The influence of electronic solid-state plasma on attenuation of transverse sound wave in a conductor

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The effect of the electron sound absorption in a conducting medium (metal) was previously considered on the assumption of the Fermi-surface deformation under the action of the sound wave. In the present work will be considered another approach to the problem based on dynamic (kinetic) interaction of the electron gas with the lattice vibrations. The analysis is carried out for the case of arbitrary degeneration degree of the solid-state plasma.

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1. Introduction

The impact of the electrons on the absorption of sound in conducting media (mostly in metals) was considered at the assumption of the Fermi–surface deformation under the action of sound waves [1], [2] [3]. However, the process of change of Fermi-surface due to the interaction of the electron gas with lattice inevitably depends on the characteristics of this interaction. This process may not strictly speaking be considered in static approximation. Dynamic and kinetic processes must be considered in the analysis of the formation of a Fermi-surface at the propagation of sound waves in the metal. In the case of semiconductors and other materials with non-degenerate electron plasma the situation it becomes even more complicated because you have to consider the deformation do not Fermi surface, and the entire spectrum of excitation of electrons. The nature of introduced "fictitious" [1] or drift [4] forces is not completely clear. Unclear is also the question about the possibility of spreading of this approach on amorphous materials.

In the present work will be considered the approach to the problem, based on dynamic (kinetic) interaction of the electron gas with the lattice vibrations.

We will consider the propagation of transverse sound in an isotropic conductor. Our goal will be consideration of the problem how the conduction electrons and the generated electric field influence the process of attenuation of the transverse sound waves.

A number of issues of propagation and attenuation of sound waves in the metal have been considered in the works [4]–[11].

1. Statement of the problem and basic equations

Transverse sound wave creates a velocity field $\mathbf{u}$ in the conductor

$$\mathbf{u}=\mathbf{u}_0e^{i(\mathbf{kr}-\omega t)},\quad \mathbf{ku}=0,\quad \omega=s_{tr}k, \tag{1.1}$$

where s_{tr} – velocity of transverse sound waves, $\mathbf{k}$ – the wave vector, k — wave number, ω – frequency of the sound wave.

Kinetic equation with the relaxation type collision integral for electrons will be as follows [2], [12]

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+e\mathbf{E}\,\frac{\partial f}{\partial \mathbf{p}}=\nu(f_{eq}-f), \tag{1.2}$$

where f_{eq} - the equilibrium Fermi distribution in a solid–state plasma

$$f_{eq}=\Big[1+\exp\frac{\mathcal{E}-\mu}{k_BT}\Big]^{-1}$$

Here $\mathcal{E}$ — the energy of the electrons, μ — chemical potential, T – temperature, k_B – Boltzmann's constant, e – the charge of the electron. The value $\nu=1/\tau$ – the electron collision rate, τ – the average time between two successive collisions of an electron. The analysis of the value of τ for different materials carried out in [13].

We assume that in the absence of a sound wave electron distribution can be considered spherically–symmetric. In this case for electron energy $\mathcal{E}_0$ we have

$$\mathcal{E}_0 = \frac{m\mathbf{v}^2}{2}.$$

Here m — the effective mass of the electron.

The sound wave breaks isotropy locally equilibrium distribution of electrons. This distribution must now be an equilibrium in the coordinate system, resting relative to the lattice. Because the local velocity of the lattice is $\mathbf{u}$ in this case will be

$$\mathcal{E} = \frac{m(\mathbf{v}-\mathbf{u})^2}{2}. \tag{1.3}$$

We assume that the velocity $\mathbf{u}$ is much less than the thermal velocity of electrons (or Fermi velocity for the case of degenerate Fermi–gas). Then the value of (1.3) can be linearized

$$\mathcal{E} \simeq \frac{m\mathbf{v}^2}{2} - m\mathbf{v}\mathbf{u} = \mathcal{E}_0 - m\mathbf{v}\mathbf{u}.$$

Through appropriate linearization of the locally equilibrium function f_{eq} we get

$$f_{eq} = f_0 - \frac{\partial f_0}{\partial \mathcal{E}} m\mathbf{v}\mathbf{u}, \quad f_0 = \Big[1 + \exp\frac{\mathcal{E}_0 - \mu}{k_B T}\Big]^{-1}. \tag{1.4}$$

Similarly, in the linear case, the term with the electric field in (1.2) has the follows form

$$e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} \simeq e\mathbf{E}\mathbf{v}\frac{\partial f_0}{\partial \mathcal{E}} \tag{1.5}$$

The linearized distribution function has the form [2]

$$f = f_0 - \frac{\partial f_0}{\partial \mathcal{E}}\psi. \tag{1.6}$$

Taking into account relations (1.5), (1.6) and (1.7) the kinetic equation (1.2) can be written for the function ψ [2] as follows

$$i\omega\psi - i\mathbf{v}\mathbf{k}\psi + e\mathbf{v}\mathbf{E} = -\nu(m\mathbf{v}\mathbf{u} - \psi). \tag{1.7}$$

Equation (1.7) can be rewritten in the form

$$-i\omega\psi + i\mathbf{v}\mathbf{k}\psi + \nu\psi - \nu\delta\mu = \mathbf{v}(e\mathbf{E} + \nu m\mathbf{u}). \tag{1.8}$$

The last term in the right-hand side of equation (1.8) corresponds to accounting of the drag effect of electrons by movement of atoms of the lattice at the scattering of electrons by lattice vibrations or defects. It is analogous to the "fictitious" force, introduced in [1] (see also [2]). Note that in this approach this term occurs naturally and does not require any additional assumptions.

Equation (1.8) coincides with the kinetic equation describing response of electrons to a assumed external transverse electric field, if, instead of the field $\mathbf{E}$ to consider the value $\mathbf{E} + \nu m\mathbf{u}/e$, that is, to replace

$$\mathbf{E} \to \mathbf{E} + \frac{\nu m\mathbf{u}}{e}.$$

Since $\mathbf{u} \sim \exp(i\mathbf{kr} - i\omega t)$, then the functions ψ, $\mathbf{E}$ have the same dependence on the coordinates and time, i.e.

$$\psi \sim \exp(i\mathbf{kr} - i\omega t), \quad \mathbf{E} \sim \exp(i\mathbf{kr} - i\omega t). \tag{1.9}$$

Then the electron current density $\mathbf{j}_e$ taking into account (1.9) is determined by the following relation [14]

$$\mathbf{j}_e = \sigma_{tr}\big(\mathbf{E} + \frac{\nu m}{e}\mathbf{u}\big). \tag{1.10}$$

Here $\sigma_{tr} = \sigma_{tr}(\mathbf{k}, \omega)$ — transverse electrical conductivity of the electron plasma.

Because of the electroneutrality volume charge density of ions (lattice) is equal to $(-eN)$, the current density is $(-eN\mathbf{u})$, $N = \text{const}$ — equilibrium concentration of electrons. Then the equation for the field $\mathbf{E}$ has the form

$$\triangle\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = -\frac{4\pi i\omega}{c^2}(\mathbf{j}_e - eN\mathbf{u}). \tag{1.11}$$

Here $\mathbf{j}_e$ — the electron current density

$$\mathbf{j}_e = e\int \mathbf{v}f\frac{2d^3p}{(2\pi\hbar)^3}.$$

By substituting the expression (1.10) into equation (1.11), we obtain

$$-k^2\mathbf{E}+\frac{\omega^2}{c^2}\Big(1+\frac{4\pi i\sigma_{tr}}{\omega}\Big)\mathbf{E}=\frac{4\pi i\nu m\omega}{c^2e}\Big(\frac{e^2N}{\nu m}-\sigma_{tr}\Big)\mathbf{u}.$$

Transverse conductivity σ_{tr} is associated with a transverse dielectric permittivity ε_{tr} by the ratio [14]

$$\varepsilon_{tr}(\mathbf{q},\omega,\nu)=1+\frac{4\pi i}{\omega}\sigma_{tr}(\mathbf{q},\omega,\nu). \tag{1.12}$$

Taking this into account, we obtain

$$\Big(\frac{\omega^2\varepsilon_{tr}}{c^2}-k^2\Big)\mathbf{E}=\frac{4\pi i\nu m\omega}{c^2e}\,(\sigma_0-\sigma_{tr})\mathbf{u}. \tag{1.13}$$

Here $\sigma_0 = Ne^2/(m\nu)$ — static conductivity of an electron solid-state plasma.

From the relation (1.13) we find the electric field $\mathbf{E}$

$$\mathbf{E}=\frac{4\pi i\nu m\omega}{e}\frac{\sigma_0-\sigma_{tr}}{\omega^2\varepsilon_{tr}-c^2k^2}\mathbf{u}. \tag{1.14}$$

Substituting the obtained expression for the field $\mathbf{E}$ (1.14) in equation (1.10), we obtain for the density of electron current $\mathbf{j_e}$ the following expression

$$\mathbf{j}_e=\sigma_{tr}\Big[\frac{4\pi i\nu m\omega}{e}\frac{\sigma_0-\sigma_{tr}}{\omega^2\varepsilon_{tr}-c^2k^2}+\frac{\nu m}{e}\Big]\mathbf{u}.$$

After some transformations we get

$$\mathbf{j}_e=\frac{\nu m\sigma_{tr}}{e}\,\frac{\omega^2\varepsilon_*-c^2k^2}{\omega^2\varepsilon_{tr}-c^2k^2}\mathbf{u}. \tag{1.15}$$

The value ε_* is introduced by analogy with ε_{tr} (1.12)

$$\varepsilon_*=1+\frac{4\pi i}{\omega}\sigma_0.$$

2. Sound wave attenuation coefficient

The energy flux density carried by a longitudinal acoustic wave is equal to [2]

$$I = \frac{\rho_0 u_0^2 s_l}{2}. \tag{2.1}$$

Here ρ_0 – the density of the substance.

The damping coefficient Γ is defined by the following expression

$$\Gamma = \frac{Q}{I}. \tag{2.2}$$

Here Q — the energy dissipation density of the sound wave. Dissipation due to the anharmonicity of the lattice vibrations of Q_l and interaction of sound waves with the electronic component and generated by the wave electric field Q_e.

Then the value Q can be represented in the form

$$Q = Q_l + Q_e. \tag{2.3}$$

We are interested in the value Q_e, that is dissipation, associated with the interaction of sound waves with solid–state plasma.

The value Q_e is calculated as [5]

$$Q_e = \frac{1}{2}\operatorname{Re}(\mathbf{F}\mathbf{u}^*) = \frac{1}{2}\operatorname{Re}((-eN\mathbf{E} + \mathbf{F}_e)\mathbf{u}^*), \tag{2.4}$$

where

$$\mathbf{F}_e = -\int \nu m(\mathbf{u} - \mathbf{v}) f \frac{2d^3p}{(2\pi\hbar)^3}. \tag{2.5}$$

Here by Re is designated the real part of a complex number.

The value $\mathbf{F}$ in (2.4) represents the force acting on the lattice. It consists of two parts. The first $(-eN\mathbf{E})$ part corresponds to the force acting on the charge of the lattice because of the presence of the electric field $\mathbf{E}$. The presence of the minus sign is due to the electroneutrality of the material. So the charge density of the lattice is opposite to the charge density of electrons and is equal to $(-eN)$.

The second term $\mathbf{F}_e$ describes the force acting on the lattice from solid–state electron plasma in the scattering of electrons on the lattice. This force is described by formula (2.5).

$$\mathbf{F}_e = -\nu m N \mathbf{u} + \frac{\nu m \mathbf{j}_e}{e}. \tag{2.6}$$

We denote the average drift velocity of electrons through $\overline{\mathbf{v}}$.

$$\mathbf{j}_e = eN\overline{\mathbf{v}}.$$

Then formula (2.6) can be rewritten in the form

$$\mathbf{F}_e = \nu m N(\overline{\mathbf{v}} - \mathbf{u}). \tag{2.7}$$

Thus according to (2.7) the force is proportional to the difference between the average drift velocity of the electrons in solid–state plasma and displacement velocity of atoms in the sound wave.

Substituting (2.6) into (2.4) and using (1.10) we come to the following expression for Q_e

$$Q_e = -\frac{1}{2}\,\mathrm{Re}((-eN\mathbf{E} - \nu m N\mathbf{u} + \frac{\nu m \mathbf{j}_e}{e})\mathbf{u}^*), \tag{2.8}$$

The electron current density $\mathbf{j}_e$ is determined by the expression (1.15). Substitute this expression in (2.8). As a result, after some transformations we get

$$Q_e = \frac{\nu^2 m^2 u_0^2}{2e^2}\,\mathrm{Re}\Big((\sigma_0 - \sigma_{tr})\frac{\omega^2\varepsilon_* - c^2k^2}{\omega^2\varepsilon_{tr} - c^2k^2}\Big). \tag{2.9}$$

In accordance with the expressions (2.2) and (2.3) the attenuation coefficient of sound wave Γ can be split into two parts

$$\Gamma = \Gamma_l + \Gamma_e, \quad \Gamma_l = \frac{Q_l}{I}, \quad \Gamma_e = \frac{Q_e}{I} \tag{2.10}$$

We will be interested in the value Γ_e, due to the influence of electron solid–state plasma and electric field. Taking into account expressions (2.1), (2.9) and (2.10) for this value we received the following result

$$\Gamma_e = \frac{\nu^2 m^2}{\rho_0 s_{tr} e^2}\,\mathrm{Re}\Big((\sigma_0 - \sigma_{tr})\frac{\omega^2\varepsilon_* - c^2k^2}{\omega^2\varepsilon_{tr} - c^2k^2}\Big). \tag{2.11}$$

Note that at low frequencies , i.e. when $\omega \to 0$ wave number $k \to 0$ too. Thus $\sigma_{tr} \to \sigma_0$ and $\varepsilon_{tr} \to \varepsilon_*$. According to formula (2.11) we find that in this limit $\Gamma_e \to 0$.

3. Conclusion

This paper presents a kinetic approach to the study of attenuation coefficient of transverse sound waves in a conductive medium. A study of the coefficient attenuation of the sound wave is based on the kinetic (dynamic) interaction of solid–state electron plasma with the lattice vibrations. The account of the influence of self–consistent electric field on the processes of dissipation and sound attenuation is carried out. The obtained results are valid for any degree of degeneracy of solid-state electron plasma.

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