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1. ABSTRACT

In this work has been calculated two new non baryonic DM density profiles inside halo region of Milky Way, MW hereafter, and it has been demonstrated that both ones are mathematically equivalents. Data have been got from rotation curve published in [17] Bhattacharjee, P.2014.

The first profile is called *Direct DM density* because it is got directly from velocity as power regression of radius in halo rotation curve. In other words velocity of rotation curve depend on radius as a power function.

The second one, *DM density as power of E*, E is gravitational field, has been introduced by author in previous papers, [8] Abarca,M.2016, where it has been used to study non baryonic DM in several galaxies. It is called “as power of E” because DM density depend on E as a power function.

Hypothesis which is the basis of theory is that non baryonic DM is generated locally by the own gravitational field according a power law. DM density = $A \cdot E^B$ where A& B are coefficients and E is gravitational intensity of field.

To find reasons that author has to do so daring statement, reader can consult [1] Abarca,M.2014. *Dark matter model by quantum vacuum*. [8] Abarca,M.2016. *Dark matter density on big galaxies depend on gravitational field as Universal law* and other papers quoted in bibliography.

Briefly will be explained method followed to develop this paper. Firstly are presented rotation curve and table with data points inside MW halo. These data come from [17] Bhattacharjee, P. Chaudary, S. Kundu, S.2014. In addition it is fitted a power regression of rotation curve points in halo region whose function is $v = a \cdot r^b$.

In fourth chapter it is developed a mathematical method to get a new DM density depending on radius called direct DM density because it is got directly from power regression of velocity depending on radius. Also it is compared Direct DM got from rotation curve [17] Bhattacharjee, P.2014 and Direct DM got from rotation curve [5] Sofue, Y.2015. It is shown that relative difference oscillates between 2.6% at 40 kpc and 3.8 % at 190 kpc which is a very exiguous difference. It is a very good news that two prominent teams of researchers got so similar results.

In fifth chapter it has been demonstrated that Direct DM profile is mathematically equivalent to DM density depending on gravitational field, as a power function i.e. DM density = $A \cdot E^B$, where A& B are cleared up depending on a & b (parameters of power regression of velocity).

In sixth chapter it has been got that for radius bigger than 40 kpc ratio baryonic density versus DM density is under 4% so it is reasonable to consider negligible baryonic density in order to develop theory introduced in this work.

In seventh chapter is compared Direct DM density got in this paper with NFW density profile fitted by Sofue in his paper. [5] Sofue, Y.2015. Throughout dominion NFW profile is bigger than Direct DM profile. Its relative difference oscillate between 25% at 40 kpc and 22% at 190 kpc.

In my opinion this remarkable fact could be explained because NFW profile is fitted with total DM enclosed inside galactic disc and as it is known inside bulge and disc there is an unknown amount of baryonic DM such as dwarf browns and cold gas clouds. However Direct DM profile is fitted with data which radius are bigger than 40 kpc where baryonic matter is negligible. It is clear that extra DM density data inside disc have influence over the whole NFW profile so it is right to conclude that relative difference between Direct DM and NFW profile might be explained by baryonic DM inside bulge and disc.

In eight chapter is compared DM density as power E in MW with DM density as power E in M31, which was published in [11] Abarca,M.2016. Results show that at a specific E, both DM densities are very similar. Relative differences are under 15 % inside main part of dominion. This fact support strongly author hypothesis about DM as power of E as Universal law.

In ninth chapter it is made dimensional analysis for DM density formula through Buckingham theorem with one pi monomial (G and h as universal constant), and two pi monomials with G, h and c as universal constant. Through a mathematical analysis it is it is rejected density formula with only one pi monomial so it is needed to consider density formula with two pi monomials. Parameters A&B are recalculated in order to be coherent with regression analysis and dimensional analysis. Namely, power of E become 5/3 instead to be decimal number coming from regression analysis and formulas acquire a new elegant aspect.

In tenth chapter is got and solved the differential equation for gravitational field E, inside solar system, accepting the same new values A& B recalculated for M31 galaxy.

It is got a formula for E, which tend to Newtonian formula if it is neglected the dark matter term. This fact is a strong back to power B=5/3 found by dimensional analysis.

Also it is calculated dark matter contained inside spherical corona definite by perihelion and aphelion radius of Mercury. It has been got 3.2 times M_{EARTH} which is a considerable amount of dark matter. By qualitative reasoning it is defended that this excessive amount of dark matter suggest that cannot be accepted the coefficient A got for M31 when is calculated dark matter inside Solar system because both gravitational systems have a tremendous different sizes.

2. INTRODUCTION

As reader knows Milky Way is the twin galaxy of M31 in Local Group of galaxies. Its disk radius is approximately 20 kpc and according [5] Sofue, Y. 2015 its baryonic mass is $M_{BARYONIC} = 1,37 \cdot 10^{11} M_{SUN}$

In previous paper [18] Abarca,M.2016 , author has calculated Direct DM and DM as power of E inside Milky Way halo through data which come from rotation curve published by [5] Sofue, Y. 2015, whereas in this paper both DM profiles have been calculated from rotation curve published in [17] Bhattacharjee, P.2014.

This new DM profile has been called direct DM density because this profile is fitted directly from data measures inside halo region. In this work radius dominion begin at 40 kpc because at this distance baryonic density is negligible as it will be shown in chapter six. Therefore the only one kind of matter in halo region it is supposed to be non baryonic dark matter or at least it is the dominant kind of matter.

It is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk and bulge. Reader can consult: [12] Nieuwenhuizen,T.M. 2010. [13] Nieuwenhuizen,T.M. 2012. [14] Nieuwenhuizen,T.M. 2010 [15] Wyrzykowski,L.2010. [16] M.R.S. Hawkins 2015.In fact there are an important amount of researchers in this way because baryonic DM and non baryonic DM are open problems still.

DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* and developed in others papers quoted in bibliography refers an original mechanism of non baryonic DM through the own gravitational field. So DM density is a power of gravitational field whose formula is $\varphi_{DM}(r) = A \cdot E^B$. Therefore it is needed to consider a radius dominion where baryonic matter would be negligible in order to study purely non baryonic DM.

In fact, according [5] Sofue, Y. 2015 data, in chapter six will be got that for radius bigger than 40 kpc baryonic matter density is under 4 % regarding DM density. This is the reason why radius dominion in this work is from 40kpc up to 190 kpc.

In chapter seven is compared DM density NFW profile with direct DM density profile and it is shown that NFW profile is bigger than Direct DM density throughout the whole dominion and its relative difference oscillates between 26 % at 40 kpc and 21% at 190 kpc. Taking into account that NFW is got through total DM enclosed inside galactic disc whereas direct DM profile is got in halo region, where baryonic density is negligible, it is possible to conclude that extra density of NFW profile come from baryonic DM enclosed in bulge and galactic halo.

Baryonic DM depend on evolution history for each galaxy however results got in my previous papers and this paper itself allows me to postulate that non baryonic DM depend on gravitational field as a universal law for giant galaxies.

In paper [1] Abarca,M.2014, it was postulated that DM density depends on gravitational field according an universal law. Further papers [2] Abarca,M.2015 and others have studied DM density as power of gravitational field in several galaxies: M31 and others galaxies. Results got support such hypothesis, because $\varphi_{DM}(r) = A \cdot E^B$ produces similar values of density in different giant galaxies through its respective A&B parameters of each galaxy.

3. OBSERVATIONAL BHATTACHARJEE DATA FOR MILKY WAY ROTATION CURVE

Table below has been got from paper [17] Bhattacharjee, P. Chaudary, S. Kundu, S.2014.

In chapter six will be shown reason why dominion data begin at 40 kpc in this work, despite the fact that it is supposed radius of galactic disk is 20 kpc.

TABLE 2
THE CIRCULAR VELOCITY, V_c , AND ITS 1- σ ERROR, ΔV_c , FOR VARIOUS VALUES OF THE GALACTOCENTRIC DISTANCE, r , FOR A RADIAL PROFILE OF THE NON-DISK TRACERS' VELOCITY ANISOTROPY PARAMETER β DERIVED FROM FIGURE 2 OF RASHKOV ET AL. (2013), WITH $\left[\frac{R_0}{\text{kpc}}, \frac{V_0}{\text{km s}^{-1}} \right] = [8.3, 244]$.

r (kpc)	V_c (km s ⁻¹)	ΔV_c (km s ⁻¹)	r (kpc)	V_c (km s ⁻¹)	ΔV_c (km s ⁻¹)
0.20	233.0	13.32	38.41	191.57	11.73
0.38	268.92	4.67	40.42	197.59	14.12
0.66	250.75	11.35	42.40	192.79	5.92
1.61	217.83	5.81	44.49	213.22	17.17
2.57	219.58	1.48	45.99	179.39	11.23
3.59	223.11	2.43	48.06	213.03	24.72
4.51	247.88	2.99	49.49	178.57	17.63
5.53	253.14	1.69	51.39	183.31	23.58
6.50	270.95	2.19	53.89	157.89	19.57
7.56	267.80	0.96	56.89	191.76	24.35
8.34	270.52	0.66	57.98	210.72	29.81
9.45	235.58	8.44	60.92	168.02	25.67
10.50	249.72	13.44	64.73	206.47	36.27
11.44	261.96	11.71	69.31	203.62	40.89
12.51	284.30	17.50	72.96	190.53	40.98
13.53	271.54	15.57	76.95	222.72	74.37
14.59	251.43	25.60	81.13	186.29	66.53
16.05	320.70	25.27	84.90	122.25	36.46
18.64	286.46	101.18	89.35	143.95	29.49
26.30	189.64	6.74	92.44	154.66	67.23
28.26	237.99	11.54	97.41	184.0	72.86
29.51	209.82	9.16	100.72	108.68	40.99
32.04	179.14	6.65	106.77	137.15	53.17
33.99	170.37	6.93	119.98	150.18	25.46
36.49	175.92	6.62	189.49	125.01	37.32

3.1 POWER REGRESSION OF VELOCITY DEPENDING ON RADIUS IN ROTATION CURVE

In order to assure that baryonic matter density is negligible versus non baryonic matter density it has been considered radius bigger than 40 kpc. In chapter six will be explained reason to get 40 kpc as starting point.

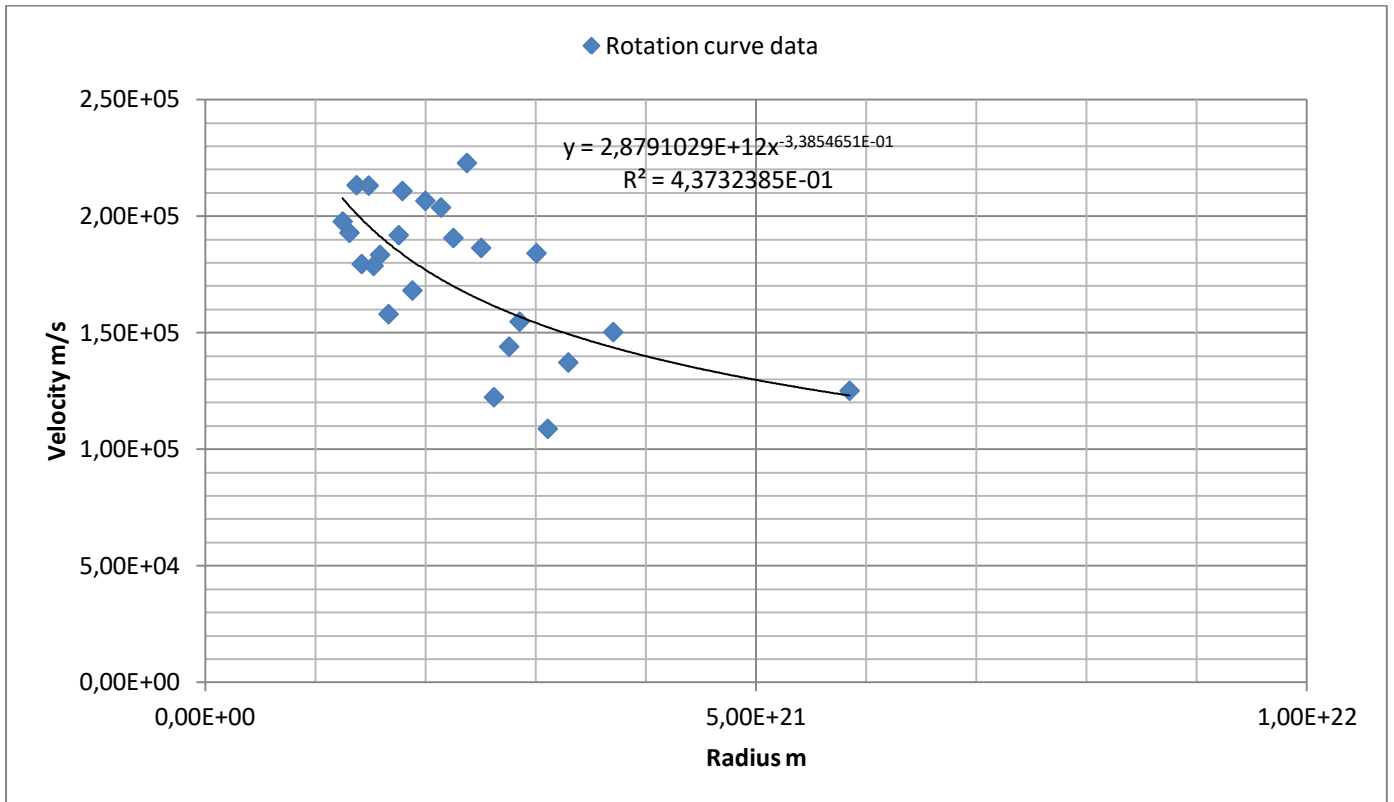
Equivalence data into International System			
Radius	Velocity	Radius	velocity
kpc	km/s	m	m/s
40,42	197,59	1,25E+21	1,98E+05
42,4	192,79	1,31E+21	1,93E+05
44,49	213,22	1,37E+21	2,13E+05
45,99	179,39	1,42E+21	1,79E+05
48,06	213,03	1,48E+21	2,13E+05
49,49	178,57	1,53E+21	1,79E+05
51,39	183,31	1,59E+21	1,83E+05
53,89	157,89	1,66E+21	1,58E+05
56,89	191,76	1,76E+21	1,92E+05
57,98	210,72	1,79E+21	2,11E+05
60,92	168,02	1,88E+21	1,68E+05
64,73	206,47	2,00E+21	2,06E+05
69,31	203,62	2,14E+21	2,04E+05
72,96	190,53	2,25E+21	1,91E+05
76,95	222,72	2,37E+21	2,23E+05
81,13	186,29	2,50E+21	1,86E+05
84,9	122,25	2,62E+21	1,22E+05
89,35	143,95	2,76E+21	1,44E+05
92,44	154,66	2,85E+21	1,55E+05
97,41	184	3,01E+21	1,84E+05
100,72	108,68	3,11E+21	1,09E+05
106,77	137,15	3,29E+21	1,37E+05
119,98	150,18	3,70E+21	1,50E+05
189,49	125,01	5,85E+21	1,25E+05

It is seen that experimental measures of rotation curve has a very good fitted curve by power regression.

Data fitted into I.S. are in grey columns below. Correlation coefficient is 0.66 which is an acceptable correlation coefficient. In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

Power regression for Milky Way rot. curve	
$V = a \cdot r^b$	
a	$2.87910294 \cdot 10^{12}$
b	$-3,385465058 \cdot 10^{-1}$
Correlation coeff.	0.6613046575

Below is shown a graphic with measures data and power regression function.



A coefficient of 0,66 is an acceptable correlation coefficient. Therefore this value support hypothesis that rotation curve of Milky Way follows a law $v = a \cdot r^b$ where a & b are written above.

4. DIRECT FORMULA FOR DM DENSITY ON MILKY WAY HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a \cdot r^b$. As $M(< r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of

$$\text{velocity results } M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In chapter 6 will be shown that for $r > 40$ kpc baryonic matter is negligible regarding DM density because baryonic matter density is lower than 4% of DM density.

$$\text{Density of D.M. is } D_{DM} = \frac{dm}{dV} \text{ where } dm = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G} \text{ and } dV = 4\pi r^2 dr \text{ so } D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G}$. results $D_{DM}(r) = L \cdot r^{2b-2}$. In case $b = -1/2$ DM density is zero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR MILKY WAY HALO

Parameters a & b from power regression of Milky Way rotation curve allow calculate easily direct DM density.

Direct DM density for Milky Way halo $40 < r < 190$ kpc
$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m ³
$L = 3,191984996 \cdot 10^{33}$
$2b - 2 = -2,677093012$

Beside is such function and table.

Below is shown results of DM density inside its dominion. Calculus are into I.S.

Radius kpc	Radius m	Direct DM kg/m ³
40	1,234E+21	1,09744975E-23
50	1,543E+21	6,03875658E-24
60	1,851E+21	3,70656792E-24
70	2,160E+21	2,45329165E-24
80	2,469E+21	1,71592912E-24
90	2,777E+21	1,25187015E-24
100	3,086E+21	9,44196151E-25
110	3,394E+21	7,31560348E-25
120	3,703E+21	5,79544336E-25
130	4,011E+21	4,67762362E-25
140	4,320E+21	3,83587002E-25
150	4,629E+21	3,18896349E-25
160	4,937E+21	2,68295907E-25
170	5,246E+21	2,28101966E-25
180	5,554E+21	1,95737477E-25
190	5,863E+21	1,69360805E-25

4.3 DIRECT DM BHATTACHARJEE DATA VERSUS SOFUE DATA

Power regression for Milky Way rot. Curve Through Sofue data	
$V = a \cdot r^b$	
a	3,492829549E+12
b	-3,425408589E-01
Correlation coeff.	0,85

In paper [18] Abarca, M. 2016 it is got parameters a & b through Sofue data published in [5] Sofue, Y. 2015.

To the left are shown parameters got through Sofue data.

Below are tabulated and compared direct DM density got through Sofue & Bhattacharjee data.

Radius kpc	Radius m	DM Sofue kg/m ³	DM Bhatacharjee kg/m ³	Relt. Diff. %
40	1,234E+21	1,06869152E-23	1,0974497E-23	2,620E+00
50	1,543E+21	5,87003998E-24	6,0387566E-24	2,794E+00
60	1,851E+21	3,59776625E-24	3,7065679E-24	2,935E+00
70	2,160E+21	2,37834768E-24	2,4532916E-24	3,055E+00
80	2,469E+21	1,66173675E-24	1,7159291E-24	3,158E+00
90	2,777E+21	1,21119348E-24	1,2518701E-24	3,249E+00
100	3,086E+21	9,12748066E-25	9,4419615E-25	3,331E+00
110	3,394E+21	7,06656209E-25	7,3156035E-25	3,404E+00
120	3,703E+21	5,59426204E-25	5,7954434E-25	3,471E+00
130	4,011E+21	4,51235967E-25	4,6776236E-25	3,533E+00
140	4,320E+21	3,69815581E-25	3,8358700E-25	3,590E+00
150	4,629E+21	3,07278024E-25	3,1889635E-25	3,643E+00
160	4,937E+21	2,58387850E-25	2,6829591E-25	3,693E+00
170	5,246E+21	2,19571888E-25	2,2810197E-25	3,740E+00
180	5,554E+21	1,88331682E-25	1,9573748E-25	3,784E+00
190	5,863E+21	1,62882611E-25	1,6936080E-25	3,825E+00

Last column shows relative difference between both set of parameters. It is clear that difference is negligible throughout whole dominion.

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E THROUGH VIRIAL THEOREM

Radius kpc	Radius m	$E = a^2 \cdot r^{2b-1}$ m/s ²
40	1,234E+21	3,518E-11
50	1,543E+21	2,419E-11
60	1,851E+21	1,782E-11
70	2,160E+21	1,376E-11
80	2,469E+21	1,100E-11
90	2,777E+21	9,028E-12
100	3,086E+21	7,566E-12
110	3,394E+21	6,448E-12
120	3,703E+21	5,573E-12
130	4,011E+21	4,873E-12
140	4,320E+21	4,303E-12
150	4,629E+21	3,833E-12
160	4,937E+21	3,440E-12
170	5,246E+21	3,107E-12
180	5,554E+21	2,823E-12
190	5,863E+21	2,579E-12

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a \cdot r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E = a^2 \cdot r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A \cdot E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by author in previous papers. [1] Abarca,M. [2] Abarca,M.

[8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power

regression formula for velocity. In previous paragraph has been shown formula for gravitational field

$E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get

A & B to find function of DM density depending on E i.e. $D_{DM} = A \cdot E^B$

Specifically formulas are $A = \frac{a^{2b-1} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$ being $b \neq 1/2$. If $b = \frac{-1}{2}$ then A= 0, which is keplerian rotation curve of velocity i.e. Density Dark matter is equal zero.

According parameters a & b got in previous chapter, A& B parameters are:

Milky Way	$D_{DM} = A \cdot E^B$
A	5,33811096426447 · 10 ⁻⁷
B	1,5962698509166

Below is tabulated DM density as power of E and direct DM density, both are identical as it was expected.

Radius	Radius	$D_{DM}(r) = L \cdot r^{2b-2}$	$E = a^2 \cdot r^{2b-1}$	$D_{DM} = A \cdot E^B$
kpc	m	Kg/m ³	m/s ²	Kg/m ³
40	1,234E+21	1,0974497E-23	3,518E-11	1,0974497E-23
50	1,543E+21	6,0387566E-24	2,419E-11	6,0387566E-24
60	1,851E+21	3,7065679E-24	1,782E-11	3,7065679E-24
70	2,160E+21	2,4532916E-24	1,376E-11	2,4532916E-24
80	2,469E+21	1,7159291E-24	1,100E-11	1,7159291E-24
90	2,777E+21	1,2518701E-24	9,028E-12	1,2518701E-24
100	3,086E+21	9,4419615E-25	7,566E-12	9,4419615E-25
110	3,394E+21	7,3156035E-25	6,448E-12	7,3156035E-25

120	3,703E+21	5,7954434E-25	5,573E-12	5,7954434E-25
130	4,011E+21	4,6776236E-25	4,873E-12	4,6776236E-25
140	4,320E+21	3,8358700E-25	4,303E-12	3,8358700E-25
150	4,629E+21	3,1889635E-25	3,833E-12	3,1889635E-25
160	4,937E+21	2,6829591E-25	3,440E-12	2,6829591E-25
170	5,246E+21	2,2810197E-25	3,107E-12	2,2810197E-25
180	5,554E+21	1,9573748E-25	2,823E-12	1,9573748E-25
190	5,863E+21	1,6936080E-25	2,579E-12	1,6936080E-25

As conclusion, in this chapter has been demonstrated that a power law for velocity.

$v = a \cdot r^b$ is mathematically equivalent a power law for DM density depending on E. $D_{DM} = A \cdot E^B$

6. RATIO BARYONIC MASS VERSUS DM MASS DEPENDING ON RADIUS FOR MILKY WAY

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in Milky Way galaxy.

[5] According Sofue, Y. data for Milky Way disk are

Milky Way	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a_d^2$		
	$M_d = 1,12 \cdot 10^{11} \text{ Msun}$	5,73 kpc	1,134683098 kg/m ²

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of

superficial density from zero to infinite. $M_d = \int_0^{\infty} 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a_d^2$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r \Sigma(r) dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r) dr$ where $D_{DM}(r) = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2} \Sigma(r) \frac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

It is defined ratio function as quotient of both differential quantities $Ratio = \frac{dM_{DISK}}{dM_{DM}} = \frac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

	m	$\frac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$	$\Sigma(r)$	Direct DM
Radius Kpc	Radius	dimensionless	kg/m ²	kg/m ³
30	9,257100E+20	1,376320E-01	6,04061960105E-03	2,3705944363E-23
32	9,874240E+20	1,081782E-01	4,26084053722E-03	1,9944436051E-23
34	1,049138E+21	8,447141E-02	3,00544700422E-03	1,6956520640E-23
36	1,110852E+21	6,557752E-02	2,11993657502E-03	1,4550626748E-23
38	1,172566E+21	5,064642E-02	1,49532867350E-03	1,2589851941E-23
40	1,234280E+21	3,893338E-02	1,05475223559E-03	1,0974497455E-23
42	1,295994E+21	2,980385E-02	7,43985117242E-04	9,6307235736E-24
44	1,357708E+21	2,272841E-02	5,24780925798E-04	8,5030077315E-24
46	1,419422E+21	1,727267E-02	3,70162001496E-04	7,5490252483E-24
48	1,481136E+21	1,308494E-02	2,61099252308E-04	6,7361086220E-24
50	1,542850E+21	9,883669E-03	1,84170226226E-04	6,0387565778E-24
52	1,604564E+21	7,445579E-03	1,29907197851E-04	5,4368542409E-24

For a radius 40 kpc ratio baryonic matter versus DM is only 3.9 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.

This is the reason why in this work dominion for radius begin at 40 kpc.

7. COMPARISON BETWEEN DIRECT DM DENSITY AND NFW DARK MATTER DENSITY

According [5] Sofue, Y., 2015. Parameters of NFW profile for Milky Way are written in table below. It is clear that NFW profile fitted through Bhattacharjee data would be a bit different. However in epigraph 4.3 has been shown that relative difference between Sofue data and Bhattacharjee data is 3% in average throughout the whole dominion. Therefore NFW profile got by Sofue is a good approximation for data got by Bhattacharjee.

Dark matter density function profile NFW
Rs = 10,7 ± 2.9 Kpc
Do = 1,2318 · 10 ⁻²¹ kg/m ³

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2} \quad \text{Where } x = \text{radius} / R_s \quad R_s \text{ is called length scale and } D_0 \text{ is density scale.}$$

Below are tabulated NFW DM density and Direct DM density depending on radius both. Third column shows relative differences, which oscillate between 28% and 15% throughout dominion.

Direct DM	DM NFW	Relt. Diff.	Radius
Kg/m ³	Kg/m ³	%	kpc
1,0974497E-23	1,467627E-23	25,22	40,00
8,0065345E-24	1,080858E-23	25,92	50,00
6,0387566E-24	8,191136E-24	26,28	60,00
4,6788105E-24	6,356208E-24	26,39	70,00
3,7065679E-24	5,031546E-24	26,33	80,00

2,9916485E-24	4,051225E-24	26,15	90,00
2,4532916E-24	3,310140E-24	25,89	100,00
2,0395523E-24	2,739483E-24	25,55	110,00
1,7159291E-24	2,292910E-24	25,16	120,00
1,4588624E-24	1,938424E-24	24,74	130,00
1,2518701E-24	1,653446E-24	24,29	140,00
1,0831739E-24	1,421733E-24	23,81	150,00
9,4419615E-25	1,231392E-24	23,32	160,00
8,2858392E-25	1,073583E-24	22,82	170,00
7,3156035E-25	9,416387E-25	22,31	180,00
6,4948401E-25	8,304684E-25	21,79	190,00

It is remarkable the fact that NFW profile is bigger than direct DM profile through the whole dominion. Its relative difference oscillate between 26% and 22%.

In my opinion this is an important fact that could be explained because NFW profile is fitted with total DM enclosed inside galactic disc.

As it is known, NFW profile is fitted over bulge, disk and galactic halo and taking in consideration that there is an unknown amount of baryonic DM in bulge and galactic disk it is right to conclude that NFW profile is fitted through a set of DM data whose values include baryonic DM and non baryonic DM, especially in bulge and disk. Therefore it is right to get a function fitted, NFW, which produce high values of DM throughout its dominion, bulge, disk and halo.

However direct DM profile, as it is got through a power regression function fitted with data velocity in halo region, it is fitted only with non baryonic DM or at least with a negligible amount of baryonic matter. Therefore it is right to deduce that Direct DM profile has to be lower than NFW profile throughout dominion inside halo.

As it was pointed at introduction, it is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk and bulge.

Reader can consult these papers about this open problem: [12] Nieuwenhuizen,T.M. 2010. [13] Nieuwenhuizen,T.M. 2012. [14] Nieuwenhuizen,T.M. 2010 [15] Wyrzykowski,L.2010. [16] M.R.S. Hawkins 2015

Conclusion got in this brief chapter is that it is necessary to split baryonic DM and non baryonic DM regarding contribution to rotation curve of galaxies because nature of both are very different.

Any theory which study DM nature without split both kind of DM in my opinion is wrong.

In fact all papers I have published try to show that non baryonic dark matter is generated by gravitational field according a Universal law. Results got in several giant galaxies back this surprising hypothesis.

8. D.M. DENSITY AS POWER OF E IN MILKY WAY VERSUS D.M. DENSITY AS POWER OF E IN M31

In this chapter will be compared DM density as power of E through A&B parameters got for Milky Way and parameters A&B got for M31. The goal this chapter is to show that both couples of parameters produce very similar DM density throughout dominion specially at medium and large distance in halo region. This results back strongly hypothesis that non baryonic DM is generated according an Universal mechanism.

In chapter 5 were developed theory of DM density as power of E. DM density = $A \cdot E^B$. In that chapter was demonstrated that mathematically it is equivalent a law of $v = a \cdot r^b$ for rotation curve to a law for DM density as

power of E. Where formula coefficients are: $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

In addition was shown formula for gravitational field through virial theorem. $E = a^2 \cdot r^{2b-1}$

Coefficients of Milky Way are below.

Power regression for Milky Way rot. curve	
V=a*r^b	
a	2,8791029E+12
b	-3,3854651E-01
Correlation coeff.	0.6613046575

Milky Way	$D_{DM} = A \cdot E^B$
A	5,338110964264·10 ⁻⁷
B	1,596269850917

In paper [11] Abarca,M.2016. Two New Dark Matter Density Profiles for M31 Halo Got from Rotation Curve were published coefficients a &b for power regression velocity depending on radius for M31 galaxy. Also were published coefficients A&B for DM density as power of E. Below are both couple of data.

Power regression for M31 through Sofue data	
V=a*r^b	
a	4,15011040E+10
b	-2,47554520E-01
Correlation coeff.	0,952254

M31 galaxy	$D_{DM} = A \cdot E^B$
A	3,766521943774E ·10 ⁻⁶
B	1,668847537702

Seemingly A& B of MW are clearly different from A&B of M31. A&B of M31 are bigger than A&B of MW. However both couples produce a DM density very similar as it will be shown below.

First and second columns refers radius regarding Milky Way. At that radius is calculated gravitational field E, with a&b parameters of Milky Way. Values of E are used to calculate DM density for Milky Way and M31 through formulas of DM as power of E to compare both series of values.

Third column shows such E. Fourth and fifth column show DM density as power of E for Milky Way and M31 and sixth column shows relative difference between both DM densities.

Surprisingly relative difference for field E belonging to 90 kpc or bigger radius, fall below 10 %. See grey row.

Radius MW data kpc	Radius MW data m	E through MW data m/s ²	DM powE MW kg/m ³	DM pow E M31 kg/m ³	Relt. Diff. MW vs M31 %
40	1,234280E+21	3,51764395E-11	1,0974497E-23	1,3496734143E-23	18,69
50	1,542850E+21	2,41949525E-11	6,0387566E-24	7,2276270284E-24	16,45
60	1,851420E+21	1,78209339E-11	3,7065679E-24	4,3389262530E-24	14,57
70	2,159990E+21	1,37611417E-11	2,4532916E-24	2,8184527216E-24	12,96
80	2,468560E+21	1,10000986E-11	1,7159291E-24	1,9395552951E-24	11,53
90	2,777130E+21	9,02836355E-12	1,2518701E-24	1,3948767776E-24	10,25
100	3,085700E+21	7,56605460E-12	9,4419615E-25	1,0386499523E-24	9,09
110	3,394270E+21	6,44837207E-12	7,3156035E-25	7,9546090336E-25	8,03
120	3,702840E+21	5,57282181E-12	5,7954434E-25	6,2352768455E-25	7,05
130	4,011410E+21	4,87276953E-12	4,6776236E-25	4,9838288736E-25	6,14
140	4,319980E+21	4,30327563E-12	3,8358700E-25	4,0502723417E-25	5,29
150	4,628550E+21	3,83308148E-12	3,1889635E-25	3,3390489121E-25	4,49
160	4,937120E+21	3,43986402E-12	2,6829591E-25	2,7872481616E-25	3,74
170	5,245690E+21	3,10731438E-12	2,2810197E-25	2,3522629802E-25	3,03
180	5,554260E+21	2,82327860E-12	1,9573748E-25	2,0045150266E-25	2,35
190	5,862830E+21	2,57853930E-12	1,6936080E-25	1,7230192544E-25	1,71

Relative differences of DM density for MW vs DM density for M31 oscillate from a maximum 18,7 % at the biggest field E and decreases up to 1,7 % at the lowest field E.

DM density as power of E regarding both galaxies are astonishing similar despite the fact that both galaxies have very different rotation curves coefficients a & b, their baryonic masses are clearly different and their galactic disks have different laws for superficial density masses.

In addition it is well known that experimental errors are not negligible despite the fact measures are made by prestigious research teams equipped with high technology tools.

Despite these facts, relative differences fall below 15% at E belonging radius bigger 60 kpc in Milky Way and fall below 10 % at E belonging radius bigger 90 kpc. See rows in grey in tables above.

In my opinion this result suggest strongly that non baryonic DM is generated by gravitational field as power of E according a Universal law. Reader can consult a similar work with different galaxies in [8] Abarca, M. 2016.

9. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER E FORMULA

9.1 POWER OF E THROUGH BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D , density of DM whose units are Kg/m^3 , E gravitational field whose units are m/s^2 , G and finally h .

In table below are developed dimensional expression for these four elements D , E , G and h .

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$$

being K a dimensionless number which may be understood as a coupling constant between field

E and DM density.

As it was shown in previous epigraph, DM density formula for Milky Way is $D_{DM} = A \cdot E^B$ where $A = 5,338110964264 \cdot 10^{-7}$ and $B = 1,596269850917$ values A and B were got by a long numerical analysis of Milky Way rotation curve data.

It is remarkable that relative difference between $1,59627$ and $10/7 = 1,42857$ is only $11,7\%$ which it is a small difference because rotation curve measures have bigger errors.

As it is shown in previous epigraph, results got for M31 parameters are $B = 1,6688475377$ and $A = 3,766521943 \cdot 10^{-6}$ In this case relative difference between $B = 1,668847$ and $10/7$ is $16,8\%$. This relative difference is acceptable because of errors of rotation curve measures.

In paragraph 9.2 will be done a dimensional analysis with two pi monomials because it will be considered velocity of light c as a universal constant involved in formula of DM density. This fact will allow to find a better theoretical formula whose power of E is very close to numerical calculus value.

Any way in epigraph 9.3 will be developed a mathematical analysis and it will demonstrated that $B = 10/7$ it is not acceptable value as power for E because is lower than $3/2$. Therefore it is right to consider a new dimensional analysis with an extract universal constant c , the speed of light in vacuum. See epigraph 9.2.

9.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G , h and c . So elements to make dimensional analysis are D , E , G , h and $c = 2.99792458 \cdot 10^8$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through

two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown

function, which can not be calculated by dimensional analysis theory.

9.2.1 IMPROVING DM DENSITY FORMULA THROUGH NUMERICAL CALCULUS IN MW &M31

In paper [19] Abarca, M.2016. Author calculated power for E through rotation curve data from [20] Huang, Y.2016 data for Milky Way. Taking in consideration A &B parameters for Milky Way, below will be calculated a theoretical formula.

MILKY WAY	$D_{DM} = A \cdot E^B$
A	$1,27687739294523 \cdot 10^{-6}$
B	1,62377420773729

As it is known dimensional analysis can not inform about $f(\pi_2)$. Supposing that $f(\pi_2)$ is a power of π_2 with rational power, then power for π_2 must be -2/3. This way power of E in formula $D_{DM} = A \cdot E^B$ will be $34/21 = 1.6190476$, which is the best

approximation to B= 1.6237742.

Finally formula for DM density is $D = \frac{J}{\sqrt[21]{G^{25} \cdot h^4 \cdot \sqrt[3]{c^2}}} E^{\frac{34}{21}}$ being J a dimensionless number.

In paper [19] Abarca, M.2016. Author calculated power for E through rotation curve data from [5] Sofue, Y.2015 for M31.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3,766521943774E \cdot 10^{-6}$
B	1,668847537702

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A \cdot E^B$ will be $5/3 = 1.666666$, which is the best approximation to B= 1.668847.

Finally formula for DM density is $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h}} E^{\frac{5}{3}}$ being M a dimensionless number.

It is clear that these formulas have been got supposing that $f(\pi_2)$ is a power of π_2 . Reason I have considered to do so daring supposition is that this way it is possible to get a power for E very close to values got by numerical calculus in Milky Way or M31. However only a quantum gravitation theory will be able to give the right and universal formula for DM density.

9.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph 5.2 $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi G A (B-1)}{2B-3} \right]^{\frac{2b-1}{2}}$ being $B \neq 1$ and $B \neq 3/2$.

In [6] Abarca, M.2016, it is got a formula for DM density through a different method which involve a Benouilli differential equation for E, gravitational field. See chapter 9, in such paper. Reader can check that solution for such differential equation is done for any value of B excepting $B \neq 1$ and $B \neq 3/2$.

It is remarkable the fact that through two methods very different, both methods give solution for DM density for any value of B excepting $B \neq 1$ and $B \neq 3/2$.

As A is a positive quantity then $2b+1 > 0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ Therefore $B \in (-\infty, 1) \cup (3/2, \infty)$.

If $B=3/2$ then $2b+1=0$ and $A=0$ so dark matter density is zero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references at the end of this paper. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2, \infty)$.

The main consequence this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is wrong because $B=10/7 = 1.428$. See epigraph 9.1.

Therefore formula $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by two pi monomials it is more suitable formula. See epigraph 9.2.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

In paragraph 9.2.1 has been got $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h}} \cdot E^{\frac{5}{3}}$ as a theoretical formula which fit perfectly empirical formula

for DM density in M31 rotation curve. Similarly has been got a theoretical formula $D = \frac{J}{\sqrt[21]{G^{25} \cdot h^4 \cdot \sqrt[3]{c^2}}} \cdot E^{\frac{34}{21}}$ for DM density in Milky Way which fit perfectly rotation curve data.

As I have said previously, only a quantum gravitation theory will be able to give the right and universal formula for DM density. Anyway, my bet is that such formula will be similar to I have got in this paper.

9.4 CALCULUS OF PARAMETERS a&b and A CONSIDERING B= 5/3

As has been shown in previous epigraph B=5/3 differ only 2 thousandth regarding B value got by regression analysis, furthermore B=5/3 is coherent with dimensional analysis which is essential for every physic formula. This two reasons has lead to recalculate the formulas with this new value B=5/3

$$b = \frac{B-2}{2B-2} \text{ and } a = \left[\frac{4\pi G A (B-1)}{2B-3} \right]^{\frac{2b-1}{2}} \text{ being } B \neq 1 \text{ and } B \neq 3/2. \text{ Therefore if } B=5/3 \text{ there is an important simplification. It is right to get } b = \frac{-1}{4} \text{ and } a = [8 \cdot \pi \cdot G \cdot A]^{\frac{-3}{4}}.$$

At this point, it is important to compare $b = -2,4755452 \cdot 10^{-1}$ got by statistical analysis of M31 rotation curve with $b = -1/4$. There is a magnificent agreement.

Similarly $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ led to $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$ whose value is $A = 4.14979028 \cdot 10^{-6}$ which is lightly different to

$A = 3,766521943774E \cdot 10^{-6}$ which was calculated with the value of b highlighted in the table, which were the original values of a & b got by regression analysis of rotation curve of M31.

Power regression for M31 through Sofue data	
V=a*r^b	
a	4,1501104*10 ¹⁰
b	-2,4755452*10 ⁻¹
Correlation coeff.	0,952254

New parameter b and A&B parameters		
M31 galaxy	$D_{DM} = A \cdot E^B$	Origin
b	-2,4755452*10 ⁻¹	Regression analysis of rotation curve of M31
B	5/3	Regression analysis and dimensional analysis
$b = \frac{B-2}{2B-2}$	New $b = -1/4$	By direct replacement of B=5/3
a (no change)	4,1501104*10 ¹⁰	Regression analysis of rotation curve of M31
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A A= 4.14979028*10 ⁻⁶	Using $b = -1/4$ in the formula of A and using $a = 4,1501104 \cdot 10^{10}$ (regressional analysis)

New parameter a&b and A&B	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a (no change)	$4,15011 \cdot 10^{10}$
$A = \frac{a^{-4}}{8\pi G}$	New parameter A $A = 4.14979 \cdot 10^{-6}$

9.5 CALCULUS OF DIMENSIONLESS NUMBER INCLUDED IN FORMULA OF DARK MATTER DENSITY

This formula got by two pi monomial $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h}} E^{\frac{5}{3}}$ contain M as dimensionless number. As $D=A \cdot E^B$

It is right that $A = \frac{a^{-4}}{8\pi G} = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h}}$ and then $M = \frac{\sqrt[6]{G \cdot h \cdot c^5}}{8 \cdot \pi \cdot \sqrt[4]{a^3}} = 2.9819168 \cdot 10^{-10}$

10. A SOLUTION FOR GRAVITATIONAL FIELD IN SOLAR SYSTEM THROUGH A&B OF M31 GOT BY BUCKINGHAM THEOREM

10.1 INTRODUCTION

In previous chapter has been got that $B=5/3 = 1.6666$ as power of gravitational field is allowed by dimensional analysis and differs less than 0.003 regarding $B_{\text{REGRESSION}} = 1,6688475377$, which was got by regression analysis of rotation curve data in M31.

In this chapter it will be shown that $B=5/3$ allows to get a solution for gravitational field which tends to Newtonian solution.

10.2 BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD

In paper [21] Abarca, M. 2017, it is developed careful and meticulously the differential equation of gravitational field. Namely, it is a first order equation type Bernoulli, not hard to solve.

In this epigraph it is shown a brief summary of paper quoted above.

The reason why it arises this type of equation is that gravitational field generates dark matter according formula $\varphi_{DM}(r) = A \cdot E^B(r)$, and in turn such amount of dark matter produces an extra gravitational field.

A&B Data [7] Abarca,M.2016	
M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3,766521943774E \cdot 10^{-6}$
B	1,668847537702

As it is shown, A and B are parameters got from rotation curve of M31 halo. This parameters were used in paper [21] Abarca,M.2017 to get the gravitational field in Solar System, whereas in the following epigraph it will be used the new parameters A&B.

The differential equation for field E got was $E'(r) = 4\pi \cdot G \cdot A \cdot E^B(r) - 2 \frac{E(r)}{r}$ which is type Bernoulli and

whose general solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B} \right)^{\frac{1}{1-B}}$ with $B \neq 1$ and $B \neq 3/2$ where C is

the parameter of initial condition of gravitational field at a specific radius and $K = 4\pi \cdot G \cdot A$

Calling $\alpha = 2B - 2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B} \right)$ formula may be written as

$$E(r) = (Cr^\alpha + Dr)^\beta$$

Calculus of Initial condition for parameter C

Suppose R_0 and E_0 are specific initial conditions for radius and gravitational field then $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^\alpha}$

As dominion radius begin at $R_{SUN} = 7 \cdot 10^8$ m. it is right to consider as initial condition $R_0 = R_{SUN} = 7 \cdot 10^8$ m and

$E_0 = E_{SUN SURFACE} = 274 \text{ m/s}^2$. To calculate E_{SUN} has been considered $M_{sun} = 1.99 \cdot 10^{30}$ kg.

Finally it is possible to write formula for gravitational field got through Bernoulli method.

In paper [21] Abarca,M.2017 was got parameter alpha, Beta, D and C.

Bernoulli Solution for Gravitational field inside Solar System $R_{SUN} < \text{Radius}$

$$E_{BER}(r) = (Cr^\alpha + Dr)^\beta \quad C = 3,47313 \cdot 10^{-14} \quad D = 6,2566 \cdot 10^{-15} \quad \alpha = 1,337695075 \quad \beta = -1,49510904$$

10.2.1 GRAVITATIONAL FIELD IN SOLAR SYSTEM WITH NEW A&B PARAMETERS

In this epigraph it will be calculated formula of gravitational field inside Solar System, with the same coefficients A&B got for M31, according dimensional analysis theory. Afterwards, it will be discussed the results.

The new parameters are highlighted in table.

$$\text{If } B=5/3 \text{ then } \alpha = \frac{4}{3} \quad \beta = \frac{-3}{2} \text{ and}$$

$$D = \left(\frac{K(1-B)}{3-2B} \right) = 2K = 8\pi GA$$

This way gravitational field arises elegantly

$$E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}} \quad \text{being } D = 8\pi GA = a^{\frac{-4}{3}}$$

New parameter a&b and A&B	
B	5/3
$b = \frac{B-2}{2B-2}$	$b = -1/4$
a (no change)	$4,15011 * 10^{10}$
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A $A = 4.14979 * 10^{-6}$
$D = 8\pi GA = a^{\frac{-4}{3}}$	$6.9596466 * 10^{-15}$

10.2.2 A PHYSICAL INTERPRETATION OF PARAMETER C IN SOLAR SYSTEM

$$C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}} \quad \text{As } D \cdot R_0 \text{ is clearly negligible because it is approximately a millionth regarding the other}$$

$$\text{addend, then } C_{APROX} = \frac{E_0^{\frac{-2}{3}}}{R_0^{\frac{4}{3}}}. \quad \text{According NASA source } R_{SUN} = 6.95508 * 10^8 \text{ m and } E_0 = 274 \text{ m/s}^2 \text{ so}$$

$C_{APROX} = 3.846775 * 10^{-14}$ and it has sub index **aprox** because it is only an approximation to the right value C, because C has to be calculated without any approximation.

$$\text{Similarly if it is neglected } D \cdot r \text{ in field formula then it is got } E_{APROX}(r) = \left(Cr^{\frac{4}{3}} \right)^{\frac{-3}{2}} = \frac{C^{\frac{-3}{2}}}{r^2}$$

$$\text{In the surface of the Sun } E_0 = \frac{GM_{SUN}}{R_0^2} \text{ and reader can check that considering } C_{APROX} = \frac{E_0^{\frac{-2}{3}}}{R_0^{\frac{4}{3}}} \text{ then}$$

$$C_{APROX}^{\frac{-3}{2}} = G \cdot M_{SUN} \text{ therefore according this approximation } E_{APROX}(r) = \frac{GM_{SUN}}{r^2}, \text{ which is highly}$$

remarkable and support strongly the fact that $B=5/3$ is definitely the right value for power of E in formula of dark mater density $D = A \cdot E^B$, because through Bernoulli formula has been got the Newtonian formula, especially regarding the power of r which is two.

The findings made in this epigraph also highlight that coefficient C is the responsible of the fraction of gravitational field generated by baryonic matter, whereas coefficient $D = 8\pi GA = a^{\frac{-4}{3}}$ is responsible of gravitational field generated by dark matter. Therefore it is crucial that if it is neglected the addend of D then formula of E tends to Newtonian formula.

10.2.3 GENERAL SOLUTION FOR GRAVITATIONAL FIELD IN SOLAR SYSTEM

As it has been got in previous epigraph $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}}$ being $D = 8\pi GA$, now it will be calculated parameter C using condition initial for gravitational field at Solar surface.

According NASA source $R_{SUN} = 6.95508 \cdot 10^8 \text{ m}$ and $E_0 = 274 \text{ m/s}^2$

$D = 8\pi GA = a^{\frac{-4}{3}} = 6.9596466 \cdot 10^{-15}$ Where parameter **a** come from regression analysis of rotation curve of M31.

It is right to get $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ and using above data $C = 3.845989 \cdot 10^{-14}$. Reader can check that the difference

between C_{EXACT} and $C_{APROX} = 3.846775 \cdot 10^{-14}$ is 8 ten-thousandth.

This way, it has been got rigorously C, and it is highly interesting to compare parameter C and D. Although D is smaller than C, D is not negligible because of $C/D = 5.5$

DISCUSSION

Firstly, considering $B=5/3$ and neglecting coefficient D, the formula $E(r)$ got by Bernoulli equation, tends to

Newtonian formula $E(r) = \frac{GM_{SUN}}{r^2}$. This fact support strongly the value $B=5/3$, which was got by dimensional analysis.

Secondly, the fact that $C/D=5.5$ it would suggest that it would not be reasonable to accept D as the same coefficient either for M31 or for Solar Sytem, because C is related with field produced by solar baryonic matter whereas D is related with field produced by Dark matter and according this values the contribution of dark matter would be too big.

It would seem that $C/D=5.5$ is too small for Solar system because it would expect that for small gravitational systems baryonic matter is far more important that dark matter. Furthermore, considering that galactic size is typically 10^9 times bigger than Solar System, the galactic gravitational interaction time is 10^9 bigger than gravitational interaction in Solar System because the speed of gravitational interaction is c. This facts suggest that formula of $D_{DM} = A \cdot E^B$ could be different for two gravitational system with sizes so different. However, according dimensional analysis coefficient B, hast to be the same $B=5/3$. Therefore, it is the coefficient A which might be different depending on size of gravitational system considered.

Finally, It is remarkable the fact that inside galactic halo region gravitational field is lower than 10^{-11} m/s^2 whereas in solar surface is 274 m/s^2 decreasing up to $4.5 \cdot 10^{-5} \text{ m/s}^2$ in Earth orbit. This fact suggests that the amount of dark matter near the Sun should be far more important than another region far away because of the power of E is $5/3$.

10.3 DARK MATTER DENSITY DEPENDING ON RADIUS IN SOLAR SYSTEM

Thanks Bernoulli solution for gravitational field is right to get DM density depending on radius through power of E formula.

In this epigraph it will be considerate the same coefficient A&B got for M31 in order to calculate total amount of dark masses, although in previous epigraph has been suggest that coefficient D, which comes from M31, it is not acceptable for Solar System.

DM Density Bernoulli profile for Solar System $R_{SUN} < \text{Radius}$
$E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}} \quad \text{being } D = 8\pi GA = a^{\frac{-4}{3}} = 6.9596466 * 10^{-15} \quad \text{and } C = 3.845989 * 10^{-14} \quad \text{into I.S. units}$
$\text{Density}_{D.M.} (r) \text{ kg/m}^3 = D_{DMB}(r) = A \cdot E^B \quad \text{Where } A = \frac{a^{\frac{-4}{3}}}{8\pi G} = 4.14979 * 10^{-6} \quad \text{and } B = \frac{5}{3}$
$D_{DM}(r) = A \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-5}{2}} \quad \text{Where } A = 4.14979 * 10^{-6} \quad C = 3.846 * 10^{-14} \quad D = 6.96 * 10^{-15}$

10.4 CALCULUS OF D.M. CONTAINED INSIDE SPHERICAL CORONA DEFINED BY PERIHELION AND APHELION RADII OF MERCURY

Thanks to Density DM got in previous epigraph, through a definite integration it is possible to calculate total matter enclosed by spherical corona defined by two specific radius R_1 and R_2

Integration is not easy as reader can check.

$$M_{DM} = \int_{R_1}^{R_2} 4\pi r^2 \cdot \rho(r) dr = \int_{R_1}^{R_2} 4\pi r^2 A E^B dr = 4\pi A \int_{R_1}^{R_2} r^2 \left[C \cdot r^{\frac{4}{3}} + D \cdot r \right]^{\frac{-5}{2}} \cdot dr = 4 \cdot \pi \cdot A \cdot I$$

Where $4\pi A = 5.21478 \cdot 10^5$ and I symbolise the definite integral.

$R_1 = \text{Mercury Perihelion} = 0.307499 \text{ U.A.}$ and $R_2 = \text{Mercury Aphelion} = 0.466697 \text{ U.A.}$

Taking equivalence units $1 \text{ U.A.} = 1,495978707 \cdot 10^{11} \text{ m}$ it is got $R_1 = 4.6 * 10^{10} \text{ m}$ and $R_2 = 6.98 * 10^{10} \text{ m}$

Thanks remarkable web site *Wolfram alpha* it is possible to calculate that definite integral to know total dark matter enclosed inside spherical corona.

Definite integral:

$$\int_{4.6 \times 10^{10}}^{6.98 \times 10^{10}} \frac{x^2}{(3.846 \times 10^{-14} x^{1.3333333} + 6.96 \times 10^{-15} x)^{2.5}} dx = 3.74522 \times 10^{29}$$

Finally M_{DM} contained inside spherical corona = $4 \cdot \pi \cdot A \cdot I = 1.953 \cdot 10^{25}$ Kg

Taking the equivalence $M_{\oplus} = M_{EARTH} = 5.972 \cdot 10^{24}$ kg then M_{DM} of the spherical corona = $3.27 M_{\oplus}$ which is a considerable amount of Dark matter.

Although this amount of dark matter is tiny compared with Solar mass, it is a considerable quantity. Probably, this fact also suggests that the coefficient $D = 6.96 \cdot 10^{-15}$, which was got in M31, is too big for Solar System.

If only it could be measured total amount of dark matter contained inside spherical corona defined by perihelion and aphelion radius then it could be calculated the coefficient A for Solar System.

10.5 CALCULUS OF D.M. CONTAINED INSIDE SPHERICAL CORONAS DEFINED BY SOLAR RADIUS AND PERIHELION AND APHELION RADII OF MERCURY

CALCULUS FOR SPHERICAL CORONA DEFINED BY SOLAR RADIUS AND MERCURY PERIHELION

Through the same formula used in previous epigraph it is got

Definite integral:

$$\int_{4.6 \times 10^{10}}^{6.95508 \times 10^8} \frac{x^2}{(3.846 \times 10^{-14} x^{1.3333333} + 6.96 \times 10^{-15} x)^{2.5}} dx = 8.78338 \times 10^{30}$$

Being $4\pi A = 5.21478 \cdot 10^5$ Solar radius $6.95508 \cdot 10^8$ m and radius of perihelion = $4.6 \cdot 10^{10}$ m

It is got a mass $M = 4.58034 \cdot 10^{26}$ kg = $76.697 M_{\oplus}$

CALCULUS FOR SPHERICAL CORONA DEFINED BY SOLAR RADIUS AND MERCURY APHELION

Definite integral:

$$\int_{6.95508 \times 10^8}^{6.98 \times 10^{10}} \frac{x^2}{(3.846 \times 10^{-14} x^{1.3333333} + 6.96 \times 10^{-15} x)^{2.5}} dx = 9.1579 \times 10^{30}$$

Being $4\pi A = 5.21478 \cdot 10^5$ Radius aphelion = $6.98 \cdot 10^{10}$ m and $M = 4.77564 \cdot 10^{26}$ kg = $79.96 M_{\oplus}$

11. CONCLUSION

This work is focused in halo region of Milky Way where baryonic density is negligible regarding non baryonic DM. Reason is that the main hypothesis all my papers is that DM non baryonic is generated locally by gravitational field. Therefore it is needed to study radius dominion where it is possible to study gravitational field propagation without interference of baryonic mass density or at least where this density is negligible.

In order to defend properly conclusion this paper is important to emphasise a result got in chapter 3 which is that correlation coefficient of velocity as power regression of radius in halo region is 0,66 which is an acceptable value.

This acceptable value of correlation between radius and velocity, support that velocity of Milky Way rotation curve follows a power law regarding radius $v = a \cdot r^b$ whose coefficient a & b were got in chapter 3.

In chapter four was mathematically demonstrated that a power law $v = a \cdot r^b$ in halo region is mathematically equivalent a DM density called Direct DM, whose formula is $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$.

Also in this chapter is compared Direct DM got from rotation curve [17] Bhattacharjee, P.2014 and Direct DM got from rotation curve [5] Sofue, Y.2015. It is shown that relative difference oscillates between 2.6% at 40 kpc and 3.8 % at 190 kpc which is a very exiguous difference.

It is a very good news that two prominent teams of researchers got so similar results.

In chapter five was demonstrated mathematically that a power law for velocity $v = a \cdot r^b$ fitted at rotation curve is mathematically equivalent a power law for DM density depending on E. $D_{DM} = A \cdot E^B$

$$\text{Being } E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1} \quad \text{and being } A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G} \quad \& \quad B = \frac{2b-2}{2b-1}.$$

Therefore joining chapters 3,4 and 5 it is concluded that a correlation coefficient as 0,66 at power regression law for rotation curve $v = a \cdot r^b$ in halo region support that DM density inside halo region is a power of gravitational field $D_{DM} = A \cdot E^B$ whose parameters A & B are written above.

In chapter seven was compared direct DM profile got in this paper with NFW fitted by [5] Sofue, Y.2015. It was checked that NFW profile is bigger than Direct DM profile a percentage which oscillates between a 25 % and 21% .

In my opinion this fact shows that inside galactic disc, non baryonic DM is not negligible whereas outside disc DM there is mainly non baryonic DM.

In eight chapter was shown that such non baryonic DM is generated according a law very similar for Milky Way and M31. Formula for non baryonic DM density is $D_{DM} = A \cdot E^B$. Where A&B are lightly different for each galaxy.

Results for both sets of values are astonishingly similar throughout halo dominion since relative differences oscillate between 18 % and 1.7 %. The less value E has, the less relative difference both profiles (MW vs M31) have. This fact back strongly hypothesis that non baryonic DM is generated by gravitational field according an Universal law in giant galaxies as I have found in previous papers with other different giant galaxies.

In ninth chapter it is made dimensional analysis for DM density formula through Buckingham theorem with one pi monomial (G and h as universal constant), and two pi monomial with G , h and c as universal constant. Through a mathematical analysis it is rejected density formula with only one pi monomial so it is needed to consider density formula with two pi monomials. Physically it is right because according my theory DM density it is generated by a quantum gravitational phenomena which include virtual particles whose speed is c .

In the formula for density with two pi monomials was found a power for E , $B = 5/3$, which is coherent with dimensional analysis theory and differs 2 thousandth from statistical B value got through rotation curve of M31.

In tenth chapter is got and solved the differential equation for gravitational field E , inside solar system, accepting the same values A & B calculated for M31 galaxy.

It is got a formula for E , which tend to Newtonian formula if it is neglected the dark matter term. This fact is a strong back to power $B = 5/3$.

Also it is calculated dark matter contained inside spherical corona definite by perihelion and aphelion radius of Mercury. It has been got 3.2 times M_{EARTH} which is a considerable amount of dark matter. By qualitative reason it is defended that this excessive amount of dark matter suggest that cannot be accepted the coefficient A got for M31 when is calculated dark matter inside Solar system because both gravitational systems have a tremendous different sizes.

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