The Physics Without Any Postulate

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Abstract: *The aim of the present work is to setup the basis of a physics theory that embeds no postulate at all. We show that only four elementary and obvious theorems, associated to very simple mathematics, are enough to forecast the main laws of the classical mechanics, including the gravitation, the thermodynamics, but also the quantum structure of the universe.*

1) Introduction

An ideal theory of physics should embed no postulate, but be based on theorems only. Are we able today to build such a perfect theory ? Although the main opinion answers no, we intend to propose here a step toward this ideal.

We will show that the classical mechanics, including the gravitation, the thermodynamics and the quantum structure of the universe can all be explained and unified by the means of only four elementary and obvious theorems, but no use of any postulate at all.

The mathematics used to achieve so are elementary, and this is satisfying as it sounds logical that the most elementary structure of the universe can be based on elementary mathematics. For instance the velocity of a Keplerian orbiter, that concerns all the astral bodies in the universe, is only the superposition of two uniform velocities, of rotation and translation. Such a simplicity must be considered as a track leading to the fundamental laws governing the universe.

As a result we show that we can forecast many physical laws that have been fully measured experimentally : Newton's second law, Kepler's laws, Galileo's equivalence principle, Newton's gravitation law, mechanical energy, Gibbs's free energy, Boltzmann's statistical entropy, law of the chemical equilibrium, law of the chemical kinetics, ideal gas law, Plank's constant, Plank-Einstein relation, de Broglie's hypothesis.

Although we prove that such an approach of the physics is fruitful, this is only a step toward a complete theory of physics free of any postulate. Much still remain to be done. This work is so intended to setup the basis of such a project, and we hope that its simplicity, with regards to what is usually the physics, will encourage many people to investigate further.

2) Toolbox

2.1) Theorems

2.1.1) Theorem of mathematicality

Theorem 1 : *The evolution in time of any measurable physical property can be modeled by the means of a mathematical function, that we call Lagrangian and note L.*

Proof : Any measure of any physical parameter with respect to time will exhibit a two dimensions plot, which abscissa represents the time and the ordinate the value of the parameter. Such a curve respecting a two dimension geometry, can be modeled by the means of the mathematics (for instance polynomials, Fourier series, …).

Consequences : A system having no measurable physical properties has a null Lagrangian. A system having a constant Lagrangian in time does not change its physical properties in time, we say that it is stable, or at equilibrium.

2.1.2) Theorem of non ubiquity

Theorem 2 : *It is impossible to measure a physical system in two different physical states simultaneously.*

Proof : No one has ever measured a physical body at rest and moving simultaneously, hot and cold simultaneously, at two different positions simultaneously, or in general in two different physical states simultaneously.

2.1.3) Theorem of bijective time dependence

Theorem 3 : *Any measurable physical quantity is related to the time by a bijection.*

Proof : Any series of measures of any physical quantity with report to the time can be split into successive intervals, in which the quantity is related to the time by a bijection (see figure 1). Being always in such a bijective interval, the quantity is always related to the time by a bijection.

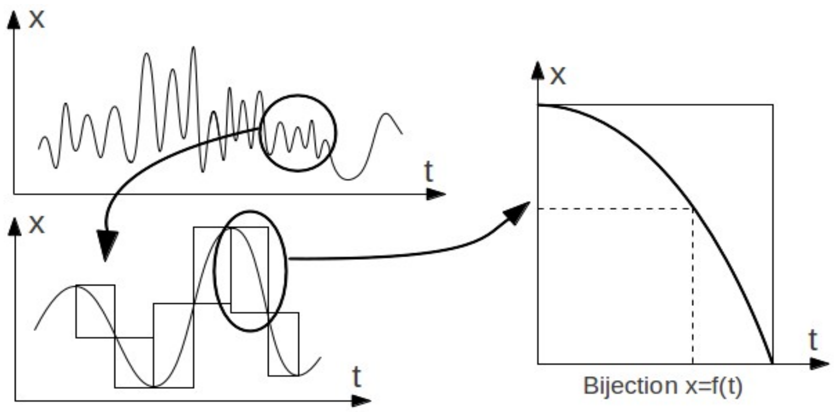

Figure 1 : *The trajectory of a classical quantity, recording its value x with respect to time t, is a succession of intervals in which x and t are related by a bijection x=f(t) and t = $f^{-1}(x)$. The trajectory might be either continuous or discontinuous.*

2.1.4) Theorem of universality

Theorem 4 : *The theorem of mathematicality, the theorem of non ubiquity and the theorem of bijective time dependence are true in all frames of reference.*

Proof : no one has ever measured, in any frame of reference, a classical system that does not respect the theorem of mathematicality, the theorem of non ubiquity and the theorem of bijective time dependence.

2.2) Definitions

We consider a physical quantity q, which derivative with respect to time is $\dot{q}$, that we will call velocity, although it is not always the kinematics velocity. Even if we define a momentum, a force and a mass, we are not referring always to the only kinematics properties, but to more general concepts applying on any quantity.

We define the momentum as the the Lagrangian divided by the velocity :

$$P=\frac{L}{\dot{q}} \tag{1}$$

We define the force as the derivative of the momentum with respect to time :

$$F = \frac{dP}{dt} \quad (2)$$

We define the “free Lagrangian” as the integral of the momentum with respect to the velocity :

$$C = \int P \, d\dot{q} \quad (3)$$

We define the mass as the Lagrangian divided by the square of the velocity :

$$m = \frac{L}{\dot{q}^2} \quad (4)$$

If the measured quantity is a vector, we state the following definitions, where the vectors are set in bold :

Momentum : $$\mathbf{P} = \left(\frac{L}{\dot{q}^2}\right) \dot{\mathbf{q}} = m \, \dot{\mathbf{q}} \quad (5)$$

Force: $$\mathbf{F} = \frac{d\,\mathbf{P}}{d\,t} \quad (6)$$

Free Lagrangian : $$C = \int \mathbf{P} \, . \, d\,\dot{\mathbf{q}} \quad (7)$$

3) Applications

3.1) Mechanics

3.1.1) Introduction

In this chapter we will study the physical properties of a classical system depending upon its kinematics quantities, i.e. its position, velocity and acceleration, which are three dimension vectors (the definitions (5), (6) and (7) therefore apply).

We will constraint our study to a stable system, so with $L = L_0 = \text{constant}$, and having a constant mass m.

3.1.2) Second law of Newton

From the relations (5) and (6), it is trivial to see that the force must be :

$$\mathbf{F} = m\,\ddot{\mathbf{q}} \quad , \quad \ddot{\mathbf{q}} \ \text{being the acceleration} \tag{8}$$

This is the second law of Newton.

3.1.3) Elementary velocities

Because the system has a constant Lagrangian and a constant mass, it must verify $\dot{q}^2 = \text{constant}$ (see definition (4)). There are only two classical motions that can verify this : the uniform rotation and the uniform translation, characterized by the following velocities :

$$\text{Uniform rotation velocity :} \quad \dot{\mathbf{q}}_{\mathbf{R}} = \boldsymbol{\omega} \times \mathbf{q} \quad , \quad \dot{q}_R = \|\dot{\mathbf{q}}_{\mathbf{R}}\| = \omega\, r = \text{constante} \tag{9}$$

$$\text{Uniform translation velocity:} \quad \dot{\mathbf{q}}_{\mathbf{T}} = \mathbf{constant} \tag{10}$$

These elementary velocities are the only two that a stable system with a constant mass can exhibit.

3.1.4) Addition of velocities

If the system has a velocity that is the sum of sub-velocities, the total stable Lagrangian is the sum of the Lagrangians corresponding to each sub-velocity :

$$L_T^0 = \sum_{n=0}^{N} L_N^0 = m \sum_{n=0}^{N} \dot{q}^2 = \text{constant} \tag{11}$$

This expression is always true if each sub-velocity respects the definitions either (9) or (10). The velocity of any classical mobile will then be given by the following general formula, where the velocities $\dot{\mathbf{q}}_{\mathbf{R}}$, $\dot{\mathbf{q}}_{\mathbf{T}}$, $\dot{\mathbf{q}}_{\mathbf{Ri}}$, $\dot{\mathbf{q}}_{\mathbf{Tj}}$ are all given by the definitions (9) and (10) :

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_{\mathbf{R}} + \dot{\mathbf{q}}_{\mathbf{T}}$$
$$\text{with} \quad \dot{\mathbf{q}}_{\mathbf{R}} = \sum_{i=1}^{I} \dot{\mathbf{q}}_{\mathbf{Ri}} \quad , \quad \dot{\mathbf{q}}_{\mathbf{T}} = \sum_{j=1}^{J} \dot{\mathbf{q}}_{\mathbf{Tj}} \quad , \text{and} \quad i\,,j = 1, 2, 3, \dots \tag{12}$$

The equation (12) is also the velocity of any Keplerian orbiter. Let us show it.

3.1.5) Keplerian motion

If we define the massless angular momentum $\mathbf{L_k}$ as $\mathbf{L_k}=\mathbf{q}\times\dot{\mathbf{q}}$. It is trivial to see that the derivation of $\mathbf{L_k}$ with respect to time, by including the relation (9), is null, thus the angular momentum is constant as expected for a central field motion.

Regarding the result (12) , the multiplication of $\mathbf{L_k}$ by $\dot{\mathbf{q}}_\mathbf{R}$ leads to :

$$\dot{\mathbf{q}}_\mathbf{R}\times\mathbf{L_k}=\dot{q}_R^2\left(1+\frac{\dot{\mathbf{q}}_\mathbf{R}\cdot\dot{\mathbf{q}}_\mathbf{T}}{\dot{q}_R^2}\right)\mathbf{q} \qquad (13)$$

Therefore the modulus of this expression is :

$$\frac{L_k}{\dot{q}_R}=\left(1+\frac{\dot{q}_T}{\dot{q}_R}\cos\theta\right)q \quad \text{or} \quad p=(1+e\cos\theta)q \qquad (14)$$

This last equation is the one of a conic where $p=L_k/\dot{q}_R$ is the semilatus rectum, $e=\dot{q}_T/\dot{q}_R$ is the eccentricity and θ is the angle between the directions of the rotation and the translation speed, i.e. the true anomaly. We see that both p and e are constants and therefore the equation (14) is nothing else but the first law of Kepler.

An other way to express the angular momentum is the use the areal velocity f :

$$L_k=2\,f \quad \text{with} \quad f=\frac{1}{2}q^2\dot{\theta} \quad \text{and} \quad \dot{\theta}=\frac{d\theta}{dt} \qquad (15)$$

As far as $\mathbf{L_k}$ is constant, the areal velocity will also be. This is the second law of Kepler.

At last, the third Kepler's law also derives simply from the constancy of the angular momentum. Indeed the integration with respect to time of the relation (15), over a complete period T of revolution, gives :

$$L_k T=\int_0^{2\pi} q^2 d\theta \qquad (16)$$

For the case where the trajectory is an ellipse, the right side of this equation is worth $2\pi ab$, where a is the major semi axis and b the minor one. Knowing that $a=p/(1-e^2)$ and $b=p/\sqrt{1-e^2}$, and remembering the definition of the semilatus rectum p given by the equations (14), it is easy to finally get the following relation :

$$L_k \dot{q}_R = 4\pi^2 a^3 / T^2 \tag{17}$$

Because L_k and $\dot{q}_R$ are constants, this last expression is nothing else but the third law of Kepler stating that the square of the period of revolution is proportional to the cube of the major semi axis.

We can conclude that a mobile respecting the relation (12) also respects the three laws of Kepler.

3.1.6) Galileo's equivalence principle

The expression (12) of the velocity is independent of the mass of the mobile. This is consistent with the Galileo's principle stating that the motion in a gravitation field is mass independent, but this is also true for all classical motions.

3.1.7) Newton's gravitation law

Deriving the equation (12) with respect to the time, we get the acceleration a of a Keplerian orbiter : $\ddot{\mathbf{q}} = \dot{\boldsymbol{\omega}} \times \mathbf{q} + \boldsymbol{\omega} \times \dot{\mathbf{q}}$. Including the solutions (9) and (10), we can write $\ddot{\mathbf{q}} = -(\boldsymbol{\omega}/q^2) \wedge [\mathbf{q} \wedge (\mathbf{q} \wedge \dot{\mathbf{q}})]$, and finally :

$$\ddot{\mathbf{q}} = -\frac{L_k \dot{q}_R}{q^3} \mathbf{q} \tag{18}$$

This is the expression of the Newton's gravitational acceleration if :

$$L_k \dot{q}_R = G M \tag{19}$$

where G is the constant of gravitation and M is the attracting mass.

We can also notice that the equation (19) is consistent with the expression (17) of the third Kepler's law.

3.1.8) Mechanical energy

If we develop the square of the equation (12), and include the result (14), it is trivial to define the massless mechanical energy E_M , i.e. the mechanical energy divided by the mass of the mobile, as follows :

$$E_M = \frac{1}{2} \dot{q}^2 - \frac{L_k \dot{q}_R}{q} = \frac{1}{2} \dot{q}_R^2 (e^2 - 1) \tag{20}$$

This expression is interesting because it describes the classical mechanical energy (divided by the mass of the orbiter) made of the addition of the usual kinetic and potential parts. It also shows, with its right member, that this energy is a negative constant for a fixed conic.

3.1.9) Conclusion

In this chapter we have shown that the motion of a classical mobile, stable with a constant mass, can only be the addition of one ore many of the two elementary uniform motions, of rotation an translation, described by the relations (9) and (10) (see equation (12)). So the classical motion is fundamentally Keplerian. The three Kepler's laws are then not only true for the astral bodies, but for all classical bodies. Even if in our day to day experience this is difficult to see, because I and J in the equation (12) are generally big integers, and changing in time.

3.2) Thermodynamics

3.2.1) Introduction

In this chapter the quantity q is not any more a position in three dimension, but can be any physical quantity encountered in thermodynamics, as the volume, the concentration or the number of particle in a special state.

We will constraint our study to stable systems, so with $L=L_0=\text{constant}$, therefore we can integrate the equation (1) to get C, as defined by (3) :

$$C=C_0+L_0\ln\left(\frac{\dot{q}}{\dot{q}_0}\right) \text{ , with } L_0\,,C_0\,,\dot{q}_0 \text{ all constant} \tag{21}$$

We can also write :

$$\dot{q}=\dot{q}_0\,e^{\frac{\Delta C}{L_0}} \text{ with } \Delta C=C-C_0 \tag{22}$$

In addition the momentum and the force, which do not have here a kinematics meaning, but a more generalized one, will verify :

$$P=\frac{L_0}{\dot{q}} \text{ and } F=-\left(\frac{L_0}{\dot{q}^2}\right)\ddot{q} \tag{23}$$

3.2.2) Gibbs's free energy and Boltzmann's entropy

Let q_n be the number of particle in the state ΔC_n , and q the number of all the particles in a global system, we shall verify :

$$q=\sum_{n=0}^{N} q_n \quad \text{thus} \quad \dot{q}=\sum_{n=0}^{N} \dot{q}_n \tag{24}$$

Here $\dot{q}_n$ is the evolution in time of the number of particle in the state n. Each state must verify the equation (22), so we can write :

$$\dot{q}=\sum_{n=0}^{N} \dot{q}_n^0 \, e^{\frac{\Delta C_n}{L_n^0}} \tag{25}$$

Here L_n^0 is the constant Lagrangian corresponding to the stable state n.

Dividing the last relation by a reference velocity $\dot{q}_0$ we can define a partition function Z as follows :

$$Z=\frac{\dot{q}}{\dot{q}_0}=\sum_{n=0}^{N} \frac{\dot{q}_n}{\dot{q}_0}=\sum_{n=0}^{N} \frac{\dot{q}_n^0}{\dot{q}_0} \, e^{\frac{\Delta C_n}{L_n^0}} \tag{26}$$

Replacing this result into (21), leads to :

$$\Delta C=L_0 \ln(Z) \tag{27}$$

This is the Gibbs's free energy, at the condition that $L_0=k\,T$, where k is the Boltzmann"s constant and T the temperature.

Consequently dividing the last expression by the temperature gives straight forward the formula of the Boltzmann's statistical entropy :

$$S_{ent}=k \ln(Z) \tag{28}$$

3.2.3) Chemical equilibrium

Let us study the following chemical reaction :

$$A+2\,B \rightarrow 3\,C+D \tag{29}$$

The total free energy of this reaction must be the sum of the free energies of the products, minus those of the reactive :

$$\Delta C_T = 3\,\Delta C_C + \Delta C_D - \Delta C_A - 2\,\Delta C_B \qquad (30)$$

According to (27), this leads to :

$$\Delta C_T = L_0 \ln\left(\frac{Z_C^3\, Z_D}{Z_A\, Z_B^2}\right) \qquad (31)$$

If the reaction is at the equilibrium ΔC_T must be constant, and we get the expression of the chemical equilibrium :

$$\frac{Z_C^3\, Z_D}{Z_A\, Z_B^2} = e^{\frac{\Delta C_T}{L_0}} = \text{constant} \qquad (32)$$

3.2.4) Chemical kinetics and ideal gas law

The theorem of bijective time dependence states that any physical quantity is related to the time by a bijection : $q = f(t)$ and $t = f^{-1}(q)$. Considering the equation (22), the most obvious way to achieve so is to have a free energy directly proportional to the time :

$$\Delta C = L_0\, \nu\, \Delta t \quad \text{with} \quad \Delta t = t - t_0 \qquad (33)$$

As far as ΔC has the same dimension as L_0 , ν must be a frequency.

Consequently q and its derivatives with respect to time become :

$$q = q_0\, e^{\nu\, \Delta t} \quad , \quad \dot{q} = \nu\, q \quad , \quad \ddot{q} = \nu^2\, q \qquad (34)$$

These formulas are typical of the chemical kinetics, when measuring the amount of reactive with respect to time (chemical reactions, radioactive decay, ...).

If q is the volume of a gas in the former equations, the corresponding force (23) is of course the pressure, and must verify :

$$F\, q = -L_0 \qquad (35)$$

This is the ideal gas law if $L_0 = N\, k\, T$, where N is the number of molecules, k the Boltzmann's constant and T the temperature.

3.2.5) Conclusion

The toolbox enables us to forecast the main laws of the thermodynamics, that have been measured experimentally, for systems at equilibrium.

3.3) Quantization

3.3.1) Introduction

In this chapter we will study the most elementary behavior of the systems. We will then consider the most elementary particles as photons, electrons, atoms, ..., in order to simplify the reasoning. We will use the word “measure” in a wide sens, as it means actually “interaction”.

3.3.2) Time quantization

Because of the theorem of non ubiquity, it impossible to measure a system in two different state at the same time. So let say that t_0 is the last time where a system is in the state L_0 , and t_1 the first time where it is in the state L_1 . The interval $\Delta t = t_1 - t_0$ can not be null, and must then at least have a non null minimum value. We call “quantum of time” this interval.

Consequently any measure can be performed at the boundaries of the quantum of time, but no measure can be achieved inside it.

As far as the theorem of universality is true, it must then exist a minimum universal quantum of time Δt_U , which is the shortest possible quantum of time in the universe.

3.3.3) Space quantization

As far as the time is quantized, any system having a kinematic velocity $\dot{q}$ will also exhibit a quantum of space :

$$\Delta q = \dot{q} \Delta t \tag{36}$$

Here again any measure can be performed at the boundaries of the quantum of space, but no measure can be achieved inside it.

As far as the theorem of universality is true, it must then exist a minimum universal quantum of space Δq_U , which is the shortest possible quantum of space in the universe.

3.3.4) Least existence

To exist a system must exhibit some physical properties, therefore its Lagrangian can not be null and must exhibit a minimum value, $L = L_Q = \text{constant}$ that we call quantum of existence.

Note that nothing forces the Lagrangian L of a system to be only the quantum Lagrangian L_Q during a quantum of time Δt .

As far as the theorem of universality is true, it must then exist a minimum universal quantum of existence L_U , which is the smallest possible quantum of existence in the universe.

3.3.5) Action quantization, relation of Plank-Einstein

The action S is the integral of the Lagrangian with respect to time. For a quantum of time its value is :

$$\Delta S = L \Delta t \tag{37}$$

We call ΔS "quantum of action".

Because L has the dimension of an energy, ΔS has the same dimension as the Plank's constant h. We can then write :

$$L = h \nu \quad \text{, with} \quad h = \Delta S \quad \text{and} \quad \nu = \frac{1}{\Delta t} \tag{38}$$

This is the relation of Plank-Einstein.

As far as the theorem of universality is true, it must then exist a minimum universal quantum of action ΔS_U , which is the smallest possible quantum of action in the universe :

$$\Delta S_U = L_U \Delta t_U \tag{39}$$

Therefore ΔS_U has all the universal characteristics to be indeed the Plank's constant.

3.3.6) Momentum quantization, de Broglie's hypothesis

Because the momentum is equal to the Lagrangian divided by the velocity (see definition (1)), as far as the system exhibits a quantum of action, it must also exhibit a quantum of momentum :

$$P_Q = \frac{\Delta S}{\Delta q} \tag{40}$$

If ΔS is the Plank's constant, this is the expression of the de Broglie's hypothesis.

If a system exists during a quantum of time, it will exhibit a quantum of action, and therefore both quanta of time and space must exist. The system must then be in motion with the velocity $v = \Delta q / \Delta t$. This explains why the quantum particles are always moving.

3.3.7) The physics inside the quantum of time

Between two measures, i.e. two interactions, a system is inside its quantum of time, and we can not get any information about it. For instance if a single particle goes from a gun to a screen, in a perfect vacuum, we have no information on its state between the gun and the screen.

If we place a measuring device between the gun and the screen, the particle will interact with the device and its quantum of time is not any more the time spent between the gun and the screen, but between the gun and the device. If the measure is non destructive, the particle can finally reach the screen, but then it will exhibit a second quantum of time, from the device to the screen. The whole trajectory will be made of two quanta of time, instead of one. Therefore we can not definitely measure the electron inside a single quantum of time.

There is however a way to get some information about what happens inside : the Young's double-slit experiment.

We place a wall pierced with a slit between the gun and the screen. If the particle hits the wall, no impact is seen on the screen, but if the particle goes through the slit, an impact appears on the screen, in front of the slit. And this is indeed what the experiment shows.

However, if we place two slits in the wall, and we push the particles one by one individually, the experiment exhibits a typical interference pattern, as shown on the figure 2.

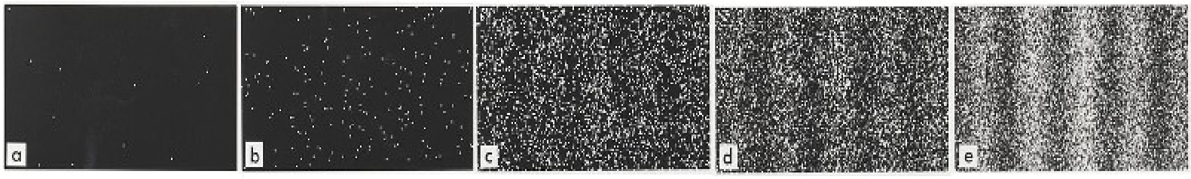

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Figure 2 : *Young's double-slit experiment with electrons pushed individually, at 5 different times of advancement*

Such an interference pattern indicates that the particle has the structure of a wave inside the quantum of time, and furthermore that it can interfere with itself. To achieve so the wave must get trough both slits at the same time. This seems to violate the theorem of non ubiquity, but it is not. Indeed as far as this theorem only applies to measurable systems, it does not apply to such a wave that is not measurable.

We observe therefore that the physical laws inside a quantum of time are very different from those dedicated to the measurable systems.

3.3.8) Conclusion

The theorem of non ubiquity is the cause of the quantization of time, and this has huge consequences as the quantization of the space, the existence of the Plank's constant, the Plank-Einstein relation, as well as the de Broglie's hypothesis.

We can give a good description of what is an elementary quantum particle : it is a wave that can be in many physical states simultaneously, that collapses into a particle when it is measured. Such a system will be alternatively a wave and a particle, but will never be both together.

4) General conclusion

The present approach describing the physics without postulate is able to forecast numbers of physical laws that have been experimentally measured, and were often difficult to unify before : Newton's second law, Kepler's laws, Galileo's equivalence principle, Newton's gravitation law, mechanical energy, Gibbs's free energy, Boltzmann's statistical entropy, law of the chemical equilibrium, law of the chemical kinetics, ideal gas law, Plank's constant, Plank-Einstein relation, de Broglie's hypothesis.

All this was obtained by the means of very simple mathematics, and no unnatural concept for a human being. This theory of physics looks like an extended classical mechanics, taking now into account the quantum and thermodynamics aspects of the world.

We only studied here the systems with a constant Lagrangian, and eventually a constant mass, so stable systems at the equilibrium. A lot of investigations are then still to be performed, with the same approach, for the unstable systems, and those with a non constant mass.

An other lack of this work is the electromagnetism. This is because it requires to state the postulate of elementary charge, but we can not accept any postulate. As we saw the mass comes with a definition (it is the Lagrangian divided by the square of the velocity), but we have no such a definition for the charge. This issue has then also still to be investigated.