Using Newtonian Law of Universal Gravitation as Approximation in Special Relativity to Study Equivalence Principle in Schwarzschild Black Hole

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A novel technique was introduced to re-investigate the "No Drama" conclusion of the Equivalence principle at the event horizon of the Schwarzschild Black Hole. It was found that the Equivalence principle has a different solution at the event horizon, on contrary to the popular belief that the gravitational effect on the observer at the horizon holds the similar properties as that to the observer travelling in the weaker gravitational field.

Keywords: Black Hole, Schwarzschild radius, Equivalence principle, Information paradox

1. Introduction

With the Hawking radiation coming into the theoretical picture; it directed a sharp conflict between quantum theory and general relativity in terms of the black hole information paradox [1], mainly due to the inconsistency between the principles of: (1) Quantum Determinism (2) Reversibility & (3) Law of Equivalence, at the event horizon.

Black Hole Complementarity (BHC)[2],[3], which is based on the strongly advocated viewpoint of ‘t Hooft [4], incorporates three postulates: (1) Hawking radiation is in pure state (2) Effective Field Theory is valid outside the stretched horizon (3) No drama i.e. the infalling observer will experience nothing unusual at the horizon, for the phenomenal description of black hole. But, AMPS [5] strongly argues that all the three postulates cannot be mutually consistent, this apparent inconsistency in the Complementarity led to the possible solution often referred as AMPS firewall [6]. Though, the firewall seems to have relatively more consistent approach to the information paradox but has a dire consequence, i.e. it gives up “No Drama”, which is believed to be in direct violation to the Equivalence principle of general relativity, leading to firewall paradox.

This paper is basically written as an illustration; the Equivalence principle has a different solution at the horizon than previously assumed, but it will also extend its discussion to the AMPS firewall; supporting it [5] as perfectly consistent with the law of Equivalence principle, thereby suggesting, any other radical viewpoints are unnecessary.

2. Incorporated Gravitation Approximation

Though, the Newtonian law of universal gravitation does not survive intact in general relativity, but we can create an approximating model of gravitation by incorporating special relativity with the Newtonian law of gravitation.

Newtonian law of universal gravitation

$$F_g = \frac{GMm_0}{R^2} \tag{1}$$

Incorporating Newtonian gravitation and special relativity yields,

$$F_g = \frac{(T^2 - T_0^2)}{2T^2R} \times moc^2 \tag{2}$$

Equating (1) and (2) gives,

$$T_0 \quad = \quad T\sqrt{1 - \frac{2GM}{c^2R}} \qquad (3)$$

Equation (2) is the new approximation of gravitation, and it is a function of time (T_o), due to mass M at distance R. T_0 is a proper time and it is measured relative to time T arbitrarily fixed at 1 (one) second in the centre of the mass M.

3. Schwarzschild radius by incorporated model of gravitation

From equation (2), gravitation of a particle $m_{o,}$ due to mass M, will become time independent at a point in space, where $T_0 = 0$, i.e. from equation (3)

$$T_0 = T\sqrt{1 - \frac{2GM}{c^2R}}$$

Or, $$T\sqrt{1 - \frac{2GM}{c^2R}} = 0 \qquad [T_0 = 0]$$

Or, $$\sqrt{1 - \frac{2GM}{c^2R}} = 0 \qquad [T = 1 \text{ second}]$$

Therefore, $R = R_B = \frac{2GM}{c^2}$ (4)

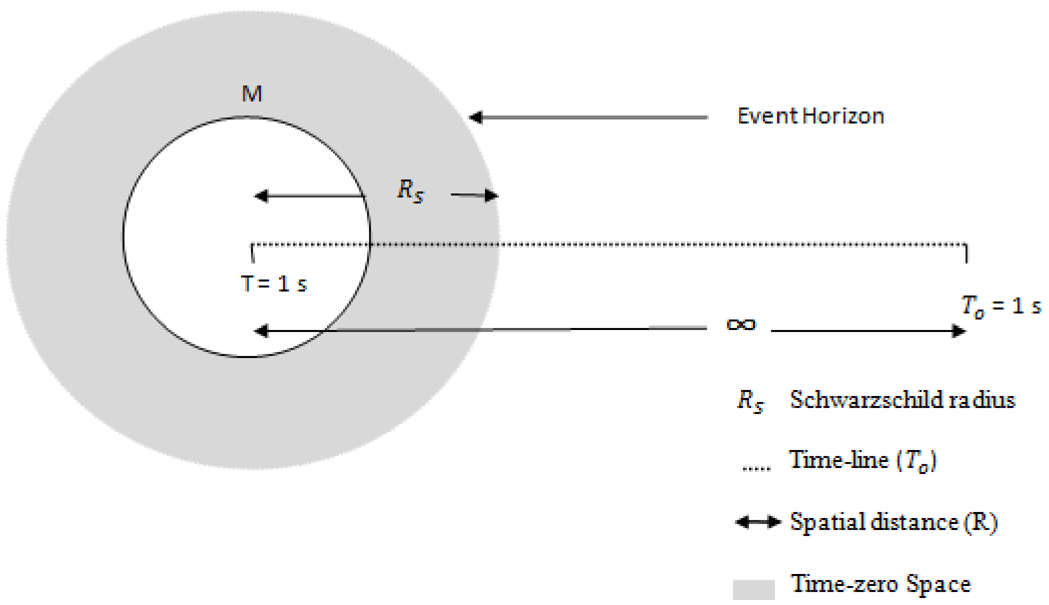

Figure (1): Schwarzschild black hole as described by approximated model of gravitation. Shaded region describes the space where T_0 equals zero. The outer boundary of the zero-time space (shaded region) is the event horizon, beyond it T_0 will take different values ranging from $0 < T_0 \leq 1$, depending upon R, as given by equation (3).

For spherical, non-rotating, non-charged mass, the radial distance R $= R_B$, given by equation (4) is the (initial) maximum radius of collapsing mass M of a black hole which gradually shrinks to (final) $R_B = 0$, giving rise to singularity on due course time. The radius $R_B = \frac{2GM}{c^2}$ locates the initial points in space where T_0 begins to equal zero. Therefore, in the conservative sense, it can be inferred to as a starting point in space, for black hole formation for mass M. The (initial) maximum radius is also known as Schwarzschild radius R_S, i.e. $R_S = R_B$ (initial). Though, R_B gradually decreases from maximum $R_B = \frac{2GM}{c^2}$ to minimum $R_B = 0$ at the singularity, but R_S will remain constant for given mass M. Also, in the shaded region (figure: 1), $T_0 \epsilon$ [0, 1] $\Rightarrow T_0 = 0, \forall [R_S - R_B] \leq R_S$, since $T_0 \nless 0$, given that $R_B \neq 0$, i.e. space confined by the event horizon has $T_0 = 0$. This has an important implication on the gravitation and on the law of equivalence. However, my deduction fails when $R_B = 0$, since T gets undetermined.

4. The Equivalence principle and the drama at the horizon

Although, approximation of Newtonian gravitation is in consistency with the equation (2); however, Newtonian gravitation is theoretically incompetent in distinguishing the event horizon. But, as we have seen, the space can be sufficiently characterized by the value of its T_0, i.e. space confined by the horizon has $T_0 = 0$, while the space outside the horizon has $T_0 \in (0, 1]$. Therefore, at the horizon, $T_0 = 0$ and R $= R_S = \frac{2GM}{c^2}$; equation (2) yields,

$$F_S = \frac{m_0 c^2}{4GM} \times c^2 \Longrightarrow F_S \, \alpha \, \frac{m_0}{M} \tag{5}$$

Since, equation (5) appears as Newtonian gravitation counterpart at the horizon, thus it may seem to lead to a similar conclusion, i.e. “for a sufficiently massive black hole, an observer in a free fall through the horizon will experience nothing out of ordinary, as the effect is similar to the free fall in any other gravitational field”; this conclusion is fairly true if we only consider its Newtonian gravitation counterpart and ignore the role of $T_0 = 0$ hidden in the equation. But, since the approximation defined as the function of T_0 is very consistent throughout the space characterized by $T_0 \in (0, 1]$; thus, by induction, we may assume that the equation (2) holds even for $T_0 = 0$ at the horizon. Therefore, keeping $T_0 = 0$ intact in the approximation, it simultaneously leads to yet another solution at the horizon, which collectively defies popular conclusion of no drama.

From the special relativity, a particle travelling at velocity v has,

$$T_0 = T \times \sqrt{1 - \frac{v^2}{c^2}} \tag{6}$$

And, from equation (3), a particle under the influence of gravitation due to mass M has,

$$T_0 = T \times \sqrt{1 - \frac{2GM}{c^2R}} \tag{7}$$

Therefore, equating them we get,

$$v = \sqrt{\frac{2GM}{R}} \tag{8}$$

Also, from Special Relativity: $\frac{T_0}{T} = \frac{m_0}{m}$ (9)

Where, T_0 is the proper time measured relative to T arbitrarily taken as one second, and m_0 and m are the gravitational mass and inertial mass of the particle respectively. Thus, from the equation (6), (7), (8), & (9), we can unequivocally associate T_0 of a system with the velocity, gravitation and inertial mass simultaneously, irrespective to the cause of its T_0.

Thus, from it, we can arrive to one logical conclusion, i.e. the Equivalence property is due to its value of T_0, and therefore, an observer placed in a small, closed system at T_0 will not be able to distinguish whether its experience is the consequence of its acceleration or gravitation due to mass M.

Therefore, for a freely falling observer through the gravitational field defined by $T_0 \in (0, 1]$, the gravitation and the acceleration (gravitation $\rightleftharpoons$ acceleration) resulting the weightlessness of a system are simultaneously associated to the same value of T_0. But, during the free fall through the horizon defined by $T_0 = 0$, this simultaneity is broken thereby leading to different paradigm. Thus, for a free fall through the horizon:

From equations (9)

$$\frac{m_0}{m} = \frac{T_0}{T}$$

Since, freely falling observer will have $T_0 = 0$ at the horizon. Therefore,

$$\frac{m_0}{m} = 0 \Longrightarrow m_0 = 0. \qquad [\because m \neq \infty]$$

And, from equation (8)

$$v = \sqrt{\frac{2GM}{R}}$$

Therefore, at the horizon, v = c.

Hence, at the horizon, where a system can be positively defined by $T_0 = 0$, due to mass M of the Schwarzschild black hole, it will have rest mass equal to zero and velocity equals to that of light, i.e. a particle cannot retain its original form on crossing the horizon, and therefore, it must transform into its photon counterparts, which is in accordance to the Equivalence property of a system defined by $T_0 = 0$. Although, the particle at $T_0 = 0$ experiences gravitation at the horizon but it is not due upon its time component, but only upon the energy it possesses and the mass of the black hole, which is clearly illustrated by equation (5). Thus, unlike in other gravitational field, velocity and gravitation decouples at the horizon ending the notion of conservative freefall. So, it is safe to say that the infalling particle experiences a drama at the horizon.

5. Discussion

As we have seen, considering only the Newtonian gravitation counterpart from equation (5), the infalling particle will observe nothing out of ordinary while passing through the horizon except for the increased in gravitational tidal force, but re-examining the equation with its time component $T_0 = 0$ intact, it can be observed that the infalling particle cannot retain its originality and must transform into its photon counterparts during its fall through the horizon; thus, extrapolating the effects of the space defined by $T_0 \in (0, 1]$ to the space defined by $T_0 = 0$, and arriving at the conclusion of sphegatification of the infalling observer is untrue. This has an important implication on the AMPS firewall resolution to the information paradox, because: (1) It provides reasoning on the immediate breaking up of entanglement between the infalling particle and the outgoing particle, at the horizon (2) and, it shows that the AMPS firewall is perfectly consistent with the Equivalence principle, despite of giving up 'No Drama' at the horizon.

6. References

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