About the Distribution of Prime Numbers

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Abstract. In this article, a prime number distribution formula is given. The formula is based on the periodic property of the sine function and an important trigonometric limit.

1. INTRODUCTION. Knowing the periodic property of sine function, a natural number a is divisible by b if

$$\sin \frac{a\,\pi}{b} = 0 \tag{1}$$

where $b \geq 1$ and $a \geq b$. On the basis of Equation 1 and the trigonometric limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

the following theorem is deduced.

Theorem 1. *Let* $P(a) = \prod_{b=2}^{\infty} \left(\frac{1}{b-a}\right) \sin\left(\frac{a\,\pi}{b}\right)$. *Then any natural number* x *is prime if* $\lim_{a \to x} P(a) \neq 0$.

Proof. For each term in the infinite product, with the factor $\frac{1}{b-a}$ the first multiple of b (a root of Eq. 1) tends to π/b as $a \to b$:

$$\begin{aligned}
\lim_{a \to b} \left(\frac{1}{b-a}\right) \sin\left(\frac{a\,\pi}{b}\right) &= \lim_{a \to b} \left(\frac{1}{b-a}\right) \sin\left(\frac{a\,\pi}{b}\right) \left(\frac{\pi}{b}\right) \left(\frac{\pi}{b}\right)^{-1} \\
&= \lim_{a \to b} \left(\frac{\pi}{b}\right) \lim_{a \to b} \frac{\sin\left(\frac{a\,\pi}{b}\right)}{\pi - \frac{a\,\pi}{b}} \\
&= \frac{\pi}{b}
\end{aligned}$$

Therefore, each term is a function with roots in multiples of b except for the first multiple, where the amplitude tends to π/b (see Figure 1)

For the infinite product P, all natural number where P tends to zero is a composite number, and where P tends to non-zero value is a prime number (product for 2 to 20 is shown in Figure 2)

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Remark. It is evident that through trigonometric identities can also achieved other functions that determine the distribution of prime numbers with the same criteria.

It is alo possible to remove pairs terms redefining the function P as follows:

$$P(a) = \left(\frac{1}{2-a}\right)\sin\left(\frac{a\,\pi}{2}\right)\prod_{b=1}^{\infty}\left(\frac{1}{(2b+1)-a}\right)\sin\left(\frac{a\,\pi}{(2b+1)}\right)$$

Finally, for computational aspects can be observed a relationship between the prime evaluated and the upper limit of the product, not yet determined.

Figure 1. Product terms for $b = 1, 2, 3, 4$

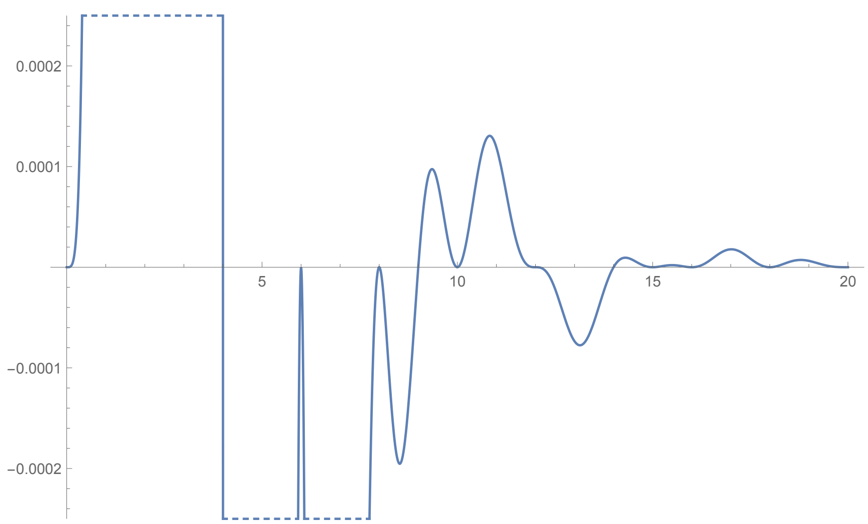

Figure 2. Product for $b = 2$ to 20