Quasi-periodic oscillations of GRO J1744-28

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ABSTRACT

Observed quasi-periodic oscillations (QPOs) of GRO J1744-28 are compared with predictions from a previously proposed three tori model. The three highest QPO frequencies are assumed to arise from three circular tori moving around the pulsar: an inner torus with charge Q_i, a torus with mass m_m in the middle and an outer torus with charge Q_o, whereas the pulsar itself bears a charge Q_s.

In addition, it follows from a special interpretation of the gravitomagnetic theory, that the three circular tori are subjected to a total number of four low-frequency precessions. The expressions of these four additional QPO frequencies are revised compared to earlier work. For GRO J1744-28 the two lowest observed QPO frequencies are attributed to the two highest of the four low-frequency QPOs. The two other frequencies of the quartet may be too low to be detected. From the two highest QPO frequencies of the quartet, lying close together, approximate values for the charges Q_s, Q_i, and Q_o are extracted. The results are compared with the observed and predicted set of seven QPOs for five other pulsars and two black holes.

The observed magnetic field is compared with the polar magnetic field, also predicted by the gravitomagnetic theory. Remarkably, the observed highly ionized iron emission lines may be compatible with the tree tori model. In order to explain the discontinuity in recently observed phase lags of GRO J1744-28, a Compton reverberation mechanism is considered, compatible with electron temperatures that depend on the radii of the tori.

1. INTRODUCTION

The accreting pulsar in the low-mass X-ray binary system GRO J1744-28 produces X-ray bursts, showing quasi-periodic X-ray oscillations (QPOs). The current understanding of these phenomena is far from complete. In this work the origin of the QPOs will further be investigated. From the observed X-ray spectrum of GRO J1744-28 Zhang *et al.* [1] deduced a spin frequency at 2.14 Hz and its harmonic at 4.2 Hz and three, clearly discernible, QPOs at 56.2, 39.0 and 20.2 Hz, respectively. In addition, Kommers *et al.* [2] found large-amplitude oscillations in the light curve of the bursts. They attributed them to a simple beating of two narrow band QPOs with comparable amplitudes at 3.75 Hz and 3.25 Hz, respectively. Attempts to interpret the QPOs of GRO J1744-28 have been given [1, 2], but no generally accepted model has emerged so far.

In this work the observed three highest QPO frequencies of GRO J1744-28 are attributed to the frequencies of the orbital motion of the three circular tori around the pulsar, whereas the two other QPOs are attributed to a gravitomagnetic precession mechanism [3−5]. In general, the latter mechanism leads to a total of four low-frequency QPOs, but it is argued that two of them may escape detection for GRO J1744-28. The full set of three high-frequency and four low-frequency QPOs has been observed for five other pulsars: SAX J1808.4–3658, XTE J1807–294, IGR J00291+5934, SGR 1806–20 and Sco X-1 and two black holes: XTE J1550–564 and Sgr A*. Alternative explanations for the latter QPOs are discussed in, e.g., refs. [3−5] and references cited therein.

Compared with other models, our model contains two new basic assumptions. Firstly, it is assumed that pulsars and black holes may bear large charges [3–5]. Secondly, for the explanation of four low-frequency QPOs another assumption may be essential: *a special interpretation of the gravitomagnetic theory*, which may be deduced from general relativity [3–9]. In this interpretation the gravitomagnetic field **B**(gm) generated by rotating mass and the electromagnetic induction field **B**(em) generated by moving charge are supposed to be equivalent. It is generally accepted that introduction of a gravitomagnetic field is consistent with orthodox general relativity, but the proposed equivalence of **B**(gm) and **B**(em) is not. Application of this special interpretation of the gravitomagnetic field, however, results in the deduction of four precession frequencies, which may be identified with four observed low-frequency QPOs [3–5].

It is noticed, that our interpretation of the gravitomagnetic field also leads to a prediction of the strength of the magnetic field of pulsars and other rotating bodies. Identification of the "magnetic-type" gravitational field with a magnetic field results into the so-called Wilson-Blackett formula. This relation applies, e.g., to a spherical star consisting of electrically neutral matter

$$\mathbf{M}(\mathrm{gm}) = -\tfrac{1}{2}\beta c^{-1}G^{1/2}\,\mathbf{S}. \tag{1.1}$$

Here **M**(gm) is the gravitomagnetic dipole moment of the star with angular momentum **S**, and β is a dimensionless constant of order unity. Recently, an indication is found that the sign of β is negative [10]. Therefore, it is adopted throughout this paper that $\beta = -1$.

Relation (1.1) appears to be approximately valid for many, widely different celestial bodies and some rotating metallic cylinders in the laboratory as well (see for a review ref. [7] and references therein). The magnetic fields of pulsars have separately been discussed [8]. The angular momentum **S** for a spherical star with mass m_s and radius r_s can be calculated from the relations

$$\mathbf{S} = I\mathbf{\Omega}_s, \quad \text{or} \quad S = I\,\Omega_s = \tfrac{2}{5} f_s m_s r_s^2 \Omega_s, \tag{1.2}$$

where $\mathbf{\Omega}_s$ is the angular velocity vector of the star ($\Omega_s = 2\pi\nu_s$ is its angular velocity and ν_s is its spin frequency), I is the moment of inertia of the star and f_s is a dimensionless factor depending on the homogeneity of the mass density in the star. For convenience sake, the value $f_s = 1$ for a homogeneous mass density will be used in this work.

The value of a gravitomagnetic dipole moment **M**(gm) or an electromagnetic dipole moment **M**(em) can be calculated from

$$\mathbf{M} = \tfrac{1}{2}R^3\mathbf{B}_\mathrm{p}, \quad \text{or} \quad M = \tfrac{1}{2}R^3 B_\mathrm{p}. \tag{1.3}$$

Here $\mathbf{B}_\mathrm{p}$ is the magnetic induction field at, say, the north pole of the star at distance R from the centre of the star to the field point where the field $\mathbf{B}_\mathrm{p}$ may be observed ($R \geq r_s$).

For $R = r_s$ combination of (1.1), (1.2) and (1.3) yields the following polar field $\mathbf{B}_\mathrm{p}$(gm)

$$\mathbf{B}_\mathrm{p}(\mathrm{gm}) = -\tfrac{2}{5}\beta c^{-1}G^{1/2} m_s r_s^{-1}\mathbf{\Omega}_s. \tag{1.4}$$

For $\beta = -1$ the directions of the vectors $\mathbf{B}_\mathrm{p}$(gm) and $\mathbf{\Omega}_s$ coincide. It is stressed that $\mathbf{B}_\mathrm{p}$(gm) at distance r_s has been derived for an ideal gravitomagnetic dipole located at the centre of the star. For a homogeneous mass distribution in the star, however, the same result for $\mathbf{B}_\mathrm{p}$(gm) can be deduced [11, 12].

Furthermore, precession phenomena are another consequence of the special interpretation of the gravitomagnetic theory. The latter theory predicts an angular precession velocity **Ω**(gm) for an angular momentum **S** of a star or a torus. The following relation then applies to $\mathbf{\Omega}$(gm)

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{dt}} = \mathbf{\Omega}(\mathrm{gm}) \times \mathbf{S}. \tag{1.5}$$

The angular precession velocity $\mathbf{\Omega}$(gm) of $\mathbf{S}$ around direction of the field $\mathbf{B}$(gm) from gravitomagnetic origin is given by [3, 7, 9]

$$\mathbf{\Omega}(\mathrm{gm}) = -2\beta^{-1}c^{-1}G^{1/2}\mathbf{B}(\mathrm{gm}), \tag{1.6}$$

where the precession frequency ν(gm) is given by $\nu(\mathrm{gm}) = \Omega(\mathrm{gm})/(2\pi)$.

As a first example, the precession of the angular momentum $\mathbf{S}_m$ of a circular torus with total mass m_m by the gravitomagnetic field of the star with angular momentum $\mathbf{S}$ will be considered. According to (1.5), an angular precession velocity $\mathbf{\Omega}$(gm) of the component $\mathbf{S}_m \sin\delta_m$ (δ_m is the angle between the directions of $\mathbf{S}$ and $\mathbf{S}_m$) will occur around $\mathbf{S}$. An approximately equatorial orbit for the torus will be adopted, so that δ_m is small. Substitution of the equatorial value of the dipolar gravitomagnetic field $\mathbf{B}_{\mathrm{eq}}(\mathrm{gm}) = -R^{-3}\mathbf{M}(\mathrm{gm})$ into (1.6) $\mathbf{\Omega}$(gm) then yields

$$\mathbf{\Omega}_{\mathrm{LT}} \approx -c^{-2}GR^{-3}\mathbf{S}, \quad \text{or} \quad \nu_{\mathrm{LT}} \approx -\tfrac{2}{5}c^{-2}Gm_s\nu_s r_s^2 R^{-3}. \tag{1.7}$$

The precession of the torus with mass m_m is an example of Lense-Thirring precession. Thus, the obtained result for $\mathbf{\Omega}$(gm) is denoted by $\mathbf{\Omega}_{\mathrm{LT}}$ and the corresponding Lense-Thirring frequency by ν_{LT}. Note that $\mathbf{B}_{\mathrm{eq}}$(gm) is approximately constant, when δ_m is small. Since $\mathbf{S}_m \sin\delta_m$ reduces to zero for $\delta_m = 0$, however, precession only occurs for $\delta_m > 0$.

Another situation occurs, when an electromagnetic field $\mathbf{B}$(em) is present besides $\mathbf{B}$(gm). Adopting that both fields are equivalent, the resulting total magnetic field $\mathbf{B}(\mathrm{tot}) = \mathbf{B}(\mathrm{gm}) + \mathbf{B}(\mathrm{em})$ has to be substituted into (1.6). In section 3 we will consider a number of different fields $\mathbf{B}$(em) contributing to $\mathbf{B}$(tot). It is noticed that the proposed equivalence of $\mathbf{B}$(gm) and $\mathbf{B}$(em) is in contradiction with the analysis of data obtained by the Gravity Probe B satellite and with reported results for the Lense-Thiring precession of the orbit of two LAGEOS satellites These results are criticized, however (see, e.g, ref. [11] and references cited therein).

The three tori model from [3], resulting in three high-frequency QPOs ν_i, ν_m and ν_o, is summarized in section 2. In section 3 a revised version of the deduction of four low-frequency QPOs from gravitomagnetic origin [3], ν_{io}, ν_{mo}, ν_{oi} and ν_{mi}, is given. In section 4 a previously introduced factor β^*, depending on the sign of β, is reconsidered. The parameter β^*, also depending on the spin frequency ν_s, determines the total polar magnetic field $\mathbf{B}_{\mathrm{p}}$(tot) of the star. In section 5 the five observed QPO frequencies of pulsar GRO J1744-28 are compared with the seven predicted QPO frequencies following from our model. In addition, the obtained radii are compared with radii deduced from other approaches. In section 6 results are summarized and conclusions are drawn.

2. THREE TORI MODEL

In this section we deal with the three highest QPO frequencies, which are assumed to arise from three circular tori around the central star. The first frequency ν_i is attributed to an *inner* torus of radius r_i containing a total electric charge Q_i, a second frequency ν_o is attributed to an *outer* torus of radius r_o with total charge Q_o, whereas a third torus of radius r_m, containing a total electrically neutral mass m_m (the subscript *m* stems from *middle*) is assumed to be present between the two other tori. Thus, it is assumed that $r_i < r_m < r_o$. In addition, it is assumed that the star bears a total charge Q_s. When the sign of Q_i

is opposite to that of the charges Q_o and Q_s, equilibrium is possible under certain restrictions.

The high-frequency QPOs ν_i and ν_o are calculated by application of Coulomb's law, the gravitation law of Newton and the centrifugal force. It can be shown [3], that all forces acting between a point mass dm_i with charge dQ_i in the inner torus can be in equilibrium with mass m_s and charge Q_s of the star and with a total charge Q_o in the outer torus, respectively. Equilibrium is only possible, if the angle $\Delta = 90^o - \theta$ between the unit vectors $\mathbf{n}_i$ and $\mathbf{n}_o$ perpendicular to the planes of the inner and outer torus, respectively, does not exceed the value $\Delta_0 \equiv 90^o - \theta_0$ (see refs. [3, 4] for the definitions of Δ, θ and θ_0 and for values of θ_0). The motion of the tori becomes unstable, when the latter condition is not fulfilled. This instability may (partially) explain the observed instability of the high-frequency QPOs ν_i and ν_o. The expression for ν_i is given by [3]

$$\nu_i = \frac{1}{2\pi}\left[\frac{Gm_s}{r_i^3}\left\{1 - \frac{m_s}{m_i}Q_i'\left(Q_s' - x^2 f\, Q_o'\right)\right\}\right]^{1/2}, \tag{2.1}$$

where m_i is the total mass in the inner torus, x is defined by $x \equiv r_i/r_o$, Q_i' is defined by the dimensionless quantity $Q_i' \equiv (G^{1/2} m_s)^{-1} Q_i$, Q_s' by $Q_s' \equiv (G^{1/2} m_s)^{-1} Q_s$ and so on. In the deduction of (2.1), it has been assumed that the mass and charge are homogeneously distributed in both tori. The value of the quantity f depends on the location of dQ_i in the inner torus with respect to the outer torus with total charge Q_o. When the inner and outer torus are lying in the same plane, f reduces to $f(x)$ in the equilibrium state

$$f(x) = \frac{-2}{\pi x}\left\{K(x) - \frac{E(x)}{1-x^2}\right\}, \tag{2.2}$$

where $K(x)$ and $E(x)$ are complete elliptic integrals of the first kind and second kind, respectively. When both tori are in a bound state but their planes do not coincide, the value of f varies along the orbit of the inner torus. When f becomes negative, the tori become unstable. At the least stable situation in the orbit f reduces to $f_0 = 0$, as has been discussed in refs. [3, 4]. When the angle Δ between the unit vectors $\mathbf{n}_i$ and $\mathbf{n}_o$ is not too small, the averaged value of f over the whole orbit may be approximated by $\overline{f(x)} = \tfrac{1}{2}\{f_0 + f(x)\} = \tfrac{1}{2}\{0 + f(x)\} = \tfrac{1}{2} f(x)$. For nearly coplanar tori the averaged value of f approaches to $\overline{f(x)} = f(x)$. The latter approximation will be used in this work.

It is to be expected that the factor m_s/m_i on the right hand side of (2.1) is very large. It will be assumed that the difference $(Q_s' - x^2 f\, Q_o')$ is very small, so that the term depending on Q_s', Q_i' and Q_o' on the right hand side of (2.1) may be small. As a result, the real radius r_i from (2.1) may then be somewhat larger or smaller than the corresponding Kepler radius $r_i = \{Gm_s/(2\pi\nu_i)^2\}^{1/3}$. In our calculations we will use the following relation

$$Q_s \approx x^2 f\, Q_o. \tag{2.3}$$

This approximation reduces the number of unknowns like x, Q_s and Q_o by one.

Following an analogous method, the same forces between a point mass dm_o with charge dQ_o in the outer torus of radius r_o can be in equilibrium with mass m_s and charge Q_s of the star and with a total charge Q_i in the inner torus of radius r_i, respectively. The following expression for the corresponding high-frequency ν_o is then obtained [3]

$$\nu_o = \frac{1}{2\pi}\left[\frac{Gm_s}{r_o^3}\left\{1 - \frac{m_s}{m_o}Q_o'\left(Q_s' + g\, Q_i'\right)\right\}\right]^{1/2}, \tag{2.4}$$

where m_o is the total mass in the outer torus and Q_o' is defined by the dimensionless quantity $Q_o' \equiv (G^{½} m_s)^{-1} Q_o$, and so on. The value of the quantity g depends on the location of dQ_o in the outer torus with respect to the inner torus with total charge Q_i (see refs. [3, 4]). When the inner and outer torus are lying in the same in the same plane, g reduces to $g(x)$ in the equilibrium state

$$g(x) = \frac{2}{\pi}\left\{\frac{E(x)}{1-x^2}\right\}, \tag{2.5}$$

where $E(x)$ is a complete elliptic integral of the second kind. When both tori are in a bound state but their planes do not coincide, the value of g varies along the orbit of the outer torus. At the least stable situation in the orbit g reduces to g_0. When the angle Δ between the unit vectors $\mathbf{n}_i$ and $\mathbf{n}_o$ is not too small, the averaged value of g over the whole orbit may be approximated by $\overline{g(x)}$ = ½ $\{g_0 + g(x)\}$ (see for definition and values of g_0 refs. [3, 4]). For nearly coplanar tori the averaged value of g approaches to $\overline{g(x)}$ = $g(x)$. The latter approximation will be used in this work.

It is to be expected that the factor m_s/m_o on the right hand side of (2.4) is large. In this work it will be assumed that the sum $(Q_s' + g\ Q_i')$ is small, so that

$$Q_s \approx -g\, Q_i. \tag{2.6}$$

This approximation also reduces the number of unknowns like g, Q_s and Q_i by one. The charge dependent term on the right hand side of (2.4), however, may differ from zero value, so that the radius r_o in (2.4) may be somewhat larger or smaller than the corresponding Kepler radius $r_o = \{Gm_s/(2\pi\nu_o)^2\}^{⅓}$.

It is noticed that Lorentz forces and the gravitational attraction between the masses m_i and m_o in the tori have been neglected in the deduction of the results (2.1) and (2.4). Moreover, general relativistic effects have not been taken into account. Starting from a Kerr-Newman space-time, Aliev and Galtsov [13] considered the latter effects for the binary system of a charged star and a charged point mass moving in a circular orbit around that star. Therefore, the results in this section have only to be considered as a first order approximation.

Furthermore, the third high-frequency QPO ν_m, due to the middle torus, is identified as the orbital frequency ν_m for a point mass dm_m in a circular orbit of radius r_m around a star with mass m_s and angular momentum S. Including the contribution due to S, the frequency ν_m for a prograde motion of dm_m in the equatorial plane around the star can be shown to be (compare with, e.g., Aliev and Galtsov [13])

$$\nu_m = \frac{1}{2\pi}\left(\frac{Gm_s}{r_m^3}\right)^{½} \frac{1}{1+\dfrac{S}{c^2 m_s}\left(\dfrac{Gm_s}{r_m^3}\right)^{½}} = \frac{1}{2\pi}\left(\frac{Gm_s}{r_m^3}\right)^{½} f_S. \tag{2.7}$$

For most pulsars the relativistic factor f_S in (2.7) depending on the angular momentum S usually approaches unity value. In that case, frequency ν_m becomes equal to the Kepler frequency ν_{K}

$$\nu_m \approx \nu_{\mathrm{K}} = \frac{1}{2\pi}\left(\frac{Gm_s}{r_{\mathrm{K}}^3}\right)^{½} \quad \text{and} \quad r_{\mathrm{K}} = \left\{\frac{Gm_s}{(2\pi\nu_{\mathrm{K}})^2}\right\}^{⅓}. \tag{2.8}$$

Note that none of the high-frequency QPOs ν_i of (2.1), ν_o of (2.4) or ν_{K} of (2.8) depend on the spin frequency ν_s of the star.

Moreover, it is noticed that the three tori model is an idealized model, since each torus is mathematically represented by a circle. In reality, the cross sectional area is no point but possesses a certain area. As an illustration, the three tori model applied to calculate Earth's net charge and the charges of the van Allen belts [14] can be considered. Comparison of the observed cross-sections of the tori (radiation belts) shows that they increase with increasing radius of the torus. Especially, the cross-section of the outer torus of radius r_o is much larger than the cross-section of the inner torus of radius r_i.

3 LOW-FREQUENCY QPOs FROM GRAVITOMAGNETIC ORIGIN

When the field **B**(gm) in (1.6) is replaced by a magnetic induction field **B**(Q), due to some charge Q, a number of different gravitomagnetic precession frequencies can be distinguished (The adjective "gravitomagnetic" has been retained, since (1.6) describes the interaction between some angular momentum (no charge) and a magnetic field (**B**(Q) in this case)). A calculation of four precession frequencies has previously been given in ref. [3, sections 3 and 4]. Here the revised results are given.

As a first example, we consider the field $\mathbf{B}(Q_o)$ generated by the total charge Q_o in the outer torus of radius r_o and acting on the torus with total mass m_m. Two precession frequencies from (1.6) can then be distinguished. Firstly, the field component $\mathbf{B}^{||}(Q_o)$ = $B(Q_o)\cos\delta_o\,\mathbf{s}$ ($\mathbf{s} \equiv \mathbf{\Omega}_s/\Omega_s$ is the direction of the rotation axis of the star) may act on the component $\mathbf{S}_m^{\perp}$ perpendicular to **s**, where $\mathbf{S}_m$ is the angular momentum of the torus with mass m_m and $S_m^{\perp}$ is defined as $S_m^{\perp} \equiv S_m \sin\delta_m$. Here δ_o and δ_m are the angles between the unit vector **s** and the unit vector $\mathbf{n}_o$ along the direction of the rotation axis of the torus with mass m_o and charge Q_o and the unit vector $\mathbf{n}_m$ along the direction of the rotation axis of the torus with mass m_m, respectively. For the field component $\mathbf{B}^{||}(Q_o)$ parallel to **s** the following expression can be calculated

$$\mathbf{B}^{||}(Q_o) = \frac{2\pi Q_o \nu_o}{c r_o} g(x_o) \cos\delta_o\, \mathbf{s}, \tag{3.1}$$

where the frequency of the charge Q_o in the torus of radius r_o is given by ν_o (see (2.4)) and x_o is defined by $x_o \equiv r_m/r_o$. The function $g(x_o)$ in (3.1) is analogously defined to $g(x)$ in (2.5). It will be assumed in this work that δ_o is small and that the field $\mathbf{B}^{||}(Q_o)$ will be approximately constant. Substitution of (3.1) into (1.6) yields for the angular precession frequency $\mathbf{\Omega}_{mo}$

$$\mathbf{\Omega}_{mo} = -4\pi\beta^{-1} \frac{G^{1/2} Q_o}{c^2 r_o} \nu_o g(x_o) \cos\delta_o\, \mathbf{s}. \tag{3.2}$$

So, for $\beta = -1$ and a positive charge Q_o the precession velocity $\mathbf{\Omega}_{mo}$ is clockwise around **s**. The following sequence with respect to the subscripts has been used in $\mathbf{\Omega}_{mo}$: the first subscript *m* stems from *middle* and the last subscript *o* from *outer*. The precession frequency ν_{mo} ($\nu_{mo} = \Omega_{mo}/(2\pi)$) is then given by

$$\nu_{mo} = +Q_o' \frac{2Gm_s}{c^2 r_o} \nu_o g(x_o) \cos\delta_o, \tag{3.3}$$

where the quantity Q_o' is again defined by $Q_o' \equiv (G^{1/2} m_s)^{-1} Q_o$. A second interaction between the field component $\mathbf{B}^{\perp}(Q_o)$ perpendicular to **s**, where $B^{\perp}(Q_o)$ is defined as $B^{\perp}(Q_o) \equiv B(Q_o)\sin\delta_o$ and the component $\mathbf{S}_m^{||}$ parallel to **s** ($S_m^{||} \equiv S_m\cos\delta_m$) may lead to an additional precession frequency ν'_{mo}. Since it is assumed that δ_o is small, the component $B^{\perp}(Q_o)$ will be small. Moreover, the field $\mathbf{B}^{\perp}(Q_o)$ may (partly) average out. Note that result (3.3) differs by a factor $\cos\delta_m$ from the incorrect expression for ν_{mo} given in ref. [3].

In an analogous way, the field component $\mathbf{B}^{||}(Q_o) = B(Q_o)\cos\delta_o\,\mathbf{s}$ may act on the component $\mathbf{S}_i^{\perp}$ perpendicular to $\mathbf{s}$, where $\mathbf{S}_i$ is the angular momentum of the *inner* torus with mass m_i and charge Q_i ($S_i^{\perp} \equiv S_i \sin\delta_i$). Here δ_i is the angle between the direction of the rotation axis of the star $\mathbf{s}$ and the unit vector $\mathbf{n}_i$ along the direction of the rotation axis of the torus with mass m_i and charge Q_i. For the resulting precession frequency ν_{io} one obtains

$$\nu_{io} = +Q_o' \frac{2Gm_s}{c^2 r_o} \nu_o\, g(x) \cos\delta_o, \tag{3.4}$$

where Q_o', r_o, ν_o and δ_o are already given in (3.3) and x is again defined by $x \equiv r_i/r_o$. Note that the quantity $g(x)$ in (3.4) equals to $g(x)$ in (2.5). A possible precession frequency ν'_{io}, analogous to ν'_{mo}, will also be neglected.

Furthermore, an electromagnetic field $\mathbf{B}(Q_i)$ generated by the total charge Q_i in the inner torus of radius r_i may act on the torus of radius r_m with total mass m_m. Two precession frequencies following from (1.6) can again be distinguished. Only the field component $\mathbf{B}^{||}(Q_i) = B(Q_i)\cos\delta_i\,\mathbf{s}$ will be considered, where δ_i is the angle between the unit vector $\mathbf{s}$ and the unit vector $\mathbf{n}_i$ along the direction of the rotation axis of the torus with mass m_i and charge Q_i. This field may act on the component $\mathbf{S}_m^{\perp}$ perpendicular to $\mathbf{s}$, where $\mathbf{S}_m$ is the angular momentum of the torus with mass m_m and $S_m^{\perp}$ is defined as $S_m^{\perp} \equiv S_m \sin\delta_m$. For the resulting precession frequency ν_{mi} one obtains

$$\nu_{mi} = -Q_i' \frac{2Gm_s}{c^2 r_m} \nu_i\, x_i\, f(x_i) \cos\delta_i, \tag{3.5}$$

where Q_i' is again defined by $Q_i' \equiv (G^{1/2} m_s)^{-1} Q_i$. The frequency of the charge Q_i in the torus of radius r_i is given by ν_i (see (2.1)) and x_i is defined by $x_i \equiv r_i/r_m$. The function $f(x_i)$ in (3.5) has analogously been defined to $f(x)$ in (2.2). In an analogous way, the field component $\mathbf{B}^{||}(Q_i) = B(Q_i)\,\cos\delta_i\,\mathbf{s}$ may act on the component $\mathbf{S}_o^{\perp}$ perpendicular to $\mathbf{s}$, where $\mathbf{S}_o$ is the angular momentum of the *outer* torus with mass m_o and charge Q_o ($S_o^{\perp} \equiv S_o \sin\delta_o$). For the resulting precession frequency ν_{oi} one obtains

$$\nu_{oi} = -Q_i' \frac{2Gm_s}{c^2 r_o} \nu_i\, x\, f(x) \cos\delta_i, \tag{3.6}$$

where all parameters have already been given before. The quantity $f(x)$ has earlier been defined in (2.2).

Note that the frequencies ν_{mo}, ν_{io} and ν_{oi} contain the same quantity $Gm_s/(c^2 r_o)$, whereas ν_{mi} contains $Gm_s/(c^2 r_m)$. In general, both dimensionless quantities are smaller than unity value for pulsars, so that the frequencies ν_{mo} and ν_{io} are usually smaller than ν_o and are therefore denoted as low-frequency QPOs. An analogous line of reasoning can be applied to the frequencies ν_{mi} and ν_{oi} with respect to ν_i. Therefore, they can also be characterized as low-frequency QPOs. So, a total of four low-frequency QPOs are predicted by the special interpretation of the gravitomagnetic theory.

It is noticed that small angles δ_m, δ_o and δ_i have always been assumed in the derivations of the precession frequencies (3.3), (3.4), (3.5) and (3.6). If all values of δ nearly reduce to zero value, prograde motion of Q_i, m_m and Q_o around $\mathbf{s} = \mathbf{\Omega}_s/\Omega_s$ takes place. Alternatively, retrograde motion of Q_i, m_m and Q_o around $\mathbf{s}$ implies that all values of δ are about 180º.

Furthermore, another remark with respect to the relative magnitudes of ν_{mo} and ν_{io} can be made. Assuming $r_o > r_m > r_i$, implies $x_o > x$. According to tables 1 in both refs. [3, 4], the quantity $g(x_o)$ is then larger than $g(x)$. As a consequence, it follows from (3.3) and (3.4) that the frequency ν_{mo} is larger than ν_{io}. Finally, no sign of any of the frequencies ν_i, ν_m, ν_o, ν_{mo}, ν_{io}, ν_{mi} and ν_{oi} is known at this moment. For convenience sake, positive signs for all frequencies have therefore been used in the calculations below.

4. PARAMETER β*

When both a magnetic field $\mathbf{B}_\mathrm{p}$(gm) from gravitomagnetic origin and a magnetic induction field $\mathbf{B}_\mathrm{p}$(em) from electromagnetic origin are present at the north pole of a star, the total polar magnetic field $\mathbf{B}_\mathrm{p}$(tot) is given by

$$\mathbf{B}_\mathrm{p}(\mathrm{tot}) = \mathbf{B}_\mathrm{p}(\mathrm{gm}) + \mathbf{B}_\mathrm{p}(\mathrm{em}). \tag{4.1}$$

According to (1.4), the direction of $\mathbf{B}_\mathrm{p}$(gm) is parallel to $\mathbf{\Omega}_s$ for $\beta = -1$. It appears helpful to define the following dimensionless quantity β* (see ref. [3])

$$\mathbf{B}_\mathrm{p}^{||}(\mathrm{tot}) = \beta^* \mathbf{B}_\mathrm{p}(\mathrm{gm}). \tag{4.2}$$

When the total field **B**(tot) is from gravitomagnetic origin only, $\mathbf{B}_\mathrm{p}$(em) = 0, and β* reduces to β* = 1. As a rule, measurements yield B(tot), so that only an estimate for β* can be obtained.

Several contributions to the field $\mathbf{B}_\mathrm{p}^{||}$(em) at the north pole of a star have been calculated in ref. [3]. First, a contribution $\mathbf{B}_\mathrm{p}^{||}(\mathrm{em}) = \mathbf{B}_\mathrm{p}^{||}(Q_s)$ is generated by the charge Q_s in the star of radius r_s and with a spin frequency ν_s. Secondly, a contribution $\mathbf{B}_\mathrm{p}^{||}(\mathrm{em}) = \mathbf{B}_\mathrm{p}^{||}(Q_i)$ is generated by the charge Q_i moving in the circular torus of radius r_i. Thirdly, a contribution $\mathbf{B}_\mathrm{p}^{||}(Q_o)$ arises from charge Q_o moving in the circular torus of radius r_o. For a value $\beta = -1$, combination of the gravitomagnetic contribution of (1.4) and the three contributions to $\mathbf{B}_\mathrm{p}^{||}$(em) leads to the following expression for the parameter β* (compare with refs. [3−5])

$$\beta^* = 1 + \beta^*_{\mathrm{current}} + Q_s' + \tfrac{5}{2} Q_i' \frac{\nu_i}{\nu_s} \frac{r_i^2/r_s^2 \cos\delta_i}{\left(1 + r_i^2/r_s^2\right)^{3/2}} + \tfrac{5}{2} Q_o' \frac{\nu_o}{\nu_s} \frac{r_o^2/r_s^2 \cos\delta_o}{\left(1 + r_o^2/r_s^2\right)^{3/2}}, \tag{4.3}$$

where δ_i and δ_o have been defined in section 3. The quantities like Q_s' are again be defined by $Q_s' \equiv (G^{1/2} m_s)^{-1} Q_s$ and so on. Note that the terms in Q_s', Q_i' and Q_o' are due to the contributions $\mathbf{B}_\mathrm{p}^{||}(Q_s)$, $\mathbf{B}_\mathrm{p}^{||}(Q_i)$ and $\mathbf{B}_\mathrm{p}^{||}(Q_o)$, respectively. Furthermore, it is noticed that the terms in (4.3) containing Q_s', Q_i' and Q_o' all possess opposite signs compared with the earlier expression for β* [3−5]. This difference is caused by the choice of $\beta = -1$ in (1.1) instead of $\beta = +1$, since recently an indication is found that the sign of β is negative [10].

The term $\beta^*_{\mathrm{current}}$ in (4.3) has been added to account for a possible contribution to the total magnetic field, due to toroidal currents in the pulsar. For $\beta^*_{\mathrm{current}} = -1$ toroidal currents completely compensate the magnetic field from gravitomagnetic origin. A striking property of (4.3) is that it provides a relation between the high-frequency QPOs ν_o and ν_i, and the spin frequency ν_s. When $r_s \ll r_i$ and $r_s \ll r_o$, the expression β* in (4.3) can be approximated by

$$\beta^* \approx 1 + \beta^*_{\mathrm{current}} + Q_s' + \tfrac{5}{2} Q_i' \frac{\nu_i r_s \cos\delta_i}{\nu_s r_i} + \tfrac{5}{2} Q_o' \frac{\nu_o r_s \cos\delta_o}{\nu_s r_o}. \tag{4.4}$$

The latter relation is used in the calculations below.

Usually, an estimate for the total polar magnetic field $B_p^{||}$(tot) of spin-down pulsars is extracted from the magnetic dipole radiation formula containing the field B_p(sd) (see, e.g., ref. [8])

$$B_p(\mathrm{sd}) = \left(\frac{3c^3 I}{8\pi^2 r_s^{\ 6}} \right)^{1/2} \left(-\frac{\dot{\nu}_s}{\nu_s^{\ 3}} \right)^{1/2} = 3.2\times 10^{19} \left(-\frac{\dot{\nu}_s}{\nu_s^{\ 3}} \right)^{1/2} . \qquad (4.5)$$

The parameter β^* may then be approximated by

$$\beta^* = B_p(\mathrm{sd})/\mathrm{B}_p(\mathrm{gm}). \qquad (4.6)$$

As has been discussed in refs. [3–5, 8], for most recycled uncharged millisecond binary pulsars the parameter $\beta^* = 1 + \beta^*_{\mathrm{current}}$ may approach zero value, so that an asymptotic value of $\beta^*_{\mathrm{current}} = -1$ follows from (4.4).

5. COMPARISON BETWEEN OBSERVATIONS AND THEORY

From observations of the pulsar in the low-mass X-ray binary system GRO J1744-28 [1, 2] a total of five QPO frequencies has been calculated. The three highest centroid frequencies at 56.2, 39.0 and 20.2 Hz of ref. [1] are identified as ν_i, ν_m and ν_o, respectively. The two low-frequency QPOs at 3.75 and 3.25 Hz, obtained from a calculation given in ref. [2], are identified as ν_{mo} and ν_{io}, respectively. The three highest QPO frequencies are characterized by Lorentzian profiles with quality factors Q defined by $Q \equiv \nu/(2\Delta)$, where Δ is the half-width at half maximum. The values of Δ follow from table 1 in ref. [1] (for example, $Q = 56.2/9.8 = 5.7$ for $\nu = 56.2$ Hz).

From the frequencies ν_i, ν_m and ν_o the corresponding Kepler radii r_i, r_m and r_o are calculated (compare with equations (2.1), (2.4) and (2.8)). In these calculations a mass $m_s = 1.4\, m_\odot = 2.7846\times10^{33}$g is adopted for the pulsar. Further, a value of $f_S = 0.9964$ and a radius $r_m = 14.5\times10^6$ cm is calculated from (2.7) by iteration using a spin frequency $\nu_s = 2.14$ Hz for the pulsar. The found radius r_m differs only slightly from the Kepler radius $r_K = 14.6\times10^6$ cm.

For the radius r_i the second term on the right hand side of (2.1), possibly containing a small factor $(Q_s' - x^2 f Q_o')$ and a large factor m_s/m_i, may lead to a deviation from the Kepler radius r_i. Likewise, deviations from the Kepler value of radius r_o may occur (see equation (2.4)). Such deviations have been calculated for five other pulsars: SAX J1808.4–3658, XTE J1807–294, IGR J00291+5934, SGR 1806–20 and Sco X-1 and two black holes: XTE J1550–564 and Sgr A* [3–5]. The more accurate values for r_i and r_o could be deduced, because a complete set of seven QPO frequencies was available for these stars. For this reason, the calculated Kepler values of r_i and r_o for GRO J1744-28 can only be considered as a first approximation.

From the obtained Kepler radii the ratios $x \equiv r_i/r_o$, $x_i \equiv r_i/r_m$ and $x_o \equiv r_m/r_o$ are calculated. Subsequently, from these ratios the values $f(x)$, $g(x)$, $f(x_i)$, $g(x_o)$, as well as the averaged values $\overline{f(x)} \approx f(x)$, $\overline{g(x)} \approx g(x)$, and so on, can be calculated (compare sections 2 and 3 of this work and ref. [3]). When an arbitrarily small value $\delta_o = 10^{\circ}$ is chosen for the outer torus, the absolute value of Q_o' can be calculated from (3.3). Substitution of x, $f(x)$ and $g(x)$, into (2.3) and (2.6) then yields the values for Q_s' and Q_i'. The results are summarized in table 1. Note that the charges Q_s' and Q_i' of GRO J1744-28 are an order of magnitude smaller than for the pulsars in refs. [3, 5]. According to relation (2.3), the ratio Q_o'/Q_s' increases for decreasing values of x. So, for GRO J1744-28 the ratio $Q_o'/Q_s' = +11.1$ is rather large.

Using the values of $\overline{g(x_o)} \approx g(x_o)$ and $\overline{g(x)} \approx g(x)$ from table 1, combination of (3.3) and (3.4) yields the following (absolute) ratio $\nu_{mo}/\nu_{io} = \overline{g(x_o)}/\overline{g(x)} = 1.51/1.25 = 1.21$, whereas the (absolute) ratio of the observed frequencies is equal to $\nu_{mo}/\nu_{io} = 3.75/3.25 = 1.15$. In order to obtain full agreement with observations, one may choose a lower value for $\overline{g(x_o)}$, whereas $\overline{g(x)}$ is left unchanged. Substitution of the value of $\overline{g(x)}$ from table 1 and the observed values of ν_{mo} and ν_{io} into $\overline{g(x_o)} = \overline{g(x)}\ \nu_{mo}/\nu_{io}$, yields a value $\overline{g(x_o)} = 1.44$. From this result follows $x_o = 0.616$, $r_o = 23.6\times10^6$ cm, $r_i = 11.9\times10^6$ cm, $x_i = 0.821$, $Q_o' = 0.744$, $Q_s' = 0.0669$ and $Q_i' = -0.0534$. All these values can be compared with the corresponding values from table 1: $x_o = 0.643$, $r_o = 22.6\times10^6$ cm, $r_i = 11.4\times10^6$ cm, $x_i = 0.786$, $Q_o' = 0.681$, $Q_s' = 0.0612$ and $Q_i' = -0.0488$, whereas x and $\overline{f(x)}$ remain the same (the value $\delta_o = 10^\circ$ is also left unchanged). Although the choice of $\overline{g(x_o)} = 1.44$ leads to agreement with observations, alternative choices are possible. For example, one may use $\overline{g(x_o)} = 1.51$ and the observed values of $\nu_{\rm mo}$ and $\nu_{\rm io}$ from table 1 and substitute them into $\overline{g(x)} = \overline{g(x_o)}\ \nu_{io}/\nu_{mo}$. Evaluation then yields other values for x, r_i, and so on. Note that replacement of the value $\overline{g(x_o)} = 1.51$ by 1.44 only slightly affects the values of r_i and r_o. Therefore, the values for r_i and r_o in our calculations will be approximated by the corresponding Kepler radii.

Table 1. Observed centroid QPO frequencies for the pulsar in GRO J1744-28. If available, quality factors Q and integrated fractional r.m.s. amplitudes are added. Relative radii x, x_i and x_o, radii r_i, r_m, and r_o, relative charges Q_s', Q_i' and Q_o' (Q' is defined by $Q' \equiv (G^{1/2}\, m_s)^{-1} Q$) and factors $\overline{f(x)}$, $\overline{f(x_i)}$, $\overline{g(x)}$ and $\overline{g(x_o)}$ are given. For comment see text.

ν (Hz)	Q	r.m.s. ampl. (%)	x	$R\times10^6$ (cm)	Q'	$\overline{f(x)}$ [h]	$\overline{g(x)}$ [h]	δ (°)
ν_i 56.2[a]	5.2[a]	0.19[b]		r_i 11.4	$-Q_i'$ 0.0488			δ_i 10
ν_m 39.0[a]	2.09[a]	5.9[b]		r_m 14.5	Q' 0			
ν_o 20.2[a]	5.7[a]	0.42[b]		r_o 22.6	Q_o' 0.681			δ_o 10
ν_s 2.14[b,c]				r_s 1	Q_s' 0.0612			
ν_{mo} 0.375[d]		5 – 15[d]	x_o 0.643	r_o 22.6	Q_o' 0.681		$\overline{g(x_o)}$ 1.51	δ_o 10
ν_{io} 0.325[d]		5 – 15[d]	x 0.506	r_o 22.6	Q_o' 0.681		$\overline{g(x)}$ 1.25	δ_o 10
ν_{mi} 0.068[e]			x_i 0.786	r_m 14.5	$-Q_i'$ 0.0488	$\overline{f(x_i)}$ 1.13		δ_i 10
ν_{oi} 0.0088[f]			x 0.506	r_o 22.6	$-Q_i'$ 0.0488	$\overline{f(x)}$ 0.352		δ_i 10
$\nu_{\rm LT}(m_i)$[g] 1.2×10^{-10}				$R = r_i$ 11.4				

[a] From table 1 in ref. [1]. [b] Ref. [1]. [c] Ref. [15]. [d] Ref. [2]. [e] Calculated from (3.5). [f] Calculated from (3.6). [g] Calculated from (1.7). [h] Definitions and a discussion of these quantities have been given in sections 2 and 3 of this work and in ref. [3].

For five pulsars and two black holes a complete set of four low-frequency QPOs has been observed [3–5]. In that case more stringent conditions can be imposed on the values of the radii r_i and r_o. However, in the case of GRO J1744-28 the low-frequency QPOs ν_{mi} and ν_{oi} have not been observed. An estimate for these frequencies can be made by substitution of all necessary parameters and an arbitrarily chosen value $\delta_i = 10^\circ$ into (3.5) and (3.6). The calculated values for ν_{mi} and ν_{oi} in table 1 show that these values may not be observable.

Furthermore, the predicted Lense-Thirring precession $\nu_{\rm LT}$ of the inner torus with radius r_i and mass m_i is calculated from (1.7). Its value appears to be negligible small. All additional results are also given in table 1. Comparison of the corotation radius $r_{\rm cor} = \{Gm_s/(2\pi\nu_s)^2\}^{1/3} = 10.1\times10^6$ cm shows that all obtained radii r_i, r_m and r_o are larger than $r_{\rm cor}$.

A value for the observed magnetic field $B_{\rm p}$(tot) for GRO J1744-28 can be obtained from *XMM-Newton* and *INTEGRAL* observations from 2014, reported by D'Aì, Di Salvo, *et al*. [15]. They attributed features in the X-ray spectrum as cyclotron resonance scattering features (CRSFs) and identified an *electron* cyclotron fundamental at ~ 4.7 keV and hints for two possible harmonics. In addition, Doroshenko *et al.* [16] reported an absorption feature at ~ 4.5 keV from *Beppo*SAX observations in 1997; they interpreted it as a cyclotron line, too.

From the value of 4.7 keV an approximate absolute value of the polar magnetic field $B_{\rm p}({\rm tot}) = 5.30\times10^{11}$ G can be calculated (see, e.g., ref. [8]). This result can be compared with the prediction by the gravitomagnetic theory (see, eq. (4) in ref. [8]), yielding a value of the polar field $B_{\rm p}({\rm gm}) = 1.16\times10^{14}$ G for $\nu_s = 2.14$ Hz and $\beta = -1$. For the parameter β^* then follows a small value $\beta^* = B_{\rm p}({\rm tot})/B_{\rm p}({\rm gm}) = \pm\ 0.0046$ from (4.2). This result shows that for the mildly-recycled pulsar of GRO J1744-28 the value of β^* already approaches to zero value. Comparison of this result with fourteen other more slowly rotating accretion-powered X-ray emitting pulsars (table 2 in ref. [8]) shows an increasing (absolute) value of β^* for a decreasing value of the spin frequency ν_s of the pulsar, ranging from $\beta^* = 0.1$ to $\beta^* = 53$. Neglecting β^* for the relative slowly rotating pulsar in GRO J1744-28 and taking the necessary parameters from table 1, the following approximate value for $\beta^*_{\rm current}$ can be calculated from (4.4)

$$\beta^*_{\rm current} = -1 - 0.06({\rm from}\,Q_s) + 0.24({\rm from}\,Q_i) - 0.62({\rm from}\,Q_o) = -1.44. \qquad (5.1)$$

So, when a positive charge Q_s is assumed for the mildly-recycled pulsar, the value for $\beta^*_{\rm current}$ becomes more negative than for than the asymptotic value $\beta^*_{\rm current} = -1$, discussed in section 4.

From simultaneous *Chandra*/HETG-*NuSTAR* observations of the X-ray spectrum of GRO J1744-28 Younes *et al.* [17] deduced an iron line complex at 6.7 keV, whereas Doroshenko *et al*. [16] also found a single fluorescence iron line at 6.7 keV. In ref. [17] the iron line complex has been resolved into three narrow Gaussian emission lines from neutral and/or near neutral Fe at 6.44 keV and from highly ionized Fe XXV and Fe XXVI at 6.65 keV and at 6.99 keV, respectively (see their table 4 and section 4.3). In addition, Degenaar *et al.* [18] also deduced related spectral lines from *Chandra*/HETG observations (see their table 1).

Following our model, the emission line at ~ 6.4 keV from neutral and lowly ionized iron may be attributed to the uncharged middle torus at $r_m = 14.5\times10^6$ cm. In addition, the inner torus of radius $r_i = 11.4\times10^6$ cm with a relatively small negative charge Q_i ($Q_i/Q_s = -\ 0.88$, see table 1) may also contribute to this line. Furthermore, the lines at rest energies 6.7 keV and at 6.9 keV from highly ionized Fe XXV and Fe XXVI, respectively, may illustrate the presence of the positive charge Q_s in the pulsar and the large charge Q_o in the outer torus of radius $r_o = 22.6\times10^6$ cm ($Q_o/Q_s = +\ 11.1$, see table 1). Especially, the combined observation of emission lines of neutral iron and highly ionized iron may confirm the simultaneous presence of an electrically neutral middle torus and a positively charged pulsar and outer torus.

When the broad emission complex at 6.7 keV results from reflection from an accretion disk, an estimate can be made for the inner radius $r_{\rm in}$ of the accretion disk. Using diskline modelling, Degenaar *et al.* [18] deduced an estimate of $r_{\rm in} = 85r_{\rm g}$ ($r_{\rm g} \equiv Gm_s/c^2$) or $r_{\rm in} = 18\times10^6$ cm. Using diskline models too, values of $r_{\rm in} = 50\text{-}115r_{\rm g}$ =

(10-24)×10^6 cm and $r_{\rm in}$ = 130$r_{\rm g}$ = 27×10^6 cm were calculated by other authors [15; 17, section 3.3.3], respectively.

Another way to probe the dimensions and electron temperatures of the tori may be possible by interpreting the observed phase lags between hard and soft photons. Such hard phase lags in the pulsed emission of GRO J1744-28 are recently discovered by D'Aì, Burderi, *et al.* [19]. As an explanation of these lags, they considered a Compton reverberation mechanism and applied the following formula to the time lag

$$t_{\rm lag} = \frac{R_{\rm cc}}{c\,(1+\tau)} \frac{\ln(E_h/E_s)}{\ln\{1+4\Theta(1+4\Theta)\}} \equiv k\ln(E_h/E_s), \tag{5.2}$$

where $R_{\rm cc}$ is the radius of the Compton cloud, τ the optical depth and Θ the electron temperature defined by $\Theta \equiv kT_e/m_ec^2$. E_h and E_s are the energies of the hard and soft photons, respectively. From their figure 4 they calculated a value k_1 = 7.0 ± 0.1 ms for the energy range E < 6.6 keV. Taking a value of 7.1 keV for kT_e and a fixed optical depth τ = 1, calculation leads to a radius $R_{\rm cc}$ of 240 km = 24×10^6 cm for size of the Compton cloud. Inserting the values of k_1, E_s = 1.3 keV and E_h = 6.6 keV into (5.2), yields a value of $t_{\rm lag}$ = 11.4 ms, in agreement with figure 4 of ref. [19].

Following our approach, we replace the radius $R_{\rm cc}$ by the radius for the inner torus r_i = 11.4×10^6 cm in the expression k_1 of (5.2) (τ = 1 is left unchanged) and then obtain kT_e = 3.4 keV. Likewise, replacement of $R_{\rm cc}$ in k_1 by the radius of the middle torus r_m = 14.5×10^6 cm leads to kT_e = 4.3 keV. In addition, for the energy range E > 6.6 keV a value k_2 = 4.5 ms has been calculated in ref. [19]. Substitution of the value of the radius of the outer torus r_o = 22.6×10^6 cm for $R_{\rm cc}$ (and of τ = 1) into the expression of k_2, then yields a value kT_e = 10 keV. Note that combination of k_2, E_h = 6.9 keV and E_s = 1.3 keV from ref. [19] yields a value $t_{\rm lag}$ = 7.5 ms, in agreement with their figure 4 for E_h = 6.9 keV. In our approach kT_e is no longer constant, but becomes dependent on radius of the torus.

It is noticed that the value kT_e = 10 keV of the outer torus is much higher than that of the inner torus with kT_e = 3.4 keV and the middle torus with kT_e = 4.3 keV. This high value is remarkably close to the weak line at 10.4 keV in the X-ray spectrum detected by D'Aì, Di Salvo *et al.* [15] and the feature at ~ 10 keV deduced by Younes *et al.* [17] (see their table 4).

A related example of such a dependence is worth mentioning. The three tori model has also been applied to calculate Earth's net charge and the charges of the van Allen belts [14]. In that case the minimum and maxima in the equatorial radial omnidirectional electron fluxes at electron energies of, e.g., 3 MeV appeared to depend on the radial distance from the Earth (see, e.g., Vette [20]). From this dependence the radii r_i, r_m and r_o have been estimated in that case.

6. SUMMARY AND CONCLUSIONS

For the pulsar in the low-mass X-ray binary system GRO J1744-28 three high-frequency QPOs have been reported by Zhang *et al.* [1]. This pulsar is a new example of a growing list of celestial bodies compatible with the so-called three tori model. Other triplets of high-frequency QPOs are previously discussed for five other pulsars, one white dwarf and two black holes [3−5]. More recently, van Doesburgh and van der Klis [21] found three high-frequency QPOs for six pulsars in low-mass X-ray binaries: 4U 1728-34, 4U 0614+09, 4U 1608-52, 4U 1636-53, 4U 1702-43 and Aquila X-1. For these pulsars the three high-frequency QPOs can clearly be distinguished (see the D rows in their figures 4 and 5): L_u (*yellow*), L_l (*magenta*) and L_{hHz} (*orange*), corresponding to ν_i, ν_m and ν_o in our notation. The three tori model has also been applied to calculate Earth's net charge and the charges of the van Allen belts [14]. It is striking that so much widely different systems can be described by the same three tori model.

In this work the three highest QPO frequencies of GRO J1744-28 at 56.2, 39.0 and 20.2 Hz reported by Zhang *et al.* [1] are put equal to the Kepler frequencies of the orbital motion of three circular tori around the pulsar. From these three QPO frequencies the approximate radii of three tori are calculated (see table 1). In addition, two narrow band QPOs with comparable amplitudes at 3.75 Hz and 3.25 Hz for GRO J1744-28 were deduced by Kommers *et al.* [2]. The latter two QPOs may be explained by a special interpretation of the gravitomagnetic theory [3−9]. From that theory follows that the three proposed tori are subjected to four low-frequency precessions. Compared to earlier work [3−5] the expressions for these four low-frequency QPOs have been revised. For the accreting pulsar of GRO J1744-28 the two low-frequency QPOs at 3.75 Hz and 3.25 Hz may be attributed to the latter precession mechanism. In a first approximation, the relative charges Q_i' and Q_o' for the inner and outer torus, respectively, and the relative charge Q_s' for the central star can be extracted from these QPOs (Q_i' is defined by $Q_i' \equiv (G^{½} m_s)^{-1} Q_i$ and so on). It appears that two additionally predicted low-frequency QPOs may be too low to be detected. A summary of these results has been given in table 1.

For five other pulsars and two black holes a complete set of four low-frequency QPOs has been observed [3−5]. From the seven observed QPOs frequencies of each star more accurate values for the three radii of the tori have been calculated. Since only five QPO frequencies have been reported for GRO J1744-28, only Kepler radii could be calculated in this case.

Another consequence of the previously proposed gravitomagnetic theory is the prediction of a polar magnetic field $\mathbf{B}_p$(gm) for electrically neutral celestial bodies. Furthermore, magnetic fields from electromagnetic origin may be generated by a rotating pulsar with charge Q_s, by toroidal currents in the pulsar, by an inner torus with charge Q_i and by an outer torus with charge Q_o. The total effect of all these fields is embodied in the parameter β^*, whereas the parameter β^*_{current} describes the toroidal currents. Both parameters for the pulsar in GRO J1744-28 are calculated, discussed and can be compared with the values of other pulsars (see section 5).

Highly ionized iron emission lines at 6.7 keV and 6.9 keV (see refs. [15–18]) may confirm the presence of the positive charge Q_s in the pulsar and the positive charge Q_o in the outer torus. The emission line at 6.4 keV from neutral/lowly ionized iron are compatible with an electrically neutral torus in the middle and with the small negative charge Q_i in the inner torus. An attempt has been given to relate these three energies to the radii of the three tori. These radii can be compared with estimates of the inner radius of the accreting disk obtained from diskline modelling in refs. [15, 17, 18]. Recently, D'Aì, Burderi, *et al.* [19] detected a discontinuity in the phase lags of GRO J1744-28. This discontinuity may be explained by a Compton cloud mechanism, in which the electron temperature depends on the radius of the torus.

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