Special relativity in complex space-time. Part 1. A choice of the domain and transformation preserving the invariance of wave equation.

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Abstract

The article shows that based on the obvious postulates, using the existing mathematical tools, an alternative theory of special relativity can be build. The results were obtained studying the invariance of a four-dimensional wave equation by the paravector calculus. As a field domain the complex space-time was selected. Its meaningfulness and implicit naturalness is shown on simple examples. The aim of the publication is to show that the physical space-time may be viewed from a different perspective, not known so far.

Keywords: *Alternative Special Relativity, complex space-time, paravectors, relativistic transformations*

Introduction

Most of the physical laws are described by differential equations. In a mathematical sense the universality of the law of physics means an invariance of formula describing this law, independently of the reference frame. Therefore the basic question in theoretical physics is to examine the transformations that keep a mathematical formula of the analysed law.

An electromagnetic wave observed from any frame is always an electromagnetic wave, so in any inertial system it should have a similar formula. The electromagnetic wave is described by a homogeneous wave equation. The equations of electric and magnetic fields in a vacuum are included in the system of inhomogeneous wave equations[1]:

$$\begin{aligned} \frac{\partial^2 \varphi(t,\mathbf{x})}{c^2 \partial t^2} - \nabla^2 \varphi(t,\mathbf{x}) = \rho(t,\mathbf{x}) \\ \frac{\partial^2 \mathbf{A}(t,\mathbf{x})}{c \partial t^2} - c\nabla^2 \mathbf{A}(t,\mathbf{x}) = \frac{\mathbf{j}(t,\mathbf{x})}{c} \end{aligned} \tag{1}$$

Both of the wave equations, homogeneous and inhomogeneous, have the same transformations keeping their shape. Arrays of these transformations form a Lorentz group. The article [3] shows that when $c = 1$, the above equations are invariant under transformation

$$t' = \frac{\alpha t + \boldsymbol{\beta}\mathbf{x}}{\sqrt{\alpha^2 - \beta^2}} \quad , \quad \mathbf{x}' = \frac{\alpha \mathbf{x} + \boldsymbol{\beta} t \pm i\boldsymbol{\beta} \times \mathbf{x}}{\sqrt{\alpha^2 - \beta^2}} \tag{2}$$

where $\alpha \in C, \quad \boldsymbol{\beta} \in C^3 \quad$ and $\quad (\alpha^2 - \beta^2) \in R_+ \setminus \{0\}$,

which in a paravector notation is as follows

$$\mathbb{X}' = \Lambda\mathbb{X} \qquad \text{or} \qquad \mathbb{X}' = \mathbb{X}\Lambda \qquad \text{and paravector } \Lambda \text{ is orthogonal.} \tag{3}$$

Transformation (2), although correct in terms of calculation, is set in an unknown structure which we will define in the future but for now, in advance, we will call it **the complex space-time**. Since the space-time is not real, transformation (2) does not fit in the Lorentz group.

Before this article the author published three devoted to paravectors, so in order to get acquainted with algebra of paravectors and to gain confidence for the used formalism it is necessary to read articles [2] and [3], and it is recommended to review [4].

In the following part in order to simplify the formulas instead of the H-L units we will use the natural system of units *(NU)*, wherein the linear velocities are relative to the speed of

[1] Equations are described in units of Heaviside-Lorentz (H-L) system

light[2]. In other words, the linear velocity is a dimensionless quantity with a value belonging to the interval $\langle 0,1\rangle$, and the value of the speed of light is $c=1$.

In the article *Algebra of paravectors*, in order to increase the transparency, there were introduced different designations and various brackets to the paravectors representing the additive and non-additive sizes. For the record:

1. The coordinates of points in the space-time are denoted by capital letters X or Y:
$$X=\begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} \tag{4}$$
2. The additive paravectors such as space-time intervals or additive field values have parentheses and traditionally we call them **four-vectors**
$$\mathbb{X}=\begin{pmatrix} \Delta t \\ \Delta\mathbf{x} \end{pmatrix} \tag{5}$$
3. The non-additive paravectors such as transformation parameters have a square brackets:
$$\Gamma:=\begin{bmatrix} \alpha \\ \boldsymbol{\beta} \end{bmatrix}=\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c} \end{bmatrix} \tag{6}$$
4. The differential operators:
$$\partial=\begin{bmatrix} \frac{\partial}{\partial t} \\ \nabla \end{bmatrix} \qquad \partial^{-}=\begin{bmatrix} \frac{\partial}{\partial t} \\ -\nabla \end{bmatrix} \tag{7}$$

In general, the values presented at pos. 2 and 3 are called paravectors.

Although the complex space-time is very similar to the Euclidean space, the main difference which is the lack of metrics in the traditional sense means that we can not transfer our habits directly on the model built by us. Therefore, we start from the description of phenomena in the real space by real paravectors to gradually receive interpretations for imaginary components. In this way we want to show that it is possible to take a look at the familiar phenomena from a different perspective.

[2]See assumptions for article [4]

1 Orthogonal paravector transformations.

Paravector orthogonal transformations can act on four-vectors in different ways, and there may be different domains to which the transformed four-vectors belong:

1. Automorphism of the complex space-time $\mathbb{X}' = \Lambda\mathbb{X}$ or $\mathbb{X}' = \mathbb{X}\Lambda$
2. Automorphism of the real/complex space $\mathbb{X}' = \Lambda\mathbb{X}\Lambda^-$
3. Automorphism of the real space-time $\mathbb{X}' = \Lambda\mathbb{X}\Lambda^*$
4. Automorphism of the quasi-real space-time, when $\mathbb{X}$ and Λ are special paravectors
5. The general case which includes all of the above cases $\mathbb{X}' = \Lambda_1\mathbb{X}\Lambda_2$

It should be noted that in all the cases, we keep the paradigm of invariance of the wave equation, and hence the speed of light, because the value of the wave speed is contained in the wave equation. The 3rd case is a classic Lorentz transformation. Using the concept of Geometric Algebra, dealt with by David Hestenes (Geometric Algebra) and William Baylis (Algebra of Physical Space). We are devoted to this subject a critical article [4]. The 5th case is the most general and includes all the previous ones, so we will disregard. The 2nd case is a purely spatial transformation in which the time component is invariant. It is therefore not of interest for the study of the phenomena concerning spatial changes occurring in time. Also, case No 4 does not seem to be interesting in terms of applications in physics, which we'll justify in the Appendix.

The most interesting, as yet unexplored was the 1st case, so we will deal with it in this and all subsequent publications. We will consider only the transformation $\mathbb{X}' = \Lambda\mathbb{X}$, because both relations are symmetrical by which the reasoning based on them is equivalent. We will explain this in more detail when we come to the problems of the electric field.

In our considerations a paravector Γ and 4-vector $\mathbb{X}$ paired in product $\Gamma^-\mathbb{X}$ play a fundamental role, so we define

Definition 1.1. We call difference of phases a **phase interval**.

$$\Delta\Theta = \begin{bmatrix} \alpha \\ -\boldsymbol{\beta} \end{bmatrix} \begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \begin{pmatrix} r \\ 0 \end{pmatrix}, \tag{8}$$

where Γ and $\mathbb{X}$ are proper paravectors, and $r \in R_+$.

The components of the phase interval will be called:

- $\Delta t'$ - phase delay
- $\Delta\mathbf{x}'$ - phase distance

Note In the issues considered by us the phase interval is a non-negative real number, and it is equivalent to interval of the universal time.

If Γ is a real orthogonal paravector, then it can be denoted in form of:

$$V = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \tag{9}$$

Vector $\mathbf{v}$ is interpreted as phase velocity. A sense of this vector determines the phase direction.

2 Velocity in the real space-time and a boost.

When analysing the transformation of preserving the invariance of the wave equation, the main problem is to determine the domain. Ideally, if the space-time be real, but the most general and contrary to appearances, mathematically the easiest is a research in the complex domain and in this direction the main course of our deliberations went.

At the beginning, to introduce transformation (2) in a more imaginable way we simplify the complex orthogonal paravector Λ to a real one in form as above (9).

Transformation (2) represented by the above paravector is interpreted as a simple velocity in the real space-time, and paravector V will be called the **velocity paravector**.:

$$\begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \Delta t + \Delta \mathbf{v} \Delta \mathbf{x} \\ \Delta \mathbf{x} + \mathbf{v} \Delta t + i \Delta \mathbf{v} \times \Delta \mathbf{x} \end{pmatrix} \tag{10}$$

We start the analysis with the simplest transformation of the time interval where the real time is converted to a real four-vector by a complex transformation. Formula (10) takes the form

$$\begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \Delta t \\ \mathbf{v} \Delta t \end{pmatrix} \tag{11}$$

In the rest frame the immovable object is aging (Δt). This same object observed from the moving frame, moves according to the formula:

$$\begin{pmatrix} \Delta t \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} t' - \mathbf{v} \Delta \mathbf{x}' \\ \Delta \mathbf{x}' - \mathbf{v} \Delta t' - i \mathbf{v} \times \Delta \mathbf{x}' \end{pmatrix} \tag{12}$$

From the vector part of the above equation we get

$$\Delta \mathbf{x}' - \mathbf{v} \Delta t' = 0 \qquad \text{and} \qquad \mathbf{v} \times \Delta \mathbf{x}' = 0, \tag{13}$$

hence $\mathbf{x}_1' = \mathbf{x}_0' + \mathbf{v} \Delta t'$ and $\left(\mathbf{x}_1' - \mathbf{x}_0'\right) \times \mathbf{v} = 0,$ which is evident for the inertial movement.

At this point we draw your attention to the fact that the obtained primed coordinates are still real, and the equation of motion in the primed frame has a form of Galilean transformation despite the fact that we used a complex transformation. We can also see that for the description of movement it does not matter whether the speed is close to light or not. The dilation factor is important at the transition from frame to frame.

At the non-relativistic approximation (which will be easier to see when we change the units to H-L system i.e. we write the speed of light outright as c) we have

$$\begin{pmatrix} c\Delta t \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1-(v/c)^2}} \begin{bmatrix} 1 \\ -\mathbf{v}/c \end{bmatrix} \begin{pmatrix} c\Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-(v/c)^2}} \begin{pmatrix} c\Delta t' - \Delta \mathbf{x}' \mathbf{v}/c \\ \Delta \mathbf{x}' - \mathbf{v} \Delta t' - i \mathbf{v} \times \Delta \mathbf{x}'/c \end{pmatrix} \tag{14}$$

For $v \ll c$ we obtain $c\Delta t = c\Delta t'$ and $\Delta\mathbf{x}' - \mathbf{v}\Delta t' = 0$, so fastening the beginning of experiment in a particular space-time point with coordinates $\left(t_0', \mathbf{x}_0'\right)$ we obtain a Galilean transformation

$$\begin{aligned} \Delta t &= \Delta t' \\ \mathbf{x}_1' &= \mathbf{x}_0' + \mathbf{v}\Delta t' \end{aligned}$$

It is evident that at low speeds the complex transformation is changed into Galilean one.

In the rest frame the observed point stands for a time Δt, which in our notation means $\mathbb{X} = \begin{pmatrix} \Delta t \\ \mathbf{0} \end{pmatrix}$. In a moving frame at a relativistic speed this point moves, and its coordinates are described by a four-vector

$$\mathbb{X}' = \begin{pmatrix} \Delta t' \\ \Delta\mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t \\ \mathbf{0} \end{pmatrix} = V\mathbb{X}, \tag{15}$$

hence $\begin{bmatrix} \frac{\Delta t'}{\Delta t} \\ \frac{\Delta\mathbf{x}'}{\Delta t} \end{bmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} = V$, or $\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1-v^2}}$ and $\frac{\Delta\mathbf{x}'}{\Delta t} = \frac{\mathbf{v}}{\sqrt{1-v^2}}$

After the elimination of Δt by substitution of the left formula into the right one we get the obvious expression for a speed of the observed point in the primed frame $\mathbf{v} = \frac{\Delta\mathbf{x}'}{\Delta t'}$.

Before moving to the topic of space-time interval transformation, we will check what the composition of simple velocities looks like, which is essentially the same issue.

The paravector which we interpret as simple **velocity** has the form of

$$V = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix}, \qquad \text{where} \qquad \mathbf{v} \in R^3 \qquad \text{and} \qquad 0 \leqslant v^2 < 1. \tag{16}$$

As a result of multiplication of this paravectors we get a complex one

$$\frac{1}{\sqrt{1-v_1^2}} \begin{bmatrix} 1 \\ \mathbf{v}_1 \end{bmatrix} \frac{1}{\sqrt{1-v_2^2}} \begin{bmatrix} 1 \\ \mathbf{v}_2 \end{bmatrix} = \frac{1}{\sqrt{1-\left(\frac{\mathbf{v}_1+\mathbf{v}_2}{1+\mathbf{v}_1\mathbf{v}_2}\right)^2 + \left(\frac{\mathbf{v}_1\times\mathbf{v}_2}{1+\mathbf{v}_1\mathbf{v}_2}\right)^2}} \begin{bmatrix} 1 \\ \frac{\mathbf{v}_1+\mathbf{v}_2+i\mathbf{v}_1\times\mathbf{v}_2}{1+\mathbf{v}_1\mathbf{v}_2} \end{bmatrix} \tag{17}$$

A **boost** will be called the compound speed represented by the complex orthogonal paravector Λ.

The above formula shows that in the one-dimension case, the resultant speed is $\mathbf{v} = \frac{\mathbf{v}_1 + \mathbf{v}_2}{1 + \mathbf{v}_1 \mathbf{v}_2}$, which is consistent with current SR. The same is with the coordinates of the object in space-time, which can be seen in the equations (2). If the transformed spatial coordinates and vector of speed are not parallel, then as a result an imaginary component is created. One hundred years ago, physicists assumed that the space-time is real, and therefore they separated the description of phenomenon into the parallel component multiplied by the Lorentz factor and invariant perpendicular one. While this treatment seems a bit artificial, it is good enough for today. Presenting the results of the study of the complex space we want to show that it is conceivable that this model better and more intuitively than a classic one reflects the relativistic phenomena.

3 Interpretations of complex vectors.

We are aware that to imagine a complex space-time can be difficult (if it is possible at all?), so it has to be approached like a mechanic does who reads a flat technical drawing and imagines a spatial structure which he has to built. Sticking to this comparison, we will try to show on a variety of simple cases like on flat figures that what we see in the available for our perception space is only a fragment of a larger whole. We note that these are only efforts of comprehension of a complex space only but not a complex time. However, if everything indicates that the space is complex, then the time must be complex too.

3.1 Vector in zero time.

The following discussion is entirely theoretical (geometrical), because the vectors about which we are going to talk exist at zero time. In the physical reality the time must be infinitesimal at least.

In the OX (rest) frame there are two different points at the same time ($\Delta t = 0$). These points define the vector $\Delta\mathbf{x}$, which is written as a 4-vector $\mathbb{X} = \begin{pmatrix} 0 \\ \Delta\mathbf{x} \end{pmatrix}$.

To make the presentation more figurative, although we talk about a single vector, we show a bundle of vectors having common origin (green point) and equal length (red ends). The ends of these vectors form a circle. After transition to the frame running with speed $-\mathbf{v}$ the above 4-vector transfers:

$$\mathbb{X}' = \begin{pmatrix} \Delta t' \\ \Delta\mathbf{x}' + i\Delta\mathbf{y}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} 0 \\ \Delta\mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \mathbf{v}\Delta\mathbf{x} \\ \Delta\mathbf{x} + i\mathbf{v}\times\Delta\mathbf{x} \end{pmatrix} \tag{18}$$

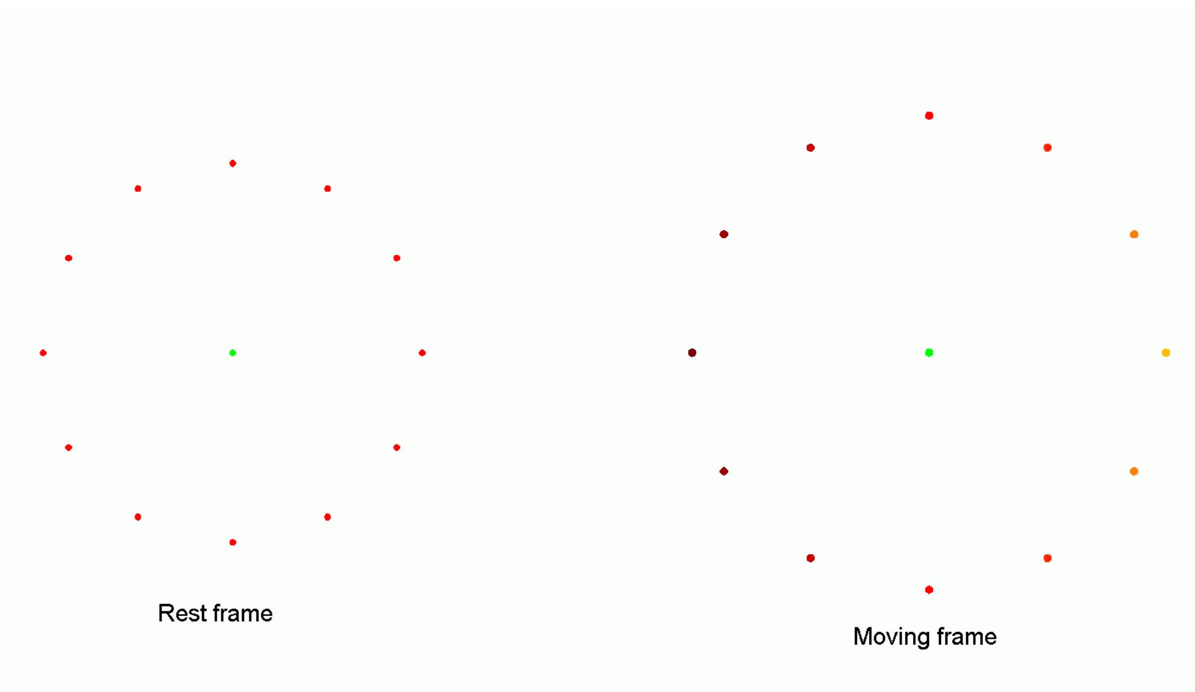

Figure 1: The image of the flashing bundle of vectors.

In this example, we do not deal with an imaginary vector, and our point is only in the interpretation of the temporal component. We assume that the imaginary vector is any auxiliary size used to balance the calculations - the dependent variable.

The last equation shows that the time $\Delta t'$ between the beginning and end of each vector $\Delta\mathbf{x}'$ is proportional to $\mathbf{v}\Delta\mathbf{x}$. The time interval is positive when the moment of vector

$\Delta\mathbf{x}'$ end is after the moment of its beginning. If the moment of the end is before of the moment of its beginning, the time interval is negative. If the vectors $\mathbf{v}$ and $\Delta\mathbf{x}$ have an opposite senses, then scalar product of their is negative. This means that the end of vector $\Delta\mathbf{x}'$ is before of its beginning. In Figure 1 times are denoted by different shades of red. The ends of the vectors that are at the front in the direction of the observer movement are before of the moment of their common beginning. If we assume that the point of reference is the origin at moment 0, then from the equation (18) we know which points are earlier and which are later. The delays of which we speak are impossible to observe because after the change to the H-L units system (19), it turns out that the image of the ends of vectors moves faster than the speed of light, which will be shown in the next example.

Below (Fig. 2) an image of the flashing disk is shown seen from the frame moving at relativistic speed. For an analysis of the image we use formula (18). Just as in the previous example the time of a drive flash is infinitesimal and $\Delta\mathbf{x}$ is a vector whose beginning is in the center of the disk, and the end lying on this disk. The real vector tells us about the shape. Therefore, we can say that the image of the circle (sphere) is a greater circle (sphere), whose points are shifted in time. The ratio is due to a dilation factor. The points on the section are at the same time. We see that the simultaneity of the frame OX does not match the simultaneity in the frame OX'. Use of the word „see" is excessive because this word is associated with receiving the light on the eye, and light moves at a finite speed. Using the word „see", we mean the mathematical formula.

From the real part of the vector part of equation (18) we conclude that, if the observed object does not change its shape over time, then the relativistic transformation scales objects by a factor of dilation without deforming it.

Here we see on the formula (18) in H-L units system.

$$\mathbb{X}' = \begin{pmatrix} c\Delta t' \\ \Delta\mathbf{x}' + i\Delta\mathbf{y}' \end{pmatrix} = \frac{1}{\sqrt{1-(v/c)^2}} \begin{pmatrix} \mathbf{v}\Delta\mathbf{x}/c \\ \Delta\mathbf{x} + i\mathbf{v}\times\Delta\mathbf{x}/c \end{pmatrix} \tag{19}$$

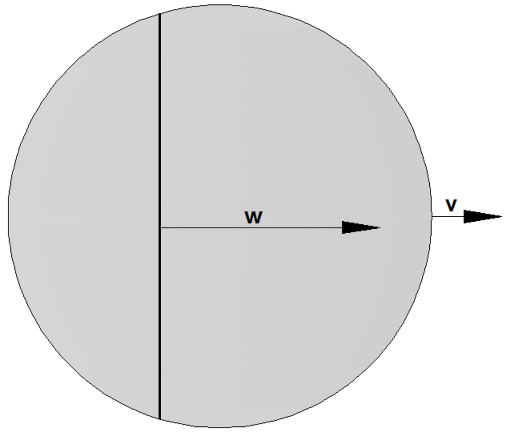

Figure 2: Mathematical image of the flashing disc.

From the above we can see that for non-relativistic speeds, the time and imaginary vector components disappear. As a representative of the section we select a point lying on the diameter of the disk parallel to the velocity vector ($\Delta\mathbf{x} \parallel \mathbf{v}$). Speed of this section is

$$\frac{\mathbf{w}}{c} = \frac{\Delta\mathbf{x}'}{c\Delta t'} = \frac{c\Delta\mathbf{x}}{\mathbf{v}\Delta\mathbf{x}}$$

which in a H-L system gives $\frac{\Delta x'}{\Delta t'} = \frac{c^2}{v} > c$, and in NU system $w = \frac{\Delta x'}{\Delta t'} = \frac{1}{v} > 1$. It is not only that the velocity of the moving section is greater than the speed of light, still it is inversely proportional to the relative speed of the disc and observer. This phenomenon does not belong to the physical concepts, but geometric ones. Considering the physical issues, we can assume that the vector corresponds to a purely spatial vector scaled by a factor of dilation. We also assume that the blurred flashing disk corresponds to flashing disk. The higher speed of the disk makes the stronger the blur.

We will take care of the imaginary vector later.

3.2 Growing vector.

In the next example, we'll complicate a consideration. We have two coincident points A and B at the initial time t_0, then point A moves at a constant relativistic speed $\mathbf{w}$ of the point

B, which remains rest.

We denote the coordinates of paravectors (Fig. 3) of the position of these points:

Point A $\quad \mathbb{X}_A = \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix}$, of which coordinates are related as follow

$$\begin{pmatrix} \Delta t^0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1-w^2}} \begin{bmatrix} 1 \\ -\mathbf{w} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix}$$

and point B $\quad \mathbb{X}_B = \begin{pmatrix} \Delta t \\ 0 \end{pmatrix}$

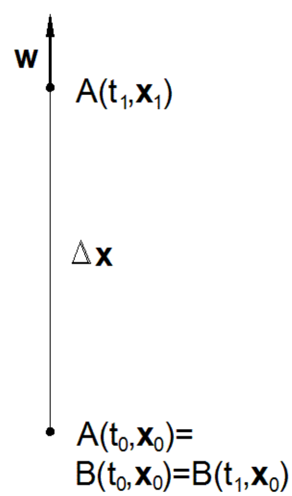

Figure 3: The vector $\Delta\mathbf{x}$ increasing with the speed of $\mathbf{w}$ in a rest frame.

Let us describe the phase interval between these points in a rest frame (OX) and the corresponding interval in the moving frame (OX'). We treat intervals in frame OX as data and we want to see what they will look like in a primed frame. In the frame OX the point A moves according to the formula

$$\begin{bmatrix} 1 \\ -\mathbf{w} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} = \begin{pmatrix} p \\ 0 \end{pmatrix}, \tag{20}$$

where p is a real positive number - the parameter proportional to time. The dilation factor is skipped, because it has a meaning only when we refer to the coordinates of second frame (in this case a local time of the object named „point A").

Point B just getting older $\qquad \mathbb{X}_B = \begin{pmatrix} \Delta t \\ 0 \end{pmatrix}$

We calculate the image of the interval $\mathbb{X} = \mathbb{X}_A - \mathbb{X}_B$. In the rest frame this is a spatial vector. Although it shifts in time and at the same time it grows, the beginning and end are at the same time (they are simultaneous,) and therefore this is a purely spatial vector, so its determinant is negative.

$$\mathbb{X}_A - \mathbb{X}_B = \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} - \begin{pmatrix} \Delta t \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \Delta \mathbf{x} \end{pmatrix} \tag{21}$$

Changing the frame to the moving one at speed of $-\mathbf{v}$ we obtain image (Fig. 4)

In frame OX' the following relationship describes the movement of point A:

$$\mathbb{X}_A = \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t'_A \\ \Delta \mathbf{x}'_A \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \Delta t'_A - \mathbf{v}\Delta \mathbf{x}'_A \\ \Delta \mathbf{x}'_A - \mathbf{v}\Delta t'_A - i\mathbf{v}\times\Delta \mathbf{x}_A \end{pmatrix} \tag{22}$$

In OX frame point B is at rest, which in OX' frame is visible as a movement in accordance with the formula:

$$\mathbb{X}_B = \begin{pmatrix} \Delta t \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t'_B \\ \Delta \mathbf{x}'_B \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \Delta t'_B - \mathbf{v}\Delta \mathbf{x}'_B \\ \Delta \mathbf{x}'_B - \mathbf{v}\Delta t'_B \end{pmatrix} \tag{23}$$

Starting from the above formulas we obtain an image of a vector in OX' frame:

$$\mathbb{X}_A - \mathbb{X}_B = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t'_A \\ \Delta \mathbf{x}'_A \end{pmatrix} - \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t'_B \\ \Delta \mathbf{x}'_B \end{pmatrix} = V^{-}\left(\mathbb{X}'_A - \mathbb{X}'_B\right), \tag{24}$$

or $\qquad \mathbb{X}' = \mathbb{X}'_A - \mathbb{X}'_B = V\left(\mathbb{X}_A - \mathbb{X}_B\right)$

$$\mathbb{X}' = \begin{pmatrix} \Delta t'_A - \Delta t'_B \\ \Delta \mathbf{x}'_A - \Delta \mathbf{x}'_B \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} 0 \\ \Delta \mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \mathbf{v}\Delta \mathbf{x} \\ \Delta \mathbf{x} + i\mathbf{v}\times\Delta \mathbf{x} \end{pmatrix} \tag{25}$$

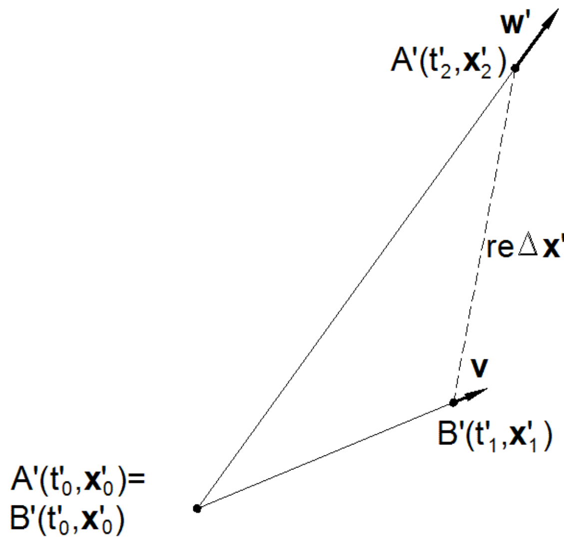

Figure 4: The real vector component $\Delta\mathbf{x}$ of Figure 3 in a frame moving at a speed of -**v**

It is seen that the image of a spatial vector, as before, is the space-time interval (the beginning and end of the vector may be at different time). This interval has a purely geometric meaning. Just as in the previous example, we emphasize that it has no physical meaning, because its determinant is negative.

If $\mathbf{v} \perp \Delta\mathbf{x}$ then:

$$\mathbb{X}' = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \mathbf{0} \\ \Delta\mathbf{x} + i\mathbf{v}\times\Delta\mathbf{x} \end{pmatrix}, \tag{26}$$

but if $\mathbf{v} \parallel \Delta\mathbf{x}$ then

$$\mathbb{X}' = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \mathbf{v}\Delta\mathbf{x} \\ \Delta\mathbf{x} \end{pmatrix} \tag{27}$$

The real part of the vector in OX' frame (Fig. 5) is interpreted traditionally as the difference between the coordinates of two points and it is proportional (with a dilation factor) to the original vector. The scalar is a time. We also see that both points are located at

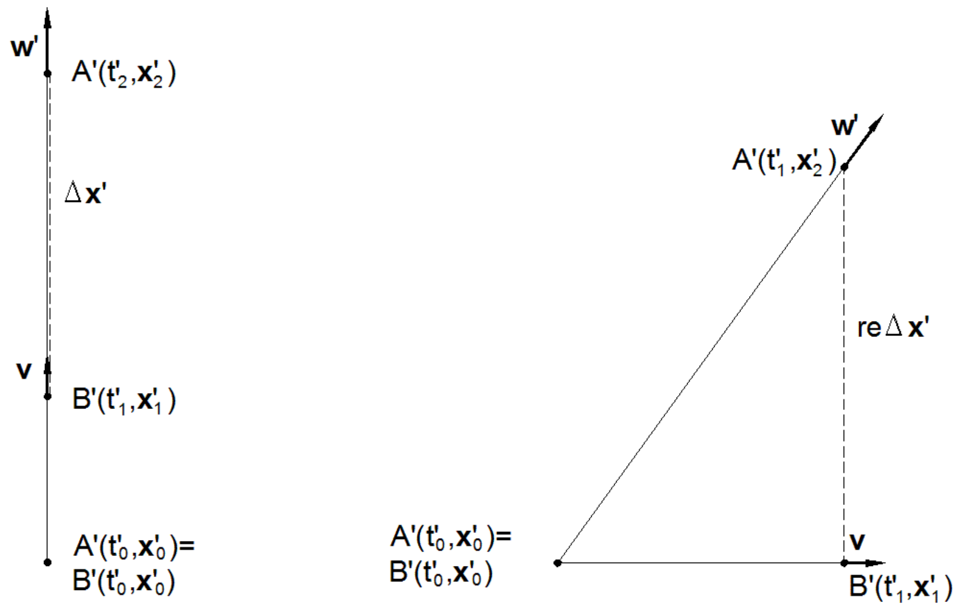

Figure 5:

the ends of vectors $\Delta \mathbf{x}'$ not always at the same time. At the same time as point B' are points lying on the plane containing the B' and the perpendicular to the movement direction of the point B'. These ends which are before the common beginning are delayed in time, and those behind are earlier. We get also an imaginary vector which is any property of movement of the real vector.

It was an image of the vector whose beginning and end were at the same time in the rest frame. But being in a moving frame OX' the observer sees the points A and B at the same time as if he took photographs, or $\Delta t' = 0$. Therefore the formulas (22) and (23) are written:

$$\mathbb{X}_A = \begin{pmatrix} \Delta t_A \\ \Delta \mathbf{x}_A \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t'_A \\ \Delta \mathbf{x}'_A \end{pmatrix} \tag{28}$$

$$\mathbb{X}_B = \begin{pmatrix} \Delta t_B \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t'_B \\ \Delta \mathbf{x}'_B \end{pmatrix} \tag{29}$$

Since now $\Delta t'_A = \Delta t'_B$ then the equation (25) takes a form

$$\mathbb{X}' = \begin{pmatrix} 0 \\ \Delta\mathbf{x}'_A - \Delta\mathbf{x}'_B \end{pmatrix} = \begin{pmatrix} 0 \\ \Delta\mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta\mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \Delta t + \mathbf{v}\Delta\mathbf{x} \\ \Delta\mathbf{x} + \mathbf{v}\Delta t + i\mathbf{v}\times\Delta\mathbf{x} \end{pmatrix} \tag{30}$$

From the scalar part of above equation it follows that $\Delta t = -\mathbf{v}\Delta\mathbf{x}$. After inserting into the vector equation we get

$$Re\Delta\mathbf{x}' = \frac{1}{\sqrt{1-v^2}}[\Delta\mathbf{x} - \mathbf{v}(\mathbf{v}\Delta\mathbf{x})] \tag{31}$$

After the dot and cross multiplication of the above formula by vector $\mathbf{v}$, we get two equations, which must be met by vectors $Re\Delta\mathbf{x}'$ and $\Delta\mathbf{x}$:

$$Re\Delta\mathbf{x}'\mathbf{v} = (\Delta\mathbf{x}\mathbf{v})\sqrt{1-v^2} \tag{32}$$

$$Re\Delta\mathbf{x}' \times \mathbf{v} = \frac{\Delta\mathbf{x}\times\mathbf{v}}{\sqrt{1-v^2}} \tag{33}$$

The above equations mean that the image of the moving object can be deformed. However, the relationship of parallelism and perpendicularity to the movement direction are kept (If $\Delta\mathbf{x} \perp \mathbf{v}$ then $Re\Delta\mathbf{x}' \perp \mathbf{v}$ and if $\Delta\mathbf{x} \parallel \mathbf{v}$ then $Re\Delta\mathbf{x}' \parallel \mathbf{v}$). Moreover, we must remember that we always have an imaginary way and shifts in time and it is possible that we see the points spherically radiating as a ball? We do not know that at this point yet, but we will explain it later.

3.3 Expanding spheres.

In a rest frame at the moment t_0 a soap bubble popped out of point O($\mathbf{x}_0$) (in Figure 6 in cross-section it's a circle) and it steadily grows at the relativistic speed $\mathbf{w}$ (w=0,4). The observer notes the coordinates at the time t.

The coordinates of the 4-vectors [point $A(t,\mathbf{x})$ - the center of circle $O(t_0,\mathbf{x}_0)$] in the rest frame (Fig. 6):

$$\mathbb{X} = \begin{pmatrix} \Delta t \\ \Delta\mathbf{x} \end{pmatrix} = \begin{pmatrix} t - t_0 \\ \mathbf{x} - \mathbf{x}_0 \end{pmatrix} \quad \text{, where } \begin{bmatrix} 1 \\ -\mathbf{w} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta\mathbf{x} \end{pmatrix} = \begin{pmatrix} p \\ 0 \end{pmatrix}. \tag{34}$$

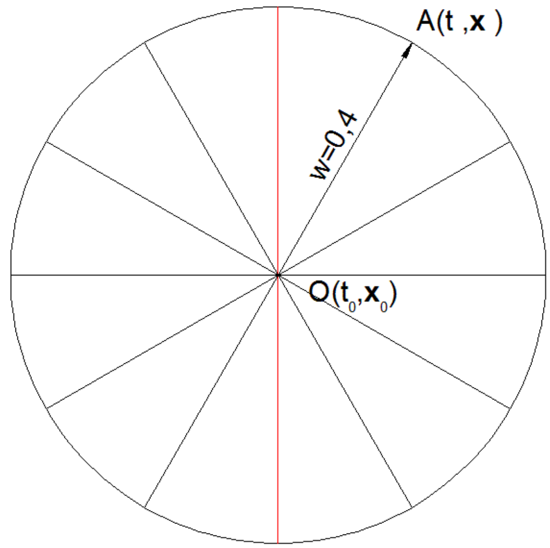

Figure 6: The expanding real sphere in the rest frame.

The right equation described in the frame OX' moving at the relativistic speed $\mathbf{v}$ (v=0,8) is:

$$\begin{bmatrix}1\\-\mathbf{w}\end{bmatrix}\begin{bmatrix}1\\-\mathbf{v}\end{bmatrix}\begin{pmatrix}\Delta t'\\\Delta\mathbf{x}'\end{pmatrix}=\begin{bmatrix}1+\mathbf{vw}\\-\mathbf{v}-\mathbf{w}+i\mathbf{w}\times\mathbf{v}\end{bmatrix}\begin{pmatrix}\Delta t'\\\Delta\mathbf{x}'\end{pmatrix}=\begin{pmatrix}p'\\0\end{pmatrix}\qquad\text{, where } p'=p\sqrt{1-v^2}\tag{35}$$

In the OX' frame we describe a circle at the moment t', so we are not interested in the scalar part of above equation, as a reference to the time t. We are interested in the real part of the vector equation. After the transfer of the complex paravector of speed to the right side we have:

$$\begin{pmatrix}\Delta t'\\\Delta\mathbf{x}'\end{pmatrix}=\frac{p'}{(1+\mathbf{vw})^2-(\mathbf{v}+\mathbf{w})^2+(\mathbf{v}\times\mathbf{w})^2}\begin{bmatrix}1+\mathbf{vw}\\\mathbf{v}+\mathbf{w}+i\mathbf{v}\times\mathbf{w}\end{bmatrix}\tag{36}$$

From the scalar part of the above formula we calculate the dependence of p' and $\Delta t'$ and we insert it into the vector formula from which we get

$$\Delta\mathbf{x}'=\frac{\mathbf{v}+\mathbf{w}+i\mathbf{v}\times\mathbf{w}}{1+\mathbf{vw}}\Delta t'\tag{37}$$

For a bunch of vectors $\mathbf{w}$ and a constant vector $\mathbf{v}$, the ends of the real part of the vectors $\Delta\mathbf{x}'$, which are proportional to $\mathbf{w}'=\frac{\mathbf{v}+\mathbf{w}}{1+\mathbf{vw}}$, create an ellipse (Fig.7).

However, it should be noted that the deformation of a real component does not mean that the complex ball is deformed because, as we remember from paravector algebra [2], the

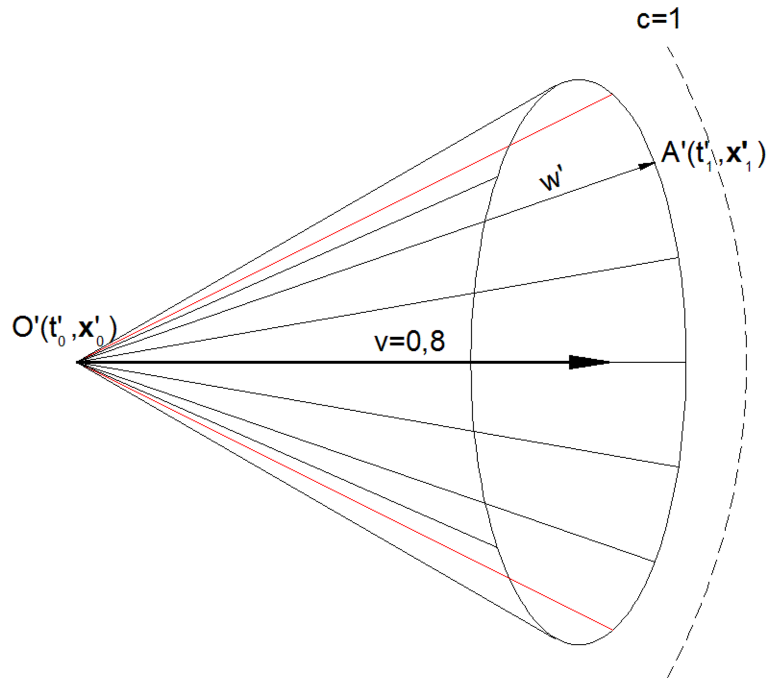

Figure 7: The real resultant velocity components in the plane X'O'Y'

scalar product is invariant. Red represents the real vectors which have not been relativistically deformed. Moreover, Figure 7 is not a photo because in photography the information carrier is the light that travels at a predetermined speed. Our figures would be photos if the information travelled infinitely fast. Therefore, it is possible that if we would took a picture we would see ... a ball. And so is the case, except that the points which we would take in the picture in the moving frame, would be not simultaneous in the rest frame.

The image of the imaginary components of the submitted velocity vectors in according to the angle between the vectors **w** and **v** is shown in Figure 8. One should note that all of the imaginary vectors are in the same direction perpendicular to the plane of the cross-section defined by vectors **v** and **w**.

Let's go even further (Fig.9). In the rest frame after a first bubble the next one popped out of the point $O(t, \mathbf{x}_0)$ and the both are rising uniformly at the same relativistic speed **w**. The first bubble (a) born at the time t_0, and the second one (b) a little later - at t_1. The observer

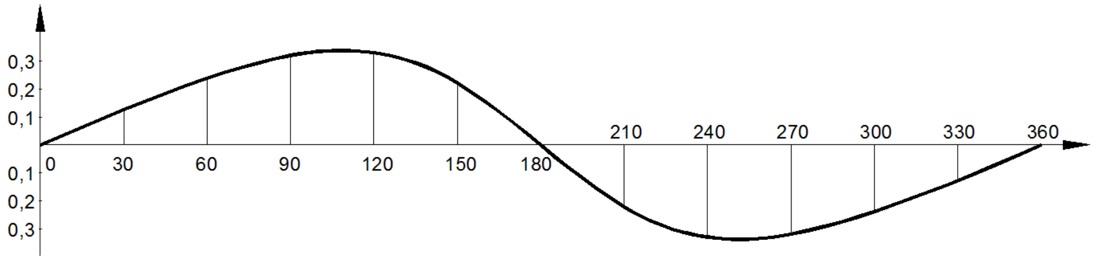

Figure 8: Imaginary components of the resultant speed on the axis O'Z'

notes the equation at the time t_2.

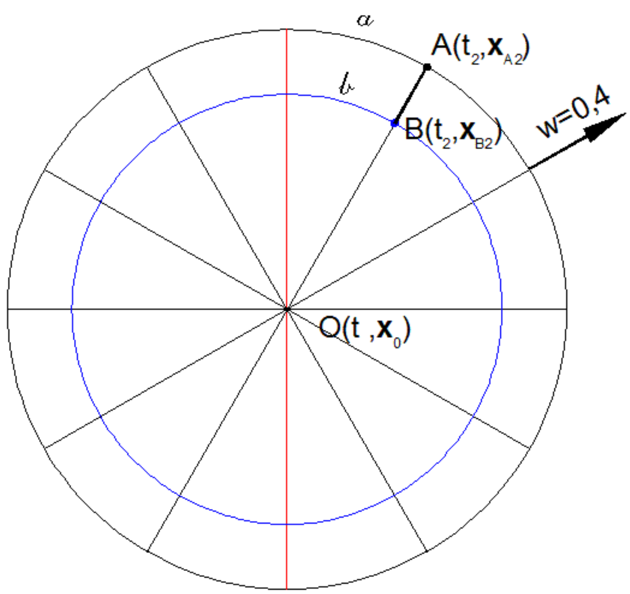

Figure 9: The concentric spheres swelling in the rest frame.

In the rest frame the first sphere (a) is described by the equation

$$\mathbb{X}_a = \begin{pmatrix} \Delta t_a \\ \Delta \mathbf{x}_a \end{pmatrix}, \qquad \text{where} \qquad \begin{bmatrix} 1 \\ -\mathbf{w} \end{bmatrix} \begin{pmatrix} \Delta t_a \\ \Delta \mathbf{x}_a \end{pmatrix} = \begin{pmatrix} p \\ 0 \end{pmatrix}, \qquad \text{and} \qquad \Delta t_a = t_2 - t_0 \tag{38}$$

The second sphere (b) is described by the analogous equation:

$$\mathbb{X}_b = \begin{pmatrix} \Delta t_b \\ \Delta \mathbf{x}_b \end{pmatrix}, \qquad \text{where} \begin{bmatrix} 1 \\ -\mathbf{w} \end{bmatrix} \begin{pmatrix} \Delta t_b \\ \Delta \mathbf{x}_b \end{pmatrix} = \begin{pmatrix} p \\ 0 \end{pmatrix}, \qquad \text{and} \qquad \Delta t_b = t_2 - t_1 \text{ i } t_0 < t_1 < t_2 \tag{39}$$

The section between points A and B which are moving in the same direction and at the same time is described by the equation

$$\begin{pmatrix} 0 \\ \mathbf{x}_{A2} - \mathbf{x}_{B2} \end{pmatrix} = \begin{pmatrix} t_2 - t_0 \\ \mathbf{x}_{A2} - \mathbf{x}_{O0} \end{pmatrix} - \begin{pmatrix} t_2 - t_1 \\ \mathbf{x}_{B2} - \mathbf{x}_{O1} \end{pmatrix} - \begin{pmatrix} t_1 - t_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \Delta t_a \\ \Delta \mathbf{x}_a \end{pmatrix} - \begin{pmatrix} \Delta t_b \\ \Delta \mathbf{x}_b \end{pmatrix} - \begin{pmatrix} t_1 - t_0 \\ 0 \end{pmatrix} \tag{40}$$

where:

$\begin{pmatrix} t_2 - t_0 \\ \mathbf{x}_2 - \mathbf{x}_0 \end{pmatrix}$ - are the coordinates of the moving point A,

$\begin{pmatrix} t_2 - t_1 \\ \mathbf{x}_1 - \mathbf{x}_0 \end{pmatrix}$ - are the coordinates of the moving point B,

$\begin{pmatrix} t_1 - t_0 \\ 0 \end{pmatrix}$ - is the time when the bubble b waits to pop out,

$\mathbf{x}_{O0} = \mathbf{x}_{O1} = \mathbf{x}_{O2} = \mathbf{x}_0$, because point O stays in place.

After transfer to the frame OX' moving at speed $-\mathbf{v}$, we transform the equation:

$$\begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{x}_{A2} - \mathbf{x}_{B2} \end{pmatrix} = \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} t_2 - t_0 \\ \mathbf{x}_{A2} - \mathbf{x}_{O0} \end{pmatrix} - \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} t_2 - t_1 \\ \mathbf{x}_{B2} - \mathbf{x}_{O1} \end{pmatrix} - \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} t_1 - t_0 \\ 0 \end{pmatrix}, \tag{41}$$

which gives

$$\begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \begin{pmatrix} \Delta t'_a \\ \Delta \mathbf{x}'_a \end{pmatrix} - \begin{pmatrix} \Delta t'_b \\ \Delta \mathbf{x}'_b \end{pmatrix} - \begin{pmatrix} \Delta t'_c \\ \Delta \mathbf{x}'_c \end{pmatrix} \tag{42}$$

where

$\begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{x}_{A2} - \mathbf{x}_{B2} \end{pmatrix}$ - the spatial section transforms into a spatio-temporal interval

$\begin{pmatrix} \Delta t'_a \\ \Delta \mathbf{x}'_a \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} t_2 - t_0 \\ \mathbf{x}_{A2} - \mathbf{x}_{O0} \end{pmatrix}$

$$\begin{pmatrix} \Delta t'_b \\ \Delta \mathbf{x}'_b \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} t_2 - t_1 \\ \mathbf{x}_{B2} - \mathbf{x}_{O1} \end{pmatrix}$$

$$\begin{pmatrix} \Delta t'_c \\ \Delta \mathbf{x}'_c \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \begin{pmatrix} t_1 - t_0 \\ 0 \end{pmatrix}$$ - expectation turns into a movement

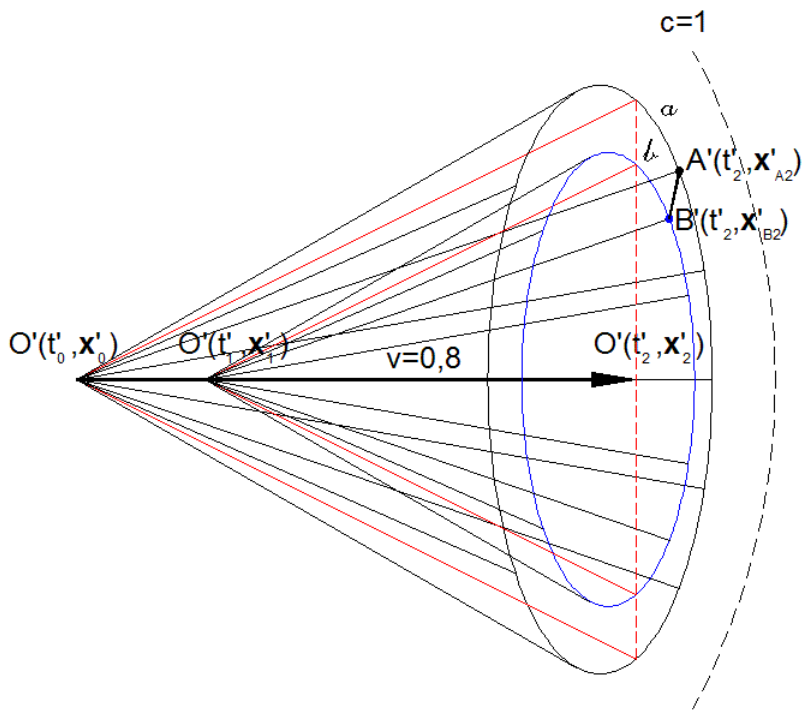

Figure 10: The image of the real components of concentric spheres from the stationary frame OX seen in a moving frame OX'.

Formula (42) shows that in the frame OX' vectors $Re\Delta\mathbf{x}'$ are the same[3] as vectors $\Delta\mathbf{x}$ in the rest frame OX (41), but the beginnings and ends of these vectors are simultaneous in frame OX , and in frame OX' are in a different time! The time difference between the end and beginning of the vector $Re\Delta\mathbf{x}'_d$ is proportional to the product $(\mathbf{x}_{A2} - \mathbf{x}_{B2})\mathbf{v}$. So, there was deformation in time. The Figure 10 shows not the picture which the observer would see taking a photo in the moving frame. The observer in the frame OX' *photographs* what he *sees*,

[3]We skip the factor of dilation, because formulas (41) and (42) are equivalent, and calculating the formula (42) the factor is reduced

and he *sees* the same points, but being at a different time than the observer of the frame OX. The picture which was taken in the moving frame shows as if there was a spatial deformation involving a mutual shift of the spheres a and b relationship, because the camera is not able to *see* simultaneously what is not simultaneous. So, ATTENTION!:

What the observers in frames OX and OX' *photograph* is not the same.

There took of place a deformation in the real space, although the shape of the complex ball is not deformed. Moreover, the deformation concerns not only space but time, too. As previously discussed, the points which we *see* simultaneously in a moving frame are not simultaneous in a rest frame. Deformation has the time and space nature, but in the complex space-time, the deformation does not occur, because the scalar product is preserved. So, what is the interpretation of the imaginary vector? Since we think in terms of real space, we can assume that the imaginary vector is associated with the magnitude of the deformation of real quantities. Observing a single point we can not see it. We see it just watching a lot of points and yet in time. In the next example it will be explained in detail.

3.4 Movement of point with the elastic collision.

Let us imagine a laboratory in the shape of a sphere. The wall is a sieve. In the center of this laboratory we placed an explosive charge. At the moment t_0 the charge explodes. Splinters uniformly scatter in all directions at a speed of w=0.4. Some of them are reflected from the walls and resiliently return to the center of the laboratory and some part freely exit the laboratory through the holes in the wall. The observation finishes at the moment when reflected splinters meet at the center of the sphere. We will show what the real image [4] of the experiment looks like, viewed from the frame moving at the speed of v=0.8. Figure 11 shows an experiment in a laboratory frame. Some of the particles elastically collide with the wall

[4] The real part of image in the mathematical sense, that is we cross out the real spatial components of the received four vectors. Our splinters are geometric points, when in fact these are the items that have energy and when passing through a sieve there is the phenomenon of dispersion, which we omit in our theoretical reasoning.

and back towards the center (purple dashed line), and others that leaked through the sieve escape into space - blue dashed line. Hitting the wall occurs at the same moment and leaves a green mark.

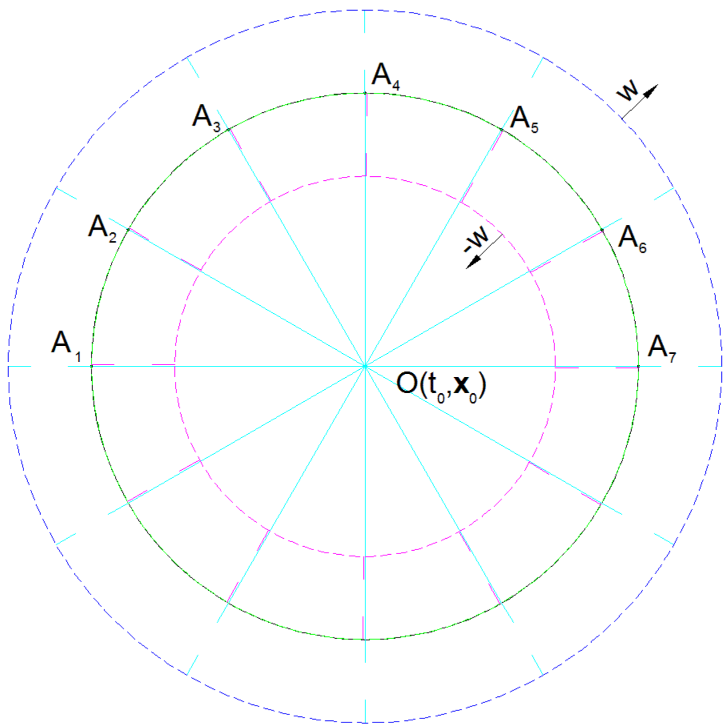

Figure 11: The explosion inside the riddled sphere viewed in the rest frame.

In Figure 11, directions distinguished every 30 degrees to show in the next drawings how in the moving frame is *viewed* the time shift of the phenomenon which is the hitting of the explosion front against the wall of the laboratory. We calculate the speed of shards from the formula (37), so in the primed frame the times and distances can be understood intuitively. Figures would correspond to photographs, if the light falling on the film had infinite speed.

Note: In figures 12 - 17 only real vectors are shown.

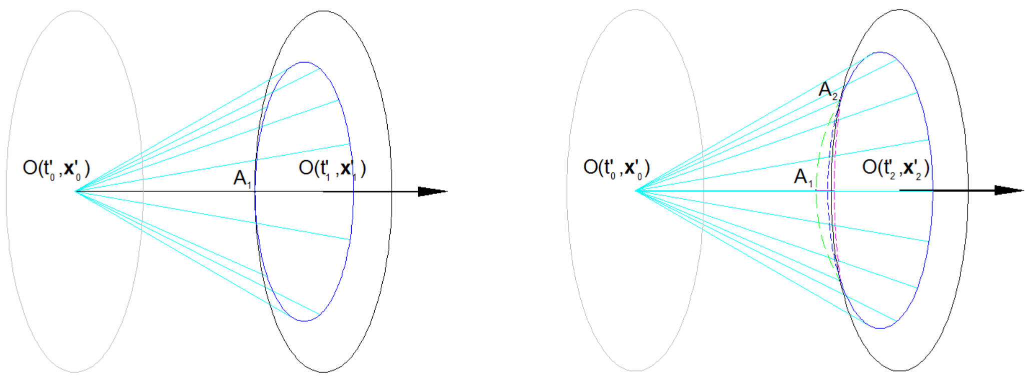

Figure 12: The first particles arrive at the wall of the laboratory. Moments t_1 and t_2

The gray ellipse on the left side represents a real part of the laboratory at the moment of explosion t_0. The black ellipses represent the laboratory at moment t_n. The azure continuous lines represent tracks of selected particles before kickback. The tracks after kickback are shown by purple dashed lines. The tracks of particles that flew through the holes are shown by blue dashed lines. A part of the solid blue ellipse represents the wave front before reaching the wall of the laboratory. The blue dashed line shows the particles which flew through the sieve. The dashed purple line shows the explosion front reflected from the wall. The green dashed line means the trace made by the particles colliding with the wall of the laboratory.

Over time the line describes a circle and there arise points of A_1 to A_7. The observation is finished at the moment when the reflected portions of the explosion front concentrate in the center of the laboratory, i.e. at t_f. The lines were plotted using AutoCAD which allows for high precision and the necessary calculations were performed using the formula (37). It should be noted that, despite the real deformation of the laboratory and the wave front, the mark which the collision left shows that our laboratory is not deformed. Therefore, we can adopt a different interpretation of the results: The shape is not deformed, only different parts of the observed laboratory move in time, or deformation of the shape can be treated as an

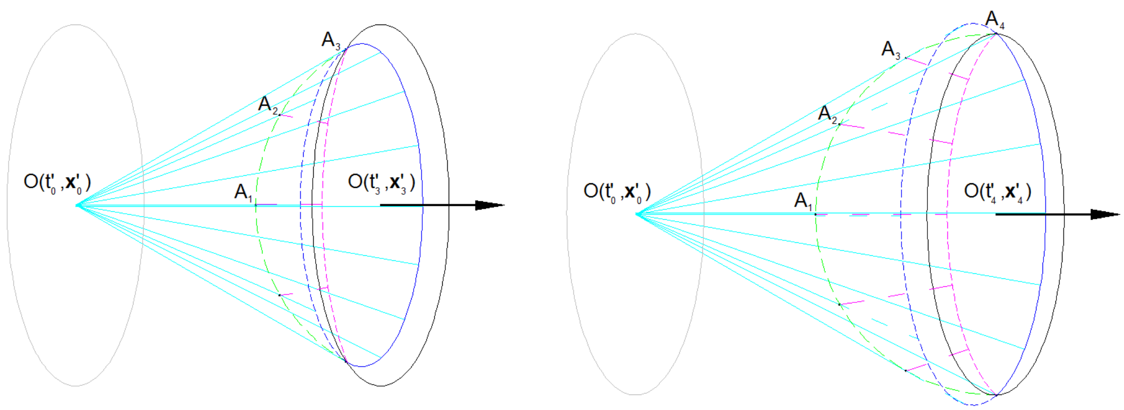

Figure 13: Moments t_3 and t_4

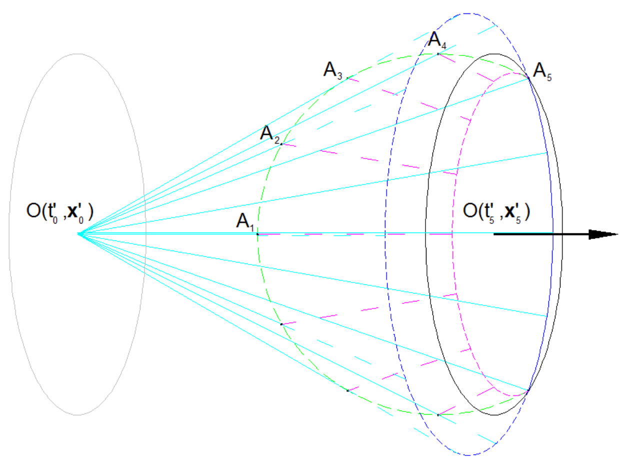

Figure 14: Moment t_5

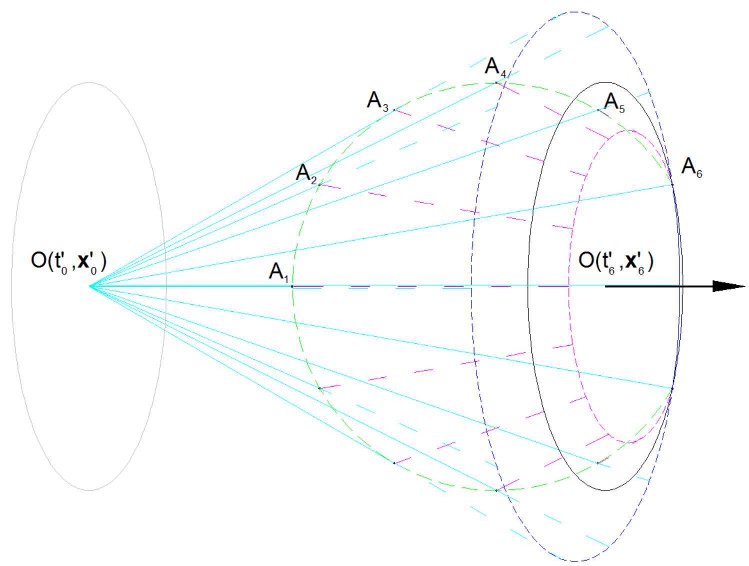

Figure 15: Moment t_6

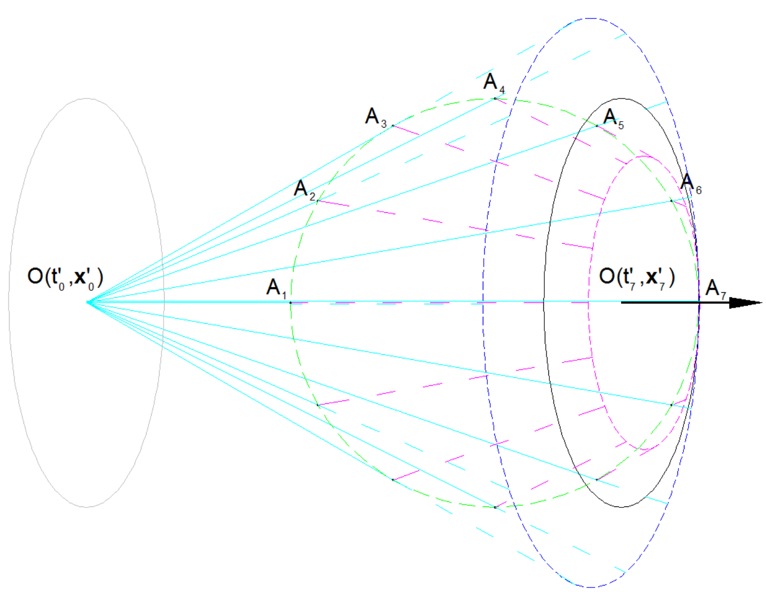

Figure 16: Moment t_7

The last particles arrive at the wall of the laboratory illusion resulting from our understanding of simultaneity. The moving observer should see the point of collision moving in time and space, but is unable to do this because this point (as shown at the beginning of this chapter) moves at a speed greater than the speed of light. We emphasize here that we are not talking about the material point, but about a geometric point. Each of the pieces (material points) moves in a straight line at a speed of less than c. What an observer is able to register is a trace of reflection which an undeformed circle. Once again, the above reasoning confirms the fact that in the complex space-time there is no deformation. This is a serious argument in favour of providing a complex model.

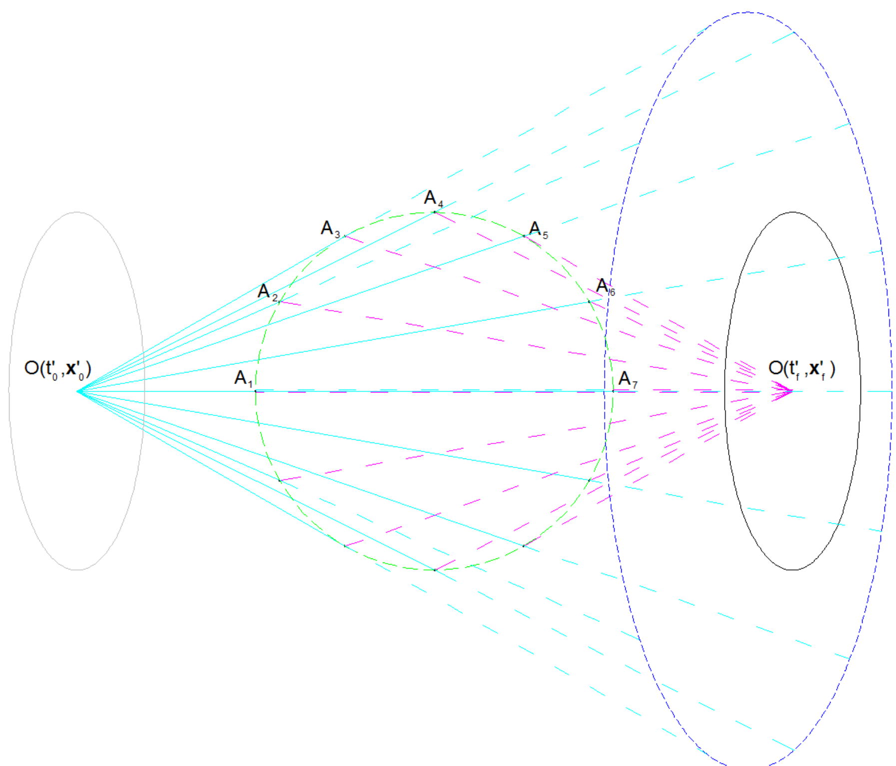

Figure 17: Moment t_f The particles reflected from the walls meet at the center of the laboratory.

4 Algebraic interpretation of the boost paravector.

Let us return once more to the formula (3), from which it follows that $\Lambda = \frac{\mathbb{X}'\mathbb{X}^-}{|\mathbb{X}|^2}$, and since the boost paravector is orthogonal, so the 4-vectors $\mathbb{X}$ and $\mathbb{X}'$ have the same module, or

$$\Lambda = \frac{\mathbb{X}'\mathbb{X}^-}{|\mathbb{X}'||\mathbb{X}|} = \angle\left(\mathbb{X}', \mathbb{X}\right) \tag{43}$$

Therefore, the boost can be interpreted as an angle ([2] def.3.5) between the four-vector in a rest frame, and its image in the moving frame. Its compliance with the assumption that the speed of physical objects is always less than the speed of light is in favour of this interpretation. This results in further consequences:

- The 4-vectors of location to the physical objects can not take arbitrary values, and only those for which these 4-vectors are proper, because otherwise they would not have the module.
- Caution is required in approach of the derivatives of various functions in the complex space-time. It seems that for some of functions (eg. The electric field) the principles of differentiation are consistent with the general principles of the classical theory of complex functions. For others, such as the Lagrange function, differentiation can be permissible only after proper coordinates.

5 Relations between the four-vectors

By studying the paravector algebra we introduced many concepts, familiar from Euclidean geometry. Now we will check how the paravector concepts are related to the classic ones. As first we will look at parallelism and we'll check how much parallelism of 4-vectors is different from the parallelism of Euclidean vectors.

We assume that 4-vectors $\mathbb{X}_1$ and $\mathbb{X}_2$ are real proper and are parallel to each other, which means that $\langle \mathbb{X}_1, \mathbb{X}_2 \rangle_V = 0$, so

$$-\Delta t_1 \Delta \mathbf{x}_2 + \Delta t_2 \Delta \mathbf{x}_1 = 0 \tag{44}$$

If we divide the above equation by $\Delta t_1 \Delta t_2$, we get that $\mathbf{v}_1 = \mathbf{v}_2$, where $\mathbf{v}_1 = \frac{\Delta \mathbf{x}_1}{\Delta t_1}$ and $\mathbf{v}_2 = \frac{\Delta \mathbf{x}_2}{\Delta t_2}$. The parallelism of 4-vectors representing the location of the two objects in the space-time means that these objects move at the same speed and in the same direction.

It is easy to see that the 4-vectors of the position of objects moving in the same direction but at different speeds are not parallel. As well as the velocity vectors of objects moving at the same speed but in opposite directions. This is obvious, because if the observer changes to a vehicle in motion, it turns out that not everything that was parallel before changing, still is. When dealing with purely geometrical vectors (component time coordinate equal to 0), parallelism of 4-vector is equivalent to Euclidean parallelism.

Perpendicularity of the 4-vectors from the previous example means that $\langle \mathbb{X}_1, \mathbb{X}_2 \rangle_S = 0$ which gives

$$\Delta t_1 \Delta t_2 - \Delta \mathbf{x}_1 \Delta \mathbf{x}_2 = 0 \tag{45}$$

If we divide this equation by $\Delta t_1 \Delta t_2$, similarly before, we get the scalar product of the velocity vectors $\mathbf{v}_1 \mathbf{v}_2 = 1$. Since the velocity of the objects carrying energy is less than 1, than the position 4-vectors of these objects are never perpendicular to each other. However, this does not mean that the 4-vectors defining the relative position of various objects in space-time may not be perpendicular, but this is no longer physics but a geometrical problem.

Conclusions

In the formula (2) it can be seen that multiplying the complex paravector of boost by the real 4-vector of position, we obtain a complex value of time. Although at first glance the complex space-time seems to be pointless, yet the fact that only this area was unexplored should be a sufficient argument for its examination. On the other hand, as well as think of it, in classical physics we can find a lot of circumstantial evidence indicating the existence in nature of the complex sizes or real ones changing its properties as if they moved to a hidden imaginary space. The best example is the magnetic field that has no source, and it exists only in connection with the moving electrical charges. From the formal side, dealing with calculus of paravectors we come to the conclusion that it perfectly fits the description of the space-time phenomena, but also it forces the treatment of space as a complex structure. Therefore, we have accepted the hypothesis that the structure of space-time is C^4.

Using the paravector calculus, we can distinguish three basic space-time automorphisms preserving the invariance of the wave equation:

$$\mathbb{X} \to \mathbb{X}': \qquad \mathbb{X}' = \Lambda\mathbb{X}$$

$$\mathbb{X}' = \Lambda\mathbb{X}\Lambda^{-1}$$

$$\mathbb{X}' = \Lambda\mathbb{X}\Lambda^{*}$$ where Λ is the orthogonal paravector.

The second transformation is a rotation that does not change a scalar, so for physics is not very interesting. The last transformation has a huge advantage - it is internal in the real space-time. This transformation has been studied by professors David Hestenes from Arizona State University and William Baylis from the University of Windsor in Canada. We devoted a critical article [4] to this issue. Our research went into the recast of the first transformation which requires complex space-time. Although we can yet distinguish the special paravectors and automorphism of quasi-real space-time (real time and imaginary space), but in our opinion it is a blind way, because a lot of results is inconsistent with the textbook knowledge, examples of which we show in the appendix.

In the complex space-time, the relativistic transformation is described by equations (2). There we get complex coordinates, and the dilation factor complies with the Lorentz factor. We also get components spatially deforming the time ($\boldsymbol{\beta}\boldsymbol{x}$) and introducing the imaginary components into the space ($\boldsymbol{\beta} \times \boldsymbol{x}$). In the real frame the time deformation is impossible to observe, because it corresponds to speed greater than the speed of light, as shown on the example of a flashing disk. The observer is not able to verify if the information received by him at the same moment was sent simultaneously. He must believe his calculations. Explaining the experience of an explosion in the laboratory and assuming a constant speed of light, in real space-time a deformation of the shape of the laboratory must have taken place. In the complex space-time deformation is apparent as well as apparent is the shortening of the mast flipped from/to the observer. The deformation of the real part of the laboratory is due to the fact that a part of it hid behind the imaginary horizon, so the laboratory as a ball has not been deformed. The transformation (2) entering the imaginary component deforms the real component, but the observer it is not able to verify it, either which is explained on the example of section 3.4. Another question arises: how to interpret the imaginary time? The answer is: I do not know, but also I don't know whether it makes sense to do it, because the local space-time of the observer is real and he can view only the projection of complex phenomena on his real frame, like in the movie the spectator can see the spatial world projected on a flat screen. For him it is important that all calculations are in balance because then they are reliable.

The local space-time of an observer (a frame of each observer) is real and it is just to describe the non-relativistic phenomena. When the observer changes to a very high speed vehicle the same phenomena are complex for him, and he wants to describe them using his real concepts. What is more, the information that he receives from the outside always and everywhere is real, because energy is an information carrier, and that as a product of mutually conjugated values is always real. Therefore, the problem is to find the rules of projection of the phenomena taking place in the complex space-time on the local real space-time of the observer.

For now, it is a hypothesis that in the next work we will try to bring closer. Most places will be devoted to the theory of the electric field, because it is complex by its nature and from this theory works began on the special theory of relativity. We must admit that so far we are not sure whether the direction of researches is correct, but many of the results confirm our hypothesis, and it seems that the space-time of the great speed is not real but its structure is complex, to which we will try to convince our readers in the next publications, because from both the intuitive and accounting sides it is much simpler than the current theory.

The complex space-time should not be bizarre for people engaged in theoretical physics, because in science for a long time we have been dealing with concepts which we do not directly experience, but which were created for the image of phenomena to be satisfactory. Such a concept is, for example, the field potential. An electric charge is affected by the electric field strength causing the force which is measured by the instrument inclining the tip and this should be enough to describe the phenomenon. Adding a constant value to the potential function does not change anything. So why was the concept of potential created? Since the potential explains a lot, so the description of the phenomena which it related to is satisfactory, and we are so accustomed to it that we cannot imagine a field theory without it. The same is true for the complex space-time: We have to get used to it, and the fact that the description of relativistic phenomena in the complex space-time does not result in paradoxes should be a sufficient reason to continue work on it.

Appendix

From the paravector algebra, we know that the set of paravectors of the form $(a, i\mathbf{c})$ (which are named special paravectors), together with the operations of summation and multiplication creates a division ring. Since the determinants of the special paravectors are positive, each of them has a module. So, on these paravectors we can build a normed space with an Euclidean metric(!). Since vectors are imaginary, we will say that this space-time is quasi-real. Although the orthogonal transformation is an operation internal in this space, then we come to the critical point, when it turns out that the equations corresponding to the Maxwell equations do not fit the theory of electricity and magnetism.

The four-vector of the position has a form

$$\mathbb{X} = \begin{pmatrix} \Delta t \\ i\Delta\mathbf{x} \end{pmatrix}, \text{ and velocity paravector } V = \frac{1}{\sqrt{1+v^2}} \begin{bmatrix} 1 \\ i\mathbf{v} \end{bmatrix},$$

and the transformation that preserves the invariance of the wave equation:

$$t' = \frac{1}{\sqrt{1+v^2}}(t - \mathbf{v}\mathbf{x}) \qquad \text{i} \quad \mathbf{x}' = \frac{i}{\sqrt{1+v^2}}(\mathbf{x} + \mathbf{v}t - \mathbf{v}\times\mathbf{x}) \tag{46}$$

Similarly as before, let's see how the time interval is transformed.

$$\begin{pmatrix} \Delta t \\ 0 \end{pmatrix} = \frac{1}{\sqrt{1+v^2}} \begin{bmatrix} 1 \\ -i\mathbf{v} \end{bmatrix} \begin{pmatrix} \Delta t' \\ i\Delta\mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1+v^2}} \begin{pmatrix} t' + \mathbf{v}\Delta\mathbf{x}' \\ i(\Delta\mathbf{x}' - \mathbf{v}\Delta t' + \mathbf{v}\times\Delta\mathbf{x}') \end{pmatrix}$$

Z części wektorowej powyższego wzoru otrzymujemy

From the vector part of the above formula we obtain

$$\Delta\mathbf{x}' - \mathbf{v}\Delta t' + \mathbf{v}\times\Delta\mathbf{x}' = 0.$$

Since the result of the vector product is perpendicular to both $\mathbf{v}$ and the vector $\Delta\mathbf{x}$, therefore there must be:

$$\Delta\mathbf{x}' - \mathbf{v}\Delta t' = 0 \qquad \text{i} \qquad \mathbf{v}\times\Delta\mathbf{x}' = 0.$$

The equation of motion of a point in the observer's space R^3 there is no dilation factor, and the movement is also described by Galileo formula. We get the resultant velocity paravector we get when we change the real vector to an imaginary one in the equation (17):

$$\frac{1}{\sqrt{1+v_1^2}}\begin{bmatrix}1\\ i\mathbf{v}_1\end{bmatrix}\frac{1}{\sqrt{1+v_2^2}}\begin{bmatrix}1\\ i\mathbf{v}_2\end{bmatrix}=\frac{1}{\sqrt{1+\left(\frac{\mathbf{v}_1+\mathbf{v}_2-\mathbf{v}_1\times\mathbf{v}_2}{1-\mathbf{v}_1\mathbf{v}_2}\right)^2}}\begin{bmatrix}1\\ i\,\frac{\mathbf{v}_1+\mathbf{v}_2-\mathbf{v}_1\times\mathbf{v}_2}{1-\mathbf{v}_1\mathbf{v}_2}\end{bmatrix}$$

We see that this defined transformation is internal in a set of equivalent with R^4 (quasi-real space-time) and everything would be OK, if not the dilation factor which is different from the current STR. As it is easy to calculate in this case the resultant velocity has no limit to the speed of light.

$$v^2=\left(\frac{\mathbf{v}_1+\mathbf{v}_2-\mathbf{v}_1\times\mathbf{v}_2}{1-\mathbf{v}_1\mathbf{v}_2}\right)^2=\frac{\left(1+v_1^2\right)\left(1+v_2^2\right)}{(1-\mathbf{v}_1\mathbf{v}_2)^2}-1$$

As a result of the submission of parallel velocities v_1= 0,8 and v_2= 0,8 we obtain the resultant velocity v = 4.44, which is inconsistent with the experimental results.

The description of the Doppler effect is different from the existing theories only in the dilation factor.

$$\Theta=\begin{bmatrix}\omega\\ -i\mathbf{k}\end{bmatrix}\begin{pmatrix}t\\ i\mathbf{x}\end{pmatrix}=\frac{1}{\sqrt{1+v^2}}\begin{bmatrix}\omega\\ -i\mathbf{k}\end{bmatrix}\begin{bmatrix}1\\ -i\mathbf{v}\end{bmatrix}\begin{pmatrix}t'\\ i\mathbf{x}'\end{pmatrix}=\begin{bmatrix}\omega'\\ -i\mathbf{k}'\end{bmatrix}\begin{pmatrix}t'\\ i\mathbf{x}'\end{pmatrix}$$

so that the new frequency is

$$\omega'=\frac{\omega-\mathbf{v}\mathbf{k}}{\sqrt{1+v^2}}.$$

Let's see yet how the electric field transforms. We want to obtain a magnetic field as the result of the transformation of the electric field . Without going into details, from the formula ... we get

$$\begin{bmatrix}1\\ -i\mathbf{v}\end{bmatrix}\begin{pmatrix}0\\ i\mathbf{E}\end{pmatrix}=\begin{pmatrix}\mathbf{v}\mathbf{E}\\ i(\mathbf{E}+\mathbf{v}\times\mathbf{E})\end{pmatrix}\quad\text{or}\quad\begin{bmatrix}1\\ -i\mathbf{v}\end{bmatrix}\begin{pmatrix}0\\ \mathbf{E}\end{pmatrix}=\begin{pmatrix}-i\mathbf{v}\mathbf{E}\\ \mathbf{E}+\mathbf{v}\times\mathbf{E}\end{pmatrix}$$

This means that, whatever embodiments are taken, by the movement of the electric field, the magnetic field is not created.

Conclusion of the appendix

The use of the special paravectors to record the coordinates of the position and speed seemed very tempting primarily because special paravectors with the summation and multiplication form a division ring. Performing multiplication and summation we are always in the quasi-real space-time, which may be regarded as real. Unfortunately, this hypothesis proved to be a dead end. Attempts to use the special paravectors to describe wave phenomena ended in failure:

- It was not possible to describe an equation of wave-front
- An assemblage of velocities allowed for obtain the superluminal speed
- Attempt to describe Maxwell's equations gave nonsense results.

Therefore, we see no point in continuing research in this direction. The topic will come back in subsequent publications in more detail.

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