Solution of the Singularity Problem of Black Hole [1]

Hyoyoung Choi * [2]

Abstract

Singularity problem is a long-standing weak point in the theory of general relativity. Most scholars assume that the solution for this singularity consists in quantum mechanics. However, waiting for quantum gravity theory to be completed to solve the singularity problem in a black hole is wrong. Hear we show that gravitational self-energy has a negative value can solve singularity problem and rescue general relativity. The black hole does not have a singularity but has a region with a total energy of zero. The distribution of mass can't be reduced to at least radius R_{gs}. Also, gravitational self-energy and R_{gs} guarantee uniform density due to repulsive gravity effect and this can be grounds for the expansion in the early universe and the uniform universe.

I. Singularity problem of black hole

Generally, stars are under gravitational contraction by their own gravity and it is known that if this is contracted within a certain radius (like Schwarzschild radius), it makes a black hole which even lights cannot escape from. [1] Since gravity is generally an attractive force, gravitational contraction continues to exist in the black hole, too. Thus, in the central part of black hole exists an area with infinite density of energy, which point we call singularity. [2] [3]

Such singularity denies application of the existing laws of physics and it is unnatural for a certain substantial object to have infinite density of energy. Besides, such singularity has never been observed as substance but is just a mathematical result of general relativity, which is considered a defect or limit of the theory. [4]

We assume that the solution for this singularity consists in quantum mechanics. [5] Though exact explanation is not available because quantum gravity theory in integration of quantum mechanics and gravity has not been completed yet.

II. Gravitational self-energy and singularity of black hole

1. Gravitational self-energy

The concept of gravitational self-energy(U_{gs}) is the total of gravitational potential energy possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass dMs, it involves the existence of gravitational potential energy among these dMs and is the value of adding up these. $M = \sum dM$.

Gravitational self-energy or Gravitational binding energy($-U_{gs}$) in case of uniform density is given by [6]

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R} \tag{1}$$

2. For black hole or singularity, never fail to consider gravitational self-energy.

In the generality of cases, the value of gravitational self-energy is small enough to be negligible, compared to mass energy mc^2. So generally, there was no need to consider gravitational self-energy. However the smaller

[1] This paper is a paper that is rearranged in a single topic (singularity problem) among my articles. (https://www.researchgate.net/publication/287217009 & https://www.researchgate.net/publication/309786718)

[2] E-mail:7icarus7@gmail.com

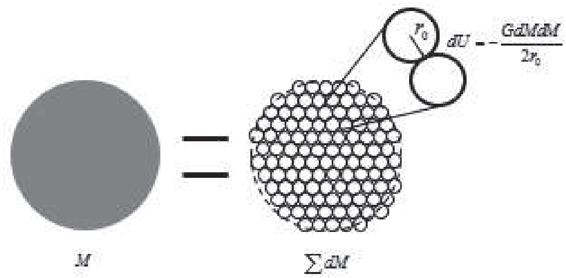

Figure 1: Since all mass M is a set of infinitesimal mass dMs and each dM is gravitational source, too, there exists gravitational potential energy among each of dMs. Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into c^2.

R becomes, the higher the absolute value of U_{gs}. For this reason, we can see that U_{gs} is likely to offset the mass energy in a certain radius.

Thus, looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,

$$U_{gs} = |-\frac{3}{5}\frac{GM^2}{R_{gs}}| = Mc^2 \tag{2}$$

$$R_{gs} = \frac{3}{5}\frac{GM}{c^2} \tag{3}$$

This equation means that if mass M is uniformly distributed within the radius R_{gs}, gravitational self-energy for such an object equals mass energy in size. So, in case of such an object, mass energy and gravitational self-energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

Comparing R_{gs} with R_S, the radius of Schwarzschild black hole,

$$R_{gs} = \frac{3}{5}\frac{GM}{c^2} < R_S = \frac{2GM}{c^2} \tag{4}$$

$$R_{gs} = 0.3R_S \tag{5}$$

This means that there exists the point where gravitational self-energy becomes equal to mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists at the point $0.3R_S$, a 30% level of the black hole radius.

Even with kinetic energy and virial theorem applied only the radius diminishes as negative energy counterbalances positive energy, but no effects at all on this point: "There is a zone which cannot be compressed anymore due to the negative gravitational potential energy." Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the virial theorem (K=-U/2)),

$$R_{gs-vir} = \frac{1}{2}R_{gs} = 0.15R_S \tag{6}$$

Since this value is on a level not negligible against the size of black hole, we should never fail to consider "gravitational self-energy" for case of black hole.

3. Black hole does not have a singularity in the center, but it has a zero energy zone.

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point R = 0. From the point R_{gs}(or R_{gs-vir}), gravity is 0, and when it enters into the area of R_{gs}(or R_{gs-vir}), total energy within R_{gs}(or R_{gs-vir}) region corresponds to negative values enabling anti-gravity to exist. This R_{gs}(or R_{gs-vir}) region comes to exert repulsive effects of gravity on the particles outside of it, therefore it interrupting the formation of singularity at the near the area R=0.

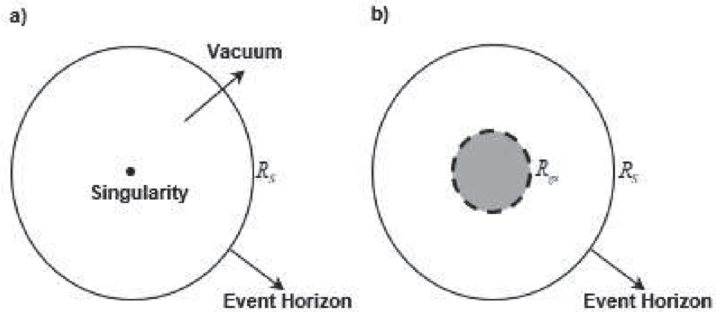

Figure 2: a) Existing Model. b) New Model. The area of within $R_{gs}(or R_{gs-vir})$ has gravitational self-energy(potential energy) of negative value, which is larger than mass energy of positive value. If r is less than R_{gs} , this area becomes negative energy(mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This area (within R_{gs}) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to r=0. Therefore the distribution of mass (energy) can't be reduced to at least radius R_{gs} .

However, it still can perform the function as black hole because R_{gs} is only 30% of R_S with a large difference in volume and, comparing total mass, it still can correspond to a very large quantity of positive mass. Therefore, it still can perform the function as black hole on the objects outside of R_S.

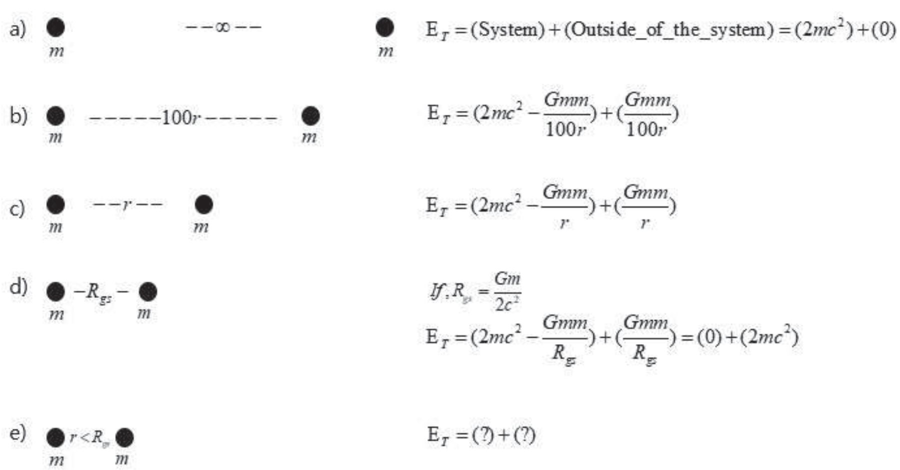

Figure 3: Explanation by the mass defect. Assumes a stationary state. if R is R_{gs}, the total energy of the system becomes 0 state. Now, in order for r to be smaller than R_{gs}, there must be energy release from the system to the outside of the system. In models that assume only positive energies, this state($r < R_{gs}$) must be prohibited because there is no positive energy to be removed from the system anymore. It is forbidden that the mass distribution of the system is within R_{gs}, and therefore no singular point is formed.

If we explain the phenomenon in terms of mass defect,

When the two particles are changed from the free state to the binded state, in order for the two particles to reach the binding state, the mass energy corresponding to the difference in the binding energy must be released to the outside of the system. We call it a mass defect. This suggests that the two particles do not reach a stable binding state without the mass defect (mass release to the outside of the system) corresponding to the difference in binding energies.

Now let's think about the inside of a black hole.

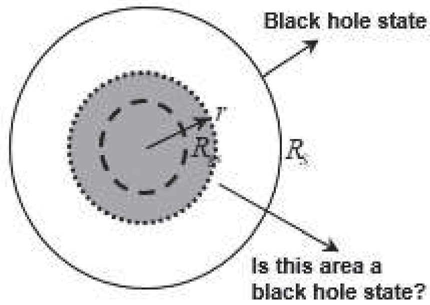

Figure 4: When a black hole is formed, the inner mass sphere(area) is not a black hole state. Thus, during the gravitational contraction process, the inner mass sphere(area) emits energy.

Let's consider the situation in which a black hole may have just been formed. For simple analysis, assume a uniform mass distribution.

$$R_S = \frac{2GM_S}{c^2} \tag{7}$$

$$M_S = \frac{R_S c^2}{2G} \tag{8}$$

Now let's consider the mass within the inside shell.

$$M(r) = \frac{4\pi}{3} r^3 \rho \tag{9}$$

When the inner mass sphere(area) forms a black hole, let's look at the size of the mass distribution.

$$R_{S-M(r)} = \frac{2GM(r)}{c^2} = (\frac{r}{R_S})^3 R_S \tag{10}$$

For example, in order for the mass within the $r = \frac{1}{2}R_S$ shell to become a black hole, the Schwarzschild radius must be $R_{S-M(r)} = \frac{1}{8}R_S$. Mass density should be compressed 64 times. The point is, when a black hole is formed, the inner mass sphere(area) is not a black hole state. Therefore, the key point is that the inner mass sphere(area) will have an energy release during the gravitational contraction. At this point, we can consider the virial theorem.

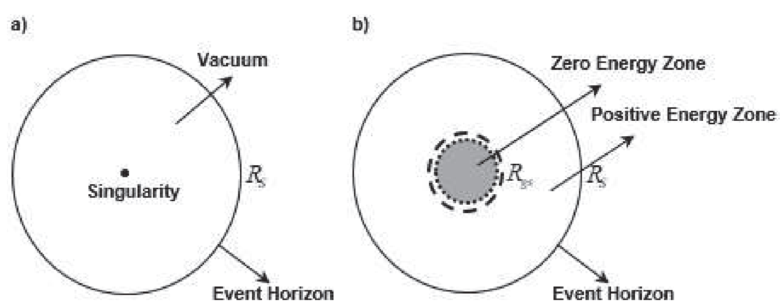

Figure 5: Internal structure of the black hole. a)Existing model b)New model. If, over time, the black hole stabilizes,the black hole does not have a singularity in the center, but it has a zero (total) energy zone.

When the mass distribution inside the black hole is reduced from R_S to R_{gs}(or R_{gs-vir}), the energy must be released from the inside of the system to the outside of the system in order to reach this binding state. Here, the system refers to the mass distribution within the radius $0 \leq r \leq R_{gs}(orR_{gs-vir})$. Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system. We need to consider the virial theorem.

At this time, the emitted energy does not go out of the black hole. This energy is distributed in the $R_{gs}(orR_{gs-vir}) < r \leq R_S$ region.

If you have only the concept of positive energy, please refer to the following explanation.

From the point of view of mass defect, $r = R_{gs}(orR_{gs-vir})$ is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than $r = R_{gs}(orR_{gs-vir})$. So black hole doesn't have singularity.

By locking horns between gravitational self-energy and mass energy, particles inside black hole or distributionof energy can be stabilized. As a final state, the black hole does not have a singularity in the center, but it has a zero (total) energy zone.

4. Waiting for quantum gravity theory to be completed to solve the singularity issue in a black hole is wrong as it was made by our stereotypes.

When there occurred a problem in singular "point", one dimensional idea that problems should be solved from λ, wavelength that has a little bigger than "point" was partially acted. Of course, we should try to establish a quantum gravity theory for other reasons.

We can think of a black hole of big size and approach this problem by reducing the mass of this black hole. In other words, we should form a certain internal structure of usual size and apply the experience that we had applied the limit.

$$\lim_{M \to small} R_{gs}$$

If you are still uncomfortable with R_{gs}, think about a black hole with the size 10 billion times bigger than the solar mass. [7] [8] Schwarzschild radius of this black hole is $R_S = 3 \times 10^{10} km$ and R_{gs} of this black hole $1 \times 10^{10} km$. Average density of this black hole is about $1.81 kg/m^3$. And average density of the Earth is about $5,200 kg/m^3$.

Is it a size that requires quantum mechanics? Is it a high density state that requires quantum mechanics? Black hole of this size is Newtonian mechanics' object and therefore, gravitational potential energy must be considered.

Let's reduce the mass of this black hole gradually and approach three times the solar mass, the smallest size of black hole where stars can be formed!

In case of the smallest black hole with three times the solar mass, [9] $R_S = 9km$. R_{gs} of this object is as far as 3km. In other words, **even in a black hole with smallest size that is made by the contraction of a star, the distribution of internal mass can't be reduced to at least radius 3km($R_{gs-vir} = 1.5km$).**

5. The minimal size of existence

[Existence = the sum of infinitesimal existences composing an existence]

A single mass M for some object means that it can be expressed as $M = \sum dM$ and, for energy, $E = \sum dE$. The same goes for elementary particles, which can be considered a set of dMs, the infinitesimal mass.

The equation (2) above means that if masses are uniformly distributed within the radius R_{gs}, the size of negative binding energy becomes equal to that of mass energy. This can be the same that the rest mass, which used to be free for the mass defect effect caused by binding energy, has all disappeared. This means the total energy value representing "some existence" coming to 0 and "extinction of the existence". **Therefore, R_{gs} is considered to act as "the minimal radius" or "a bottom line" of existence with some positive energy.**

III. Expansion of the general relativity and cosmology

1. In all existing solutions, the mass term M must be replaced by $(M - M_{gs})$.

We can solve the problem of singularity by separating the term$(-M_{gs})$ of gravitational self-energy from mass and including it in the solutions of field equation.

$M \to (M) + (-M_{gs})$, $-M_{gs}$ is the equivalent mass of gravitational self-energy. In all existing solutions(Schwarzschild, Kerr, Reissner-Nordström, ...), the mass term M must be replaced by $(M - M_{gs})$.

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{11}$$

Schwarzschild-Choi solution is

$$ds^2 = -(1 - \frac{2G(M - M_{gs})}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2G(M-M_{gs})}{c^2 r})} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{12}$$

For the sphere with uniform density,

$$-M_{gs} = -\frac{3}{5}\frac{GM^2}{Rc^2} \tag{13}$$

1) If $M \gg |-M_{gs}|$, in other words if $r \gg R_S$, we get the Schwarzschild solution.

2) If $M = |-M_{gs}|$

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{14}$$

3) If $M \ll |-M_{gs}|$, in other words if $0 \leq r \ll R_{gs}$,

$$ds^2 \simeq -(1 + \frac{2GM_{gs}}{c^2 r})c^2 dt^2 + \frac{1}{(1 + \frac{2GM_{gs}}{c^2 r})} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{15}$$

In the domain of $0 \leq r \ll R_{gs}$,

The area of within R_{gs} has gravitational self-energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other. [10] So, we can assume that $-M_{gs}$ is almost evenly distributed. Therefore ρ_{gs} is constant. And we must consider the Shell Theorem.

$$-M_{gs} = -\frac{4\pi r^3}{3}\rho_{gs} \tag{16}$$

$$(1 + \frac{2GM_{gs}}{c^2 r}) = 1 + \frac{2G(\frac{4\pi}{3}r^3\rho_{gs})}{c^2 r} = 1 + \frac{8\pi G\rho_{gs} r^2}{3c^2} \tag{17}$$

$$ds^2 \simeq -(1 + \frac{8\pi G\rho_{gs} r^2}{3c^2})c^2 dt^2 + \frac{1}{(1 + \frac{8\pi G\rho_{gs} r^2}{3c^2})} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{18}$$

If $r \to 0$,

$$ds^2 \simeq -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{19}$$

There is no singularity.

In practice, mass contraction must be stopped at the point where $M_{shell} = M_{shell-gs}$.

If, after the formation of a black hole, it has been stabilized for a long time, the model of Fig.5 should be taken into account. The black hole does not have a singularity in the center, but it has a zero energy zone.

2. Kretschmann scalar

For Schwarzschild black hole, the Kretschmann scalar [11] is,

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{48G^2M^2}{c^4r^6} \tag{20}$$

In the preexisting model, if $r \to 0$, it does diverse.

However, this model is

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{48G^2(M - M_{gs})^2}{c^4r^6} \approx \frac{48G^2(-M_{gs})^2}{c^4r^6} = \frac{(16\pi G\rho_{gs})^2}{3c^4} \tag{21}$$

It does not diverge. Therefore, black hole doesn't have singularity.

3. Gravitational self-energy can provide the concept of minimal size, one of the reasons for introducing string theory.

In quantum mechanics we could think of De Broglie's matter wave theory about length of some objects.

However this formula does restrict the upper limit of velocity to c, but does not have the upper limit for mass. In other words this implies there is no lower limit of wavelength and it can go to 0 in quantum mechanics. It's the same with Heisenberg's Uncertainty Principle.

Furthermore, 0.02 milligram of Planck mass($m_p = \sqrt{\frac{c\hbar}{G}}$) they introduced is not even the size for the role of upper or lower limit among "$0 \sim \infty$". Since Planck mass cannot perform a role as the upper limit, Planck length, inversely related to the Planck mass, cannot do a role of lower limit either.

Planck mass and length means that "within such a size we should consider the quantum mechanical effects." but does not indicate "no more or no less can exist."

To remove singularity, considering gravitational self-energy is only enough without need to assume some minimal unit like a string. Thus, the existing relations need to be transformed so that they may include the minimal length by dint of gravitational self-energy.

$$\Delta x \sim \frac{\hbar}{\Delta p} \tag{22}$$

$$\Delta x \sim \frac{\hbar}{\Delta p} + R_{gs} \geq \frac{3}{5}\frac{GM}{c^2} = \frac{3}{5}\frac{GE}{c^4} \tag{23}$$

Since the increase of momentum is put into the increase of energy and mass, Δx cannot go to 0. Therefore, we can introduce the minimal length naturally and work out the problems of singularity or infinity.

4. The source of uniformity and expansion

The mainstream physics assumes inflation to explain about the uniformity, flatness and horizon of the universe. [12] [13]

However, if the universe starts from the high density state ($0 < r < R_{gs}$), as the negative gravitational potential energy is bigger than the positive mass energy in the area of $0 < r < R_{gs}$, there exists a anti-gravity effect. Therefore, matter distribution is essentially closer to a uniform density within the radius of R_{gs}. Thus inevitably the universe expands in uniform density. This can be a strong explanation for the fact that the universe that we observe has almost a uniform density.

Also, the universe that was born in high density state does not have a singularity by gravitational collapse, but can be one explanation for the fact that the universe expands.

5. R_{gs} of the present universe

From the equation (3),

$$R_{gs} = \sqrt{\frac{5c^2}{4\pi G\rho}} \tag{24}$$

Use to the Planck data('2013, h=0.678), [14]

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 8.64 \times 10^{-27} kgm^{-3} \tag{25}$$

$$R_{gs} = \sqrt{\frac{5c^2}{4\pi G\rho}} = 2.49 \times 10^{26} m = 26.3Gly \tag{26}$$

If the universe that was born in high density state, universe does not a singularity by gravitational collapse, but must expands to the R_{gs}(26.3Gly) at least.

IV. Conclusions

If, over time, the black hole stabilizes, the black hole does not have a singularity in the center, but it has a zero total energy zone. The distribution of mass (energy) can't be reduced to at least radius $R_{gs}(or R_{gs-vir})$. Gravitational self-energy (or Gravitational potential energy) can solve singularity problem and rescue general relativity that collapses itself. Also it can be grounds for the expansion in the early universe and the uniform universe.

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