Five Part Harmony

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Abstract

This text demonstrates that the complex *i* can be combined with a Hamilton style quaternion to produce a 5-D mathematical structure. Essentially, the complex plane is combined with an arbitrary unit vector. The complex *i* is shown to anti-commute with the unit vectors **i**, **j**, and **k**. The resulting geometry is shown to be an extension of Hamilton's quaternions based upon the complex plane rather than real numbers. This new geometric structure is presented in Figure 1 and Equations 3 through 3.3. This configuration makes it possible to calculate the diameter of the proton at rest with the estimated value being 1.668×10^{-15} meter. This is within the accepted measured range of the proton diameter at $1.755(102) \times 10^{-15}$ meter as given by the NIST, and it is very close to the proton diameter at $1.68174(78) \times 10^{-15}$ meter as measured at the Paul Scherrer Institute in 2010 by using muonic hydrogen.

Preface

Knowledge of quaternions and Linear Algebra is required. This essay was written for the 2016 FQXi Essay Contest. This work contains excerpts from other works by the author. All of the work is original. None of the work is formally published. However, the works are posted to the website viXra.org.

"Absorb what is useful. Discard what is useless. Add what is specifically your own." - Bruce Lee

Discussion

Summary:

The topic of this essay contest concerns the link – if any – between mindless mathematics and the time evolution of goal oriented systems. This requires that there be a method of assigning a "direction" to time.

Time is often viewed as a "dimension" in both Mathematics and Physics. Herein, the author will respectfully disagree. Herein, time is viewed as a scalar value and the complex *i* is viewed as a dimension. This facilitates the combination of Hamilton's quaternions with the complex plane.

Background:

The center-piece of this essay is the following wave-function:

Equation 1:

$$\boldsymbol{\Psi} = e^{i\omega}e^{\mathbf{Q}} = e^{i\omega+q_0+\mathbf{q}} = e^{i\omega}e^{q_0}e^{\mathbf{q}} = e^{\mathbf{P}};\ \mathbf{P} = i\omega + \mathbf{Q}$$

Equation 1.1:

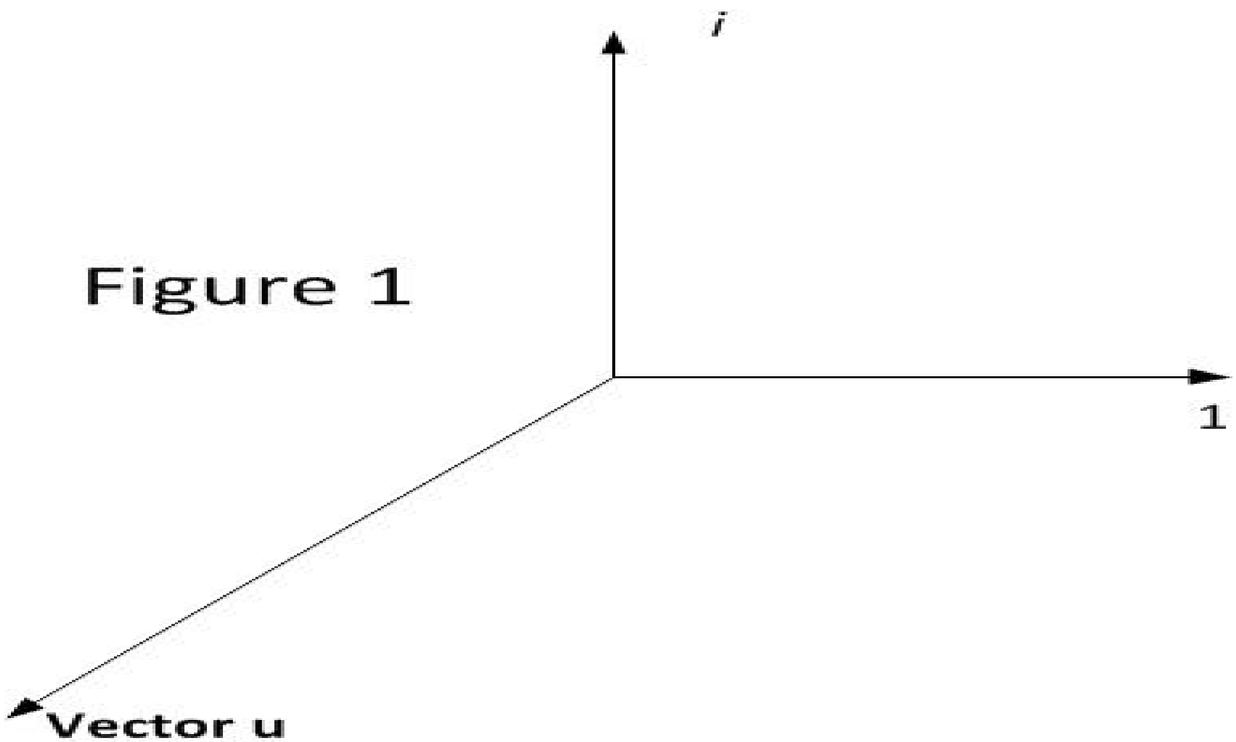
$$\mathbf{Q} = q_0 + \mathbf{q} = q_0 + q_i\mathbf{i} + q_j\mathbf{j} + q_k\mathbf{k}$$

The five unit dimensions are the scalar value one, the complex *i*, and the three unit vectors **i**, **j**, and **k**. To visualize the space created by these dimensions, it is necessary to go back several hundred years to when mathematicians first attempted to represent a two-dimensional plane. They placed scalar values along the x-axis. They then associated the y-axis with the complex *i* and thereby created the complex plane. The geometry presented here places an arbitrary unit vector **u** at the origin of the complex plane. This arbitrary unit vector is oriented perpendicular to the complex plane in accordance with the right-hand rule. These five axes now constitute a five-dimensional space. This is represented as Figure 1.

Equation 1 has several interesting features. It can never be zero. It can be as large as desired. It can be as small as desired. But it can never be zero. The real terms can be zero or the complex terms can be zero. But they cannot simultaneously both be zero. The vector terms can be zero or the scalar term can be zero. But they cannot simultaneously all be zero.

Also, **Equation 1** is in the form necessary to apply the separation of variables method. Since it uses exponentials, it automatically satisfies a generic wave equation and many other differential equation forms. The usual approach is to make the complex phase angle ω a function of time thereby giving the wave-function the following form:

$$\mathbf{\Psi} = T(t)Q(\mathbf{Q})$$



Equation 1 is simply Euler's Equation multiplied by the exponential of a quaternion. Euler’s Equation is:

Equation 1.2:

$$e^{i\omega} = \cos(\omega) + \sin(\omega)\, i$$

Euler’s Equation is represented by a line segment of unit length in the complex plane. One end-point is at the origin and the other end-point is at the point x=cos(ω), y=sin(ω)*i*.

The exponential of the quaternion can be expressed as follows:

Equation 1.3:

$$e^{\mathbf{Q}} = e^{q_0}\left[\cos(\gamma_0) + \sin(\gamma_i)\,\mathbf{i} + \sin(\gamma_j)\mathbf{j} + \sin(\gamma_k)\mathbf{k}\right] = e^{q_0}[\cos(\gamma_0) + L\mathbf{u}\,]$$

Equation 1.3 was developed by the author in reference [1] pages 13-15. The exponential of the quaternion is equal to a vector perpendicular to the complex plane plus a scalar value.

Equation 1.3.1:

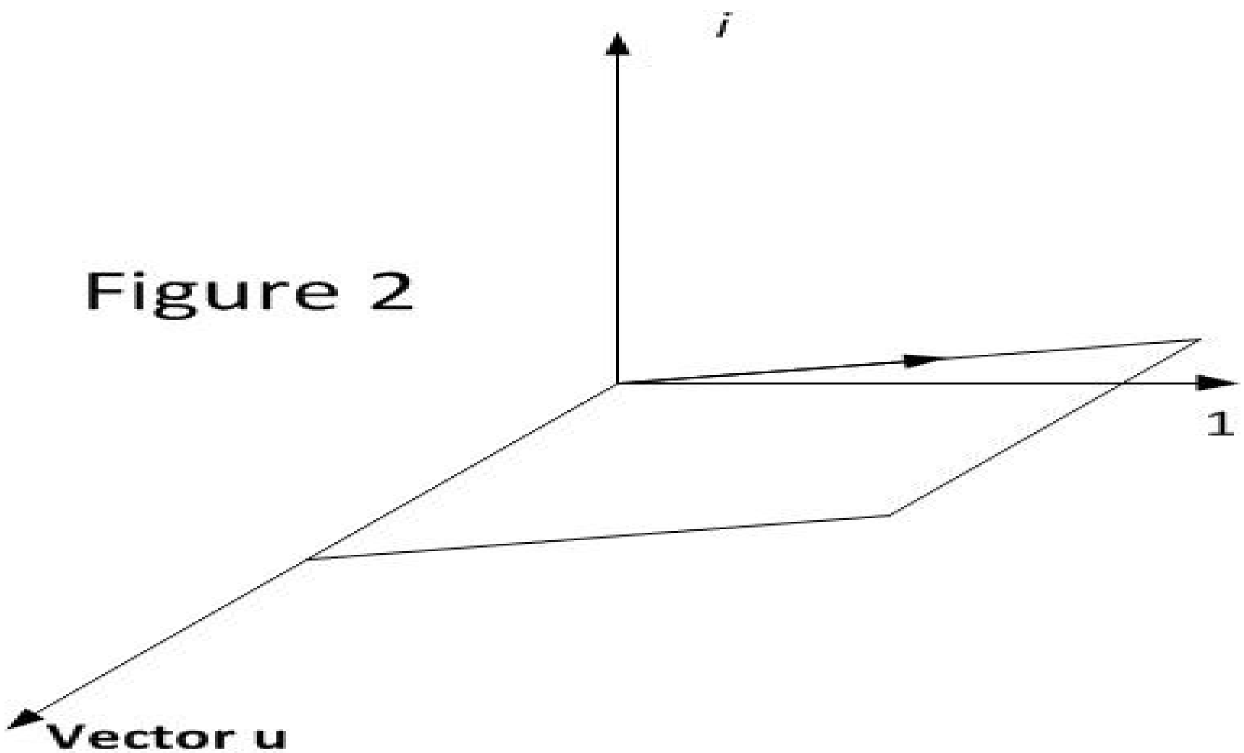
$$L\mathbf{u} = \sin(\gamma_i)\,\mathbf{i} + \sin(\gamma_j)\mathbf{j} + \sin(\gamma_k)\mathbf{k}\ ; L = \sqrt{sin^2(\gamma_i) + sin^2(\gamma_j) + sin^2(\gamma_k)}$$

Here, L is the length of the vector portion of **Equation 1.3** and **u** is a unit vector in the direction of the vector portion of **Equation 1.3**. Therefore, **Equation 1** can be written as:

Equation 1.4:

$$\boldsymbol{\Psi} = [\cos(\omega) + \sin(\omega)\,i]e^{q_0}[\cos(\gamma_0) + L\mathbf{u}\,]$$

Equation 1.4 is the sum of a line segment in the complex plane plus a rectangle in the 5-D space of Figure 1. The line segment is equal to Euler's Equation in the complex plane multiplied by the scalar portion of the quaternion. The rectangle is perpendicular to the complex plane. Its edges are specified by Euler's Equation in the complex plane and by the quaternion's vector portion along the other edge. This is illustrated in Figure 2. The exponential of q_0 can be distributed between these terms.



The rectangle that is perpendicular to the complex plane has four dimensions since it has one fewer dimension than the five-dimensional space. Given that time is frequently combined with the complex *i*, the author will ***speculate*** that this rectangle can be made to represent four-dimensional space-time. It will be shown below that the complex *i* anti-commutes with the unit vectors **i**, **j**, and **k**. The author will also ***speculate*** that this anti-commutation combined with the line segment in the complex plane is closely associated with the "arrow of time". The only question concerns the phase angle ω in the complex plane. The author will address this question next.

The following ***hypothesis*** is proposed:

Equation 2:

$$e^{i\omega} = \cos(\omega) + \sin(\omega)\,i = \sqrt{1 - \frac{v^2}{c^2}} + \left(\frac{v}{c}\right) i\,;\, v\, represents\, motion$$

Anti-Commutation:

When a quaternion is multiplied by its conjugate, the result is a scalar value. Also, when a complex number is multiplied by its conjugate, the result is a scalar value. Therefore, let us multiply a **pentuple** and its conjugate. This should also produce a scalar value.

$$(a_0 + ai + \mathbf{a})(a_0 - ai - \mathbf{a})$$

$$a_0 a_0 + ai a_0 + \mathbf{a} a_0 +$$

$$-a_0 ai - (ai)(ai) - \mathbf{a} ai +$$

$$-a_0 \mathbf{a} - ai\mathbf{a} - \mathbf{aa}$$

The red terms cancel and the green terms cancel since the scalar value a_0 commutes normally. This simplifies to the following:

$$a_0^2 + a^2 - \mathbf{a}^2 - \mathbf{a}ai - ai\mathbf{a}$$

Notice the order of multiplication between the complex *i* and the vector **a** in the two blue terms. **The author will now invoke a radical concept. If the complex *i* anti-commutes with the unit vectors contained within vector a, then the two blue terms will sum to zero**! The above expression will then simply be a scalar equal to the sum of the five squares. The author presently has no reason to think that the scalar axis anti-commutes. The pentuple used above is a simplified case since the complex vector terms are zero. This makes the anti-commutation behavior more obvious.

Matrix Form:

Since **Equation 1.2** has two terms and **Equation 1.3** has four terms, it follows that 8 terms will be produced when they are multiplied together. Therefore, **Equation 1** leads to octonions, and **Equation 1** can be thought of as a five-term subset of the octonions. It is fairly straight-forward to express this octonion system in matrix form. This is done in reference [2] pages 26-29. An alternative to that 8x8 matrix system is presented next.

The author will now introduce a new structure that will alter the presentation of this subset of octonions **O**.

$$\mathbf{O} = \mathbf{A} + i\mathbf{B}\,; \mathbf{A} \in \mathbf{Q}, \mathbf{B} \in \mathbf{Q}$$

$$\mathbf{A} = a_0 + a_i\mathbf{i} + a_j\mathbf{j} + a_k\mathbf{k}\,;\ \mathbf{B} = b_0 + b_i\mathbf{i} + b_j\mathbf{j} + b_k\mathbf{k}$$

$$\mathbf{O} = (a_0 + a_i\mathbf{i} + a_j\mathbf{j} + a_k\mathbf{k}) + i(b_0 + b_i\mathbf{i} + b_j\mathbf{j} + b_k\mathbf{k})$$

$$\mathbf{O} = (a_0 + b_0 i) + (a_i + b_i i)\mathbf{i} + (a_j + b_j i)\mathbf{j} + (a_k + b_k i)\mathbf{k}$$

Equation 3:

$$\mathrm{A}_0 = \begin{bmatrix} a_0 - \Delta a_0 & \Delta b_0 \\ \Delta a_0 & b_0 - \Delta b_0 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix}\,;\ \mathrm{A}_i = \begin{bmatrix} a_i - \Delta a_i & \Delta b_i \\ \Delta a_i & b_i - \Delta b_i \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\mathrm{A}_j = \begin{bmatrix} a_j - \Delta a_j & \Delta b_j \\ \Delta a_j & b_j - \Delta b_j \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix}\,;\ \mathrm{A}_k = \begin{bmatrix} a_k - \Delta a_k & \Delta b_k \\ \Delta a_k & b_k - \Delta b_k \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

Equation 3.1:

$$\mathbf{P_A} = \mathrm{A}_0 + \mathrm{A}_i\mathbf{i} + \mathrm{A}_j\mathbf{j} + \mathrm{A}_k\mathbf{k} = \mathbf{O}$$

Equation 3.1 contains all of the information of the octonion **O**. The only assumption that is built into **Equation 3.1** is that the various “a” and “b” scalar values commute normally with the complex *i*. The column matrix composed of [1 + *i*] now represents the complex plane. Multiplication of this column matrix by one of the unit vectors **i**, **j**, or **k** produces a quasi 3-D building block as presented in Figure 2. Therefore, **Equation 3.1** represents a method of constructing a five-dimensional space using 3 quasi 3-D building blocks ($\mathrm{A}_i\mathbf{i}$, $\mathrm{A}_j\mathbf{j}$, $\mathrm{A}_k\mathbf{k}$) and the complex plane (A_0). **Equation 3.1** is essentially a Hamilton style quaternion based upon the complex plane rather than real numbers. To be consistent with the form of **Equation 1**, the various A_x terms must be the following:

Equation 3.2:

$$a_0 = \cos(\omega)\, e^{q_0} \cos(\gamma_0)\,;\ a_i = \cos(\omega)\, e^{q_0} \sin(\gamma_i)\,;\ a_j = \cos(\omega)\, e^{q_0} \sin(\gamma_j)\,;\ a_k = \cos(\omega)\, e^{q_0} \sin(\gamma_k)$$

$$b_0 = \sin(\omega)\, e^{q_0} \cos(\gamma_0)\,;\ b_i = \sin(\omega)\, e^{q_0} \sin(\gamma_i)\,;\ b_j = \sin(\omega)\, e^{q_0} \sin(\gamma_j)\,;\ b_k = \sin(\omega)\, e^{q_0} \sin(\gamma_k)$$

Multiplication of two pentuples takes the following matrix form:

Equation 3.3:

$$\mathbf{P_A P_C} = \begin{bmatrix} +\mathrm{A}_0 & 0 & 0 & 0 \\ 0 & +\mathrm{A}_0 & 0 & 0 \\ 0 & 0 & +\mathrm{A}_0 & 0 \\ 0 & 0 & 0 & +\mathrm{A}_0 \end{bmatrix}\begin{bmatrix} \mathrm{C}_0 \\ \mathrm{C}_i \\ \mathrm{C}_j \\ \mathrm{C}_k \end{bmatrix} + \begin{bmatrix} 0 & -\mathrm{A}_i & -\mathrm{A}_j & -\mathrm{A}_k \\ +\mathrm{A}_i & 0 & -\mathrm{A}_k & +\mathrm{A}_j \\ +\mathrm{A}_j & +\mathrm{A}_k & 0 & -\mathrm{A}_i \\ +\mathrm{A}_k & -\mathrm{A}_j & +\mathrm{A}_i & 0 \end{bmatrix}\begin{bmatrix} \mathrm{C}_0^* \\ \mathrm{C}_i^* \\ \mathrm{C}_j^* \\ \mathrm{C}_k^* \end{bmatrix}$$

Equation 3.3 is developed by the author in reference [2] pages 36-38.

After a few moments of consideration, the following identity becomes clear:

Equation 3.3.1:

$$\mathbf{P_A P_C} + \mathbf{P_A P_{C^*}} = \begin{bmatrix} +A_0 & -A_i & -A_j & -A_k \\ +A_i & +A_0 & -A_k & +A_j \\ +A_j & +A_k & +A_0 & -A_i \\ +A_k & -A_j & +A_i & +A_0 \end{bmatrix} \begin{bmatrix} C_0 \\ C_i \\ C_j \\ C_k \end{bmatrix} + \begin{bmatrix} +A_0 & -A_i & -A_j & -A_k \\ +A_i & +A_0 & -A_k & +A_j \\ +A_j & +A_k & +A_0 & -A_i \\ +A_k & -A_j & +A_i & +A_0 \end{bmatrix} \begin{bmatrix} C_0^* \\ C_i^* \\ C_j^* \\ C_k^* \end{bmatrix}$$

Furthermore, the complex *i* terms associated with C_x will cancel out in **Equation 3.3.1** leaving two copies of pentuple **A** multiplied by the real portion of pentuple **C**.

The author is not yet prepared to state the inverse of this relationship. In reference [2] pages 24-25 the author demonstrated that the octonion coefficient matrix could be inverted by pre-multiplying by the complex conjugate and by post-multiplying by the quaternion conjugate. Something similar might be applicable here. The problem is complicated by the anti-commutation of the complex *i* and by the question of the validity of multiplication associativity.

Four-Vectors:

Now let us produce a four-vector from the wave-function given in **Equation 1**. This can be done by taking the natural logarithm of **Equation 1**:

Equation 4:

$$\ln(\boldsymbol{\Psi}) = \mathbf{P} = i\omega + \mathbf{Q} = i\omega + q_0 + \mathbf{q}$$

It is very tempting to compare this to a four-vector from Relativity by equating ω with ct and setting q_0 equal to 0. However, that would be incorrect because the units for ω and for the coefficients of **Q** are radians since they were in the exponential. Instead, a four-vector can be produced as follows:

Equation 4.1:

$$\frac{n\lambda}{2\pi}[\ln(\boldsymbol{\Psi}) - q_0] = \frac{n\lambda}{2\pi}(i\omega + \mathbf{q}) = ict + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})\,; \frac{n\lambda}{2\pi}\omega = ct\,; \frac{n\lambda}{2\pi}\mathbf{q} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

or

Equation 4.2:

$$\ln(\boldsymbol{\Psi}) = i\omega + \mathbf{Q} = \frac{2\pi}{n\lambda}[ict + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})] + q_0\,; \omega = \frac{2\pi}{n\lambda}ct\,; \mathbf{q} = \frac{2\pi}{n\lambda}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

The value λ is a length that is used to convert between the pentuple form and the four-vector form. The λ value is essentially a wavelength. The "n" term is simply the number of wavelengths. Therefore, the quantity nλ is the length associated with one cycle of 2π radians. In principle, "n" should be an integer. However, there is no rigid mathematical requirement that this be true. The q_0 term must be added to the four-vector to produce an object or structure that fits into the pentuple format. **Therefore, to be compatible with the wave-function presented in Equation 1, both SR and GR must be written using a four-vector combined with a scalar term**. Of course, the scalar term can be zero. It is worth mentioning that Einstein was aware of something similar to this when he studied the Kaluza-Klein Equation and the master rejected the idea of a scalar field. The author will ***speculate*** that the scalar term is related to the vacuum energy and/or the cosmological constant. The author will also ***speculate*** that it is possible for the scalar term to be a function of time. For example, consider the confusion that would result if the following ***speculation*** were true:

$$let\ q_0 = \frac{2\pi}{n\lambda} ct$$

The concepts of scalar time and complex time would be completely confused!!! Nature would never be so devious - or would it?

Proton Diameter:

Support work for this argument is presented in references [3] and [4]. That work is based upon a simplified version of **Equation 1**. The following could simply be a coincidence. However, this coincidence allows for a calculation that is hard to ignore. The mass ratio (M_p/M_e) between the proton and the electron is 1836.15267245(75) as given by the NIST. This is one of the most accurately known parameters in Physics. This value is very close to $6\pi^5$ (1836.118109). The observed deviation between the measured value and $6\pi^5$ is attributed to ***absolute motion*** of the observer by application of **Equation 2**. The calculated magnitude for the associated motion is .006136 c. This velocity is typical of a stellar explosion. It would be easy enough to dismiss this as nonsense or numerology except for one thing. The same model makes a prediction for the size of the proton. The calculated value for the diameter of the proton is 1.668 x 10^{-15} meter. This is within the accepted measured range of the proton diameter at 1.755(102) x 10^{-15} meter as given by the NIST and it is very close to the proton diameter at 1.68174(78) x 10^{-15} meter as measured at the Paul Scherrer Institute in 2010 by using muonic hydrogen. The equation used here for the proton diameter calculation is equation 13.4 from reference [4]:

$$d = 4\sqrt{3}\left(\frac{1}{mc}\right)\left(\frac{h}{2\pi}\right)\ln(\pi) = \frac{2\sqrt{3}}{\alpha} q_0$$

Therefore, the author will place the burden upon the reader. How is it possible that a coincidence can make such a surprisingly good estimation for the diameter of the proton using such a simple equation? Hopefully, there is a better answer than "another coincidence".

Predictions:

The model presented here allows for several testable predictions.

The simplest prediction is that the mass ratio (M_p/M_e) between the proton and the electron is dependent upon the ***absolute velocity*** of the observer. A velocity with respect to the Earth of 432 km/hr is sufficient to produce values that would be outside the accepted range for the mass ratio (M_p/M_e). A satellite in orbit could easily accomplish this. The observed affect would be to increase the ***variance*** in the measured data. The average value would be expected to remain unchanged since the satellite would still be in Earth's orbit and would therefore share the Earth's reference frame on average.

A more difficult test involves the Stern-Gerlach experiment. The prediction is that the distance between the two streams of atoms is dependent upon the ***absolute velocity*** of the test apparatus.

If the Earth is in absolute motion, then the observed cosmological red-shift data should have a bias in the direction of motion. The prediction is that the red-shift in the direction of motion should be decreased and the red-shift in the opposite direction should be increased.

Conclusions

There are three conclusions that are supported by this work. The first is that physical reality has five dimensions. This is supported by the accuracy of the theoretical calculation for the proton diameter based upon the general wave equation presented herein. The second is that the reference frame of the Earth is moving at 0.006136 c with respect to a true rest frame. This conclusion is necessitated by the empirical accuracy of the measured proton to electron mass ratio (M_p/M_e). Essentially, the deviation of M_p/M_e from $6\pi^5$ must be explained and **Equation 2** provides a degree of freedom to achieve this. The third conclusion is that the complex *i* anti-commutes with the unit vectors **i**, **j**, and **k**. This is a feature of the 5-D geometry. It is certainly possible – perhaps even likely – that these conclusions are false. However, if that is the case then the reader is left to explain the proton diameter calculation.

Acknowledgements

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