$i\sqrt{3}\lambda_7$	$-i\sqrt{3}\lambda_6$	0

Figure A2-2: Product table for the λ_i matrices.

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