Gravity, Anomaly Cancellation, Anomaly Matching, and the Nucleus

Syed Afsar Abbas

Centre for Theoretical Physics, JMI University, New Delhi - 110025, India

and

Jafar Sadiq Research Institute
AzimGreenHome, NewSirSyed Nagar, Aligarh - 202002, India
(e-mail : drafsarabbas@gmail.com)

Abstract

In the Standard Model there has been the well known issue of charge quantization arising from the anomalies and with or without spontaneous symmetry breaking being brought into it. It is well known that in the purely anomalies case, unexpectedly there pops in a so called "bizarre" solution. We discuss this issue and bring in the 't Hooft anomaly matching condition to find a resolution of the above bizarreness conundrum. We find a completely consistent solution with a unique single nucleon-lepton chiral family. We find that at low energies, the nucleus should be understood as made up of fundamental proton and neutron and where quarks play no role whatsoever. In addition it provides a new understanding and consistent solutions of some long standing basic problems in nuclear physics, like the quenching of the Gamow-Teller strength in nuclei and the issue of the same "effective charge" of magnitude 1/2 for both neutron and proton in the nucleus. Fermi kind of four-fermion-point-interaction appears as an exact (non-gauge) result.

Keywords: Gravity, mixed gauge gravitational anomaly, Standard Model, anomaly cancellation, 't Hooft anomaly matching condition, nuclear effective charge, Gamow-Teller strength in nuclei, Fermi four-fermion-point-interaction

Using the anomalies only and with or without spontaneous symmetry breaking, in the Standard Model of particle physics, there arises the well known issue of charge quantization or lack thereof. There pops in a so called "bizarre" solution in the pure anomalies situation (to be discussed below). We focus on this issue by invoking 't Hooft anomaly matching condition to find a resolution of the above bizarreness puzzle. This provides a new understanding and solution of some long standing basic problems in nuclear physics.

The Standard Model (SM) of particle phyiscs is based on the group structure $SU(N_C) \otimes SU(2)_L \otimes U(1)_Y$ ($N_C = 3$). This symmetry is spontaneously broken (SSB) to $SU(N_C) \otimes U(1)_{em}$ by an Englert-Brout-Higgs (EBH) field taken to be an $SU(2)_L$ group doublet [1,2],

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1}$$

The first generation is special (i.e. non-repetitive) in a manner that we shall explain below. These are assigned to the following representations in the SM:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (N_c, 2, Y_q) \tag{2}$$

$$u_R, (N_c, 1, Y_u), \quad d_R, (N_c, 1, Y_d) \tag{3}$$

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (1, 2, Y_l) \tag{4}$$

$$e_R, (1, 1, Y_e) \tag{5}$$

There are five unknown hypercharges plus the above unknown Y_ϕ of the EBH doublet.

For the group $SU(2)_L \otimes U(1)_{Y_W}$ we define the electric charge operator as

$$Q = T_3 + b\, Y \tag{6}$$

In the SM we have three massless generators W_1, W_2, W_3 of $SU(2)_L$ and X of $U(1)_Y$. SSB by EBH mechanism provides mass to the $W^\pm$ and Z^0 gauge particles while ensuring zero mass for photons [1,2]

Let $T_3 = -\frac{1}{2}$ of the EBH field develop a nonzero vacuum expectation value $< \phi >_0$. One of the four generators $(W_1 W_2 W_3, X)$ is thereby left unbroken, (meaning that we ensure a massless photon as a generator of the $U(1)_{em}$ group), we demand:

$$Q < \phi >_0 = 0 \tag{7}$$

This fixes the unknown b and we obtain,

$$Q = T_3 + (\frac{1}{2Y_\phi})Y \tag{8}$$

Anomalies play a very significant role in quantum field theories. As we require the SM to be renormalisable, we have to ensure that all the anomalies vanish. Thus we have three anomalies listed as A, B and C as below [1,2]

$$\mathit{Anomaly\ A:}\quad TrY[SU(N_C)]^2 = 0\ \ ;\ 2Y_q = Y_u + Y_d \tag{9}$$

$$\mathit{Anomaly\ B:}\quad TrY[SU(2)_L]^2 = 0\ \ ;\ Y_l + N_c\, Y_q = 0 \tag{10}$$

giving,

$$Y_q = -\frac{Y_l}{N_c} \tag{11}$$

And

$$\mathit{Anomaly\ C:}\quad Tr[Y^3] = 0 \tag{12}$$

giving

$$2N_cY_q^3 - N_cY_u^3 - N_cY_d^3 + 2Y_l^3 - Y_e^3 = 0 \tag{13}$$

We still need to have more terms to determine all the hypercharges. The Yukawa mass terms in the SM is.

$$\mathcal{L} = -\phi^\dagger \bar{q}_L \bar{u}_R + \phi q_L \bar{d}_R + \phi e_L \bar{e}_R \tag{14}$$

which on demand of gauge invariance yields,

$$Y_u = Y_q + Y_\phi, \quad Y_d = Y_q - Y_\phi, \quad Y_e = Y_e - Y_\phi \tag{15}$$

Now substituting Y_q and Y_u, Y_d, Y_e in eq. (13),

$$(Y_l + Y_\phi)^3 = 0 \tag{16}$$

Then

$$Y_l = -Y_\phi \tag{17}$$

and putting this above,

$$Y_q = \frac{Y_\phi}{N_c} \tag{18}$$

These yields expressions for all the hypercharges. Finally, we get quantized electric charges [1,2] in the SM:

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}), \quad Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{19}$$

$$Q(\nu_e) = 0, \quad Q(e) = -1 \tag{20}$$

In the above we saw that the three anomalies A,B and C, together with SSB through the EBH mechanism and generation of Yukawa masses for the matter particles, do give consistent and complete charge quantization in the SM.

Chirality ensures that the fermions are massless. So composites of fundamental entities in the chiral limit may match each other through the 't Hooft anomaly matching condition [3]. We took the first generation as unique. It is unique as the coloured massless u-, d- quarks form an isospin doublet in the SM. Then the only colourless composites that we can create in the ground state, the spin-half fermions, are proton (uud - quarks) and neutron (udd - quarks). Now (p,n) do form a massless chiral isopin-doublet. Thus the 't Hooft matching condition is indeed saitsfied. (Note the same argument does not go though for the three flavours - u,d and s quarks).

Thus due to the above, the first generation of quark-lepton (as given above in eqns. (2-5)) goes over to a unique single (non-repetitive) generation of massless chiral nucleon-lepton as follows:

$$N_L = \begin{pmatrix} p \\ n \end{pmatrix}_L, (1, 2, Y_N) \tag{21}$$

$$p_R, (1, 1, Y_p), \quad n_R, (1, 1, Y_n) \tag{22}$$

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (1, 2, Y_l) \tag{23}$$

$$e_R, (1, 1, Y_e) \tag{24}$$

Now the same logic as for the quarks goes through. As above the electric charge is given by eqn.(8). The three anomaly conditions lead to,

$$Anomaly\ A: \quad TrY[SU(N_C)]^2 = 0 \ ; \ 2Y_N = Y_p + Y_n \tag{25}$$

$$Anomaly\ B: \quad TrY[SU(2)_L]^2 = 0 \ ; \ giving \ \ Y_N = -Y_l \tag{26}$$

$$Anomaly\ C: \quad Tr[Y^3] = 0 \tag{27}$$

giving

$$2Y_N^3 - Y_p^3 - Y_n^3 + 2Y_l^3 - Y_e^3 = 0 \tag{28}$$

The Yukawa mass terms (fixing the nucleon mass to be about 940 MeV) as above bring in

$$Y_p = Y_N + Y_\phi, \quad Y_n = Y_N - Y_\phi, \quad Y_e = Y_e - Y_\phi \tag{29}$$

Thus we find

$$Y_l = -Y_\phi \tag{30}$$

Finally, we get quantized electric charges for this unique nucleon-lepton single generation as,

$$Q(p) = 1, \quad Q(n) = 0 \tag{31}$$

$$Q(\nu_e) = 0, \quad Q(e) = -1 \tag{32}$$

In the above we saw that three anomalies A,B and C together with SSB through the EBH mechanism and generation of Yukawa masses for the matter particles, do give consistent and complete charge quantization for this unique single generation of nucleon-lepton. Most important to see that these nucleons are taken as fundamental particles and not composites of quarks - the 't Hooft anomaly matching had made these nucleons massless and point-like chiral fermions as fundamental particles belonging to a single generation.

Note that now the nucleon as isospin doublet of fundamental particles - proton and neutron, provides a basis for Charge Independence (CI) in nuclear physics. This thus requires Generalized Pauli Exclusion Principle (GPEP) and with the use of the Bruckner-Hartree-Fock analysis brings about and justifies the Independent Particle Model (IPM) as the most successful shell model of the nucleus. Thus this is clear that the above single generation with proton and neutron provides a chiral basis and gets 940 MeV mass through Yukawa coupling. All this provides a consistent rationale for the IPM success in understanding the structure of the nucleus in terms of nucleons treated as fundamental particles - like in the pre-meson and pre-quark days.

Next one may ask whether it is possible to obtain charge quantization in the SM entirely with the anomalies with no SSB through EBH kind of mechanism, This means no Yukawa mass also. One notices that the three anomaly conditions A,B and C above are not constraining enough for this purpose.

However, in addition to the above three anomalies A, B and C there is another triangular anomaly with a mixture of one chiral current vertex with two enegy-momentum tensor (gravitational) vertices. This is the way that gravity is brought into play in the long wave length limit [4,5]. Thus a necessary consequence that the SM couples to the gravity consistently is that the sum of $U(1)_Y$ hypercharges of the Weyl fermions, $TrY = 0$ must vanish. This is called the mixed-gravitation-gauge anomaly.

This was brought into play by Geng and Marshak [6]. Calling the fourth new Anomaly D, leads to the following constraint

$$\textit{Anomaly D}: \quad TrY = 0 \ ; \ 2 \, N_c \, Y_q - N_c \, Y_u - N_c \, Y_d + 2 \, Y_l - Y_e = 0 \tag{33}$$

They [6] claimed that just by using these four anomalies (A,B,C and E) they were able to obtain charge quantization in the SM without the requirement of the SSB mechanism. This was unfortunately based on a fallacious argument as was pointed out in ref. [2]. Thus actually there is no charge quantization based entirely on the basis of the anomalies only. The four anomalies are not constraining enough to bring about charge quantization in the SM [2].

However here we are interested in the additional independent solution, the so called "bizarre" solution due to the imposition of just the four anomaliy conditions, as pointed out by Minahan, Raymond and Warner [7]. This "bizarre" case is

$$Y_q = Y_l = Y_e = 0; \ \ Y_u = -Y_d \tag{34}$$

Later Geng and Marshak pointed out [8] as to why this "bizarre" solution was ignored by them in their work. They showed that this bizarre solution has inconsistencies and because of which it should be rejected.

In this paper we go beyond the above bizarreness and find a way around the above conundrum. This we do in terms of the chiral single nucleon-lepton generation given above. Now entirely within the structure of the four anomalies and without any SSB mechanism, we find the following solutions.

The Amomalies A,B and C give constraints as given above in eqns. (25), (26) and (28). The fourth new Anomaly D leads to the following constraint

$$Anomaly\ D: \quad TrY = 0 \ ; \ 2\,Y_N - \ Y_p - \ Y_n + 2\,Y_l - Y_e = 0 \tag{35}$$

Putting it together

$$Y_p + Y_n + Y_e = 0 \tag{36}$$

Then we get a simple and unique solution as

$$Y_p = -Y_n \tag{37}$$

and further we get,

$$Y_N = Y_l = Y_e = 0 \tag{38}$$

Now note that eqns.(37) and (38) are clearcut solutions for the nucleon-lepton unique single generation above. This is counterpart of the above inconsistent "bizzare" solution of the u-d quarks first generation. There is nothing bizarre in the nucleon-lepton single generation all-anomalies solution above. However to identify the solution and to distinguish it from the others we call it the "BIZ" solution

Note that we defined the single generation electric charge in the most general manner as given in eqn.(6). Now the parameter 'b' is an undetermined parameter. Due to the EBH field and SSB arising thereby we could fix 'b' as in eqn.(8) in the single nucleon-lepton picture above, in terms of the EBH hypercharge. In the BIZ model we do not have that luxury. So using eqn.(6) the charges in the BIZ model are:

$$Q(\dot{p}_L) = \frac{1}{2}\ ; \quad Q(\dot{n}_L) = -\frac{1}{2} \tag{39}$$

$$Q(\dot{\nu}_L) = \frac{1}{2}\ ; \quad Q(\dot{e}_L) = -\frac{1}{2} \tag{40}$$

$$Q(\dot{p}_R) = bY_p\ ; \quad Q(\dot{n}_R) = -bY_p \tag{41}$$

$$Q(\dot{e}_R) = 0 \tag{42}$$

Note that to distinguish the states here with respect to the nucleon and leptons in eqns. (31) and (32) we have put a dot on the states here and call them "dot-nucleons" and "dot-leptons".

Let us renormalize the dot-nucleon charges in eqn. (41) by dividing with Y_p and getting

$$Q'(\dot{p}_R) = b \; ; \quad Q'(\dot{n}_R) = -b \tag{43}$$

Putting it in matrix form

$$Q' \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = \begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = b \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} \tag{44}$$

Now first let us take "b" as a simple real number. As a number, it is completely undetermined and there is no justification in putting it as 1 or 1/2 or 1/3 or .295 etc. However "b" is actually quantized if treated as the diagonal generator of SU(2). Then for the R-handed dot-nucleons the quantized charges arise as

$$Q' \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} \tag{45}$$

These are the R-handed charges. For the sake of clarity let us write all the charges now for the dot-nucleons and the dot-leptons in the BIZ model as

$$Q(\dot{p}_L) = \frac{1}{2} \; ; \quad Q(\dot{n}_L) = -\frac{1}{2} \tag{46}$$

$$Q(\dot{\nu}_L) = \frac{1}{2} \; ; \quad Q(\dot{e}_L) = -\frac{1}{2} \tag{47}$$

$$Q(\dot{p}_R) = \frac{1}{2} \; ; \quad Q(\dot{n}_R) = \frac{1}{2} \tag{48}$$

$$Q(\dot{e}_R) = 0 \tag{49}$$

Now both the L-handed and the R-handed doublets exist and for the dot-nucleons have electric charges as given above. Note that for the nucleon case the L-handed doublets are different from the R-handed singlets. However in the dot-nucleon case both the L-handed and the R-handed doublets are present in equal measure. What does it mean?

Note that the Dirac equation separates out into the L- and R- chiral parts. But the negative energy sea associated with the L- and R- fermions are not separately defined in a gauge invariant way, for the L- and R- fermions [9 p. 423, 10] . But above we saw that the cancellation of the four anomalies ensures that this classical property of the Dirac equation is indeed recovered (see eqns. (46) and (48)) for the case of the dot-nucleons. This is indeed a satisfying consistency check on the dot-nucleons solution of the BIZ model.

Now above we saw that the nucleon-lepton single generation with the EBH field and the SSB, gave a consistent descrition of the IPM (shell-model) of the nucleus. Then how does this new solution as given above without any Yukawa mass fit into the nucleus. Clearly, this new picture should be simultaneously and dually valid for the description of the nucleus. In addition, it should also sit outside and be independent of the IPM model structure of the nucleus.

Now we know that the electric charge of the nucleus, and as extensively used in IPM, it is given as,

$$Q = \frac{1}{2}(Z - N) + \frac{1}{2}(Z + N) \tag{50}$$

Note that the above expresssion of electric charge of the IPM of the nucleus does not distinguish between the L- and the R-handedness of the nucleons. But the dot-nucleons model of the nucleus says that the electric charge of the nucleus should have handedness associated with it in an intrinsic manner. Using charges in eqns.(46) and (48) we get,

$$Q = [\frac{1}{2}(Z - N)]_L + [\frac{1}{2}(Z + N)]_R \tag{51}$$

Now we see that,

$$T_z = [\frac{1}{2}(Z - N)]_L \tag{52}$$

This model is clearly so very different from the IPM model of the nucleus. This should therfore be taken to mean that this description is simulatneously valid for the nucleus. How can we understand its existence?

During the early period of nuclear physics, it was well known, and as pointed out by Sachs [11, p.11], that there are two kind of two-body forces in the nucleus (note below n-n means interaction between two neutrons etc.)

$$OPTION\ 1\ :\ n - n = p - p = n - p \tag{53}$$

$$OPTION\ 2\ :\ n - p \gg n - n\ \ and\ \ n - p \gg p - p \tag{54}$$

The first option explained the p-p, n-n and n-p being identical as seen in the scattering data. It could also account for deuteron being a bound state of n-p by incorporating an extra concept of spin-dependence of the nuclear force. It meant that the nuclear force is Charge Independent (CI) within the framework of the SU(2) nuclear-isospin model. By extending it to include GPEP, this model became the basis of the presently very sucessful IPM while getting a justification from the Brueckmer-Hartree-Fock analysis.

The second option arose as a consequence of the fact that for light and medium heavy nuclei, almost invariably one finds that $A \approx 2Z$ where A is the atomic mass number and Z is the atomic number. So here N=Z for these nuclei. Thus it appears that the light nuclei prefer to add nucleons in n-p pairs. Thus one type of nucleons are not favoured here in light nuclei. Thus there should exist strong interaction between neutrons and protons. This is dominant for light and medium-heavy nuclei and also has the virtue of explaing two nucleon bound system as that of the n-p pair rather than the n-n or the p-p pair. And this is done without invoking an extra spin-dependent effect as done in the first case. Thus as per this picture, light N=Z nuclei have the following structure,

$$(N = Z)_A = (n - p) + (n - p) + (n - p) + (n - p) + ... \tag{55}$$

Here for every (n-p) pair the isospin is zero. Thus for these nuclei the nuclear isospin T_z sinks to the lowest value - that is zero. As per the IPM model, it is not possible to understand why the isospin always sinks to the lowest value of $|T_z| = T$. It is just taken as a phenomenological fact.

This property of lowest T_z and hence lowest isopin value $|T_z| = T$ is given by the first term of the electric charge in eqns. (50) and (51). This is not explained by IPM model. But this uniqoe property of isospin of light nuclei is a clear prediction of the second option and is a unique predition of our BIZ model.

Now this BIZ model is saying that T_z of light nuclei is entirely L-handed in nature as in eqn. (52). So T_z is arising from the L-handed nature of BIZ charge of the nucleus. However as simultaneously, the IPM also is a good model of the nucleus, it attaches itself to this minimum value of T_z by picking its own isospin by taking $|T_z| = T$. So note that T_z arises as L-handed term from the BIZ model and the isospin T from the IPM. This is the way that they co-exist peacefully to bring about what we call muclear physics. Thus there is this fundamental duality connection between the BIZ model and the IPM of the nucleus.

Thus the weak interaction in the nucleus should indicate this property. For the weak interaction the nuclear third-component of isospin on dot-nucleon should be given as:

$$T_z|\dot{p}_L >= \frac{1}{2}|\dot{p}_L >, \quad T_z|\dot{n}_L >= -\frac{1}{2}|\dot{n}_L > \tag{56}$$

Indeed we see that the charge exchange (p,n) scattering should be given by weak interaction and thus be a model to explain the giant Gamow-Teller resonances in nuclei [12,13]. Thus in the following example, the charge exchange occurs, due to the weak interaction being L-handed, only in the first T_z term of their electric charges,

$$^{90}_{40}Zr_{50}(p,n)^{90}_{41}Nb_{39} \ : \ \ only \ in \ L-part \ (40-50)/2 \ to \ (41-49)/2 \tag{57}$$

Thus in the BIZ model, the chiral L-electric charge only partakes in the weak interaction and thus in the charge changing reaction (p,n), it brings about the Gamow- Teller Giant resonance. However the difference of the GT strengths of the β^- and β^+ in the IPM is [12 p. 116, 13 p. 425] , $S^{GT}_{\beta-} - S^{GT}_{\beta+} = -3g_A^2\tau_z = 3g_A^2(N - Z)$. It is well known that this value overestimates by a factor of two with respect to the experimental value. However for the BIZ model we see that the IPM τ_z term in the sum rule should get renormalized to the term T_z as given in eqns. (52 and (56)). Thus in the BIZ model the end resut is $3g_A^2\frac{(N-Z)}{2}$. This is matching the experimental result exactly. This is a clear vindication of our BIZ model of the nucleus.

Next there are further indications in nuclear physics which give more support to our new model structures i.e. both BIZ and IPM holding simultaneously in the nucleus. de-Shalit [14] has pointed out that in the nuclear shell model (i.e. the IPM) even number of protons or neutrons couple to zero angular momentum both in the ground states of even-even nuclei as well as for the odd-A nuclei for the ground states as well as the low excited states. So in the IPM the average moments of odd-even nuclei depend whether it is a proton or a neutron. So the IPM clearly ditinguishes protons from neutrons.

However there are many electromagnetic properties of nuclei which do not depend upon whether the particle is proton or neutron. Thus the static electric quadrupole moments (EQM) and electric quandrupole (E2) transition, in odd-A nuclei, do not distinguish whether these occur in odd-Z or odd-N nuclei, Experimental data requires that these protons and neutrons both have equal, and 1/2 in magnitude, of the "effective charge". We point out that this indicates that there is an indepenndent and simultaneously holding model to IPM and which should be identified with our BIZ model. So let us see how BIZ explains the issue of the effective charges in the nucleus.

Now for example for the nucleus ${}^{17}_{8}O_9$ as per IPM, there is a single neutron outside a shell-closed magic nucleus ${}^{16}_{8}O_8$. Now as neutron is charge neutral, its EQM and E2 should be zero. However it is found that these quantities are as large as that of ${}^{17}_{9}F_8$ due to the extra proton charge outside the magic nucleus. Amazingly its EQM and E2 both require the proton to have an extra effective charge of magnitute about 1/2 unit of its free charge and which is also the same as what neutron displays in ${}^{17}_{8}O_9$. Within IPM this is understood phenomenologically as arising from mass and charge polarization of the closed shell core by the extra proton and neutron. This polarization is however surprisingly pretty large and also independent of whether the particle is proton or neutron [14,15].

However this is the natural electric charge in the BIZ model. Electromagnetically both L- and R-handed electric charges play identical roles. Thus as per the electric charge in eqn.(51) of the BIZ model, the closed shell at N=Z=8, the first L-handed charge term is zero and the R-handed term for the extra dot-nucleon for both proton and neutron gives exactly the same charge unit of 1/2 in magnitude. Note that as there is a core of closed shell the orbital angular momemntum zero for the valence nucleon is not possible and thus to conserve parity it should contribute to the EQM and the E2 opertor going as $\sim\ e^2 r^2 Y_2$. This is an amazing success of the BIZ model. Note that the experimenatlly measured values may differ somewhat from the exact value of 1/2, due to the coexistence of the IPM with the BIZ model

Let us now include the dot-leptons in the BIZ solution in eqns. (47) and (49). Along with the dot-nucleons these should arise only within the nucleus because of their 1/2 fractional charges. In fact their currents would be exactly like the original Fermi currents for the beta decay and they too should have four fermion point interaction. Here the charges in a current too change by one unit. These current-current interactions in say $\dot{\nu}_e + \dot{n} \rightarrow \dot{p} + \dot{e}^-$ procees is given in the BIZ model as,

$$M = GJ^{weak}_{dot-nucleon}J^{weak}_{dot-lepton} = G(\bar{\psi}_{\dot{p}}\gamma_\mu\psi_{\dot{n}})(\bar{\psi}_{\dot{e}}\gamma_\mu\psi_{\dot{\nu}}) \quad (58)$$

Note that the above is an exact point interaction of four point massless fermions. There is no gauge particle in this model. Hence it is different from the SM beta decay theory where in the limit of heavy mass of the gauge particle M_{W^-}, the matrix M goes over to Fermi-like interaction. The BIZ solution of the Fermi-like interaction is not an approximation of any kind, but as exact solution. The basis of this difference has to be understood in future.

However the neutrinos arising from the nucleus in a beta decay has to be provided by the SM Fermi model limit at low energies. The BIZ solution weak processes however should be confined in the nucleus itself. How this is possible and what effects it implies has also to be studied in future.

Though the shell model IPM of the nucleus has been a successfull model of the nucleus, there has been strong indications that this cannot be the whole story. The liquid drop model has been a contender of this additional structure for long a time. Cook has summarized the situation nicely in his book [16]. The best way to understand the issue is to look at the mean free path (MFP) of nucleon in the nucleus. To understand the structure of the IPM, one has to demand a large value of the MFP. On the other hand the liquid drop character of the nucleus demands a short MFP. Within the framework of shell model these differences are impossible to reconcile [16]. Here we have a completely new structure - the BIZ model, which is simulataneously providing an independent mathematical structure to the IPM, in providing a complete understanding of the nucleus. Hence we expect that the BIZ model should also provide an explanation of the liquid drop character of the nucleus. This has to be studies in the future too.

Here we see that 't Hooft anomaly matching condition provides a single non-repetitive generation of massless chiral nucleons and chiral leptons. Three triangular anomaly matching conditions along with the EBH field provinding SSB, massless photon and fermion masses and thus giving consistent charge quantization to the single generation nucleon-lepton family. This then leads to a rationale for the successfull shell model, the IPM of the nucleus treating nucleons as fundamental entities in an exact manner (rather as an approximation valid at low energies of nucleons basically built up of quarks). There is in addition a new solution - the so called BIZ model - which provides a new structure of nucleons and leptons which exist entirely within the confines of the nucleus. This provides an additional model of the nucleus and which solves the issue of the quenching of the Gamow-Teller strengths in nuclei and also of the puzzling "effective charges" of neutron and proton in nuclei. It also provides a model of a Fermi kind of four-fermion-point interaction as an exact state of massless and point like fermions. The crucial role of gravity in the case of the BIZ solution is basic and decisive.

References

(1). A. Abbas, Phys. Lett **B 238**, 344 (1990)

(2). A. Abbas, J. Phys. **G 16**, L163 (1990)

(3). G. 't Hooft, NATO Sci. Ser. **B 59** 135 (1980)

(4). R. Delbourgo and A. Salam, Phys. Lett. **40B**, 381 (1972)

(5). L. Alvarez-Gaume and E. Witten, Nucl. Phys. **B234**, 269 (1983)

(6). C. Q. Geng and R. E. Marshak, Phys. Rev. D **39**, 693 (1989)

(7). J. A. Minahan, P. Ramond and R. C. Warner, Phys. Rev D **41** 715 (1990)

(8). C. Q. Geng and R. E. Marshak, Phys. Rev. D **41**, 717 (1990)

(9). R. E. Marshak, "Conceptual Foundations of Modern Particle Physics", World Scientific, Singapore, 1993

(10). R. J. Jackiw, in Proc. Symp. Anomalies, Geometry and Topology, Ed. W. A. Bardeen and A. R. White, World Scientific, Singapore, 1985

(11). R. G. Sachs, "Nuclear Theory", Addison Wesley, Cambridge, 1955

[12]. K. Grotz and H. V. Klapdor, "The Weak Interaction in Nuclear, Particle and Astrophysics", Adam Hilger, Bristol, 1990

[13]. Syed Afsar Abbas, "Group Theory in Particle, Nuclear and Hadron Physics", CRC Press, London, 2016

(14). A. de-Shalit, Phys. Rev. **113**, 547 (1959)

(15). A. Abbas and L. Zamick, Phys. Rev. C **21**, 718 (1980)

(16). N. D. Cook, "Models of the Atomic Nucleus", Springer-Verlag, Berlin, 2005