Does a toy shadow encompass more information than the toy itself?

Arturo Tozzi
Center for Nonlinear Science, University of North Texas
1155 Union Circle, #311427
Denton, TX 76203-5017, USA, and
Computational Intelligence Laboratory, University of Manitoba, Winnipeg, Canada
Winnipeg R3T 5V6 Manitoba
00390815602913
tozziarturo@libero.it

James F. Peters
Department of Electrical and Computer Engineering, University of Manitoba
75A Chancellor's Circle, Winnipeg, MB R3T 5V6, Canada and
Department of Mathematics, Adıyaman University, 02040 Adıyaman, Turkey,
Department of Mathematics, Faculty of Arts and Sciences, Adıyaman University
02040 Adıyaman, Turkey and Computational Intelligence Laboratory, University of
Manitoba, WPG, MB, R3T 5V6, Canada
00393358447399
james.peters3@umanitoba.ca

An observed two-dimensional shadow might encompass more information than its corresponding three-dimensional object. By changing the orientation of a three-dimensional object or the observer's position, we detect different shadows from diverse perspectives, therefore increasing our available information. Starting from this simple observation and extending it to the Einstein's four-dimensional spacetime and to Bekenstein and Hawking equations, we show how, in terms of special and general relativity, information content is not a stationary and fixed quantity as currently believed, but rather depends on the observer's standpoint. This has deep implications in digital physics, information theory, computer vision, shape theory and cosmology.

INTRODUCTION

Information is a measurable physical quantity that currently stands for the most general paradigm able to investigate physical, cosmological and biological systems. It has been claimed that the physical world is made up of information itself *(1)*, so that our Universe is assessable in pure terms of information. The idea that information is the fundamental physical quantity dates back to F.W. Kantor *(2)*. By then, different information-related perspectives have been developed, from the hypothesis that the Universe is a giant digital computer *(3,4)*, to the suggested link among information theory, statistical thermodynamics and the probabilistic nature of quantum mechanics *(5,6,7,8)*, from computational loop quantum gravity *(9)* to connections between information and the Bekenstein and Hawking entropy *(10,11)*. Therefore, information sits at the core of physics and an "it from bit" dogma has been proposed *(12)*: every field or particle exists because of its observation. Here we ask: are the tenets of the information paradigm true?

Counterintuitively, a three-dimensional object might encompass less information that a two-dimensional one. When watched from a given standpoint, an object could be less informative than its own shadow (**Figure 1A**). Is there a way to achieve an increase in shadow's information content, in order to detect and measure more details about the toy? The answer is positive, if we rotate clockwise (or anticlockwise) the toy along its central axis (**Figures 1B, 1C**). In this case, we achieve different shadows with different angulations and enhance our incoming information about the object. Therefore, for an observer, objects are more informative when they move, or when the observer moves around them. Or, in other words, it is easier to extract information when, in an inertial system, objects and observers are not static.

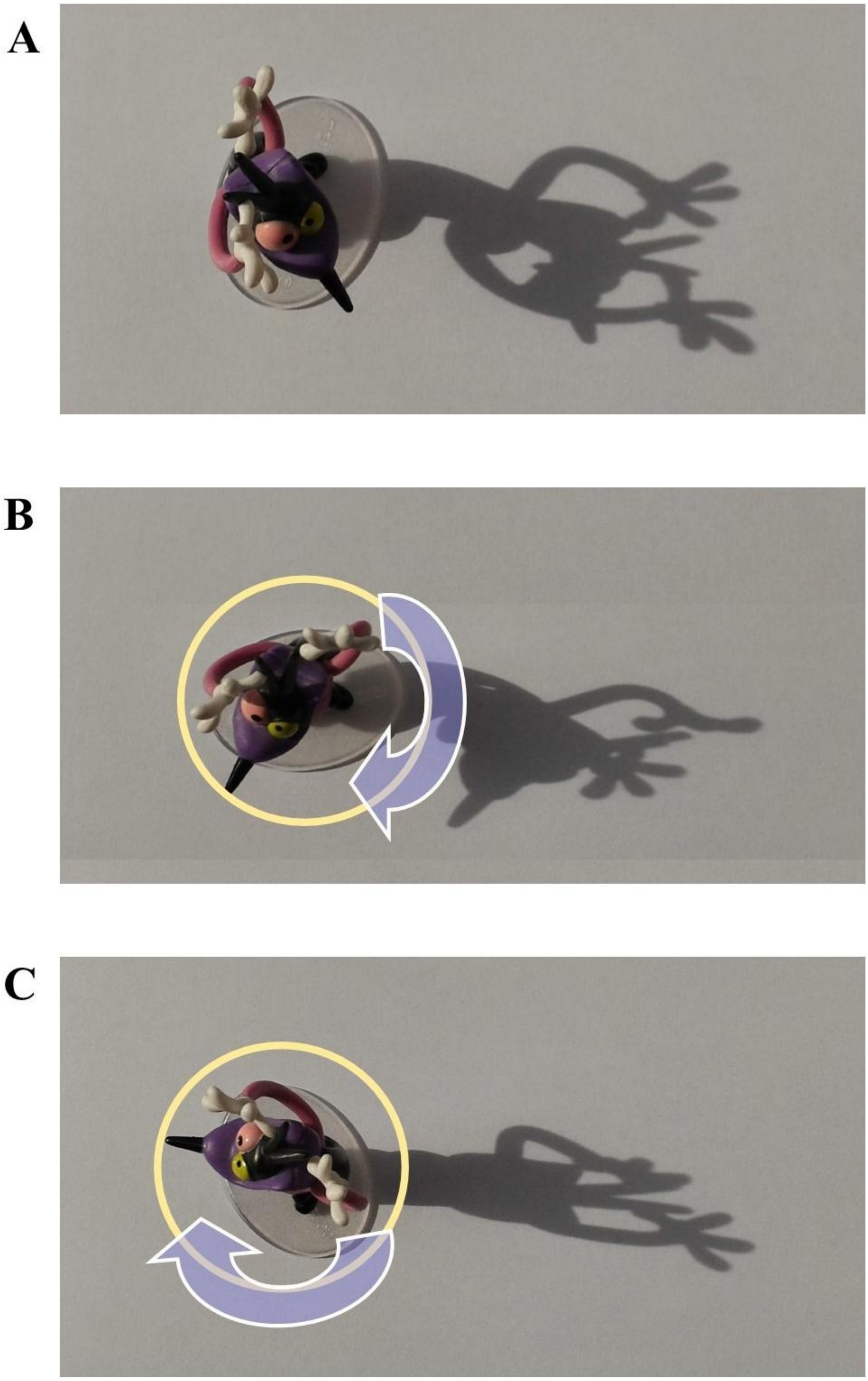


Figure 1. A toy under a light source. In **Figure 1A**, the two-dimensional shadow offers more details of the three-dimensional object (its varying prolonged and contracted shape) than its view from above. **Figures 1B** and **1C** show how toy rotations lead to shadows with different shapes, and, consequently, with different information content. The same holds true, if the observer, and not the toy, moves. If the observer and the object form a system in uniform translatory motion relative to one each other, we are allowed to use the Einstein's equations of special and general relativity.

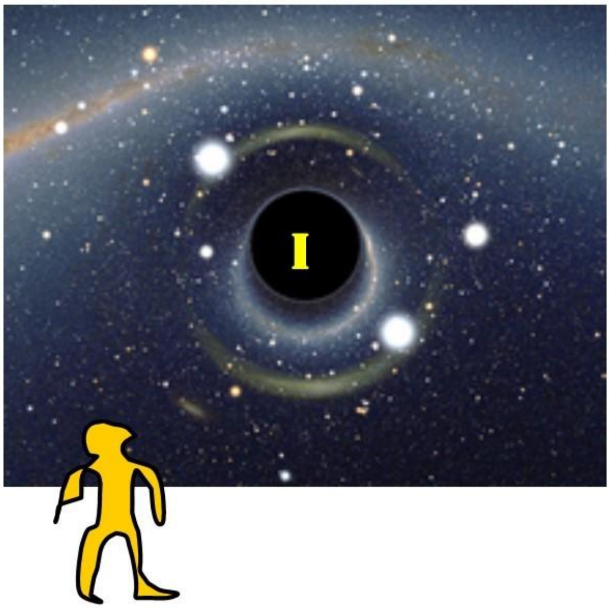
This lead us into the field of special relativity *(13)*. By the standpoint of an observer in uniform translatory motion relative to the object, it seems to be modified when its speed increases. In particular, for an inertial observer, when the object approaches the light speed, its mass seems to increase and its length decrease along one of its spatial dimensions. However, the Einstein formulas do not state that the mass changes according to the different standpoints of the observer, rather it modifies due to the gravitational deformation of the spacetime. Indeed, the rest mass is unchanged, while the reported increase of the inertial mass when the object approaches the light speed is due just to the increased momentum. Therefore, masses are invariant in special relativity, while the increase of the momentum leads to their changes in general relativity. Due to the highly debated and still controversial issue of the mass invariance, here we prefer to focus on an easier detectable change predicted by Einstein's equations: the modifications in objects length along one dimension. On one side, according to Einstein's equations for special relativity *(13)*, one of the three spatial dimensions changes, depending either on the observer's standpoint, or the observed object speed. On the other side, Einstein's field equation of general relativity *(14)* correlates the spacetime curvature (expressed in terms of Ricci tensor and scalar curvature) in a point x of a pseudo-riemannian spacetime (the observer's position, in our case) with the energy-pulse tensor, that describes the density of matter and energy in x. In terms of object length, if the inertial observer changes his position and speed relative to the object, the spatial dimension he is able to detect modifies. This leads him to detect changes in the surface of the observed object. In Einstein's words, the length of a regulus (in this case, our object) is not invariant, because its dimension X shortens together with increases in its speed. This means that a spherical object at rest, say a black hole, appears more or less squeezed by the standpoint of two hypothetical observers, one at rest, and another traveling at the speed light. Therefore, according to the different observer's standpoint, motion and speed, the black hole's spherical surface is more or less deformed.

It might be objected that the statement about changes in length doesn't mean anything, because the latter depends just on the frame. In any description, only the quantities that are invariant under the assessed transformations make sense. Consequently, the Minkowski norm of 4-vectors makes sense in special relativity, while only certain global quantities make sense in general relativity. However, what's non-trivial is that such quantities (such as the length of our toy's shadow), constructed out of non-invariant quantities, can be shown to exist. Here we show how it can be demonstrated that such apparently non-invariant quantities display different information content. The Bekenstein inequality comes into play. Let k be Boltzmann's constant, R the radius of a sphere that encompasses a given system (in this case, our black hole), E the total mass energy (including any rest masses), $\hbar$ the reduced Planck constant and c the speed of light. The Bekenstein bound is an upper limit on the thermodynamic entropy S (or the information I, according to Shannon *(15)*) endowed in a space region equipped with a given amount of energy. In other words, the Bekenstein bound stands for the maximum quantity of information required to describe a physical system down to the quantum level. The universal form of the bound can be described as follows *(16,17)*:

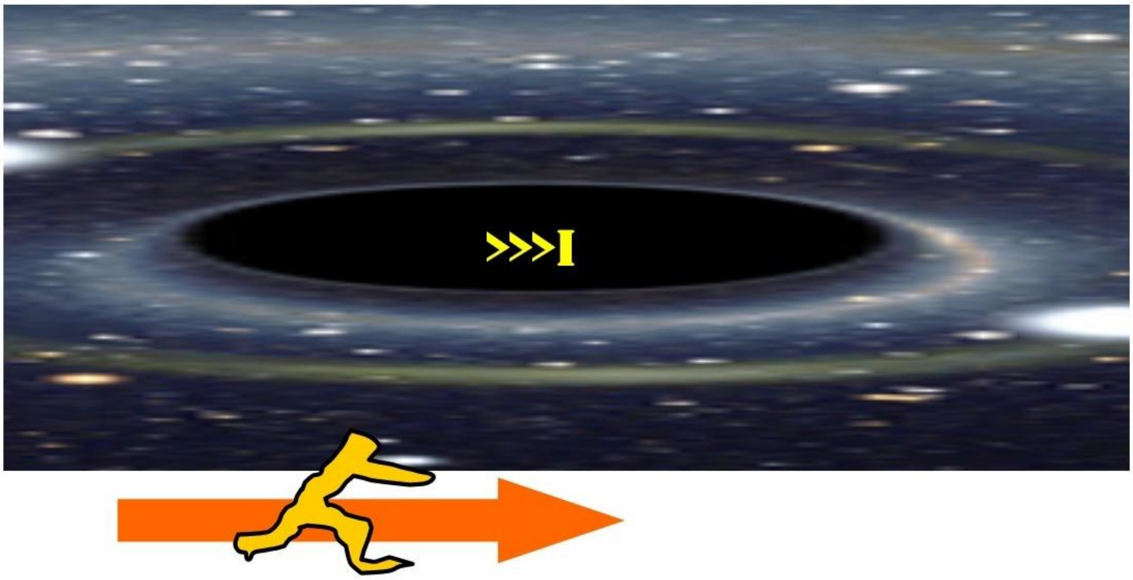
$$S \leq \frac{2\pi RkE}{\hbar c} = \frac{2\pi Rk(hf)}{\hbar c} = \frac{2\pi Rk\left(\frac{hc}{\lambda}\right)}{\hbar c},$$

where f is the wave frequency of the particulate photon energy E and h is the Planck constant. The Bekenstein and Hawking formulas state that the black hole entropy is proportional to the area of its event horizon *(18)*. A noteworthy consequence of the Berkenstein bound is that the entropy encompassed in the black hole is proportional to the two-dimensional border of a sphere-like object enclosing it, i.e., to its horizon surface. Indeed, black hole thermodynamics conjectures that the maximal entropy scales with the radius squared, and not cubed as might be expected. This led to further developments, such as the theory of the world as hologram *(19,20)*.

By joining the two issues of Einstein's special and general relativity and black hole thermodynamics, we are allowed to make the following statement: according to the different observer's standpoints, the information encompassed in the black hole varies (**Figure 2**). Therefore, when the observer moves along a different spacetime curvature, the information he detects about the analyzed black hole is different. This means that, in an inertial system, the objects' information content must change when the observer's speed modifies, because also the object's surface, correlated with thermodynamic and information entropy, modifies.



OBSERVER AT REST
(INERTIAL TO THE BLACK HOLE)



OBSERVER AT HALF OF LIGHT SPEED

Figure 2. When an observer analyzes a black hole, he detects different amounts of information, according to his relative speed. Indeed, for the Beckenstein and Hawking theorems, the entropy of the black hole, and therefore the encompassed information I, is proportional to its surface.

It might be objected that the example above (an observer close at speed light) is just a "gedankenexperiment", because nobody of us may reach such speeds. However, in terms of information, the example is feasible. Take, e.g., a photons beam, traveling at speed light, that crosses a particles cloud. The beam enters in the cloud from the right and leaves it from the left. Which kind of information does the photons beam exchange with the cloud? By one side, he comes in touch with a very elongated cloud with a huge surface, due to the Einstein relativity's dictates. Therefore, he assesses a very high amount of Berkenstein and Hawking information. By another side, the photons beam, despite its relative time is hugely dilated and almost immovable, is able to detect information from the cloud, even if the latter displays instead a a very short, brief relative time. Therefore, we are in front of a paradox: an entity (the photons beam) merged in a very long, almost eternal, fixed time, is able to interact with an huge amount of information produced in a very short relative timescale (the cloud).
In sum, the entropy endowed in an object is not an invariant, static, stationary and fixed physical quantity as currently believed, but fully depends on the observer. Consequently, the amount of information encompassed in an object depends on the observer: the faster he moves in an inertial frame, the more information he gains about the object at rest being analyzed.

If information is a relative and not an absolute quantity, therefore digital physics, i.e., the family of theoretical frameworks based on the premise that our Universe is describable by information, does not allow and guarantee an objective scientific perspective. On the other hand, the above described "relativity of information", being a general feature of thermodynamic entropy and information, might be extended in order to evaluate countless different systems. To make an example, despite physical information is quite different from the one displayed in biological communication endowed with *meaning*, our findings provide a connection with the enacting subject, *de-absolutizing* the quantifiable measurements of informational physics. This makes easier (and new) the connection with the biological and social concepts of information *(21,22)*. Furthermore, the emphasis on information content's differences, that are correlated with objects shapes variations, paves the way to novel approaches to different branches, such as, e.g., computational topology and computer vision. The last, but not the least, our findings also provide an ultimate connection with Big Bang. The amount of energy (and therefore of information) detected in our Universe depends on the observer. Our current tenet: "the energy was very high at the Big Bang and decreased in the following phases of the Universe", might depend just on our cosmic framework of inertial observers at rest. Indeed, the statement might not hold true for an hypothetical observer external to our Universe: he would probably ascribe to us different values of entropy and information, depending just on his speed relative to our cosmos.

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REFERENCES

1) J.D. Bekenstein, Black holes and information theory. (2003). arXiv:quant-ph/0311049
2) F. W. Kantor, Information Mechanics. Wiley-Interscience. (1977). ISBN 10: 0471029688 / ISBN 13: 9780471029687
3) J. Schmidhuber, Algorithmic Theories of Everything. (2000). arXiv:quant-ph/0011122
4) H. Zenil, A Computable Universe: Understanding and Exploring Nature As Computation with a Foreword by Sir Roger Penrose. (2012). Singapore: World Scientific Publishing Company
5) E. T. Jaynes, Information Theory and Statistical Mechanics. Phys. Rev. 106: 620 (1957).
6) S. Lloyd, Ultimate physical limits to computation. Nature 406, 1047-1054 (2000). doi:10.1038/35023282
7) A. Marzuoli,M. Rasettim, Computing Spin Networks. Annals of Physics 318: 345–407 (2005).
8) I. Fuentes-Guridi,F. Girelli, E.Livine, Holonomic Quantum Computation in the Presence of Decoherence. Phys. Rev. Lett. 94, 020503 (2005).
9) P. A. Zizzi,Quantum computation toward quantum gravity - Gen.Rel.Grav. 33, 1305-1318 gr-qc/0008049 (2001).
10) C .F. von Weizsäcker, The Structure of Physics. Editors: Görnitz, Thomas, Lyre, Holger (Eds.). Springer Netherlands. (2006). ISBN 978-1-4020-5235-4
11) T. Görnitz, Abstract Quantum Theory and Space-Time Structure I. Ur Theory and Bekenstein-Hawking Entropy. Int. J. Theoret. Phys. 27 (5): 527–542 (1988).

12) J. A. Wheeler, Information, physics, quantum: The search for links. In Zurek, Wojciech Hubert. Complexity, Entropy, and the Physics of Information. Redwood City, California: Addison-Wesley. (1990). ISBN 9780201515091. OCLC 21482771
13) A. Einstein, ZurElektrodynamikbewegterKörper. Annalen der Physik (Berlin) (in German), Hoboken, NJ (published 10 March 2006), 322 (10), 891–921, (1905) Bibcode:1905AnP...322..891E
14) A. Einstein, Die Feldgleichungen der Gravitation. KöniglichPreussische Akademie der Wissenschaften (in German), Berlin, Germany, 844–847 (1915).
15) C. E. Shannon, A Mathematical Theory of information. The Bell System Technical Journal, 27, 379-423, 623-656 (1948).
16) J. D. Bekenstein, Black Holes and Entropy. Phys Rev D, 7(8), 2333-2346 (1973).
17) J. D. Bekenstein, Generalized second law of thermodynamics in black-hole physics. Phys Rev D, 9(12) 3292-3300 (1974).
18) Hawking, S. Information loss in black holes. Phys. Rev. D. 72 (8): 084013. (2005). doi:10.1103/PhysRevD.72.084013
19) G. 't Hooft, Dimensional Reduction in Quantum Gravity. (1993). arXiv:gr-qc/9310026
20) L. Susskind, The World as a Hologram. (1994). arXiv:hep-th/9409089
21) M. Eldred, The Digital Cast of Being: Metaphysics, Mathematics, Cartesianism, Cybernetics, Capitalism, Communication ontos, Frankfurt (2009). ISBN 978-3-86838-045-3
22) L. Floridi, Informational Realism. In Weckert, J., and Al-Saggaf, Y, eds., Computing and Philosophy Conference, vol. 37 (2004).