A Proof of Goldbach's Conjecture

Chongxi Yu

Techfields Inc.

1679 S Dupont HYW, Dover, Delaware, USA

Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach's conjecture is about all numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, we divided any even numbers into 10 groups and primes into 4 groups, Goldbach's conjecture becomes much simpler. Here we give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture, fundamental theorem of arithmetic, Euclid's proof of infinite primes

Introduction

Prime numbers[1] are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved.

I believe that prime numbers are "basic building blocks" of the natural numbers and they must follow some very simple basic rules and do not need "advanced mathematics tools" to solve them. Two of the basic rules are the "fundamental theorem of arithmetic" and Euclid's proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, [1] which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.[2] Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof[2] that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just **n** primes, then the biggest prime will be labeled $\mathbf{P_n}$. Now we can form the number Q by multiplying together all these primes and adding 1, so

$$\mathbf{Q = (P_1 \times P_2 \times P_3 \times P_4 ... \times P_n) + 1}$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n.

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

If N is an even integer:

N= N/2 + N/2 = (N/2+m) + (N/2-m); m = 0, 1, 2, 3,……M. We need to prove [(N/2+m] and [N/2-m] can be primes at same time.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach’s conjecture is about all numbers, the pattern of prime numbers likes a “kaleidoscope” of numbers, if we divide all even numbers into 10 groups and primes into 4 groups, Goldbach’s conjecture will be much simpler.

If a number (N>3) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 11, it have 1/3 chance is divisible by 3 and 1/7 chance is divisible by 7, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, and 1/11 chance to be divisible by 11, so on, so we have terms: 1/3, 1/7x2/3, 1/11x2/3x6/7, 1/13x2/3x6/7x10/11…,

Let N_o represent any odd number, the chance of N_o to be a non-prime is: [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + …]} ----------Formula 1

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5.

Let ∑ represent the sum of the infinite terms and Δ=1-∑, according to Euclid's proof[3] that the set of prime numbers is endless. Δ is the chance of any odd number to be a prime. ∑ may be very close to 1 when N is growing to ∞, but always less than 1. Let Δ=1-∑, when N is growing to ∞, Δ may be very close to 0, but always more than 0 according to Euclid's proof that the set of prime numbers is endless. If Δ is 0, then there is no prime, that is not true.

The sum of first 20 terms = [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +
(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +
(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +
(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +
(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) +
(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +
(1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) +
(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +
(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73)
=[0.333333 + 0.095238 + 0.051948 + 0.039960 +0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.010328 + 0.009370 + 0.008436 + 0.007538 + 0.006543 + 0.005766 + 0.005483 + 0.004910 + 0.004564 + 0.004377 + 0.003831] =0.689015

For the first 20 term: ∑ = 0.689015, Δ=1-∑ =0.310985

The chance of N_o to be a prime is: Δ=1-[[(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + …]} ------------Formula 2

Let us consider the following cases:

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 110, 120, 1120, 1130,…, then N/2 has only 0 or 5 as its last digit:

1a. Except 5 and 2, any prime must have 1, 3, 7, or 9 as its last digit. When both N and N/2 have 0 as their last digit, then N must be 20, 40, 60, 80, 100, 120,…, N. For enough large number N, Let's consider $N=O_1+O_2$ = (N/2+L+3) + (N/2-L-3), O_1 and O_2 is an odd number. $O_1>O_2$, $O_1-O_2=2L+6$, L=0, 5, 10, 15, 20, 25, 30….L, $O_1-O_2=2L+6$=6, 26, 46, 66, 86, 106, 126,…,(2L+6), then O_1 is an odd number with 3 as its last digit, O_2 is an odd number with 7 as its last digit.

Also we can have $N=O_1+O_2$=(N/2+L+7)+(N/2-L-7), O_1 and O_2 is an odd number. $O_1>O_2$, $O_1-O_2=2L+14$, L=0, 10, 20, 30….L, $O_1-O_2=2L+14$=14, 34, 54, 74, 94, 114, 134,…(2L+14), then O_1 is an odd number with 7 as its last digit, O_2 is an odd number with 3 as its last digit.

Then, we have odd number pairs as listed in table 1:

Table 1. The odd number pairs in $N=O_1+O_2$=(N/2+L+3)+(N/2-L-3) and $N=O_1+O_2$=(N/2+L+7)+(N/2-L-7)

N-7	N-17	N-37	N-47	N-67	N-97	…	N/2+L+3	N/2-L-7	…	83	73	53	43	23	13	3
7	17	37	47	67	97	…	N/2-L-3	N/2+L+7	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let $1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,…; $3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193….; $7 represents a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197…; and $9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,….

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,…; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,….; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77…; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,….

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number with 3 as its last digit is a product of $3x$1 or $7x$9; $1 is decided by $3 or $3 is decided by $1and $9 is decided by $7 or $7 is decided by 9, so we need to consider only $3 and $7, $1 and $9, $3 and $9, or $1 and $7.

For number N, there are 5 x N/10 odd numbers, N/10 odd numbers with 1 as its last digit, N/10 odd numbers with 3 as its last digit, N/10 odd numbers with 5 as its last digit, N/10 odd numbers with 7 as its last digit, and N/10 odd numbers with 9 as its last digit. Odd numbers with 5 as its last digit is not primes except 5. According to Euclid's proof, primes are endless and it is easy to prove that prime with 1, 3, 7, or 9 as its last digit is also endless.

Let's select $3 to be a product of $3 and $1 or $7 and $9. If a number (N>3) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime; any number is divisible by 7, it have 1/3 chance is divisible by 3; any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, so on, so we have terms: 1/3, 1/7x2/3, 1/13x2/3x6/7…. For number N, there are N/10 odd number with 1 as its last digit, N/10 odd number with 3 as its last digit, N/10 odd number with 7 as its last digit, and N/10 odd number with 9 as its last digit.

The chance of any odd number with 3 as its last digit to be a non-prime is: [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]} ----------(Formula 3)

The number (n) of primes in N/10 odd number with 3 as its last digit: n_3=N/10 -{N/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]} ----------(Formula 4)

For infinite terms, the number will grow slowly and will be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid's proof of endless prime numbers. Let $\sum_3$ represents the sum of the above infinite terms and Δ_3 represents the chance of any odd number to be a prime. When N is growing to ∞, and $\Delta_3=1-\sum_3$ may be close to 0, but never be 0.

The sum of first 20 terms = [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23)+(1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) +

(1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) +
(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) +
(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83) +
(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) +
(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) +
(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) +
(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) +
(1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) +
(1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137) +
(1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157) =
[1/3+1/10.5+1/22.75+1/32.23+1/46.33+1/77.92+1/93.07+1/104.15+1/120+1/164.61+1/171.01+1/197.14+1/233.2+1/250.2+1/262.47+1/279.80+1/317.27+1/344.97+1/398.24+1/416.11=[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333+0.006075+0.005848+0.0050773+0.004288+0.003997+0.003810+0.003574+0.003152+0.002899+0.002511+0.002403+0.002331] =0.6102883

For the first 20 term: $\sum_3$ = 0.6103, Δ_3=1-$\sum_3$ =0.3897

For N=600, the smallest prime \$1 is 11, it decided the possible largest prime \$3 is 53, the smallest prime \$9 is 19, but 3 x 3 is 9, so the possible largest prime \$7 is 47, 47 x 3 x3=423 (the next will 67 x 3 x3=603>600), so we have: Prime number with 3 as its last digit=600/10 –{600/10[(1/3)+ (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47)] = 600/10-600/10[0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333 = 60-60x0.566654 = 60-34 = 26, it is 3 less than 29 primes (in total 60 odd number) with 3 as their last digit from 1 to 600 because 600 is not a big enough number, when N is big enough, the calculated number will be very close to the real number of primes . For N=600, we have Δ_3=1-$\sum_3$=1-0.566654 = 0.433346, every odd number with 3 as its last digit has almost 43% chance to be a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime; for a number bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime.

Every odd number with 7 as its last digit is a product of $3x$9 or $7x$1; $1 is decided by $7 and $9 is decided by $3, so we need to consider only $3 and $7 (we have other selections too, $9 and $7, $9 and $1, or $3 and $1).

The chance of any odd number with 7 as its last digit to be a non-prime is: [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]} ----------(Formula 3)

The number (n) of primes in N/10 odd number with 7 as its last digit is: n=N/10 -{N/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + …]} --------Formula 4.

That mean we have almost same number of primes with 7 as their last digit as the number of primes with 3 as their last digit. From above formula, we can know smaller number N has high percentage to be primes than bigger number N.

Let $\sum_7$ represent the sum of the above infinite terms. $\sum_7 = \sum_3$, when N is growing to ∞, and $\Delta_7 = 1 - \sum_7$ may be close to 0, but never be 0.

For simple, to find out at least one pair primes of (N/2+m) + (N/2-m), we need to fix (N/2+m) or (N/2-m) to be a prime as the list (half in left side and half in right side) in table 1. Let see $7=7 first (left side of table 1), we can know there is a bigger chance for (N-$7) to be a prime with 3 as its last digit than (N-O7). If N-7 can be divisible by 7, then (N-7)+7 [7a + 7 = 7(a+1), 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 0 as its last digit and only 70, 140, 210, 280, 350, 420, 490, 560, 630,… are divisible by 7, but we worked on only N and N/2 with 0 as their last digit, only 140, 280, 420, 560,…,(1 in 14) can be divisible by 7, so the term (1/7x13/14) should be taken off from Formula 3.

For the next prime $7=17, (N-17) cannot be divisible by 17 except 340, 680,…, so (1/17x33/34) should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($7) with 7 as their last digit in any number N, the chance of every N-$7 with 3 as its last digit to be a prime is: $\Delta_3 = 1 - \sum_3$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/7x13/14)] + (1/17x33/34) +(1/37x73/74)….]}---------Formula 5

When the number of $7 is 5 or more, 5 x[(1/7x13/14) + (1/17x33/34) + (1/37x73/74)…]>1, so every 5 primes with 7 as their last digit ($7) will have at least 1 prime of N-$7 to form 1 pair of primes in which one has 7 as its last digit and another has 3 as its last digit and their sum is any number N in which N and N/2 have 0 as its last digit.

For enough large number N, Let's consider $N=O_1+O_2=(N/2+L+1)+(N/2-L-1)$, O_1 and O_2 are odd numbers. $O_1>O_2$, $O_1-O_2=2L+2$, L=0, 10, 20, 30….L, $O_1-O_2=2L+2=2$, 22, 42, 62, 82, 102, 122,…(2L+2), then O_1 is an odd number with 1 as its last digit, O_2 is an odd number with 9 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+9)+(N/2-L-9)$, O_1 and O_2 are odd numbers. $O_1>O_2$, $O_1-O_2=2L+18$, L=0, 10, 20, 30….L, $O_1-O_2=2L+18=18$, 38, 58, 78, 98, 118, 138,…, (2L+18), then O_1 is an odd number with 9 as its last digit, O_2 is an odd number with 1 as its last digit.

These odd number pairs are listed in table 2:

Table 2. The odd number pairs in $N = O_1 + O_2 = (N/2+L+1) + (N/2-L-1)$ and $N = O_1+O_2=(N/2+L+9)+(N/2-L-9)$

N-9	N-19	N-29	N-39	N-59	N-79	N-89	…	N/2+L+1	N/2-L-9	…	101	71	61	41	31	11
9(3x3)	19	29	39	59	79	89	…	N/2-L-1	N/2+L+9	…	N-101	N-71	N-61	N-41	N-31	N-11

Every odd number (O1) with 1 as its last digit is a product of $1x$1, $3x$7, and $9x$9. The first $9 is 19, but the odd number 9 = 3x3, so 3 is the smallest prime for $9.

The chance of any odd number with 1 as its last digit to be a non-prime is: [(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]------------Formula 6

The number (n) of primes in N/10 odd number with 1 as its last digit is: n=N/10 -{N/10[(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]}------------Formula 7

The sum of first 20 terms = [(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+ (1/53x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43) + (1/59x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53) + (1/61x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59) + (1/71x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61) + (1/73x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71)+ (1/79x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73) + (1/83x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79) + (1/89x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83) + (1/101x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89) + (1/103x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101) + (1/109x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101x102/103)] = 0.333333 +0.060606 + 0.046620 + 0.029444 +0.046086 + 0.01748 + 0.01568 + 0.011966 + 0.010747 + 0.008517 + 0.007506 + 0.006484 + 0.006032 + 0.005783 + 0.00529 + 0.004953 + 0.004564 + 0.003976 + 0.003861 + 0.003612 = 0.63254

Every odd number (O9) with 9 as its last digit is a product terms of $1 x$9, $3x$3 or $7x$7. We need to consider only $1, $3, and $7.

The chance of any odd number with 9 as its last digit to be a non-prime is: [(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)…]--------------Formula 8

the number (n) of primes in N/10 odd number with 9 as its last digit: n=N/10 -{N/10[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)…]}-----------Formula 9

The sum of first 20 terms = [(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13)] +(1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) +(1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47) + (1/61x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53) + (1/67x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61) + (1/71x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67) + (1/73x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71) + (1/83x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73) + (1/97x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83)] + (1/101x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97) + 1/103x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101)] = = 0.333333 + 0.095238 + 0.051948 + 0.039960+ 0.030558 + 0.021258 + 0.013926 + 0.011291 + 0.010740 + 0.009222 + 0.008928 + 0.007323 + 0.006097 + 0.005460 + 0.005076 + 0.004867 + 0.004323 + 0.003569 + 0.003393 + 0.003294 = 0.669804

Let see $9=9 (3 x 3) first (left side of table 2), we can know there is a bigger chance for (N-$9) to be a prime with 1 as its last digit than (N-O9). If N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, but N with 0 as its last digit and only 30, 60, 90, 120, 150, 180, 210, 240, 260,… are divisible by 3 or 9,…, and we worked on only N and N/2 with 0 as their last digit, only 60, 120, 180, 240,…, (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 6.

For the next prime $9=19, (N-19) cannot be divisible by 19 except 380, 760,…, so (1/19x37/38) should be taken off from Formula 6, so on.

Let n_9 represents the total number of primes ($9) with 9 as their last digit in any number N, the chance of every N-$9 with 1 as its last digit to be a prime is: $\Delta_1=1-\sum_1$ = 1– {[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]– [(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)….]}---------Formula 10

When the number of $9 is 4 or more, 4 x[(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)….] >1, so every 4 $9 will have at least 1 prime of N-$9 to form 1 pair of primes in which one has 9 as its last digit and another has 1 as its last digit and their sum is any number N in which N and N/2 have 0 as its last digit.

For N = 600 (see table 3), 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit and 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime with 9 as its last digit.

7 Prime	17 Prime	27 3x9	37 Prime	47 prime	57 3x19	67 Prime	77 7x11	87 3x29	97 Prime	107 prime	117 3x3x13	127 Prime	137 prime	147 3x7x7	157 prime	167 prime	177 3x59	187 11x17
593 Prime	583 11x53	573 3x191	563 Prime	553 7x79	543 3x181	533 13x41	523 prime	513 3x3x3x19	503 Prime	493 17x29	483 3x7x23	473 11x43	463 prime	453 3x151	443 Prime	433 prime	423 3x3x47	413 7x59

223 prime	233 prime	243 3x3x3x3x3	253 11x23	263 prime	273 3x7x13	283 prime	293 prime	303 3x101	313 Prime	323 17x19	333 3x111	343 7x7x7	353 Prime	363 3x11x11	373 Prime	383 Prime	393 3x131	403 13x31
377 13x29	367 prime	357 3x7x17	347 prime	337 prime	327 3x109	317 Prime	307 prime	297 3x3x33	287 7x41	277 Prime	267 3x89	257 Prime	247 13x19	237 3x79	227 Prime	217 7x31	207 3x3x23	197 Prime
387 3x3x43	397 prime	407 11x37	417 3x139	427 7x61	437 23x19	447 3x149	457 prime	467 prime	477 3x3x53	487 prime	497 7x71	507 3x13x13	517 11x47	527 17x31	537 3x179	547 prime	557 prime	567 3x3x3x3x7
213 3x71	203 7x29	193 prime	183 3x61	173 Prime	163 prime	153 3x3x17	143 11x13	133 7x19	123 3x41	113 3x3x13	103 prime	93 3 x31	83 prime	73 7x11	63 3x3x7	53 prime	43 prime	33 3x11
																3 Prime	13 Prime	23 prime
																597 3x199	587 prime	577 prime
589 19x31	579 3x193	569 Prime	559 13x43	549 3x3x61	539 7x7x11	529 23x23	519 3x178	509 Prime	499 Prime	489 3x163	479 Prime	469 7x67	459 3x3x3x17	449 Prime	439 Prime	429 3x11x13	419 Prime	409 Prime
11 Prime	21 3x7	31 Prime	41 Prime	51 3x17	61 Prime	71 Prime	81 3x3x3	91 7x13	101 Prime	111 3x37	121 11x11	131 Prime	141 3x47	151 Prime	161 7x23	171 3x3x19	181 Prime	191 Prime

381 3x127	371 7x53	361 19x19	351 3x3x3x13	341 11x31	331 Prime	321 3x107	311 Prime	301 7x43	291 3x97	281 Prime	271 Prime	261 3x3x29	251 Prime	241 Prime	231 3x7x11	221 13x17	211 Prime	201 3x67
219 3x73	229 Prime	239 Prime	249 3x83	259 7x37	269 Prime	279 3x3x31	289 17x17	299 13x23	309 3x103	319 11x29	329 7x47	339 3x113	349 Prime	359 Prime	369 3x3x41	379 prime	389 Prime	399 3x7x19
209 11x19	199 Prime	189 3x3x7	179 Prime	169 13x13	159 3x53	149 Prime	139 Prime	129 3x43	119 7x17	109 Prime	99 3x3x11	89 Prime	79 Prime	69 3x23	59 Prime	49 7x7	39 3x13	29 Prime
391 17x23	401 Prime	411 3x137	421 Prime	431 Prime	441 3x3x7x7	451 11x41	461 Prime	471 3x157	481 13x37	491 Prime	501 3x167	511 7x73	521 Prime	531 3x3x59	541 Prime	551 19x29	561 3x11x17	571 Prime
																	591 3x197	581 7x81
																	9 3x3	19 Prime

1b. When both N has 0 as its last digit, and N/2 has 5 as its last digit.

Table 3. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L-2)$

N-7	N-17	N-37	N-47	N-67	N-97	…	N/2+L-2	N/2-L-2	…	83	73	53	43	23	13	3
7	17	37	47	67	97	…	N/2-L+2	N/2+L+2	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see \$7=7 first (left side of table 3), we can know there is a bigger chance for (N-\$7) to be a prime with 3 as its last digit than (N-O7). If N-7 can be divisible by 7, then (N-7)+7 (=N) [7a + 7 = 7(a+1), 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 0 as its last digit and only 70, 140, 210, 280, 350, 420, 490, 560, 630,… are divisible by 7, but we worked on only N with 0 as their last digit and N/2 with 5 as their last digit, so only 70, 210, 350, 490,…,(1 in14) can be divisible by 7, so the term (1/7x13/14) should be taken off from Formula 3.

For the next prime \$7=17, (N-17) cannot be divisible by 17 except 170, 510,…, so (1/17x33/34) should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes (\$7) with 7 as their last digit in any number N, the chance of every N-\$7 with 3 as its last digit to be a prime is: Δ_3=1-$\sum_3$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/7x13/14)] + (1/17x33/34) +(1/37x73/74)….]}

When the number of \$7 is 5 or more, 5 x[(1/7x13/14) + (1/17x33/34) + (1/37x73/74)…]>1, so every 5 \$7 will have at least 1 prime of N-\$7 to form 1 pair of primes in which one has 7 as its last digit and another has 3 as its last digit and their sum is any number N in which N have 0 as its last digit and N/2 have 5 as its last digit.

Table 4. The odd number pairs in N=O_1+O_2=(N/2+L-4)+(N/2-L+4) and N=O_1+O_2=(N/2+L-6)+(N/2-L+6)

N-9	N-19	N-29	N-59	N-79	N-89	…	N/2+1-4	N/2-L+6	…	101	71	61	41	31	11
9(3x3)	19	29	59	79	89	…	N/2-L+4	N/2+L-6	…	N-101	N-71	N-61	N-41	N-31	N-11

Let see $9=9 (3 x 3) first (left side of table 4), we can know there is a bigger chance for (N-$9) to be a prime with 1 as its last digit than (N-O9). If N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, but N with 0 as its last digit and only 30, 60, 90, 120, 150, 180, 210, 240, 260,… are divisible by 3 or 9,…, but we worked on only N with 0 as their last digit and N/2 with 5 as its last digit, only 30, 90, 150, 210,…, (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 6.

For the next prime $9=19, (N-19) cannot be divisible by 19 except 190, 570,…, so (1/19x37/38) should be taken off from Formula 6, so on.

Let n_9 represents the total number of primes ($9) with 9 as their last digit in any number N, the chance of every N-$9 with 1 as its last digit to be a prime is: Δ_1=1-$\sum_1$ = 1– {[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]– [(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)….]}

When the number of $9 is 4 or more, 4 x[(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)….] >1, so every 4 $9 will have at least 1 prime of N-$9 to form 1 pair of primes in which one has 9 as its last digit and another has 1 as its last digit and the sum is any number N in which N has 0 as its last digit and N/2 has 5 as its last digit.

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 112, 122, 1122, 1132,…, then N/2 has only 6 or 1 as its last digit.

 2a. When any even integer (N) has 2 as its last digit, such as 12, 32, 52, 112, 1132,…, then N/2 has 6 as its last digit:

Table 5. The odd number pairs in $N=O_1+O_2$=(N/2+L-3)+(N/2-L+3) and $N=O_1+O_2$=(N/2+L+3)+(N/2-L-3)

N-9	N-19	N-29	N-59	N-79	N-89	…	N/2+L-3	N/2-L-3	…	83	73	53	43	23	13	3
9(3x3)	19	29	59	79	89	…	N/2-L+3	N/2+L+3	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see \$9 = 9 first (left side of table 5), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, but N with 2 as its last digit and only 12, 42, 72, 102, 132, 162, 192, 222, 252, 282, 312, 342, 372, 402, 432, 462, 492, 522, 552,… are divisible by 3 or 9,…, but we worked on only N with 2 as their last digit and N/2 with 6 as their last digit, only 12, 72, 132, 192, 252,…, (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 552, 932,…, so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 3 as its last digit to be a prime is: $\Delta_3 =1-\sum_3 =1-$ {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)….]}

When the number of \$9 is 4 or more, 4 x[(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)…]>1, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which one has 9 as its last digit and another has 3 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 6 as its last digit.

Table 6. The odd number pairs in $N=O_1+O_2$=(N/2+L-5)+(N/2-L+5) and $N=O_1+O_2$=(N/2+L-5)+(N/2-L+5)

N-11	N-31	N-41	N-61	N-71	N-101	…	N/2+L-5	N/2-L+5	…	101	71	61	41	31	11
11	31	41	61	71	101	…	N/2-L+5	N/2+L-5	…	N-101	N-71	N-61	N-41	N-31	N-11

Let see \$1=11 first (left side of table 6), we can know there is a bigger chance for (N-\$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11, [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, but N with 2 as its last digit and only 132, 242, 352, 462, 572, 682, 792, 902,… are divisible by 11,…, but we worked on only N with 2 as their last digit and N/2 with 6 as its last digit, only 132, 352, 572, 792,…,(1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 6.

For the next prime \$1=31, (N-31) cannot be divisible by 31 except 372, 992,…, so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes (\$1) with 1 as their last digit in any number N, the chance of every N-\$1 with 1 as its last digit to be a prime is: Δ_1=1-$\sum_1$ = 1– {[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) + (1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…] – [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]}

When the number of \$1 is 11 or more, [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….] >1, so every 11 \$1 will have at least 1 prime of N-\$1 to form 1 pair of primes in which both have 1 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 6 as its last digit.

2b. When any even integer (N) has 2 as its last digit, such as 22, 42, 62, 82, 102, 1122,…, then N/2 has 1 as its last digit:

Table 7. The odd number pairs in N=O_1+O_2=(N/2+L+2)+(N/2-L-2) and N=O_1+O_2=(N/2-L+2)+(N/2+L-2)

N-9	N-19	N-29	N-39	N-49	N-59	…	N/2+L+2	N/2-L+2	…	73	63	53	43	33	23	13	3
9	19	29	59	79	89	…	N/2-L-2	N/2+L-2	…	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Let see \$9=9 first (left side of table 7), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 2 as its last digit and only 12, 42, 72, 102, 132, 162, 192, 222, 252, 282, 312, 342, 372, 402, 432, 462, 492, 522, 552,… are divisible by 3 or 9,…, but we worked on only N with 2 as their last digit and N/2 with 1 as their last digit, only 42, 102, 162, 222, 282, 342, 402,…(1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$9=19, (N-19) cannot be divisible by 19 except 552, 932,…, so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes (\$9) with 9 as their last digit in any number N, the chance of every N-\$9 with 3 as its last digit to be a prime is: Δ_3 =1-$\sum_3$ =1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23) + (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58)….]}

When the number of \$9 is 4 or more, 4 x[(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)…]>1, so every 4 \$9 will have at least 1 prime of N-\$9 to form 1 pair of primes in which one has 9 as its last digit and another has 3 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 1 as its last digit.

Table 8. The odd number pairs in N=O_1+O_2=(N/2+L+0)+(N/2-L-0) and N=O_1+O_2=(N/2+L-0)+(N/2-L+0)

N-11	N-21	N-31	N-41	N-51	N-61	N-71	…	N/2+L+0	N/2-L+0	…	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	…	N/2-L-0	N/2+L-0	…	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Let see \$1=11 first (left side of table 8), we can know there is a bigger chance for (N-\$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 2 as its last digit and only 132, 242, 352, 462, 572, 682, 792, 902,… are divisible by 11,…, but we worked on only N with 2 as their last digit and N/2 with 1 as its last digit, only 22, 242, 462, 682,…, (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 6.

For the next prime \$1=31, (N-31) cannot be divisible by 31 except 62, 682,…, so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes (\$1) with 1 as their last digit in any number N, the chance of every N-\$1 with 1 as its last digit to be a prime is: $\Delta_1=1-\sum_1$ = 1– {[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]– [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]}

When the number of \$1 is 11 or more, [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]>1, so every 11 \$1 will have at least 1 prime of N-\$1 to form 1 pair of primes in which both have 1 as its last digit and their sum is any number N in which N has 2 as its last digit and N/2 has 1 as its last digit.

3a. When any even integer (N) has 4 as its last digit, such as 24, 44, 64, 84, 104, 1124,…, then N/2 has 2 as its last digit:

Table 9. The odd number pairs in $N=O_1+O_2$=(N/2+L+1)+(N/2-L+7) and $N=O_1+O_2$=(N/2-L+1)+(N/2+L+7)

N-7	N-17	N-37	N-47	N-67	N-97	N-107	…	N/2+L-5	N/2-L-5	…	107	97	67	47	37	17	7
7	17	37	47	67	97	107	…	N/2-L+5	N/2+L+5	…	N-107	N-97	N-67	N-47	N-37	N-17	N-7

Let see $7=7 first (left side of table 9), we can know there is a bigger chance for (N-$7) to be a prime with 7 as its last digit than (N-O7). If N-7 can be divisible by 7, then (N-7)+7 (=N) [7a + 7 = 7(a+1), 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 4 as its last digit and only 14, 84, 154, 224, 294, 364, 434, 504, 574,… are divisible by 7, but we worked on only N with 4 as their last digit and N/2 with 2 as their last digit, only 84, 224, 364, 504,…, (1 in 14) can be divisible by 7, so the term (1/7x13/14) should be taken off from Formula 3.

For the next prime $7=17, (N-17) cannot be divisible by 17 except 204, 544, 884,…, so (1/17x33/34) should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($7) with 7 as their last digit in any number N, the chance of every N-$7 with 7 as its last digit to be a prime is: Δ_7=1-$\sum_7$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/7x13/14)] + (1/17x33/34) +(1/37x73/74)….]}

When the number of $7 is 5 or more, 5 x[(1/7x13/14) + (1/17x33/34) + (1/37x73/74)…]>1, so every 5 $7 will have at least 1 prime of N-$7 to form 1 pair of primes in which both have 7 as its last digit and the sum is any number N in which N have 4 as its last digit and N/2 have 2 as its last digit.

Table 10. The odd number pairs in $N=O_1+O_2$=(N/2+L+1)+(N/2-L-1) and $N=O_1+O_2$=(N/2+L-1)+(N/2-L+1)

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L+1	N/2-L+1	…	103	83	53	43	23	13	3
11	21	31	41	51	61	71	81	…	N/2-L-1	N/2+L-1	…	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Let see $1=11 first (left side of table 10), we can know there is a bigger chance for (N-$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 4 as its last digit and only 44, 154, 264, 374, 484, 594, 704, 814,… are divisible by 11,…, but we worked on only N with 4 as their last digit and N/2 with 2 as its last digit, only 44, 264, 484, 704,…, (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 3.

For the next prime $1=31, (N-31) cannot be divisible by 31 except 124, 744,…, so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes ($1) with 1 as their last digit in any number N, the chance of every N-$1 with 3 as its last digit to be a prime is: $\Delta_3=1-\sum_3$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]}

When the number of $1 is 11 or more, [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]>1, so every 11 $1 will have at least 1 prime of N-$1 to form 1 pair of primes in which one prime has 1 as its last digit and another has 3 as its last digit and their sum is any number N in which N has 4 as its last digit and N/2 has 2 as its last digit.

3b. When any even integer (N) has 4 as its last digit, such as 14, 34, 54, 74, 94, 1114,…, then N/2 has 7 as its last digit:

Table 11. The odd number pairs in $N=O_1+O_2$=(N/2+L+0)+(N/2-L+0) and $N=O_1+O_2$=(N/2-L+0)+(N/2+L+0)

N-7	N-17	N-37	N-47	N-67	N-97	N-107	…	N/2+L+0	N/2-L+0	…	107	97	67	47	37	17	7
7	17	37	47	67	97	107	…	N/2-L+0	N/2+L+0	…	N-107	N-97	N-67	N-47	N-37	N-17	N-7

Let see $7=7 first (left side of table 11), we can know there is a bigger chance for (N-$7) to be a prime with 7 as its last digit than (N-O7). If N-7 can be divisible by 7, then (N-7)+7 (=N) [7a + 7 = 7(a+1), 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 4 as its last digit and only 14, 84, 154, 224, 294, 364, 434, 504, 574,… are divisible by 7, but we worked on only N with 4 as their last digit and N/2 with 7 as their last digit, only 14, 154, 294, 434, 574,…, (1 in 14) can be divisible by 7, so the term (1/7x13/14) should be taken off from Formula 3.

For the next prime $7=17, (N-17) cannot be divisible by 17 except 34, 374, 714,…, so (1/17x33/34) should be taken off from Formula 3, so on.

Let n_7 represents the total number of primes ($7) with 7 as their last digit in any number N, the chance of every N-$7 with 7 as its last digit to be a prime is: $\Delta_7=1-\sum_7$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/7x13/14)] + (1/17x33/34) +(1/37x73/74)….]}

When the number of $7 is 5 or more, 5 x[(1/7x13/14) + (1/17x33/34) + (1/37x73/74)…]>1, so every 5 $7 will have at least 1 prime of N-$7 to form 1 pair of primes in which both have 7 as its last digit and the sum is any number N in which N have 4 as its last digit and N/2 have 7 as its last digit.

Table 12. The odd number pairs in $N=O_1+O_2$=(N/2+L-4)+(N/2-L+4) and $N=O_1+O_2$=(N/2+L+4)+(N/2-L-4)

N-11	N-31	N-41	N-51	N-61	N-71	…	N/2+L-4	N/2-L-4	…	73	53	43	23	13	3
11	31	41	51	61	71	…	N/2-L+4	N/2+L+4	…	N-73	N-53	N-43	N-23	N-13	N-3

Let see $1=11 first (left side of table 12), we can know there is a bigger chance for (N-$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 4 as its last digit and only 44, 154, 264, 374, 484, 594, 704, 814,… are divisible by 11,…, but we worked on only N with 4 as their last digit and N/2 with 7 as its last digit, only 154, 374, 594, 814,… (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 3.

For the next prime $1=31, (N-31) cannot be divisible by 31 except 434, 1054,…, so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes ($1) with 1 as their last digit in any number N, the chance of every N-$1 with 3 as its last digit to be a prime is: Δ_3=1-$\sum_3$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]}

When the number of $1 is 11 or more, [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]>1, so every 11 $1 will have at least 1 prime of N-$1 to form 1 pair of primes in which one prime has 1 as its last digit and another have 3 as its last digit and their sum is any number N in which N has 4 as its last digit and N/2 has 7 as its last digit.

4a. When any even integer (N) has 6 as its last digit, such as 26, 46, 66, 86, 106, 1126,…, then N/2 has 3 as its last digit:

Table 13. The odd number pairs in N=O_1+O_2=(N/2+L+0)+(N/2-L+0) and N=O_1+O_2=(N/2-L+0)+(N/2+L+0)

N-3	N-13	N-23	N-43	N-53	N-73	N-83	…	N/2+L+0	N/2-L+0	…	83	73	53	43	23	13	3
3	13	23	43	53	73	83	…	N/2-L+0	N/2+L+0	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see $3=3 first (left side of table 13), we can know there is a bigger chance for (N-$3) to be a prime with 3 as its last digit than (N-O3). If N-3 can be divisible by 3, then (N-3)+3 (=N) [3a + 3 = 3(a+1), 3a and 3(a+1) must be divisible by 3] will be divisible by 3, but N with 6 as its last digit and only 6, 36, 66, 96, 126, 156, 186, 216, 246,… are divisible by 3, but we worked on only N with 6 as their last digit and N/2 with 3 as their last digit, only 6, 66, 126, 186, 246,…,(1 in 6) can be divisible by 3, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime $3=13, (N-13) cannot be divisible by 13 except 26, 286, 546,…, so (1/13x25/26) should be taken off from Formula 3, so on.

Let n_3 represents the total number of primes ($3) with 3 as their last digit in any number N, the chance of every N-$3 with 3 as its last digit to be a prime is: Δ_3=1-$\sum_3$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/3x5/6)] + (1/13x25/26) +(1/23x45/46) +….]}

When the number of $3 is 3 or more, 3 x[([(1/3x5/6)] + (1/13x25/26) +(1/23x45/46)….]>1, so every 3 $3 will have at least 1 prime of N-$3 to form 1 pair of primes in which both have 3 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 has 3 as its last digit.

Table 14. The odd number pairs in N=O_1+O_2=(N/2+L-4)+(N/2-L+4) and N=O_1+O_2=(N/2+L+4)+(N/2-L-4)

N-7	N-17	N-37	N-47	N-67	N-97	…	N/2+L-4	N/2-L-4	…	89	79	59	29	19
7	17	37	47	67	97	…	N/2-L+4	N/2+L+4	…	N-89	N-79	N-59	N-29	N-19

Let see $7=7 first (left side of table 11), we can know there is a bigger chance for (N-$7) to be a prime with 7 as its last digit than (N-O7). If N-7 can be divisible by 7, then (N-7)+7 (=N) [7a + 7 = 7(a+1), 7a and 7(a+1) must be divisible by 7] will be divisible by 7, but N with 6 as its last digit and only 56, 126, 196, 266, 336, 406, 476, 546, 616,… are divisible by 7, but we worked on only N with 6 as their last digit and N/2 with 3 as their last digit, only 126, 266, 406, 546,…, (1 in 14) can be divisible by 7, so the term (1/7x13/14) should be taken off from Formula 8.

For the next prime $7=17, (N-17) cannot be divisible by 17 except 226, 566, 906,…, so (1/17x33/34) should be taken off from Formula 8, so on.

Let n_7 represents the total number of primes ($7) with 7 as their last digit in any number N, the chance of every N-$7 with 9 as its last digit to be a prime is: Δ_9=1-$\sum_9$ = 1– {[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) +…,]– [(1/7x13/14)] + (1/17x33/34) +(1/37x73/74) +….]}

When the number of $7 is 5 or more, 5 x[(1/7x13/14) + (1/17x33/34) + (1/37x73/74)…]>1, so every 5 $7 will have at least 1 prime of N-$7 to form 1 pair of primes in which 1 prime has 7 as its last digit and another has 9 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 have 3 as its last digit.

4b. When any even integer (N) has 6 as its last digit, such as 16, 36, 56, 76, 96, 1116,…, then N/2 has 8 as its last digit:

Table 15. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L+1)$ and $N=O_1+O_2=(N/2-L-1)+(N/2+L+1)$

N-9	N-19	N-29	N-59	N-79	N-89	N-109	…	N/2+L-1	N/2-L-1	…	107	97	67	47	37	17	7
9(3x3)	19	29	59	79	89	109	…	N/2-L+1	N/2+L+1	…	N-107	N-97	N-67	N-47	N-37	N-17	N-7

Let see $9=9 first (left side of table 15), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 6 as its last digit and only 36, 66, 96, 126, 156, 186, 216, 246, 276, 306, 336, 366, 396, 426, 456, 486, 516, 546,… are divisible by 3 or 9,…, but we worked on only N with 6 as their last digit and N/2 with 8 as their last digit, only 36, 96, 156, 216, 276, 336, 396, 456, 516,…(1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime $9=19, (N-19) cannot be divisible by 19 except 76, 456, 836,…, so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes ($9) with 9 as their last digit in any number N, the chance of every N-$9 with 7 as its last digit to be a prime is: $\Delta_7 = 1-\sum_7 = 1-$ {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23) + (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58)….]}

When the number of $9 is 4 or more, 4 x[(1/3x5/6)] + (1/19x37/38) +(1/29x57/58)…]>1, so every 4 $9 will have at least 1 prime of N-$9 to form 1 pair of primes in which one has 9 as its last digit and another has 7 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 has 8 as its last digit.

Table 16. The odd number pairs in $N=O_1+O_2$=(N/2+L-5)+(N/2-L+5) and $N=O_1+O_2$=(N/2+L-5)+(N/2-L+5)

N-3	N-13	N-23	N-43	N-53	N-73	N-83	…	N/2+L-5	N/2-L+5	…	83	73	53	43	23	13	3
3	13	23	43	53	73	83	…	N/2-L+5	N/2+L-5	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let see \$3=3 first (left side of table 16), we can know there is a bigger chance for (N-\$3) to be a prime with 3 as its last digit than (N-O3). If N-3 can be divisible by 3, then (N-3)+3 (=N) [3a + 3 = 3(a+1), 3a and 3(a+1) must be divisible by 3] will be divisible by 3, but N with 6 as its last digit and only 6, 36, 66, 96, 126, 156, 186, 216, 246,… are divisible by 3, but we worked on only N with 6 as their last digit and N/2 with 8 as their last digit, only 36, 96, 156, 216,…,(1 in 6) can be divisible by 3, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime \$3=13, (N-13) cannot be divisible by 13 except 156, 416, 676,…, so (1/13x25/26) should be taken off from Formula 3, so on.

Let n_3 represents the total number of primes (\$3) with 3 as their last digit in any number N, the chance of every N-\$3 with 3 as its last digit to be a prime is: $\Delta_3=1-\sum_3$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/3x5/6)] + (1/13x25/26) +(1/23x45/46) +….]}

When the number of \$3 is 3 or more, 3 x[([(1/3x5/6)] + (1/13x25/26) +(1/23x45/46)….]>1, so every 3 \$3 will have at least 1 prime of N-\$3 to form 1 pair of primes in which both have 3 as its last digit and their sum is any number N in which N has 6 as its last digit and N/2 has 8 as its last digit.

5a. When any even integer (N) has 8 as its last digit, such as 28, 48, 68, 88, 108, 1128,…, then N/2 has 4 as its last digit:

Table 17. The odd number pairs in N=O_1+O_2=(N/2+L-1)+(N/2-L+5) and N=O_1+O_2=(N/2-L-1)+(N/2+L+5)

N-9	N-19	N-29	N-59	N-79	N-89	…	N/2+L-5	N/2-L+5	…	89	79	59	29	19	9(3x3)
9(3x3)	19	29	59	79	89	…	N/2-L+5	N/2+L-5	…	N-89	N-79	N-59	N-29	N-19	N-9

Let see $9=9 first (left side of table 17), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 8 as its last digit and only 18, 48, 78, 108, 138, 168, 198, 228, 258, 288, 318,… are divisible by 3 or 9,…, but we worked on only N with 8 as their last digit and N/2 with 4 as their last digit, only 48, 108, 168, 228, 288,…(1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime $9=19, (N-19) cannot be divisible by 19 except 228, 608, 988…, so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes ($9) with 9 as their last digit in any number N, the chance of every N-$9 with 9 as its last digit to be a prime is: Δ_9=1-$\sum_9$ = 1– {[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) +…,]– [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58)….]}

When the number of $9 is 4 or more, 4 x [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58)….] >1, so every 4 $9 will have at least 1 prime of N-$9 to form 1 pair of primes in which both have 9 as its last digit and their sum is any number N in which N has 8 as its last digit and N/2 have 4 as its last digit.

Table 18. The odd number pairs in $N=O_1+O_2$=(N/2+L+3)+(N/2-L-3) and $N=O_1+O_2$=(N/2+L-3)+(N/2-L+3)

N-11	N-31	N-41	N-61	N-71	N-101	…	N/2+L+3	N/2-L+3	…	97	67	47	37	17	7
11	31	41	61	71	101	…	N/2-L-3	N/2+L-3	…	N-97	N-67	N-47	N-37	N-17	N-7

Let see $1=11 first (left side of table 18), we can know there is a bigger chance for (N-$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 8 as its last digit and only 88, 198, 308, 418, 528, 638, 748, 858, 968,… are divisible by 11,…, but we worked on only N with 8 as their last digit and N/2 with 4 as its last digit, only 88, 308, 528, 748, 968,… (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 3.

For the next prime $1=31, (N-31) cannot be divisible by 31 except 248, 868,…, so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes ($1) with 1 as their last digit in any number N, the chance of every N-$1 with 7 as its last digit to be a prime is: $\Delta_7=1-\sum_7$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]}

When the number of $1 is 11 or more, [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]>1, so every 11 $1 will have at least 1 prime of N-$1 to form 1 pair of primes in which one prime has 1 as its last digit and another have 7 as its last digit and their sum is any number N in which N has 8 as its last digit and N/2 has 4 as its last digit.

5b. When any even integer (N) has 8 as its last digit, such as 18, 38, 58, 78, 98, 1118,…, then N/2 has 9 as its last digit:

Table 19. The odd number pairs in $N=O_1+O_2$=(N/2+L-0)+(N/2-L+0) and $N=O_1+O_2$=(N/2-L-0)+(N/2+L+0)

N-9	N-19	N-29	N-59	N-79	N-89	…	N/2+L-0	N/2-L+0	…	89	79	59	29	19	9(3x3)
9(3x3)	19	29	59	79	89	…	N/2-L+0	N/2+L-0	…	N-89	N-79	N-59	N-29	N-19	N-9

Let see $9=9 first (left side of table 19), if N-9 can be divisible by 9, then (N-9)+9 (=N) [9a + 9 = 9(a+1), 9a and 9(a+1) must be divisible by 3 or 9] will be divisible by 3 or 9, N with 8 as its last digit and only 18, 48, 78, 108, 138, 168, 198, 228, 258, 288, 318,… are divisible by 3 or 9,…, but we worked on only N with 8 as their last digit and N/2 with 9 as their last digit, only 18, 78, 138, 198, 258, 318,…, (1 in 6) can be divisible by 3 or 9, so the term (1/3x5/6) should be taken off from Formula 3.

For the next prime $9=19, (N-19) cannot be divisible by 19 except 38, 418, 798…, so (1/19x37/38) should be taken off from Formula 3, so on.

Let n_9 represents the total number of primes ($9) with 9 as their last digit in any number N, the chance of every N-$9 with 9 as its last digit to be a prime is: $\Delta_9=1-\sum_9$ = 1– {[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) +…,]– [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58)….]}

When the number of $9 is 4 or more, 4 x [(1/3x5/6)] + (1/19x37/38) + (1/29x57/58)….] >1, so every 4 $9 will have at least 1 prime of N-$9 to form 1 pair of primes in which both have 9 as its last digit and their sum is any number N in which N has 8 as its last digit and N/2 have 9 as its last digit.

Table 20. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L-2)$

N-11	N-31	N-41	N-61	N-71	N-101	…	N/2+L-2	N/2-L-2	…	97	67	47	37	17	7
11	31	41	61	71	101	…	N/2-L+2	N/2+L+2	…	N-97	N-67	N-47	N-37	N-17	N-7

Let see $1=11 first (left side of table 20), we can know there is a bigger chance for (N-$1) to be a prime with 1 as its last digit than (N-O1). If N-11 can be divisible by 11, then (N-11)+11 (=N) [11a + 11 = 11(a+1), 11a and 11(a+1) must be divisible by 11] will be divisible by 11, N with 8 as its last digit and only 88, 198, 308, 418, 528, 638, 748, 858, 968,… are divisible by 11,…, but we worked on only N with 8 as their last digit and N/2 with 9 as its last digit, only 198, 418, 638, 858,… (1 in 22) can be divisible by 11, so the term (1/11x21/22) should be taken off from Formula 3.

For the next prime $1=31, (N-31) cannot be divisible by 31 except 558, 1178…, so (1/31x61/62) should be taken off from Formula 6, so on.

Let n_1 represents the total number of primes ($1) with 1 as their last digit in any number N, the chance of every N-$1 with 7 as its last digit to be a prime is: $\Delta_7=1-\sum_7$ = 1– {[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…,]– [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]}

When the number of $1 is 11 or more, [(1/11x21/22)] + (1/31x61/62) +(1/41x81/82)….]>1, so every 11 $1 will have at least 1 prime of N-$1 to form 1 pair of primes in which one prime has 1 as its last digit and another have 7 as its last digit and their sum is any number N in which N has 8 as its last digit and N/2 has 9 as its last digit.

For any even number, Goldbach's conjecture is true.

References:

1. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
2. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Lemma 5.
3. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., p. 10, section 2.
4. Zhang, Yitang (2014). "Bounded gaps between primes".Annals of Mathematics. 179 (3): 1121–1174