TRUE GRAVITATIONAL CONSTANT, SCHWARZSCHILD RADIUS, BLACK HOLES, AND RELATED ISSUES

Jaswant Rai Mahajan

While all objects having mass also possess energy, energy itself has no rest mass. But both mass and energy interact gravitationally. Therefore, the real gravitational constant is G/c^4, instead of simple G, while the interacting partners are represented by their respective energy (Mc^2, mc^2). The new formulation leads to some interesting results, which are derived and discussed in the pdf of this communication.

Introduction

In late 17th century (~1687), Sir Isaac Newton inter-related inertia, force, and mass in his Laws of Motion, while also described a mathematical formula for predicting the universal gravitational force between the interacting masses: $F = G\ M\ m/\ R^2$. These laws have proved very successful even after the advent of Einstein's **Relativity** (1905 – 1915), which takes over when the interacting bodies are moving with very high velocity ($v \rightarrow c$) and possess total energy (Mc^2) far above their respective **Rest Mass Energy** (M_0c^2; m_0c^2). Moreover, during Relativity studies, Einstein also established the equivalence of mass and energy ($E = Mc^2$) and connected it to the rest mass energy of the object:
$E = Mc^2 = M_0\ c^2\ /\ (1– v^2/c^2)^{1/2}$. Furthermore, **Einstein's Field Equations (EFE)** and its variations employ mass/energy density, the energy tensors, and two modifications of the Universal Gravitational Constant, G/c^2 and G/c^4, instead of mass and simple G:

$$R_{uv} -1/2\ R\ g_{uv} + \Lambda\ g_{uv} = 8\pi\ \mathbf{G/c^4}\ T_{uv}$$

Λ (lambda) is the famous **Cosmological Constant**, while other symbols have special technical meanings as established in the Relativity equations. But despite the imperative use of the modified parameters in Relativity, the true nature and the origin of G have not been stressed and made clear to the students of physical sciences and much less to the public at large. Just check for yourself the great popularity of $E = Mc^2$ against the obscurity of G/c^4 [1, 2].

Well, this lapse and lack of popularity is possibly due to the fact that while human beings have been comparing the weights (masses) of miscellaneous objects for millenniums, the concept of energy (potential, kinetic, binding, etc.) and especially the rest mass energy content are just limited to the scholars in the scientific fields. Moreover, the vast majority of scientists - except the specialists in Relativity and High Energy Physics – are just dealing with usual objects under ordinary conditions, where their total energy is equal or very close to their rest mass energy ($Mc^2 = M_0c^2$) and the modified gravitational expression, $F = Mc^2\ x\ mc^2\ x\ G/c^4\ R^2$, becomes equal to the much simpler classical Newtonian formulation: $F = G\ M\ m/\ R^2$.

The True Nature of the Gravitational Constant

Nevertheless, the fact remains that while energy without any rest mass is known (EMR or Photons), there is no known object possessing mass, but lacking energy! Consequently, the real gravitational constant is G/c^4, while the interacting bodies are represented by their respective energy (Mc^2 and mc^2). Thus, when we substitute masses in Newton's gravitational force or the potential energy formulas, we commit omissions, both in the numerator and denominator ($c^2 \times c^2 / c^4 = 1$), which fortunately are not fatal, but lead to the correct result.

At this point, let us recall that Einstein's fame and his Relativity got a real boost, when an English team lead by Sir Arthur Eddington, during a total solar eclipse in 1919, proved that the light (EMR: Energy) coming from a star behind our sun was really bent by sun's gravitational field. Furthermore, it is now well established that the gravitational attraction of EMR caused by the intervening galaxies, under the proper conditions, produces the phenomenon of **Gravitational Lensing**.

The True Gravitational Constant (G/c^4) and its Implications

It is interesting to note some of the important information conveyed by the classical Newtonian and its equivalent new gravitational formulations. For instance, $Mc^2 \times G/c^4 = M \times G/c^2 = R_C$, which in classical terms would represent the radius of the circular orbit for an object moving with velocity c (e.g. photons, v = c), *if the gravitational source (M) was contained within this limiting sphere*. Moreover, the above expression immediately conveys that $M \times G = R_C \times c^2$, which is very explicit in the new formulations, but is just hidden in the classical notation. Similarly, extrapolation of $MG/R_C = c^2$ to $MG/R = v^2$, indicates that each and every gravitational source interacts with (through) G/c^4 and creates around itself potential energy gradient (curvature), which is directly proportional to mass/energy, but varies inversely with the distance from the gravitational source. And the same is true for the force field or the centripetal gravitational acceleration: $G M/ R^2 = v^2 /R$. Thus, these gravitational gradients, though expressed in the classical language, guide the celestial objects in their trajectories and correspond to the modern concept of space-time curvature. For instance, a journey towards higher potential energy (P. E.) represents an uphill task and the celestial body spends some of its kinetic energy (K. E.) to traverse it; while the opposite is true for rolling downhill towards the gravity source and more negative P. E. In short, though space-time do not show us the familiar hills and valleys of our homeland, the spatial upward and downward potential energy gradients guide the celestial objects in their journey. *And this is even true for photons, which undergo gravitational **red shift** or **blue shift**, depending on whether these are travelling away from or towards the gravitational source.*

Furthermore, the interaction between two objects, $Mc^2 \times mc^2 \times G/c^4 = M\ m\ G$, may be written as $(Mc^2\ G/c^4)\ mc^2 = R_C \times mc^2$ or $Mc^2\ (mc^2\ G/c^4) = Mc^2 \times r_c$, which is not available from the classical

expression. Just check these observations for the interactions of Sun (M = 2 x 10^{30} kg) and Earth (m = 6 x 10^{24} kg); G = 6.6735 x 10^{-11} m^3/s^2 kg.

Sun: M G = 1.3347 x 10^{20} m^3/s^2 = 9 x 10^{16} m^2/s^2 x 1483 m = c^2 x R_C, where last item is Sun's R_C.
Earth: m G = 4.0041 x10^{14} m^3/s^2 = 9 x 10^{16} m^2/s^2 x 4.449 x10^{-3} m = c^2 x r_c, where last item is Earth's r_c.
Sun and Earth: M G m = 8.0082 x 10^{44} J m = 1.8 x 10^{47} J x 4.449 x 10^{-3} m = 5.4 x 10^{41} J x 1483 m.

Yet, in spite of the clarity and exactness of the above mathematical formulations, photons' circular orbit radius (R_C) sounds unfamiliar in Relativity, possibly due to the present knowledge of the Rest Mass and the prohibitive energy cost of the **Relativistic Effects** at very high velocities, which make it mathematically clear ($E = Mc^2 = M_0 c^2 / (1– v^2/c^2)^{1/2}$) that no material object can attain the velocity c, which is the natural speed of light (EMR) in vacuum. Thus, due to the relativistic effects suffered by the material objects, the so called **"photon sphere"** has been calculated [3] to be 3 times larger than R_C and one and a half (1.5) times bigger than the very famous **Schwarzschild radius,** $R_S = 2\ MG/c^2 = 2\ R_C$, which delimits the sphere from which no projectile – ***not even photons - can escape***! *Consequently, if the source of gravitation (M) could be compressed within this limiting sphere, such a sphere would be invisible and has been denominated a* ***Black Hole***.
Incidentally, R_C being buried within the Schwarzschild radius would be invisible / un-detectable.

Schwarzschild Radius and Black Holes

Historically, Schwarzschild radius was the 1st exact solution to Einstein's Field Equations providing the gravitational field (or the curvature of space-time) *outside a non-rotating, spherically symmetric body*. However, at the time of its derivation (1916) it was just a mathematical curiosity, because Black Holes were not fully admitted till the 2nd half of the 20th century, when these objects became a hot topic for theoretical research. Its derivation employing the Schwarzschild metric and tensors is rather involved and complicated, but in the classical formulation ($R_S = 2\ MG/c^2$) it determines the limiting radial distance beyond which c becomes the **escape velocity** of an object (EMR).
With the passage of some decades, the advent of **space age** and the ensuing **space race**, coupled with the great advances of **observational cosmology -** made possible by sophisticated instruments, computer technology, and ever larger, more potent and diverse types of telescopes (optical, radio, IR, UV, X – ray, and especially the orbiting **Hubble Telescope**, etc.) - have greatly enlarged the horizons of the observable universe, providing fascinating images of both the near as well as the very far away galaxies. These observations have revealed spectacular cosmological phenomenon never seen before: multiple images of distant **Quasars** due to **gravitational lensing**, merging

galaxies, jets of energy and particles shooting out of the galactic centers with speeds approaching that of light, very distant Supernovas, etc... Consequently, Black Holes are now thought to be ubiquitous and **Super Massive Black Holes** (SMBH) are believed to be present at the center of almost all galaxies, where *an imaginary spherical surface with the Schwarzschild radius is believed to represent their* **Event Horizon,** which delimits the outermost boundary of a Black Hole [1, 2].

But after all this elaborate description about the existence of Black Holes, the question arises: Whether in our Universe, where everything is spinning and moving, authentic Schwarzschild Black Holes - *spherically symmetrical and non-rotating* - can really exist? So it is comforting to find that the specialists and experts in the field agree that the properties of the spinning **Kerr Black Holes** are not much different from the hypothetical stationary ones.

However, after this encouraging news about the Black Holes, another doubt has been crossing my mind. I wonder if the formulas derived for the *orbiting and escape velocities of material objects* would apply to EMR. Material objects have rest mass and well-defined kinetic energy component (K. E. = $mv^2/2$), which varies as the square of its velocity. To stay in an orbit, the K. E. should be one half of its gravitational P. E. and counterbalanced by the binding energy (B. E.). Thus, to escape from this orbit, the object must overcome its B. E. by increasing its velocity and raising the K. E. just above its P. E. Hence, v^2 -escape ≥ 2 v^2 -orbit or v-escape $\geq \sqrt{2}$ v-orbit. Alternatively, an object with the orbiting velocity at R_C could escape from a distance just beyond 2 R_C (R_S), where the P. E. is reduced to below one half of its value at R_C.

But in a sharp contrast to the material objects, EMR has a constant velocity (c, in vacuum) and the energy of a photon is given by $E = hf = h\,c/\lambda = mc^2$. Consequently, as h and c are the Universal Constants, photons can change their energy only by increasing or decreasing their wavelength (λ). Furthermore, the total energy of a photon appears and functions as kinetic (mc^2). In fact, it is the famous "**quantum**" of energy. *And needless to add that EMR knows no 'Rest' and thus has no Rest Mass; it is born from action upon the Universal Medium (Absolute or un-polarized Vacuum) and comprises movement, which carries energy and momentum at the top speed 'c' permitted in vacuum. Thus, to fulfill its destiny, the undulating EM fields of a photon wiggle on and on till it encounters a partner to whom it can pass on its "quantum" of energy and finally rest in peace - or 'a state of relative non-existence'!*

Now, let us check some of the facts. At R_S, the gravitational potential is just $c^2/2$, while photons' energy is given by mc^2, that is double the P. E. value and far above the escape energy! Thus, photons are free to escape and would lose only one half of its energy to attain infinite distance from the G –source (λ-final = 2 λ-initial). And at R_C, the P. E. (mc^2) is just equal to the energy of photons, leaving no room for binding energy (B. E.). Thus, EMR should escape from just above R_C,

instead of waiting for R_S. However, as mentioned earlier, EMR may not be able to escape from this region, as it is buried under the Schwarzschild radius (R_S) - populated by plasma and energetic particles. But it could very well interact with plasma and the matter particles present in the $R_C - R_S$ girdle, giving rise to some detectable phenomenon, such as the recently observed energy and plasma jets shooting out from the galactic center towards its poles on the flatter side of the galactic disc.

But, what about the relativistic effects on the **Effective Mass** of photons ($m = E/c^2 = hf/c^2 = h/c\ \lambda$), which is responsible for their gravitational interaction (G – interaction)? Well, according to the true G-constant (G/c^4), photons' interaction with the gravitating mass/energy is given by: $Mc^2 \times G/c^4 \times hf = M\ G \times hf/c^2$, which amounts to the respective "effective mass" interaction with G/c^4, as already mentioned in the Abstract and discussed during the the implications of the true gravitational constant. Thus, any changes in the energy of a photon are also reflected in its G –interactions.

In short, this is the puzzle or dilemma about R_C, R_S, and R-escape, as applied to EMR. Consequently, the eagerly awaited findings of the **Event Horizon Telescope – EHT**, would be very instructive and could shed some light on the projected properties of a Black Hole: It's **Schwarzschild radius**, or the extent of its **Event Horizon** and its Edge; an estimate of its energy density; the presence or absence of the **Photon Sphere**; the plasma whirlpools and jets and their composition; and any clue about the presence / absence of the long-sought **Singularity**, etc.

Nevertheless, in spite of these lingering doubts, but encouraged by the observational evidence for the presence of Black Holes, let us examine some physical characteristics of a few illustrative, but hypothetical Schwarzschild Radius Black Holes (Table 1).

Preamble:

But before visiting the Table(s), I would like to point out that the intention of this study is to develop a logical argument, instead of finding the exact figures for the 'vital statistics' of our universe. Thus, the approximate values of the Universal Constants and the assumed variables used herein are listed below:

Planck constant, $h = 6.63 \times 10^{-34}$ J s / cycle

Reduced Planck constant, $\hbar = h/2\pi = 1.055 \times 10^{-34}$ J s / radian

Speed of light (EMR) in vacuum, $c = 3 \times 10^8$ m / s

Interaction parameter between EMR and its Inner Vacuum, $\hbar c = 3.165 \times 10^{-26}$ J m / radian

Light year = 365.25 days = 3.15576×10^7 s = 9.46728×10^{15} m

Mass of Sun: 2×10^{30} kg; Mass of Earth: 6×10^{24} kg

Assumed Mass of each Galaxy: 10^{12} solar masses = 2×10^{42} kg

Assumed Number of Galaxies in our Universe: 10^{12}

Assumed Total Mass and Energy (EMR + Matter) of the Universe = 2×10^{54} kg; (1.8×10^{71} J)

Conventional Gravitational constant, $G = 6.6735 \times 10^{-11}$ m^3/s^2 kg

Semi Schwarzschild radius gravitational constant, $G^* = G/c^2 = 7.415 \times 10^{-28}$ m/kg

True gravitational constant, $G^{\#} = G/c^4 = 8.2389 \times 10^{-45}$ s^2 / kg m (1/Force; 1/N; m/J)

Inverse of $G^{\#}$, $1/G^{\#} = c^4/G = 1.21375 \times 10^{44}$ kg m/s^2 (Force; N; J/m)

Table 1: Physical Properties of some Hypothetical Schwarzschild-Radius (R_S) Black Holes

Example	Mass (kg)	$R_S = 2GM/c^2$ (m)	Volume (m^3)	Density (kg / m^3)
1. Earth	6×10^{24}	8.898×10^{-3}	2.951×10^{-6}	2.03×10^{30}
2. Sun	2×10^{30}	2.966×10^{3}	1.093×10^{11}	1.83×10^{19}
3. Sun x 10^3	2×10^{33}	2.966×10^{6}	1.093×10^{20}	1.83×10^{13}
4. Sun x 10^6	2×10^{36}	2.966×10^{9}	1.093×10^{29}	1.83×10^{7}
5. Sun x 10^9	2×10^{39}	2.966×10^{12}	1.093×10^{38}	18.3
6. Sun x 10^{12}	2×10^{42} (**Galaxies**)	2.966×10^{15}	1.093×10^{47}	1.83×10^{-5}
7. Sun x 10^{24}	2×10^{54} (**Universe**)	2.966×10^{27}	1.093×10^{83}	1.83×10^{-29}

Comments: The physical characteristics of Black Holes are usually illustrated with our Sun, which provides a very compact (R_S = 2.966 km) and extremely dense (1.83×10^{19} kg/m^3) example (Row 2). But as R_S is directly proportional to mass, while the volume varies as the cube of the radius, the mass density of black holes drops very rapidly with the increase of its mass, becoming extremely low for the higher members of the above Table. And on lowering the mass, the trend is reversed. The other very important purpose of the above Table is to draw your attention to the fact that almost all the material objects, including those not shown in the Table – atoms, molecules, aggregated matter, Moon, Earth, Sun, stellar systems, galaxies, and even the galaxy clusters - have physical dimensions several orders of magnitude larger than their respective Schwarzschild radius – *that is, their total mass is not contained within this limiting radius*. Just compare the estimated radius of Earth (6.4×10^6 m) with its calculated R_S (8.898×10^{-3} m; Row 1). Now, let us jump to a galaxy-size super massive example. Our **Milky Way Galaxy**, which is estimated to contain 100 – 400 billion (10^{11} to 4×10^{11}) stars, is spread across 100 – 180 thousand light years. If its total mass amounted to one trillion solar masses (Row 6), its *calculated* R_S (2.966×10^{15} m) would be a mere 0.3133 lightyear! Therefore, the Black Hole at the center of the Milky Way (**Sagittarius A***), incorporating a few million solar masses, would be just a spec as viewed from Earth, which is estimated to be 30 – 50 thousand light years from the galaxy's center. And as there are about a trillion galaxies in our Universe, there is expected a similar number of Black Holes, both big and small.

Now, in sharp contrast to the examples examined so far, when we extrapolate to the total assumed (estimated) mass/energy of the Universe (2 x 10^{54} kg; Row 7) – ***which on a large scale is observed to be homogeneous and uniform in all directions*** – its calculated R_S (2.966 x 10^{27} m; 313.29 billion light years) and even its *Semi Schwarzschild Radius* (R_C = 1.483 x 10^{27} m = 156.645 x 10^9 light years) are respectively 22.7 and 11.35 times larger than the presently estimated age and extent (13.8 x 10^9 light years = 1.3065 x 10^{26} m) of our Universe! Furthermore, the Cosmologists are puzzled to verify that the Cosmos is undergoing an accelerated expansion, obliging them to propose the presence of **"Dark Energy", *having anti-gravity properties***!

Thus the inescapable conclusion is that though we live far away from our local Black Hole (Sagittarius A*), all of us are living inside the Universal Black Hole. Furthermore, the gigantic "Merry-Go-Round" of the Cosmos is still expanding to reach its Semi Schwarzschild Radius (R_C = 1.483 x 10^{27} m) and possibly even to attain its Schwarzschild radius (R_S = 2.966 x 10^{27} m).

But as a matter of fact, we do not experience any bizarre effects of being inside a Black Hole! And the Universe as a whole is expanding and even accelerating, instead of collapsing under the influence of its gravitational field. Furthermore, the space within the gravitationally bound groups forming the galaxies and the stellar systems does not expand; only the intergalactic space (Void) between the unbound galaxies increases, as the Universe as a whole is expanding. Consequently, either the criteria and definition of a Black Hole need some revision or there must be some other explanation for this peculiar behavior.

Well, one possible explanation for the observed facts is that although the Universe appears statistically homogeneous and uniform when examined on a very large scale, it is not so homogeneous at the smaller scale of its component galaxies and the stellar systems, where the local gravitational fields paint a different picture. Thus, though all objects attract each other mutually, the local strong fields predominate over the long range weak fields and determine the behavior of objects in their near vicinity. Thus, we are all gravitationally bound to Earth, while the Moon – about 380,000 km away – balances its negative gravitational potential energy with its positive kinetic energy by going around the Earth. And both combined as a system orbit the Sun to avoid their fall from the heavens into the nuclear furnaces of the sun! Other planets and their satellites behave in a similar manner. In their turn, the stellar systems revolve around their respective partners (if any) and the galactic center. Thus, there are hierarchies of gravitational fields within fields, which govern the individual movements of the trillions of components of a galaxy, resulting in a coherent, but an extremely dynamic system. And the same is true for the galaxy clusters, when the influence of gravity can bring their members close enough for a mutual embrace. But even the far flung galaxies - which are unable to throw their gravitational arms around each other - form a fairly well-knit physical structure of the Cosmos. However, there are differences between the closely-knit

and the loosely-knit members of the Universe. Thus, as pointed out earlier, when the cosmologists examine the Cosmos at a very large scale, they verify that the space within the galaxies and their stellar components does not expand, conserving their overall dynamic structures. But in a sharp contrast, the intergalactic space (Void) among the weakly interacting galaxies undergoes expansion, increasing their separation, while maintaining the radial (commoving) configuration of the Universe.

*Combining all these very significant observations collected during the past several decades brings to mind the image of an independently expanding "**Balloon of Space**" in which the materialized dynamic components - galaxies and their stellar systems, gathered and governed by their mutual local gravitational fields - are suspended and dispersed, enjoying an almost independent existence.* Furthermore, it points out that while the gravitational fields within the galaxies and among the tightly bound galaxy-clusters are stronger than the push of the expanding space, the intergalactic gravitational pull among the loosely-bound galaxies is weaker than the push of the space expansion!

Finally, the true nature of the gravitational field, coupled with the presence of two independent but opposing fields – the centripetal G-field trying to aggregate matter and compact it into smaller and smaller volume, versus the centrifugal Vacuum-field expanding our Universe to larger and larger volume – has led to an alternative model for our Cosmos, which will be shared as soon as some pending issues have been satisfactorily addressed.

References and Notes

1. The topics discussed herein are familiar to the professionals and students of Physics and Cosmology. Important items are often **highlighted** in the text for internet search for any further and up-to-date information. Additional references are pinpointed only in some special cases, when deemed necessary / desirable. Similarly, the new ideas, novel concepts, and important conclusions are also highlighted and / or shown in *italics* to draw reader's attention.
2. "100 Years of General Relativity – Einstein", *Scientific American*, Special Issue, September 2015.
3. Wikipedia: Schwarzschild radius; see Relativistic circular orbits and the photon sphere.