Does Thomas-Wigner rotation show the fallacy of „Lorentz rotation”?

JÓZEF RADOMAŃSKI
e-mail: radomanski@wikakwa.pl
48-300 Nysa, ul. Bohaterów Warszawy 9, Poland

Abstract

As a result of the superposition of non-collinear relativistic velocities, apart from the resultant velocity, a component of turn is obtained which is called the Thomas-Wigner rotation. The paper discusses the Lorentz transformation using the paravector calculus. It has been shown that any „Lorentz rotation” is a combination of real velocity and a Euclidean rotation, and that as a result of the superposition of appropriately selected velocities any object can rotate in place, which may indicate the fallacy of the idea of the „Lorentz rotation”.

Keywords: *Lorentz transformation, Maxwell equations, Thomas-Wigner rotation, complex space-time, Special Relativity, paravectors*

Introduction

This article is the third one on paravectors and, like the previous ones, refers to the work of William Baylis, a professor of theoretical physics at the University of Windsor in Canada. The current work is written in the same way as the previous ones, so to understand it **it's necessary to carefully read the article** *Algebra of paravectors* [1]. To show that the formalism is correct and so that the reader gets accustomed to it, the article starts with showing a convergence of our and W.Baylis' results. However, our conclusions lead to doubts about the correctness of the transformation called by W.Baylis „Lorentz rotation” or to the correctness of the Lorentz transformation (if both transformations are equivalent?).

It is commonly known that the superposition of relativistic velocities gives the effect of rotation, which is called Thomas-Wigner rotation. Studying the Baylisean version of the Lorentz transformation we came to the detail, which probably had not been noticed by W.Baylis: Giving duly selected straight boosts to his frame, the observer can... turn in place. This turn is not a movement of rotation. That is, in consequence of superposition of duly selected boosts we receive a stationary frame but rotated in relation to the original one. This raises our serious doubts as to the correctness of the transformation $\mathbb{X}' = \Lambda\mathbb{X}\Lambda^*$.

In the article *Algebra of paravectors* [3] paravectors are presented in such a way so as to show their similarity to vectors. Paravectors with relations of summation and multiplication form a division ring, so they have the properties of integers. A set of paravectors creates a linear space over the field of complex numbers. After the introduction of the concept of the integrated product they become like vectors, because they create an „almost-unitar” space. This is not the unitary space in the concept known from linear algebra, because the positive-definiteness does not apply to the scalar product of paravectors. The result is such that there is no metric in paravector space. However, there is a relation having properties of the square of norm: it is the determinant of paravector. Although for the proper and singular paravectors the module can be defined (not to be confused with the norm), yet a set of these paravectors does not create a unitary space, either because the result of summation is not closed in their set.

The second publication *Four-divergence as a paravector operator* [4] presents transformation formulas of expressions containing spatio-temporal differentiation operators: 4-gradient and a 4-divergence. Using these formulas the invariance of wave equation under the paravector orthogonal transformations has been proved and the transformation of selected differential equations under rotation of the coordinate system has been shown.

Paravectors are generally described by multi-vector or tensor calculus so at the beginning the reader may be a little confused due to the formalism used by us. However, it is not complicated because our notation bases on the matrix calculus and has one great advantage compared with classic formalisms: it clearly separates the scalar components of spatial ones, making reasoning and self-control over the calculations very easy. The simplicity of calculations shows that the paravector calculus is suitable for use in a natural way in the field theory.

Both [3] and [4] articles refer to purely mathematical matters. This work explores themes of physics, therefore it is necessary to put the assumption for the used units system. To simplify the formulas (involving among other things, getting rid of the material constants appearing in Maxwell's equations) we assume as follows:

1. Physical issues are dealt with in a vacuum.
2. Formulas of electricity theory are given in Natural Units system (NU) i.e. Linear velocities are dimensionless quantities, and their values are relative to the speed of light. In NU system the module of the speed of light is equal to 1, namely the transition to units of Heaviside-Lorentz (H-L) follows by the replacement of:

	units of NU system	units of H-L system
linear velocity	v	v/c
time	t	ct
current density	j	j/c
paravector of light speed	(1,**c**), where |**c**|=1	(c,**c**)

3. The discussed problems concern inertial systems only.
4. In the whole work we sought to retain the rule that the real sizes (numbers, vectors, paravectors) are denoted by Roman characters and the complex ones by Greek ones. This does not refer to physical quantities whose letter designations are commonly used in the literature.

1 Lorentz transformation

W.Baylis describes the Lorentz transformation as:

$$\mathbb{X}' = \Lambda \mathbb{X} \Lambda^* \tag{1}$$

where Λ is an orthogonal paravector, and 4-vector $\mathbb{X} = \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix}$ belongs to real space-time.

The above transformation is called *Lorentz rotation*, which is the ordinary Euclidean rotation when the scalar part of paravector Λ is a real number, and the spatial part of paravector is an imaginary vector. When both scalar and vector parts are real, Lorentz rotation is the boost. Generally, Lorentz rotation (Lorentz transformation) is described by complex orthogonal paravector. Although Λ is represented by complex paravector, Lorentz rotation is an internal operation in the set R^4:

$$\begin{gathered}\Lambda \mathbb{X} \Lambda^* = \frac{\Gamma}{|\Gamma|} \mathbb{X} \frac{\Gamma^*}{|\Gamma|} = \frac{1}{\alpha^2 - \beta^2} \begin{bmatrix} \alpha \\ \boldsymbol{\beta} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} \begin{bmatrix} \alpha^* \\ \boldsymbol{\beta}^* \end{bmatrix} = \\ = \frac{1}{a^2 - b^2 + c^2 - d^2} \begin{bmatrix} a + id \\ \mathbf{b} + i\mathbf{c} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} \begin{bmatrix} a - id \\ \mathbf{b} - i\mathbf{c} \end{bmatrix} = \\ = \frac{1}{\det\Gamma} \begin{pmatrix} (a^2 + b^2 + c^2 + d^2)\Delta t + 2(a\mathbf{b} + \mathbf{c}d - \mathbf{b} \times \mathbf{c})\Delta \mathbf{x} \\ 2(a\mathbf{b} + \mathbf{c}d + \mathbf{b} \times \mathbf{c})\Delta t + (a^2 - b^2 - c^2 + d^2)\Delta \mathbf{x} + 2(a\mathbf{c} + d\mathbf{b}) \times \Delta \mathbf{x} + 2[\mathbf{b}(\mathbf{b}\Delta \mathbf{x}) + \mathbf{c}(\mathbf{c}\Delta \mathbf{x})] \end{pmatrix}.\end{gathered} \tag{2}$$

From the above it follows that if $\mathbb{X} \in R^4$, then $\mathbb{X}' \in R^4$.

Now, we will analyse the transformation of the simplest paravector - the time interval. The image of time interval (stationary point in the original frame) after Lorentz transformation is the space-time interval (a moving point in the primed frame):

$$\frac{1}{\alpha^2-\beta^2}\begin{bmatrix} \alpha \\ \beta \end{bmatrix}\begin{pmatrix} \Delta t \\ 0 \end{pmatrix}\begin{bmatrix} \alpha^* \\ \beta^* \end{bmatrix}=\begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix}, \tag{3}$$

which means that the observed point shifted to the distance $\Delta\mathbf{x}'$ in the time $\Delta t'$ at a speed of $\mathbf{v}$ such that:

$$\Delta t\frac{(a^2+b^2+c^2+d^2)}{a^2-b^2+c^2-d^2}\begin{bmatrix} 1 \\ \frac{2(a\mathbf{b}+\mathbf{c}d+\mathbf{b}\times\mathbf{c})}{a^2+b^2+c^2+d^2} \end{bmatrix}=\Delta t'\begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix}, \tag{4}$$

Hence the speed is

$$\mathbf{v}=\frac{2(a\mathbf{b}+\mathbf{c}d+\mathbf{b}\times\mathbf{c})}{a^2+b^2+c^2+d^2} \qquad \text{and} \qquad \Delta t'=\frac{(a^2+b^2+c^2+d^2)}{a^2-b^2+c^2-d^2}\Delta t \tag{5}$$

Value $\frac{a^2+b^2+c^2+d^2}{a^2-b^2+c^2-d^2}$ is the so-called time-dilation factor $\gamma=1/\sqrt{1-v^2}$, which is shown below

$$\frac{1}{\sqrt{1-v^2}}=\frac{1}{\sqrt{1-\left(2\frac{a\mathbf{b}+\mathbf{c}d+\mathbf{b}\times\mathbf{c}}{a^2+b^2+c^2+d^2}\right)^2}}=\frac{a^2+b^2+c^2+d^2}{a^2-b^2+c^2-d^2} \tag{6}$$

Again for the formalities we check that the resulting speed is always less than the speed of light.

$$v^2=\frac{4(a\mathbf{b}+\mathbf{c}d+\mathbf{b}\times\mathbf{c})^2}{(a^2+b^2+c^2+d^2)^2}=4\frac{a^2\mathbf{b}^2+\mathbf{c}^2d^2+(\mathbf{b}\times\mathbf{c})^2+2a\mathbf{bc}d}{(a^2+b^2+c^2+d^2)^2} \tag{7}$$

Since $(\mathbf{b}\times\mathbf{c})^2=b^2c^2-(\mathbf{bc})^2$ and $\mathbf{bc}=ad$ (the assumption that the paravector Λ is proper one) we obtain

$$v^2=4\frac{a^2b^2+c^2d^2+b^2c^2+a^2d^2}{(a^2+b^2+c^2+d^2)^2}=4\frac{(a^2+c^2)(b^2+d^2)}{(a^2+b^2+c^2+d^2)^2}=$$

$$=\frac{[(a^2+c^2)+(b^2+d^2)]^2-[(a^2+c^2)-(b^2+d^2)]^2}{(a^2+b^2+c^2+d^2)^2}=1-\frac{(a^2-b^2+c^2-d^2)^2}{(a^2+b^2+c^2+d^2)^2}<1$$

Now, we will show that the transformation (1) does not change the spatial components of the transformed vectors that are perpendicular to the direction of movement. For simplicity, we assume that Λ is a real paravector ($c=0$, $d=0$), or we have $\mathbf{v}\parallel\mathbf{b}$. We check what is the image of vector $\Delta\mathbf{x}$ if $\Delta\mathbf{x}\perp\mathbf{b}$.

$$\Lambda\Delta X\Lambda^*=\frac{1}{a^2-b^2}\begin{pmatrix} (a^2+b^2)\Delta t \\ 2a\mathbf{b}\Delta t+(a^2-b^2)\Delta\mathbf{x} \end{pmatrix}=\begin{pmatrix} \frac{a^2+b^2}{a^2-b^2}\Delta t \\ \frac{2a\mathbf{b}}{a^2-b^2}\Delta t+\Delta\mathbf{x} \end{pmatrix}=\begin{pmatrix} \Delta t' \\ \Delta\mathbf{x}' \end{pmatrix} \tag{8}$$

As can be seen, the transformation (1) does not change the spatial components perpendicular to the direction of movement (when $\Delta\mathbf{x}\perp\mathbf{b}$), so this is a classic Lorentz transformation.

We have yet to check what non-relativistic approximation of Lorentz transformation looks like. To simplify the calculation, in the same way as before, we assume that the paravector Λ is a real one ($c=0$ and $d=0$)

$$\begin{pmatrix} \Delta t' \\ \Delta\mathbf{x}' \end{pmatrix}=\frac{1}{a^2-b^2}\begin{bmatrix} a \\ \mathbf{b} \end{bmatrix}\begin{pmatrix} \Delta t \\ 0 \end{pmatrix}\begin{bmatrix} a \\ \mathbf{b} \end{bmatrix} \tag{9}$$

which gives

$$\begin{pmatrix} \Delta t' \\ \Delta\mathbf{x}' \end{pmatrix}=\frac{1}{a^2-b^2}\begin{bmatrix} a \\ \mathbf{b} \end{bmatrix}\begin{bmatrix} a \\ \mathbf{b} \end{bmatrix}\begin{pmatrix} \Delta t \\ 0 \end{pmatrix}=\frac{a^2+b^2}{a^2-b^2}\begin{bmatrix} 1 \\ \frac{2a\mathbf{b}}{a^2+b^2} \end{bmatrix}\begin{pmatrix} \Delta t \\ 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{v}=\frac{2\mathbf{b}/a}{1+(\mathbf{b}/a)^2} \tag{10}$$

For non-relativistic velocities we have $a\gg b$, whence in H-L system we get

$$\begin{pmatrix} c\Delta t' \\ \Delta\mathbf{x}' \end{pmatrix}=\begin{bmatrix} 1 \\ \mathbf{v}/c \end{bmatrix}\begin{pmatrix} c\Delta t \\ 0 \end{pmatrix}=\begin{pmatrix} c\Delta t \\ \mathbf{v}\Delta t \end{pmatrix} \tag{11}$$

what is the Galilean transformation:

$$\begin{aligned}\Delta t' &= \Delta t\\ \mathbf{x}' &= \mathbf{v}\Delta t + \mathbf{x}'_0\end{aligned}$$

Since any orthogonal paravector can be represented as the product of three real orthogonal paravectors, then the general conclusion is true:

Conclusion 1.1. For non-relativistic velocities a Lorentz transformation becomes a Galilean transformation.

Conclusion 1.2. The Lorentz rotation (1) is an orthogonal transformation, ie. it preserves the scalar product of paravectors (def. 2.2.3 [3]).

Proof. Under the transformation (1) the scalar product of paravectors has a form

$$\left\langle A'_1, A'_2\right\rangle = \left[\Lambda A_1 \Lambda^* (\Lambda A_2 \Lambda^*)^-\right]_S = \left[\Lambda A_1 A_2^- \Lambda^-\right]_S = \langle A_1, A_2\rangle$$

Since Λ is an orthogonal paravector, so $\Lambda(A_1 A_2^-)\Lambda^-$ is the rotation of the integrated product, and as we know from Theorem 2.3.6 [3], the rotation does not change the scalar. □

The last request is equivalent with the statement that the Lorentz transformation does not change the shape of objects/phenomena in space-time.

Everything confirms the compliance of our calculation of William Baylisean Algebra of Physical Space (APS).

2 Maxwell equations.

Now, we will look at the Maxwell equations closer. Based on the article [4], differentiation operators (4-gradient ∂^- and 4-divergence ∂) under transformation $\mathbb{X}' = \Lambda\mathbb{X}\Lambda^*$ will change to:

$$\begin{aligned}\partial A(\mathbb{X}) &= \Lambda^* \partial' \Lambda A(\Lambda^- \mathbb{X}' \Lambda^{*-}) &(12)\\ \partial^- A(\mathbb{X}) &= \Lambda^- \partial'^- \Lambda^{*-} A(\Lambda^- \mathbb{X}' \Lambda^{*-}) &(13)\end{aligned}$$

We transform equations of electrostatics according to the first identity

$$\begin{bmatrix} \frac{\partial}{\partial t} \\ \nabla \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{E}(\mathbb{X}) \end{pmatrix} = \begin{pmatrix} \rho(\mathbb{X}) \\ 0 \end{pmatrix} \quad \Leftrightarrow \quad \begin{bmatrix} \frac{\partial}{\partial t'} \\ \nabla' \end{bmatrix} \Lambda \begin{pmatrix} 0 \\ \mathbf{E}(\Lambda^- \mathbb{X}' \Lambda^{*-}) \end{pmatrix} = \Lambda^{*-} \begin{pmatrix} \rho(\Lambda^- \mathbb{X}' \Lambda^{*-}) \\ 0 \end{pmatrix} \tag{14}$$

By Theorem 2.5 [4], we can right-side multiply the resulting equation by any orthogonal paravector, for example Λ^-

$$\begin{bmatrix} \frac{\partial}{\partial t'} \\ \nabla' \end{bmatrix} (\Lambda \begin{pmatrix} 0 \\ \mathbf{E}(\Lambda^- \mathbb{X}' \Lambda^{*-}) \end{pmatrix} \Lambda^-) = \Lambda^{*-} \begin{pmatrix} \rho(\Lambda^- \mathbb{X}' \Lambda^{*-}) \\ 0 \end{pmatrix} \Lambda^-, \tag{15}$$

hence, on the left side of the above equation we have

$$\begin{pmatrix} 0 \\ \mathbf{E}' + i\mathbf{B}' \end{pmatrix} = \frac{1}{a^2 - b^2 + c^2 - d^2} \begin{bmatrix} a + id \\ \mathbf{b} + i\mathbf{c} \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{E} \end{pmatrix} \begin{bmatrix} a + id \\ -\mathbf{b} - i\mathbf{c} \end{bmatrix}, \tag{16}$$

and on the right side

$$\begin{pmatrix} \rho' \\ -\mathbf{j}' \end{pmatrix} = \frac{\rho}{a^2 - b^2 + c^2 - d^2} \begin{bmatrix} a - id \\ -\mathbf{b} + i\mathbf{c} \end{bmatrix} \begin{bmatrix} a + id \\ -\mathbf{b} - i\mathbf{c} \end{bmatrix} \tag{17}$$

The equation (15) is a system of Maxwell's equations in the „primed" space.

$$\begin{bmatrix} \frac{\partial}{\partial t'} \\ \nabla' \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{E}' + i\mathbf{B}' \end{pmatrix} = \begin{pmatrix} \rho' \\ -\mathbf{j}' \end{pmatrix} \tag{18}$$

or

$$\begin{array}{ll} \nabla'\mathbf{E}' = \rho' & \nabla'\mathbf{B}' = 0 \\ \nabla' \times \mathbf{B}' = \frac{\partial \mathbf{E}'}{\partial t'} + \mathbf{j}' & \nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'} \end{array} \tag{19}$$

By identity (13) we obtain the conditions for the field to meet the wave equation.

$$\left[\begin{array}{c} \frac{\partial}{\partial t} \\ -\nabla \end{array}\right]\left(\begin{array}{c} \varphi(\mathbb{X}) \\ 0 \end{array}\right) = \left(\begin{array}{c} 0 \\ \mathbf{E}(\mathbb{X}) \end{array}\right) \quad \Leftrightarrow \quad \left[\begin{array}{c} \frac{\partial}{\partial t'} \\ -\nabla' \end{array}\right]\Lambda^{*-}\left(\begin{array}{c} \varphi(\Lambda^{-}\mathbb{X}'\Lambda^{*-}) \\ 0 \end{array}\right) = \Lambda\left(\begin{array}{c} 0 \\ \mathbf{E}(\Lambda^{-}\mathbb{X}'\Lambda^{*-}) \end{array}\right) \tag{20}$$

Just as previously we multiply right-side the received equation by Λ^{-}, and we obtain

$$\left[\begin{array}{c} \frac{\partial}{\partial t'} \\ -\nabla' \end{array}\right](\Lambda^{*-}\left(\begin{array}{c} \varphi(\Lambda^{-}\mathbb{X}'\Lambda^{*-}) \\ 0 \end{array}\right)\Lambda^{-}) = \Lambda\left(\begin{array}{c} 0 \\ \mathbf{E}(\Lambda^{-}\mathbb{X}'\Lambda^{*-}) \end{array}\right)\Lambda^{-}. \tag{21}$$

which gives

$$\left(\begin{array}{c} \varphi' \\ \mathbf{A}' \end{array}\right) = \Lambda^{*-}\left(\begin{array}{c} \varphi \\ 0 \end{array}\right)\Lambda^{-} \qquad \text{and} \qquad \left(\begin{array}{c} 0 \\ \mathbf{E}' + i\mathbf{B}' \end{array}\right) = \Lambda\left(\begin{array}{c} 0 \\ \mathbf{E} \end{array}\right)\Lambda^{-} \tag{22}$$

So, the equation (21) can be written

$$\left[\begin{array}{c} \frac{\partial}{\partial t'} \\ -\nabla' \end{array}\right]\left(\begin{array}{c} \varphi' \\ -\mathbf{A}' \end{array}\right) = \left(\begin{array}{c} 0 \\ \mathbf{E}' + i\mathbf{B}' \end{array}\right) \tag{23}$$

or

$$\frac{\partial \varphi'}{\partial t'} + \nabla'\mathbf{A}' = 0 \tag{24}$$

$$-\frac{\partial \mathbf{A}'}{\partial t'} - \nabla'\varphi' = \mathbf{E}' \tag{25}$$

$$\nabla' \times \mathbf{A}' = \mathbf{B}' \tag{26}$$

The results confirm the compatibility of our considerations and the theory applicable over a century, because the field is transformed in such a way that in Maxwell's equations we obtain the current density and the Lorenz gauge condition is maintained. The field is transformed by rotation and by Theorem (2.3.6) [3] we know that rotation does not change the scalar component of the rotated paravector which is equivalent to the Lorenz gauge invariance. Since this is not a Euclidean rotation, the real vector of the electric field becomes a complex vector, whose imaginary component is interpreted as the magnetic field. Assuming that paravector Λ is the real one (for the sake of simplicity), by equations (16) (17) we get:

$$\mathbf{E}' = \frac{a^2+b^2}{a^2-b^2}\mathbf{E} - 2\frac{\mathbf{b}(\mathbf{b}\mathbf{E})}{a^2-b^2} + 2ia\frac{\mathbf{b}\times\mathbf{E}}{a^2-b^2} = \frac{a^2+b^2}{a^2-b^2}(\mathbf{E} + i\mathbf{v}\times\mathbf{E}) - 2\frac{\mathbf{b}(\mathbf{b}\mathbf{E})}{a^2+b^2} \tag{27}$$

$$\rho' = \frac{a^2+b^2}{a^2-b^2}\rho \qquad \text{oraz} \qquad \mathbf{j}' = \mathbf{v}\rho, \qquad \text{gdzie} \qquad \mathbf{v} = \frac{2a\mathbf{b}}{a^2+b^2} \tag{28}$$

For unrelativistic velocities we have $b^2 \ll a^2$, so the time-dilation factor is equal to 1 and the last component of the equation (27) disappears.

Now we calculate the fields that appear around an infinite straight wire which always flows constant current. The intensity of the electric field of the load placed at a distance of $\mathbf{r}$ is

$$\mathbf{E} = \frac{Q\mathbf{r}}{r^3} = \frac{\mathbf{r}}{r^3}\rho(\mathbf{s}d\mathbf{l}) \tag{29}$$

Since always the electrical charges are distributed equally along an infinite straight wire, so the electric field of negative charges at a point of $\mathbf{h}$ of the wire equals:

$$\mathbf{E}_{-} = s\rho_{-}\int_{-\infty}^{+\infty}\frac{\mathbf{l}+\mathbf{h}}{\left(\sqrt{l^2+h^2}\right)^3}dl, \qquad \text{because } \mathbf{r} = \mathbf{l}+\mathbf{h}, \mathbf{l}\perp\mathbf{h} \text{ and } \mathbf{s}\parallel\mathbf{l} \tag{30}$$

If electrons move along the wire at a speed $\mathbf{v} = \frac{2\mathbf{b}}{a^2+b^2} \parallel \mathbf{l}$ then by equation (22) the vectors of the electric field and magnetic induction are the spatial components of 4-vector:

$$E_- = \frac{\rho_- s}{a^2-b^2} \int_{-\infty}^{+\infty} \begin{bmatrix} a \\ \mathbf{b} \end{bmatrix} \begin{pmatrix} 0 \\ \frac{\mathbf{l}+\mathbf{h}}{\left(\sqrt{l^2+h^2}\right)^3} dl \end{pmatrix} \begin{bmatrix} a \\ -\mathbf{b} \end{bmatrix} = \begin{pmatrix} 0 \\ \mathbf{E}+i\mathbf{B} \end{pmatrix} \tag{31}$$

$$\mathbf{E}+i\mathbf{B} = \frac{\rho s}{a^2-b^2} \int_{-\infty}^{+\infty} \frac{\left(a^2+b^2\right)(\mathbf{l}+\mathbf{h}) - 2\mathbf{b}(\mathbf{bl}) + 2i\mathbf{b}\times\mathbf{h}}{\left(\sqrt{l^2+h^2}\right)^3} dl =$$

$$= \frac{a^2+b^2}{a^2-b^2}\left(-\left.\frac{\rho s \mathbf{l}_1}{\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} + \left.\frac{\rho s \mathbf{h} l}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} + \frac{\mathbf{b}b}{a^2+b^2} \left.\frac{2\rho s}{\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} + 2i\frac{\mathbf{b}\times\mathbf{h}}{a^2+b^2} \left.\frac{\rho s l}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} \right) \tag{32}$$

where before the parenthesis we have the dilation coefficient. Vector $\mathbf{l}_1$ is an unitary vector whose direction is in line with the direction of wire so $\mathbf{l}_1 = \mathbf{b}/b$. The first and third integrals reset, and since $\frac{2\mathbf{b}\rho s}{a^2+b^2} = \mathbf{j}$ is the current density then we get

$$\mathbf{E}+i\mathbf{B} = \frac{a^2+b^2}{a^2-b^2}\left(\left.\frac{2\rho s \mathbf{h} l}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} + i \left.\frac{\mathbf{j}\times\mathbf{h} l}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} \right) \tag{33}$$

The real component is the electric field of moving electrons, while the imaginary component is the magnetic field proportional to the current flowing in the wire.

$$\mathbf{E}_- = \frac{a^2+b^2}{a^2-b^2} \left.\frac{2\rho s \mathbf{h} l}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} = \frac{a^2+b^2}{a^2-b^2}\frac{2\rho s \mathbf{h}}{h^2} \tag{34}$$

$$\mathbf{B} = \frac{a^2+b^2}{a^2-b^2} \left.\frac{\mathbf{j}\times\mathbf{h} l}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} = \frac{a^2+b^2}{a^2-b^2}\frac{2\mathbf{j}\times\mathbf{h}}{h^2} \tag{35}$$

The second formula shows the Biot-Savart law, so it is another confirmation of the correctness of the theory created by professor Baylis. In the wire there is the same number of positive and negative charges, but only negative ones move. Integrating formula (27) by the wire but getting positive charges only, we get

$$\mathbf{E}_+ = \left.\frac{\rho_+ s l \mathbf{h}}{h^2\sqrt{l^2+h^2}}\right|_{-\infty}^{+\infty} = \frac{2\rho_+ s \mathbf{h}}{h^2} \tag{36}$$

Since the electrons in the wire move at the non-relativistic speed then there is no resultant electric field.

Using the theorems 2.3 - 2.5 in the article [4] it was demonstrated that the wave equation

$$\begin{bmatrix} \frac{\partial}{\partial t} \\ \nabla \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial t} \\ -\nabla \end{bmatrix} \begin{pmatrix} \varphi(\mathbb{X}) \\ 0 \end{pmatrix} = \rho(\mathbb{X}) \tag{37}$$

transforms to

$$\begin{bmatrix} \frac{\partial}{\partial t'} \\ \nabla' \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial t'} \\ -\nabla' \end{bmatrix} \Lambda^{*-} \begin{pmatrix} \varphi\left(\Lambda^- \mathbb{X}' \Lambda^{*-}\right) \\ 0 \end{pmatrix} \Lambda^- = \Lambda^{*-} \rho\left(\Lambda^- \mathbb{X}' \Lambda^{*-}\right) \Lambda^- \tag{38}$$

From the considerations above follows that Baylisean version of the Lorentz transformation does not look bad:

- is compatible with the Lorentz transformation
- is compatible with the valid rules of the theory of electricity and magnetism, that is:
  - by transforming the scalar potential there appears the vector potential
  - by transforming the charge density there appears the current density
  - the Lorentz gauge condition is invariant
- Although the Lorentz rotation is a complex transformation, it transforms the real space-time at itself.

Unfortunately, some details cause concern:

1. In the formula (15) there is a discrepancy between the direction of charges movement (coordinate of a charge $\Lambda^{-}\mathbb{X}\Lambda^{*-}$), and the direction of electric current ($\Lambda^{*-}\rho\Lambda^{-}$).
2. As a result of the submission of inertial traversing movements we can get a rotation in place (which will be subject of the next chapter).

3 From attempts of realignment of unitary paravector to critique of Lorentz rotation.

In the first chapter, we assumed that paravectors representing boosts are real just to simplify the calculations and to make paravectors more intuitive to interpret. However, by submitting real paravectors we get complex ones. The question arises: Is there a way to bring any complex paravector representing boost into a real one? This question can be put another way: Does there exist a real orthogonal paravector L for any orthogonal paravector Λ such that $\Lambda\mathbb{X}\Lambda^{*} = L\mathbb{X}L$? Although the answer is negative, then by checking this hypothesis it appeared that another theorem was true:

Theorem 3.1. For any orthogonal paravector Λ there is an orthogonal real one L such that the product ΛL^{-} (or $L^{-}\Lambda$) is an orthogonal special paravector.

Proof.

A special paravector is $R = \frac{1}{\sqrt{r^2+s^2}}\begin{bmatrix} r \\ i\mathbf{s}\end{bmatrix}$, and a real paravector is $L = \frac{1}{\sqrt{k^2-l^2}}\begin{bmatrix} k \\ \mathbf{l}\end{bmatrix}$.

We need to find such a paravector L that

$$\frac{1}{\sqrt{r^2+s^2}}\begin{bmatrix} r \\ i\mathbf{s}\end{bmatrix} = \frac{1}{\sqrt{k^2-l^2}}\begin{bmatrix} k \\ -\mathbf{l}\end{bmatrix}\frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c}\end{bmatrix}, \tag{39}$$

that is, as a result of the multiplication we must get an imaginary scalar component equal to zero and zero as the real vector, which gives the conditions:

$$\begin{cases} kd-\mathbf{cl} &= 0 \\ k\mathbf{b}-a\mathbf{l}+\mathbf{l}\times\mathbf{c} &= 0 \end{cases} \tag{40}$$

The above system of equations can be presented by one paravector equation

$$\begin{bmatrix} 0 \\ \mathbf{l}\end{bmatrix}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix} = k\begin{bmatrix} id \\ \mathbf{b}\end{bmatrix} \tag{41}$$

Since paravector $\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}$ is proper one, choosing number k as the parameter we calculate the vector $\mathbf{l}$

$$\mathbf{l} = k\frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2}, \tag{42}$$

or the paravector

$$L_0 = k\begin{bmatrix} 1 \\ \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2}\end{bmatrix}^{-} \quad \text{and} \quad |L_0| = k\sqrt{\frac{a^2-b^2+c^2-d^2}{a^2+c^2}}. \tag{43}$$

After substituting the above results to (39) and using the conditions which proper paravectors meet ($ad = \mathbf{bc}$), we obtain

$$R = \frac{\sqrt{a^2+c^2}}{a^2-b^2+c^2-d^2}\begin{bmatrix} 1 \\ \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2}\end{bmatrix}^{-}\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c}\end{bmatrix} = \frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix} \tag{44}$$

We denote the equation (39) in the form

$$\frac{1}{\sqrt{a^2+c^2}}\left[\begin{array}{c} a \\ i\mathbf{c} \end{array}\right]=\frac{1}{\sqrt{1-\frac{b^2+d^2}{a^2+c^2}}}\left[\begin{array}{c} 1 \\ \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2} \end{array}\right]^{-}\frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\left[\begin{array}{c} a+id \\ \mathbf{b}+i\mathbf{c} \end{array}\right], \tag{45}$$

or

$$R=\frac{1}{\sqrt{a^2+c^2}}\left[\begin{array}{c} a \\ i\mathbf{c} \end{array}\right] \quad \text{and} \quad L=\frac{1}{\sqrt{1-\frac{b^2+d^2}{a^2+c^2}}}\left[\begin{array}{c} 1 \\ \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2} \end{array}\right] \tag{46}$$

If we reverse the order of paravectors ($R=\Lambda L$) and repeat the above proof then we get

$$R=\frac{1}{\sqrt{a^2+c^2}}\left[\begin{array}{c} a \\ i\mathbf{c} \end{array}\right] \quad \text{and} \quad L=\frac{1}{\sqrt{1-\frac{b^2+d^2}{a^2+c^2}}}\left[\begin{array}{c} 1 \\ \frac{a\mathbf{b}+d\mathbf{c}-\mathbf{b}\times\mathbf{c}}{a^2+c^2} \end{array}\right] \tag{47}$$

□

Conclusion 3.1. Each orthogonal paravector can be presented in the form of:

$$\Lambda=RL_1 \qquad \text{or} \qquad \Lambda=L_2R \tag{48}$$

where L is an orthogonal real paravector, and R is an orthogonal special one.

Since for special paravectors the relation of conjugation is equivalent to the reverse relationship then for the Lorentz rotations it means that

$$\Lambda\mathbb{X}\Lambda^*=RL_1\mathbb{X}L_1R^{-1} \qquad \text{or} \qquad \Lambda\mathbb{X}\Lambda^*=L_2R\mathbb{X}R^{-1}L_2, \tag{49}$$

From the above it follows

Conclusion 3.2. Each Lorentz rotation is a combination of boost (timelike rotation [2]) and Euclidean rotation (spacelike rotation [2]).

The result of the superposition of transformations represented by Λ and L^- is an Euclidean rotation represented by paravector

$$L^-\Lambda=\frac{1}{\sqrt{a^2+c^2}}\left[\begin{array}{c} a \\ i\mathbf{c} \end{array}\right]=\left[\begin{array}{c} \cos(\alpha/2) \\ i\mathbf{n}\sin(\alpha/2) \end{array}\right] \tag{50}$$

We calculate the image of the vector $\mathbf{x}$ from the dependence

$$\left(\begin{array}{c} 0 \\ \mathbf{x}' \end{array}\right)=L^-\Lambda\left(\begin{array}{c} 0 \\ \mathbf{x} \end{array}\right)\Lambda^*L^-=\frac{1}{a^2+c^2}\left[\begin{array}{c} a \\ i\mathbf{c} \end{array}\right]\left(\begin{array}{c} 0 \\ \mathbf{x} \end{array}\right)\left[\begin{array}{c} a \\ -i\mathbf{c} \end{array}\right]=\left[\begin{array}{c} \cos(\alpha/2) \\ i\mathbf{n}\sin(\alpha/2) \end{array}\right]\left(\begin{array}{c} 0 \\ \mathbf{x} \end{array}\right)\left[\begin{array}{c} \cos(\alpha/2) \\ -i\mathbf{n}\sin(\alpha/2) \end{array}\right] \tag{51}$$

hence we obtain

$$\mathbf{x}'=\mathbf{x}\cos\alpha-\mathbf{n}\times\mathbf{x}\sin\alpha+2\mathbf{n}(\mathbf{n}\mathbf{x})\sin^2(\alpha/2) \tag{52}$$

The mutual relationship between the vectors $\mathbf{x}$ and $\mathbf{x}'$ is specified by the dot product $\mathbf{x}\mathbf{x}'=x^2[\cos\alpha+2\cos^2\beta\sin^2(\alpha/2)]$, where α is an angle of rotation and β is an angle between the rotated vector $\mathbf{x}$ and the axis of rotation $\mathbf{n}$.

The conclusion 3.2 is a serious accusation at „Lorentz rotations". It turns out that we can piece together the boosts in such a way that the result is a turnover. In other words, the pilot of the rocket **can chose boosts of the engine of his vehicle in such a way that as a result of this superposition he will rotate in place doing no rotation**. It seems that this is a serious grind in the theory of W.Baylis, and perhaps in the classical theory ... as long as these theories are equivalent?

W.Baylis defines the Lorentz transformation as:

$$\mathbb{X}'=\Lambda\mathbb{X}\Lambda^*=\frac{1}{a^2-b^2+c^2-d^2}\left[\begin{array}{c} a+id \\ \mathbf{b}+i\mathbf{c} \end{array}\right]\left(\begin{array}{c} \Delta t \\ \Delta\mathbf{x} \end{array}\right)\left[\begin{array}{c} a-id \\ \mathbf{b}-i\mathbf{c} \end{array}\right] \tag{53}$$

Paravector Λ is the result of multiplication of real orthogonal paravectors. Passing from the frame moving relatively to the initial reference system with speed described by paravector $\Lambda = \frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c} \end{bmatrix}$ to the frame with speed $\mathbf{v} = 2\frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2-b^2+c^2-d^2}$, which corresponds to paravector L, by eq. (46) we obtain the rotation:

$$\mathbb{X}'' = \frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c} \end{bmatrix}\mathbb{X}\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ -i\mathbf{c} \end{bmatrix} \tag{54}$$

To illustrate the above overall results we calculate an example.

Example 3.1. The vehicle moves at a speed which is the result of two perpendicular boosts 0.8c successively in direction E and N. One should find such a boost, after which the vehicle will not move, but turn into place with respect to his starting position.

We couple two perpendicular boosts of the reference frame corresponding to speeds v=0.8 in the directions OX and OY (This are directions East and North on the map). To calculate the length of the vectors **b** we use the formula (10) and assume that the parameter $a = 1$. After rejecting the results $b > 1$ (unproper paravectors), and taking into account that the directions of vectors **b** and **v** are consistent, we get paravectors corresponding to the assumed speeds:

$$L_1 = \frac{1}{\sqrt{0.75}}\begin{bmatrix} 1 \\ (0.5 \quad 0 \quad 0) \end{bmatrix} \qquad \text{and} \qquad L_2 = \frac{1}{\sqrt{0.75}}\begin{bmatrix} 1 \\ (0 \quad 0.5 \quad 0) \end{bmatrix} \tag{55}$$

So, the resultant speed corresponds to the paravector

$$\Lambda = L_2 L_1 = \frac{1}{0.75}\begin{bmatrix} 1 \\ (0.5 \quad 0.5 \quad -0.25i) \end{bmatrix} \tag{56}$$

So we have $a = 1$, $\quad \mathbf{b} = (0.5 \quad 0.5 \quad 0)$, $\quad \mathbf{c} = (0 \quad 0 \quad -0.25)$ $\quad$ and $\quad d = 0$.

On the basis of the formula (46) the searched boost is described by the paravector

$$L_3 = \frac{1}{\sqrt{0.5252...}}\begin{bmatrix} 1 \\ (-0.3529... \quad -0.5882... \quad 0) \end{bmatrix} \tag{57}$$

After the boost towards SSW by a value of $v_3 = (0.48; 0.8; 0)$ the vehicle will be stationary in relation to the system from which he started, but it turns out that he will be rotated in the plane XOY an angle corresponding to the paravector

$$R = \begin{bmatrix} 0.970143 \\ 0 \quad 0 \quad 0.24254i \end{bmatrix} \tag{58}$$

Any vector $\Delta\mathbf{x}$ observed by pilot of the vehicle will rotate around the vertical according to the relation

$$\begin{bmatrix} 0 \\ \Delta\mathbf{x}' \end{bmatrix} = R\begin{bmatrix} 0 \\ \Delta\mathbf{x} \end{bmatrix}R^-, \tag{59}$$

which using formulas (50) and (58) corresponds to the angle about $\alpha = 28°$.

It should be noted that the obtained rotation depends only on the change of the car's reference system, in which the rotation refers to any property described by the pilot. So we deal with the rotation of the observer. It is obvious that the observer changing the direction of his flight can freely change the direction of the relative motion of the observed object, but he should not do this at the same time in the same way with all the observed objects.

Conclusions

The conclusion of the transformation (1) saying that **we can always find a way of advancing forward, which will allow us to rotate in place** raises serious doubts in relation to the theory promoted by Professor William Baylis. The question is: What does the „Lorentz rotation" represent? If we divide the numerator and denominator of the fraction (10) (describing the velocity $\mathbf{v} = \frac{2a\mathbf{b}}{a^2+b^2}$) by a^2 we get $\mathbf{v} = \frac{2\mathbf{w}}{1+w^2}$. From here it is close to receiving the association with the quotient of momentum and kinetic energy. And here we come to the essence of the Lorentz transformation proposed by Professor Baylis (and perhaps the classical Lorentz transformation?). According to W.Baylis **the definition of speed results from the relationship of the kinetic energy and momentum**. We want to define the speed traditionally as a way covered in time, and some signs indicate that these two definitions of speed are not equivalent. Energy always should be the product of conjugated paravectors, and so it should be described by a real paravector. This energy-momentum paravector of the object in motion will have always a positive scalar component (kinetic energy) and a real vector of momentum. So here the rotation, or the vanish of vector component makes sense when submitting the speeds of the observer but the result of multiplication of mutually conjugated paravectors will be different for each observed energy-momentum paravector. In the formula (54) we deal with the same rotation for each 4-vector $\mathbb{X}$, so that the observer makes an Euclidean rotation in space.

According to the theorem 3.1, exactly two orthogonal real paravectors L and one orthogonal special paravector R it can be assigned to any orthogonal paravector. These real paravectors have very interesting properties and therefore a separate publication will be devoted to them. Moreover, the discovered properties of real boost paravectors L suited better to describe energy than geometry.

Another issue to which we would like to draw the reader's attention is the formalism of two-element arrays used to record paravectors, which is a simple and very effective tool for study the phenomena in space-time. Since the elements of the complex matrix (dimension 4x4) are grouped into two components with different but, characteristic for them selves properties (scalar and vector), the calculation becomes transparent, and physical and geometrical interpretations are more evident. Many authors have long been described the physics of space-time in this way. Four-vectors have the timelike and spacelike components. Unfortunately, the current literature is dominated by very general and unintuitive tensor calculus, in which behind indicators a fundamental difference between the scalar (time) component and spatial one is invisible. Our way of describing paravectors is very similar to that used by W.Baylis. We modified only the developed form of paravector for vertical layout because it makes the calculations more transparent. In this way, the four Maxwell's equations can be written as a single equation. They do not obscure the differences between its various components. The same is with conditions for the electric field. Proof of the invariance of the wave equation under the Lorentz transformation also becomes simple without losing its generality [4].

The attentive reader should have noticed that the paravector calculus is a useful tool with the help of which anyone can take a look at the Special Relativity from a different perspective, and most importantly, that it is possible to use it to develop this theory in another direction. The results that we have obtained are very promising and we will present them gradually in subsequent publications.

Acknowledgments

I would like to thank my friend, Jacek Lewinski, for the support that I received from him many times, and without which my works would not arise.

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