The Four-Dimensional Spacetime with the Mass Density[1]

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Abstract: *Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1907, German mathematician H. Minkowski connected together space and time into single idea, creating a new the four-dimensional spacetime. We proposed the extension of this idea by the connection together the four-dimensional spacetime and the mass density into* ***the single idea****, creating* ***a new entity****: the four-dimensional spacetime with the mass density. In this paper we discussed the physical consequences of this a new idea in terms of the gravitational phenomena.*

1. Introduction

Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1907 German mathematician H. Minkowski connected a space and time into single idea, creating the four-dimensional spacetime [1]. This idea enjoyed success in the *Special Relativity* (SR) and the *General Relativity* (GR), correctly describing a range of physical phenomena.

In this paper we proposed extension of Minkowski's idea by the connection together the four-dimensional spacetime and *the mass density* into **the single idea**, creating a new mathematical structure: *the four-dimensional spacetime with the mass density*.

2. The bare spacetime

This idea is a mathematically defined:

$$\rho_{\mu\nu}^{bare} \overset{def}{=} \rho^{bare} \cdot \eta_{\mu\nu} = \begin{bmatrix} \rho^{bare} & 0 & 0 & 0 \\ 0 & -\rho^{bare} & 0 & 0 \\ 0 & 0 & -\rho^{bare} & 0 \\ 0 & 0 & 0 & -\rho^{bare} \end{bmatrix} \tag{1}$$

where: $\rho_{\mu\nu}^{bare}$ is *the bare mass density tensor*, ρ^{bare} is the bare mass density, $\eta_{\mu\nu}$ is *the Minkowski tensor*, $\mu, \nu = 0, 1, 2, 3$. Such a defined the homogeneous and isotropic a four-dimensional spacetime and the bare mass density ρ^{bare} are not now separate entities, but they form one whole – *the bare spacetime*. The equation (1) describes *the tensor field of inertia*, which assigns of each point of a bare spacetime the tensor $\rho_{\mu\nu}^{bare}$. This field of inertia is a special case of the gravitational field. The bare mass density has ceased to be the scalar and became a tensor. Particles behave in accordance with *the principle of inertia*, i.e. they are at rest or moving in a straight line at constant speed with respect to the bare spacetime (not with respect to the massless spacetime itself, see to chapter 4).

[1]The main theses of this paper has been presented during the conference *First Hermann Minkowski Meeting on the Foundations of Spacetime Physics*, 15-18 May 2017, Albena, Bulgaria.

The metric of the bare spacetime is *the Minkowski metric*

$$ds^2(\eta_{\mu\nu}) = \eta_{\mu\nu} \cdot dx^\mu dx^\nu \,. \tag{2}$$

The metric (2) does not depend *explicitly* on the bare mass density tensor and well suited to describe all the physical phenomena occurring in SR.

3. The effective spacetime[2]

From GR we know, that under influence outer gravitational field the Minkowski spacetime become the pseudo-Riemannian spacetime: $\eta_{\mu\nu} \to g_{\mu\nu}(x)$, where $g_{\mu\nu}(x)$ is a symmetric and position dependent *the metric tensor*.

We suppose that under influence outer gravitational field the bare density mass tensor $\rho_{\mu\nu}^{bare}$ become the symmetric and position dependent *the effective density mass tensor*[3]: $\rho_{\mu\nu}^{bare} \to \rho_{\mu\nu}(x)$. This is **the hypothesis** requiring an empirical confirmation (see chapter 8). The spacetime with the effective mass density we will call *the effective spacetime*. The process in which the spacetime acquires the effective or bare mass density we will call *the massification of spacetime*.

The basic mathematical object characterizing the geometrical properties of the effective spacetime is the mathematically defined

$$ds^2(\rho_{\mu\nu}(x)) \overset{def}{=} \frac{\rho_{\mu\nu}(x)}{\rho^{bare}} \cdot dx^\mu dx^\nu \tag{3}$$

where the effective mass density tensor $\rho_{\mu\nu}(x)$ describes the physical and geometrical properties of the effective spacetime and also the mathematical relationship between the effective spacetime and the bare medium under the influence of a gravitational fields. Note that ρ^{bare} never reaches zero, although it may be very close (see to chapter 7). In the contrast to the vacuum, the bare or effective spacetime **is a never empty**.

The effective mass density tensor $\rho_{\mu\nu}(x)$ and the metric (3) **are very similar** to the metric tensor $g_{\mu\nu}(x)$ and the metric

$$ds^2(g_{\mu\nu}(x)) = g_{\mu\nu}(x) \cdot dx^\mu dx^\nu \tag{4}$$

known from GR.

Let's analyze the motion of the body in an effective spacetime and we compare this equation with the Newtonian equation.

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[2] To describes the physical phenomena under influence outer gravitational field we will use the mathematical formalism in the frame of GR.

[3] The concept of the effective mass tensor to describe gravitational phenomena, instead of the usual the metric tensor, for the first time was discussed in [2].

4. The equation of motion in the effective spacetime

The Lagrangian function for the free body in the effective spacetime has form

$$L = \frac{1}{2}\rho_{\mu\nu}(x)\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$$

The equation of motion

$$\frac{d}{d\tau}\left(\rho_{\gamma\mu}(x)\cdot\frac{dx^{\mu}}{d\tau}\right)-\frac{1}{2}\frac{\partial\rho_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=0 \tag{5}$$

where: $p_{\gamma}(x)=\rho_{\gamma\mu}(x)\cdot\frac{dx^{\mu}}{d\tau}$ is *the effective density of the four-momentum*, τ is *the proper time*. This is *the geodesic equation* in the effective spacetime (see also the equation (8)).

The equation of motion (5) *explicitly* refers to the effective spacetime, which is described by the effective mass density tensor $\rho_{\mu\nu}(x)$. So the motion of the body takes place **only** in relation to the effective spacetime, not to the relation of the empty spacetime itself or all bodies in the Universe (*Mach's Principle* [3], see the chapter 8). The new quality of the understanding has been reached.

When $\rho_{\gamma\mu}(x)$ does not depends *explicitly* on τ, the equation (5) takes the form

$$\rho_{\gamma\mu}(x)\cdot\frac{d^{2}x^{\mu}}{d\tau^{2}}+\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x))\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=0 \tag{6}$$

where:

$$\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x)) \overset{def}{=} \frac{1}{2}\left(\frac{\partial\rho_{\gamma\mu}(x)}{\partial x^{\nu}}+\frac{\partial\rho_{\gamma\nu}(x)}{\partial x^{\mu}}-\frac{\partial\rho_{\mu\nu}(x)}{\partial x^{\gamma}}\right). \tag{7}$$

The equation (7) is very similar to *the Christoffel symbols of the first kind*, where instead metric tensor $g_{\mu\nu}(x)$, we apply the effective mass density tensor $\rho_{\mu\nu}(x)$.

The geodesic equation in GR has the form

$$\frac{d}{d\tau}\left(g_{\gamma\mu}(x)\frac{dx^{\mu}}{d\tau}\right)-\frac{1}{2}\frac{\partial g_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=0 \tag{8}$$

Note that equations (5) and (8) are very similar to themselves.

If the surrounding bodies consist only with the bare masses, i.e. $\rho_{\mu\nu}(x)=\rho_{\mu\nu}^{bare}$, $\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}^{bare})=0$ then the equation of motion (6) takes the form:

$$\rho_{\mu\nu}^{bare}\cdot\frac{d^{2}x^{\nu}}{d\tau^{2}}=0\,. \tag{9}$$

The body with the bare mass density $\rho_{\mu\nu}^{bare}$ is in the rest or moves in a straight line with the constant velocity in the respect to the bare spacetime. The principle of inertia has gained a new meaning and the equation (9) determines a new inertial reference frame – *the bare reference frame*. This reference frame is determined by the bare spacetime property only. The measuring instruments: clocks and roots now have the bare mass density. Old traditional reference system consisting with the massless rods and clocks **loses its physical sense**.

During any change in state of motion of the body appears the inertia, which source is the massification of spacetime. The inertia becomes an intrinsic property of the massification of spacetime. The magnitude of the inertia of any body is also determined by the massification of spacetime. This is the opposite of that, than previously thought. Until now it was thought that inertia is determined by the masses of the Universe and by their distribution [4]. In our model an isolated the body in the Universe always has inertia, because the spacetime with the bare mass density tensor $\rho_{\mu\nu}^{bare}$ formed an inseparable whole. The spacetime ceased to be empty (see chapter 5).

In the particular case the equation (9) becomes

$$\rho^{bare} \cdot \frac{d^2 x^i}{d\tau^2} = 0 . \tag{9a}$$

where: i = 1, 2, 3. The body with the bare mass density ρ^{bare} moves at constant velocity with respect to the bare spacetime.

5. The weak gravitational field

In the weak gravitational field the effective mass density tensor $\rho_{\mu\nu}(x)$ we can decompose to sum of two fields: the inertial field $\rho_{\mu\nu}^{bare}$ and a very small perturbation in the gravitational field $\rho_{\mu\nu}^{*}(x) << 1$. Mathematically we can write it in the following simple form: $\rho_{\mu\nu}(x) = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^{*}(x)$.

The metric (1) takes form:

$$ds^2\left(\rho_{\mu\nu}^{*}(x)\right) = \left(\eta_{\mu\nu} + \frac{\rho_{\mu\nu}^{*}(x)}{\rho^{bare}}\right) \cdot dx^{\mu} dx^{\nu} \tag{10}$$

5.1. The equation of motion

At slow motion speeds, in a static, a weak and the spherically symmetric field the equation of motion (6) reduces to

$$\rho^{bare}\left(1 + \frac{\rho_{rr}^{*}(r)}{\rho^{bare}}\right) \cdot \frac{d^2 r}{dt^2} \cong -\frac{c^2}{2} \cdot \nabla \rho_{00}^{*}(r) \tag{11}$$

where: c is the speed of light.

The equation (11) is a **different** than well-known Newton's equation of the motion for the gravity. It what currently we consider to be *the inertial mass density*, really is the sum of the bare mass density ρ^{bare} and $\rho^{*}_{rr}(x)$ - *rr*-component of the very small perturbation in the effective mass density. Note that *the gravitational mass density* does not appear *explicitly* in the equation (11). Does it mean that during massification of the spacetime *the Equivalence Principle* lost its *raison d'être*?

Dividing both sides of the equation (11) by ρ^{bare} (on the assumption that $\rho^{*}_{rr}(r) \to 0$), we get

$$\frac{d^2 r}{dt^2} \cong -\frac{c^2}{2} \cdot \nabla \left(\frac{\rho^{*}_{00}(r)}{\rho^{bare}} \right) \tag{11a}$$

This is a new form of the equation of motion for the body in the gravitational field. This equation is a very similar to the equation

$$\frac{d^2 r}{dt^2} \cong -\frac{c^2}{2} \cdot \nabla h_{00}(r) \tag{11b}$$

known from GR, where $h_{00}(r)$ is the very small perturbation in the metric tensor $g_{\mu\nu}(r)$.

According to *the Correspondence Principle* (*CP*) we expect that there is a relationship between the component $\rho^{*}_{00}(r)$ and the gravitational potential $V(r)$ in the following form [5]

$$\frac{\rho^{*}_{00}(r)}{\rho^{bare}} = 1 + \frac{2V(r)}{c^2} \tag{12}$$

where: $V(r) = -\frac{GM}{r}$, G is the gravitational constant, M is the mass and r is the distance.
After substituting (12) to (11a), we obtain

$$\frac{d^2 r}{dt^2} = -\frac{\partial V(r)}{\partial r} \tag{13}$$

the well-known Newtonian equation of motion in the gravitational potential $V(r)$.

5.2. The rotating body

Let' s consider the slowly rotating body in a static and a weak gravitational field. The equation of motion have the form

$$\rho^{bare} \cdot \frac{d^2 x^i}{dt^2} = -\frac{c^2}{2} \frac{\partial \rho^{*}_{00}(x)}{\partial x^i} + c \cdot \left(\frac{\partial \rho^{*}_{0k}(x)}{\partial x^j} - \frac{\partial \rho^{*}_{0j}(x)}{\partial x^k} \right) \frac{dx^i}{dt} \tag{14}$$

The following mathematical expressions are responsible for **the real** sources of inertia: $\frac{\partial \rho^{*}_{00}(x)}{\partial x^i}$ and $\frac{\partial \rho^{*}_{0k}(x)}{\partial x^j} - \frac{\partial \rho^{*}_{0j}(x)}{\partial x^k}$. The first term is a force, which is appearing due to the existence of a gradient in the small perturbation of the 00-component in the effective mass tensor $\frac{\partial \rho_{00}^{*}(x)}{\partial x^{i}}$. The second one is velocity-dependent $\frac{dx^{i}}{dt}$ and the rotation $\frac{\partial \rho_{0k}^{*}(x)}{\partial x^{j}} - \frac{\partial \rho_{0j}^{*}(x)}{\partial x^{k}}$.

For the Newtonian approximation the suitable components we can determine from the matrix [6]

$$\rho_{\mu\nu}^{*}(x) = \rho^{bare} \cdot \begin{bmatrix} -\frac{\omega^{2}(x^{2}+y^{2})}{c^{2}} & \frac{\omega y}{c} & -\frac{\omega x}{c} & 0 \\ \frac{\omega y}{c} & 0 & 0 & 0 \\ -\frac{\omega x}{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where: $\frac{c^{2}}{2}\frac{\partial \rho_{00}^{*}(x)}{\partial x^{i}} = -\frac{1}{2}\omega^{2} r^{2}$ and $r^{2} = x^{2} + y^{2}$.

Finally, we get well-known equation for the slowly rotating body in a static and weak gravitational field, which includes *the centrifugal* and *Coriolis acceleration*.

$$\frac{d^{2}r}{dt^{2}} = -\omega^{2} r - 2\omega \frac{dr}{dt} \qquad (15)$$

In the Newtonian approximation the equations of motion (13) and (15) does not depend *explicitly* on the effective mass density and the centrifugal and Coriolis forces are the fictitious forces.

6. The rotating bucket with water problem

There are two entirely different measurements of the Earth's angular velocity, *astronomical* (from upper culmination to upper culmination of the star) and *dynamic* (by means of Foucault's pendulum experiment), which give *the same results* (in the limit of the experimental errors). In both cases the motion of the body is described with respect to the effective spacetime and the coincidence of these measurements is the result of massification of the spacetime.

In the famous experiment with *the rotating bucket with water* [7, 8] the motion of water takes place also to relative of the effective spacetime, therefore the surface of water takes the shape of the parabolic. So, massification of the spacetime explains both these physical phenomena.

7. What is the bare and effective mass density?

Each theoretical model must correspond with the real of the physical world. We suppose that the bare mass density ρ^{bare} for the Universe **probably corresponds** with *the critical density* $\rho_{c} = \frac{3H^{2}}{8\pi G}$ [6], where: H is the Hubble constant. This term is use in the modern cosmology to determine the spatial geometry of the Universe, where ρ_{c} is the critical density for which the spatial geometry is flat (or Euclidean).

8. The experimental confirmation

The idea of massification of spacetime requires **the experimental confirmation**. According to our model, each body moving in the gravitational field of a star in an elliptical orbit, should demonstrate a change in the effective mass. Predicted the annual relative change in the effective mass, as resulting from ellipticity of the orbit for the Earth, is equal to 6.6 x 10^{-10} [9]. GR does not predicts a such fluctuations.

9. Summary

In this paper we applied an alternative attempt to describe gravitational phenomena, using a new idea of the massification of spacetime. If our model will experimentally confirmed, then we will get the following benefits:

1. During any change in state of motion of the body appears the inertia, which source is the massification of the spacetime.
2. The inertia becomes an intrinsic property of massification of the spacetime.
3. The magnitude of the inertia of any body is determined by massification of the spacetime.
4. Inertial forces, appearing in the non-inertial frames of reference, there are no longer fictitious forces.
5. In the gravitational field clocks and roots indicate the different time and length, than in the absence of the field. This different result is caused from the change of the effective mass density in a gravitational field [9].

References

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Appendix

A1. In the search of a field equations

In his book [3] A. Einstein wrote: *This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in not accelerated motion relatively to space, but relatively to the centre of all the other masses in the Universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo. In order to develop this idea within the limits of the modern theory of action through a medium,* ***the properties of the space-time continuum which determine inertia must be regarded as field properties***

***of space**, analogous to the electromagnetic field. The concepts of classical mechanics afford no way of expressing this. For this reason Mach's attempt at a solution failed for the time being.*

In this paper we presented that the inertia is shown as a field properties of the effective spacetime.

Tensor $\rho_{\mu\nu}(x)$ is the geometrical and the physical object. Geometrically determines the metric of the effective spacetime, physically is responsible for the gravitational fields.
In a weak field the component of $\frac{\overset{*}{\rho}_{00}(r)}{\rho^{bare}}$ is expressed by the equation

$$\nabla^2\left(\frac{\overset{*}{\rho}_{00}(r)}{\rho^{bare}}\right)=\frac{8\pi G}{c^2}\cdot\rho(x) \tag{A1}$$

where $\rho(x)$ is the mass density of the body under influence the gravitational field. According to the *CP* (see the equation (12)), the equation (A1) becomes the Poisson equation for the gravity.

$$\nabla^2 V(x)=4\pi G\rho(x) \tag{A2}$$

Having the effective mass tensor $\rho_{\mu\nu}(x)$, we can find *the curvature tensor* $R_{\mu\nu\lambda\eta}(\rho_{\mu\nu}(x))$ of this effective spacetime. The influence of the gravitational field on the bare spacetime can be very complicated, thus the mathematical structure of $R_{\mu\nu\lambda\eta}(\rho_{\mu\nu}(x))$ can be a very sophisticated. We must therefore find the differential equations that govern of the effective spacetime under the influence of the variable gravitational fields.

Here is the field equation without mathematical proof ($\rho^{bare}=1$)

$$R_{\mu\nu}(\rho_{\mu\nu})-\frac{1}{2}\rho_{\mu\nu}\cdot R(\rho_{\mu\nu})+\lambda\cdot\rho_{\mu\nu}=-\frac{8\pi G}{c^4}\cdot T_{\mu\nu}(\rho_{\mu\nu}) \tag{A3}$$

where: tensor $R_{\mu\nu}(\rho_{\mu\nu})$ and scalar $R(\rho_{\mu\nu})$ are the essentially unique contractions of the curvature tensor[4], $\lambda\cdot\rho_{\mu\nu}$ contains only alone the effective spacetime with the mass density, λ is (the cosmological) constant, $T_{\mu\nu}(\rho_{\mu\nu})$ *the energy-momentum tensor*, which now depends on the effective mass tensor $\rho_{\mu\nu}$ and never reaches zero, *i. e.* $T_{\mu\nu}(\rho_{\mu\nu})\neq 0$, although it may be very close.

The left side of the equation (A3) (for $\lambda=0$) describes the geometry of the effective spacetime under influence of a gravitational fields and the right side describes the distribution of the effective sources, also being under influence of a gravitational fields.

[4] $R_{\mu\nu}(\rho_{\mu\nu})\equiv\rho^{\lambda\kappa}\cdot R_{\mu\lambda\nu\kappa}(\rho_{\mu\nu})$, $R(\rho_{\mu\nu})\equiv\rho^{\mu\nu}\cdot R_{\mu\nu}(\rho_{\mu\nu})$.