About Neutrinos Masses

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ABSTRACT – Inspired in Dimitar Valev proposal that the masses of some elementary particles are proportional to their interaction couplings evaluated at very low energies, we give estimates for the masses of the three flavors of neutrinos. A procedure analogous to the see-saw mechanism is also used to do a second estimate of the electron-neutrino mass. From the flavors neutrino masses, we get the differences in the squared eigenstates masses, used to fit the solar and atmospheric neutrinos observations.

1 – Introduction

The failure on the energetic balance of an experiment on beta decay conducted by Chadwick, led Pauli to propose in1930 the existence of an undetected particle of spin one-half. Four years later, Fermi incorporated with success this new particle in a theoretical account of the beta decay. (Please see the paper by Etienne Dreyer [1] and references cited therein).

Almost three decades later, Reines and Cowan [2] detected antineutrinos emitted from a nuclear reactor at USA.

For a long time neutrinos were considered as mass-less particles, and this feature fits nicely in the Standard Model of Particles Physics [3A, 3B]. However discovery of neutrinos oscillations by the Super-Kamiokande experiment in 1998 [4], has proven that neutrinos have a nonzero mass.

Dimitar Valev [5] has verified that the ratio between the proton and electron masses is close to the strength of the strong coupling evaluated at low energies divided by the electromagnetic coupling. Then Valev [5] extended this concept in order to estimate the mass of the electron-neutrino. It is one of the aims of this work to pursue further on this subject as a means to estimate the masses of mu and tau neutrinos.

Besides this an alternative treatment is proposed here, and the obtained neutrino mass is compared with that found by Dermisek [6], which interpreted it through see-saw mechanism [7]. We got two sets of masses for the three flavors of neutrinos, and these masses are used to compare with the observational findings of solar and atmospheric neutrinos [18, 21].

2 – Valev hypothesis in a sound basis

Bag models [8,9] seems to be a good starting point as a means to estimate the strong coupling at very low energies. The MIT bag model [8,9] considers that in nucleons, quarks and gluons are confined inside a bag. The interfaces between hadronic matter and the vacuum are represented by the walls of the bag. As was pointed out by Jaffe [8] it is possible to distinguish a certain region of space in a way consistent with the relativity theory, by submitting the frontier of that region to a constant pressure B exerted by the neighborhood vacuum over the interface of the hadron. In an idealized picture the nucleon looks similar to a gas bubble immersed in an isotropic and uniform perfect fluid. The bubble's dynamics is determined by the balance between the external pressure exerted by the fluid (vacuum) and the thermodynamic pressure of the confined gas of quarks and gluons. However the number of particles in the bag is small, rather than the great number of molecules of gas contained in the bubble.

Next we propose a potential to describe this model. First consider a confining potential represented by a harmonic oscillator. We suppose that the averaged quark kinetic energy corresponds to half of the mass-energy of the quark. Therefore the other half will account in average for its potential energy. We write (with $\hbar = c = 1$)

$$\tfrac{1}{2}\, m_q = \alpha_s / r. \qquad (1)$$

In (1) m_q is the constituent mass of the quark, r is the bubble's radius, being α_s the strong coupling. Each quark contributes to the potential with a term

$$V_q = 2\alpha_s / r. \qquad (2)$$

Meanwhile the contribution of the vacuum to the potential reads

$$V_{vac} = (4/3)\,\pi\, r^3\, B. \tag{3}$$

Taking in account that the nucleon is constituted by three quarks we finally obtain

$$V_{bag} = (4/3)\,\pi\, r^3\, B + 6\,\alpha_s/r. \tag{4}$$

Minimizing (4) with respect to r, we get at the minimum of radius R

$$V_{bag}\,(R) = 8\,\alpha_s/R = M. \tag{5}$$

In (5) we have identified the bag potential evaluated at its minimum value with the mass-energy of the nucleon.

On the other hand X. Ji [10] considered three quarks extremely relativistic with energy $3/r$ and confined by the vacuum pressure given by (3). Proceeding in an analogous way we have done before we can write

$$V_{X\text{-}Ji}\,(R) = 4/R = M. \tag{6}$$

Comparing (5) and (6), we find $\alpha_s = 0.5$. This value of the strong coupling at energy scale of the nucleon mass, gives an accepted value for the nucleon radius, namely 0.84 fm. The strength of the strong coupling here evaluated can be compared with $\alpha_s = 0.465$ (from reference [11]) and $\alpha_s = 4/9$ (from ref. [12]).

An expression for the running coupling constant of the strong interaction was obtained in [13,14], through a heuristic approach. It is given by

$$\alpha_s(\mu) = \alpha_{s0} / [1 + (\alpha_{ref} / 2) \ln(\mu / \mu_0)]. \qquad (7)$$

In (7), μ is the energy scale which probes the strong interaction and we take

$$\mu_0 = 940 \text{ MeV}, \qquad \text{and} \qquad \alpha_{ref} = 2\, \alpha_{s0}. \qquad (8)$$

Therefore (7) can be rewritten as

$$\alpha_s(\mu) = \alpha_{s0} / [1 + \alpha_{s0} \ln(\mu / \mu_0)]. \qquad (9)$$

Because it was deduced through a heuristic approach [13,14], we think that relation (9) will well represent the lower energy sector of the strong interaction. This reasoning cannot be warranted in the case of using perturbation theory.

Now let us go back to Valev [5] proposal. Thinking in terms of very low energies, we may consider the uncharged pion (π^0):- the lowest massive hadron, having mass-energy equal to 135 MeV. Therefore we write

$$m_p / m_e = 1836 = \alpha_s(135) / \alpha. \qquad (10)$$

Inserting (10) into (9), we obtain

$$1836 / 137 = \alpha_{s0} / [1 + \alpha_{s0} \ln(135 / 938)]. \qquad (11)$$

Solving equation (11) for α_{s0}, we find

$$\alpha_{s0} = 0.4967. \tag{12}$$

As can be verified in (12), the obtained value of the strong coupling at the energy scale of the nucleon is very close to 0.5, a number we get when eqs. (5) and (6) are compared. We think that this argumentation strongly supports the assumption raised by Dimitar Valev [5].

3 – Mass of electron-neutrino

The Valev proposal [5] relating mass of elementary particles to Extremely Low Energy (ELE) coupling can be written as

$$m_x / m_e = \alpha_x(0) / \alpha. \tag{13}$$

Indeed, the $\alpha_x(0)$ coupling refers to estimate the interaction involving the x-particle at lowest energy available. For the electron-neutrino case, it seems that this corresponds to the energy delivery in the neutron beta decay reaction, namely

$$n \rightarrow p + e^- + \bar{\upsilon}_e. \tag{14}$$

The energy delivery in this process reads

$$E_{\upsilon e} = m_n - m_p - m_e = .779 \text{ MeV}. \tag{15}$$

The neutron beta decay is driven by the weak interaction and its coupling strength is given by, being M_w the mass of the W-boson

$$\alpha_w(E) = (1 / \xi^2)\ \alpha\ (E / M_w)^2. \tag{16}$$

The unitarity is violated [15] when $\alpha_w = 1$, and this occurs at [16]

$$E = M_F = 1.84\ \text{TeV}. \tag{17}$$

Unitarity limit imposed to (16) yields $\xi^2 = 3.823$.

Now let us evaluate the weak coupling at E = .779 MeV. We have

$$\alpha_w(.779\ \text{MeV}) = (1 / 3.823)\ \alpha\ (.779 / 80{,}400)^2 = \alpha\ 2.4556 \times 10^{-11}. \tag{18}$$

Pursuing further we get

$$\alpha_w(.779\ \text{MeV}) / \alpha = m_{\upsilon e} / m_e = 2.4556 \times 10^{-11}. \tag{19}$$

Solving (19) for the electron-neutrino mass, we find

$$m_{\upsilon e} = 1.25 \times 10^{-5}\ \text{eV}. \tag{20}$$

This value is somewhat out the range predicted by Dermisek [6] for the mass of the lightest neutrino, namely between 5×10^{-5} eV and 5×10^{-4} eV.

The number we have found, also shows a discrepancy with the value of

2.1×10^{-4} eV, estimated by Valev [5] for the mass of the electron-neutrino.

4 – The mass of the muon-neutrino

It is seems that the best way of evaluating the mu-neutrino mass is by considering the charged pion weak decay. The reaction reads

$$\pi^- \longrightarrow \mu^- + \bar{\upsilon}_\mu. \qquad (21)$$

The energy delivery is

$$E_{\upsilon\mu} = m_\pi - m_\upsilon = 33.9 \text{ MeV}. \qquad (22)$$

Working in an analogous way we have done in the electron-neutrino case we get

$$\alpha_w(33.9 \text{ MeV}) / \alpha = m_{\upsilon\mu} / m_e = 4.65 \times 10^{-8}. \qquad (23)$$

Solving for the mass of the mu-neutrino we find

$$m_{\upsilon\mu} = 2.37 \times 10^{-2} \text{ eV}. \qquad (24)$$

5 – The mass of the tau-neutrino

Below, we show the tau lepton decay mode which delivers the lowest energy [17]

$$\tau^- \rightarrow \pi^+ + 2\,\pi^- + \pi^0 + \upsilon_\tau. \tag{25}$$

The energy delivery is

$$E_{\upsilon\tau} = m_\tau - 3m_\pi(\text{charged}) - m_\pi(\text{neutral}) = 1223\ \text{MeV}. \tag{26}$$

In an analogous way we have worked before we can write

$$\alpha_W(1223\ \text{MeV}) / \alpha = m_{\upsilon\tau} / m_e = 6.052\ \text{x}10^{-5}. \tag{27}$$

Finally solving for the mass of the tau-neutrino, we get

$$m_{\upsilon\tau} = 30.9\ \text{eV}. \tag{28}$$

6 – Neutrinos oscillations

Neutrinos oscillations are treated in a very friendly fashion in a paper by Chris Waltham [18]. An idea advanced by Pontecorvo [19] proposes that if neutrinos had small but different masses, during the propagation of a neutrino wave, an electron-neutrino could change in a muon-neutrino, due to a mixing of different flavors. Neutrinos oscillations were detected in the Super-Kamiokande collaboration experiment [4, 20A, 20B], and is considered as a proof that neutrinos have mass.

However neutrinos flavors are not eigen states of the wave function, and the mass terms inferred of the experiment are given as linear combination of these flavors. Next we propose a way to verify if the masses of neutrinos flavors we have estimated in previous sections are consistent with the observational findings.

We start by proposing a 2X2 non-diagonal symmetric matrix **M**, whose elements are associated to the mass of the neutrinos flavors. We define

$$M_{11} = p \quad , \quad M_{22} = q \quad , \quad \text{and} \quad M_{12} = M_{21} = w. \tag{29}$$

The matrix **M** can be diagonalized by imposing

$$\det [\, \mathbf{M} - \lambda \mathbf{I} \,] = 0. \tag{30}$$

Relation (30), where **I** is the identity matrix, leads to

$$(p - \lambda)\,(q - \lambda) = w^2. \tag{31}$$

Solving equation (31) for λ, we get

$$2\,\lambda_{\pm} = (q + p) \pm [(q - p)^2 + 4\,w^2]^{1/2}. \tag{32}$$

Atmospheric Neutrinos

As was pointed out by Waltham [18], the measured flux of υ_μ's is about half the expected value, while that for υ_e's is about right. There are strong indications from Super-Kamiokande that the missing υ_μ's are showing up as υ_τ's. Inspired in this idea we make the choice

$$p = m_{\upsilon e} = 1.25 \times 10^{-5} \text{ eV}, \qquad q = m_{\upsilon\mu} = 2.37 \times 10^{-2} \text{ eV}, \tag{33}$$

$$m_{\mu\tau} = 30.9 \text{ eV}, \quad \text{and} \quad w = (m_{\upsilon e}\, m_{\mu\tau})^{1/2} = 1.965 \times 10^{-2} \text{ eV}. \tag{34}$$

Inserting these numbers in eq. (32), we find

$$\lambda_+ \equiv m_1 = 3.48 \times 10^{-2} \text{ eV}, \tag{35}$$

$$\lambda_- \equiv m_2 = -\ 1.11 \times 10^{-2} \text{ eV}. \tag{36}$$

From (35) and (36), we find

$$\Delta m_A^2 = m_1^2 - m_2^2 = 1.1 \times 10^{-3} \text{ eV}^2. \tag{37}$$

The above value must be compared with 3 x10 $^{-3}$ eV ^{2} and 2.5 x10 $^{-3}$ eV ^{2}, quoted in references [18] and [21], respectively.

Solar Neutrinos

For the solar neutrino case, we assume a bold hypothesis. We suppose that in the presence of matter (the solar core) the two neutrinos flavors with the large and the small masses, namely the tau and the electron neutrinos, enter the non-diagonal matrix through the geometric average of their masses. We define

$$<m> = (m_{\upsilon e}\, m_{\mu\tau})^{1/2}. \qquad (38)$$

Next step, we take the four elements of the matrix at equal footing and write

$$p = q = w = (1/4) <m>. \qquad (39)$$

With these considerations eq. (32) takes the form

$$\lambda_{\pm} = w \pm w, \qquad (40)$$

which gives

$$\lambda_{+} = 2\, w, \qquad \text{and} \qquad \lambda_{-} = 0. \qquad (41)$$

Therefore we have

$$m_2^2 = 0, \quad \text{and} \quad m_1^2 = \lambda_+^2 = 4\, w^2 = (1/4)\, (m_{\upsilon e}\, m_{\mu\tau}). \qquad (42)$$

Eq. (42) implies

$$\Delta m_S^2 = m_1^2 = (1/4)\, (30.9 \text{ x} 1.25 \text{ x} 10^{-5}) \text{ eV}^2 = 9.66 \text{ x} 10^{-5} \text{ eV}^2. \qquad (43)$$

This value must be compared with 6 x10^{-5} eV^2 quoted in [18], and that reported in [21] equal to 7.5 x10^{-5} eV^2.

7 – Heuristic evaluation of neutrinos masses

Motivated by quark-lepton symmetry, Dermisek [6] uses the masses of the top quark and that of the grand unification theory (GUT) scale (M_x) as a means to estimate the mass of the lightest neutrino (m_υ).

In this section, inspired in Dermisek idea [6], we are going to use a heuristic approach in order to evaluate m_υ. However, besides using the mass scale M_X as threshold energy, we will also consider that the weak force represented by the W-boson is at work.

As a starting point let us consider a non-linear Dirac-like equation. We write (with $\hbar = c = 1$)

$$\partial \Psi / \partial x - \partial \Psi / \partial t = m_\upsilon \Psi - M_x \Psi^2. \qquad (44)$$

By imposing that $\partial \Psi / \partial x = \partial \Psi / \partial t = 0$, and solving for Ψ, we find after a little algebra

$$\Psi^2 = (m_\upsilon / M_x)^2. \qquad (45)$$

Meanwhile let us consider in a 4-d momentum space a hypercube which edge has a size M_w. The fraction F of the 4-volume of this cube occupied by a particle of momentum m_υ is given by

$$F = (m_\upsilon / M_w)^4. \qquad (46)$$

Now we make the requirement that $\Psi^2 = F$. Doing this and solving for m_υ, we get

$$m_\upsilon = M_w{}^2 / M_x. \qquad (47)$$

Putting numbers in (47), $M_w = 80.4$ GeV, $M_x = 2 \times 10^{16}$ GeV[6], yieds

$$m_\upsilon = 3.23 \times 10^{-4} \text{ eV}. \qquad (48)$$

This is the new estimate for the mass of the electron neutrino, an order of magnitude greater than the value previously estimated by us.

On the other hand Dermisek [6] has obtained the relation

$$m_{\upsilon 1} = [(m_{top})^2 / M_{GUT}] \, |U_{\tau 1}|^2. \qquad (49)$$

Making the identification of (47) and (49), we find

$$|U_{\tau 1}| = M_w / m_{top} = 80.4 / 172 = .467. \qquad (50)$$

According to Dermisek [6], $.20 \leq U_{\tau 1} \leq .58$.

8 – Discussion

The solar neutrino parameter given by the difference of squared masses, either in the case of theoretical calculations (eq. (43)), or in the case of fitting it to the observed neutrino oscillations (refs.[18], [21]), can be summarized as

$$\Delta m_s^2 = f\ m_{\upsilon e}\, m_{\upsilon\tau}. \tag{51}$$

We notice that f is a fraction between the limits $1/4$ and $1/(2\pi)$.

We propose that relation (51) is always valid and that a better determination of the mass of a first neutrino (the electron-neutrino for instance), leads automatically to determination of the mass of the second one. With this idea in mind we can write

$$m_{\upsilon e}\ m_{\upsilon\tau} = (m_{\upsilon e})^{new}\ (m_{\upsilon\tau})^{new}. \tag{52}$$

If we adopt $(m_{\upsilon e})^{new}$ as the value we get in the previous section (eq. (48)), we obtain

$$(m_{\upsilon\tau})^{new} = 1.20\ \text{eV}. \tag{53}$$

Estimations of the mass of the tau-neutrino can be found in references [22] and [23], and there a value for few electronvolts for it, can also be contemplated.

Finally, if we maintain the first result obtained for the muon-neutrino mass (eq.(24)), we can display two sets of masses evaluated in this paper for the three generations of neutrinos, namely:

$$\text{FIRST SET:}\quad 1.25\text{x}10^{-5}\ \text{eV};\quad 2.37\text{x}10^{-2}\ \text{eV};\quad 30.9\ \text{eV}. \tag{54}$$

$$\text{SECOND SET:}\quad 3.23\text{x}10^{-4}\ \text{eV};\quad 2.37\text{x}10^{-2}\ \text{eV};\quad 1.20\ \text{eV}. \tag{55}$$

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