

Quantum algorithms are determining the property of a certain function

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(Dated: May 20, 2017)*

We discuss a character of quantum algorithms. In fact, all of them determine the property of a certain function. The function under study must have the property $f(x) = f(-x)$ when $f(x) \neq 0$.

PACS numbers: 03.67.Lx, 03.67.Ac

Keywords: Quantum computation architectures and implementations, Quantum algorithms, protocols, and simulations

I. INTRODUCTION

The quantum theory (cf. [1–6]) gives approximate and at times remarkably accurate numerical predictions. Much experimental data approximately fits to the quantum predictions for the past some 100 years. We do not doubt the correctness of the quantum theory. The quantum theory, in these days, keeps saying modern science with respect to information theory, where the science is called the quantum information theory [6]. Therefore, the quantum theory gives us another very useful theory in order to create new information science and to explain the handling of raw experimental data in our physical world.

As for foundations of the quantum theory, Leggett-type non-local variables theory [7] is experimentally investigated [8–10]. The experiments report that the quantum theory does not accept Leggett-type non-local variables interpretation. However, there are debates for the conclusions of the experiments. See Refs. [11–13].

Meanwhile, as for applications of the quantum theory, implementation of a quantum algorithm to solve Deutsch’s problem [14–16] on a nuclear magnetic resonance quantum computer is reported firstly [17]. An implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is also reported [18]. There are several attempts to use single-photon two-qubit states for quantum computing. Oliveira *et al.* implement Deutsch’s algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [19]. Single-photon Bell states are prepared and measured [20]. Also the decoherence-free implementation of Deutsch’s algorithm is reported by using such single-photon and by using two logical qubits [21]. More recently, a one-way based experimental implementation of Deutsch’s algorithm is reported [22]. In 1993, the Bernstein-Vazirani algorithm was reported [23, 24]. It can be considered

as an extended Deutsch-Jozsa algorithm. In 1994, Simon’s algorithm was reported [25]. Implementation of a quantum algorithm to solve the Bernstein-Vazirani parity problem without entanglement on an ensemble quantum computer is reported [26]. Fiber-optics implementation of the Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms with three qubits is discussed [27]. Quantum learning robust against noise is studied [28]. A quantum algorithm for approximating the influences of Boolean functions and its applications is recently reported [29]. Quantum computation with coherent spin states and the close Hadamard problem is also discussed [30]. Transport implementation of the Bernstein-Vazirani algorithm with ion qubits is more recently reported [31]. Quantum Gauss-Jordan elimination and simulation of accounting principles on quantum computers are discussed [32]. Finally, we mention that dynamical analysis of Grover’s search algorithm in arbitrarily high-dimensional search spaces is studied [33].

On the other hand, the earliest quantum algorithm, the Deutsch-Jozsa algorithm, is representative to show that quantum computation is faster than classical counterpart with a magnitude that grows exponentially with the number of qubits. In 2015, it was discussed that the Deutsch-Jozsa algorithm can be used for quantum key distribution [34]. In 2017, it was discussed that secure quantum key distribution based on Deutsch’s algorithm using an entangled state [35].

In this paper, we discuss a character of quantum algorithms. In fact, all of them determine the property of a certain function. The function under study must have the property $f(x) = f(-x)$ when $f(x) \neq 0$.

II. QUANTUM ALGORITHMS ARE DETERMINING THE PROPERTY OF A CERTAIN FUNCTION

Suppose

$$f : \{-2^N - 1, -(2^N - 2), \dots, 2^N - 2, 2^N - 1\} \\ \rightarrow \{-2^N - 1, -(2^N - 2), \dots, 2^N - 2, 2^N - 1\}. \quad (1)$$

is a function. We introduce a function with $g(x)$ the transformation from binary to natural representation. We define $g^{-1}(f(g(x))) = F(x)$. We assume

$$F(x) = F(-x) \in \{0, 1\}^N, \\ x \in \{0, 1\}^N. \quad (2)$$

In (1) we define a function f from a set of discrete values to the same set. The argument of (1) appears to be from a number to a number. In (2) we assume that x is a binary representation of such a number. The write-up suggests that x is a binary, i.e., 0,1 vector,

a Cartesian product $\overbrace{\{0, 1\} \times \{0, 1\} \times \dots \times \{0, 1\}}^N$, instance, $x = (0, 1, 1, 0, 0, 1, \dots, 1)$. And we define $-x = -(0, 1, 1, 0, 0, 1, \dots, 1)$.

Our discussion combines quantum parallelism with a property of quantum mechanics known as interference.

Let us follow the quantum states through some algorithm. Throughout the paper, we omit the normalization factor. We define $|-x\rangle = -|x\rangle$. The input state is

$$|\psi_1\rangle = |\overbrace{0, 0, \dots, 1}^N\rangle |\overbrace{1, 1, \dots, 1}^N\rangle \\ = \sum_{y=-(2^N-1)}^{-2} |x\rangle |\overbrace{1, 1, \dots, 1}^N\rangle + \sum_{y=+1}^{2^N-1} |x\rangle |\overbrace{1, 1, \dots, 1}^N\rangle. \quad (3)$$

Now $y = g^{-1}(x)$ is a natural representation of binary x . For example, $y = 3$ implies $x = 11$.

Next, the function F is evaluated using

$$U_F : |x, z\rangle \rightarrow |x, z + F(x)\rangle, \quad (4)$$

and

$$U_F : |x, z\rangle \rightarrow |x, z + F(x)\rangle \\ \Leftrightarrow -|x, z\rangle \rightarrow -|x, z + F(x)\rangle \\ \Leftrightarrow |-x, z\rangle \rightarrow |-x, z + F(x)\rangle \\ \Leftrightarrow |-x, z\rangle \rightarrow |-x, z + F(-x)\rangle, \quad (5)$$

by using $F(x) = F(-x)$. We employ z . It is a binary. Here, $z + F(x) = (z_1 \oplus F_1(x), z_2 \oplus F_2(x), \dots, z_N \oplus F_N(x))$. The symbol \oplus indicates addition modulo 2.

We have the fact;

$$U_F |\overbrace{0, 0, \dots, 1}^N\rangle |\overbrace{1, 1, \dots, 1}^N\rangle = |\overbrace{0, 0, \dots, 1}^N\rangle |\overline{F(0, 0, \dots, 1)}\rangle. \quad (6)$$

Here, for example, if $F(0, 0, \dots, 1) = (0, 1, 1, 0, 0, 1, \dots, 1)$, then $\overline{F(0, 0, \dots, 1)} = (1, 0, 0, 1, 1, 0, \dots, 0)$. Surprisingly, the condition $F(x) = F(-x)$ is necessary for the condition (6) when $F(x) \neq 0$.

Let us start from the following:

$$U_F |\psi_1\rangle = |\psi_2\rangle \\ = \sum_{y=-(2^N-1)}^{-2} |x\rangle |\overline{F(x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle |\overline{F(x)}\rangle. \quad (7)$$

Hence we have

$$|\psi_2\rangle = \sum_{y=-(2^N-1)}^{-2} |x\rangle |\overline{F(x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle |\overline{F(x)}\rangle \\ = \sum_{y=+2}^{2^N-1} |-x\rangle |\overline{F(-x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle |\overline{F(x)}\rangle \\ = \sum_{y=+2}^{2^N-1} |-x\rangle |\overline{F(x)}\rangle + \sum_{y=+1}^{2^N-1} |x\rangle |\overline{F(x)}\rangle \\ = |0, 0, \dots, 1\rangle |\overline{F(0, 0, \dots, 1)}\rangle, \quad (8)$$

by using $\overline{\overline{F(x)}} = F(x)$.

We cannot avoid the following property of the function in order to maintain the consistency between (6) and (8) when $F(x) \neq 0$;

$$\overline{\overline{F(x)}} = \overline{F(-x)}. \quad (9)$$

That is,

$$F(x) = F(-x). \quad (10)$$

III. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have discussed a character of quantum algorithms. In fact, all of them have determined the property of a certain function. The function under study must have had the property $f(x) = f(-x)$ when $f(x) \neq 0$.

1. Quantum computer determines a function. The property of a function, is that it transforms an element from the domain into an element of the codomain.
2. The property of the ‘‘quantum determined function’’, call it g , is as close as possible to the function itself.
3. So x in the domain, $y = f(x)$, in the codomain.
4. So if $|g(x) - f(x)|$ is less than epsilon, a small real number, then quantum computing determines a function close to $f(x)$.

We may say: Quantum algorithms are determining the property of a certain function when the property is close,

as much as possible, to the one for the evaluation of the function.

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