How a Minimum time step based in Pre- Planckian space-time if Friedman Equation H set equal to zero Leads to the Arrow of Time.

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Abstract

Our view point is to assume that the cosmological constant is indeed invariant. And also done where we use an inflaton value due to use of a scale factor $a \sim a_{\min} t^{\gamma}$ if we furthermore use $\delta g_{tt} \sim a_{\min}^{2} \cdot \phi_{initial}$ as the variation of the time component of the metric tensor g_{tt} in Pre-Planckian Space-time up to the Planckian space-time initial values. In doing so, we come up with a polynomial expression for a minimum time step, we can call Δt which would be likely smaller than the Planck time interval/ We then discuss how the development of Δt leads to a development of the arrow of time, and the preservation of information, of essential type, in cosmological early universe dynamics. In doing so, the goal of an 'arrow of time' is intrinsically linked to the utility of causal structure. Our goal of identification of causal structure, which we bring up is essential to the arrow of time, and the use of the Friedman equation, which in itself gives little information, is motivation for how we form the arrow of time, and assumed causal structure. Without this sort of additional information, the Friedman equation and the use of the Heisenberg Uncertainty principle are actually mathematical structures with no real content, in themselves. And the existence of Causal structure only commences right after the regime of Space-time for which H > 0,. Not in when H = 0. The aims of this study is to configure extremely general Friedman equations into a specific inquiry as to how to form a minimum time step, and its relations to how the arrow of time (linked to initial generation of entropy) arises through what are initially vague Newtonian conservation of energy laws for an expanding universe and this due to specific structured inquiry as delineated in this paper.

Key words Inflaton physics, causal structure, non Linear Electrodynamics.

1. Examination of the minimum time step, in Pre-Planckian Space-time

The basic idea is that there be a use of the Friedman Equation treatment of Hubble expansion which can be written according to Roos [1] as in terms of a De Sitter universe as

$$H^2 = \frac{8\pi}{3} \cdot \rho_0 + \frac{\Lambda}{3} \tag{1}$$

In itself , this is assuming a treatment which is seen in accordance to the following assumptions, as given namely [2], that there is a linkage between the following Newtonian version of the Newtonian version of the Friedman Equation and its relativistically correct form, due to a substation as also written in, as

$$\begin{aligned} H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} \cdot \rho_0 + \frac{2\kappa}{r_0^2 a^2}\Bigg|_{Newtonian} \\ H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \cdot u_{Energy-Density} - \frac{\kappa c^2}{r_{c,0}^2 a^2} + \frac{\Lambda}{3}\Bigg|_{relativistically-correct} \\ &\xrightarrow[\kappa \to 0]{} \frac{8\pi G}{3c^2} \cdot u_{Energy-Density} + \frac{\Lambda}{3}\Bigg|_{relativistically-correct-flat-space} \end{aligned} \tag{2}$$

In the third line of Eq. (2) the essential substitution is to go from a κ in the Newtonian case where we have the universe as bounded if $\kappa < 0$ in a gravitational sense, or unbounded in a gravitational sense, if $\kappa > 0$ to the question of, in relativity of negative curvature, $\kappa < 0$, or positive curvature, with $\kappa > 0$. Here what we do, in our own adaptation of Eq. (2) is to realize that the Newtonian case involves the conservation of energy for an expanding universe, while we conflate the 2nd and 3rd relativistic case involves changing from a mass density, ρ_0, by an energy Density, $u_{Energy-Density}$ which is a generalized version of what we are attempting to analyze.

So, in the Pre Planckian regime of Space-time, our initial assumption is twofold, i.e. we assume that we cannot reference either $\kappa < 0$ or $\kappa > 0$. The default choice we will pick is, in the Pre Planckian space-time to simplify our analysis, is to then set $\kappa = 0$. And then we will re image the energy density which will then be in conjunction with a revitalized version of a modified early universe version of the Heisenberg Uncertainty principle, we will give as stated in [3] which leads to a modification of the HUP which is due to writing

$$\begin{aligned} &\Delta E \Delta t \sim \left[\hbar \big/ \left(\delta g_{tt} \sim a_{\min}^2 \cdot \phi_{initial}\right)\right]\Big|_{\Pr e-Planckian} \\ &\xrightarrow[(\Pr e-Planckian) \to (Planckian)]{} \Delta E \Delta t \sim \hbar\Big|_{Planckian} \end{aligned} \tag{3}$$

The seemingly bizarre math for this modification of Pre-Planckian to Planckian adaptation of the HUP, is in tandem as given in [3] with division of the Planck's constant by what is a fluctuation of time based metric tensors as given by writing , if $a_{\min}^{2}$ is the non zero square of a minimum scale factor, and ϕ is an inflaton field, with Eq. (4) dimensionless, that we have

$$g_{tt} \sim \delta g_{tt} \approx a_{\min}^{2}\phi \tag{4}$$

The specifics of this adaptation is in the idea of an arrow of time which will be important as to the formation of our Δt term, as well as going from a Pre Causal space-time structure for when $\Delta E \Delta t \sim \left[\hbar \middle/ \left(\delta g_{tt} \sim a_{\min}^{2} \cdot \phi_{initial}\right)\right]\Big|_{\Pr e-Planckian}$ to the causal structure inherent in the formation of $\Delta E \Delta t \sim \hbar\big|_{Planckian}$. I.e. in our case, the genesis of the arrow of time, will be conflated with our procedure as to formation of the setup of causal structure, but this will be before the Universe begins to expand. So our next seemingly nonsensical mathematical step is to examine Eq. (1) where we will be examining what happens if the Left hand Side of Eq. (1) is set equal to zero, and we make the following substitution, namely

$$\begin{aligned}
&H^{2} = \frac{8\pi}{3} \cdot \rho_{0} + \frac{\Lambda}{3} \xrightarrow[(\Pr e-Planckian)\to(Just-before-Planckian)]{} \\
&0 = \frac{8\pi G}{3c^{2}} \cdot u_{Energy-Density} + \frac{\Lambda}{3}\Bigg| \\
&\& \\
&0 = \frac{8\pi G}{3c^{2}} \cdot u_{Energy-Density} + \frac{\Lambda}{3}\Bigg| \\
&\xrightarrow[\left(u_{Energy-Density}\right) \to \left(\frac{\Delta E}{Volume} \sim \left[\hbar / Volume \cdot \Delta t \cdot \left(\delta g_{tt} \sim a_{\min}^{2} \cdot \phi_{initial}\right)\right]\Big|_{\Pr e-Planckian}\right)]{} \\
&\frac{8\pi G}{3c^{2}} \cdot \left[\hbar \middle/ \Delta t \cdot Volume \cdot \left(\delta g_{tt} \sim a_{\min}^{2} \cdot \phi_{initial}\right)\right]\Big|_{\Pr e-Planckian} + \frac{\Lambda}{3} \approx 0
\end{aligned} \tag{5}$$

What we have done, using what appears to be random mathematics, is to assume that we can, if we specify a minimum scale factor, non-zero, for $a_{\min}^{2}$, and an inflaton ϕ_{init} initial value which if it contains Δt that we can recover a minimum time step. But this $a_{\min}^{2}$ involves a nonsingular bounce parameter for the beginning of the Universe which is a violation of the Penrose singularity theorem, i.e. see [4], and a replacement of the singular point of the start of the expansion of the Universe with a small sphere of space-time physics, which we will cite as far as the NLED (nonlinear dynamics) picture of cosmology for which $a_{\min}^{2}$ is of the order of scaled value say as of about 10^-110, or so, which is unbelievably small, but non zero, and which is commensurate with the results given in [5] , and which is akin to the given results of [6] as well of Loop quantum gravity. So then our task is to come up with a polynomial description of Δt which permits an iterative description of a minimal time step, and perforce, then the start of the Arrow of time in cosmology.

2. Basic idea, The Padmabhan approximation of ϕ_{init} so we can have a polynomial solution to solve Δt

Our objective is to use Eq. (5) with [7]

$$a \sim a_{\min} t^{\gamma} \tag{6}$$

And [7]

$$\begin{aligned} & a \approx a_{\min} t^{\gamma} \\ & \Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \\ & \Leftrightarrow V \approx V_0 \cdot \exp\left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\} \end{aligned} \tag{7}$$

And [7]

$$\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \tag{8}$$

And[3,7,8]

$$g_{tt} \sim \delta g_{tt} \approx a_{\min}^2 \phi \tag{9}$$

The end result is that we can make the following approximation for solving for Δt where we assume that Planck length is normalized to 1, and that our volume of space-time for the creation of Δt commences just before a Planck length, cubed volume of space-time for parameterization of our defining equation for Δt

$$\left|\phi_{initial}\right| \sim \left| \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t \right\} \right| \sim \frac{24\pi\hbar}{\Delta t \cdot a_{\min}^2 \cdot \Lambda} \tag{10}$$

It is best in this situation to use Planck like Units for which we can set the values equal to 1 for h bar, G, and all that, so we can with a bit of work simplify Eq. (10) into being a nonlinear Polynomial equation for Δt, and this can be done via use of a logarithmic expansion of the following form as given by [9] , on page 299, when we have the following for when 0 < x < 2 of using

$$\log_e x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \ldots \tag{11}$$

We then get the following nonlinear equation for Δt and this would have to be iterated numerically, after we set h bar, G , and other things normalized to 1, with attendant treatment of suitable choice of Δt

$$\Delta t \cdot \left| \left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right) - \frac{\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right)^2}{2} + \frac{\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right)^3}{3} - \ldots \right| \approx \left(\sqrt{\frac{\gamma}{\pi G}} \right)^{-1} \frac{48\pi\hbar}{a_{\min}^2 \cdot \Lambda} \tag{12}$$

So in coming up with this iterative procedure for finding Δt we will then commence in discussion of, especially if this is within a very small initial "bubble' of space time try to understand if we have then actually started an arrow of time. And what is its relationship to causal structure?

3. Conclusion. What would be sufficient to state we have an arrow of time? If Δt is isolated by solving Eq. (12)?

An arrow of time, as we have postulated we state is due to a non zero, initial time step, i.e. Δt, and this in a "bubble" of nonzero but nearly singular space-time as a starting point for our expansion, as given in Eq. (12) above. Using [10], we can ascertain that our modification of the Friedman Equation as given in, and commented upon in Eq. (5) is, indeed a 'quantum bounce' as given in [10] in Eq. (2) and Eq. (6) of pp 152 and 153 of that [10] document. And our novelty feature in our work is to ascertain, via use of the research convention that we are assuming a nonsingular bubble of space-time in which we define Δt in. , our definition of an initial entropy is flavored by the convention that the arrow of time starts with the generation of nonzero entropy. Our convention, using [11] is to use the entropy as given by

$$S_\Lambda = \pi \cdot \left(\frac{R_c}{l_{Planck}} \right)^2 \tag{13}$$

Here, in the case of maximally generated entropy, we would have R_c as a cosmological event horizon, with the result that in the present era, the above Eq. (13) is an astonishing 1.99 times 10^122 in value. So, how can this entropy generate an arrow of time, initially? Our convention is to then use, say

$$S_\Lambda \big|_{Arrow-of-time} = \pi \cdot \left(\frac{R_c\big|_{initial} \sim c \cdot \Delta t}{l_{Planck}} \right)^2 \neq 0 \tag{14}$$

When Eq.(14) is > 1, due to forming Δt from Eq. (12) above, we have the beginning of the arrow of time. It's relationship to Dowker defined Causal structure [12] is simplicity itself, namely that should the distance relations approximately adhere to

$$\left(\frac{R_c\big|_{initial} \sim c \cdot \Delta t}{l_{Planck}} \right) \sim \vartheta(1) \tag{15}$$

Or at least be close to the boundary of the bubble of space-time, that in terms of Space-time, we are beginning the regime of Causal structure, but that if we have the ratio of Eq. (15) significantly less than 1, and within the bubble of space-time that the causal structure has not formed yet. I.e. it all depends upon the way Δt iterates out. I.e. the strangest situation would be to have, say that Eq. (14) was true, which is our beginning of the arrow of time, in terms of generation of entropy, but that this would be divorced from Eq. (15). An open question to ask would be say is the following possible? Namely

$$S_{\Lambda}\big|_{Arrow-of-time} = \pi \cdot \left(\frac{R_c\big|_{initial} \sim c \cdot \Delta t}{l_{Planck}} \right)^2 \neq 0$$
$$\left(\frac{R_c\big|_{initial} \sim c \cdot \Delta t}{l_{Planck}} \right) << 1 \tag{16}$$

This assumes also that issues, and more over the formalism of [13,14,15,16, 17] are answerable. I.e. do we have a trace of the inflaton as postulated by Corda in [13]? It is crucially important to know this. As to [14,15, 16] if the entropy so generated is commensurate with gravitons being generated, by primordial processes, according to conflating graviton count with the entropy via infinite quantum statistics, [17] we may have additional physics to analyze, due to an extremely simple starting model.

Note that all of this, depends upon an arrow of time, as defined through Eq. (14) which gives content to what is otherwise physically inconsequential mathematical equations. As well as allow us to investigate [18] . We thereby conclude as to how we can configure extremely general Friedman equations into a specific inquiry as to how to form a minimum time step, and its relations to how the arrow of time (linked to initial generation of entropy) arises through what are initially vague Newtonian conservation of energy laws for an expanding universe and this due to specific structured questions arranged one after the other as delineated in this paper. i.e. the initial Newtonian equations, as given , then the general relativistic equations and then the minimum time step, with the final conclusion being Eq. (14) and Eq. (15). With Eq. (16) being an open research question.

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