Aims and Intention from Mindful Mathematics:
The Encompassing Physicality of Geometric Clifford Algebra

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The emergence of sentience in the physical world - the ability to sense, feel, and respond - is central to questions surrounding the mind-body problem. Cloaked in the modern mystery of the wavefunction and its many interpretations, the search for a solid fundamental foundation to which one might anchor a model trails back into antiquity. Given the rather astounding presumption that abstractions of the mathematician might somehow inform this quest, we examine the role of geometric algebra of 3D space and 4D spacetime in establishing the foundation needed to resolve contentions of quantum interpretations. The resulting geometric wavefunction permits gut-level intuitive visualization, clarifies confusion regarding observables and observers, and provides the solid quantum foundation essential for attempts to address emergence of the phenomenon of sentience.

INTRODUCTION

The human eye [1, 2] has single photon sensitivity, renders the collapse of a single wavefunction as a visual sensation. The experiment bridges the full gap from Maxwell's equations and the wavefunction to the sentience that prompts the observer's aim and intent to report the photon's arrival. It echoes the theme of the Foundational Questions Institute's 2017 essay contest[3].

"Wandering Towards a Goal: How can mindless mathematical laws give rise to aims and intention?"

With some consideration one might conclude they can't, and with further consideration might take this to be obvious. At the least, the task requires both math and physics. If one chooses to keep focus on the mathematics, the question might better be phrased as

"What mathematical laws are most useful in modelling the physics that gives rise to aims and intention?"

This rephrasing appears to have the significant advantage of moving the hard problem, that of origins of the sentience that manifests aims and intention, from math to physics, to ground it more solidly in the tangible physical foundation of real 3D space and 4D spacetime.

In his essay on the making of the Standard Model[4], Professor Weinberg shares that

"The study of what was not understood by scientists, or was understood wrongly, seems to me often the most interesting part of the history of science."

That essay makes no mention of the geometric interpretation of Clifford algebra[5], a tool absent from the dialog of particle physicists during the decades preceeding its writing. It makes no mention of this background independent algebra of interactions of geometric primitives of physical space - interactions of the point, line, plane, and volume elements of Euclid.

What follows places this omission in historical context, presents a model of wavefunctions and their interactions based upon the Pauli and Dirac algebras of 3D space and 4D spacetime, and explores consequences of its inclusion in worldviews of physicist and philosopher. With emphasis upon the measurement problem, it examines the role of the observer in quantum mechanics and emergence of the sentience essential for aims and intention in our physical world.

GEOMETRIC CLIFFORD ALGEBRA

Figure 1 illustrates an important point - geometric algebra (and its extension into geometric calculus) claims to encompass the better part of the particle physicist's mathematical toolkit[6–8].

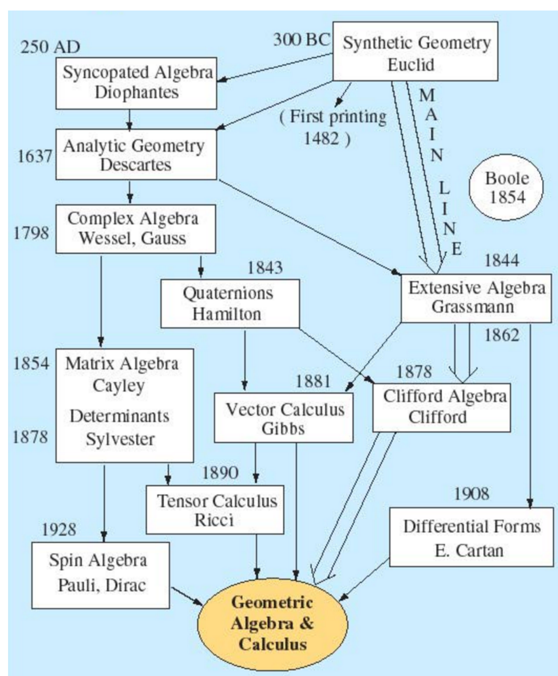

FIG. 1. Evolution of Geometric Algebra [9]

It would seem that there is a certain profundity to this, that the physicist's essential set of mathematical tools shown in the figure are a subset of the interaction algebra of fundamental geometric objects - point, line, plane, and volume elements - of our physical space.

One can argue that a possible measure of this profundity might eventually be found in the truth (or not) of the assertion that, if a mathematical model is ever to begin to approach the mind-body problem, that model will have its roots in the language of geometric algebra. Indeed, this might be taken as the point of this essay.

Clifford Algebra as originally conceived is the algebra of interactions between geometric objects[10–12]. Grassman was "...a pivotal figure in the historical development of a universal geometric calculus for mathematics and physics... He formulated most of the basic ideas and... anticipated later developments. His influence is far more potent and pervasive than generally recognized."[13]

Grassman's work lay fallow until Clifford "...united the inner and outer products into a single *geometric* product. This is associative, like Grassman's product, but has the crucial extra feature of being *invertible*, like Hamilton's quaternion algebra."[14]

While Clifford algebra attracted considerable interest, with his early death in 1879 the absence of an advocate to balance the powerful Gibbs contributed to its eventual neglect. It was "...largely abandoned with the introduction of what people saw as a more straightforward and generally applicable algebra, the *vector algebra* of Gibbs... This was effectively the end of the search for a unifying mathematical language and the beginning of a proliferation of novel algebraic systems..."[8].

Geometric algebra resurfaced, unrecognized, as algebra without geometric meaning in the Pauli and Dirac matrices of the 1920s.

Forty years passed until the original geometric intent [10–12] was rediscovered by David Hestenes, expanded, and introduced to physics [5], and yet another forty until he was awarded the 2002 Oersted Medal by the American Physical Society for "Reformulating the Mathematical Language of Physics"[6]. It remains that the power of geometric interpretation has for the most part been lost.

When realized, the algebra suggests an intuitive understanding in which all of physics is geometry[15]. According to Wheeler, "There is nothing in the world except empty curved space. Matter, charge, electromagnetism, and other fields are only manifestations of the curvature of space."[16]

However, the geometry of point particles (quarks and leptons) is static, their attributes taken to be intrinsic, internal. It is only with the external gauge fields that dynamics enters geometry and the phase coherence defining quantum system boundaries is manifested. 'Internal' coherence is geometrically inaccessible.

While string theory moves beyond dimensionless points to mode structures of 1D strings and 2D branes, it is not unreasonable to suggest that a satisfactory model will ultimately require fundamental geometric objects corresponding to the full three dimensions of physical space.

As jumping to strings led to innumerable landscapes, and yet more so with branes, it would seem that stepping up to the full 3D Pauli algebra of our physical space would yield dynamics of landscapes upon landscapes upon landscapes, burying insight under the intractable wealth of possibilities.

However, with that jump the dynamics are now those of the 4D Dirac algebra of flat Minkowski spacetime. Couldn't be simpler. Dimensions of string theory become a subset of the degrees of freedom of the model. The perspective shifts from abstract higher dimensions to interactions of objects one can visualize in 3D space. Within the more limited constraints of the Standard Model, the perspective shifts from point particles to the structure of spacetime. The perspective shifts.

THE WAVEFUNCTION

The wavefunction presented here is comprised of two constructs - geometry and fields [17]. For geometry it adopts the minimally complete 3D Pauli algebra of physical space - one scalar, three vectors, three bivector pseudovectors, and one trivector pseudoscalar - point, line, plane, and volume elements of Euclid, with the additional attribute of being orientable [18]. For fields it endows them with quantized electric and magnetic fields [19].

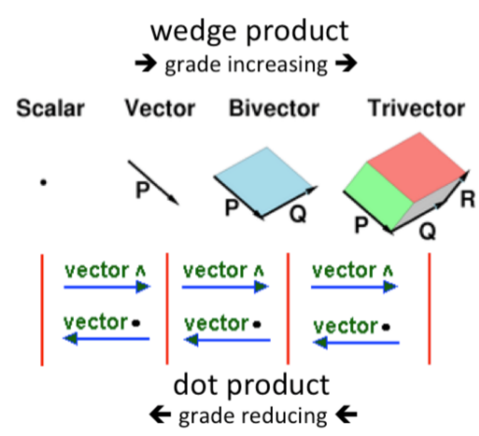

FIG. 2. Geometric algebra components in 3D Pauli algebra of space. The term grade is preferred to dimension, whose meaning is sometimes ambiguous and confused with degrees of freedom. The two products (dot and wedge or inner and outer) comprising the geometric product lower and raise the grade. Mixing of grades makes geometric algebra unique in the ability to handle geometric concepts in any dimension[20]

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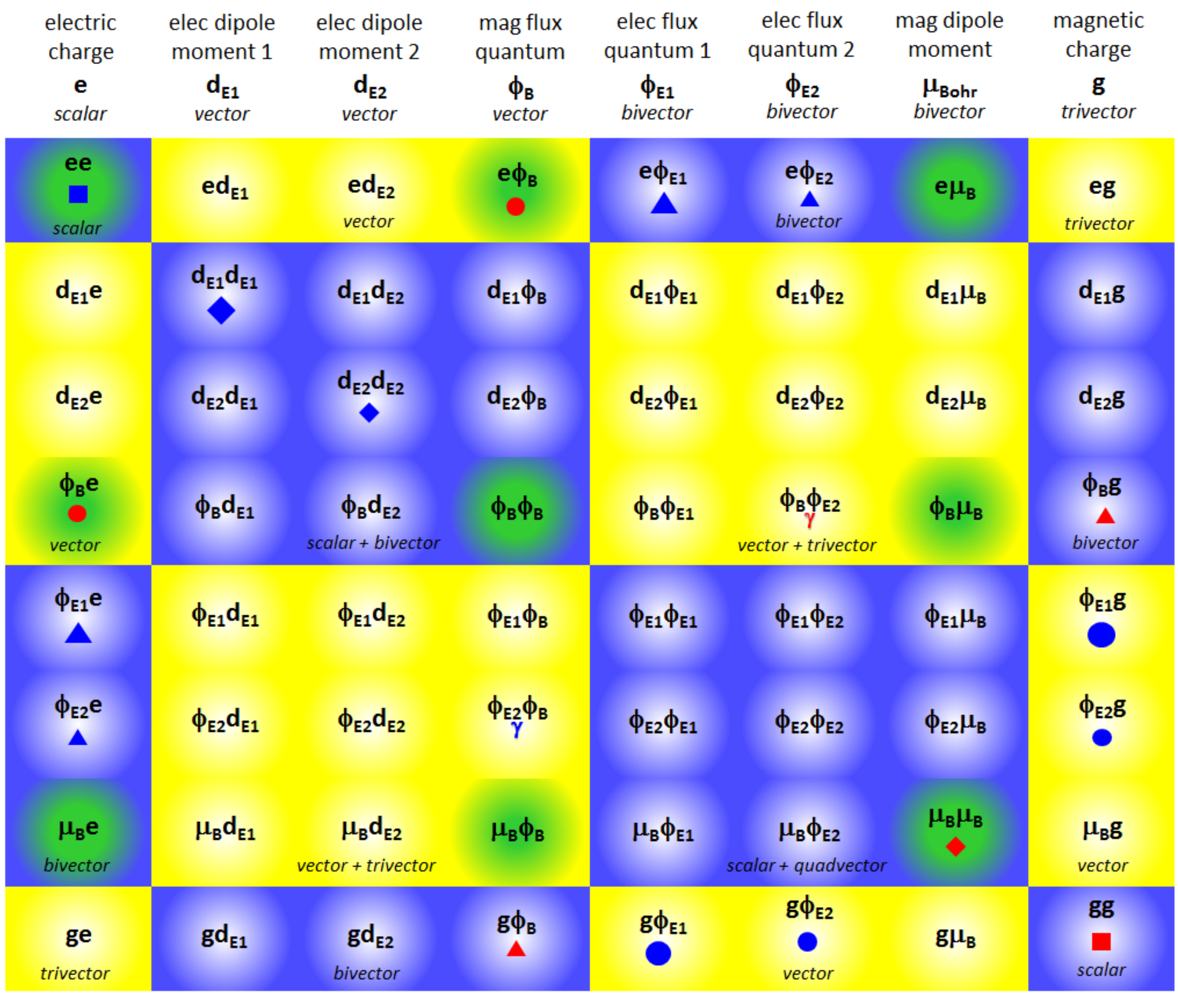

FIG. 3. **The S-matrix** As shown at top and left, a minimally complete Pauli algebra of 3D space is comprised of one scalar, three each vectors and bivectors, and one trivector. Attributing electric and magnetic fields to these fundamental geometric objects (FGOs) yields the wavefunction model [19]. In the manner of the Dirac equation, taking those at top to be the electron wavefunction suggests those at left correspond to the positron. Their geometric product generates the background independent 4D Dirac algebra of flat Minkowski spacetime, arranged in odd transition modes (yellow) and even eigenmodes (blue) by grade. Time (relative phase) emerges from the interactions. **Modes of the stable proton are highlighted in green**[37, 38].

While this wavefunction can be easily and intuitively visualized, it is not an observable[21, 22]. Observables are interactions, represented in geometric algebra by geometric products of wavefunctions. As shown in figure 3, these geometric products generate a 4D Dirac algebra of flat Minkowski spacetime. Time (relative phase) emerges from the interactions.

Topological symmetry breaking is implicit in geometric algebra. As shown in figure 2, given two vectors a and b, the geometric product ab mixes products of different dimension, or *grade*. In the product $ab = a \cdot b + a \wedge b$, two 1D vectors have been transformed into a point scalar and a 2D bivector.

"The problem is that even though we can transform the line continuously into a point, we cannot undo this transformation and have a function from the point back onto the line..." [23].

Interactions of wavefunctions are represented by the geometric product. They break topological symmetry due to this property of grade increasing operations. Topological duality[24–27] is evident in the differing geometric grades of electric and magnetic charges of figure 3. Electric charge is a scalar, magnetic charge its topological dual and the highest grade element of the Pauli algebra, the pseudoscalar (see appendix).

Knowing geometries and fields of modes shown in figure 3, one can calculate the mode impedances, an equivalent representation[28] of the complete scattering matrix description of observables of particle physics[29–38].

Absent electric and magnetic fields, the geometric model represents the vacuum impedance structure. Excitation of the lowest order mode, the Coulomb mode shown at the upper left of figure 3, yields the 377 ohm vacuum impedance seen by the photon [39].

QUANTUM INTERPRETATIONS

Interpretations of the formalism and phenomenology of quantum mechanics address distinctions between knowledge and reality, between epistemic and ontic, between how we know and what we know. It's a pursuit that straddles the boundary between philosophy and physics. There are many areas of contention, including reality and observability of the wavefunction and wavefunction collapse, determinism and the probabilistic character of wavefunction collapse, entanglement and non-locality, hidden variables, realism versus the instrumentalism of 'shut up and calculate', observer role,...[21, 22, 38].

In each of these areas quantum interpretations seek to address the same basic question - how to understand the measurement problem?[40, 41] How does one get rid of the shifty split[42] of the quantum jump[43], develop a smooth and continuous real-space visualization of state reduction dynamics?[44] What governs the flow of energy and information in wavefunction collapse?

The point here is that, unlike other interpretations, the present approach has a working electromagnetic geometric model. The wavefunction can be visualized in our 3D physical space. It is this that permits resolution of the contentions of quantum interpretations, providing the solid foundation for modeling towards sentience.

The Measurement Problem

"The measurement problem in quantum mechanics is the problem of how (or whether) wavefunction collapse occurs. The inability to observe this process directly has given rise to many different interpretations of quantum mechanics, and poses a key set of questions that each interpretation must answer."[45]

At root the confusion arises from modeling electrons and quarks as point particles. Points cannot collapse. One cannot understand the decoherence of wavefunction collapse without understanding self-coherence. Presence of the point particle in the Standard Model leaves self-coherence lost in mathematical abstraction, rather than presenting the impedance-driven coherence and decoherence of interacting electromagnetic modes visualized in 4D spacetime.

Reality and Observability of the Wavefunction

The wavefunction is comprised of fundamental geometric objects of geometric algebra. The wavefunction is not observable. Interactions of wavefunctions generates the observable S-matrix of the elementary particle spectrum[36, 37]. By conservation of energy, the reality of observable interactions would seem to require that the things that interact, the wavefunctions, are real.

Index	Interpretation	Authors	non-local?	probabilistic?	hidden variables?	wavefcn real?	wavefcn collapse?	universal wavefcn?	observer role?	unique history?
30	Objective Collapse	GRW 1986, Penrose 1989	Yes	Yes	No	Yes	Yes	No	No	Yes
30	Transactional	Cramer 1986	Yes	Yes	No	Yes	Yes	No	No	Yes
30	Quantum Impedances	Cameron & Suisse 2013	Yes	Yes	No	Yes	Yes	No	No	Yes
25	Relational	Rovelli 1994	No	Yes	No	No	Yes	No	No	agnostic
23	Quantum Logic	Birkhoff 1936	agnostic	agnostic	No	agnostic	No	No	No	Yes
17	Ithaca	Mermin 1996	No	Yes	No	No	No	No	No	No
15	Consistent Histories	Griffiths 1984	No	agnostic	No	agnostic	No	No	No	No
15	Copenhagen	Bohr & Heisenberg 1927	No	Yes	No	No	Yes	No	Yes	Yes
9	Qbism	Caves, Fuchs, Schack 2002	No	Yes	No	No	Yes	No	Yes	No
6	Orthodox	von Neumann 1932	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
-3	Many Worlds	Everett 1957	No	No	No	Yes	No	Yes	No	No
-18	de Broglie – Bohm	de Broglie 1927, Bohm 1952	Yes	No	Yes	Yes	No	Yes	No	Yes

FIG. 4. Comparison of Interpretations. The Index parameter quantifies strength of agreement between a given interpretation and the rest. Values in the Index column are calculated by adding a point for entries that agree with a given interpretation, subtracting for entries that disagree, and giving half values for agnostics. Appearance over nearly a century of growing numbers of quantum interpretations demonstrates the lack of proper physical understanding of fundamental phenomena[21].

Reality and Observability of Wavefunction Collapse

Collapse of the wavefunction follows from decoherence[46, 47], from differential phase shifts between the coupled modes of a given quantum system. The phase shifts are generated by interaction impedances of wave functions [44]. What emerges from collapses are observables. The reality of observables would seem to require that the collapse is real, however the smooth and continuous dynamics of wavefunction collapse are not observable, only the end result.

Determinism and Probabilistic Wave Function Collapse

"... the Schrodinger wave equation determines the wavefunction at any later time. If observers and their measuring apparatus are themselves described by a deterministic wave function, why can we not predict precise results for measurements, but only probabilities?" [48]

The probabilistic character of quantum mechanics follows from the fact that phase is not a single measurement observable. The measurement extracts the amplitude. The internal phase information of the coherent quantum state is lost as the wave function decoheres. For quantum mechanics to be deterministic would require phase to be a single measurement observable, a global symmetry rather than local.

Deterministic aspects are present in the sense that ensemble probabilities are determined by the impedance matches[49]. This *unobservable determinism*, as required by gauge invariance, removes some of the mystery from 'probabilistic' behavior.

Superposition of Quantum States

Investigating the meaning of the newly discovered quantum states of Heisenberg and Schrodinger, Dirac led the way in introducing state space (later to be identified with Hilbert space) to the theory. He defines states as "...the collection of all possible measurement outcomes." [50] According to Dirac,

"*The superposition that occurs in quantum mechanics is of an essentially different nature from any occurring in the classical theory*" (italics in original) [51].

What distinguishes quantum superposition from classical is linear superposition of states, of wavefunctions, as opposed to superposition of fields. The wavefunction is comprised of coupled electromagnetic modes, their fields sharing the same energy at different times. The state into which they collapse is determined by time/phase shifts of impedances they see.

Entanglement

"Entanglement is simply Schrodinger's name for superposition in a multiparticle system." [52] For wavefunctions to be entangled means they are quantum phase coherent, that they share that unobservable property.

non-Locality

Scale invariant impedances (photon far-field, quantum Hall/vector Lorentz, centrifugal, chiral, Coriolis, three body,...) are non-local. Excepting the massless photon, which has both scale invariant far-field and scale dependent near-field impedances, invariant impedances cannot do work, cannot transmit energy or information. The resulting motions are perpendicular to the applied forces. They only communicate phase, not a single measurement observable. They are the channels linking the entangled eigenstates of non-local state reduction. They cannot be shielded[53, 54]. The invariant impedances are topological. The associated potentials are inverse square.

Hidden Variables

Early on in quantum theory, the probabilistic character prompted Born[55, 56] to comment "...anybody dissatisfied with these ideas may feel free to assume that there are additional parameters not yet introduced into the theory which determine the individual event."

If one takes the 'hidden' variables to be quantum phases (not observable), then it follows that the "...additional parameters not yet introduced into the theory..." are the phase shifters, the quantum impedances.

Observer Role

Both geometric algebra and quantized impedances are background independent[57], the one 'first person', the other two body[58]. No independent observers.

There is no role for an observer within coherent quantum systems, within a wavefunction. To 'observe', to make a measurement, one must perturb the wavefunction. To extract the amplitude one must collapse the wavefunction.

One might define an observer as both the impedance that decoheres the wavefunction and that system which accepts the energy/information of the collapsing wavefunction. Or one might take the impedance for granted and consider the observer simply as that which accepts the energy. Either way the term is redundant, the conceptual artifice unnecessary and confusing, implying as it does aims and intention. The description in terms of wavefunction decoherence is adequate.

SUMMARY AND CONCLUSION

Taking the wavefunction to be comprised of the fundamental geometric objects of physical 3D space yields an approach that appears to ‘illuminate the Standard Model from within’ [17], a remarkable example of the unreasonable effectiveness of math in describing physics. It leaves one in yet further awe of the architecture and curious about the ancient architects, the old ones, and just how subtle their construction might prove to be.

It brings us back to the point of this essay, the assertion that if a mathematical model is ever to begin to approach the mind-body problem, that model will have its roots in the language of geometric algebra, the mathematical language of the physical spacetime in which we exist.

It appears that only with such an approach, with a geometric wave function, can the foundational contentions of the quantum interpretations community be resolved. It seems that such a resolution is essential to permit one to begin to coherently address the much more subtle emergence of sentience from the interactions of wavefunctions.

Nowhere in this do we find an observer unless we put one there. It brings to mind the Buddhist view of the illusion of self, of the absence of an observer in full awareness of this moment. “...grasping is not something done by the self, but rather self is something done by grasping.” [59]

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APPENDIX

Clarifying Terminology in Geometric Algebra

There is possibility for confusion in the terminology of geometric algebra.

terminology difference between Pauli and Dirac algebras					
3D Pauli	scalar	vector	pseudovector	pseudoscalar	_
	e	$\mathbf{d}_E$, $\mathbf{\phi}_B$	$\mathbf{\phi}_E$, $\mathbf{\mu}_B$	**g**	_
GA grade	0 - scalar	1 - vector	2 - bivector	3 - trivector	4 - quadvector
4D Dirac	**e**	$\mathbf{d}_E$, $\mathbf{\phi}_B$	$\mathbf{\phi}_E$, $\mathbf{\mu}_B$	**g**	**I**
	scalar	vector	bivector	pseudovector	pseudoscalar

FIG. 5. Bivector and trivector are pseudovector and pseudoscalar of the Pauli algebra. Trivector and quadvector are pseudovector and pseudoscalar of the Dirac algebra.

As shown in the figure, the highest grade element of an algebra is the pseudoscalar of that algebra. In the Dirac algebra, this results in the bivector being interposed between vector and pseudovector of the Pauli algebra, and opens possibilities for endless confusion. For this reason we favor the scalar/vector/bivector/trivector/quadvector nomenclature, but at times the use of conventional pseudovector or pseudoscalar tags seems well advised. At such times both appelations will be shown.

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