Conjecture on a subset of Woodall numbers divisible by Poulet numbers

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Abstract. The Woodall numbers are defined by the formula W(n) = n*2^n – 1 (see the sequence A003261 in OEIS). In this paper I conjecture that any Woodall number of the form 2^k*2^(2^k) – 1, where k ≥ 3, is either prime either divisible by a Poulet number.

Conjecture:

Any Woodall number of the form 2^k*2^(2^k) – 1, where k ≥ 3, is either prime either divisible by a Poulet number.

Note: see the sequence A003261 in OEIS for Woodall numbers n*2^n – 1 up to n = 300).

Verifying the conjecture:
(for the first seven such Woodall numbers)

: W(2^3) = W(8) = 2047 (= 23*89) which is a Poulet number;

: W(2^4) = W(16) = 1048575 (= 3*5^2*11*31*41) which is divisible by 341 (= 11*31) and 13981 (= 11*31*41), both Poulet numbers;

: W(2^5) = W(32) = 137438953471 (= 223*616318177) which is a Poulet number;

: W(2^6) = W(64) = 1180591620717411303423 (= 3*11*31*43*71*127*281*86171*122921) which is divisible at least by 341 (= 11*31), 5461 (= 43*127), 19951 (= 71*281), 24214051 (= 281*86171), all four Poulet numbers;

: W(2^7) = W(128) = 43556142965880123323311949751266331066367 (= 7*31*73*151*271*631*23311*262657*348031*49971617830801) which is divisible at least by 4681 (= 31*151) and 15841 (= 7*31*73), both Poulet numbers;

: W(2^8) = W(256) = 29642774844752946028434172162224104410437116074403984394101141506025761187823615 (=

3^2*5*7*13*17*23*67*89*241*353*397*683*2113*7393*20857*312709*599479*4327489*1761345169*2931542417*98618273953) which is divisible at least by 2047 (= 23*89), 137149 (= 23*67*89), 745889 (= 353*2113), 8280229 (= 397*20857), 15621409 (= 2113*7393), all five Poulet numbers;

: W(2^9) = W(512) is a number with 157 digits which is prime (see the sequence A002234 in OEIS: "Numbers n such that the Woodall number n*2^n – 1 is prime").