Free Dirac Current for Superposed States

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Abstract

The article aims to investigate the four current due to the superposition of free Dirac states with a special view towards the terms containing oscillatory factors responsible for “the trembling motion of electrons”----Zitterbewegung. As we shall see that these terms simply disappear. Zitterbewegung is not possible with free Dirac States

Introduction

The trembling motion of electrons-Zitterbewegung[1]—is a fact that emerges from the current formed from the superposition of free Dirac states. From mathematical considerations in this article it emerges as a fact that the terms containing the relevant oscillatory factors reduce to zero value leading to the complete disappearance of Zitterbewegung

Superposition of Dirac States

Due to the superposition[2] of Dirac States we have,

(1.1)

$$\chi = \sum_{r,s} \int \frac{1}{\sqrt{E_{\vec{p}}}} \left[C(E,\vec{p},r) u_r(\vec{x},t,\vec{p},E) exp[-ip_\mu x^\mu] + D(-E,\vec{p}',s) v_r(\vec{x},t,-E,\vec{p}')\right] exp[ip_\mu x^\mu] d^3p$$

(1.2)

$$\chi = \sum_{r,s} \int \frac{1}{\sqrt{E_{\vec{p}}}} [C(E,\vec{p},r) u_r(\vec{x},t,\vec{p},E) exp[-i[Et - \vec{p}.\vec{x}]] + D(-E,\vec{p}',s) v_s(\vec{x},t,-E,-\vec{p})] \, exp[i[Et - \vec{p}'.\vec{x}] d^3p$$

$[E > 0]$

[u_r $r$efers to positive energy states:u_1 to +½ spin states and u_2 to -1/2 spin states;

v_r refers to negative energy states:v_1 to ½ spin states and v_2 to -1/2 spin states]

To note that $E^2 = \vec{p}^2 + {m_0}^2$

The value of $\vec{p}$ fixes up the value of E:for each $\vec{p}^2$ we have $\pm E$. We integrate over momentum statesp_x, p_y and p_z as indicated by d^3p

Equivalently ,

(1.3) $\chi = \sum_{r',s'} \int \frac{1}{\sqrt{E_{\vec{p}}}} [C(E',\vec{p}',r') u_{r'}(\vec{x},t,E',\vec{p}') exp[-i[E't - \vec{p}'.\vec{x}]] +$

$D(-E',-\vec{p}',s') v_{s'}(\vec{x},t,-E',-\vec{p}')] \, exp[i[E't - \vec{p}'.\vec{x}] d^3p' ;[E' > 0]$

(2) $\bar{\chi} = \sum_{r,s} \int \frac{1}{\sqrt{E_{\vec{p}}}} \left[C^*(E,\vec{p},r) \bar{u}_r(\vec{x},t,E,\vec{p}) exp[ip_\mu x^\mu] +\right.$

$\left. D^*(-E,-\vec{p},s) \bar{v}_s(\vec{x},t,-E,-\vec{p})\right] exp[-ip_\mu x^\mu] d^3p$

$$\bar{\chi} = \sum_{r,s} \int \frac{1}{\sqrt{E_{\vec{p}}}} [C^*(E,\vec{p},r) \bar{u}_r(\vec{x},t,E,\vec{p}) exp[i[Et - \vec{p}.\vec{x}]] + D^*(-E,-\vec{p},s) \bar{v}_s(\vec{x},t,-E,-\vec{p})] \, exp[-i[Et - \vec{p}.\vec{x}]] d^3p$$

Standard relations for normalized Dirac solutions

$\bar{u}_r(p) u_s(p) = \delta_{rs}; \bar{v}_r(p) v_s(p) = -\delta_{rs}$

$\bar{v}_r(p) u_s(p) = 0; \bar{u}_r(p) v_s(p) = 0$

The Four Current

There are three types of possible combinations in the Dirac current for the wave packet, $j^\mu = \int \bar{\chi} \gamma^\mu \chi \, d^3x$

1) Particle-particle states in combination
2) Antiparticle-antiparticle states in combination
3) Particle –antiparticle or antiparticle- particle combinations of states which yield zero for the integral

Particle-Particle states in Combination

(3.1) $I = \int C(E',\vec{p}',r') C^*(E,\vec{p},r) \frac{1}{\sqrt{E_{\vec{p}}}} \bar{u}_r(E,\vec{p},s') \exp[i[Et - \overline{\vec{p}.\vec{x}}] \gamma^\mu u_{r'}(E',\vec{p}',s') \exp[-i[E't -$

$\vec{p}'.\vec{x}'] \frac{1}{\sqrt{E_{\vec{p}'}}} d^3x d^3p d^3p'$

$[E' > 0, E > 0]$

$$= \int C(E',\vec{p}',r') C^*(E,\vec{p},r) \frac{1}{\sqrt{E_{\vec{p}}}} \frac{1}{\sqrt{E_{\vec{p}'}}} \bar{u}_r(E,\vec{p}) \gamma^\mu u_{r'}(E',\vec{p}') \exp[-i[(E' - E)t - (\vec{p}' - \vec{p}).\vec{x}'] d^3x d^3p d^3p'$$

$$= \int C(E',\vec{p}',r')C^*(E,\vec{p},r)\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}'}}}\bar{u}_r(E,\vec{p})\gamma^\mu u_r(E',\vec{p},s')\exp[-i(E'-E)\delta^3(\vec{p}-\vec{p}')d^3pd^3p'$$

$$= \int C(E'\vec{p},r')C^*(E,\vec{p},r)\frac{1}{E_{\vec{p}}}\bar{u}(E,\vec{p},r)\gamma^\mu u(E',\vec{p},r')\exp[-i(E'-E)t]d^3p$$

$$= \int C(E,\vec{p},r')C^*(E,\vec{p},r)\frac{1}{E_{\vec{p}}}\bar{u}_r(E,\vec{p})\gamma^\mu u_{r'=r}(E,\vec{p})\,d^3p$$

NB: $E > 0$; $E' > 0$ as stated earlier,$\exp[-i(E'-E)t] = 1\ when\ \vec{p}' = \vec{p}\ since : E = E' from\ E^2 = \vec{p}^2 + {m_0}^2. Also\ \ \bar{u}_r(E,\vec{p})\gamma^\mu u_{r'\neq r}(E,\vec{p}) = 0$]When $\vec{p}' \neq \vec{p}$, $\delta^3(\vec{p}-\vec{p}') = 0$. The exponential factor $\exp[-i(E'-E)t]$gets multiplied by zero from delta function to result in zero value of te product

$$I = \int |C(E,\vec{p},s)|^2\frac{1}{E_{\vec{p}}}\bar{u}_r(E,\vec{p})\gamma^\mu u_r(E,\vec{p})\,d^3p$$

(3.2)

$I = \int |C(E,\vec{p},s)|^2\frac{1}{E_{\vec{p}}}\frac{p^\mu}{m}d^3p \to A\ real\ quantity$and a four vector:$\frac{p^\mu}{m}\ is\ a\ four\ vector$;$\frac{d^3p}{E_{\vec{p}}}$and $|C(E,\vec{p},s)|^2$ are invariants

To Note[3]:

For evaluating $\bar{u}_r(E,\vec{p})\gamma^\mu u_r(E,\vec{p})$ we take $\mu = 1$ as an instance that is we consider:$\bar{u}_r(E,\vec{p})\gamma^1 u_r(E,\vec{p})$

From Dirac equation

$$(\gamma^\mu p_\mu - m)u_r = 0$$

$$u_r = \frac{\gamma^\mu p_\mu}{m}u_r$$

From conjugate Dirac equation $\bar{u}_r(\gamma^\mu p_\mu - m) = 0$

$$\bar{u}_r = \bar{u}_r\frac{\gamma^\mu p_\mu}{m}$$

Therefore

$$\bar{u}_r(E,\vec{p})\gamma^1 u_r(E,\vec{p}) = \frac{1}{2}\bar{u}_r(E,\vec{p})\gamma^1 u_r(E,\vec{p}) + \frac{1}{2}\bar{u}_r(E,\vec{p})\gamma^1 u_r(E,\vec{p})$$

$$= \bar{u}_r\frac{\gamma^\mu p_\mu}{2m}\gamma^1 u_r + \bar{u}_r\gamma^1\frac{\gamma^\mu p_\mu}{2m}u_r$$

$$\bar{u}_r\frac{\{\gamma^\mu p_\mu,\gamma^1\}}{2m}u_r$$

By direct calculation $\{\gamma^{\mu}p_{\mu},\gamma^{1}\}=2p^{x}$

$$\bar{u}_r(E,\vec{p})\gamma^{1}u_r(E,\vec{p})=\bar{u}_r\frac{p^{x}}{m}u_r=\bar{u}_r u_r\frac{p^{x}}{m}=\frac{p^{x}}{m}$$

Since $\bar{u}_r u_r=1$

Antiparticle-Antiparticle States in Combination

(4.1)

$$I=\int D(-E',\vec{p}',s')D^{*}(-E,\vec{p},s)\frac{1}{\sqrt{E_{\vec{p}}}}\bar{v}_s(-E,\vec{p})\exp[-i[Et-\overrightarrow{\vec{p}.\vec{x}}]\gamma^{\mu}v_{s'}(-E',\vec{p}')\exp[i[E't-\vec{p}'.\vec{x}']\frac{1}{\sqrt{E_{\vec{p}'}}}d^{3}xd^{3}pd^{3}p'$$

$$[E>0;E'>0]$$

$$=\int D(-E',\vec{p}',s')D^{*}(-E,\overrightarrow{p},s)\frac{1}{\sqrt{E_{\vec{p}}}}\bar{v}_s(-E,\vec{p})\gamma^{\mu}v_{s'}(-E',\vec{p}')\exp[i[(E'-E)t-(\vec{p}'-\vec{p}).\vec{x}']\frac{1}{\sqrt{E_{\vec{p}'}}}d^{3}xd^{3}pd^{3}p'$$

$$=\int D(-E',\vec{p}',s')D^{*}(-E,\vec{p},s)\frac{1}{\sqrt{E}}\bar{v}_s(-E,\vec{p})\gamma^{\mu}v_{s'}(-E',\vec{p}')\frac{1}{\sqrt{E'}}\exp[i(E'-E)t]\delta^{3}(\vec{p}-\vec{p}')d^{3}pd^{3}p'$$

$$=\int D(-E,\vec{p},s')D^{*}(-E,\vec{p},s)\frac{1}{E_{\vec{p}}}\bar{v}_s(-E,\vec{p})\gamma^{\mu}v_{s'=s}(-E,\vec{p}))d^{3}p$$

When $\vec{p}\neq\vec{p}'$ the contribution of the integrand to the integral is zero since the delta function yields zero value . When $\vec{p}=\vec{p}',E'=\pm E\Longrightarrow|E'|=|E|$. The the above integral E' and E imply theabsolute values of energy. Thus when $\vec{p}=\vec{p},\exp[i(E'-E)t=1$

We have,

$$I=\int D(-E,\vec{p},s')D^{*}(-E,\vec{p},s)\frac{1}{E_{\vec{p}}}\bar{v}_s(-E,\vec{p})\gamma^{\mu}v_{s'=s}(-E,\vec{p}))d^{3}p$$

$$I=\int|D(-E,\vec{p},s)|^{2}\frac{1}{E_{\vec{p}}}\bar{v}_s(E,\vec{p})\gamma^{\mu}v_s(E,\vec{p})\,d^{3}p$$

(4.2)

$I=-\int|D(-E,\vec{p},s)|^2\frac{1}{E_{\vec{p}}}\frac{p^\mu}{m}d^3p\rightarrow$ A real quantity and a four vector:$\frac{p^\mu}{m}\ is\ a\ four\ vector$; $\frac{d^3p}{E_{\vec{p}}}$and $|D(E,\vec{p},s)|^2$ are invariants

Proof of Formula[4]

For evaluating $\bar{u}_r(E,\vec{p})\gamma^\mu u_r(E,\vec{p})$ we take $\mu=1$ as an instance that is we consider:$\bar{u}_r(E,\vec{p})\gamma^1 u_r(E,\vec{p})$

From Dirac equation

$$(\gamma^\mu p_\mu+m)v_r=0$$

$$v_r=-\frac{\gamma^\mu p_\mu}{m}v_r$$

From conjugate Dirac equation $v_r(\gamma^\mu p_\mu-m)=0$

$$\bar{v}_r=\bar{v}_r\frac{\gamma^\mu p_\mu}{m}$$

Therefore

$$\bar{v}_r(E,\vec{p})\gamma^1 v_r(E,\vec{p})=\frac{1}{2}\bar{v}_r(E,\vec{p})\gamma^1 v_r(E,\vec{p})+\frac{1}{2}\bar{v}_r(E,\vec{p})\gamma^1 v_r(E,\vec{p})$$

$$=-\bar{v}_r\frac{\gamma^\mu p_\mu}{2m}\gamma^1 v_r-\bar{v}_r\gamma^1\frac{\gamma^\mu p_\mu}{2m}v_r$$

$$=\bar{v}_r\frac{\{\gamma^\mu p_\mu,\gamma^1\}}{2m}v_r$$

By direct calculation $\{\gamma^\mu p_\mu,\gamma^1\}=2p^x$

$$\bar{v}_r(E,\vec{p})\gamma^1 v_r(E,\vec{p})=-\bar{v}_r\frac{p^x}{m}v_r=-\bar{v}_r v\frac{p^x}{m}=-\frac{p^x}{m}$$

Combination of Particle Antiparticle States in the Dirac Current

$$(5.1)\ I=\int C(E',\vec{p}')D^*(-E,\vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}}{}'}}\bar{v}_s(-E,\vec{p})\exp[-i[Et-\vec{p}.\vec{x}]]\gamma^\mu u_r(E',\overrightarrow{p'})]\exp[-i[E't-\vec{p}'.\vec{x}]d^3xd^3pd^3p'$$

(5.2) $I'=\int C^*(E',\vec{p}')D(-E,\vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}'}}}\bar{u}_r(E',\vec{p}')\exp[i[E't-\vec{p}'.\vec{x}]]\gamma^\mu v_s(E,\vec{p})]\exp[i[Et-\vec{p}.\vec{x}]d^3xd^3pd^3p'\ ;E>E'>0$

$$I = \int C(E', \vec{p}')D^*(-E, \vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}}'}}\bar{v}_s(-E, \vec{p})\gamma^\mu u_r(E', \overrightarrow{p'})]\exp[-i[(E + E')t - (\vec{p} + \vec{p}').\vec{x}]d^3xd^3pd^3p'$$

$$I = \int C(E', \vec{p}')D^*(-E, \vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}}'}}\bar{v}_s(-E, \vec{p})\gamma^\mu u_r(E', \overrightarrow{p'})]\exp[-i[(E + E')t]\delta(\vec{p} + \vec{p}').\vec{x}]d^3pd^3p'$$

$$(6) I = \int C(E, \vec{p})D^*(-E, \vec{p})\frac{1}{E_{\vec{p}=\vec{p}'}}\bar{v}_s(-E, \vec{p})\gamma^\mu u_r(E, \vec{p})]\exp[-2\mathrm{iEt}]\, d^3p$$

For particles moving in opposite directions $\vec{p} = -\vec{p}'$, contribution from the delta function is non trivial else contribution from it makes the integrand zero.

For $|\vec{p}| \neq |\vec{p}'|$, contribution from the delta function is again zero

Evaluation of

$$\bar{v}_s(-E, \vec{p})\gamma^\mu u_r(E, \vec{p})$$

Dirac Equation

$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad (A)(7)$

$$\bar{\psi}(i\gamma^\mu \partial_\mu + m) = 0 \ \ (8)\ (A')$$

Trial Solutions

1) $\psi^{(+)} = u_r \exp[-ip_\mu x^\mu] = u_r \exp[-i[p_0x^0 + p_1x^1 + p_2x^2 + p_3x^3]] = u_r \exp[-i[p^0x^0 - p^1x^1 - p^2x^2 - p^3x^3] = u_r \exp[-i[p^0x^0 - \vec{p}.\vec{x}]; \psi^{(+)} = u_r \exp[-i[p^0x^0 - \vec{p}.\vec{x}]; E = p_0 = p^0 > 0$[Metric signature:$(1, -1, -1, -1)$]
2) $\psi^{(-)} = v_r \exp[ip_\mu x^\mu] = v_r \exp[i[p_0x^0 + p_1x^1 + p_2x^2 + p_3x^3]] = v_r \exp -i[p^0x^0 - p^1x^1 - p^2x^2 - p^3x^3] = v_r \exp[i[p^0x^0 - \vec{p}.\vec{x}]; \psi^{(-)} = v_r \exp[i[p^0x^0 - \vec{p}.\vec{x}; E = p_0 = p^0 > 0$
   I n the above trial solutions energy and momentum always have opposite signs
   Also: Energy operator:$\hat{E} = i\frac{\partial}{\partial t}$, Momentum operator:$\hat{p}^\alpha - \ i\frac{\partial}{\partial x^\alpha}; \hbar = 1$
   $\hat{E}\psi^{(+)} = +p^0\psi^{(+)}; \hat{E}\psi^{(-)} = -p^0\psi^{(+)}$;
   The crucial point is that in formulating the Klein Gordon equation from the relation $E^2 = p^2 + {m_0}^2$ the same operator formalism is used: therefore this operator formalism is a valid one for Relativistic quantum mechanics

3) $\bar{\psi}^{(+)} = \bar{u}_r \exp[i[p^0x^0 - \vec{p}.\vec{x}]$
4) $\bar{\psi}^{(-)} = \bar{u}_r \exp[-i[k^0x^0 - \vec{p}.\vec{x}]$

Substituting (1) and (2) into (7) we have

$$(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3 - m)\psi^{(+)} = 0$$

$$\bar{\psi}^{(+)}(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3 - m) = 0$$

$$\psi^{(+)} = \frac{p^0\gamma^0 - k^1\gamma^1 - k^2\gamma^2 - k^3\gamma^3}{m}\psi^{(+)} (9.1)$$

$$\bar{\psi}^{(+)} = \bar{\psi}^{(+)}\frac{-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3}{m} \; (9.2)$$

Substituting (1) and (2) into (8) we have

$$(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3 - m)\psi^{(-)} = 0$$

$$\bar{\psi}^{(-)}(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3 - m) = 0$$

$$\psi^{(-)} = \frac{(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)}{m}\psi^{(-)} \; (10.1)$$

$$\bar{\psi}^{(-)} = \bar{\psi}^{(-)}\frac{p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3}{m} \; (10.2)$$

Evaluation of :

$\bar{u}_r\gamma^0 v_r$ and $\bar{u}_r\gamma^i v_r; i = x, y, z$

For that we first evaluate

$\bar{\psi}^{(+)}\gamma^0\psi^{(-)} = \bar{u}_r\gamma^0 v_r \times exponential\ part$ and $\bar{\psi}^{(+)}\gamma^i\psi^{(-)} = \bar{u}_r\gamma^i v_r$ ×exponential part

$$\bar{\psi}^{(+)}\gamma^0\psi^{(-)} = \frac{1}{2}\bar{\psi}^{(+)}\gamma^0\psi^{(-)} + \frac{1}{2}\bar{\psi}^{(+)}\gamma^0\psi^{(-)}$$

$$= \bar{\psi}^{(+)}\frac{-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3}{2m}\gamma^0\psi^{(-)} + \bar{\psi}^{(+)}\gamma^0\frac{(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)}{m}\psi^{(-)}$$

[using 9.2 and (10.1)]

$$= \bar{\psi}^{(+)}\frac{(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)\gamma^0 + \gamma^0(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p\gamma^3)}{2m}\psi^{(-)} = \bar{\psi}^{(+)}(-)\frac{k_0}{m}\psi^{(-)}$$
$$= -\frac{p_0}{m}\bar{\psi}^{(+)}\psi^{(-)} = -\frac{p_0}{m} \times 0$$

Again

$$\bar{\psi}^{(+)}\gamma^1\psi^{(-)} = \frac{1}{2}\bar{\psi}^{(+)}\gamma^1\psi^{(-)} + \frac{1}{2}\bar{\psi}^{(+)}\gamma^1\psi^{(-)}$$

$$= \bar{\psi}^{(+)}\frac{-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3}{2m}\gamma^1\psi^{(-)} + \bar{\psi}^{(+)}\gamma^1\frac{(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)}{m}\psi^{(-)}$$

[using 9.2and 10.1]

$$= \bar{\psi}^{(+)}\frac{(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)\gamma^1 + \gamma^1(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)}{2m}\psi^{(-)}$$

$$= \bar{\psi}^{(+)}\frac{(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)\gamma^1 + \gamma^1(-p^0\gamma^0 + p^1\gamma^1 + p^2\gamma^2 + p^3\gamma^3)}{2m} = -\bar{\psi}^{(+)}\frac{p_1}{m}\psi^{(-)}$$

$$= -\frac{p_1}{m}\bar{\psi}^{(+)}\psi^{(-)} = 0$$

$\bar{\psi}^{(+)}\gamma^0\psi^{(-)}$ and $\bar{\psi}^{(+)}\gamma^i\psi^{(-)}$

Being zero each we do have

$\bar{u}_r\gamma^0 v_r = 0$ and $\bar{u}_r\gamma^i v_r = 0$

Similar conclusions for I'

$$I' = \int C^*(E', \vec{p}')D(-E, \vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}}{}'}}\bar{u}_r(E', \vec{p}')\exp[i[E't - \vec{p}'.\vec{x}]]\gamma^\mu v_s(E, \vec{p})]\exp[i[Et - \vec{p}.\vec{x}]d^3x d^3p d^3p'$$

$$I' = \int C^*(E', \vec{p}')D(-E, \vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}}{}'}}\bar{u}_r(E', \vec{p}')\gamma^\mu v_s(E, \vec{p})\exp[i[(E + E')t - (\vec{p} + \vec{p}').\vec{x}]d^3x d^3p d^3p'$$

$$I' = \int C^*(E', \vec{p}')D(-E, \vec{p})\frac{1}{\sqrt{E_{\vec{p}}}}\frac{1}{\sqrt{E_{\vec{p}}{}'}}\bar{u}_r(E', \vec{p}')\exp[i[E't - \vec{p}'.\vec{x}]]\gamma^\mu v_s(E, \vec{p})]\exp[i[(E + E')t]\delta^3(\vec{p} + \vec{p}').\vec{x}]d^3p d^3p'$$

$$I' = \int \frac{1}{E_p}C^*(E, \vec{p})D(-E, \vec{p})\bar{u}_r(E, \vec{p})\gamma^\mu v_s(E, \vec{p})\exp[2iEt]$$

Evaluation of :

$\bar{v}_s\gamma^0 u_r$ and $\bar{v}_s\gamma^i u_r$; $i = 1,2,3$ spatial components

For that we first evaluate

$\bar{\psi}^{(-)}\gamma^0\psi^{(+)} = \bar{v}_s\gamma^0 u_r \times exponential\ part$ and $\bar{\psi}^{(-)}\gamma^i\psi^{(+)} = v_s\gamma^i u_r$ ×exponential part

$$\bar{\psi}^{(-)}\gamma^0\psi^{(+)} = \frac{1}{2}\bar{\psi}^{(-)}\gamma^0\psi^{(+)} + \frac{1}{2}\bar{\psi}^{(-)}\gamma^0\psi^{(+)}$$

$$= \bar{\psi}^{(-)}\frac{p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3}{2m}\gamma^0\psi^{(+)} + \bar{\psi}^{(-)}\gamma^0\frac{p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3}{2m}\psi^{(+)}$$

$$= \bar{\psi}^{(-)}\frac{(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3)\gamma^0 + \gamma^0(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3)}{2m}\psi^{(+)}$$

[using (9.1) and (10.2)]

$$= \bar{\psi}^{(+)}(-)\frac{p_0}{m}\psi^{(-)} = -\frac{p_0}{m}\bar{\psi}^{(-)}\psi^{(+)} = -\frac{p_0}{m}\times 0$$

Again

$$\bar{\psi}^{(-)}\gamma^1\psi^{(+)} = \frac{1}{2}\bar{\psi}^{(-)}\gamma^i\psi^{(+)} + \frac{1}{2}\bar{\psi}^{(-)}\gamma^1\psi^{(+)}$$

$$= \bar{\psi}^{(-)}\frac{p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3}{2m}\gamma^1\psi^{(+)} + \bar{\psi}^{(-)}\gamma^{`1}\frac{p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3}{2m}\psi^{(+)}$$

[using (9.1) and (10.2)]

$$= \bar{\psi}^{(-)}\frac{(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3)\gamma^1 + \gamma^1(p^0\gamma^0 - p^1\gamma^1 - p^2\gamma^2 - p^3\gamma^3)}{2m}\psi^{(+)} = -\frac{p^1}{m}\bar{\psi}^{(-)}\psi^{(+)}$$

$$= -\frac{p^1}{m}\times 0 = 0$$

Being zero each we do have

$\bar{u}_r\gamma^0 v_r = 0$ and $\bar{u}_r\gamma^i v_r = 0$

An interesting Observation:

We know that the Dirac Current for wave packet is a real current.

Therefore:

$$J^k = \left(J^k\right)^*$$

This result may be used with nay standard formula[4] for the Dirac current[wave packet considering superposition of Dirac states].

You will 0btain: $J^k = A + B$

$$\left(J^k\right)^* = A - B$$

Implying B=0

$B$is the part that contains Zitterbewegung[oscillatory part]. So there is no Zitterbewegung

Conclusion

Thus we observe from theoretical considerations that for free particle superposition of Dirac states it is not possible to have admixture terms[particle-antiparticle or antiparticle particle combinations] that contain oscillatory factors responsible for Zitterbewegung---trembling motion of electrons. It is important to keep in mind that Zitterbewegung by definition is expected from superposition of *free* Dirac states. But for pragmatic reasons we always do have some field effect, of major type or of minor type. The presence of such a field potential could modify our conclusions.

References

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