Final version. The final formulation of the TSV.

The Corpuscular Structure of Matter, the Interactions between Particles, Quantum Phenomena, and Cosmological Data as a Consequence of Selfvariations.

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ABSTRACT

With the term “Law of Selfvariations” we mean an exactly determined increase of the rest mass and the absolute value of the electric charge of material particles. In this article we present the basic theoretical investigation of the law of selfvariations. We arrive at the central conclusion that the interaction of material particles, the corpuscular structure of matter, and the quantum phenomena can be justified by the law of Selfvariations. We predict a unified interaction between particles with a unified mechanism (the Unified Selfvariation Interaction, USVI). Every interaction is described by three distinct terms with distinct consequences in the USVI. The theory predicts a wave equation whose special cases are the Maxwell equations, the Schrödinger equation and the related wave equations. The theory provides a mathematical expression for any conserved physical quantity, and the current density 4-vector in every case. The corpuscular structure and wave behaviour of matter and the relation between them emerge clearly and the theory also predicts the rest masses of material particles. We prove an «internal symmetry» theorem which justifies the cosmological data. The study we present can be the basis for further investigation of the Selfvariations and its consequences.

Keywords: Particles and Fields, Quantum Physics, Cosmology.

CONTENTS p.

1. INTRODUCTION

The theoretical foundation of physics developed in the last century is mainly summed up in the special and general theory of relativity and in quantum mechanics. The great advances in theoretical physics are mainly attributed to these two theories. There are however good reasons to consider seriously that these two theories may not be the fundamental theories of physics. The incompatibility between them, the failure to find a deeper cause of quantum phenomena, and the multitude of assumptions, imposed by the experimental data, for the development of quantum mechanics, are just some of these reasons. Mainly, though, they are far from a theoretical foundation of Physics.

These weaknesses of theoretical physics have led to the confident belief that a deeper understanding of physical reality is impossible. Spearheading this argument was the lack of understanding of quantum phenomena. Over the years Einstein's view that we should seek and understand the cause of quantum phenomena was ignored and marginalized.

A question that arises is whether there is a prominent fundamental law in nature. A law which has the potential to reproduce our basic knowledge in physics. If indeed there is such a law in nature then a continuing reduction of the axioms of theoretical physics is expected to converge to this law. We present such a study below.

The present study is founded on three axioms: The principle of conservation of the four-vector of momentum, the equation of the Theory of Special Relativity for the rest mass of the material particles and the law of Selfvariations.

With the term “Law of Selfvariations” we mean an exactly determined increase of the rest mass and the absolute value of the electric charge of material particles. The law is consistent with the principles of conservation of energy, momentum, angular momentum and electric charge. It is also invariant under the Lorentz-Einstein transformations.

The most direct consequence of the law of Selfvariations is that energy, momentum, angular momentum and electric charge (when the material particle is electrically charged) of particles are distributed in the surrounding spacetime. For example, to compensate for the increase (in absolute value) of the electric charge of the electron, the particle emits a corresponding positive electric charge into the surrounding spacetime. Otherwise, the conservation of the electric charge would be violated. Similarly, the increase of the rest mass of the material particle involves the “emission” of negative energy as well as momentum in the 3 space-time surrounding the material particle (spacetime energy-momentum, STEM).

Later we will see that STEM contains charge when the particle is charged. The law of Selfvariations quantitatively describes the interaction of material particles with the STEM.

Every material particle interacts both with the STEM emitted by itself due to the selfvariations, and with the STEM originating from other material particles. The material particle and the STEM with which it interacts, comprise a dynamic system which we call “generalized particle”. In the present article we study this continuous interaction. The conclusions resulting from the law of Selfvariations will be referred to as "the Theory of Selfvariations" (TSV).

The main conclusion reached is that the three axioms we use reproduce all of our basic knowledge in physics. In particular they predict and justify the particle structure of matter, the interactions of particles, the quantum effects, and the cosmological data. Moreover an exceptionally large number of new statements about physical reality can be derived from these axioms.

The TSV predicts a common mechanism for the interaction of particles, the Unified Selfvariation Interaction (USVI). The USVI implies that each interaction consists of three components with different characteristics. One of these components corresponds to our familiar Lorentz force as known from electromagnetism, another component corresponds to the curvature of space-time, while the existence of the third component was totally unknown to us before the formulation of the TSV.

The TSV predicts a wave equation whose special cases are Maxwell's equations, the Schrödinger equation and the associated wave equations. We determine a unified mathematical expression for all conserved physical quantities and calculate the corresponding 4-vector for the current density. Both the density and the current density of conserved physical quantities have a ‘crystalline’ structure which is a reference to the quantum behavior of matter.

The equations of the TSV predict a strictly determined structure of matter. They highlight both the particulate structure and the wave behavior of matter and the relationship between them.

We prove an «internal symmetry» theorem which justifies the cosmological data. We will show that for observations done at cosmological scale, our observation instruments directly record the consequenses of Selfvariations.

2. THE BASIC STUDY OF THE STRUCTURE OF THE GENERALIZED PARTICLE

2.1. Introduction

In this chapter we give the mathematical formulation of the law of selfvariations for the rest mass and we determine the fundamental physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ which are obtained from the law. For the formulation of the equations the following notation is used:

$W =$ the energy of the particle

$\mathbf{J} =$ the momentum of the particle

$m_0 =$ the rest mass of the particle

$E =$ the energy of the STEM interacting with the particle

$\mathbf{P} =$ the momentum of the STEM interacting with the particle

$E_0 =$ the rest energy of the STEM interacting with the particle .

With the above symbolism, the law of Selfvariations for the rest mass is given by equations

$$\frac{\partial m_0}{\partial t} = -\frac{b}{\hbar} E m_0$$

$$\nabla m_0 = \frac{b}{\hbar} \mathbf{P} m_0$$

for $m_0 \neq 0$ in every system of reference $0(\mathrm{t}, x, y, z)$, $\hbar = \dfrac{h}{2\pi}$, h is the reduced Planck's constant, b is a constant, $b \neq 0, b \in \mathbb{C}$ and $\nabla = \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{pmatrix}$.

The conclusions resulting from the law of Selfvariations will be referred to as "the Theory of Selfvariations" (TSV). In the beginning we present the TSV in inertial frames of reference.

2.2. The basic study of the internal structure of the generalized particle

We consider a particle with rest mass $m_0 \neq 0$ and we denote E_0 the rest energy of the STEM interacting with the particle. The rest mass m_0 and the rest energy E_0 given by equations (2.1) and (2.2) respectively, according to special relativity [1-4]

$$m_0^2 c^4 = W^2 - c^2 \mathbf{J}^2 \tag{2.1}$$

$$E_0{}^2 = E^2 - c^2 \mathbf{P}^2 \, . \tag{2.2}$$

We now denote the four-vectors

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ict \\ x \\ y \\ z \end{bmatrix} \tag{2.3}$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{iW}{c} \\ J_x \\ J_y \\ J_z \end{bmatrix} \tag{2.4}$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_x \\ P_y \\ P_z \end{bmatrix} \tag{2.5}$$

where c is the light constant (vacuum velocity of light) and i is the imaginary unit, $i^2 = -1$.

Using this notation, the law of Selfvariations and equations (2.1) and (2.2) are written in the form of equations (2.6), (2.7) and (2.8)

$$\frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k m_0 , k = 0,1,2,3 , \; m_0 \neq 0 \tag{2.6}$$

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 + m_0{}^2 c^2 = 0 \tag{2.7}$$

$$P_0^2+P_1^2+P_2^2+P_3^2+\frac{E_0^2}{c^2}=0. \tag{2.8}$$

However equations (2.7), (2.8) remain valid in the case where $m_0=0$, $E_0=0$.

After differentiating equation (2.7) with respect to $x_k, k=0,1,2,3$ we obtain

$$J_0\frac{\partial J_0}{\partial x_k}+J_1\frac{\partial J_1}{\partial x_k}+J_2\frac{\partial J_2}{\partial x_k}+J_3\frac{\partial J_3}{\partial x_k}+m_0c^2\frac{\partial m_0}{\partial x_k}=0$$

and with equation (2.6) we obtain

$$J_0\frac{\partial J_0}{\partial x_k}+J_1\frac{\partial J_1}{\partial x_k}+J_2\frac{\partial J_2}{\partial x_k}+J_3\frac{\partial J_3}{\partial x_k}+\frac{b}{\hbar}P_k m_0^2c^2=0$$

and with equation (2.7) we obtain

$$J_0\frac{\partial J_0}{\partial x_k}+J_1\frac{\partial J_1}{\partial x_k}+J_2\frac{\partial J_2}{\partial x_k}+J_3\frac{\partial J_3}{\partial x_k}-\frac{b}{\hbar}P_k\left(J_0^2+J_1^2+J_2^2+J_3^2\right)=0$$

$$\begin{aligned}&J_0\left(\frac{\partial J_0}{\partial x_k}-\frac{b}{\hbar}P_kJ_0\right)+J_1\left(\frac{\partial J_1}{\partial x_k}-\frac{b}{\hbar}P_kJ_1\right)\\&+J_2\left(\frac{\partial J_2}{\partial x_k}-\frac{b}{\hbar}P_kJ_2\right)+J_3\left(\frac{\partial J_3}{\partial x_k}-\frac{b}{\hbar}P_kJ_3\right)=0, k=0,1,2,3\end{aligned}. \tag{2.9}$$

We now symbolize

$$\frac{\partial J_i}{\partial x_k}-\frac{b}{\hbar}P_kJ_i=\lambda_{ki},\ \ k,i=0,1,2,3. \tag{2.10}$$

With this notation, equation (2.9) can be written in the form

$$J_0\lambda_{k0}+J_1\lambda_{k1}+J_2\lambda_{k2}+J_3\lambda_{k3}=0,\ \ k=0,1,2,3. \tag{2.11}$$

We now need the 4×4 matrix T as given by equation

$$T=\begin{bmatrix}\lambda_{00}&\lambda_{01}&\lambda_{02}&\lambda_{03}\\\lambda_{10}&\lambda_{11}&\lambda_{12}&\lambda_{13}\\\lambda_{20}&\lambda_{21}&\lambda_{22}&\lambda_{23}\\\lambda_{30}&\lambda_{31}&\lambda_{32}&\lambda_{33}\end{bmatrix}. \tag{2.12}$$

With this notation, equation (2.11) can be written in the form

$$TJ=0. \tag{2.13}$$

From $m_0 = 0$ in equation (2.7) we get again equations (2.11) and (2.13).

Proof. For $m_0 = 0$ in equation (2.7) we get

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 = 0$$

and differentiating with respect to $x_k, k = 0,1,2,3$ we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} = 0$$
$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} - 0 = 0$$
$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar} P_k \left(J_0^2 + J_1^2 + J_2^2 + J_3^2 \right) = 0$$

$$J_0 \left(\frac{\partial J_0}{\partial x_k} - \frac{b}{\hbar} P_k J_0 \right) + J_1 \left(\frac{\partial J_1}{\partial x_k} - \frac{b}{\hbar} P_k J_1 \right)$$
$$+ J_2 \left(\frac{\partial J_2}{\partial x_k} - \frac{b}{\hbar} P_k J_2 \right) + J_3 \left(\frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar} P_k J_3 \right) = 0, k = 0,1,2,3$$

and symbolizing

$$\frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar} P_k J_i = \lambda_{ki}, \ \ k, i = 0,1,2,3$$

we get again equations (2.11) and (2.13) .□

We now prove the following theorem:

Theorem 2.1. ′′ For $k \neq i, k, i \in \{0,1,2,3\}$ it hold that

1. $m_0 \left(\frac{\partial P_i}{\partial x_k} - \frac{\partial P_k}{\partial x_i} \right) = 0, k \neq i, k, i = 0,1,2,3$ (2.14)

2. $m_0 \neq 0 \Rightarrow \frac{\partial P_i}{\partial x_k} = \frac{\partial P_k}{\partial x_i}, \forall k \neq i, k, i = 0,1,2,3$ (2.15)

3. When

$$\frac{\partial P_i}{\partial x_k} \neq \frac{\partial P_k}{\partial x_i}$$

for at least one pair $(k,i), k \neq i, k,i \in \{0,1,2,3\}$ it holds that

$$m_0 = 0\,.'' \tag{2.16}$$

Proof. Indeed, by differentiating equation (2.6) with respect to x_i, $i = 0,1,2,3$ we get

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right) = \frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

and using the identity

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right) = \frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right)$$

we get

$$\frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right) = \frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

and with equation (2.6) we have

$$\frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_i m_0\right) = \frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

$$P_i\frac{\partial m_0}{\partial x_k} + m_0\frac{\partial P_i}{\partial x_k} = P_k\frac{\partial m_0}{\partial x_i} + m_0\frac{\partial P_k}{\partial x_i}$$

and with equation (2.6) we have

$$P_i\frac{b}{\hbar}P_k m_0 + m_0\frac{\partial P_i}{\partial x_k} = P_k\frac{b}{\hbar}P_i m_0 + m_0\frac{\partial P_k}{\partial x_i}$$

$$m_0\left(\frac{\partial P_i}{\partial x_k} - \frac{\partial P_k}{\partial x_i}\right) = 0$$

from which we obtain relations (2.15) and (2.16). □

3. PHYSICAL QUANTITIES λ_{ki}, $k,i = 0,1,2,3$ AND THE CONSERVATION PRINCIPLES OF ENERGY AND MOMENTUM

3.1. Introduction

The physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ are related to the conservation of energy and momentum of the generalized particle. This investigation we will present in this section.

We present the internal symmetry, which expresses the isotropy of spacetime, and the external symmetry which expresses the anisotropy of spacetime. In this chapter we two of the fundamental theorems of TSV: the theorem of internal symmetry and the first theorem of external symmetry.

3.2. Physical quantities λ_{ki}, $k,i = 0,1,2,3$ and the conservation principles of energy and momentum

We start our study with the proof of the following theorem:

Theorem 3.1. ´´For $m_0 \neq 0$ and when the generalized particle conserves its momentum along the axes $x_i, i = 0,1,2,3$, that is

$$J_i + P_i = c_i = \text{constant} \tag{3.1}$$

then the following equation holds

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{\hbar}\left(c_k P_i - c_i P_k\right) \tag{3.2}$$

for every $k, i = 0,1,2,3,\ k \neq i.$ ´´

Proof. Combining relation (2.15) with equation (3.1) we obtain

$$\frac{\partial}{\partial x_k}\left(c_i - J_i\right) = \frac{\partial}{\partial x_i}\left(c_k - J_k\right)$$

$$\frac{\partial J_i}{\partial x_k} = \frac{\partial J_k}{\partial x_i}$$

and with equation (2.10) we get

$$\frac{b}{\hbar}P_k J_i + \lambda_{ki} = \frac{b}{\hbar}P_i J_k + \lambda_{ik}$$

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right)$$

which is equation (3.2). The rest of equations (3.2) are derived taking into account equation (3.1). Equations (3.2) holds for $k \neq i,\ k,\mathrm{i} = 0,1,2,3$, since equation (2.14), from which equations (3.2) results is an identity for $k = i$ and gives no information in this case.□

We now prove the following theorem:

Theorem 3.2. TSV theorem for the symmetry of indices:

′′ For $m_0 \neq 0$ and when the generalized particle conserves its momentum along the axes x_i and x_k with $k \neq i$, the following equivalences hold

1. $\lambda_{ik} = \lambda_{ki} \Leftrightarrow J_k P_i = J_i P_k \Leftrightarrow c_i J_k = c_k J_i \Leftrightarrow c_k P_i = c_i P_k$. (3.3)

2. $\lambda_{ik} = -\lambda_{ki} \Leftrightarrow \lambda_{ki} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{2\hbar}\left(c_k P_i - c_i P_k\right)$. (3.4)

$k,i = 0,1,2,3,\ k \neq i$.′′

Proof. The theorem is an immediate consequence of equation 3.2.□

We now consider the four-vector $C,$ as given by equation

$$C = J + P = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}. \tag{3.5}$$

When the generalized particle conserves its momentum along every axis, then the four-vector C is constant. Also, we denote M_0 the total rest mass of the generalized particle, as given by equation

$$C^T C = c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2\,\mathrm{c}^2 \tag{3.6}$$

where C^T is the transposed of the column vector C.

For reasons that will become apparent later in our study, we give the following definitions: We name the symmetry $\lambda_{ik} = \lambda_{ki},\ k \neq i,\ k, i = 0,1,2,3$ internal symmetry, and the symmetry $\lambda_{ik} = -\lambda_{ki},\ k \neq i,\ k, i = 0,1,2,3$ external symmetry. We now prove the following theorem:

Theorem 3.3. Internal Symmetry Theorem:

$''$ For $m_0 \neq 0$ and when the generalized particle conserves its momentum in every axis, the following hold:

1. $\lambda_{ik} = \lambda_{ki}$ for every $k, i = 0,1,2,3 \iff J$, P and C are parallel

$$\iff P = \Phi J \text{ where } \Phi \in \mathbb{C}. \tag{3.7}$$

2. For $\Phi = -1$ or $\Phi = 0$ the following equations hold

$$\begin{aligned} E_0 &= \pm m_0 c^2 \wedge M_0 = 0 \\ m_0 &= \pm M_0 \wedge E_0 = 0 \end{aligned} \tag{3.8}$$

respectively.

3. For $\Phi \neq -1$ and $\Phi \neq 0$ the following equations hold:

$$\Phi = K \exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right] \tag{3.9}$$

$$m_0 = \pm \frac{M_0}{1+\Phi} \tag{3.10}$$

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi} \tag{3.11}$$

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3 \tag{3.12}$$

$$P_i = \frac{\Phi c_i}{1+\Phi}, \mathrm{i} = 0,1,2,3 \tag{3.13}$$

where K is a dimensionless constant physical quantity.

For every $k, i \in \{0,1,2,3\}$ it holds that

$$\lambda_{ki} = \lambda_{ik} \iff \lambda_{ki} = 0\,'' \tag{3.14}$$

Proof. Equivalence (3.7) result immediately from equivalence (3.3). For $\Phi = -1$ from the last of equivalence (3.7) we obtain $P = -J$ and from equations (2.7), (2.8) and (3.5), (3.6) we obtain

$$E_0^2 = m_0^2 c^2 \wedge M_0 = 0$$

which is the first of the equations (3.8). For $\Phi = 0$ from the last of equivalence (3.7) we obtain $P = 0$ and from equations (3.5), (3.6) and (2.7) we obtain

$$m_0^2 = M_0^2 \wedge E_0 = 0$$

which is the second of the equations (3.8).

For $\Phi \neq -1$ and $\Phi \neq 0$ from the last of equivalence (3.7) we obtain $P_i = \Phi J_i$ for every $i = 0,1,2,3$ and with equation (3.1) $J_i + P_i = c_i$ we initially obtain equations (3.12) and (3.13). Then, combining equations (2.7) and (3.12) we get

$$m_0^2 c^2 + \frac{1}{(\Phi+1)^2}\left(c_0^2 + c_1^2 + c_2^2 + c_3^2\right) = 0$$

and with equation (3.6) we obtain equation

$$m_0^2 c^2 - \frac{M_0^2 c^2}{(\Phi+1)^2} = 0 \tag{3.15}$$

and we finally have

$$m_0 = \pm \frac{M_0}{1+\Phi}$$

which is equation (3.10). Similarly, combining equations (2.8) and (3.13) we obtain equation (3.11). We now prove that function Φ is given by equation (3.9).

Differentiating equation (3.15) with respect to x_v, $v = 0,1,2,3$ and considering equation (2.6) we obtain

$$\frac{2b}{\hbar} P_v m_0^2 c^2 + \frac{2M_0^2 c^2}{(\Phi+1)^3} \frac{\partial \Phi}{\partial x_v} = 0$$

and with equation (3.15) we have

$$\frac{b}{\hbar}P_v\frac{M_0^2c^2}{(\Phi+1)^2}+\frac{M_0^2c^2}{(\Phi+1)^3}\frac{\partial\Phi}{\partial x_v}=0$$

$$\frac{\partial\Phi}{\partial x_v}=-\frac{b}{\hbar}P_v(\Phi+1)$$

and with equation (3.13) for $i=v$ we arrive at equation

$$\frac{\partial\Phi}{\partial x_v}=-\frac{b}{\hbar}c_v\Phi,\ \ v=0,1,2,3. \tag{3.16}$$

By integration of equation (3.16) we obtain

$$\Phi=K\exp\left[-\frac{b}{\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]$$

where K is the integration constant, which is equation (3.9).

Combining equations (2.10), (3.12) and (3.13) for $k=0,1,2,3$ we obtain

$$\lambda_{ki}=\frac{\partial J_i}{\partial x_k}-\frac{b}{\hbar}P_kJ_i$$

$$\lambda_{ki}=\frac{\partial}{\partial x_k}\left(\frac{c_i}{1+\Phi}\right)-\frac{b}{\hbar}\frac{\Phi c_k}{1+\Phi}\frac{c_i}{1+\Phi}$$

$$\lambda_{ki}=-\frac{c_i}{(1+\Phi)^2}\frac{\partial\Phi}{x_k}-\frac{b}{\hbar}\frac{\Phi c_kc_i}{(1+\Phi)^2}$$

and with equation (3.16) for $v=k$ we obtain

$$\lambda_{ki}=\frac{c_i}{(1+\Phi)^2}\frac{b}{\hbar}c_k\Phi-\frac{b}{\hbar}\frac{\Phi c_kc_i}{(1+\Phi)^2}$$

$$\lambda_{ki}=0.\ \square$$

We formulated internal symmetry theorem for $J\neq 0$ in order for the material particle to exist. If we formulate the theorem for $P\neq 0$, the material particle and the STEM exchange places in the equations and the conclusions of the TSV.

Following we do the study based on case 3. of the internal symmetry theorem. That is in the case where $\Phi\neq -1$ and $\Phi\neq 0$. The study of the cases $\Phi\neq -1$ and $\Phi\neq 0$, i.e. of equations (3.8) are not considered in the present publication.

According to the previous theorem, internal symmetry is equivalent to the parallelism of the four-vectors J, P. Starting from this conclusion we can determine the physical content of the internal symmetry.

In an isotropic space the spontaneous emission of STEM by the material particle is isotropic. Due to the linearity of the Lorentz-Einstein transformations, this isotropic emission has as a consequence the parallelism of the four-vectors J, P ([5] par. 5.3). Thus, the theorem of internal symmetry 3.3 holds for the spontaneous emission of STEM by the material particle due to Selfvariations .

In the following chapters, we will make clear that the internal symmetry refers to a spontaneous internal increase of the rest mass and the electrical charge of the material particles, independent of any external causes. The consequences of this increase is the cosmological data, as we'll see in Chapter 16. Also, the internal symmetry is associated with Heisenberg's uncertainty principle.

We start the investigation of the external symmetry with the proof of the following theorem:

Theorem 3.4. First theorem of the TSV for the external symmetry: ´´ For $m_0 \neq 0$ and when the generalized particle conserves its momentum along every axis, and the symmetry $\lambda_{ik} = -\lambda_{ki}$ holds for every $k \neq i, k, i = 0,1,2,3$, then:

1.

$$\begin{aligned} &c_i \lambda_{vk} + c_k \lambda_{iv} + c_v \lambda_{ki} = 0 \\ &J_i \lambda_{vk} + J_k \lambda_{iv} + J_v \lambda_{ki} = 0 \\ &P_i \lambda_{vk} + P_k \lambda_{iv} + P_v \lambda_{ki} = 0 \end{aligned} \tag{3.17}$$

for every $i \neq v, v \neq k, k \neq i,\ k, i, v = 0,1,2,3$.

2. $$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \lambda_{ki} - \frac{bc_v}{2\hbar} \lambda_{ki} = -\frac{b}{\hbar} J_v \lambda_{ki} + \frac{bc_v}{2\hbar} \lambda_{ki} \tag{3.18}$$

for every $k \neq i, k, i, v = 0,1,2,3$.

3. $$\lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21} = 0 \text{ . ´´} \tag{3.19}$$

Proof. From equivalence (3.4) we obtain

$$\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), k \neq i, k, i = 0,1,2,3 \tag{3.20}$$

Considering equation (3.20) we get

$$c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = \frac{b}{2\hbar}\left[c_i\left(c_k J_v - c_v J_k\right) + c_k\left(c_v J_i - c_i J_v\right) + c_v\left(c_i J_k - c_k J_i\right)\right] = 0$$

$$c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = 0$$

after the calculations. Thus, we get the first of equations (3.17). Similarly, from the other two equalities of equivalence (3.4) we obtain the second and the third equation of (3.17). Since $k \neq i$ in equivalence (3.4), the physical quantities $\lambda_{vk}, \lambda_{iv}, \lambda_{ki}$ in equations (3.17) are defined for $v \neq k, i \neq v, k \neq i, k, i, v = 0,1,2,3$.

Differentiating equation (3.20) with respect to $x_v, v = 0,1,2,3$ we obtain

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left(c_i\frac{\partial J_k}{\partial x_v} - c_k\frac{\partial J_i}{\partial x_v}\right)$$

and with equation (2.10) we get

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left[c_i\left(\frac{b}{\hbar}P_v J_k + \lambda_{vk}\right) - c_k\left(\frac{b}{\hbar}P_v J_i + \lambda_{vi}\right)\right]$$

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left[\frac{b}{\hbar}P_v\left(c_i J_k - c_k J_i\right) + c_i\lambda_{vk} - c_k\lambda_{vi}\right]$$

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{\hbar}P_v\frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) + \frac{b}{2\hbar}\left(c_i\lambda_{vk} - c_k\lambda_{vi}\right)$$

and with equation (3.20) we obtain

$$\frac{\partial\lambda_{ki}}{\partial x_v} = \frac{b}{\hbar}P_v\lambda_{ki} + \frac{b}{2\hbar}\left(c_i\lambda_{vk} - c_k\lambda_{vi}\right)$$

and with the first of equations (3.17) we obtain

$$c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = 0$$
$$c_i\lambda_{vk} - c_k\lambda_{vi} + c_v\lambda_{ki} = 0$$

$$c_i\lambda_{vk} - c_k\lambda_{vi} = -c_v\lambda_{ki}$$

we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \lambda_{ki} - \frac{bc_v}{2\hbar} \lambda_{ki}$$

which is equation (3.18). The second equality in equation (3.18) emerges from the substitution

$$P_v = c_v - J_v, v = 0,1,2,3$$

according to equation (3.5).

Taking into account equation (3.20) we obtain

$$\lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21} =$$
$$\frac{b^2}{4\hbar^2}\left[\left(c_1 J_0 - c_0 J_1\right)\left(c_2 J_3 - c_3 J_2\right) + \left(c_2 J_0 - c_0 J_2\right)\left(c_3 J_1 - c_1 J_3\right) + \left(c_3 J_0 - c_0 J_3\right)\left(c_1 J_2 - c_2 J_1\right)\right] = 0$$

after the calculations. □

From equation (2.14) it follows that, for $m_0 = 0$, we don’t know if it is

$$\frac{\partial P_i}{\partial x_k} = \frac{\partial P_k}{\partial x_i}$$

or

$$\frac{\partial P_i}{\partial x_k} \neq \frac{\partial P_k}{\partial x_i},\ k \neq i, k, i = 0,1,2,3.$$

Next we study the external symmetry based on equation (3.4), which holds for $m_0 \neq 0$. In chapter 7 we will see the equation of the TSV that holds whether it is $m_0 \neq 0$ or $m_0 = 0$ (see equations (7.77) and (14.16)).

We accept that for the generalized particle the momentum conservation law holds for every axis $x_i, i = 0,1,2,3$. Therefore, in our study the equations of the present chapter hold. With this assumption, TSV is based on three axioms: The law of Selfvariations, the special relativity equation for the rest mass and the energy-momentum conservation of the generalized particle.

4. THE UNIFIED SELFVARIATIONS INTERACTION (USVI)

4.1. Introduction

The most direct consequence of the law of selfvariations is the emission of STEM in spacetime. Through STEM the TSV predicts a common mechanism, a common cause for the interactions of the material particles (Unified Selfvariations Interaction, USVI).

In this chapter we prove the secod theorem of external symmetry which enables us to determine the potential field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of USVI. The field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is defined for any interaction and not only for the electromagnetic and the gravitational interaction. It also satisfies four equations which correspond to the four Maxwell equations. These equations, as well as the Maxwell equations, are special cases of more general equations as we shall see in the next chapter. At the end of the chapter we calculate the field potential.

The USVI consists of the sum of three terms. The first term is demonstrated by a force parallel to the 4 dimensional momentum of the material particle. This term is always non-zero. The second term demonstrates the spacetime curvature and the third the familiar from electromagnetism, Lorentz force.

4.2. The unified selfvariations interaction (USVI)

According to the law of selfvariations every material particle interacts both with the STEM emitted by itself due to the selfvariations, and with the STEM originating from other material particles. In the second case, an indirect interaction emerges between material particles through the STEM. STEM emitted by one material particle interact with another material particle. Through this mechanism the TSV predicts a unified interaction between material particles. The individual interactions only emerge from the different, for each particular case, physical quantity Q which selfvariates, resulting in the emission of the corresponding STEM.In this chapter we study the basic characteristics of the USVI. We suppose that for the generalized particle the conservation of energy-momentum holds, hence the equations of the preceding chapter also hold. For the rate of change of the four-vector $\frac{1}{m_0}J$ we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right)=-\frac{J_i}{m_0^2}\frac{\partial m_0}{\partial x_k}+\frac{1}{m_0}\frac{\partial J_i}{\partial x_k}$$

and with equations (2.6) and (2.10) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right)=-\frac{J_i}{m_0^2}\frac{b}{\hbar}P_k m_0+\frac{1}{m_0}\left(\frac{b}{\hbar}P_k J_i+\lambda_{ki}\right)$$

and we finally obtain

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right)=\frac{\lambda_{ki}}{m_0},\ k,i=0,1,2,3. \qquad (4.1)$$

According to equation (4.1), when $\lambda_{ki}\neq 0$ for at least two indices $k,i,\ \ k,i=0,1,2,3,$ the kinetic state of the material particle is disturbed. According to equivalence (3.14) in the internal symmetry it is $\lambda_{ki}=0$ for every $k,i=0,1,2,3.$ Therefore, in the internal symmetry the material particle maintains its kinetic state. In an isotropic spacetime we expect that the spontaneous emission of STEM by the material particle cannot disturb its kinetic state. Consequently, the internal symmetry concerns the spontaneous emission of STEM by the material particle in an isotropic spacetime.

In contrast, in the case of the external symmetry it can be $\lambda_{ki}\neq 0$ for some indices $k,i,\ \ k,i=0,1,2,3$. Therefore, the external symmetry must be due to STEM with which the material particle interacts, and which originate from other material particles. The distribution of STEM depends on the position in spacetime of the material particle relative to other material particles. This leads to the destruction of the isotropy of spacetime for the material particle. The external symmetry factor will emerge in the study that follows.

The initial study of the Selfvariations concerned the rest mass and the electric charge. The study we have presented up to this point allows us to study the Selfvariations in their most general expression.

We consider a physical quantity Q which we shall call selfvariating "charge Q ", or simply charge Q, unaffected by every change of reference frame, therefore Lorentz-Einstein invariant, and obeys the law of Selfvariations, that is equation

$$\frac{\partial Q}{\partial x_k}=\frac{b}{\hbar}P_k Q,\ k=0,1,2,3. \qquad (4.2)$$

In equation (4.2) the momentum $P_k, k=0,1,2,3$, i.e. the four-vector P, depends on the selfvariating charge Q. Two material particles carrying a selfvariating charge of the same nature interact with each other when the STEM emitted by the charge Q_1 of one of them interacts with the charge Q of the other. In this particular case, we denote with Q the charge of the material particle we are studying.

The rest mass m_0 is defined as a quantity of mass or energy divided by c^2, which is invariant according to the Lorentz-Einstein transformations. The 4-vector of the momentum J of the material particle is related to the rest mass m_0 through equation (2.7). The charge Q contributes to the energy content of the generalized particle and, therefore, also contributes to its rest mass. Furthermore, the charge Q modifies the 4-vector of momentum J of the material particle. Consequently, for the change of the four-vector J of the material particle due to the charge Q the four-vector P of equation (2.10) enters into equation (4.2). The consequences of this conclusion become evident when we calculate the rate of change of the four-vector $\frac{1}{Q}J$.

Theorem 4.1. Second theorem of the TSV for the external symmetry:

′′1. The rate of change of the four-vector $\frac{1}{Q}J$ due to the Selfvariations of the charge Q is given by equation

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q},\quad k,i=0,1,2,3 \tag{4.3}$$

2. For $k \neq i$ the physical quantities $\frac{\lambda_{ki}}{Q}$ are given by

$$\frac{\lambda_{ki}}{Q}=za_{ki},\ a_{ki}=\text{constants}, k\neq i, k,i=0,1,2,3 \tag{4.4}$$

where z is the function

$$z=\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right] \tag{4.5}$$

3. For the constants a_{ki} the following equations hold

$$\begin{aligned} c_i a_{vk}+c_k a_{iv}+c_v a_{ki}&=0\\ J_i a_{vk}+J_k a_{iv}+J_v a_{ki}&=0\\ P_i a_{vk}+P_k a_{iv}+P_v a_{ki}&=0 \end{aligned} \tag{4.6}$$

for every $i\neq v, v\neq k, k\neq i,\ i,k,v=0,1,2,3$.

4. $\alpha_{ik}=-\alpha_{ki}, \forall k\neq i, k,i=0,1,2,3$ (4.7)

5. $\alpha_{01}\alpha_{32}+\alpha_{02}\alpha_{13}+\alpha_{03}\alpha_{21}=0$.′′ (4.8)

Proof. In order to prove the theorem, we take

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=-\frac{J_i}{Q^2}\frac{\partial Q}{\partial x_k}+\frac{1}{Q}\frac{\partial J_i}{\partial x_k}$$

and with equations (4.2) and (2.10) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=-\frac{J_i}{Q^2}\frac{b}{\hbar}P_kQ+\frac{1}{Q}\left(\frac{b}{\hbar}P_kJ_i+\lambda_{ki}\right)$$

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q}$$

which is equation (4.3). Equations (4.2) and (2.10) hold for every $k,\mathrm{i}=0,1,2,3.$ Therefore, equation (4.3) also holds for every $k,i=0,1,2,3.$

For $k\neq i,\ k,i=0,1,2,3$ and $v=0,1,2,3$ equation (3.18) holds and, since $Q\neq 0$, we obtain

$$Q\frac{\partial\lambda_{ki}}{\partial x_v}=\frac{b}{\hbar}P_vQ\lambda_{ki}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

and with equation (4.2) we get

$$Q\frac{\partial\lambda_{ki}}{\partial x_v}=\lambda_{ki}\frac{\partial Q}{\partial x_v}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

$$\frac{1}{Q^2}\left(Q\frac{\partial\lambda_{ki}}{\partial x_v}-\lambda_{ki}\frac{\partial Q}{\partial x_v}\right)=-\frac{bc_v}{2\hbar}\frac{\lambda_{ki}}{Q}$$

$$\frac{\partial}{\partial x_v}\left(\frac{\lambda_{ki}}{Q}\right)=-\frac{bc_v}{2\hbar}\frac{\lambda_{ki}}{Q}$$

and integrating we obtain

$$\frac{\lambda_{ki}}{Q}=a_{ki}\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]$$

where $a_{ki},k\neq i,k,i=0,1,2,3$ are the integration constants, and with (4.5) we get equation (4.4). Equations (4.6) are derived from the combination of equations (3.17) and (4.4), taking into account that $zQ\neq 0$. Equation (4.7) is derived from the combination of equation

$\lambda_{ik} = -\lambda_{ki}, k \neq i, k, i = 0,1,2,3$ with equation (4.4). Simirarly, equation (4.8) is derived from the combination of equations (3.19) and (4.4).□

We will also use equation

$$\frac{\partial z}{\partial x_k} = -\frac{bc_k}{2\hbar} z, k = 0,1,2,3 \tag{4.9}$$

which results immediately from equation (4.5).

For $k = i, k, i = 0,1,2,3$ equation (4.4) does not hold. So we define the physical quantities T_k as given by equation

$$T_k = \alpha_{kk} = \frac{\lambda_{kk}}{zQ}, k = 0,1,2,3. \tag{4.10}$$

Taking into account the notation of equation (4.10) the main diagonal of matrix T of equation (2.12) is given from matrix Λ

$$\Lambda = \frac{1}{zQ}\begin{bmatrix} \lambda_{00} & 0 & 0 & 0 \\ 0 & \lambda_{11} & 0 & 0 \\ 0 & 0 & \lambda_{22} & 0 \\ 0 & 0 & 0 & \lambda_{33} \end{bmatrix} = \begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}. \tag{4.11}$$

We now define the three-vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, as given by equations (4.12) and (4.13) respectively

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \frac{1}{Q}\begin{pmatrix} ic\lambda_{01} \\ ic\lambda_{02} \\ ic\lambda_{03} \end{pmatrix} \tag{4.12}$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} = \frac{1}{Q}\begin{pmatrix} \lambda_{32} \\ \lambda_{13} \\ \lambda_{21} \end{pmatrix}. \tag{4.13}$$

Vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ contain all of the physical quantities λ_{ki} for $k \neq i, k, i = 0,1,2,3$ since $\lambda_{ik} = -\lambda_{ki}$.

Combining equations (4.12) and (4.13) with equation (4.4), the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are written in the form of equations (4.14) and (4.15), respectively

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = icz \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \tag{4.14}$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} = z \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} . \tag{4.15}$$

We write equation (2.10) in the form

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + \lambda_{ki}, k, i = 0,1,2,3 \ . \tag{4.16}$$

The rate of change of the momentum of the material particle equals the sum of the two terms in the right part of equation (4.16). For $k = 0$, and since $x_0 = ict$, equation (83) gives the rate of change of the particle momentum with respect to time t, i.e. the physical quantity we call "force". By using the concept of force, as defined by Newton, we also have to use the concept of velocity. For this reason we symbolize $\mathbf{u}$ the velocity of the material particle, as given by equation

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} . \tag{4.17}$$

Also, we define the 4-vector of the four-vector u, as given by equation

$$u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} ic \\ u_x \\ u_y \\ u_z \end{bmatrix} . \tag{4.18}$$

We now prove the following theorem:

Theorem 4.2. ´´ The rates of change with respect to time $t\left(x_0 = ict\right)$ of the four-vectors J and P of the momentum of the generalized particle carrying charge Q are given by equations

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J - \frac{i}{c} zQ\Lambda u - \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} \tag{4.19}$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Qdx_0} J + \frac{i}{c} zQ\Lambda u + \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} .'' \tag{4.20}$$

Proof. The matrix Λ is given in equation (4.11). By $\mathbf{u}\times\boldsymbol{\beta}$ we denote the outer product of vectors $\mathbf{u}$ and $\boldsymbol{\beta}$.

We now prove the first of equations (4.19):

$$\frac{d}{dt}\left(\frac{J_0}{Q}\right) = \frac{\partial}{\partial t}\left(\frac{J_0}{Q}\right) + u_1 \frac{\partial}{\partial x}\left(\frac{J_0}{Q}\right) + u_2 \frac{\partial}{\partial y}\left(\frac{J_0}{Q}\right) + u_3 \frac{\partial}{\partial z}\left(\frac{J_0}{Q}\right)$$

and using the notation of equation (2.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right) = ic\frac{\partial}{\partial x_0}\left(\frac{J_0}{Q}\right) + u_1 \frac{\partial}{\partial x_1}\left(\frac{J_0}{Q}\right) + u_2 \frac{\partial}{\partial x_2}\left(\frac{J_0}{Q}\right) + u_3 \frac{\partial}{\partial x_3}\left(\frac{J_0}{Q}\right)$$

and with equation (4.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right) = ic\frac{\lambda_{00}}{Q} + u_1\frac{\lambda_{10}}{Q} + u_2\frac{\lambda_{20}}{Q} + u_3\frac{\lambda_{30}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right) = \frac{\lambda_{00}}{Q} - \frac{i}{c}\left(u_1\frac{\lambda_{10}}{Q} + u_2\frac{\lambda_{20}}{Q} + u_3\frac{\lambda_{30}}{Q}\right)$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right) = \frac{\lambda_{00}}{Q} + \frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q} + u_2\frac{\lambda_{02}}{Q} + u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{1}{Q}\frac{dJ_0}{dx_0} - \frac{J_0}{Q^2}\frac{dQ}{dx_0} = \frac{\lambda_{00}}{Q} + \frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q} + u_2\frac{\lambda_{02}}{Q} + u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{dJ_0}{dx_0} = \frac{dQ}{Qdx_0} J_0 + \lambda_{00} + \frac{i}{c}\left(u_1\lambda_{01} + u_2\lambda_{02} + u_3\lambda_{03}\right)$$

and with equations (4.10) and (4.12) we have

$$\frac{dJ_0}{dx_0}=\frac{dQ}{Qdx_0}J_0+zQT_0-\frac{i}{c}Q\left(\frac{i}{c}u_1\alpha_1+\frac{i}{c}u_2\alpha_2+\frac{i}{c}u_3\alpha_3\right)$$

which is the first of equations (4.19) since

$$-\frac{i}{c}zQT_0u_0=-\frac{i}{c}zQT_0ic=zQT_0\,.$$

We prove the second of equations (4.19) and we can similarly prove the third and the fourth:

$$\frac{d}{dt}\left(\frac{J_x}{Q}\right)=\frac{\partial}{\partial t}\left(\frac{J_x}{Q}\right)+u_1\frac{\partial}{\partial x}\left(\frac{J_x}{Q}\right)+u_2\frac{\partial}{\partial y}\left(\frac{J_x}{Q}\right)+u_3\frac{\partial}{\partial z}\left(\frac{J_x}{Q}\right)$$

and using the notation of equations (2.3) and (2.4) we obtain

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right)=\frac{ic\partial}{\partial x_0}\left(\frac{J_1}{Q}\right)+u_1\frac{\partial}{\partial x_1}\left(\frac{J_1}{Q}\right)+u_2\frac{\partial}{\partial x_2}\left(\frac{J_1}{Q}\right)+u_3\frac{\partial}{\partial x_3}\left(\frac{J_1}{Q}\right)$$

and with equation (4.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right)=ic\frac{\lambda_{01}}{Q}+u_1\frac{\lambda_{11}}{Q}+u_2\frac{\lambda_{21}}{Q}+u_3\frac{\lambda_{31}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_1}{Q}\right)=-\frac{iu_1}{c}\frac{\lambda_{11}}{Q}+\frac{\lambda_{01}}{Q}-\frac{iu_2}{c}\frac{\lambda_{21}}{Q}+\frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{1}{Q}\frac{dJ_1}{dx_0}-\frac{J_1}{Q^2}\frac{dQ}{dx_0}=-\frac{iu_1}{c}\frac{\lambda_{11}}{Q}+\frac{\lambda_{01}}{Q}-\frac{iu_2}{c}\frac{\lambda_{21}}{Q}+\frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{dJ_1}{dx_0}=\frac{dQ}{Qdx_0}J_1-\frac{iu_1}{c}\lambda_{11}+\lambda_{01}-\frac{iu_2}{c}\lambda_{21}+\frac{iu_3}{c}\lambda_{13}$$

and with equations (4.10), (4.12) and (4.13), we obtain

$$\frac{dJ_1}{dx_0}=\frac{dQ}{Qdx_0}J_1-\frac{i}{c}zQT_1u_1-\frac{i}{c}Q\alpha_1-\frac{i}{c}Q\left(u_2\beta_3-u_3\beta_2\right)$$

which is the second of equations (4.19). Equation (4.20) results from the combination of equations (4.19) and (3.5). □

Using the symbol $\mathbf{J}$ for the momentum vector of the material particle

$$\mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

and taking into account equations (2.3) and (2.4) and (4.11) the set of equations (4.19) can be written in the form

$$\begin{aligned} &\frac{dW}{dt} = \frac{dQ}{Qdt}W + zQc^2T_0 + Q\mathbf{u}\cdot\boldsymbol{\alpha} \\ &\frac{d\mathbf{J}}{dt} = \frac{dQ}{Qdt}\mathbf{J} + zQ\begin{pmatrix} T_1u_1 \\ T_2u_2 \\ T_3u_3 \end{pmatrix} + Q\left(\boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta}\right) \end{aligned}. \tag{4.21}$$

Equations (4.21) are a simpler form of equation (4.19) with which they are equivalent.

The rate of change of the four-vector J of the momentum of the material particle is given by the sum of the three terms in the right part of equation (86). The USVI and its consequences for the material particle depend on which of these terms is the strongest and which is the weakest.

The first term expresses a force parallel to four-vector J which is always different than zero due to the Selfvariations. As we will see next, the second term is related to the curvature of spacetime. The third term on the right of equation (4.19) is known as the Lorentz force, in the case of electromagnetic fields. In many cases a term or some of the terms on the right of equation (4.19) are zero, with the exception of the first term which is always different than zero.

From equation (4.19) we conclude that the pair of vectors $(\boldsymbol{\alpha},\boldsymbol{\beta})$ expresses the intensity of the field of the USVI according to the paradigm of the classical definition of the field potential. From equation (2.10) we derive that the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ have units (dimensions) of $kg\cdot s^{-1}$. Thus, from equation (4.12) we derive that if Q is the rest mass, the intensity $\boldsymbol{\alpha}$ has unit of $m\cdot s^{-2}$. If Q is the electric charge, the intensity $\boldsymbol{\alpha}$ has unit of $N\cdot C^{-1}$. Now we will prove that for field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ the following equations (4.22) hold:

Theorem 4.3. ´´ For the vector pair $(\boldsymbol{\alpha},\boldsymbol{\beta})$ the following equations hold:

$$\nabla\cdot\boldsymbol{\alpha}=-\frac{icbz}{2\hbar}\left(c_1\alpha_{01}+c_2\alpha_{02}+c_3\alpha_{03}\right) \quad \text{(a)}$$

$$\nabla\cdot\boldsymbol{\beta}=0 \quad \text{(b)}$$

$$\nabla\times\boldsymbol{\alpha}=-\frac{\partial\boldsymbol{\beta}}{\partial t} \quad \text{(c)} \qquad (4.22)$$

$$\nabla\times\boldsymbol{\beta}=-\frac{bz}{2\hbar}\begin{pmatrix} c_0\alpha_{01}+c_2\alpha_{21}+c_3\alpha_{31}\\ c_0\alpha_{02}+c_2\alpha_{12}+c_3\alpha_{32}\\ c_0\alpha_{03}+c_2\alpha_{13}+c_3\alpha_{23}\end{pmatrix}+\frac{\partial\boldsymbol{\alpha}}{c^2\partial t}\,.´´ \quad \text{(d)}$$

Proof. Differentiating equations (4.14) and (4.15) with respect to x_k, $k=0,1,2,3$ and considering equation (4.9), we obtain equations

$$\frac{\partial\boldsymbol{\alpha}}{\partial x_k}=-\frac{bc_k}{2\hbar}\boldsymbol{\alpha} \qquad (4.23)$$

$$\frac{\partial\boldsymbol{\beta}}{\partial x_k}=-\frac{bc_k}{2\hbar}\boldsymbol{\beta}\,. \qquad (4.24)$$

From equations (4.23) and (4.24) we can easily derive equations (4.22). Indicatively, we prove equation (4.22b). From equation (4.15) we obtain

$$\nabla\cdot\boldsymbol{\beta}=\alpha_{32}\frac{\partial z}{\partial x_1}+\alpha_{13}\frac{\partial z}{\partial x_2}+\alpha_{21}\frac{\partial z}{\partial x_3}$$

and with equation (4.9) we get

$$\nabla\cdot\boldsymbol{\beta}=-\frac{bz}{2\hbar}\left(c_1\alpha_{32}+c_2\alpha_{13}+c_3\alpha_{21}\right)$$

and with the first of equations (4.6) for $(i,v,k)=(1,3,2)$ we get

$$\nabla\cdot\boldsymbol{\beta}=0\,.$$

The first of equations (4.6) should be taken into account for the proof of the rests of equations of (4.22). □

Considering equations (4.22) we define the scalar quantity ρ and the vector quantity $\mathbf{j}$, as given by equations

$$\begin{aligned} &\rho = \sigma \nabla \cdot \boldsymbol{\alpha} = -\sigma \frac{icbz}{2\hbar}\left(c_1 a_{01} + c_2 a_{02} + c_3 a_{03}\right) \\ &\mathbf{j} = \sigma \frac{c^2 bz}{2\hbar}\begin{pmatrix} -c_0 a_{01} - c_2\, a_{21} + c_3 a_{13} \\ -c_0 a_{02} + c_1\, a_{21} - c_3 a_{32} \\ -c_0 a_{03} - c_1\, a_{13} + c_2 a_{32} \end{pmatrix} \end{aligned} \tag{4.25}$$

where $\sigma \neq 0$ is a constant. We now prove that for the physical quantities ρ and $\mathbf{j}$ the following continuity equation holds:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0\,. \tag{4.26}$$

Proof. : From the first of equations (4.25) we obtain

$$\begin{aligned} &\rho = \sigma \nabla \cdot \boldsymbol{\alpha} \\ &\frac{\partial \rho}{\partial t} = \sigma \frac{\partial}{\partial t}\left(\nabla \cdot \boldsymbol{\alpha}\right) \\ &\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\sigma \frac{\partial \boldsymbol{\alpha}}{\partial t}\right) \end{aligned}$$

and with the second of equations (4.25) and equation (4.22d) we get

$$\begin{aligned} &\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\sigma c^2 \nabla \times \boldsymbol{\beta} - \mathbf{j}\right) \\ &\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j} \end{aligned}$$

which is equation (4.26). □

According to equation (4.26), the physical quantity ρ is the density of a conserved physical quantity q with current density $\mathbf{j}$. The conserved physical quantity q is related to field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ through equations (4.22).We will revert to the issue of sustainable physical quantities in the next chapters.

The density ρ and the current density $\mathbf{j}$ have a rigidly defined internal structure as derived from equations (4.25). We now consider the four-vector of the current density j of the conserved physical quantity q, as given by equation

$$j=\begin{bmatrix} j_0 \\ j_1 \\ j_2 \\ j_3 \end{bmatrix}=\begin{bmatrix} i\rho c \\ j_x \\ j_y \\ j_z \end{bmatrix} \tag{4.27}$$

and the 4×4 matrix M

$$M=\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix}. \tag{4.28}$$

Using matrix M equations (4.25) can be written in the form of equation

$$j=\frac{\sigma c^2 bz}{2\hbar}MC\,. \tag{4.29}$$

From equations (4.22b,c) we conclude that the potential is always defined in the $(\boldsymbol{\alpha},\boldsymbol{\beta})$- field of the USVI. That is, the scalar potential

$$V=V(t,x,y,z)=V(x_0,x_1,x_2,x_3)$$

and the vector potential $\mathbf{A}$

$$\mathbf{A}=\mathbf{A}(t,x,y,z)=\mathbf{A}(x_0,x_1,x_2,x_3)=\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}=\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

are defined through the equations

$$\begin{aligned} &\boldsymbol{\beta}=\nabla\times\mathbf{A} \\ &\boldsymbol{\alpha}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t}=-\nabla V-\frac{ic\partial\mathbf{A}}{\partial x_0}\quad. \end{aligned}$$

We can introduce in the above equations the gauge function f. That is, we can add to the scalar potential V the term

$$-\frac{\partial f}{\partial t}=-\frac{ic\partial f}{\partial x_0}$$

and to the vector potential $\mathbf{A}$ the term

∇f

for an arbitrary function f

$$f = f\left(t, x, y, z\right) = f\left(x_0, x_1, x_2, x_3\right)$$

without changing the intensity $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of the field. The proof of the above equations is known and trivial and we will not repeat it here. For the field potential of the USVI the following theorem holds:

Theorem 4.4.

′′1. In the $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-field of USVI the pair of scalar-vector potentials $(V, \mathbf{A})$ is always defined through equations

$$\begin{aligned} \boldsymbol{\beta} &= \nabla \times \mathbf{A} \\ \boldsymbol{\alpha} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = ic\nabla A_0 - \frac{ic\partial \mathbf{A}}{\partial x_0} \end{aligned} \quad . \tag{4.30}$$

2. The four-vector A of the potential

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} \frac{iV}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix} \tag{4.31}$$

is given by equation

$$A_i = \begin{cases} \frac{2\hbar}{b} \frac{\alpha_{ki}}{c_k} z + \frac{\partial f_k}{\partial x_i}, \text{if } i \neq k \\ \frac{\partial f_k}{\partial x_i}, \text{if } i = k \end{cases} \tag{4.32}$$

where $c_k \neq 0, k \in \{0,1,2,3\}, i = 0,1,2,3$ and f_k is the gauge function.

3. For $c_k c_i \neq 0, k \neq i, k, i \in \{0,1,2,3\}$ equation (4.33) holds

$$f_k = f_i + \frac{4\hbar^2 z}{b^2} \frac{\alpha_{ki}}{c_k c_i}, c_k c_i \neq 0, k \neq i,\ k, i = 0,1,2,3. \,'' \tag{4.33}$$

Proof. Equations (4.30) are equivalent to equations (4.22b, c) as we have already mentioned. The proof of equation (4.32) can be performed through the first of equations (4.6). The proof is lengthy and we omit it (see chapter 11). You can verify that the potential of equation (4.32) gives equations (4.14) and (4.15) through equations (4.30) taking also into account the first of equations (4.6).

From equation (4.32) the following four sets of the potentials follow:

$$\begin{aligned} & c_0 \neq 0 \\ & A_0 = \frac{\partial f_0}{\partial x_0} \\ & A_1 = \frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3} \end{aligned} \tag{4.34}$$

$$\begin{aligned} & c_1 \neq 0 \\ & A_0 = \frac{2\hbar z}{b}\frac{\alpha_{10}}{c_1} + \frac{\partial f_1}{\partial x_0} \\ & A_1 = \frac{\partial f_1}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_1}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{13}}{c_1} + \frac{\partial f_1}{\partial x_3} \end{aligned} \tag{4.35}$$

$$\begin{aligned} & c_2 \neq 0 \\ & A_0 = \frac{2\hbar z}{b}\frac{\alpha_{20}}{c_2} + \frac{\partial f_2}{\partial x_0} \\ & A_1 = \frac{2\hbar z}{b}\frac{\alpha_{21}}{c_2} + \frac{\partial f_2}{\partial x_1} \\ & A_2 = \frac{\partial f_2}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{23}}{c_2} + \frac{\partial f_2}{\partial x_3} \end{aligned} \tag{4.36}$$

$$c_3 \neq 0$$

$$A_0 = \frac{2\hbar z}{b}\frac{\alpha_{30}}{c_3} + \frac{\partial f_3}{\partial x_0}$$

$$A_1 = \frac{2\hbar z}{b}\frac{\alpha_{31}}{c_3} + \frac{\partial f_3}{\partial x_1} \quad . \tag{4.37}$$

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{32}}{c_3} + \frac{\partial f_3}{\partial x_2}$$

$$A_3 = \frac{\partial f_3}{\partial x_3}$$

Indicatively, we calculate the components α_1 and β_1 of the intensity $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of the USVI field from the potentials (4.34). From the second of equations (4.30) we obtain

$$\alpha_1 = ic\left(\frac{\partial A_0}{\partial x_1} - \frac{\partial A_1}{\partial x_0}\right)$$

and with equations (4.34) we get

$$\alpha_1 = ic\left[\frac{\partial}{\partial x_1}\left(\frac{\partial f_0}{\partial x_0}\right) - \frac{\partial}{\partial x_0}\left(\frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1}\right)\right]$$

$$\alpha_1 = -ic\frac{2\hbar}{b}\frac{\alpha_{01}}{c_0}\frac{\partial z}{\partial x_0}$$

and with equation (4.9) we get

$$\alpha_1 = icz\alpha_{01}$$

that is we get the intensity α_1 of the field, as given by equation (4.14).

From the first of equations (4.30) we have

$$\beta_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}$$

and with equations (4.34) we get

$$\beta_1 = \frac{\partial}{\partial x_2}\left(\frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3}\right) - \frac{\partial}{\partial x_3}\left(\frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2}\right)$$

$$\beta_1 = \frac{2\hbar}{b}\frac{\alpha_{03}}{c_0}\frac{\partial z}{\partial x_2} - \frac{2\hbar}{b}\frac{\alpha_{02}}{c_0}\frac{\partial z}{\partial x_2}$$

and with equation (4.9) we get

$$\beta_1 = -\frac{c_2\alpha_{03}}{c_0}z + \frac{c_3\alpha_{02}}{c_0}z$$

and considering that $\alpha_{02} = -\alpha_{20}$, we get

$$\beta_1 = -\frac{z}{c_0}\left(c_2\alpha_{03} + c_3\alpha_{20}\right). \tag{4.38}$$

From the first of equations (4.6) for $(i,v,k) = (2,0,3)$ we obtain

$$c_2a_{03} + c_3a_{20} + c_0a_{32} = 0$$
$$c_2a_{03} + c_3a_{20} = -c_0a_{32}$$

and substituting into equation (4.38), we see that

$$\beta_1 = z\alpha_{32}$$

that is, we get the intensity β_1 of the field, as given by equation (4.15).

The gauge functions f_k, $\mathrm{k} = 0,1,2,3$ in equations (4.34)-(4.37) are not independent of each other. For $c_k \neq 0$ and $c_i \neq 0$ for $k \neq i, k, i = 0,1,2,3$ equation (4.39) holds

$$f_k = f_i + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{ki}}{c_k c_i}, c_k c_i \neq 0, k \neq i,\ k, i = 0,1,2,3. \tag{4.39}$$

The proof of equation (4.39) is through the first of equations (4.6). The proof is lengthy and we omit it. Indicatively, we will prove the third of equations (4.34) from the third of equations (4.35) for $k = 1$ and $i = 0$ in equation (4.39).

For $c_0 \neq 0$ and $c_1 \neq 0$ both equations (4.34) and equations (4.35) hold. From equation (4.39) for $k = 1$ and $i = 0$ we get equation

$$f_1 = f_0 + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_1 c_0}. \tag{4.40}$$

From the third of equations (4.35) and equation (4.40) we get

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial}{\partial x_2}\left(f_0 + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0 c_1}\right)$$
$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_0}{\partial x_2} + \frac{4\hbar^2}{b^2}\frac{\alpha_{10}}{c_0 c_1}\frac{\partial z}{\partial x_2}$$

and with equation (4.9) we obtain

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_0}{\partial x_2} - \frac{2\hbar z}{b}\frac{c_2\alpha_{10}}{c_0 c_1}$$

$$A_2 = \frac{2\hbar z}{b c_0 c_1}\left(c_0\alpha_{12} - c_2\alpha_{10}\right) + \frac{\partial f_0}{\partial x_2}$$

and since $\alpha_{10} = -\alpha_{01}$, we get equation

$$A_2 = \frac{2\hbar z}{b c_0 c_1}\left(c_0\alpha_{12} + c_2\alpha_{01}\right) + \frac{\partial f_0}{\partial x_2}. \tag{4.41}$$

From the first of equations (4.6) for $(i,v,k)=(0,1,2)$ we obtain

$$c_0 a_{12} + c_2 a_{01} + c_1 a_{20} = 0$$
$$c_0 a_{12} + c_2 a_{01} = -c_1 a_{20}$$
$$c_0 a_{12} + c_2 a_{01} = c_1 a_{02}$$

and substituting into equation (4.41) we obtain equation

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2}. \tag{4.42}$$

Equation (4.42) is the third of equations (4.34).

According to equation (4.39), if $c_k \neq 0$ for more than one of the constants $c_k, k = 0,1,2,3$, the sets of equations of potential resulting from equation (4.32) have in the end a gauge function. In the application we presented assuming $c_0 \neq 0$ and $c_1 \neq 0$ for a specific gauge function f_0 in equations (4.34), the gauge function f_1 in equations (4.35) is given by equation (4.40). □

We conclude the investigation of the potential of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of USVI by proving the following corollary:

Corollary 4.1. ′′In the external symmetry, the 4-vector C of the total energy content of the generalized particle cannot vanish:

$$C = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.'' \tag{4.43}$$

Proof. Indeed, for $C = 0$ we obtain $J = -P$ from equation (3.5). Therefore, the four-vectors J and P are parallel. According to equivalence (3.7) the parallelism of the four-vectors J and P is equivalent to the internal symmetry. Therefore, in the external symmetry it is $C \neq 0$.
□

A direct consequence of these findings is that the potential of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of USVI is always defined, as given from equation (4.43). This conclusion is derived from the fact that at least one of the constants $c_k, k \in \{0,1,2,3\}$ is always different than zero.

5. THE CONSERVED PHYSICAL QUANTITIES OF THE GENERALIZED PARTICLE AND THE WAVE EQUATION OF THE TSV

5.1. Introduction

The TSV predicts a wave equation whose special case are the Maxwell equations, the Schrödinger equation and other relevant equations. The wave equation Ψ of the TSV is related to the conserved physical quantities. We determine a mathematical expression for the total of the conservable physical quantities, and we calculate the current density 4-vector j.

The density ρ and the current density $\mathbf{j}$ of the conserved physical quantities have a strictly determined 'crystalline' structure which relates with the quantum behavior of matter. The physical quantities ρ and $\mathbf{j}$ are related with an entirely different way than given by the equation $\mathbf{j} = \rho\mathbf{u}$ used by the theories of the previous century.

5.2. The conserved physical quantities of the generalized particle and the wave equation of the TSV

The generalized particle has a set of conserved physical quantities q which we determine in this chapter. At first, we generalize the notion of the field, as it is derived from the equations of theTSV. We prove the following theorem:

Theorem 5.1.

′′1. For the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$ of the pair of vectors

$$\boldsymbol{\xi} = ic\Psi \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} \tag{5.1}$$

$$\boldsymbol{\omega} = \Psi \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.2}$$

where $\Psi = \Psi(x_0, x_1, x_2, x_3)$ is a function satisfying equation

$$\frac{\partial \Psi}{\partial x_k} = \frac{b}{\hbar}\left(\lambda J_k + \mu P_k\right)\Psi \tag{5.3}$$

$k = 0,1,2,3,\quad (\lambda,\mu) \neq \left(0,0\right),\ \ \lambda,\mu \in \mathbb{C}$ are functions of x_0, x_1, x_2, x_3, the following equations holds

$$\begin{aligned} &\nabla \cdot \boldsymbol{\omega} = 0 \\ &\nabla \times \boldsymbol{\xi} = -\frac{\partial \boldsymbol{\omega}}{\partial t} \end{aligned} \quad . \tag{5.4}$$

2. The generalized particle has a set of conserved physical quantities q with density ρ and current density $\mathbf{j}$

$$\begin{aligned} &\rho = \sigma \nabla \cdot \boldsymbol{\xi} \\ &\mathbf{j} = \sigma c^2 \left(\nabla \times \boldsymbol{\omega} - \frac{\partial \boldsymbol{\xi}}{c^2 \partial t} \right) \end{aligned} \tag{5.5}$$

where $\sigma \neq 0$ are constants, for which conserved physical quantities the following continuity equation holds

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 \,. \tag{5.6}$$

3. The four-vectors of the current density j are given by equation

$$j = -\frac{\sigma c^2 b}{\hbar} \Psi M \left(\lambda J + \mu P \right) \ .'' \tag{5.7}$$

Proof. Matrix M in equation (5.7) is given by equation (4.28). We denote **J** and **P** the three-dimensional momentums as given by equations

$$\mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} \tag{5.8}$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} . \tag{5.9}$$

For the proof of the theorem we first demonstrate the following auxiliary equations (5.10)-(5.15)

$$\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{5.10}$$

$$\mathbf{P}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{5.11}$$

$$\mathbf{J}\times\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -J_0\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.12}$$

$$\mathbf{P}\times\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -P_0\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.13}$$

$$\mathbf{J}\times\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = \begin{pmatrix} J_2 a_{21} - J_3 a_{13} \\ J_3 a_{32} - J_1 a_{21} \\ J_1 a_{13} - J_2 a_{32} \end{pmatrix} \tag{5.14}$$

$$\mathbf{P}\times\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = \begin{pmatrix} P_2 a_{21} - P_3 a_{13} \\ P_3 a_{32} - P_1 a_{21} \\ P_1 a_{13} - P_2 a_{32} \end{pmatrix} . \tag{5.15}$$

In order to prove equation (5.10) we get

$$\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = J_1 a_{32} + J_2 a_{13} + J_3 a_{21}$$

and with the second of equations (4.6) for $(i,v,k)=(1,3,2)$, we have

$$\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0$$

Similarly, from the third of equations (4.6) we obtain equation (5.11). We now get

$$\mathbf{J}\times\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix}=\begin{pmatrix}J_2a_{03}-J_3a_{02}\\J_3a_{01}-J_1a_{03}\\J_1a_{02}-J_2a_{01}\end{pmatrix}=\begin{pmatrix}J_2a_{03}+J_3a_{20}\\J_3a_{01}+J_1a_{30}\\J_1a_{02}+J_2a_{10}\end{pmatrix}$$

and with the second of equations (4.6) we obtain

$$\mathbf{J}\times\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix}=\begin{pmatrix}-J_0a_{32}\\-J_0a_{13}\\-J_0a_{21}\end{pmatrix}$$

$$\mathbf{J}\times\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix}=-J_0\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}$$

which is equation (5.12). Similarly, by considering the third of equations (4.6) we derive equation (5.13). Equations (5.14) and (5.15) are derived by taking into account equations (5.8) and (5.9).

Equations (5.4) are proven with the use of equations (5.10)-(5.15). We prove the first as an example. From equation (5.2) we obtain

$$\nabla\cdot\boldsymbol{\omega}=\nabla\Psi\cdot\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}$$

and with equation (5.3) we get

$$\nabla\cdot\boldsymbol{\omega}=\frac{b}{\hbar}\lambda\Psi\mathbf{J}\cdot\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}+\frac{b}{\hbar}\mu\Psi\mathbf{P}\cdot\begin{pmatrix}a_{32}\\a_{13}\\a_{21}\end{pmatrix}$$

and with equations (5.10) and (5.11) we obtain

$$\nabla\cdot\boldsymbol{\omega}=0.$$

From equations (5.4) and (5.5), the continuity equation (5.6) results. The proof is similar to the one for equation (4.26). The proof of equation (5.7) is done with the use of equations (5.10)-(5.15), and equation (4.28). □

Field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ presented in the previous chapter is a special case of the field $(\boldsymbol{\xi},\boldsymbol{\omega})$ for $\lambda=\mu=-\frac{1}{2}$. For these values of the parameteres λ,μ we obtain from equations (5.3)

$$\frac{\partial\Psi}{\partial x_k}=\frac{b}{\hbar}\left(-\frac{1}{2}J_k-\frac{1}{2}P_k\right)\Psi$$
$$\frac{\partial\Psi}{\partial x_k}=-\frac{b}{2\hbar}\left(J_k+P_k\right)\Psi$$

and with equation (3.5) we obtain

$$\frac{\partial\Psi}{\partial x_k}=-\frac{bc_k}{2\hbar}\Psi$$

and finally we obtain

$$\Psi=z=\exp\left(-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right)$$

and from equations (5.1),(5.2) and (4.14),(4.15) we obtain $\boldsymbol{\xi}=\boldsymbol{\alpha}$ and $\boldsymbol{\omega}=\boldsymbol{\beta}$.

From equation (2.10) it emerges that the dimensions of the physical quantities $\lambda_{ki},k,i=0,1,2,3$ are

$$\left[\lambda_{ki}\right]=kgs^{-1},k,i=0,1,2,3.$$

Thus, from equations (4.12), (4.13) and (4.14), (4.15) we obtain the dimensions of the physical quantities $Q\alpha_{ki},k,i=0,1,2,3$. Furthermore, from equation (4.11) we obtain the dimensions of the physical quantities $QT_k,k=0,1,2,3$. Thus, we get the following relationships

$$\begin{aligned}&\left[Q\alpha_{ki}\right]=kgs^{-1},k\neq i,k,i=0,1,2,3,\\&\left[QT_k\right]=kgs^{-1},k=0,1,2,3.\end{aligned}\tag{5.16}$$

Using the first of equations (5.16) we can determine the units of measurement of the $(\boldsymbol{\xi},\boldsymbol{\omega})$-field for every selfvariating charge Q. When Q is the electric charge, we can verify that the field units are $(V\cdot m^{-1},T)$. When Q is the rest mass, the field units are $\left(m\cdot s^{-2},s^{-1}\right)$.

The dimensions of the field depend solely on the units of measurement of the selfvariating charge Q.

From equation (5.7) and taking into account that $\lambda, \mu \in \mathbb{C}$ we can define the dimensions of the physical quantities q through the first of equations (5.16). When Q is the electric charge, and for $\sigma = \varepsilon_0$, where ε_0 is the electric permittivity of the vacuum, q is a conserved physical quantity of electric charge. For $\sigma = \frac{\hbar \varepsilon_0}{e}$, where e the constant value we measure in the lab for the electric charge of the electron, q is a conserved physical quantity of angular momentum. For $\sigma = \frac{\varepsilon_0}{e}$, q is a dimensionless conserved physical quantity, that $q \in \mathbb{C}$. When Q is the rest mass, and for $\sigma = \frac{1}{4\pi G}$, where G is the gravitational constant, q is a conserved physical quantity of mass. Theorem 5.1 reveals the conserved physical quantities of the generalized particle.

One of the most important corollaries of the theorem 5.1 is the prediction that the generalized particle has wave-like behavior. We prove the following corollary:

Corollary 5.1. ´´For function Ψ the following equation holds

$$
\begin{aligned}
&\sigma c^2 \alpha_{ki} \left(\nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2} \right) = \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \\
&\sigma c^2 \alpha_{ki} \left(\nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} \right) = \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i}
\end{aligned}
\tag{5.17}
$$

$k \neq i, \quad k, i = 0,1,2,3$.´´

Proof. To prove the corollary, considering that $x_0 = ict$, we write equations (5.4) and (5.5) in the form

$$
\begin{aligned}
&\nabla \cdot \boldsymbol{\xi} = -\frac{i}{\sigma c} j_0 \\
&\nabla \cdot \boldsymbol{\omega} = 0 \\
&\nabla \times \boldsymbol{\xi} = -\frac{ic \partial \boldsymbol{\omega}}{\partial x_0} \\
&\nabla \times \boldsymbol{\omega} = \frac{1}{\sigma c^2} \mathbf{j} + \frac{i \partial \boldsymbol{\xi}}{c \partial x_0}
\end{aligned}
\quad . \tag{5.18}
$$

We will also use the identity (5.19) which is valid for every vector $\boldsymbol{\alpha}$

$$\nabla \times \nabla \times \boldsymbol{\alpha} = \nabla\left(\nabla \cdot \boldsymbol{\alpha}\right) - \nabla^2 \boldsymbol{\alpha}. \tag{5.19}$$

From the third of equations (5.18) we obtain

$$\nabla \times \nabla \times \boldsymbol{\xi} = -\nabla \times \left(\frac{ic\partial \boldsymbol{\omega}}{\partial x_0}\right)$$

$$\nabla \times \nabla \times \boldsymbol{\xi} = -\frac{ic\partial}{\partial x_0}\left(\nabla \times \boldsymbol{\omega}\right)$$

and using the identity (5.19) we get

$$\nabla\left(\nabla \cdot \boldsymbol{\xi}\right) - \nabla^2 \boldsymbol{\xi} = -\frac{ic\partial}{\partial x_0}\left(\nabla \times \boldsymbol{\omega}\right)$$

and with the first and fourth of equations (5.18) we get

$$\nabla\left(-\frac{i}{\sigma c} j_0\right) - \nabla^2 \boldsymbol{\xi} = \frac{\partial^2 \boldsymbol{\xi}}{\partial x_0^2} - \frac{i}{\sigma c}\frac{\partial \mathbf{j}}{\partial x_0}$$

and we finally get

$$\nabla^2 \boldsymbol{\xi} + \frac{\partial^2 \boldsymbol{\xi}}{\partial x_0^2} = \frac{i}{\sigma c}\left(\frac{\partial \mathbf{j}}{\partial x_0} - \nabla j_0\right). \tag{5.20}$$

Working similarly from equation (5.18) we obtain

$$\nabla^2 \boldsymbol{\omega} + \frac{\partial^2 \boldsymbol{\omega}}{\partial x_0^2} = -\frac{1}{\sigma c^2}\nabla \times \mathbf{j}. \tag{5.21}$$

Combining equations (5.20) and (5.21) with equations (5.1) and (5.2), we get

$$\alpha_{ki}\left(\nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2}\right) = \frac{i}{\sigma c^2}\left(\frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i}\right),\ k \neq i,\quad k,i = 0,1,2,3$$

which is equation (5.17).□

Equation (5.17) can be characterized as *“the wave equation of the TSV”*. The basic characteristics of equation (5.17) depend on whether the physical quantity

$$F=\nabla^2\Psi+\frac{\partial^2\Psi}{\partial^2 x_0^2}=\nabla^2\Psi-\frac{\partial^2\Psi}{c^2\partial t^2} \tag{5.22}$$

is zero or not.

This conclusion is drawn through the following theorem:

Theorem 5.2. ´´For the generalized particle the following equivalences hold

$$\nabla^2\Psi-\frac{\partial^2\Psi}{c^2\partial t^2}=0 \tag{5.23}$$

if and only if for each $k\neq i,\ k,i=0,1,2,3$ it is

$$\frac{\partial j_i}{\partial x_k}=\frac{\partial j_k}{\partial x_i} \tag{5.24}$$

if and only if

$$\begin{aligned}\nabla^2\boldsymbol{\xi}-\frac{\partial^2\boldsymbol{\xi}}{c^2\partial t^2}&=0\\ \nabla^2\boldsymbol{\omega}-\frac{\partial^2\boldsymbol{\omega}}{c^2\partial t^2}&=0\end{aligned}\quad .'' \tag{5.25}$$

Proof. In the external symmetry there exists at least one pair of indices $(k,i),\ k\neq i,\ k,i\in\{0,1,2,3\}$ for which $\alpha_{ki}\neq 0$. Therefore, when equation (5.24) holds, then equation (5.23) follows from equation (5.17), and vice versa. Thus, equations (5.23) and (5.24) are equivalent. When equation (5.24) holds, then the right hand sides of equations (5.24) and (5.25) vanish, that is, equations (5.25) hold. The converse also holds, thus equations (5.24) and (5.25) are equivalent. Therefore, equations (5.23), (5.24), and (5.25) are equivalent. □

In case that $F=0$, that is in case that equivalences (5.23), (5.24) and (5.25) hold, we shall refer to the state of the generalized particle as the “generalized photon”. According to equations (5.25), for the generalized photon the $(\boldsymbol{\xi},\boldsymbol{\omega})$-field is propagating with velocity c in the form of a wave. For the generalized photon, the following corollary holds:

Corollary 5.2. ´´ For the generalized photon, the four-vector j of the current density of the conserved physical quantities q, varies according to the equations

$$\nabla^2 j_k - \frac{\partial^2 j_k}{c^2 \partial t^2} = 0,\ k = 0,1,2,3\,.´´ \tag{5.26}$$

Proof. We prove equation (5.26) for $k = 0$, and we can similarly prove it for $k = 1,2,3$. Considering equation (4.27), we write equation (5.6) in the form

$$\frac{\partial j_0}{\partial x_0} + \frac{\partial j_1}{\partial x_1} + \frac{\partial j_2}{\partial x_2} + \frac{\partial j_3}{\partial x_3} = 0\,. \tag{5.27}$$

Differentiating equation (5.27) with respect to x_0 we get

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_0}\left(\frac{\partial j_1}{\partial x_1}\right) + \frac{\partial}{\partial x_0}\left(\frac{\partial j_2}{\partial x_2}\right) + \frac{\partial}{\partial x_0}\left(\frac{\partial j_3}{\partial x_3}\right) = 0$$

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_1}\left(\frac{\partial j_1}{\partial x_0}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial j_2}{\partial x_0}\right) + \frac{\partial}{\partial x_3}\left(\frac{\partial j_3}{\partial x_0}\right) = 0$$

and with equation (5.24) we get

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_1}\left(\frac{\partial j_0}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial j_0}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\frac{\partial j_0}{\partial x_3}\right) = 0$$

$$\frac{\partial^2 j_0}{\partial x_0^2} + \nabla^2 j_0 = 0$$

which is equation (5.26) for $k = 0$, since $x_0 = ict$.□

The way in which equations (5.25) emerge in the TSV is completely different from the way in which the electromagnetic waves emerge in Maxwell's electromagnetic theory [6-10]. Maxwell's equations predict the equations (5.25) for $j = 0$. The TSV predicts $(\boldsymbol{\alpha},\boldsymbol{\beta})$ waves for $j \neq 0$, when equation (5.24) is valid. Moreover the current density j in this case varies according to equation (5.26).

We now prove the following corollary of theorem 5.1:

Corollary 5.3. ´´For the 4-vector

$$\left[\frac{\partial\Psi}{\partial x}\right]=\begin{bmatrix}\dfrac{\partial\Psi}{\partial x_0}\\ \dfrac{\partial\Psi}{\partial x_1}\\ \dfrac{\partial\Psi}{\partial x_2}\\ \dfrac{\partial\Psi}{\partial x_3}\end{bmatrix} \tag{5.28}$$

it is

$$M\left[\frac{\partial\Psi}{\partial x}\right]=-\frac{1}{\sigma c^2}j \tag{5.29}$$

where

$$M=\begin{bmatrix}0 & \alpha_{01} & \alpha_{02} & \alpha_{03}\\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13}\\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32}\\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0\end{bmatrix}$$

and j the 4-vector of the current density of the conserved physical quantities of the generalized particle.´´

Proof. From equation (5.3) and with the notation of equation (5.28) we have

$$\left[\frac{\partial\Psi}{\partial x}\right]=\frac{b}{\hbar}\Psi\left(\lambda J+\mu P\right)$$

and multiplying from the left with the matrix M we get

$$M\left[\frac{\partial\Psi}{\partial x}\right]=\frac{b}{\hbar}\Psi M\left(\lambda J+\mu P\right)$$

and with equation (5.7) we have

$$M\left[\frac{\partial\Psi}{\partial x}\right]=-\frac{1}{\sigma c^2}j$$

which is equation (5.29).□

The equations (5.3), (5.7) and (5.29) give the relation of the wave function Ψ with the physical quantities J, P and j of the generalized particle.

One of the most important conclusions of the theorem 5.1 is that it gives the degrees of freedom of the equations of the TSV. In equation (5.7) the parameters

$\lambda, \mu \in \mathbb{C}, (\lambda, \mu) \neq (0,0)$ can have arbitrary values or can be arbitrary functions of x_0, x_1, x_2, x_3. The TSV has *two degrees of freedom*. Therefore, the investigation of the wave equation of the TSV takes place through the parameters λ and μ of equation (5.7).

If we set $(\lambda, \mu, b) = (1, 0, i)$ or $(\lambda, \mu) = \left(\frac{i}{b}, 0\right)$ in equation (5.7), we get equations

$$\begin{aligned} \frac{\partial \Psi}{\partial x_0} &= \frac{i}{\hbar} J_0 \Psi \\ \nabla \Psi &= \frac{i}{\hbar} \mathbf{J} \Psi \end{aligned} . \tag{5.30}$$

For $(\lambda, \mu, b) = (0, 1, i)$ or $(\lambda, \mu) = \left(0, \frac{i}{b}\right)$ we have

$$\begin{aligned} \frac{\partial \Psi}{\partial x_0} &= \frac{i}{\hbar} P_0 \Psi \\ \nabla \Psi &= \frac{i}{\hbar} \mathbf{P} \Psi \end{aligned} . \tag{5.31}$$

For $\lambda = \mu$ we have

$$\frac{\partial \Psi}{\partial x_k} = \frac{b}{\hbar} \mu \Psi \left(J_k + P_k\right), k = 0,1,2,3$$

and with equation (3.5) we have

$$\frac{\partial \Psi}{\partial x_k} = \frac{b c_k}{\hbar} \mu \Psi, k = 0,1,2,3$$

and equivalently we have

$$\begin{aligned} \frac{\partial \Psi}{\partial x_0} &= \frac{b c_0}{\hbar} \mu \Psi \\ \nabla \Psi &= \frac{b}{\hbar} \mu \Psi \mathbf{C} \end{aligned} . \tag{5.32}$$

Taking into account that $x_0 = ict$ and $J_0 = \frac{iW}{c}$, we recognize in equations (5.30) the Schrödinger operators. Using the macroscopic mathematical expressions of the momentum $\mathbf{J}$ and energy W of the material particle, we get the Schrödinger equation [11-15]. The Schrödinger equation is a special case of the wave equation of the TSV. The designation of the degrees of freedom λ and μ determines in a large extend the form of equation (5.7).

6. THE LORENTZ-EINSTEIN-SELFVARIATIONS SYMMETRY

6.1. Introduction

In this chapter we calculate the Lorentz-Einstein transformations of the physical quantities λ_{ki}, $k,i = 0,1,2,3.$ The part of spacetime occupied by the generalized particle can be flat or curved. The Lorentz-Einstein transformations give us information about this subject.

The spacetime curvature depends on the elements $\lambda_{00}, \lambda_{11}, \lambda_{22}, \lambda_{33}$ of the main diagonal of the matrix T of the TSV. We prove that if $\lambda_{kk} \neq 0$ for at least one $k \in \{0,1,2,3\}$ spacetime is curved. For $\lambda_{00} = \lambda_{11} = \lambda_{22} = \lambda_{33} = 0$ spacetime may be either curved or the flat spacetime of special relativity.

6.2. The Lorentz-Einstein-Selfvariations symmetry

We consider an inertial frame of reference $O'(t', \mathrm{x}', \mathrm{y}', \mathrm{z}')$ moving with velocity $(u,0,0)$ with respect to another inertial frame of reference $O(t, \mathrm{x}, \mathrm{y}, \mathrm{z})$, with their origins O' and O coinciding at $t' = t = 0$. We will calculate the Lorentz-Einsteintransformations for the physical quantities λ_{ki}, $k,i = 0,1,2,3$. We begin with transformations (6.1) and (6.2)

$$
\begin{aligned}
\frac{\partial}{\partial t'} &= \gamma\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right) \\
\frac{\partial}{\partial x'} &= \gamma\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right) \\
\frac{\partial}{\partial y'} &= \frac{\partial}{\partial y} \\
\frac{\partial}{\partial z'} &= \frac{\partial}{\partial z}
\end{aligned}
\tag{6.1}
$$

$$
\begin{aligned}
W' &= \gamma\left(W - uJ_x\right) & E' &= \gamma\left(E - uP_x\right) \\
J_x{}' &= \gamma\left(J_x - \frac{u}{c^2}W\right) & P_x' &= \gamma\left(P_x - \frac{u}{c^2}E\right) \\
J_y{}' &= J_y & P_y' &= P_y \\
J_z{}' &= J_z & P_z' &= P_z
\end{aligned}
\tag{6.2}
$$

where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$.

We then use the notation (2.3), (2.4), (2.5) and obtain the transformations (6.3) and (6.4)

$$\begin{aligned}
\frac{\partial}{\partial x_0{}'} &= \gamma\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right) \\
\frac{\partial}{\partial x_1{}'} &= \gamma\left(\frac{\partial}{\partial x_1} + i\frac{u}{c}\frac{\partial}{\partial x_0}\right) \\
\frac{\partial}{\partial x_2{}'} &= \frac{\partial}{\partial x_2} \\
\frac{\partial}{\partial x_3{}'} &= \frac{\partial}{\partial x_3}
\end{aligned} \tag{6.3}$$

$$\begin{aligned}
J_0{}' &= \gamma\left(J_0 - i\frac{u}{c}J_1\right) & P_0{}' &= \gamma\left(P_0 - i\frac{u}{c}P_1\right) \\
J_1{}' &= \gamma\left(J_1 + i\frac{u}{c}J_0\right) & P_1{}' &= \gamma\left(P_1 + i\frac{u}{c}P_0\right). \\
J_2{}' &= J_2 & P_2{}' &= P_2 \\
J_3{}' &= J_3 & P_3{}' &= P_3
\end{aligned} \tag{6.4}$$

We now derive the transformation of the physical quantity λ_{00}. From equation (2.10) for $k = i = 0$ we get for the inertial reference frame $O'(t', \mathrm{x}', \mathrm{y}', \mathrm{z}')$

$$\lambda_{00}{}' = \frac{\partial J_0{}'}{\partial x_0{}'} - \frac{b}{\hbar}P_0{}'J_0{}'$$

and with transformations (6.3) and (6.4) we obtain

$$\lambda_{00}{}' = \gamma^2\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right)\left(J_0 - i\frac{u}{c}J_1\right) - \frac{b}{\hbar}\gamma^2\left(P_0 - i\frac{u}{c}P_1\right)\left(J_0 - i\frac{u}{c}J_1\right)$$

$$\lambda_{00}{}' = \gamma^2\left(\frac{\partial J_0}{\partial x_0} - i\frac{u}{c}\frac{\partial J_1}{\partial x_0} - i\frac{u}{c}\frac{\partial J_0}{\partial x_1} - \frac{u^2}{c^2}\frac{\partial J_1}{\partial x_1} - \frac{b}{\hbar}P_0J_0 + i\frac{u}{c}\frac{b}{\hbar}P_0J_1 + i\frac{u}{c}\frac{b}{\hbar}P_1J_0 + \frac{u^2}{c^2}\frac{b}{\hbar}P_1J_1\right)$$

and replacing physical quantities

$$\frac{\partial J_0}{\partial x_0}, \frac{\partial J_1}{\partial x_0}, \frac{\partial J_0}{\partial x_1}, \frac{\partial J_1}{\partial x_1}$$

from equation (2.10) we get

$$\lambda_{00}{}' = \gamma^2 (\frac{b}{\hbar} P_0 J_0 + \lambda_{00} - i\frac{u}{c}\frac{b}{\hbar} P_0 J_1 - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\frac{b}{\hbar} P_1 J_0 - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\frac{b}{\hbar} P_1 J_1$$
$$- \frac{u^2}{c^2}\lambda_{11} - \frac{b}{\hbar} P_0 J_0 + i\frac{u}{c}\frac{b}{\hbar} P_0 J_1 + i\frac{u}{c}\frac{b}{\hbar} P_1 J_0 + \frac{u^2}{c^2}\frac{b}{\hbar} P_1 J_1)$$

and we finally obtain equation

$$\lambda_{00}{}' = \gamma^2 \left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11} \right).$$

Following the same procedure for $k, \mathrm{i} = 0,1,2,3$ we obtain the following 16 equations for the Lorentz-Einstein transformations of the physical quantities λ_{ki} :

$$\lambda_{00}{}' = \gamma^2 \left(\lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11} \right)$$
$$\lambda_{01}{}' = \gamma^2 \left(\lambda_{01} + i\frac{u}{c}\lambda_{00} - i\frac{u}{c}\lambda_{11} + \frac{u^2}{c^2}\lambda_{10} \right)$$
$$\lambda_{02}{}' = \gamma \left(\lambda_{02} - i\frac{u}{c}\lambda_{12} \right)$$
$$\lambda_{03}{}' = \gamma \left(\lambda_{03} - i\frac{u}{c}\lambda_{13} \right)$$

$$\lambda_{10}{}' = \gamma^2 \left(\lambda_{10} - i\frac{u}{c}\lambda_{11} + i\frac{u}{c}\lambda_{00} + \frac{u^2}{c^2}\lambda_{01} \right)$$
$$\lambda_{11}{}' = \gamma^2 \left(\lambda_{11} + i\frac{u}{c}\lambda_{10} + i\frac{u}{c}\lambda_{01} - \frac{u^2}{c^2}\lambda_{00} \right) \qquad (6.5)$$
$$\lambda_{12}{}' = \gamma \left(\lambda_{12} + i\frac{u}{c}\lambda_{02} \right)$$
$$\lambda_{13}{}' = \gamma \left(\lambda_{13} + i\frac{u}{c}\lambda_{03} \right)$$

$$
\begin{aligned}
\lambda_{20}' &= \gamma\left(\lambda_{20} - i\frac{u}{c}\lambda_{21}\right) \\
\lambda_{21}' &= \gamma\left(\lambda_{21} + i\frac{u}{c}\lambda_{20}\right) \\
\lambda_{22}' &= \lambda_{22} \\
\lambda_{23}' &= \lambda_{23} \\
\lambda_{30}' &= \gamma\left(\lambda_{30} - i\frac{u}{c}\lambda_{31}\right) \\
\lambda_{31}' &= \gamma\left(\lambda_{31} + i\frac{u}{c}\lambda_{30}\right) \\
\lambda_{32}' &= \lambda_{32} \\
\lambda_{33}' &= \lambda_{33}
\end{aligned}
.
$$

The first two of equations (6.5) is self-consistent when equation

$$\lambda_{00} = \lambda_{11} . \tag{6.6}$$

Then by the second of equations (6.5) we obtain

$$\lambda_{01}' = \lambda_{01} .$$

According to equivalence (3.14) these transformations relate to the external symmetry, in which it holds that $\lambda_{ik} = -\lambda_{ki}$ for $i \neq k, i, k = 0,1,2,3$. Thus, we obtain the following transformations for the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$

$$
\begin{aligned}
\lambda_{00}' &= \lambda_{00} \\
\lambda_{11}' &= \lambda_{11} \\
\lambda_{22}' &= \lambda_{22} \\
\lambda_{33}' &= \lambda_{33}
\end{aligned}
\qquad
\begin{aligned}
\lambda_{01}' &= \lambda_{01} \\
\lambda_{02}' &= \gamma\left(\lambda_{02} + i\frac{u}{c}\lambda_{21}\right) \\
\lambda_{03}' &= \gamma\left(\lambda_{03} - i\frac{u}{c}\lambda_{13}\right) \\
\lambda_{32}' &= \lambda_{32} \\
\lambda_{13}' &= \gamma\left(\lambda_{13} + i\frac{u}{c}\lambda_{03}\right) \\
\lambda_{21}' &= \gamma\left(\lambda_{21} - i\frac{u}{c}\lambda_{02}\right)
\end{aligned}
. \tag{6.7}
$$

Taking into account equations (4.4), (4.10) and that the physical quantity $zQ \neq 0$ is invariant under the Lorentz-Einstein transformations, we obtain the following transformations for the constants $\alpha_{ki}, k \neq i, k, i = 0,1,2,3$ and the physical quantities $T_k, k = 0,1,2,3$

$$\begin{aligned} T_0' &= T_0 \\ T_1' &= T_1 \\ T_2' &= T_2 \\ T_3' &= T_3 \end{aligned} \qquad \begin{aligned} \alpha_{01}' &= \alpha_{01} \\ \alpha_{02}' &= \gamma\left(\alpha_{02} + i\frac{u}{c}\alpha_{21}\right) \\ \alpha_{03}' &= \gamma\left(\alpha_{03} - i\frac{u}{c}\alpha_{13}\right) \\ \alpha_{32}' &= \alpha_{32} \\ \alpha_{13}' &= \gamma\left(\alpha_{13} + i\frac{u}{c}\alpha_{03}\right) \\ \alpha_{21}' &= \gamma\left(\alpha_{21} - i\frac{u}{c}\alpha_{02}\right) \end{aligned} . \tag{6.8}$$

Equation (6.6) correlates the physical quantities λ_{00} and λ_{11} in the same inertial frame of reference. Taking into account equation (4.10) we obtain $T_0 = T_1$. Thus, when transformations (6.8) hold, $T_0 = T_1$ also holds. The reference frame $\mathrm{O}'(t', x', y', z')$ moves with respect to the reference frame $\mathrm{O}(t, x, y, z)$ with constant velocity along the $x.$-axis. If we assume that the motion is along the y- or z-axis, the generalization of equation $T_0 = T_1$ follows; the Lorentz-Einstein transformations lead to the following equation $T_0 = T_1 = T_2 = T_3 = 0$. Thus, we derive the following two corollaries.

Corollary 6.1. ´´ When the portion of spacetime occupied by the generalized particle is flat, it is

$$T_0 = T_1 = T_2 = T_3 = 0 .´´ \tag{6.9}$$

Corollary 6.2.´´When

$$T_k \neq 0 \tag{6.10}$$

for at least one $k \in \{0,1,2,3\}$ the portion of spacetime occupied by the generalized particle is curver and not flat.´´

Notice that from the way of proof of corollary 6.1 it follows that the converse is not true. For external symmetries which have $T_0 = T_1 = T_2 = T_3 = 0$, spacetime may be either flat or curved. In chapter 9 we have shown how to check if spacetime is flat or curved for external symmetries with $T_0 = T_1 = T_2 = T_3 = 0$.

In the external symmetry it is $\alpha_{ki} \neq 0$ for at least on pair of indices $k,i \in \{0,1,2,3\}$. Thus, in external symmetry it is $\alpha_{ki} = 0$ only for some pairs of indices $k,i \in \{0,1,2,3\}$. The Lorentz-Einstein transformations reveal that in flat spacetime this cannot be arbitrary. Let's assume that it is

$$\alpha_{02} = 0$$

for every inertial frame of reference. Then, we obtain

$$\alpha_{02}' = 0$$

and with transformations (6.8) we obtain

$$\gamma\left(\alpha_{02} + i\frac{u}{c}\alpha_{21}\right) = 0$$

and since it is $\alpha_{02} = 0$ we obtain that it also holds

$$\alpha_{21} = 0 .$$

Working similarly with all of the transformations (6.8) we end up with the following four sets of equations of external symmetry in the flat spacetime:

$$\begin{aligned}
&T_0 = T_1 = T_2 = T_3 = 0 \\
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\
&\alpha_{02} \neq 0 \\
&\alpha_{03} \neq 0 \\
&\alpha_{32} \neq 0 \\
&\alpha_{13} \neq 0 \\
&\alpha_{21} \neq 0
\end{aligned} \qquad (6.11)$$

$$
\begin{aligned}
&T_0 = T_1 = T_2 = T_3 = 0\\
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0\\
&\alpha_{02} = 0\\
&\alpha_{03} = 0\\
&\alpha_{32} \neq 0 \vee \alpha_{32} = 0\\
&\alpha_{13} = 0\\
&\alpha_{21} = 0
\end{aligned}
\tag{6.12}
$$

$$
\begin{aligned}
&T_0 = T_1 = T_2 = T_3 = 0\\
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0\\
&\alpha_{02} \neq 0 \vee \alpha_{02} = 0\\
&\alpha_{03} = 0\\
&\alpha_{32} \neq 0 \vee \alpha_{32} = 0\\
&\alpha_{13} = 0\\
&\alpha_{21} \neq 0 \vee \alpha_{21} = 0
\end{aligned}
\tag{6.13}
$$

$$
\begin{aligned}
&T_0 = T_1 = T_2 = T_3 = 0\\
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0\\
&\alpha_{02} = 0\\
&\alpha_{03} \neq 0 \vee \alpha_{03} = 0 \qquad .\\
&\alpha_{32} \neq 0 \vee \alpha_{32} = 0\\
&\alpha_{13} \neq 0 \vee \alpha_{13} = 0\\
&\alpha_{21} = 0
\end{aligned}
\tag{6.14}
$$

As we will see the number of external symmetries in four-dimensional spacetime is 59. From these $2+4+16+16=38$ cases, 9 are discarded and only 29 are external symmetries which belong to the set of 59 external symmetries. The symmetry that equations (6.11)-(6.14) express will be referred to as the symmetry of the Lorentz-Einstein-Selfvarlations. These symmetries hold only in case that the part of spacetime occupied by the generalized particle is flat.

7. THE FUNDAMENTAL STUDY FOR THE CORPUSCULAR STRUCTURE OF MATTER IN EXTERNAL SYMMETRY. THE Π-PLANE. THE $SV-T$ METHOD

7.1. Introduction

The material particles are in a constant interaction between them (via the USVI) because of STEM. This interaction has consequences in the internal structure of the generalized particle, including the distribution of its total energy and momentum between the material particle and the surrounding spacetime.

In the external symmetry, the internal structure of the generalized particle is determined by the relations among the elements of the matrix T. The same holds for the four-vector C, the rest mass m_0 of the material particle and the rest energy E_0 of STEM, with which the material particle interacts. In this chapter, we study this relation among the elements of the matrix T.

We present the proofs of six fundamental theorems which determine the structure of particles which accompany the USVI. In parallel with the theorem proofs we show the $SV-T$ method of the TSV (the Selfvariations Test). The $SV-T$ method enables us to check the validity of any mathematical equation of the TSV, or other theories, as well as the self-consistency of the TSV.

7.2. The fundamental study for the corpuscular structure of matter in external symmetry. The Π-plane. The $SV-T$ method

From equations (2.12) and (4.4), (4.10) we have

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & T_2 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & T_3 \end{bmatrix}. \qquad (7.1)$$

Equation (7.1) gives the external symmetry matrices as a function of the constants $\alpha_{ki}, k \neq i, k, i = 0,1,2,3$ and the physical quantities zQ and $T_k = \alpha_{kk}, k = 0,1,2,3$.

We start our study with the proof of the following theorem:

Theorem 7.1. ′′In the external symmetry and for the elements of the matrix T it holds that:

$$\begin{aligned} &T_i a_{\nu k}=0 \\ &i\neq\nu,\nu\neq k,k\neq i,i,\nu,k=0,1,2,3 \end{aligned} \;.'' \tag{7.2}$$

Proof. We differentiate the second equation of the set of equations (4.6)

$$\begin{aligned} &J_i\alpha_{\nu k}+J_k\alpha_{i\nu}+J_\nu\alpha_{ki}=0 \\ &i\neq\nu,\nu\neq k,k\neq i,i,\nu,k=0,1,2,3 \end{aligned}$$

with respect to x_j, $j=0,1,2,3$. Considering equations (2.10) and (4.4), we have

$$\alpha_{\nu k}\left(\frac{b}{\hbar}P_jJ_i+zQ\alpha_{ji}\right)+\alpha_{i\nu}\left(\frac{b}{\hbar}P_jJ_k+zQ\alpha_{jk}\right)+\alpha_{ki}\left(\frac{b}{\hbar}P_jJ_\nu+zQ\alpha_{j\nu}\right)=0$$

$$\frac{b}{\hbar}P_j\left(J_i\alpha_{\nu k}+J_k\alpha_{i\nu}+J_\nu\alpha_{ki}\right)+zQ\left(\alpha_{\nu k}\alpha_{ji}+\alpha_{i\nu}\alpha_{jk}+\alpha_{ki}\alpha_{j\nu}\right)=0$$

and with the second equation of the set of equations (4.6), and taking into account that $zQ\neq 0$, we obtain

$$\begin{aligned} &\alpha_{\nu k}\alpha_{ji}+\alpha_{i\nu}\alpha_{jk}+\alpha_{ki}\alpha_{j\nu}=0 \\ &i\neq\nu,\nu\neq k,k\neq i,i,\nu,k,j=0,1,2,3 \end{aligned} . \tag{7.3}$$

Inserting into equation (7.3) successively $(i,\nu,k)=(0,1,2),(0,1,3),(0,2,3),(1,2,3)$ and $j=0,1,2,3$, we arrive at the set of equations

$$\begin{aligned} &T_0\alpha_{32}=0 \\ &T_0\alpha_{13}=0 \\ &T_0\alpha_{21}=0 \\ &T_1\alpha_{02}=0 \\ &T_1\alpha_{03}=0 \\ &T_1\alpha_{32}=0 \\ &T_2\alpha_{01}=0 \\ &T_2\alpha_{03}=0 \\ &T_2\alpha_{13}=0 \\ &T_3\alpha_{01}=0 \\ &T_3\alpha_{02}=0 \\ &T_3\alpha_{21}=0 \end{aligned} . \tag{7.4}$$

The set of equations (7.4) is equivalent to equation (7.2).□

Theorem 7.1 is one of the most powerful tools for investigating the external symmetry. This results from corollary 7.1:

Corollary 7.1. ´´For the elements of the matrix T of the external symmetry the following hold:

1. For every $k \neq i, \nu \neq k, \nu \neq i, k, i, \nu \in \{0,1,2,3\}$ it holds that

$$\left.\begin{array}{l}\alpha_{ki} \neq 0 \\ k \neq i \\ \nu \neq k, i\end{array}\right\} \Rightarrow \mathrm{T}_{\nu} = 0 . \tag{7.5}$$

2.

$$\begin{array}{l}T_0 \neq 0 \Rightarrow \alpha_{32} = \alpha_{13} = \alpha_{21} = 0 \\ T_1 \neq 0 \Rightarrow \alpha_{02} = \alpha_{03} = \alpha_{32} = 0 \\ T_2 \neq 0 \Rightarrow \alpha_{01} = \alpha_{03} = \alpha_{13} = 0 \\ T_3 \neq 0 \Rightarrow \alpha_{01} = \alpha_{02} = \alpha_{21} = 0\end{array} .'' \tag{7.6}$$

Proof. Corollary 7.1 is an immediate consequence of theorem 7.1.□

We now prove theorem 7.3, which intercorrelates the elements T_0, T_1, T_2, T_3 of the matrix T :

Theorem 7.2. ´´ For the elements T_0, T_1, T_2, T_3 of the T matrix it holds that:

$$T_0 T_1 T_2 T_3 = 0 .'' \tag{7.7}$$

Proof. We develop equation (2.13), obtaining the set of equations

$$\begin{array}{l}J_0\lambda_{00} + J_1\lambda_{01} + J_2\lambda_{02} + J_3\lambda_{03} = 0 \\ -J_0\lambda_{01} + J_1\lambda_{11} - J_2\lambda_{21} + J_3\lambda_{13} = 0 \\ -J_0\lambda_{02} + J_1\lambda_{21} + J_2\lambda_{22} - J_3\lambda_{32} = 0 \\ -J_0\lambda_{03} - J_1\lambda_{13} + J_2\lambda_{32} + J_3\lambda_{33} = 0\end{array}$$

and from equations (4.4) and (4.10) we have

$$\begin{array}{l}J_0 zQT_0 + J_1 zQ\alpha_{01} + J_2 zQ\alpha_{02} + J_3 zQ\alpha_{03} = 0 \\ -J_0 zQ\alpha_{01} + J_1 zQT_1 - J_2 zQ\alpha_{21} + J_3 zQ\alpha_{13} = 0 \\ -J_0 zQa_{02} + J_1 zQa_{21} + J_2 zQ\mathrm{T}_2 - J_3 zQa_{32} = 0 \\ -J_0 zQ\alpha_{03} - J_1 zQ\alpha_{13} + J_2 zQ\alpha_{32} + J_3 zQT_3 = 0\end{array}$$

and since it holds that $zQ \neq 0$, we take the set of equations

$$\begin{array}{l} J_0T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \\ -J_0\alpha_{01} + J_1T_1 - J_2\alpha_{21} + J_3\alpha_{13} = 0 \\ -J_0\alpha_{02} + J_1\alpha_{21} + J_2T_2 - J_3\alpha_{32} = 0 \\ -J_0\alpha_{03} - J_1\alpha_{13} + J_2\alpha_{32} + J_3T_3 = 0 \end{array} . \tag{7.8}$$

The set of equations given in (7.8) comprise a 4×4 homogeneous linear system of equations with unknowns the momenta J_0, J_1, J_2, J_3. In order for the material particle to exist, the system of equations (7.8) must obtain non-vanishing solutions. Therefore, its determinant must vanish. Thus, we obtain equation

$$\begin{array}{l} T_0T_1T_2T_3 + T_0T_1\alpha_{32}^2 + T_0T_2\alpha_{13}^2 + T_0T_3\alpha_{21}^2 + T_1T_2\alpha_{03}^2 + T_1T_3\alpha_{02}^2 + T_2T_3\alpha_{01}^2 \\ +(\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21})^2 = 0 \end{array}$$

and with equation (4.8) we get

$$T_0T_1T_2T_3 + T_0T_1\alpha_{32}^2 + T_0T_2\alpha_{13}^2 + T_0T_3\alpha_{21}^2 + T_1T_2\alpha_{03}^2 + T_1T_3\alpha_{02}^2 + T_2T_3\alpha_{01}^2 = 0$$

$$T_0T_1T_2T_3 + T_0\alpha_{32}T_1\alpha_{32} + T_0\alpha_{13}T_2\alpha_{13} + T_0\alpha_{21}T_3\alpha_{21} + T_1\alpha_{03}T_2\alpha_{03} + T_1\alpha_{02}T_3\alpha_{02} + T_2\alpha_{01}T_3\alpha_{01} = 0$$

and with equations (7.4) we get

$T_0T_1T_2T_3 = 0$.□

From theorem 7.2 the following corollary follows, regarding the elements of the main diagonal of the matrices of the external symmetry:

Corollary 7.2. ′′At least one of the elements of the main diagonal of the matrix T is equal to zero.′′

Proof. Corollary 7.2 is an immediate consequence of theorem 7.2.□

We consider the 4×4 N matrix, given as:

$$N = \begin{bmatrix} 0 & \alpha_{32} & \alpha_{13} & \alpha_{21} \\ -\alpha_{32} & 0 & -\alpha_{03} & \alpha_{02} \\ -\alpha_{13} & \alpha_{03} & 0 & -\alpha_{01} \\ -\alpha_{21} & -\alpha_{02} & \alpha_{01} & 0 \end{bmatrix} . \tag{7.9}$$

Using the matrix N , we now write equation (4.6) in the form of

$$
\begin{aligned}
&NC = 0 \\
&NJ = 0\ . \\
&NP = 0
\end{aligned}
\tag{7.10}
$$

We now prove Lemma 7.1:

Lemma 7.1. ´´The four-vectors C, J, P satisfy the set of equations

$$
\begin{aligned}
&N^2C = 0 \\
&N^2J = 0\ .´´ \\
&N^2P = 0
\end{aligned}
\tag{7.11}
$$

Proof. We multiply the set of equations (7.10) from the left with the matrix N, and equations (7.11) follow. □

Using lemma 7.1 we prove theorem 7.2:

Theorem 7.3. ´´For $M \neq 0$ it holds that:

1. $MN = NM = 0$. (7.12)

2. $M^2 + N^2 = -\alpha^2 I$ (7.13)

$$\alpha^2 = \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 + \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2\ . \tag{7.14}$$

Here, I is the 4×4 identity matrix.

3. For $\alpha \neq 0$ the matrix M has two eigenvalues τ_1 and τ_2, with corresponding eigenvectors v_1 and v_2, given by:

$$
\begin{aligned}
&\tau_1 = i\alpha \\
&v_1 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} - \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \\ \alpha_{03}\alpha_{13} - \alpha_{02}\alpha_{21} \\ \alpha_{01}\alpha_{21} - \alpha_{03}\alpha_{32} \\ \alpha_{02}\alpha_{32} - \alpha_{01}\alpha_{13} \end{bmatrix}
\end{aligned}
\tag{7.15}
$$

$$\tau_2 = -i\alpha$$

$$v_2 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} + \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \\ \alpha_{03}\alpha_{13} - \alpha_{02}\alpha_{21} \\ \alpha_{01}\alpha_{21} - \alpha_{03}\alpha_{32} \\ \alpha_{02}\alpha_{32} - \alpha_{01}\alpha_{13} \end{bmatrix}. \tag{7.16}$$

4. For $\alpha \neq 0$ the matrix N has the same eigenvalues with the matrix M, and two corresponding eigenvectors n_1 and n_2, given by:

$$\tau_1 = i\alpha$$

$$n_1 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{bmatrix} - \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \\ \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{bmatrix} \tag{7.17}$$

$$\tau_2 = -i\alpha$$

$$n_2 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{bmatrix} + \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \\ \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{bmatrix}. \tag{7.18}$$

5. When $\alpha^2 \neq \alpha_{ki}^2, k \neq i, k, i \in \{0,1,2,3\}$ is

$$\alpha^2 = \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 + \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0 \tag{7.19}$$

$$\begin{aligned} M^2C &= 0 \\ M^2J &= 0 \\ M^2P &= 0 \end{aligned} \quad . \tag{7.20}$$

6. For $\alpha^2 = \alpha_{ki}^2, k \neq i, k, i \in \{0,1,2,3\}$ it can be

$\alpha^2 \neq 0$.''

Proof. The matrices M and N are given by equations (4.28) and (7.9). The proof of equations (7.12), (7.13), (7.14), (7.15), (7.11) and (7.16) can be performed by the appropriate mathematical calculations and the use of equation (4.8).

We multiply equation (7.13) from the right with the column matrices C, J, P , and obtain

$$M^2C + N^2C = -\alpha^2 C$$
$$M^2J + N^2J = -\alpha^2 J$$
$$M^2P + N^2P = -\alpha^2 P$$

and from equations (7.11) we obtain

$$\begin{aligned} M^2C &= -\alpha^2 C \\ M^2J &= -\alpha^2 J \quad . \\ M^2P &= -\alpha^2 P \end{aligned} \qquad (7.21)$$

According to the set of equations (7.21), and for $\alpha \neq 0$, $\alpha^2 \neq \alpha_{ki}^2, k \neq i, k, i \in \{0,1,2,3\}$, the matrix $M^2 \neq 0$ has as eigenvalue $\alpha^2 \neq 0$ with corresponding eigenvector $v \neq 0$. From equations (7.21) it is evident that the four-vectors C, J, P are parallel to the four-vector V, hence they are also parallel to each other. This is impossible in the case of the external symmetry, according to Theorem 3.3. Therefore, $\alpha^2 = 0$, so that the matrix $M^2 \neq 0$ does not have the four-vector V as an eigenvector. If the case it is $M^2 = 0$ from equations (7.21) we get

$$\alpha^2 C = 0$$
$$\alpha^2 J = 0$$
$$\alpha^2 P = 0$$

and because is $J \neq 0$ we again have $\alpha^2 = 0$. Thus, we arrive at equation (7.19). Then, from equations (7.21) we arrive at equations (7.20), since it holds that $\alpha^2 = 0$.

For $\alpha^2 = \alpha_{ki}^2, k \neq i, k, i \in \{0,1,2,3\}$ it could be $\alpha^2 \neq 0$ and the 4-vectors C, J, P are not parallel. The general proof is tedious and is omitted. We will only refer to the reason why for $\alpha^2 = \alpha_{ki}^2, k \neq i, k, i \in \{0,1,2,3\}$ it can be $\alpha^2 \neq 0$.

Matrix M^2 derives from the equation

$$M^2=\begin{bmatrix} -\alpha_{01}^2-\alpha_{02}^2-\alpha_{03}^2 & \alpha_{02}\alpha_{21}-\alpha_{03}\alpha_{13} & -\alpha_{01}\alpha_{21}+\alpha_{03}\alpha_{32} & \alpha_{01}\alpha_{13}-\alpha_{02}\alpha_{32} \\ \alpha_{02}\alpha_{21}-\alpha_{03}\alpha_{13} & -\alpha_{01}^2-\alpha_{21}^2-\alpha_{13}^2 & -\alpha_{01}\alpha_{02}+\alpha_{32}\alpha_{13} & -\alpha_{01}\alpha_{03}+\alpha_{32}\alpha_{21} \\ -\alpha_{01}\alpha_{21}+\alpha_{03}\alpha_{32} & -\alpha_{01}\alpha_{02}+\alpha_{32}\alpha_{13} & -\alpha_{02}^2-\alpha_{21}^2-\alpha_{32}^2 & -\alpha_{02}\alpha_{03}+\alpha_{13}\alpha_{21} \\ \alpha_{01}\alpha_{13}-\alpha_{02}\alpha_{32} & -\alpha_{01}\alpha_{03}+\alpha_{32}\alpha_{21} & -\alpha_{02}\alpha_{03}+\alpha_{13}\alpha_{21} & -\alpha_{03}^2-\alpha_{13}^2-\alpha_{32}^2 \end{bmatrix}.$$

In case $\alpha^2=\alpha_{ki}^2, k\neq i, k,i\in\{0,1,2,3\}$ (see chapter 12) all the non diagonal elements of matrix M^2 are equal to zero. A consequence of this is that the eigenvalue equation $M^2u=\nu u$ of M^2 becomes an identity, hence the equations (7.21) are valid also for $\alpha^2\neq 0$. On the contrary for $\alpha^2\neq\alpha_{ki}^2, k\neq i, k,i\in\{0,1,2,3\}$ at least one non diagonal element of the matrix M^2 is not zero. A consequence of this is that the eigenvalue equation $M^2u=\nu u$ of M^2 is not an identity and that equations (7.21) are valid only for $\alpha^2=0$, for the above mentioned reasons.□

From theorem 7.3 it follows:

Corollary 7.3. ´´For the four-vector j of the conserved physical quantities q it holds that:

1. $Mj=0$, if $\alpha^2\neq\alpha_{ki}^2, k\neq i, k,i\in\{0,1,2,3\}$. (7.22)

2. $Nj=0$.´´ (7.23)

Proof. We multiply equation (5.7) by matrix M from the left and obtain

$$Mj=-\frac{\sigma c^2 b}{\hbar}\Psi\left(\lambda M^2J+\mu M^2P\right)$$

and with the second and the third of equations (7.20) we have

$$Mj=0.$$

We multiply the terms of equation (5.7) from the left with the matrix N, and obtain

$$Nj=-\frac{\sigma c^2 b}{\hbar}\Psi NM\left(\lambda J+\mu P\right)$$

and with equation (7.12) we take

$Nj = 0$. □

In the equations of the TSV there appear sums of squares that vanish, like the ones appearing in equations (3.6) and (7.19). Writing these equations in a suitable manner, we can introduce into the equations of the TSV complex numbers. From equation (3.6) , and for $M_0 \neq 0$, we obtain

$$\left(\frac{c_0}{M_0 c}\right)^2 + \left(\frac{c_1}{M_0 c}\right)^2 + \left(\frac{c_2}{M_0 c}\right)^2 + \left(\frac{c_3}{M_0 c}\right)^2 + 1 = 0 .$$

Therefore, the physical quantities

$$\frac{c_0}{M_0 c}, \frac{c_1}{M_0 c}, \frac{c_2}{M_0 c}, \frac{c_3}{M_0 c}$$

belong in general to the set of complex numbers $\mathbb{C}$. This transformation of the equations of the TSV is not necessary. It suffices to remember that within the equations of the TSV there are sums of squares that vanish.

We consider now the three-dimensional vectors

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} \tag{7.24}$$

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} . \tag{7.25}$$

In the case of the T matrices with $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$, we define the vector $\boldsymbol{\mu} \neq \mathbf{0}$ from equation

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{pmatrix} . \tag{7.26}$$

Combining equations (5.1), (5.2) with equations (7.24) and (7.25) we obtain

$$\boldsymbol{\xi} = ic\Psi\mathbf{n} \tag{7.27}$$

$$\boldsymbol{\omega} = \Psi \boldsymbol{\tau} \ . \tag{7.28}$$

The field $\boldsymbol{\xi}$ is parallel to the vector $\mathbf{n}$ and the field $\boldsymbol{\omega}$ is parallel to the vector $\boldsymbol{\tau}$. Moreover the only variable quantity of the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$ is the function $\Psi = \Psi(x_0, x_1, x_2, x_3)$.

For every vector

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

which is determined by the physical quantities of the TSV, we define the physical quantity

$$\| \boldsymbol{\alpha} \| = \left(\boldsymbol{\alpha}^T \boldsymbol{\alpha} \right)^{\frac{1}{2}} = \left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 \right)^{\frac{1}{2}} . \tag{7.29}$$

Here, the matrix $\boldsymbol{\alpha}^T$ is the transposed matrix of the column matrix $\boldsymbol{\alpha}$.

From equations (7.24) and (7.25) we obtain

$$\boldsymbol{\tau} \cdot \mathbf{n} = \alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21} \ .$$

Also, from equation (4.8) we have

$$\boldsymbol{\tau} \cdot \mathbf{n} = 0 . \tag{7.30}$$

Therefore, the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$ are perpendicular to each other. Considering also equation (7.26), we see that the triple of the vectors $\{ \boldsymbol{\mu}, \mathbf{n}, \boldsymbol{\tau} \}$ forms a right-handed vector basis.

From equation (7.19) we have

$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = -\left(\alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \right)$$

and with equations (7.24), (7.25), and using the notation of equation (7.29), we obtain

$$\| \mathbf{n} \|^2 = - \| \boldsymbol{\tau} \|^2$$

and finally we obtain

$$\| \mathbf{n} \| = \pm i \, \| \boldsymbol{\tau} \| . \tag{7.31}$$

From equation (7.27) we have

$$\boldsymbol{\mu}^2 = (\mathbf{n} \times \boldsymbol{\tau})^2$$

and since the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$ are perpendicular to each other, we obtain from equation (7.30) that

$$\boldsymbol{\mu}^2 = \mathbf{n}^2 \boldsymbol{\tau}^2$$

and using the notation of equation (7.29) we have

$$\|\boldsymbol{\mu}\|^2 = \|\mathbf{n}\|^2 \|\boldsymbol{\tau}\|^2$$

$$\|\boldsymbol{\mu}\| = \pm \|\mathbf{n}\| \|\boldsymbol{\tau}\|$$

and from equation (7.31) we take

$$\|\boldsymbol{\mu}\| = \pm i \|\mathbf{n}\|^2 = \mp i \|\boldsymbol{\tau}\|^2 \ . \tag{7.32}$$

In the case of the T matrices, where $\|\mathbf{n}\| \neq \mathbf{0}$, and from equation (7.31), it follows that $\|\boldsymbol{\tau}\| \neq 0, \|\boldsymbol{\mu}\| \neq 0$. In these cases we can define the set of unit vectors { $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3$ }, given by

$$\begin{aligned} \boldsymbol{\varepsilon}_1 &= \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \\ \boldsymbol{\varepsilon}_2 &= \frac{\mathbf{n}}{\|\mathbf{n}\|} \\ \boldsymbol{\varepsilon}_3 &= \frac{\boldsymbol{\tau}}{\|\boldsymbol{\tau}\|} \\ \|\mathbf{n}\| &\neq 0 \end{aligned} \quad . \tag{7.33}$$

The triple of vectors { $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3$ } forms a right-handed orthonormal vector basis.

In the cases of the T matrices with $\boldsymbol{\tau} \neq \mathbf{0}$, we define with Π the plane perpendicular to the vector $\boldsymbol{\tau} \neq \mathbf{0}$. In the cases where moreover $\mathbf{n} \neq \mathbf{0}$, we obtain from equation (7.26) that $\boldsymbol{\mu} \neq \mathbf{0}$. In these cases the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$ are perpendicular to the vector $\boldsymbol{\tau}$, as implied by equations (7.26) and (7.30). Therefore, the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$ belong to the plane Π, and they also form an orthogonal basis of this plane. We note that the vectors of the TSV, which may belong to the plane Π, are given as a linear combination of the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$. Therefore, the condition for $\boldsymbol{\tau} \neq \mathbf{0}$ is not sufficient, in order for the plane Π to acquire a physical meaning. Also, we note that because of equation (7.19), the plane Π, when it is defined, *is not a vector subspace of* $\mathbb{R}^3$.

We now prove theorem 7.4:

Theorem 7.4. ′′In the case of the T matrices with $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$ and $\boldsymbol{\tau} \neq \pm\mathbf{n} \neq \mathbf{0}$, the vectors

$\mathbf{J}, \mathbf{P}, \mathbf{C}, \mathbf{j}, \nabla\Psi$ belong to the same plane Π.′′

Proof. From equations (4.6), for $(i, \nu, k) = (1, 3, 2)$, we obtain

$$
\begin{aligned}
&c_1\alpha_{32} + c_2\alpha_{13} + c_3\alpha_{21} = 0 \\
&J_1\alpha_{32} + J_2\alpha_{13} + J_3\alpha_{21} = 0 \\
&P_1\alpha_{32} + P_2\alpha_{13} + P_3\alpha_{21} = 0
\end{aligned}
$$

and from equations (5.8),(5.9) and (7.24) we get

$$
\begin{aligned}
&\boldsymbol{\tau} \cdot \mathbf{C} = 0 \\
&\boldsymbol{\tau} \cdot \mathbf{J} = 0 \\
&\boldsymbol{\tau} \cdot \mathbf{P} = 0
\end{aligned}
\tag{7.34}
$$

where

$$\mathbf{C} = \mathbf{J} + \mathbf{P} \tag{7.35}$$

as implied by equation (3.5). From equation (7.34) we conclude that the vectors $\mathbf{C}, \mathbf{J}, \mathbf{P}$, being perpendicular to vector $\boldsymbol{\tau}$, belong to the plane Π. From equation (5.3) and equations (5.8) and (5.9) we obtain

$$\nabla\Psi = \frac{b}{\hbar}\Psi\left(\lambda\mathbf{J} + \mu\mathbf{P}\right).$$

Therefore, the vector $\nabla\Psi$, as a linear combination of the vectors $\mathbf{J}, \mathbf{P}$, belongs to the plane Π. By developing the terms of equation (7.23), the first obtained equation is

$$\alpha_{32} j_1 + \alpha_{13} j_2 + \alpha_{21} j_3 = 0$$

and using equation (7.24) we have

$$\boldsymbol{\tau} \cdot \mathbf{j} = 0. \tag{7.36}$$

Therefore, the vector $\mathbf{j}$, being perpendicular to the vector $\boldsymbol{\tau}$, belongs to the plane Π. The vectors $\mathbf{J}, \mathbf{P}, \mathbf{C}, \mathbf{j}, \nabla\Psi$ vary according to the equations of the TSV, while staying on the plane Π. □

We now prove theorem 7.5:

Theorem 7.5.

1. ´´The 4-vector j of the current density of the conserved physical quantities q of the generalized particle is given by the equation

$$j = -\frac{\sigma b c^2}{\hbar}\Psi\left(\left(\mu - \lambda\right)\Lambda J + \mu M C\right) \tag{7.37}$$

where λ and μ the two degrees of freedom of the TSV and

$$\Lambda = \begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix}$$

the fundamental matrices Λ and M of the TSV.

2. $\Lambda J = 0 \Rightarrow j = -\frac{\sigma b c^2}{\hbar}\mu \Psi M C$.´´ (7.38)

Proof. From equation (3.5) we have

$$P = C - J$$

and replacing the momentum P in equation (5.7) we have

$$j = -\frac{\sigma c^2 b}{\hbar}\Psi M\left(\lambda J + \mu\left(C - J\right)\right)$$

$$j = -\frac{\sigma c^2 b}{\hbar}\Psi M\left(\left(\lambda - \mu\right)J + \mu C\right)$$

$$j = -\frac{\sigma c^2 b}{\hbar}\Psi\left(\left(\lambda - \mu\right)MJ + \mu M C\right). \tag{7.39}$$

From equations (7.1) and (4.11), (4.28) we have

$T = zQ\left(M + \Lambda\right)$

and from equation (2.13) we have

$zQ\left(M + \Lambda\right)J = 0$

and since $zQ \neq 0$ we have

$\left(M + \Lambda\right)J = 0$

$MJ + \Lambda J = 0$

and finally we get

$$MJ = -\Lambda J \tag{7.40}$$

From equations (7.39) and (7.40) we get equation (7.37). The equation (7.38) follows from the equation (7.37) for $\Lambda J = 0$.□

As we shall see next the equation $\Lambda J = 0$ is valid for a large number of external symmetry matrices. For these matrices Nr. 2. of theorem 7.5 is valid.

We now prove the following corollary of theorem 7.5:

Corollary 7.4. ´´In flat spacetime the 4-vector j of the current density of the conserved physical quantities q of the generalized particle is given by equation

$$j = -\frac{\sigma b c^2}{\hbar}\mu \Psi M C .´´$$

Proof. From corollary 6.1 and equation (4.11) it follows that in flat spacetime we have $\Lambda = 0$ and therefore Nr. 2. of theorem 7.5 is true.□

The next theorem 7.6 relates the four-vector J with the elements of the main diagonal of the external symmetry matrix T .

Theorem 7.6. ´´For every external symmetry matrix T it holds that

$$T_0 J_0^2 + T_1 J_1^2 + T_2 J_2^2 + T_3 J_3^2 = 0 .´´ \tag{7.41}$$

Proof. Since the material particle exists, at least one component of the four-vector J is nonzero. We prove the theorem for $J_0 \neq 0$. The proof for $J_i \neq 0, i = 1,2,3$ follows similar lines. For $J_0 \neq 0$, we obtain from equations (7.8)

$$
\begin{aligned}
&J_0T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0\\
&\alpha_{01} = \frac{1}{J_0}\left(J_1T_1 - J_2\alpha_{21} + J_3\alpha_{13}\right)\\
&\alpha_{02} = \frac{1}{J_0}\left(J_1\alpha_{21} + J_2T_2 - J_3\alpha_{32}\right)\\
&\alpha_{03} = \frac{1}{J_0}\left(-J_1\alpha_{13} + J_2\alpha_{32} + J_3T\right)
\end{aligned}
\tag{7.42}
$$

and replacing the terms $\alpha_{01}, a_{02}, a_{03}$ in the first of equations (7.42) we obtain

$$
\begin{aligned}
&J_0T_0 + \frac{J_1}{J_0}\left(J_1T_1 - J_2\alpha_{21} + J_3\alpha_{13}\right) + \frac{J_2}{J_0}\left(J_1\alpha_{21} + J_2T_2 - J_3\alpha_{32}\right)\\
&+\frac{J_3}{J_0}\left(-J_1\alpha_{13} + J_2\alpha_{32} + J_3T_3\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
&J_0^2T_0 + J_1^2T_1 - J_1J_2\alpha_{21} + J_1J_3\alpha_{13} + J_2J_1\alpha_{21} + J_2^2T_2\\
&-J_2J_3\alpha_{32} - J_3J_1\alpha_{13} + J_3J_2\alpha_{32} + J_3^2T_3 = 0
\end{aligned}
$$

$$
T_0J_0^2 + T_1J_1^2 + T_2J_2^2 + T_3J_3^2 = 0\ .\square
$$

The internal structure of every generalized particle depends on the corresponding matrix T. We show the $SV-T$ method of the TSV (the Selfvariations Test). The $SV-T$ method enables us to check the validity of any mathematical equation of the TSV, or other theories, as well as the self-consistency of the TSV. The $SV-T$ method consists of the following steps:

The $SV-T$ - method ;

From equations (2.10), (4.4) and (4.10) we obtain

$$
\begin{aligned}
&\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar}P_kJ_i + zQ\alpha_{ki}\,.\\
&k,i = 0,1,2,3
\end{aligned}
\tag{7.43}
$$

We choose an equation (E_1), which holds for the matrix T, and for which there exist at least two different components of the four-vector J, or one component and the rest mass m_0. By differentiating equation (E_1) with respect to $x_k, k = 0,1,2,3$ we obtain a second equation (E_2).

Due to equation (7.43)

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki}$$
$$k, i = 0,1,2,3$$

the constants $\alpha_{ki}, k \neq i, k, i = 0,1,2,3$ and the physical quantities $T_k = \alpha_{kk}, k = 0,1,2,3$ are introduced into equation (E_2). Equation (E_2) has to be compatible with the elements of the matrix T. In the case equation (E_1) contains the rest mass m_0 we apply equation (2.6)

$$\frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k m_0, k = 0,1,2,3.$$

We calculate the number of the external symmetry matrices. This number is determined by theorems 7.1, 7.2 and corollaries 7.1 and 7.2. Also notice that the external symmetry matrices are non-zero. Applying simple combinatorial rules, we see that altogether there exist

$$N_0 = 14$$

external symmetry matrices with $\alpha_{ki} = 0$ for every $k \neq i, k, i = 0,1,2,3$. These matrices contain non-zero elements only on the main diagonal. The number N_1 of matrices with one element $\alpha_{ki} \neq 0, k \neq i, k, i \in \{0,1,2,3\}$ is

$$N_1 = 6.$$

The number of matrices with two elements, $\alpha_{ki} \neq 0, k \neq i, k, i \in \{0,1,2,3\}$ is

$$N_2^{'} = 27$$

with three elements it is

$$N_3^{'} = 20$$

with four elements it is

$$N_4^{'} = 15$$

with five elements it is

$N_5 = 6$

with six elements it is

$N_6 = 1$.

From equation (2.13) and the second of the equations (4.6) we can prove that some of these matrices give the four-vector $J = 0$, thus are rejected. Therefore, we obtain

$$\begin{aligned} N_0 &= 14 \\ N_1 &= 6 \\ N_2 &= N_2^{'} - 4 = 24 \\ N_3 &= N_3^{'} - 8 = 12 \\ N_4 &= N_4^{'} - 12 = 3 \\ N_5 &= 6 \\ N_6 &= 1 \end{aligned}.$$

Thus the total number N_T of external symmetry matrices is

$$N_T = N_0 + N_1 + N_2 + N_3 + N_4 + N_5 + N_6 = 66. \qquad (7.44)$$

The matrix $T = 0$ is unique

$N_{\mathrm{O}} = 1$

and according to theorem 3.3 this matrix expresses the internal symmetry. Therefore, the total number of the matrices of the internal and external symmetry predicted by the Law of Selfvariations is

$$N_{\mathrm{O}T} = N_{\mathrm{O}} + N_T = 67. \qquad (7.45)$$

There exist

$$N_J = N_T - 16 = 50 \qquad (7.46)$$

external symmetry matrices with different four-vectors J, P, C, j.

We now prove for example that the following matrix

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & T_2 & 0 \\ -\alpha_{03} & -\alpha_{13} & 0 & T_3 \end{bmatrix}$$

is not an external symmetry matrix. Applying theorem 7.3 for the above matrix we have

$$T_0 = T_1 = T_2 = T_3 = 0$$

and therefore it takes the form

$$T = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & 0 & 0 \\ -\alpha_{03} & -\alpha_{13} & 0 & 0 \end{bmatrix}$$

and with equation (2.13) we obtain

$$\begin{aligned} &J_1\alpha_{01} + J_3\alpha_{03} = 0 \\ &-J_0\alpha_{01} - J_2\alpha_{21} + J_3\alpha_{13} = 0 \\ &J_1\alpha_{21} = 0 \\ &-J_0\alpha_{03} - J_1\alpha_{13} = 0 \end{aligned}$$

and since

$$\alpha_{01}\alpha_{03}\alpha_{13}\alpha_{21} \neq 0$$

we have

$$J_0 = J_1 = J_2 = J_3 = 0$$

which is impossible since there is no material particle in this case.

We present now a notation for the matrices of the external symmetry. In every matrix T we use an upper and a lower index. As lower indices we use the pairs $(k,i), k \neq i, k,i = 0,1,2,3$ of the constants $\alpha_{ki} \neq 0$, which are nonzero. These indices, which appear always in pairs, are placed in the order of the following constants: $\alpha_{01}, \alpha_{02}, \alpha_{03}, \alpha_{32}, \alpha_{13}, \alpha_{21}$, which are nonzero. As upper indices we use the indices of the nonzero elements of the main diagonal, in the following order: T_0, T_1, T_2, T_3. With this notation, the external symmetry matrices are given from the following seven sets Ω:

$$\Omega_0 = \left\{T^0, T^1, T^2, T^3, T^{01}, T^{02}, T^{03}, T^{12}, T^{13}, T^{23}, T^{012}, T^{013}, T^{023}, T^{123}\right\}$$

$$\Omega_1 = \left\{T_{01}^{01}, T_{02}^{02}, T_{03}^{03}, T_{32}^{23}, T_{13}^{13}, T_{21}^{12}\right\}$$

$$\Omega_2 = \left\{T^0_{0102}, T_{0102}, T^0_{0103}, T_{0103}, T^0_{0203}, T_{0203}, T^3_{3213}, T_{3213}, T^2_{3221}, T_{3221}, T^1_{1321}, T_{1321},\right.$$

$$\left.T^1_{0113}, T_{0113}, T^1_{0121}, T_{0121}, T^2_{0232}, T_{0232}, T^2_{0221}, T_{0221}, T^3_{0332}, T_{0332}, T^3_{0313}, T_{0313}\right\}$$

$$\Omega_3 = \left\{T^0_{010203}, T_{010203}, T^1_{011321}, T_{011321}, T^2_{023221}, T_{023221}, T^3_{033213}, T_{033213},\right.$$
$$\left.T_{010221}, T_{010313}, T_{020332}, T_{321321}\right\}$$

$$\Omega_4 = \left\{T_{01023213}, T_{01033221}, T_{02031321}\right\}$$

$$\Omega_5 = \left\{T_{0102033221}, T_{0102033213}, T_{0102031321}, T_{0102321321}, T_{0103321321}, T_{0203321321}\right\} \quad . \tag{7.47}$$

$$\Omega_6 = \left\{T_{010203321321}\right\}$$

We note that the matrix T_{321321} of the set Ω_3 is discarded by the $SV-T$ method. The application of the $SV-T$ method to set Ω_5 is of particular interest, as we will see in chapter 14.

Based on the theorems of the TSV we can study all external symmetry matrices. As an example we study the symmetry T^1_{0121} of set Ω_2 . From equation (7.1) for $\alpha_{01}\alpha_{21} \neq 0$ and $\alpha_{02} = \alpha_{03} = \alpha_{32} = \alpha_{13} = 0$ we get

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}. \tag{7.48}$$

From equations (7.4), and since $\alpha_{01} \neq 0$ and $\alpha_{21} \neq 0$, we have $T_0 = T_2 = T_3 = 0$, and the matrix (7.48) becomes

$$T^1_{0121} = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{7.49}$$

For $T_1 \neq 0$, according to corollary 6.2 the portion of spacetime occupied by the generalized particle is curved. In the case the portion of spactime occupied by the generalized particle is flat, we obtain from corollary 6.1 that $T_1 = 0$.

From equation (2.13) or, equivalently, from equations (7.8) we obtain

$$\begin{aligned} &J_1\alpha_{01} = 0 \\ &-J_0\alpha_{01} + J_1T_1 - J_2\alpha_{21} = 0 \\ &J_1\alpha_{21} = 0 \end{aligned}$$

and since $\alpha_{01}\alpha_{21} \neq 0$, we have that

$$\begin{aligned} &J_1 = 0 \\ &J_2 = -\frac{\alpha_{01}}{\alpha_{21}}J_0 \end{aligned}. \tag{7.50}$$

From the second of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get

$$\begin{aligned} &J_0a_{12} + J_2a_{01} + J_1a_{20} = 0 \\ &J_0a_{13} + J_3a_{01} + J_1a_{30} = 0 \\ &J_0a_{23} + J_3a_{02} + J_2a_{30} = 0 \\ &J_1a_{23} + J_3a_{12} + J_2a_{31} = 0 \end{aligned}$$

and taking into account that $\alpha_{ki} = -\alpha_{ik}, k \neq i, k, i = 0,1,2,3$, and the elements of matrix T^1_{0121} as given by equation (7.49) we get

$$\begin{aligned} &J_0\alpha_{12} + J_2\alpha_{01} = 0 \\ &J_3 = 0 \end{aligned}$$

$$\begin{aligned} &-J_0\alpha_{21} + J_2\alpha_{01} = 0 \\ &J_3 = 0 \end{aligned}$$

$$J_2 = \frac{\alpha_{21}}{\alpha_{01}} J_0$$
$$J_3 = 0 \tag{7.51}$$

From equations (7.50) and (7.51) we get

$$-\frac{\alpha_{01}}{\alpha_{21}} = \frac{\alpha_{21}}{\alpha_{01}}$$

$$\begin{aligned}&\alpha_{01}^2 + \alpha_{21}^2 = 0\\&\alpha_{21} = \pm i\alpha_{01}\end{aligned}. \tag{7.52}$$

From equations (7.50) and (7.51), and from equation (2.4), we get the four-vector J

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{21}}{\alpha_{01}} \\ 0 \end{bmatrix} = J_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix} = \frac{W}{c} \begin{bmatrix} i \\ 0 \\ \mp 1 \\ o \end{bmatrix}. \tag{7.53}$$
$$J_0 \neq 0$$

For the second equality in equation (7.53) we applied the second equation of equations (7.52).

We apply the $SV-T$ method for the matrix T_{0121}^{1}. From equation (7.53) we obtain

$$J_2 = \pm i J_0 . \tag{7.54}$$

This equation contains the components J_0, J_2 of the four-vector J. We differentiate equation (7.54) with respect to $x_k, k = 0,1,2,3$ and, taking into account equation (7.43) we get

$$\frac{b}{\hbar} P_k J_2 + zQ\alpha_{k2} = \pm i \left(\frac{b}{\hbar} P_k J_0 + zQ\alpha_{k0} \right)$$

and using equation (7.54) we have

$$zQ\alpha_{k2} = \pm izQ\alpha_{k0}$$

and since $zQ \neq 0$ we get

$$\alpha_{k2} = \pm i\alpha_{k0}, k = 0,1,2,3 . \tag{7.55}$$

In equation (7.55) we insert successively $k = 0,1,2,3$

For $k = 0$ we obtain

$$\alpha_{02} = \pm i\alpha_{00} = \pm iT_0$$

which holds, since $\alpha_{02} = 0, T_0 = 0$.

For $k = 1$ we get

$$\alpha_{12} = \pm i\alpha_{10}$$

and since $\alpha_{10} = -\alpha_{01}$, we get

$$\alpha_{12} = \pm i\alpha_{01}$$
$$\alpha_{01}^2 + \alpha_{21}^2 = 0$$

which are equations (7.52).

For $k = 2$ we obtain

$$a_{22} = \pm ia_{20}$$
$$T_2 = \pm ia_{20}$$

which holds for the matrix T_{0121}^1, since $a_{02} = 0, T_2 = 0$.

For $k = 3$ we have

$$\alpha_{32} = \pm i\alpha_{30}$$
$$\alpha_{32} = \mp i\alpha_{03}$$

which holds for the matrix T_{0121}^1, since $\alpha_{32} = 0, \alpha_{03} = 0$.

From equation (3.4) we have

$$\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), k, i = 0,1,2,3$$

and with equation (4.4) we have

$$zQ\alpha_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), k, i = 0,1,2,3 \tag{7.56}$$

For $k = 0, i = 1$ in equation (7.56) we have

$$zQa_{01} = \frac{b}{2\hbar}\left(c_1 J_0 - c_0 J_1\right)$$

and because of $J_1 = 0$ according to equation (7.50) we have

$$J_0 = \frac{2\hbar}{bc_1} zQ\alpha_{01}. \qquad (7.57)$$

Similarly for $k = 2, i = 1$ in equation (7.56) we have

$$J_2 = \frac{2\hbar}{bc_1} zQ\alpha_{21}. \qquad (7.58)$$

Considering that $J_1 = J_3 = 0$ according to equations (7.50), (7.51) from equations (7.57) and (7.58) we have

$$J = J\left(Q\right) = \frac{2\hbar}{bc_1} zQ \begin{bmatrix} \alpha_{01} \\ 0 \\ \alpha_{21} \\ 0 \end{bmatrix}. \qquad (7.59)$$

Equation (7.59) expresses the contribution of the charge Q to the 4-vector of momentum of the generalized particle.

From the first of the equations (4.6), for $\left(i, \nu, k\right) = \left(0,1,2\right), \left(0,1,3\right), \left(0,2,3\right), \left(1,2,3\right)$ we have

$$\begin{aligned} &c_0\alpha_{12} + c_2\alpha_{01} + c_1\alpha_{20} = 0 \\ &c_0\alpha_{13} + c_3\alpha_{01} + c_1\alpha_{30} = 0 \\ &c_0\alpha_{23} + c_3\alpha_{02} + c_2\alpha_{30} = 0 \\ &c_1\alpha_{23} + c_3\alpha_{12} + c_2\alpha_{31} = 0 \end{aligned}$$

and taking into account the elements of the matrix T^1_{0121} we have

$$\begin{aligned} &-c_0\alpha_{21} + c_2\alpha_{01} = 0 \\ &c_3\alpha_{01} = 0 \\ &c_3\alpha_{12} = 0 \end{aligned}$$

and since $\alpha_{01}\alpha_{21} \neq 0$ we obtain

$$\begin{aligned} &c_2 = \frac{\alpha_{21}}{\alpha_{01}} c_0 \\ &c_3 = 0 \end{aligned}. \qquad (7.60)$$

From equations (3.5) and (7.52), (7.60) we obtain the four-vector C

$$C=\begin{bmatrix} c_0 \\ c_1 \\ \dfrac{\alpha_{21}}{\alpha_{01}}c_0 \\ 0 \end{bmatrix}=\begin{bmatrix} c_0 \\ c_1 \\ \pm ic_0 \\ 0 \end{bmatrix}. \qquad (7.61)$$

From equation (2.10) for $k=0$ and taking into account equation

$$\lambda_{00}=zQT_0=0$$

we obtain

$$\frac{\partial J_0}{\partial x_0}=\frac{b}{\hbar}P_0J_0$$

and with equation (7.59) we obtain

$$\frac{\partial(zQ)}{\partial x_0}=\frac{b}{\hbar}P_0zQ$$

$$z\frac{\partial Q}{\partial x_0}+Q\frac{\partial z}{\partial x_0}=\frac{b}{\hbar}P_0zQ$$

and with equation (2.6) we obtain

$$z\frac{b}{\hbar}P_0Q+Q\frac{\partial z}{\partial x_0}=\frac{b}{\hbar}P_0zQ$$

$$\frac{\partial z}{\partial x_0}=0$$

and with equation (4.9) we obtain

$$c_0=0$$

and the equation (7.61) written in the form

$$C=\begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}. \qquad (7.62)$$

$$c_1\neq 0$$

For the four-vector $P(Q)$ we obtain

$$P(Q) = -\frac{2\hbar}{bc_1} zQ \begin{bmatrix} \alpha_{01} \\ 0 \\ \alpha_{21} \\ 0 \end{bmatrix}. \tag{7.63}$$

After having determined the four-vectors $J(Q), P(Q)$ and C, we can calculate the rest masses $m_{0,USVI}$, $\frac{E_{0,USVI}}{c^2}$ of the USVI particle, and the total rest mass M_0 of the generalized particle. From equations (2.7) and (7.54) we get

$$m_{0,USVI} = 0. \tag{7.64}$$

From equations (2.8) and (7.63) we have

$$E_{0,USVI} = 0. \tag{7.65}$$

From equations (7.62), (3.6) and (3.9), (3.10), (3.11) we also have

$$\begin{aligned} &c_1 = \pm iM_0 c \neq 0 \\ &c_0 = c_2 = c_3 = 0 \\ &\Phi = K \exp\left(-\frac{bc_1}{\hbar} x_1\right) \\ &m_0 = \pm \frac{M_0}{1+\Phi} \\ &E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi} \end{aligned} \tag{7.66}$$

for the generalized particle, for the duration of the interaction T_{0121}^1.

From equations (4.11) and (7.49) we get

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and with equation (7.53) we get

$$\Lambda J = J_0 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{21}}{\alpha_{01}} \\ 0 \end{bmatrix}$$

$$\Lambda J = 0$$

and with relation (7.38) we get

$$j = -\frac{\sigma b c^2}{\hbar} \mu \Psi M C \,. \tag{7.67}$$

From equations (4.28) and (7.49) we get

$$M = \begin{bmatrix} 0 & a_{01} & 0 & 0 \\ -a_{01} & 0 & -a_{21} & 0 \\ 0 & a_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7.68}$$

and with equations (7.62) and (7.67) we get

$$j = -\frac{\sigma b c^2}{\hbar} \mu \Psi \begin{bmatrix} 0 & a_{01} & 0 & 0 \\ -a_{01} & 0 & -a_{21} & 0 \\ 0 & a_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}$$

$$j = -\frac{\sigma c^2 b c_1}{\hbar} \mu \Psi \begin{bmatrix} \alpha_{01} \\ 0 \\ \alpha_{21} \\ 0 \end{bmatrix} . \tag{7.69}$$

From equations (5.17), (7.69), (7.52), and taking into account the elements of the matrix T^1_{0121} we have

$$
\begin{aligned}
&\frac{\partial j_0}{\partial x_1} = -\sigma c^2 F\alpha_{01} = \pm i\sigma c^2 F\alpha_{21}\\
&\frac{\partial j_0}{\partial x_2} = \pm i\frac{\partial j_0}{\partial x_0}\\
&\frac{\partial j_0}{\partial x_3} = 0\\
&F = \nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2}
\end{aligned}
\quad . \qquad (7.70)
$$

From equations (5.29) and (7.69) we get

$$
M\left[\frac{\partial\Psi}{\partial x}\right] = \frac{bc_1}{\hbar}\mu\Psi\begin{bmatrix}\alpha_{01}\\ 0\\ \alpha_{21}\\ 0\end{bmatrix}
$$

and with equations (7.68) and (5.28) we get

$$
\begin{bmatrix}0 & a_{01} & 0 & 0\\ -a_{01} & 0 & -a_{21} & 0\\ 0 & a_{21} & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dfrac{\partial\Psi}{\partial x_0}\\ \dfrac{\partial\Psi}{\partial x_1}\\ \dfrac{\partial\Psi}{\partial x_2}\\ \dfrac{\partial\Psi}{\partial x_3}\end{bmatrix} = \frac{bc_1}{\hbar}\mu\Psi\begin{bmatrix}\alpha_{01}\\ 0\\ \alpha_{21}\\ 0\end{bmatrix}
$$

$$
\begin{aligned}
&\alpha_{01}\frac{\partial\Psi}{\partial x_1} = \frac{bc_1}{\hbar}\alpha_{01}\mu\Psi\\
&-\alpha_{01}\frac{\partial\Psi}{\partial x_0} - \alpha_{21}\frac{\partial\Psi}{\partial x_2} = 0\\
&\alpha_{21}\frac{\partial\Psi}{\partial x_1} = \frac{bc_1}{\hbar}\alpha_{21}\mu\Psi
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial\Psi}{\partial x_1} = \frac{bc_1}{\hbar}\mu\Psi\\
&\frac{\partial\Psi}{\partial x_2} = -\frac{\alpha_{01}}{\alpha_{21}}\frac{\partial\Psi}{\partial x_0}
\end{aligned}
$$

and with equation (7.52) we get

$$\begin{aligned}&\frac{\partial \Psi}{\partial x_1}=\frac{bc_1}{\hbar}\mu\Psi\\&\frac{\partial \Psi}{\partial x_2}=-\frac{\alpha_{01}}{\alpha_{21}}\frac{\partial \Psi}{\partial x_0}=\mp i\frac{\partial \Psi}{\partial x_0}\end{aligned}\ . \qquad (7.71)$$

In equations (7.71) there is the degree of freedom μ of the TSV.

Method with which we studied the symmetry T_{0121}^{1} can be applied to any external symmetry. This method based on equations (7.4), (7.8), (4.6), on the $SV-T$ method, on equation (5.7), on corollary 5.1 and on corollary 5.3.

The internal symmetry expresses the spontaneous isotropic STEM emission from a material particle because of the selfvariations. The external symmetry emerges when the material particle interacts via the USVI with other material particles and this is equivalent with the destruction of the spacetime isotropy. The rest mass of the material particle and of the STEM in the first case, as well as the rest mass which stems from the USVI in the second case, is given by the equations (2.7) and (2.8). The equations (7.59) and (7.63)

$$\begin{aligned}&J(Q)=\frac{2\hbar}{bc_1}zQ\begin{bmatrix}\alpha_{01}\\0\\\alpha_{21}\\0\end{bmatrix}\\&P(Q)=-\frac{2\hbar}{bc_1}zQ\begin{bmatrix}\alpha_{01}\\0\\\alpha_{21}\\0\end{bmatrix}\end{aligned} \qquad (7.72)$$

give the 4-vectors $J(Q)$ and $P(Q)$ of USVI, and via equations (2.7) and (2.8) we get equations (7.64), (7.65) and (7.66), for the symmetry T_{0121}^{1} we have studied.

In equations (7.72) we see the term

$$J(Q)=\frac{2\hbar}{bc_1}zQ\begin{bmatrix}\alpha_{01}\\0\\\alpha_{21}\\0\end{bmatrix}$$

which is responsible for the external symmetry. That is the momentum of the USVI that is added to the momentum of the internal symmetry destroying the parallel property of the 4-

vectors J, P and C. This term is zero if and only if, it is $Q=0$, i.e. in the case where the material particle does not curry some charge Q of the interaction. For $Q=0$ and from equation (7.72) it follows that $J(Q)=0$ and $P(Q)=0$.

With the knowledge of the external symmetry term we can express the 4-vectors J and P of the material particle-STEM system (i.e. of the generalized particle) when the material particle is involved in an interaction . From equations (3.12), (3.13) and (7.62), (7.59), (7.63) we get the 4-vectors J and P as given by the equations

$$\begin{aligned} J &= \frac{1}{1+\Phi}\begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix} + \frac{2\hbar}{bc_1} zQ \begin{bmatrix} \alpha_{01} \\ 0 \\ \alpha_{21} \\ 0 \end{bmatrix} \\ P &= \frac{\Phi}{1+\Phi}\begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix} - \frac{2\hbar}{bc_1} zQ \begin{bmatrix} \alpha_{01} \\ 0 \\ \alpha_{21} \\ 0 \end{bmatrix} \end{aligned} \tag{7.73}$$

for the symmetry T^{1}_{0121}.

It is easy to find out via equation (3.4) that the equations (7.73) correctly give the physical quantities $\lambda_{ki}, k \neq i, k,i = 0,1,2,3$. This is expected since internal symmetry cannot affect the physical quantities $\lambda_{ki}, k \neq i, k,i = 0,1,2,3$. Thus we can calculate the constants $\alpha_{ki}, k \neq i, k,i = 0,1,2,3$ and the physical quantities $T_k, k = 0,1,2,3$ either through equations (7.72), (7.43)

$$\begin{aligned} &\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki} \\ &k,i = 0,1,2,3 \end{aligned}$$

or through equations (7.73), (3.4).

Every external symmetry has its own 4-vector C and its own term $J(Q)$. In every external symmetry there exist equations corresponding to equations (7.72) and to equations (7.73)

$$
\begin{aligned}
&P(Q) = -J(Q) \\
&J = \frac{1}{1+\Phi} C + J(Q) \\
&P = \frac{\Phi}{1+\Phi} C - J(Q)
\end{aligned}
\tag{7.74}
$$

for the USVI particle and for the generalized particle. Equation

$$P(Q) = -J(Q)$$

expresses the fact that the momentum added to the material particle is subtracted from STEM, and vice versa.

For $Q = 0$ the consequences of the interaction are negated , and from equations (7.74) we obtain

$$
\begin{aligned}
&Q = 0 \\
&P(Q) = -J(Q) = 0 \\
&C = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \\
&\Phi = K \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3 \right) \right) \\
&J = \frac{1}{1+\Phi} C = \frac{1}{1+\Phi} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \\
&P = \frac{\Phi}{1+\Phi} C = \frac{\Phi}{1+\Phi} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}
\end{aligned}
$$

that is we get equations (3.9), (3.12) and (3.13). Therefore when the material particle does not interact with other material particles, internal symmetry arises. Theorem 3.3 of the internal symmetry expresses the state to which the generalized particle tends spontaneously.

From equations (2.7) and (2.8) another important conclusion follows. In internal symmetry the material particle and STEM exchange roles if mutually exchanged

$$\left(J_0, J_1, J_2, J_3, m_0, \frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k m_0\right) \longleftrightarrow \left(P_0, P_1, P_2, P_3, \frac{E_0}{c^2}, \frac{\partial E_0}{\partial x_k} = \frac{b}{\hbar} J_k E_0\right), k = 0,1,2,3. \quad (7.75)$$

According to theorem 3.3 in internal symmetry the 4-vectors J and P are parallel which implies that they have the same form. Hence the mutual exchange (7.75) has no consequences in internal symmetry. If we assume that one of the 4-vectors J and P corresponds to the material particle, then the other corresponds to STEM. This fact can also be seen from equations (3.9)-(3.13) of the theorem 3.3, which can be written in an equivalent form.

$$\begin{aligned}
&\Phi^* = \frac{1}{\Phi} = \frac{1}{K}\exp\left(\frac{b}{\hbar}\left(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3\right)\right) = K^*\exp\left(\frac{b}{\hbar}\left(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3\right)\right)\\
&m_0 = \pm\frac{\Phi^* M_0}{1+\Phi^*}\\
&E_0 = \pm\frac{M_0c^2}{1+\Phi^*}\\
&J_i = \frac{\Phi^* c_i}{1+\Phi^*}, i = 0,1,2,3\\
&P_i = \frac{c_i}{1+\Phi^*}, i = 0,1,2,3
\end{aligned} \quad . (7.76)$$

The different appearance of the 4-vectors J and P, and the rest masses m_0 and $\frac{E_0}{c^2}$ in theorem 3.3 is superficial. Their form depends on whether we use equation Φ or equation Φ^* to write them.

In contrast to internal symmetry, in external symmetry we cannot interchange the physical content of the 4-vectors J and P. If we repeat the procedure for the determination of the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ starting from equation (2.8) instead of equation (2.7), we do not result in equations (3.4) (while internal symmetry is not affected due to the equivalence (3.14)). In external symmetry the mutual exchange (7.75) is not enough for the role exchange of the material particle and STEM. From equations (7.73) it follows that the role exchange of the material particle and STEM via equations (7.75), (7.76) can only be realized with the simultaneous change of sign of the charge Q ($Q \to -Q$).

Combining equations (2.10), (3.5) and (4.4), and with equation (2.13) we have

$$
\begin{aligned}
&\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki}, k, i = 0,1,2,3 \\
&J + P = C \\
&TJ = 0
\end{aligned} \tag{7.77}
$$

It is easy to find out that the TSV can be formulated starting from equations (7.77). The equations (7.77) give the Selfvariations of the 4-vectors J and P. They are more general than equation (2.6) since they give the TSV equations independent of whether the rest mass m_0 of the material particle is zero or not. We have chosen to start the formulation of the TSV from equation (2.6), which gives the equations of the TSV for $m_0 \neq 0$, for the reason that there is no other way to approach equations (7.77). Moreover their physical content would not be clear.

8. THE SET Ω_0

8.1. Introduction

In this chapter we study the T matrices, which have all their elements equal to zero, except the elements on the main diagonal. Thus we study matrices of the form

$$T = zQ\Lambda = zQ\begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}$$

i.e. the elements of the set $\Omega_0 = \left\{T^0, T^1, T^2, T^3, T^{01}, T^{02}, T^{03}, T^{12}, T^{13}, T^{23}, T^{012}, T^{013}, T^{023}, T^{123}\right\}$.

In the symmetries of the set Ω_0 the rest mass of the particle which accompanies the USVI may be non-zero. In all external symmetries this property can be found only in the sets Ω_0 and Ω_1.

8.2. The symmetries T=zQΛ

From equations (4.11) we obtain

$$T = zQ\Lambda = zQ\begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}. \tag{8.1}$$

From equations (4.28) ,(7.9) and (8.1) we have

$$\begin{aligned} M &= 0 \\ N &= 0 \end{aligned}. \tag{8.2}$$

The matrices M and N are zero; as a consequence the matrices of the symmetries $T = zQ\Lambda$ share common properties, which we shall study in the following.

According to corollary 7.2 , at least one of the diagonal elements of the matrices of equation (8.1) is zero. Also they cannot be all zero, since in the case of the external symmetry it holds that $T \neq 0$. Therefore, there is a number of

$$N_0 = \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 14$$

different matrices for which the relation $T = zQ\Lambda$ holds.

A common characteristic for the 14 kinds of symmetries $T = zQ\Lambda$ is that $\boldsymbol{\tau} = \mathbf{0}$, and therefore the plane Π is not defined. Similarly, the vectors $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3$ of equations (7.33) are not defined.

A fundamental characteristic of the symmetries $T = zQ\Lambda$ is that the four-vector j of the conserved physical quantites q vanishes. Combining the first of equations (8.2) with equation (5.7) we obtain

$$j = 0 . \tag{8.3}$$

Therefore, in the part of spacetime occupied by the generalized particle, there is no flow of conserved physical quantities q.

Another common characteristic is that the rest mass m_0 of the material particle can be diferent from zero

$$m_{0,USVI} = m_0 = 0 \vee m_{0,USVI} = m_0 \neq 0 \tag{8.4}$$

for all 14 matrices of the symmetry. The form of the four-vector J is different for each matrix of the symmetry.

We calculate now the four-vector of momentum J of the matrix T^{12}. According to our notation we have

$$T^{12} = zQ\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8.5}$$

From equation (2.13), and since $T_0 = T_3 = 0$ and $T_1T_2 \neq 0$, we obtain for the four-vector J, in the form

$$J = \begin{bmatrix} J_0 \\ 0 \\ 0 \\ J_3 \end{bmatrix}. \tag{8.6}$$

Combining equations (2.7) and (8.6), we obtain for the rest mass m_0 the equation

$$-m_0^2c^2 = J_0^2 + J_3^2 . \qquad (8.7)$$

We apply now the $SV-T$ method :

We differentiate equation (8.7) with respect to $x_k, k = 0,1,2,3$ and taking into account equations (2.6) and (7.43) we obtain

$$-\frac{b}{\hbar}P_k m_0^2 c^2 = J_0\left(\frac{b}{\hbar}P_k J_0 + zQ\alpha_{k0}\right) + J_3\left(\frac{b}{\hbar}P_k J_3 + zQ\alpha_{k3}\right)$$

and from equation (8.7) we have

$$zQJ_0\alpha_{k0} + zQJ_3\alpha_{k3} = 0$$

and since $zQ \neq 0$, we have

$$J_0\alpha_{k0} + J_3\alpha_{k3} = 0, k = 0,1,2,3 . \qquad (8.8)$$

We insert successively $k = 0,1,2,3$ into equation (8.8), hence:

For $k = 0$ we have

$$J_0T_0 + J_3\alpha_{03} = 0$$

which holds since for the matrix T^{12} it is $T_0 = \alpha_{03} = 0$.

For $k = 1$ we have

$$J_0\alpha_{10} + J_3\alpha_{13} = 0$$

which holds since for the matrix T^{12} it is $\alpha_{10} = \alpha_{13} = 0$.

For $k = 2$ we have

$$J_0\alpha_{20} + J_3\alpha_{23} = 0$$

which holds since for the matrix T^{12} it is $\alpha_{20} = \alpha_{32} = 0$.

For $k = 3$ we have

$$J_0\alpha_{30} + J_3T_3 = 0$$

which holds since for the matrix T^{12} it is $\alpha_{30} = T_3 = 0$.

According to the proof of equation (8.7) it is possible that $J_0 = 0$ or $J_3 = 0$, but it is not possible that $J_0 = J_3 = 0$, since in this case the material particle does not exist. Therefore from equation (8.7) we conclude that

$$m_0 \neq 0 \vee \left\{ m_0 = 0 \wedge J_3 = \pm i J_0 \right\}. \tag{8.9}$$

Simirarly we can prove that relations analogous to relation (8.10), hold for all matrices of the symmetry $T = zQ\Lambda$.

For the matrix T^{12} is $T_1 T_2 \neq 0$. Therefore the part of spacetime occupied by the generalized particle in the symmetry T^{12} is curved, according to corollary 6.2.

Because of equation (8.3) the wave equation (5.17) holds identically $(0 = 0)$. Therefore for the symmetries $T = zQ\Lambda$ the study of the wave behavior of matter is done via equation (5.3).

Starting from equation (8.7) and applying the same method of proof as for equations (4.19) and (4.20) we obtain

$$\frac{dJ}{dx_0} = \frac{dQ}{Q dx_0} J \tag{8.10}$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Q dx_0} J \tag{8.11}$$

for the symmetry T^{12}. From equations (8.6) and (8.10) we obtain

$$\begin{aligned} \frac{dJ_0}{dx_0} &= \frac{dQ}{Q dx_0} J_0 \\ \frac{dJ_3}{dx_0} &= \frac{dQ}{Q dx_0} J_3 \end{aligned}$$

and finally we obtain

$$\begin{aligned} & J_0 = \sigma_0 Q \\ & J_3 = \sigma_3 Q \\ & (\sigma_0, \sigma_3) \neq (0,0) \\ & \sigma_0, \sigma_3 = \text{constants} \end{aligned} \quad . \tag{8.12}$$

Thus the four-vector J is given by equation

$$J = J(Q) = Q\begin{bmatrix} \sigma_0 \\ 0 \\ 0 \\ \sigma_3 \end{bmatrix} \tag{8.13}$$

$$(\sigma_0, \sigma_3) \neq (0,0)$$
$$\sigma_0, \sigma_3 = \text{constants}$$

as implied by equation (8.6). Therefore, for the symmetry T^{12} the momentum of the material particle is proportional to the charge Q. This feature is a common characteristic for all matrices of the symmetry $T = zQ\Lambda$.

Combining equations (3.5) and (8.13) we have

$$P = P(Q) = \begin{bmatrix} c_0 - \sigma_0 Q \\ c_1 \\ c_2 \\ c_3 - \sigma_3 Q \end{bmatrix}. \tag{8.14}$$

$$(\sigma_0, \sigma_3) \neq (0,0)$$
$$\sigma_0, \sigma_3 = \text{constants}$$

Now from equations (4.2) and (8.14) we have

$$\begin{aligned} \frac{\partial Q}{\partial x_0} &= \frac{b}{\hbar}(c_0 - \sigma_0 Q)Q \\ \frac{\partial Q}{\partial x_1} &= \frac{b}{\hbar}c_1 Q \\ \frac{\partial Q}{\partial x_2} &= c_2 Q \\ \frac{\partial Q}{\partial x_3} &= \frac{b}{\hbar}(c_3 - \sigma_3 Q)Q \end{aligned} \quad . \tag{8.15}$$

From the identity

$$\frac{\partial}{\partial x_k}\left(\frac{\partial Q}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(\frac{\partial Q}{\partial x_k}\right), k \neq i, k, i = 0,1,2,3$$

and equations (8.15) we have after the calculations

$$\begin{aligned}&\sigma_0 c_1 = 0\\&\sigma_0 c_2 = 0\\&\sigma_3 c_1 = 0\\&\sigma_3 c_2 = 0\\&c_0\sigma_3 = c_3\sigma_0\end{aligned}$$

and because of

$$(\sigma_0, \sigma_3) \neq (0,0)$$

we finally get

$$\begin{aligned}&c_1 = c_2 = 0\\&c_0\sigma_3 = c_3\sigma_0\end{aligned}. \tag{8.16}$$

From equations (8.15) and (8.16) we have

$$\begin{aligned}&Q = Q(x_0, x_3)\\&\frac{\partial Q}{\partial x_0} = \frac{b}{\hbar}(c_0 - \sigma_0 Q)Q\\&\frac{\partial Q}{\partial x_3} = \frac{b}{\hbar}(c_3 - \sigma_3 Q)Q\end{aligned}. \tag{8.17}$$

From equation (8.17) we have

$$\begin{aligned}&Q = \frac{c_0}{\sigma_0}\frac{1}{1 - K_{12}\exp\left(-\frac{b}{\hbar}(c_0 x_0 + c_3 x_3)\right)}\\&c_0\sigma_3 = c_3\sigma_0\\&\sigma_0 \neq 0\end{aligned} \tag{8.18}$$

$$\begin{aligned}&Q = \frac{c_3}{\sigma_3}\frac{1}{1 - K_{12}\exp\left(-\frac{b}{\hbar}(c_0 x_0 + c_3 x_3)\right)}\\&c_0\sigma_3 = c_3\sigma_0\\&\sigma_3 \neq 0\end{aligned} \tag{8.19}$$

where $K_{12} \in \mathbb{C}, K_{12} \neq 0$ constant. For $\sigma_0\sigma_3 \neq 0$ the equations (8.18) and (8.19) are equivalent, because of the second equation of (8.16).

From equation (8.14) and the first equation of (8.16) we have

$$P = P(Q) = \begin{bmatrix} c_0 - \sigma_0 Q \\ 0 \\ 0 \\ c_3 - \sigma_3 Q \end{bmatrix}. \tag{8.20}$$

From equation (3.5) and the first equation of (8.16) we have

$$C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_3 \end{bmatrix}. \tag{8.21}$$

From equations (8.18), (8.19) and (8.13) we have

$$J = J(Q) = J(x_0, x_3, c_0, c_3) = \frac{1}{1 - K_{12} \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3\right)\right)} \begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_3 \end{bmatrix} \tag{8.22}$$

and from equations (8.21), (8.13) and (8.18), (8.19) we have

$$P = P(Q) = P(x_0, x_3, c_0, c_3) = \left(1 - \frac{1}{1 - K_{12} \exp\left(-\frac{b}{\hbar}\left(c_0 x_0 + c_3 x_3\right)\right)} \right) \begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_3 \end{bmatrix}. \tag{8.23}$$

From equations (8.21), (8.22) and (8.23) it follows that the 4-vectors J, P, C are parallel. According to the equivalence (3.4) and equation (4.4) this parallelism is expected for the symmetries $T = zQ\Lambda$, since it is $\alpha_{ki} = 0, \forall k \neq i, k, i = 0,1,2,3$. However the parallelism of the 4-vectors J, P, C we have met in the theorem 3.3 as a characteristic of internal symmetry. Hence we will finish the chapter for the symmetries $T = zQ\Lambda$ with the refutation of this apparent inconsistency.

From equation (8.13) we get $J_1 = J_2 = 0$ for the symmetry T^{12}, hence the initial equation (2.7) is written

$$J_0^2 + J_3^2 + m_0^2 c^2 = 0\,. \tag{8.24}$$

Subsequently we perform the same procedure as for the proof of equation (2.10), from equation (2.7). After the calculations and because in symmetry T^{12} it holds that $\alpha_{ki} = 0, \forall k \neq i, k, i = 0,1,2,3$ equation (8.5) follows from equation (8.24). During the procedure of proof, the physical quantities T_1 and T_2 do not follow from equation (8.24). In contrast from equation (2.7) for $J_1 \neq 0, J_2 \neq 0$ and $\alpha_{ki} = 0, \forall k \neq i, k, i = 0,1,2,3$ we get $T_1 = T_2 = 0$, as is predicted from the internal symmetry theorem 3.3. Exactly at this point we find the differences of the symmetries $T = zQ\Lambda$ with internal symmetry. In internal symmetry it is $T_0 = T_1 = T_2 = T_3 = 0$, and according to corollary 6.1 the part of spacetime occupied by the generalized particle may be a plane. Moreover space is isotropic, in the part of spacetime occupied by the generalized particle. The momentum vectors $\mathbf{J}$, $\mathbf{P}$ and $\mathbf{C}$ are 3-dimensional, and it is not possible to let vanish some component J_1, J_2, J_3 of the momentum from equation (2.7), with an appropriate rotation of the reference system we use. There is a very specific inertial reference frame in which $J_1 = J_2 = J_3 = 0$ ([5], chapter 5.3). In contrast with the symmetries $T = zQ\Lambda$ spacetime is curved as implied by the corollary 6.2. Moreover in symmetries $T = zQ\Lambda$ spacetime is *intensely anisotropic*, in the part of spacetime which is occupied by the generalized particle. According to equations (8.21), (8.22) and (8.23) the momentums $\mathbf{C}$, $\mathbf{J}$ and $\mathbf{P}$ in symmetry T^{12} are 1-dimensional, towards the direction of the axis $x_3 = z$. The intense anisotropy of space, in the part of spacetime which is occupied by the generalized particle, is a basic characteristic of the symmetry $T = zQ\Lambda$. This anisotropy varies for the symmetries of the set Ω_0 in equations (7.47). One symmetry $T = zQ\Lambda$ is characterized by the symmetries of the 4-vector J which are absend in the equation (2.7). For symmetry T^{12} the components are J_1 and J_2.

From equations (8.13) and (8.24) we have

$$\left(\sigma_0^2+\sigma_3^2\right)Q^2+m_0^2c^2=0$$
$$m_0=m_0\left(Q\right)=\pm\frac{i}{c}\left(\sigma_0^2+\sigma_3^2\right)^{\frac{1}{2}}Q\,. \qquad (8.25)$$
$$\left(\sigma_0,\sigma_3\right)\neq\left(0,0\right)$$

Equation (8.25) gives the contribution of charge Q to the rest mass m_0 of the generalized particle.

We now calculate the distribution of the total rest mass M_0 of the generalized particle between the material particle and STEM. From equations (8.22) and (8.24) we have

$$\frac{c_0^2+c_3^2}{\left(1-K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)\right)^2}+m_0^2c^2=0$$

and from equations (8.21) and (3.5) we have

$$-\frac{M_0^2c^2}{\left(1-K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)\right)^2}+m_0^2c^2=0$$

and finally we get

$$m_0=\pm\frac{M_0}{1-K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)} \qquad (8.26)$$

Analogous from equations (8.23), (2.8), and (3.5) we have

$$E_0=\pm\frac{M_0c^2K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)}{1-K_{12}\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_3x_3\right)\right)}\,. \qquad (8.27)$$

Equations (8.26) and (8.27) give the distribution of rest mass M_0 between the material particle and STEM. The study of the remaining 13 symmetries $T=zQ\Lambda$ is done in the same way as the one we demonstrated for symmetry T^{12}.

We now set $K_{12}=-K$ in equations (8.22) and (8.23), where K the constant of equation (3.9). Comparing equations (8.22), (8.23) and (3.9), (3.12), (3.13) we come to the conclusion that the external symmetry T^{12} can emerge from the internal symmetry for $J_1 = J_2 = 0$. This can occur when an external cause blocks the emmision of STEM along the axes x_1 and x_2. In this way the isotropic emmision of the internal symmetry is converted into the anisotropic external symmetry T^{12}. In general the following corollary of theorem 3.3 holds:

Corollary 8.1. ′′The external symmetry $T = zQ\Lambda$ can emerge from the internal symmetry when the components of the momentum J of the material particle are in less than four axes $x_i, i \in \{0,1,2,3\}$ in four-dimensional spacetime. These axes define the kind of external symmetry $T = zQ\Lambda$ that results.′′

We present the method which can produce the symmetry T^{12} from internal symmetry. We consider a case where an external cause can block the STEM emission in external symmetry, on the axes x_1 and x_2. In this case we have

$$P_1 = P_2 = 0 \,. \tag{8.28}$$

Now from equations (3.13), (3.12) and (8.28) we have

$$\begin{aligned} c_1 = c_2 = 0 \\ J_1 = J_2 = 0 \end{aligned} \,. \tag{8.29}$$

From the combination of equations (8.28), (8.29) with equations (3.12), (3.13), (3.10), (3.11) there arise the corresponding equations (8.22), (8.23), (8.26), (8.27) with $K_{12} = -K$.

Using function Φ of equation (3.9) for $c_1 = c_2 = 0$, and $K_{12} = -K$ equations (8.22), (8.23), (8.26) and (8.27) are written in the form

$$\begin{aligned} & J_i = \frac{c_i}{1+\Phi}, \mathrm{i} = 0,1,2,3 \\ & c_1 = c_2 = 0 \end{aligned} \tag{8.30}$$

$$\begin{aligned} & P_i = \frac{\Phi c_i}{1+\Phi}, \mathrm{i} = 0,1,2,3 \\ & c_1 = c_2 = 0 \end{aligned} \tag{8.31}$$

$$\begin{aligned} & m_0 = \pm \frac{M_0}{1+\Phi} \\ & c_1 = c_2 = 0 \end{aligned} \tag{8.32}$$

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi} \quad . \qquad (8.33)$$
$$c_1 = c_2 = 0$$

Corollary 8.1 gives us a mechanism through which the symmetry $T = zQ\Lambda$ can emerge. The external cause is necessary, since the internal symmetry expresses the spontaneous isotropic emmision of STEM due to the selfvariations.

From the combination of equations (3.5), (5.3) and (8.30), (8.31) we get

$$\frac{\partial \Psi}{\partial x_0} = \frac{b}{\hbar} \frac{(\lambda + \mu\Phi) c_0}{1+\Phi} \Psi$$
$$\nabla \Psi = \frac{b}{\hbar} \frac{\lambda + \mu\Phi}{1+\Phi} \Psi \mathbf{C} \quad . \qquad (8.34)$$

By setting

$$G = G(x_0, x_1, x_2, x_3) = \frac{\lambda + \mu\Phi}{1+\Phi} \qquad (8.35)$$

equation (8.34) is written in the form

$$\frac{\partial \Psi}{\partial x_0} = \frac{b}{\hbar} G c_0 \Psi$$
$$\nabla \Psi = \frac{b}{\hbar} G \Psi \mathbf{C} \quad . \qquad (8.36)$$

From identity

$$\nabla \times \nabla \Psi = 0$$

and with the second of equations (8.36), we get

$$\nabla G \times \mathbf{C} = 0$$

and consequently vector ∇G is written in the form

$$\nabla G = \frac{b}{\hbar} g \mathbf{C} \qquad (8.37)$$

where $g = g(x_0, x_1, x_2, x_3)$.

From equations (8.36) and (8.37) we get the wave equation of the TSV for the symmetry $T = Q\Lambda$, as given by equations

$$
\begin{aligned}
&\frac{\partial \Psi}{\partial x_0}=\frac{b}{\hbar}Gc_0\Psi \\
&\nabla^2\Psi=\frac{b^2\,\|\mathbf{C}\|^2}{\hbar^2}\left(G^2+g\right)\Psi\,. \\
&\nabla G=\frac{b}{\hbar}g\mathbf{C}
\end{aligned}
\tag{8.38}
$$

The third of the equations (8.38) correlates the functions G and g. One of the pairs of functions G and g is given by the equations

$$
\begin{aligned}
&G=\left(\frac{b\mathbf{r}\cdot\mathbf{C}}{\hbar}\right)^k \\
&g=k\left(\frac{b\mathbf{r}\cdot\mathbf{C}}{\hbar}\right)^{k-1}
\end{aligned}
\tag{8.39}
$$

where $\mathbf{r}=\left(x_1,x_2,x_3\right)$ and $k\in\mathbb{C}$ constant. From equations (8.38) and (8.39) we have

$$
\begin{aligned}
&\frac{\partial \Psi}{\partial x_0}=\frac{bc_0}{\hbar}\left(\frac{b\mathbf{r}\cdot\mathbf{C}}{\hbar}\right)^k\Psi \\
&\nabla^2\Psi=\frac{b^2\,\|\mathbf{C}\|^2}{\hbar^2}\left(\left(\frac{b\mathbf{r}\cdot\mathbf{C}}{\hbar}\right)^{2k}+k\left(\frac{b\mathbf{r}\cdot\mathbf{C}}{\hbar}\right)^{k-1}\right)\Psi\,. \\
&k\in\mathbb{C}
\end{aligned}
\tag{8.40}
$$

Equations (8.38) have general validity in the symmetry $T=zQ\Lambda$. Every symmetry $T=zQ\Lambda$ is defined by the constants $c_i, i=0,1,2,3$, which go to zero. The same holds for function $\Psi=\Psi\left(x_0,x_1,x_2,x_3\right)$. In the symmetry T^{12} it is $\Psi=\Psi\left(x_0,x_3\right)$. The symmetries of the set Ω_1 have $j=0$ and therefore the wave equation Ψ does not relate to any flow of conserved physical quantities q.

9. THE SET Ω_3

9.1. Introduction

In this chapter we study the generalized particle of the matrices

$$T^{0}_{010203} = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}$$

and

$$T_{010203} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}$$

of the set

$\Omega_3 = \left\{T^{0}_{010203}, T_{010203}, T^{1}_{011321}, T_{011321}, T^{2}_{023221}, T_{023221}, T^{3}_{033213}, T_{033213}, T_{010221}, T_{010313}, T_{020332}, T_{321321}\right\}$. The study of the remaining symmetries of the set Ω_3 is done in the same way with the study we present in this chapter. We note that the matrix T_{321321} of the set Ω_3 is discarded by the $SV-T$ method.

9.2. The symmetries T^{0}_{010203} and T_{010203}.

From equation (7.1) for $\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$ and $\alpha_{32} = \alpha_{13} = \alpha_{21} = 0$ we get

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & 0 & 0 \\ -\alpha_{02} & 0 & T_2 & 0 \\ -\alpha_{03} & 0 & 0 & T_3 \end{bmatrix}. \tag{9.1}$$

$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$

From theorem 7.1 we have that for this matrix it is

$T_1 = T_2 = T_3 = 0$

and thus it is written in the form

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}. \tag{9.2}$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

From the matrix in equation (9.2) we obtain the symmetries

$$T_{010203}^{0} = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \tag{9.3}$$

$$T_{010203} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}. \tag{9.4}$$

First we study the symmetry T_{010203}^{0}. From equation (2.13) or, equivalently, from equations (7.8) we obtain

$$\begin{aligned} &J_0 T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \\ &J_0\alpha_{01} = 0 \\ &J_0\alpha_{02} = 0 \\ &J_0\alpha_{03} = 0 \end{aligned}$$

and since $\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$ and $T_0 \neq 0$ we have

$$\begin{aligned} &J_0 = 0 \\ &J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \end{aligned}. \tag{9.5}$$

From the second of the equations (4.6), and for $(i,\nu,\kappa) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we obtain

$$\begin{aligned} &J_0\alpha_{12} + J_2\alpha_{01} + J_1\alpha_{20} = 0 \\ &J_0\alpha_{13} + J_3\alpha_{01} + J_1\alpha_{30} = 0 \\ &J_0\alpha_{23} + J_3\alpha_{02} + J_2\alpha_{30} = 0 \\ &J_1\alpha_{23} + J_3\alpha_{12} + J_2\alpha_{31} = 0 \end{aligned}$$

$$\begin{aligned} J_2\alpha_{01} - J_1\alpha_{02} &= 0 \\ J_3\alpha_{01} - J_1\alpha_{03} &= 0\ . \\ J_3\alpha_{02} - J_2\alpha_{03} &= 0 \end{aligned} \tag{9.6}$$

From equations (9.5), (9.6), and since it holds that $a_{01}a_{02}a_{03} \neq 0$, we have

$$\begin{aligned} J_0 &= 0 \\ J_2 &= \frac{\alpha_{02}}{\alpha_{01}} J_1\ . \\ J_3 &= \frac{\alpha_{03}}{\alpha_{01}} J_1 \end{aligned} \tag{9.7}$$

From equations (9.7) we obtain the four-vector $J = J\left(Q\right)$

$$J = J\left(Q\right) = J_1 \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} \\ \dfrac{\alpha_{03}}{\alpha_{01}} \end{bmatrix} \tag{9.8}$$

$$J_1 \neq 0$$

From equations (9.5) and (9.7) we get

$$J_1\alpha_{01} + J_1\frac{\alpha_{02}^2}{\alpha_{01}} + J_1\frac{\alpha_{03}^2}{\alpha_{01}} = 0$$

$$J_1\left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2\right) = 0$$

and since $J_1 \neq 0$ we get

$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = 0\,. \tag{9.9}$$

From the first of the equations (4.6), and for $\left(i,\nu,\kappa\right) = \left(0,1,2\right),\left(0,1,3\right),\left(0,2,3\right),\left(1,2,3\right)$ we obtain

$$\begin{aligned} c_0\alpha_{12} + c_2\alpha_{01} + c_1\alpha_{20} &= 0 \\ c_0\alpha_{13} + c_3\alpha_{01} + c_1\alpha_{30} &= 0 \\ c_0\alpha_{23} + c_3\alpha_{02} + c_2\alpha_{30} &= 0 \\ c_1\alpha_{23} + c_3\alpha_{12} + c_2\alpha_{31} &= 0 \end{aligned}$$

$$
\begin{aligned}
&c_2\alpha_{01} - c_1\alpha_{02} = 0 \\
&c_3\alpha_{01} - c_1\alpha_{03} = 0 \ . \\
&c_3\alpha_{02} - c_2\alpha_{03} = 0
\end{aligned}
\tag{9.10}
$$

From equations (9.10) and taking into account the elements of the matrix T^{0}_{010203} we have

$$
\begin{aligned}
c_2 &= \frac{\alpha_{02}}{\alpha_{01}} c_1 \\
c_3 &= \frac{\alpha_{03}}{\alpha_{01}} c_1
\end{aligned}
\ .
\tag{9.11}
$$

From equations (9.11) we obtain the four-vector C

$$
C = \begin{bmatrix} c_0 \\ c_1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} c_1 \\ \dfrac{\alpha_{03}}{\alpha_{02}} c_1 \end{bmatrix} .
\tag{9.12}
$$

From equivalence (3.4) we obtain

$$
\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), k \neq i, k, i = 0,1,2,3
$$

and with equation (4.4) we have

$$
zQ\alpha_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right)
$$

and for $k = 0, i = 0,1,2,3$ we obtain

$$
\begin{aligned}
zQ\alpha_{01} &= \frac{b}{2\hbar}\left(c_1 J_0 - c_0 J_1\right) \\
zQ\alpha_{02} &= \frac{b}{2\hbar}\left(c_2 J_0 - c_0 J_2\right) \\
zQ\alpha_{03} &= \frac{b}{2\hbar}\left(c_3 J_0 - c_0 J_3\right)
\end{aligned}
$$

and with equations (9.8) we have

$$zQ\alpha_{01} = -\frac{bc_0}{2\hbar} J_1$$
$$zQ\alpha_{02} = -\frac{bc_0}{2\hbar} J_2 \,. \tag{9.13}$$
$$zQ\alpha_{03} = -\frac{bc_0}{2\hbar} J_3$$

We apply the $SV-T$ method to equations (9.13). We differentiate the first

$$zQ\alpha_{01} = -\frac{bc_0}{2\hbar} J_1 \tag{9.14}$$

with respect to $x_\nu, \nu = 0,1,2,3$ and, taking into account equations (4.9), (4.2) and (7.43), we get

$$\alpha_{01} \frac{\partial(zQ)}{\partial x_\nu} = -\frac{bc_0}{2\hbar} \frac{\partial J_1}{\partial x_\nu}$$
$$\alpha_{01}\left(-\frac{bc_\nu}{2\hbar} + \frac{b}{\hbar} P_\nu\right) zQ = -\frac{bc_0}{2\hbar}\left(\frac{b}{\hbar} P_\nu J_1 + zQ\alpha_{\nu 1}\right)$$

and with equation (9.14) we get

$$-\frac{bc_\nu}{2\hbar} zQ\alpha_{01} = -\frac{bc_0}{2\hbar} zQ\alpha_{\nu 1}$$

and since $zQ \neq 0$ we get

$$c_\nu \alpha_{01} = c_0 \alpha_{\nu 1} \,. \tag{9.15}$$

Setting successively $\nu = 0,1,2,3$ into equation (9.15) we get:

For $\nu = 0$,

$$c_0 \alpha_{01} = c_0 \alpha_{01} \,.$$

For $\nu = 1$,

$$c_1 \alpha_{01} = c_0 \alpha_{11}$$
$$c_1 \alpha_{01} = c_0 T_1$$
$$c_1 \alpha_{01} = 0$$

and since $\alpha_{01} \neq 0$ we get

$$c_1 = 0. \tag{9.16}$$

For $\nu = 2$,

$$c_2\alpha_{01} = c_0\alpha_{21}$$
$$c_2\alpha_{01} = 0$$

$$c_2 = 0. \tag{9.17}$$

For $\nu = 3$,

$$c_3\alpha_{01} = c_0\alpha_{31}$$
$$c_3\alpha_{01} = 0$$

$$c_3 = 0. \tag{9.18}$$

Similarly, the application of the $SV-T$ method to the second and third of equations (9.13), again gives equations (9.16), (9.17) and (9.18).

From equations (9.12) and (9.16), (9.17), (9.18) we obtain

$$C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{9.19}$$

$$c_0 \neq 0$$

From equations (9.13) and (9.19) we obtain

$$J_1 = -\frac{2\hbar}{bc_0} zQ\alpha_{01}$$
$$J_2 = -\frac{2\hbar}{bc_0} zQ\alpha_{02}$$
$$J_3 = -\frac{2\hbar}{bc_0} zQ\alpha_{03}$$

and taking into account that $J_0 = 0$, we have

$$J = J\left(Q\right) = -\frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \mathbf{n} \end{bmatrix} = -\frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix}. \tag{9.20}$$

$c_0 \neq 0, c_1 = c_2 = c_3 = 0$

In equation (9.20) the function z is given by equation (4.5). Equation (9.20) expresses the dependence of the four-vector J on the charge Q in the case of the external symmetry T_{010203}^{0}.

For the four-vector $P\left(Q\right)$ we obtain

$$P = P\left(Q\right) = -J\left(Q\right) = \frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix}. \tag{9.21}$$

$c_0 \neq 0, c_1 = c_2 = c_3 = 0$

After having determined the four-vectors $J\left(Q\right), P\left(Q\right)$ and C, we can calculate the rest masses $m_{0,USVI}$, $\frac{E_{0,USVI}}{c^2}$ of the USVI particle, and the total rest mass M_0 of the generalized particle. From equations (2.7) and (9.8) we get

$$-m_{0,USVI}^2 c^2 = J_1^2\left[1 + \left(\frac{\alpha_{02}}{\alpha_{01}}\right)^2 + \left(\frac{\alpha_{03}}{\alpha_{01}}\right)^2\right]$$

$$-m_{0,USVI}^2 c^2 = \frac{J_1^2}{\alpha_{01}^2}\left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2\right)$$

and using equation (9.9) we obtain

$$m_{0,USVI} = 0. \tag{9.22}$$

From equations (2.8) and (9.21) we have

$$E_{0,USVI} = 0. \tag{9.23}$$

For the proof of equation (9.23) we used also equation (9.9). From equations (3.6), (9.19) and (3.9), (3.10), (3.11) we have

$$
\begin{aligned}
&c_0 = \pm iM_0 c \neq 0 \\
&c_1 = c_2 = c_3 = 0 \\
&C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&\Phi = K \exp\left(-\frac{bc_0}{\hbar} x_0 \right) \\
&m_0 = \pm \frac{M_0}{1+\Phi} \\
&E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi}
\end{aligned}
\tag{9.24}
$$

for the duration of the interaction T^0_{010203}.

From equations (4.11) and (9.3) we get

$$
\Lambda = \begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

and with equation (9.8) we get

$$
\Lambda J = J_1 \begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} \\ \dfrac{\alpha_{03}}{\alpha_{01}} \end{bmatrix}
$$

$$
\Lambda J = 0
$$

and with relation (7.38) we get

$$
j = -\frac{\sigma bc^2}{\hbar} \mu \Psi MC . \tag{9.25}
$$

From equations (4.28) and (9.3) we get

$$M = \begin{bmatrix} 0 & a_{01} & \alpha_{02} & \alpha_{03} \\ -a_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \tag{9.26}$$

and with equations (9.25) and (9.19) we get

$$j = -\frac{\sigma b c^2}{\hbar}\mu\Psi \begin{bmatrix} 0 & a_{01} & \alpha_{02} & \alpha_{03} \\ -a_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$j = \frac{\sigma b c^2 c_0}{\hbar}\mu\Psi \begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix}. \tag{9.27}$$

Combining equations (5.3) and (3.5) we get

$$\frac{\partial \Psi}{\partial x_k} = \frac{b}{\hbar}\Psi\left(\left(\lambda-\mu\right)J_k + \mu c_k\right)$$
$$k = 0,1,2,3$$

and with equations (9.8) and (7.25) we get

$$\frac{\partial \Psi}{\partial x_0} = \frac{b c_0 \mu}{\hbar}\Psi$$
$$\nabla\Psi = \frac{b}{\hbar}\left(\frac{\lambda-\mu}{\alpha_{01}}J_1 + \frac{\mu c_0}{\alpha_{02}}\right)\Psi\mathbf{n}\ . \tag{9.28}$$
$$\lambda,\mu \in \mathbb{C}, \left(\lambda,\mu\right) \neq \left(0,0\right)$$

Let us remind that the parameters λ,μ appearing in equation (9.28) express the two degrees of freedom of the TSV.

Setting

$$G = G\left(x_0, x_1, x_2, x_3\right) = \frac{\lambda-\mu}{\alpha_{01}}J_1 + \frac{\mu c_0}{\alpha_{02}} \tag{9.29}$$

equation (9.28) is written in the form

$$\begin{aligned}\frac{\partial\Psi}{\partial x_0} &= \frac{bc_0\mu}{\hbar}\Psi \\ \nabla\Psi &= \frac{b}{\hbar}G\Psi\mathbf{n}\end{aligned}\quad . \tag{9.30}$$

From identity

$$\nabla\times\nabla\Psi = 0$$

and the second of equations (9.30) we get

$$\nabla G\times\mathbf{n} = 0$$

and, therefore, the vector ∇G is written as

$$\nabla G = \frac{b}{\hbar}g\mathbf{n} \tag{9.31}$$

where $g = g\left(x_0, x_1, x_2, x_3\right)$.

From the second of equations (9.30) we get

$$\nabla^2\Psi = \frac{b}{\hbar}\left(G\nabla\Psi + \Psi\nabla G\right)\cdot\mathbf{n}$$

and with the second of equations (9.30), and equation (9.31) we get

$$\nabla^2\Psi = \frac{b^2}{\hbar^2}\left(G^2\Psi\mathbf{n} + \Psi g\mathbf{n}\right)\cdot\mathbf{n}$$

$$\nabla^2\Psi = \frac{b^2}{\hbar^2}\left(G^2 + g\right)\Psi\mathbf{n}^2$$

and with equation (7.25) we get

$$\nabla^2\Psi = \frac{b^2}{\hbar^2}\left(G^2 + g\right)\left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2\right)\Psi$$

and with equation (9.9) we finally get

$$\nabla^2\Psi = 0. \tag{9.32}$$

In the symmetry T_{010203}^{0} the wave function Ψ of the TSV satisfies the Laplace equation. The first of equations (9.30) and equation (9.32)

$$\begin{aligned}\frac{\partial\Psi}{\partial x_0} &= \frac{bc_0\mu}{\hbar}\Psi \\ \nabla^2\Psi &= 0\end{aligned} \tag{9.33}$$

constitute the wave equation of the TSV for the symmetry T_{010203}^{0}.

The portion of space-time occupied by the generalized particle is curved, since $T_0 \neq 0$, according to corollary 6.2. Also from the combination of equations (9.8) and (4.19), (4.20) and taking into account that $T_1 = T_2 = T_3 = 0$ and $J_0 = 0$, $P_0 = c_0 - J_0 = c_0$, we obtain

$$
\begin{aligned}
&\frac{d\mathbf{J}}{dx_0} = \frac{dQ}{Qdx_0}\mathbf{J} - \frac{i}{c}Q\boldsymbol{\alpha} = \frac{dQ}{Qdx_0}\mathbf{J} + zQ\mathbf{n} \\
&J_0 = 0 \\
&\frac{d\mathbf{P}}{dx_0} = -\frac{dQ}{Qdx_0}\mathbf{J} + \frac{i}{c}Q\boldsymbol{\alpha} = -\frac{dQ}{Qdx_0}\mathbf{J} - zQ\mathbf{n} \\
&P_0 = c_0
\end{aligned}
\tag{9.34}
$$

for the *USVI* of the external symmetry T^0_{010203}.

In symmetry T_{010203} it holds that $T_0 = T_1 = T_2 = T_3 = 0$. Hence from theorem 7.6 we could have that $J_0 \neq 0$, and the four-vector J could take the form

$$
\begin{aligned}
&J = \begin{bmatrix} J_0 \\ J_1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} J_1 \\ \dfrac{\alpha_{03}}{\alpha_{01}} J_1 \end{bmatrix} \\
&J_0 \neq 0
\end{aligned}
\tag{9.35}
$$

in the symmetry T_{010203}. However equation (9.35) is rejected. Following the same procedure as the one for proving equation (9.22), we obtain from equation (9.35) that

$$m_0^2 c^2 = -J_0^2 \neq 0. \tag{9.36}$$

Applying the $SV - T$ method, we conclude that equation (9.36) cannot hold. Therefore, the symmetries T^0_{010203} and T_{010203} have the same four-vectors J, P, C and j. The only difference lies in the vanishing or non-vanishing of the physical quantity T_0.

The symmetry T^0_{010203} has $T_0 \neq 0$ and therefore the spacetime part occupied by the generalized particle is curved according to corollary 6.2. In symmetry T_{010203} it is $T_0 = 0$ and following from corollary 6.1 the spacetime could be either flat or curved. We shall prove that in symmetry T_{010203} spacetime is curved.

From equation (3.5) it follows that the components of the 4-vector C transform under Lorentz-Einstein as well as the components of the 4-vectors J and P. That is they transform under Lorentz-Einstein according to equations (6.4). The first parts of equations (9.23) and (9.24) are invariant under Lorentz-Einstein while the second parts are not. Thus the Lorentz-Einstein transformations are not valid for the symmetry T_{010203}, therefore spacetime is curved. With the observation that spacetime is curved we conclude the study of the symmetries T^{0}_{010203} and T_{010203}.

10. THE GENERALIZED PARTICLE OF THE FIELD $(\boldsymbol{\alpha},\boldsymbol{\beta})$ AND THE CONFINEMENT EQUATION

10.1. Introduction

In this chapter we study the generalized particle of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$, for which the function Ψ is known ($\Psi = z = \exp\left(-\frac{b}{2\hbar}\left(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3\right)\right)$). This shall allow us to perform a particular application of theorem 5.1. The generalized photon in the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ exists if and only if $M_0 = 0$. That is in the case the total rest mass of the generalized particle is zero. For $M_0 \neq 0$ the generalized particle appears.

We present a short study for the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of the symmetry T^2_{0221}. This study focuses on the 4-vector j of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$. That is we study the 4-vector j in case where $(\boldsymbol{\xi},\boldsymbol{\omega}) = (\boldsymbol{\alpha},\boldsymbol{\beta})$.

The energy content of the generalized particle is quantized when it is confined to a finite portion of spacetime. We prove the condition of the confinement for the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$. Following we prove the general condition of the confinement for the field $(\boldsymbol{\xi},\boldsymbol{\omega})$.

10.2. The generalized particle of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ and the confinement equation

Initially we prove that the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is a special case of the field $(\boldsymbol{\xi},\boldsymbol{\omega})$. For $\lambda = \mu = -\frac{1}{2}$ in equation (5.3) we obtain

$$\frac{\partial \Psi_k}{\partial x_k} = -\frac{b}{2\hbar}\left(J_k + P_k\right)\Psi, k = 0,1,2,3$$

and with equation (3.5) we have

$$\frac{\partial \Psi_k}{\partial x_k} = -\frac{bc_k}{2\hbar}\Psi, k = 0,1,2,3$$

and using the notation of equation (4.9) we have

$$\Psi = z = \exp\left(-\frac{b}{2\hbar}\left(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3\right)\right). \tag{10.1}$$

From equation (10.1) and equations (5.1), (5.2) and (4.14), (4.15), we obtain

$$\begin{aligned} \boldsymbol{\xi} = \boldsymbol{\alpha} = icz\begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} = icz\mathbf{n} \\ \boldsymbol{\omega} = \boldsymbol{\beta} = z\begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} = z\boldsymbol{\tau} \end{aligned} . \tag{10.2}$$

The field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is a special case of the field $(\boldsymbol{\xi},\boldsymbol{\omega})$ for $\lambda = \mu = -\frac{1}{2}$.

The fact that the function Ψ of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is known allows us to derive two impotant results about the total rest mass M_0 of the generalized partcle. The first concerns the relation between the total rest mass M_0 of the generalized particle with theorem 5.2, in case of field $(\boldsymbol{\alpha},\boldsymbol{\beta})$. From equation (10.1) we obtain

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = \frac{b^2}{4\hbar^2}\left(c_0^2 + c_1^2 + c_1^2 + c_1^2\right)$$

and with equation (3.6) we have

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = -\frac{b^2}{4\hbar^2}M_0^2c^2\Psi \ . \tag{10.3}$$

According to equation (10.3) and theorem 5.2 the generalized photon in the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ exists, if and only if

$$M_0 = 0, \tag{10.4}$$

that is in the case the total rest mass of the generalized particle is zero. For $M_0 \neq 0$ the generalized particle appears.

Setting $\lambda = \mu = -\frac{1}{2}$ in the equations of chapter 5, we arrive at the equations of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. For example, by setting $\lambda = \mu = -\frac{1}{2}$ into equation (5.7) we obtain

$$j = \frac{\sigma c^2 bz}{2\hbar} MC .$$

This is equation (4.29), as we have proved in chapter 4, for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. On the other hand, equation (10.3) results only because the function Ψ is known, as given by equation (10.1) for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$.

The field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ exists in all external symmetries, except the ones with $T = zQ\Lambda$ of the set Ω_0 of equations (7.47). Every external symmetry of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$ expresses one of the possible states of the generalized particle. For the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ every external symmetry expresses one of the possible states of the field.

We are now presenting a short study for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of the symmetry T^2_{0221} of the set Ω_2. This study focuses on the 4-vector j of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. We consider the case where Q is electric charge, hence $\boldsymbol{\alpha}$ is the electric field and $\boldsymbol{\beta}$ the magnetic.

The symmetry T^2_{0221} is given by the matrix

$$T = zQ \begin{bmatrix} T_0 & 0 & \alpha_{02} & 0 \\ 0 & T_1 & -\alpha_{21} & 0 \\ -\alpha_{02} & \alpha_{21} & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}$$

$$\alpha_{02}\alpha_{21} \neq 0$$

according to equation (7.1), and considering equation (7.4) we have

$$T = T^2_{0221} = zQ \begin{bmatrix} 0 & 0 & \alpha_{02} & 0 \\ 0 & 0 & -\alpha_{21} & 0 \\ -\alpha_{02} & \alpha_{21} & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\alpha_{02}\alpha_{21} \neq 0$$

$$T^2_{0221} = zQ\begin{bmatrix} 0 & 0 & \alpha_{02} & 0 \\ 0 & 0 & -\alpha_{21} & 0 \\ -\alpha_{02} & \alpha_{21} & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad T^2_{0221} = zQ\begin{bmatrix} 0 & 0 & \alpha_{02} & 0 \\ 0 & 0 & -\alpha_{21} & 0 \\ -\alpha_{02} & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10.5}$$

From equations (10.5), (4.28), (7.9) we get the matrices M and N

$$\left.\begin{aligned} M &= \begin{bmatrix} 0 & 0 & \alpha_{02} & 0 \\ 0 & 0 & -\alpha_{21} & 0 \\ -\alpha_{02} & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ N &= \begin{bmatrix} 0 & 0 & 0 & \alpha_{21} \\ 0 & 0 & 0 & \alpha_{02} \\ 0 & 0 & 0 & 0 \\ -\alpha_{21} & -\alpha_{02} & 0 & 0 \end{bmatrix} \end{aligned}\right\}. \tag{10.6}$$

From equations (4.14), (4.15) and (10.5) we have

$$\boldsymbol{\alpha} = icz\begin{pmatrix} 0 \\ \alpha_{02} \\ 0 \end{pmatrix} \tag{10.7}$$

$$\boldsymbol{\beta} = z\begin{pmatrix} 0 \\ 0 \\ \alpha_{21} \end{pmatrix}. \tag{10.8}$$

The electric field is on axis $x_2 = y$ and the magnetic on axis $x_3 = z$. The function z in equations (10.7) and (10.8) expresses the wave form of the field for

$$b = i \tag{10.9}$$

according to equation (10.1).

Applying the method which we have used for the study of the symmetry T^1_{0121} in chapter 7, we get the equations

$$\alpha_{02}^2 + \alpha_{21}^2 = 0$$

$$\alpha_{21} = \pm i \alpha_{02}$$

$$J = J\left(Q\right) = J_0 \begin{bmatrix} 1 \\ \pm i \\ 0 \\ 0 \end{bmatrix} \tag{10.10}$$

$$C = \begin{bmatrix} 0 \\ 0 \\ c_2 \\ 0 \end{bmatrix}$$

$$P = P\left(Q\right) = -J\left(Q\right)$$

for the symmetry T_{0221}^2.

The material particle exists for $J \neq 0$, hence from the third of equations (10.10) we have

$$J_0 \neq 0. \tag{10.11}$$

From the fourth of equations (10.10) it follows that for $c_2 = 0$ is $C = 0$, which is impossible. Therefore it is

$$c_2 \neq 0. \tag{10.12}$$

Considering the second of equations (10.10) the equation (10.7) can be written in the form

$$\boldsymbol{\alpha} = \pm cz \begin{pmatrix} 0 \\ \alpha_{21} \\ 0 \end{pmatrix} \quad . \tag{10.13}$$

$$c_2 \neq 0, c_0 = c_1 = c_3 = 0$$

From equations (10.8) and (10.13) it follows that the electric and the magnetic field have equal norm:

$$\| \boldsymbol{\alpha} \| = \| \boldsymbol{\beta} \|. \tag{10.14}$$

From aquations (7.22), (7.23), (10.6), and taking into account the second of equations (10.10) we obtain

$$j = j_0 \begin{bmatrix} 1 \\ \mp i \\ 0 \\ 0 \end{bmatrix} = \rho c \begin{bmatrix} i \\ \pm 1 \\ 0 \\ 0 \end{bmatrix}. \tag{10.15}$$

The density ρ for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by the first of the equations (4.25)

$$\rho = -\sigma \frac{icbz}{2\hbar}\left(c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03}\right). \tag{10.16}$$

From equations (10.16), (10.5) and (10.9) we have

$$\rho = \frac{\sigma c c_2 \alpha_{02} z}{2\hbar}$$

and replacing density ρ in equation (10.15) we have

$$j = \frac{\sigma c^2 c_2 \alpha_{02}}{2\hbar} z \begin{bmatrix} i \\ \pm 1 \\ 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{10.17}$$

$c_2 \neq 0, c_0 = c_1 = c_3 = 0$

According to the third of equations (10.10) the vector

$$\mathbf{J} = \mathbf{J}(Q) = J_0 \begin{pmatrix} \pm i \\ 0 \\ 0 \end{pmatrix} = \frac{W}{c} \begin{pmatrix} \mp 1 \\ 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{10.18}$$

of the momentum of the particle is on the axis $x_1 = x$. Similarly from equation (10.17) it follows that the vector

$$\mathbf{j} = \frac{\sigma c^2 c_2 \alpha_{02}}{2\hbar} z \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{10.19}$$

$c_2 \neq 0, c_0 = c_1 = c_3 = 0$

is also on the axis $x_1 = x$.

Let $\sigma = \varepsilon_0$, where ε_0 is the dielectric constant of the vacuum, then in equation (10.17) we have

$$j = \frac{\varepsilon_0 c^2 c_2 \alpha_{02}}{2\hbar} z \begin{bmatrix} i \\ \pm 1 \\ 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (10.20)$$

$$z = \exp\left(-\frac{bc_2}{2\hbar} x_2\right)$$

Inserting $\sigma = \frac{\hbar\varepsilon_0}{e}$ in equation (10.17) we get

$$j = \frac{\varepsilon_0 c^2 c_2 \alpha_{02}}{2e} z \begin{bmatrix} i \\ \pm 1 \\ 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (10.21)$$

$$z = \exp\left(-\frac{bc_2}{2\hbar} x_2\right)$$

Equation (10.20) gives the 4-vector of the current density of a conserved quantity of electric charge (see chapter 5). Similarly equation (10.21) gives the 4-vector of the current density of a conserved quantity of angular momentum. In both cases the vector $\mathbf{j}$ is on axis $x_1 = x$, as follows from equation (10.19). Finally from the third, fourth and fifth of equations (10.10) we have

$$m_{0,USVI} = 0$$

$$E_{0,USVI} = 0$$

$$c_2 = \pm M_0 c \neq 0$$

$$C = \begin{bmatrix} 0 \\ 0 \\ c_2 \\ 0 \end{bmatrix}$$

$$(10.22)$$

$$\Phi = K \exp\left(-\frac{ic_2}{\hbar} x_2\right)$$

$$m_0 = \pm\frac{M_0}{1+\Phi}$$

$$E_0 = \pm\frac{\Phi M_0 c^2}{1+\Phi}$$

$$(b = i)$$

for the rest masses of the particle which emerges as a consequence of the USVI, and for the total rest mass M_0 of the generalized particle, for the duration of the interaction T_{0221}^2, and for $b = i$.

The above equations completely describe the state of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ in the case of the symmetry T_{0221}^2. Working in the same way we can study the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for the symmetries of the sets $\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5, \Omega_6$.

The second conclusion concerns the consequences for the energy content of the generalized particle when it is trapped in a fixed volume V. By knowing the function Ψ we can study the consequences for a material particle that is confined within a constant volume V. The conserved physical quantity q, is constant within the volume V occupied by the generalized particle. Therefore, it holds that

$$\begin{aligned} &\frac{dq}{dt} = \frac{icdq}{dx_0} = 0 \\ &V = \text{constant} \end{aligned} . \tag{10.23}$$

The total conserved physical quantity q contained within the volume V occupied by the generalized particle is

$$q = \int_V \rho dV . \tag{10.24}$$

The equation (10.24) holds independently of the fact, whether the volume V of the generalized particle variates or not. The density ρ for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by the equation (10.16)

$$\rho = -\sigma \frac{icbz}{2\hbar} \left(c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03} \right).$$

In the case of

$$c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03} = 0 ,$$

that is in the case of

$$\mathbf{n} \cdot \mathbf{C} = 0 ,$$

as derived from equations (3.5) and (7.25), we obtain from equation (10.16) that $\rho = 0$. That is, for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ the following equivalence holds

$$\rho = 0 \Leftrightarrow \mathbf{n} \cdot \mathbf{C} = 0 \Leftrightarrow c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03} = 0 . \tag{10.25}$$

In the case of $\rho \neq 0$, and from the combination of equations (10.24) and (10.16), we have

$$q = -\sigma \frac{icb(\mathbf{n} \cdot \mathbf{C})}{2\hbar} \int_V zdV . \tag{10.26}$$

The integration in the second part of equation (10.26) is performed within the total volume V occupied by the generalized particle. Therefore, in the case the volume V is constant, the integral in the second part of equation (10.26) is independent of the quantiites $x_1 = x, x_2 = y, x_3 = z$. Therefore, in the case volume V is constant, the physical quantity q in equation (10.26) depends only on time. Thus by combining equations (10.23) and (10.26) for a constant volume V, we obtain

$$\begin{aligned} &\frac{d}{dt}\int_V zdV = 0 \\ &\mathbf{n} \cdot \mathbf{C} \neq 0 \\ &V = \text{constant} \end{aligned} . \tag{10.27}$$

Working with equation (10.27) in the general case presents some mathematical difficulties. Therefore in the present work we will restrict our study on the simplest case. We shall study the case for which the total momentum $\mathbf{C}$ of the generalized particle is aligned on the direction of the x - axis, that is for the case of $c_1 \neq 0, c_2 = c_3 = 0$. In this case we obtain from equation (3.6) that $M_0^2c^2 = -c_0^2 - c_1^2$. Furthermore it must also hold that $\rho \neq 0$, that is $c_1\alpha_{01} \neq 0$, according to equivalence (10.25), and since $c_1 \neq 0$, it must also hold that $\alpha_{01} \neq 0$. Therefore our study refers to the particular case where

$$\begin{aligned} &c_1 \neq 0 \\ &c_2 = c_3 = 0 \\ &\alpha_{01} \neq 0 \\ &M_0^2c^2 = -c_0^2 - c_1^2 \end{aligned} . \tag{10.28}$$

We suppose that the generalized particle occupies the constant volume V defined by the relations (10.29) in a frame of reference $O(t, x_1 = x, x_2 = y, x_3 = z)$

$$\begin{aligned}&\alpha \le x_1 \le \beta\\&0 \le x_2 \le L_2\\&0 \le x_3 \le L_3\\&\alpha < \beta\\&L = \beta - \alpha > 0\\&L_2, L_3 > 0, L_2, L_3 = \text{constants}\end{aligned} \quad . \tag{10.29}$$

For the quantities α, β it holds that

$$\frac{d\alpha}{dt} = \frac{d\beta}{dt} = u < c \tag{10.30}$$

where u is the velocity with which the volume V is moving in the chosen frame of reference.

From equation (10.1), relations (10.28), (10.29) and since $x_0 = ict$, we have

$$z = \exp\left(-\frac{icbc_0}{2\hbar} t\right) \exp\left(-\frac{bc_1}{2\hbar} x_1\right)$$

$$\int_V zdV = \int_0^{L_3} \int_0^{L_2} \int_\alpha^\beta \exp\left(-\frac{icbc_0}{2\hbar} t\right) \exp\left(-\frac{bc_1}{2\hbar} x_1\right) dx_1 dx_2 dx_3$$

$$\int_V zdV = \exp\left(-\frac{icbc_0}{2\hbar} t\right) \int_0^{L_3} \int_0^{L_2} \int_\alpha^\beta \exp\left(-\frac{bc_1}{2\hbar} x_1\right) dx_1 dx_2 dx_3$$

$$\int_V zdV = -\frac{2\hbar L_2 L_3}{bc_1} \exp\left(-\frac{icbc_0}{2\hbar} t\right) \left[\exp\left(-\frac{bc_1\beta}{2\hbar}\right) - \exp\left(-\frac{bc_1\alpha}{2\hbar}\right)\right] . \tag{10.31}$$

From equation (10.31) we see that equation equation (10.27) holds, if and only if

$$\exp\left(-\frac{bc_1\beta}{2\hbar}\right) - \exp\left(-\frac{bc_1\alpha}{2\hbar}\right) = 0$$

$$\exp\left(-\frac{bc_1(\beta - \alpha)}{2\hbar}\right) = 1$$

$$\exp\left(\frac{bc_1(\beta - \alpha)}{2\hbar}\right) = 1$$

$$\exp\left(\frac{bc_1 L}{2\hbar}\right)=1 \quad . \qquad (10.32)$$
$$L=\beta-\alpha$$

Equation (10.32) holds only in the case the constant b of the Law of Selfvariations is an imaginary number, $b=i\,||\,b\,||\,,||\,b\,||\in\mathbb{R}$. The confinement of the generalized particle renders imaginary the constant b of the law of Selfvariations. Next, from equation (10.32) we obtain

$$\exp\left(\frac{i\,||\,b\,||\,c_1 L}{2\hbar}\right)=1$$
$$b=i\,||\,b\,||,||\,b\,||\in\mathbb{R}$$

$$\cos\left(\frac{||\,b\,||\,c_1 L}{2\hbar}\right)=1$$
$$\sin\left(\frac{||\,b\,||\,c_1 L}{2\hbar}\right)=0$$
$$b=i\left\|b\right\|,||\,b\,||\in\mathbb{R}$$

and finally, we get

$$c_1=n\frac{4\pi\hbar}{L\,||\,b\,||},n=\pm1,\pm2,\pm3,... \qquad (10.33)$$

Combining equation (10.33) with the last of the equations (10.28) we have

$$M_0^2c^2=-c_0^2-n^2\frac{16\pi^2\hbar^2}{L^2\,||\,b\,||^2},n=1,2,3... \qquad (10.34)$$

Therefore the momentum c_1 and the rest mass M_0 of the confined generalized particle are quantized.

For $c_0=0$ from equation (10.34) we get

$$M_0=n\frac{i4\pi\hbar}{cL\,||\,b\,||},n=\pm1,\pm2,\pm3,... \quad . \qquad (10.35)$$
$$c_0=0$$

Combining equations (10.1), (10.33) and (10.35) we have

$$\begin{aligned}
&\Psi = z = \exp\left(-n\frac{2\pi i}{L}x\right) \\
&c_0 = c_2 = c_3 = 0 \\
&c_1 = n\frac{4\pi\hbar}{L\,||\,b\,||} \qquad . \\
&M_0 = n\frac{i4\pi\hbar}{cL\,||\,b\,||} = \frac{ic_1}{c} \\
&n = \pm 1, \pm 2, \pm 3, \ldots
\end{aligned} \tag{10.36}$$

The function Ψ expresses a standing harmonic wave of wavelength

$$\lambda = \frac{L}{n}, n = 1, 2, 3\ldots$$

on the x -axis. One of the symmetries which give the standing wave of equations (10.36) is T^1_{0121} (see equations (7.49)-(7.71)).

We studied the 4-vector j of symmetry T^2_{0221} assuming validity of equation (10.9), i.e. for $b = i$. In contrast, from the confinement of the generalized particle it follows that $b = i\,||\,b\,||, ||\,b\,|| \in \mathbb{R}$. The values that can be taken by the constant $b \in \mathbb{C}$ of the law of Selfvariations are directly related with the phenomenon we are studying, in every case where the equations of TSV are applied.

We now calculate the equation corresponding to the equation (10.27) for the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$, in general. The reason of not having calculated the general equation already in chapter 5 is that the relation of the confinement of the generalized particle with the appearance of the quantization would not have become obvious.

From equations (4.27), (4.28) and (5.7), and since it holds that $j_0 = i\rho c$, we obtain:

$$\rho = \frac{icb}{\hbar}\Psi\left[\lambda\left(\alpha_{01}J_1 + \alpha_{02}J_2 + \alpha_{03}J_3\right) + \mu\left(\alpha_{01}P_1 + \alpha_{02}P_2 + \alpha_{03}P_3\right)\right]$$

and together with equations (5.8), (5.9) and (7.25) we have

$$\rho = \frac{icb}{\hbar}\Psi\left(\lambda\mathbf{J}\cdot\mathbf{n} + \mu\mathbf{P}\cdot\mathbf{n}\right). \tag{10.37}$$

From equations (10.23), (10.37) for the generalized particle occupying a constant volume V we obtain

$$\begin{aligned}&\frac{d}{dt}\int_V \Psi\left(\lambda \mathbf{J}\cdot\mathbf{n}+\mu\mathbf{P}\cdot\mathbf{n}\right)dV=0\\&\lambda \mathbf{J}\cdot\mathbf{n}+\mu\mathbf{P}\cdot\mathbf{n}\neq 0\\&V=\text{constant}\end{aligned} \qquad . \tag{10.38}$$

For $\lambda=\mu=-\frac{1}{2}$ equation (10.38) gives equation (10.27), after considering equations (3.5) and (10.1).

For the internal symmetry $T=0$ it holds that $M=0$, and from equation (5.7) we obtain $j=0$. Hence equation (10.23) degenerates into the identity, $0=0$, therefore the confinement equation (10.38) does not hold. The same holds also for all the external symmetries $T=zQ\Lambda$ as follows from equation (8.3). The confinement equation (10.38) is valid for the generalized particles of the external symmetries of the sets $\Omega_1,\Omega_2,\Omega_3$ of equations (7.47). In symmetries of these sets is $j\neq 0$ (see chapters 12, 7, 9). In symmetries of the sets Ω_4,Ω_6 is $j=0$ (see chapters 13, 15).

The confinement equation (10.38) implies the quantization of particular physical quantities of the generalized particle. However quantization can also emerge from different causes related with the state of the generalized particle. One of them are the boundary conditions applied to the solutions of the differential equations of the TSV, for each particular state of the generalized particle.

11. THE EXTERNAL SYMMETRY FACTOR. THE PHYSICAL CONTENT OF THE GAUGE FUNCTION

11.1. Introduction

In this chapter we study the factor which generates external symmetry. That is the momentum which emerges from the USVI, which is added to the internal symmetry momentum and eliminates the parallel property of the 4-vectors J, P and C.

In the symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$, where the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is defined, the external symmetry factor is determined by the potential of the field. In the symmetries of the set Ω_0, where $(\boldsymbol{\alpha}, \boldsymbol{\beta}) = (0,0)$ valid, the potential is constant.

The study we present demonstrates the physical content of the gauge function f of the potential. The gauge function f is related to the theorem of internal symmetry, i.e. with the isotropy of spacetime.

11.2. The external symmetry factor. The physical content of the gauge function

To isolate and eventually to work out the external symmetry factor it is necessary to use all fundamental theorems of chapter 7, as well as theorem 4.4 of chapter 4. Next we give the first corollary:

Corollary 11.1 ´´In the external symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$ the factor $J(Q)$ which eliminates the parallel property of the 4-vectors J, P and C of external symmetry during the involvement of a material particle in an interaction (USVI) is given by the equation

$$J = J(Q) = -QA = -Q\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = -Q\begin{bmatrix} \dfrac{iV}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix} \tag{11.1}$$

where A is the 4-vector of the potential of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ with gauge function $f_k = 0, k \in \{0,1,2,3\}$, as given by the equation

$$A_i = \begin{cases} \dfrac{2\hbar}{b} \dfrac{\alpha_{ki}}{c_k} z, \text{if } i \neq k \\ 0, \text{if } i = k \end{cases} \tag{11.2}$$
$c_k \neq 0$

where $c_k \neq 0, k \in \{0,1,2,3\}, i = 0,1,2,3$. ′′

Proof. The field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is defined for external symmetries of the sets $\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_5,\Omega_6$ of equations (7.47). The potential of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is given by equation (4.32) of theorem 4.4. The potential is not uniquely defined. Therefore, the determination of the external symmetry factor was possible only after the study of every one of the external symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. The external symmetry factor for this set is given by equations (11.1) and (11.2). According to corollary 4.1 in external symmetry there is $c_k \neq 0$ for at least one index $k \in \{0,1,2,3\}$. Therefore the potential (11.2) is defined in all symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. In cases where $c_k \neq 0$ for more than one index $k \in \{0,1,2,3\}$ the potentials which emerge from equation (11.2) are equivalent, according to equation (4.33) of theorem 4.4. The factor of external symmetry is that what, in the field «language» we would name *potential momentum* of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of the USVI. □

With the knowledge of the 4-vector $J(Q)$, as given by equations (11.1) and (11.2) we get the 4-vector $P(Q)$ of USVI from the first of equations (7.74)

$$P = P(Q) = -J(Q) = QA. \tag{11.3}$$

From equations (11.3) and (11.2) it follows that the rest mass $m_{0,USVI} = m_0$ and the rest energy $E_{0,USVI} = E_0$ of the USVI particle is not the same in every symmetry of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$, in any case however we have the equations

$$(J(Q))^2 = (P(Q))^2 \tag{11.4}$$

$$E_{0,USVI} = \pm m_{0,USVI} c^2 \tag{11.5}$$

as follows from equations (11.3) and (2.7), (2.8).

From equations (11.1) and (11.2) it follows that in all symmetries of the set $\Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_6$ (see chapters 7, 9, 13, 15) the rest mass m_0 and the rest energy E_0 of the USVI particle is zero:

$$\begin{aligned} m_{0,USVI} &= m_0 = 0 \\ E_{0,USVI} &= E_0 = 0 \end{aligned}.$$

For the $N_1 = 6$ symmetries of the set Ω_1 it can be

$$\begin{aligned} m_{0,USVI} &= m_0\left(Q\right) \neq 0 \vee m_{0,USVI} = m_0 = 0 \\ E_{0,USVI} &= E_0\left(Q\right) \neq 0 \vee E_{0,USVI} = E_0 = 0 \end{aligned}$$

(see chapter 12).

In internal symmetry, i.e. the spontaneous STEM emission of the material particle as a consequence of the Selfvariations, the 4-vectors J, P and C are parallel. When the material particle involves in an interaction the direction of the 4-vector $J\left(Q\right)$ of the potential momentum of the USVI in spacetime depends not only on the material particle. Thus the addition to the internal symmetry 4-vector destroys the parallel property of the 4-vectors J, P and C of the generalized particle. The momentum 4-vector of internal symmetry is given by theorem 3.3, and considering equation (11.1) we have

$$J = \frac{1}{1+\Phi}C + J\left(Q\right) = \frac{1}{1+\Phi}C - QA = \frac{1}{1+\Phi}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} - Q\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} \tag{11.6}$$

for the momentum 4-vector J of the generalized particle. From equations (3.5) and (11.1), (11.3) we also have

$$P = \frac{\Phi}{1+\Phi}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} + Q\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} \tag{11.7}$$

for the momentum 4-vector P of STEM.

In the following chapters we do not discriminate between the 4-vectors $J(Q), P(Q)$ of potential momentum as a consequence of the USVI, and the total momentums J, P of the generalized particle, by generally using the symbols J, P . The presence of the function Φ in equations (11.6) and (11.7) eliminates any cause of confusion. Also we use the same symbol m_0 for the rest mass of the particle which emerges as a consequence of the USVI as for the rest mass of the material particle. For the rest masses also there is not case of confusion since the USVI particles is

$$m_{0,USVI} = 0 \vee m_{0,USVI} = m_0(Q) \neq 0 \ ,$$

while the material particle's rest masses is

$$m_0 = \frac{M_0}{1+\Phi}$$

for the external symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$.

We now apply corollary 11.1 to symmetry T^0_{010203}, whose elements we know from chapter 9. According to equation (9.19) we have $c_0 \neq 0$, hence from equations (11.2) we have

$$\begin{gathered} A = \frac{2\hbar z}{bc_0} \begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} \\ c_0 \neq 0, c_1 = c_2 = c_3 = 0 \\ z = \exp\left(-\frac{bc_0}{2\hbar} x_0 \right) \end{gathered} \tag{11.8}$$

and with equation (11.1) we have

$$\begin{gathered} J = J(Q) = -\frac{2\hbar z Q}{bc_0} \begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} \\ c_0 \neq 0, c_1 = c_2 = c_3 = 0 \\ z = \exp\left(-\frac{bc_0}{2\hbar} x_0 \right) \end{gathered} \tag{11.9}$$

and with equation (11.3) we have

$$P = P(Q) = -J(Q) = \frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix}$$
$$c_0 \neq 0, c_1 = c_2 = c_3 = 0 \qquad (11.10)$$
$$z = \exp\left(-\frac{bc_0}{2\hbar}x_0\right)$$

for the momentum of the USVI particle.

Now from equations (11.6), (11.7), (11.8) and (9.19) we have

$$J = \frac{1}{(1+\Phi)}\begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} \qquad (11.11)$$

$$P = \frac{\Phi}{(1+\Phi)}\begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} \qquad (11.12)$$

for the momentum of the generalized particle.

From equations (2.7) and (11.9) we have

$$\frac{4\hbar^2 z^2 Q^2}{b^2 c_0^2}\left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2\right) + m_0^2 c^2 = 0$$

and with equation (9.9) we have

$$0 + m_0^2 c^2 = 0$$
$$m_0 = 0$$

for the rest mass of the USVI particle. From equations (2.8) and (11.10) we have

$$\frac{4\hbar^2 z^2 Q^2}{b^2 c_0^2}\left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2\right) + \frac{E_0^2}{c^2} = 0$$

and with equation (9.9) we have

$$\frac{E_0^2}{c^2} = 0$$

$$E_0 = 0$$

which is equation (9.23).

From equations (2.7) and (11.11) we have

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 = \frac{c_0^2}{(1+\Phi)^2} + \frac{4\hbar^2 z^2 Q^2}{b^2 c_0^2}\left(\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2\right)$$

and with equation (9.9) we have

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 = \frac{c_0^2}{(1+\Phi)^2}$$

and with equation (2.7) we have

$$-m_0^2 c^2 = \frac{c_0^2}{(1+\Phi)^2}\,. \tag{11.13}$$

From equation (9.19) we have

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = c_0^2$$

and with equation (3.6) we have

$$-M_0^2 c^2 = c_0^2$$

and with equation (11.13) we have

$$m_0^2 c^2 = \frac{M_0^2 c^2}{(1+\Phi)^2}$$

and finally we have

$$m_0 = \pm\frac{M_0}{1+\Phi}$$

which is equation (3.10). Likewise from the equations (2.8) and (11.12) we get (3.11)

$$E_0 = \pm\frac{\Phi M_0}{1+\Phi}\,.$$

From the pairs of equations (11.11) and (11.12) we have

$$\begin{aligned} J_0 &= \frac{c_0}{1+\Phi} \\ J_1 &= -\frac{2\hbar z Q \alpha_{01}}{bc_0} \\ P_0 &= \frac{\Phi c_0}{1+\Phi} \\ P_1 &= \frac{2\hbar z Q \alpha_{01}}{bc_0} \end{aligned} \quad . \tag{11.14}$$

From equation (3.4) and equations (11.14) we have

$$\lambda_{01} = \frac{b}{2\hbar}\left(J_0 P_1 - J_1 P_0\right)$$

$$\lambda_{01} = \frac{b}{2\hbar}\left(\frac{c_0}{1+\Phi}\frac{2\hbar z Q}{bc_0}\alpha_{01} - \left(-\frac{2\hbar z Q \alpha_{01}}{bc_0}\right)\frac{\Phi c_0}{1+\Phi}\right)$$

$$\lambda_{01} = \left(\frac{zQ}{1+\Phi} + \frac{\Phi z Q}{1+\Phi}\right)\alpha_{01}$$

$$\lambda_{01} = zQ\alpha_{01} \, . \tag{11.15}$$

Equation (11.15) is the equation (4.4) for $k=0$ and $i=1$. In the same way we can proof that the pair of equations (11.6), (11.7) give equations (4.4) for the symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$.

A question which arises from corollary 11.1 is if it can be generalized with the gauge function $f_k, k \in \{0,1,2,3\}, c_k \neq 0$ to be different than zero ($f_k \neq 0$) in equation (4.32). The answer is given by the following corollary:

Corollary 11.2 ′′For the factor $J(Q) = -QA$ of external symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. the 4-vector A of the potential of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by equation

$$A_i = \begin{cases} \frac{2\hbar}{b}\frac{\alpha_{ki}}{c_k}z + \frac{\partial f_k}{\partial x_i}, \text{if } i \neq k \\ \frac{\partial f_k}{\partial x_i}, \text{if } i = k \end{cases} \tag{11.16}$$

where $c_k \neq 0, k \in \{0,1,2,3\}, i = 0,1,2,3$ and the gauge function $f_k \neq 0$ satisfies the condition

$$\begin{aligned} & c_k \neq 0, k \in \{0,1,2,3\} \\ & \frac{\partial f_k}{\partial x_i} = \frac{c_i}{c_k}\frac{\partial f_k}{\partial x_k}, i = 0,1,2,3 \end{aligned}. \tag{11.17}$$

In the case where $c_k \neq 0$ for more than one index $k \in \{0,1,2,3\}$, i.e. where $c_k c_i \neq 0, k \neq i, k, i \in \{0,1,2,3\}$, the potentials which arise from equation (11.16) are equivalent, as follows from equation (4.33)

$$f_k = f_i + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{ki}}{c_k c_i}, c_k c_i \neq 0, k \neq i,\ k, i = 0,1,2,3 \tag{11.18}$$

of the theorem 4.4.´´

Proof. From equations (11.16) and (11.7) we have

$$\begin{aligned} & J_k = \frac{c_k}{1+\Phi} - QA_k \\ & P_i = \frac{\Phi c_i}{1+\Phi} + QA_i \\ & J_i = \frac{c_i}{1+\Phi} - QA_i \\ & P_k = \frac{\Phi c_k}{1+\Phi} + QA_k \\ & k, i = 0,1,2,3 \end{aligned} . \tag{11.19}$$

Initially we ask that the combination of the equations (3.4) and (11.19) should give equation (4.4):

$$\lambda_{ki} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right)$$

$$zQ\alpha_{ki} = \frac{b}{2\hbar}\left(\left(\frac{c_k}{1+\Phi} - QA_k\right)\left(\frac{\Phi c_i}{1+\Phi} + QA_i\right) - \left(\frac{c_i}{1+\Phi} - QA_i\right)\left(\frac{\Phi c_k}{+\Phi} + QA_k\right)\right)$$

$$zQ\alpha_{ki} = \frac{b}{2\hbar}\left(\frac{\Phi c_k c_i}{(1+\Phi)^2} + \frac{Qc_k A_i}{1+\Phi} - \frac{\Phi Q c_i A_k}{1+\Phi} - Q^2 A_k A_i - \frac{\Phi c_i c_k}{(1+\Phi)^2} - \frac{Qc_i A_k}{1+\Phi} + \frac{\Phi Q c_k A_i}{1+\Phi} + Q^2 A_i A_k\right)$$

$$zQ\alpha_{ki} = \frac{b}{2\hbar}\left(\frac{Qc_k A_i}{1+\Phi} - \frac{\Phi Qc_i A_k}{1+\Phi} - \frac{Qc_i A_k}{1+\Phi} + \frac{\Phi Qc_k A_i}{1+\Phi}\right)$$

$$zQ\alpha_{ki} = \frac{b}{2\hbar}\left(\left(\frac{1}{1+\Phi} + \frac{\Phi}{1+\Phi}\right)Qc_k A_i - \left(\frac{1}{1+\Phi} + \frac{\Phi}{1+\Phi}\right)Qc_i A_k\right)$$

$$zQ\alpha_{ki} = \frac{b}{2\hbar}Q\left(c_k A_i - c_i A_k\right)$$

and since $Q \neq 0$ we get

$$\frac{2\hbar}{b} z\alpha_{ki} = c_k A_i - c_i A_k . \tag{11.20}$$

Combining equations (11.20) and (11.16) for $c_k \neq 0, k \in \{0,1,2,3\}, i \neq k, i = 0,1,2,3$ we have

$$\frac{2\hbar}{b} z\alpha_{ki} = c_k\left(\frac{2\hbar}{b}\frac{\alpha_{ki}}{c_k} z + \frac{\partial f_k}{\partial x_i}\right) - c_i \frac{\partial f_k}{\partial x_k}$$

$$0 = c_k \frac{\partial f_k}{\partial x_i} - c_i \frac{\partial f_k}{\partial x_k}$$

and finally we have

$$\frac{\partial f_k}{\partial x_i} = \frac{c_i}{c_k}\frac{\partial f_k}{\partial x_k}$$

which is condition (11.17). The proof is completed by showing that the corollary is valid for everyone of the external symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. The last step of the proof is necessary because in the procedure of the proof we used equation (11.1) of corollary 11.1. □

Via equations (11.6), (11.7) and (11.1), (11.3) corollary 11.2 gives the 4-vectors J and P of the generalized particle and the USVI particle respectively for the symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. It is easy to prove that the rest mass of the material particle is given by equation (3.10), and the rest mass of the USVI particle is zero for the rest mass of the set $\Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. In the symmetries of set Ω_1 the rest mass m_0 of the USVI can also be non-zero (see chapter 12).

The condition (11.17) of the corollary 11.2 is not contained in corollary 11.1 since we have the identity, $0 = 0$. This has the following consequence: The potential of equations (11.2) is valid for all symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. The potential of equations (11.16), i.e. when the gauge function is nonzero ($f_k \neq 0$ for at least one $k \in \{0,1,2,3\}$) is valid only for one subset of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$ (see chapter 13).

We now calculate the 4-vector $J = -QA$ including the potential A and the gauge function f, when $f \neq 0$. From equations (4.34)-(4.37) we have

$c_0 \neq 0$

$$J = -Q\begin{bmatrix} \frac{\partial f_0}{\partial x_0} \\ \frac{\partial f_0}{\partial x_1} \\ \frac{\partial f_0}{\partial x_2} \\ \frac{\partial f_0}{\partial x_3} \end{bmatrix} - Q\frac{2\hbar z}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{02} \end{bmatrix}$$

$c_1 \neq 0$

$$J = -Q\begin{bmatrix} \frac{\partial f_1}{\partial x_0} \\ \frac{\partial f_1}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_1}{\partial x_3} \end{bmatrix} - Q\frac{2\hbar z}{bc_1}\begin{bmatrix} -a_{01} \\ 0 \\ -\alpha_{21} \\ \alpha_{13} \end{bmatrix}$$

$c_2 \neq 0$

$$J = -Q\begin{bmatrix} \frac{\partial f_2}{\partial x_0} \\ \frac{\partial f_2}{\partial x_1} \\ \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_2}{\partial x_3} \end{bmatrix} - Q\frac{2\hbar z}{bc_2}\begin{bmatrix} -a_{02} \\ \alpha_{21} \\ 0 \\ -\alpha_{32} \end{bmatrix}$$

$$c_3 \neq 0$$

$$J = -Q\begin{bmatrix} \frac{\partial f_3}{\partial x_0} \\ \frac{\partial f_3}{\partial x_1} \\ \frac{\partial f_3}{\partial x_2} \\ \frac{\partial f_3}{\partial x_3} \end{bmatrix} - Q\frac{2\hbar z}{bc_3}\begin{bmatrix} -a_{03} \\ -\alpha_{13} \\ \alpha_{32} \\ 0 \end{bmatrix}$$

and with equation (11.17) written in the form

$$c_0 \neq 0$$

$$\frac{\partial f_0}{\partial x_i} = \frac{c_i}{c_0}\frac{\partial f_0}{\partial x_0}, i = 0,1,2,3$$

$$c_1 \neq 0$$

$$\frac{\partial f_1}{\partial x_i} = \frac{c_i}{c_1}\frac{\partial f_1}{\partial x_1}, i = 0,1,2,3$$

$$c_2 \neq 0$$

$$\frac{\partial f_2}{\partial x_i} = \frac{c_i}{c_2}\frac{\partial f_2}{\partial x_2}, i = 0,1,2,3 \qquad (11.21)$$

$$c_3 \neq 0$$

$$\frac{\partial f_3}{\partial x_i} = \frac{c_i}{c_3}\frac{\partial f_3}{\partial x_3}, i = 0,1,2,3$$

we obtain

$$c_0 \neq 0$$

$$J = -Q\frac{\partial f_0}{c_0 \partial x_0}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} - Q\frac{2\hbar z}{bc_0}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{02} \end{bmatrix} \qquad (11.22)$$

$c_1 \neq 0$

$$J = -Q\frac{\partial f_1}{c_1\partial x_1}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} - Q\frac{2\hbar z}{bc_1}\begin{bmatrix} -a_{01} \\ 0 \\ -\alpha_{21} \\ \alpha_{13} \end{bmatrix} \tag{11.23}$$

$c_2 \neq 0$

$$J = -Q\frac{\partial f_2}{c_2\partial x_2}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} - Q\frac{2\hbar z}{bc_2}\begin{bmatrix} -a_{02} \\ \alpha_{21} \\ 0 \\ -\alpha_{32} \end{bmatrix} \tag{11.24}$$

$c_3 \neq 0$

$$J = -Q\frac{\partial f_3}{c_3\partial x_3}\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} - Q\frac{2\hbar z}{bc_3}\begin{bmatrix} -a_{03} \\ -\alpha_{13} \\ \alpha_{32} \\ 0 \end{bmatrix} \tag{11.25}$$

Comparing equations (11.6) and (11.22)-(11.25) we get

$$-\frac{Q}{c_0}\frac{\partial f_0}{\partial x_0} = -\frac{Q}{c_1}\frac{\partial f_1}{\partial x_1} = -\frac{Q}{c_2}\frac{\partial f_2}{\partial x_2} = -\frac{Q}{c_3}\frac{\partial f_3}{\partial x_3} = \frac{1}{1+\Phi}$$

and equivalently we obtain

$$\begin{aligned} \frac{\partial f_0}{\partial x_0} &= -\frac{1}{1+\Phi}\frac{c_0}{Q} \\ \frac{\partial f_1}{\partial x_1} &= -\frac{1}{1+\Phi}\frac{c_1}{Q} \\ \frac{\partial f_2}{\partial x_2} &= -\frac{1}{1+\Phi}\frac{c_2}{Q} \\ \frac{\partial f_3}{\partial x_3} &= -\frac{1}{1+\Phi}\frac{c_3}{Q} \end{aligned} \tag{11.26}$$

The equations (11.26) and (11.22)-(11.25) illustrate the physical content of the gauge function. When $f \neq 0$ the first term of the right hand side of the equations (11.22)-(11.25), which is the term depending on the gauge function, expresses the momentum

$$J = \frac{1}{1+\Phi} C$$

of the material particle in internal symmetry, as follows from theorem 3.3 of internal symmetry. The gauge function expresses the internal symmetry, i.e. the spontaneous STEM emission of the material particles due to the Selfvariations. When $f \neq 0$ the gauge function is not arbitrary, as initially entered in equations (4.32) and as considered by the theories of the 20^{th} century.

For symmetry T_{010203}^{0} it is $c_0 \neq 0, c_1 = c_2 = c_3 = 0$ and $a_{01}a_{02}a_{03} \neq 0$ hence from equation (11.22) we have

$$J = -Q \frac{\partial f_0}{\partial x_0} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \frac{2\hbar z Q}{b c_0} \begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix}. \tag{11.27}$$

Equation (11.27) is equivalent with equation (11.11), because of equation (11.17).

The method we presented results in equation (11.9)

$$J = J(Q) = -\frac{2\hbar z Q}{b c_0} \begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix}$$

in a different way than from the $SV-T$ method (equation (9.20)). Equation (11.1) written in the form

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = -\frac{1}{Q} J(Q) \tag{11.28}$$

gives the 4-vector of the potential A via the 4-vector of momentum $J(Q)$, independently of equations (4.32). In the symmetries of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$ it is

$(\boldsymbol{\alpha},\boldsymbol{\beta}) \neq (0,0)$. Hence in these symmetries the potential A can be calculated either from equations (4.32) or from equation (11.28).

For the symmetries $T = zQ\Lambda$ of the set Ω_0 it is $(\boldsymbol{\alpha},\boldsymbol{\beta}) = (0,0)$ and $J(Q) \neq 0$. This is also valid for symmetry T^{12} which was studied in chapter 8. From equation (8.13) we now get

$$\begin{aligned} &J(Q) = Q\begin{bmatrix} \sigma_0 \\ 0 \\ 0 \\ \sigma_3 \end{bmatrix} \\ &(\sigma_0,\sigma_3) \neq (0,0) \quad . \\ &\sigma_0,\sigma_3 = \text{constants} \end{aligned} \tag{11.29}$$

From equations (11.28) and (11.29) we have

$$\begin{aligned} &A = -\begin{bmatrix} \sigma_0 \\ 0 \\ 0 \\ \sigma_3 \end{bmatrix} \\ &(\sigma_0,\sigma_3) \neq (0,0) \\ &\sigma_0,\sigma_3 = \text{constants} \end{aligned} \tag{11.30}$$

and from equation (4.30) we have $(\boldsymbol{\alpha},\boldsymbol{\beta}) = (0,0)$ valid for T^{12}. An analogous equation with the equation (11.30) is valid for every external symmetry of the set Ω_0. The potential of the symmetries of the set Ω_0 is constant.

From equations (4.32) and (8.16) we now get

$$\begin{aligned} &c_0 \neq 0 \\ &A_0 = \frac{\partial f_0}{\partial x_0} \\ &A_1 = \frac{\partial f_0}{\partial x_1} \\ &A_2 = \frac{\partial f_0}{\partial x_2} \\ &A_3 = \frac{\partial f_0}{\partial x_3} \end{aligned} \tag{11.31}$$

$$
\begin{aligned}
& c_3 \neq 0 \\
& A_0 = \frac{\partial f_3}{\partial x_0} \\
& A_1 = \frac{\partial f_3}{\partial x_1} \\
& A_2 = \frac{\partial f_3}{\partial x_2} \\
& A_3 = \frac{\partial f_3}{\partial x_3}
\end{aligned}
\tag{11.32}
$$

for the symmetry T^{12}. Equations (11.31) are equivalent with equations (11.32), because of equations (11.26), (11.17).

From equations (11.31), (11.32) and (11.30) we have

$$
\begin{aligned}
& c_0 \neq 0 \\
& A_0 = \frac{\partial f_0}{\partial x_0} = -\sigma_0 \neq 0 \\
& A_1 = \frac{\partial f_0}{\partial x_1} = 0 \\
& A_2 = \frac{\partial f_0}{\partial x_2} = 0 \\
& A_3 = \frac{\partial f_0}{\partial x_3} = -\sigma_3 \neq 0
\end{aligned}
\tag{11.33}
$$

$$
\begin{aligned}
& c_3 \neq 0 \\
& A_0 = \frac{\partial f_3}{\partial x_0} = -\sigma_0 \neq 0 \\
& A_1 = \frac{\partial f_3}{\partial x_1} = 0 \qquad . \\
& A_2 = \frac{\partial f_3}{\partial x_2} = 0 \\
& A_3 = \frac{\partial f_3}{\partial x_3} = -\sigma_3 \neq 0
\end{aligned}
\tag{11.34}
$$

From equations (11.33) and (11.34) we get

$$
\begin{aligned}
&f_0 = -\sigma_0 x_0 - \sigma_3 x_3 + S_0\\
&f_3 = -\sigma_0 x_0 - \sigma_3 x_3 + S_3\\
&\sigma_0, \sigma_3 = \text{constants}\\
&S_1, S_3 = \text{constants}
\end{aligned}
\tag{11.35}
$$

for the symmetry T^{12}. From equations (4.33) we get

$$
\begin{aligned}
&f_0 = f_3\\
&c_0 c_3 \neq 0\\
&c_1 = c_2 = 0
\end{aligned}
\tag{11.36}
$$

for the symmetry T^{12}. From equations (11.35), (11.36) and (8.16) we get

$$
\begin{aligned}
&f_0 = f_3 = -\sigma_0 x_0 - \sigma_3 x_3 + S_{03}\\
&S_{03} = \text{constant}\\
&\sigma_0, \sigma_3 = \text{constants}\\
&\sigma_0 \sigma_3 \neq 0\\
&c_0 c_3 \neq 0\\
&c_1 = c_2 = 0\\
&c_0 \sigma_3 = c_3 \sigma_0
\end{aligned}
\tag{11.37}
$$

for the symmetry T^{12}. An analogous equation with the equation (11.37) is valid for every external symmetry of the set Ω_0.

12. THE SET Ω_1

12.1. Introduction

In this chapter we determine the basic characteristics of the symmetries of set Ω_1. In the symmetries of set $\Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$ the rest mass m_0 of the USVI is equal to zero. In addition, the second term of the USVI on the right-hand side of equations (4.19) and (4.20) vanishes in the symmetries of set $\Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. In the symmetries of set Ω_1 the rest mass m_0 of the USVI can also be non-zero ($m_0 \neq 0$). In addition, the second term of the USVI is non-zero

$$\frac{i}{c} zQ\Lambda u \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

on the right-hand side of equations (4.19) and (4.20). We present the detailed study of the symmetry T_{01}^{01} . The study of the remaining symmetries of set

$\Omega_1 = \{T_{01}^{01}, T_{02}^{02}, T_{03}^{03}, T_{32}^{23}, T_{13}^{13}, T_{21}^{12}\}$ is performed along similar lines.

12.2. The symmetry T_{01}^{01}

From equation (7.1) for $\alpha_{01} \neq 0$ and $\alpha_{02} = \alpha_{03} = \alpha_{32} = \alpha_{13} = \alpha_{21} = 0$ we get

$$T = zQ \begin{bmatrix} T_0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}. \tag{12.1}$$

$\alpha_{01} \neq 0$

From theorem 7.1 or, equivalently, from equations (7.4), and taking into account that $\alpha_{01} \neq 0$ we get

$$T_2 = T_3 = 0 \tag{12.2}$$

therefore, equation (12.1) becomes

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12.3}$$

$\alpha_{01} \neq 0$

From the second of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get

$$\begin{aligned} J_0 a_{12} + J_2 a_{01} + J_1 a_{20} &= 0 \\ J_0 a_{13} + J_3 a_{01} + J_1 a_{30} &= 0 \\ J_0 a_{23} + J_3 a_{02} + J_2 a_{30} &= 0 \\ J_1 a_{23} + J_3 a_{12} + J_2 a_{31} &= 0 \end{aligned}$$

and taking into account that $\alpha_{ki} = -\alpha_{ik}, k \neq i, k, i = 0,1,2,3$, and the elements of matrix T as given by equation (12.3) we get

$$\begin{aligned} J_2 \alpha_{01} &= 0 \\ J_3 \alpha_{01} &= 0 \end{aligned}$$

and since $\alpha_{01} \neq 0$ we get

$$J_2 = J_3 = 0 \tag{12.4}$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ 0 \\ 0 \end{bmatrix}. \tag{12.5}$$

Equation (12.5) gives the four-vector $J(Q)$. As we have already mentioned, we will use the symbol J in equation (11.1) as well as in equation (11.6), since there is no issue of confusion between the four-vectors J and $J(Q)$.

From equations (2.13) and (12.3), (12.5), and considering that $zQ \neq 0$ we get

$$\begin{bmatrix} T_0 & a_{01} & 0 & 0 \\ -a_{01} & T_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} J_0 \\ J_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and therefore

$$\begin{aligned} &T_0 J_0 + \alpha_{01} J_1 = 0 \\ &-\alpha_{01} J_0 + T_1 J_1 = 0 \end{aligned} . \tag{12.6}$$

Equations (12.6) constitute a 2×2 homogeneous system with the momenta J_0 and J_1 as unknowns. Since it is $J_2 = J_3 = 0$ and $J \neq 0$ system (12.6) has a non-zero solution and , therefore, its determinant is equal to zero, that is

$$T_0 T_1 + \alpha_{01}^2 = 0 . \tag{12.7}$$

From equation (12.7) and because $\alpha_{01} \neq 0$ we get

$$T_0 T_1 \neq 0 . \tag{12.8}$$

Thus, from equation (12.3) and relation (12.8) we get

$$T_{01}^{01} = zQ \begin{bmatrix} T_0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{12.9}$$

In accordance with our symbolism the elements α_{01} , T_0 and T_1 of the matrix (12.9) are nonzero.

From the first of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get

$$\begin{aligned} &c_0 a_{12} + c_2 a_{01} + c_1 a_{20} = 0 \\ &c_0 a_{13} + c_3 a_{01} + c_1 a_{30} = 0 \\ &c_0 a_{23} + c_3 a_{02} + c_2 a_{30} = 0 \\ &c_1 a_{23} + c_3 a_{12} + c_2 a_{31} = 0 \end{aligned}$$

and taking into account that $\alpha_{ki} = -\alpha_{ik}, k \neq i, k, i = 0,1,2,3$, and the elements of matrix T_{01}^{01} as given by equation (12.9) we get

$$\begin{aligned} &c_2 \alpha_{01} = 0 \\ &c_3 \alpha_{01} = 0 \end{aligned}$$

and since $\alpha_{01} \neq 0$ we get

$$c_2 = c_3 = 0 \tag{12.10}$$

$$C = \begin{bmatrix} c_0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}. \tag{12.11}$$

From equations (3.4) and (4.4) we get

$$zQ\alpha_{01} = \frac{b}{2\hbar}\left(c_1 J_0 - c_0 J_1\right). \tag{12.12}$$

Given that $\alpha_{01} \neq 0$ we get from equation (12.12) relation

$$c_0 J_1 \neq c_1 J_0 . \tag{12.13}$$

From relation (12.13) it follows that the four-vectors J and C of equations (12.5) and (12.11) are not parallel.

Applying the $SV-T$ method it is easy to determine that equation (12.12) is compatible with the symmetry T_{01}^{01} only when $c_0 \neq 0 \wedge c_1 = 0$ or $c_0 = 0 \wedge c_1 \neq 0$ in the equation (12.11). Therefore, the four-vector C is expressed in the following two ways

$$C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12.14}$$

$$c_0 \neq 0$$

$$C = \begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}. \tag{12.15}$$

$$c_1 \neq 0$$

When equation (12.14) holds, it is $z = z\left(x_0\right)$, as follows from equations (4.5) and (12.14), it is also $Q = Q\left(x_0\right)$. Taking also into account equations (12.5) and (12.12) we get

$$c_0 \neq 0$$

$$C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J = J\left(x_0\right) = \begin{bmatrix} J_0\left(x_0\right) \\ J_1\left(x_0\right) \\ 0 \\ 0 \end{bmatrix} \quad . \tag{12.16}$$

$$J_1 = J_1\left(x_0\right) = -\frac{2\hbar}{bc_0} zQ\alpha_{01}$$

$$z = \exp\left(-\frac{bc_0}{2\hbar} x_0\right)$$

$$Q = Q\left(x_0\right)$$

Similarly, when equation (12.15) holds, we get

$$c_1 \neq 0$$

$$C = \begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}$$

$$J = J\left(x_1\right) = \begin{bmatrix} J_0\left(x_1\right) \\ J_1\left(x_1\right) \\ 0 \\ 0 \end{bmatrix} \quad . \tag{12.17}$$

$$J_0 = J_0\left(x_1\right) = \frac{2\hbar}{bc_1} zQ\alpha_{01}$$

$$z = \exp\left(-\frac{bc_1}{2\hbar} x_1\right)$$

$$Q = Q\left(x_1\right)$$

From equations (2.7) and (12.5) we get

$$J_0^2 + J_1^2 + m_0^2 c^2 = 0 \,. \tag{12.18}$$

We now prove that the rest mass m_0 of the USVI particle can be non-zero in the symmetry T_{01}^{01}, that is

$$m_0 \neq 0 \,. \tag{12.19}$$

For $m_0 \neq 0$ in equation (12.18) we apply the $SV-T$ method differentiating with respect to $x_\nu, \nu = 0,1,2,3$, and considering equations (7.43) and (2.6) we get

$$J_0 \frac{\partial J_0}{\partial x_\nu} + J_1 \frac{\partial J_1}{\partial x_\nu} + m_0 c^2 \frac{\partial m_0}{\partial x_\nu} = 0$$

$$J_0 \left(\frac{b}{\hbar} P_\nu J_0 + zQ\alpha_{\nu 0} \right) + J_1 \left(\frac{b}{\hbar} P_\nu J_1 + zQ\alpha_{\nu 1} \right) + m_0 c^2 \frac{b}{\hbar} P_\nu m_0 = 0$$

$$\frac{b}{\hbar} P_\nu \left(J_0^2 + J_1^2 + m_0^2 c^2 \right) + zQ\left(J_0 \alpha_{\nu 0} + J_1 \alpha_{\nu 1} \right) = 0$$

and with equation (12.18) we get

$$zQ\left(J_0 \alpha_{\nu 0} + J_1 \alpha_{\nu 1} \right) = 0$$

and because $zQ \neq 0$ we have

$$J_0 \alpha_{\nu 0} + J_1 \alpha_{\nu 1} = 0 \,. \tag{12.20}$$

For succesive $\nu = 0,1,2,3$ in equation (12.20) we get:

For $\nu = 0$,

$$J_0 T_0 + J_1 \alpha_{01} = 0$$

which is the first of equations (12.6).

For $\nu = 1$,

$$J_0 \alpha_{10} + J_1 T_1 = 0$$

and equivalently

$$-J_0 \alpha_{01} + J_1 T_1 = 0$$

which is the second of equations (12.6).

For $\nu = 2$,

$$J_0 \alpha_{20} + J_1 \alpha_{21} = 0$$

which holds since

$\alpha_{20} = \alpha_{21} = 0$

for symmetry T_{01}^{01}.

For $\nu = 3$,

$J_0\alpha_{30} + J_1\alpha_{31} = 0$

which holds since

$\alpha_{30} = \alpha_{21} = 0$

for symmetry T_{01}^{01}.

We prove the equivalence

$$m_0 \neq 0 \Leftrightarrow T_1 \neq \pm T_0 \tag{12.21}$$

for symmetry T_{01}^{01}.

Proof. We prove that it is impossible to have

$$m_0 \neq 0 \wedge T_1 = \pm T_0 . \tag{12.22}$$

For $T_1 = \pm T_0$ from equation (7.41) of theorem 7.6 we have

$T_0^2\left(J_0^2 + J_1^2\right) = 0$

and with equation (2.7) we have

$T_0^2 m_0^2 c^2 = 0$

and from the relation (12.8) we get $m_0 = 0$ which is absurd since we have assumed that $m_0 \neq 0$. Therefore we have the equivalence (12.21). □

We prove now that for $m_0 \neq 0$ the physical quantities T_0 and T_1 are not constant and are given by the equations

$$c_0 \neq 0, c_1 = c_2 = c_3 = 0$$
$$m_0 \neq 0$$
$$T_0 = \pm a_{01}\left(\gamma_0 \exp\left(\frac{bc_0x_0}{\hbar}\right) - 1\right)^{\frac{-1}{2}} \tag{12.23}$$
$$T_1 = -\frac{\alpha_{01}^2}{T_0} = \mp a_{01}\left(\gamma_0 \exp\left(\frac{bc_0x_0}{\hbar}\right) - 1\right)^{\frac{1}{2}}$$
$$\gamma_0 = constant, \gamma_0 \in \mathbb{C}$$

$$c_1 \neq 0, c_0 = c_2 = c_3 = 0$$
$$m_0 \neq 0$$
$$T_1 = \pm a_{01}\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right) - 1\right)^{-\frac{1}{2}} . \tag{12.24}$$
$$T_0 = -\frac{\alpha_{01}^2}{T_1} = \mp a_{01}\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right) - 1\right)^{\frac{1}{2}}$$
$$\gamma_1 = constant, \gamma_1 \in \mathbb{C}$$

Proof. We prove the equation (12.23) and similarly equation (12.24) is proved. For $c_0 \neq 0 \wedge c_1 = 0$ from equations (3.4) and (4.4) we have

$$zQ\alpha_{01} = \frac{b}{2\hbar}\left(c_1J_0 - c_0J_1\right)$$
$$zQ\alpha_{01} = \frac{b}{2\hbar}\left(0 - c_0J_1\right)$$

$$J_1 = -\frac{2\hbar\alpha_{01}}{bc_0} zQ . \tag{12.25}$$

Form equation (12.25) and the first of equations (12.6) we have

$$T_0J_0 + \alpha_{01}J_1 = 0$$
$$J_0 = -\frac{\alpha_{01}}{T_0} J_1$$

$$J_0 = J_0\left(x_0\right) = \frac{2\hbar\alpha_{01}^2}{bc_0}\frac{zQ}{T_0} . \tag{12.26}$$

Differentiating equation (12.26) with respect to x_0 we get

$$\frac{dJ_0}{dx_0} = -\frac{2\hbar\alpha_{01}^2}{bc_0}\frac{zQ}{T_0^2}\frac{dT_0}{dx_0} + \frac{2\hbar\alpha_{01}^2}{bc_0}\frac{1}{T_0}\frac{d\left(zQ\right)}{dx_0}$$

and with equations (2.10), (4.10), (4.9) and (4.2), for $c_0 \neq 0 \wedge c_1 = c_2 = c_3 = 0$ we have

$$\frac{b}{\hbar}P_0 J_0 + zQT_0 = -\frac{2\hbar\alpha_{01}^2}{bc_0}\frac{zQ}{T_0^2}\frac{dT_0}{dx_0} + \frac{2\hbar\alpha_{01}^2}{bc_0}\frac{1}{T_0}\left(-\frac{bc_0}{2\hbar} + \frac{b}{\hbar}P_0\right)zQ$$

and with equation (12.26) we have

$$zQT_0 = -\frac{2\hbar\alpha_{01}^2}{bc_0}\frac{zQ}{T_0^2}\frac{dT_0}{dx_0} - \frac{zQ\alpha_{01}^2}{T_0}$$

and because of $zQ \neq 0$ we have

$$T_0^3 = -\frac{2\hbar\alpha_{01}^2}{bc_0}\frac{dT_0}{dx_0} - \alpha_{01}^2 T_0$$

and solving the differential equation we get

$$T_0^2 = \frac{\alpha_{01}^2}{\gamma_0 \exp\left(\frac{bc_0}{\hbar}x_0\right) - 1}$$

$$T_0 = \pm a_{01}\left(\gamma_0 \exp\left(\frac{bc_0 x_0}{\hbar}\right) - 1\right)^{-\frac{1}{2}}$$

and with equation (12.7) we get equation (12.23).□

Consequently, the rest mass m_0 of the USVI particle can be non-zero in the symmetry T_{01}^{01}.

From equations (12.9), (12.6), (12.16) and (12.23) we get

$c_0 \neq 0, c_1 = c_2 = c_3 = 0$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} \pm\left(\gamma_0 \exp\left(\frac{bc_0 x_0}{\hbar}\right) - 1\right)^{-\frac{1}{2}} & 1 & 0 & 0 \\ -1 & \mp\left(\gamma_0 \exp\left(\frac{bc_0 x_0}{\hbar}\right) - 1\right)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\gamma_0 \in \mathbb{C}, \gamma_0 \neq 0$

$$J_0 = J_0(x_0) = \pm\left(\gamma_0 \exp\left(\frac{bc_0 x_0}{\hbar}\right) - 1\right)^{\frac{1}{2}} \frac{2\hbar\alpha_{01}}{bc_0} zQ \tag{12.27}$$

$$J_1 = J_1(x_0) = -\frac{2\hbar\alpha_{01}}{bc_0} zQ$$

$$J = \frac{2\hbar\alpha_{01}}{bc_0} zQ \begin{bmatrix} \pm\left(\gamma_0 \exp\left(\frac{bc_0 x_0}{\hbar}\right) - 1\right)^{\frac{1}{2}} \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$m_0 = m_0(x_0) = \pm(-\gamma_0)^{\frac{1}{2}} \cdot \frac{2\hbar\alpha_{01}}{bcc_0} Q$$

$$Q = Q(x_0)$$

and from equations (12.9), (12.6), (12.17) and (12.23) we get

$$c_1 \neq 0, c_0 = c_2 = c_3 = 0$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} \pm\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right) - 1\right)^{-\frac{1}{2}} & 1 & 0 & 0 \\ -1 & \mp\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right) - 1\right)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma_1 \in \mathbb{C}, \gamma_1 \neq 0$$

$$J_0 = J_0(x_1) = \frac{2\hbar\alpha_{01}}{bc_1} zQ$$

$$J_1 = \pm\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right) - 1\right)^{\frac{1}{2}} \frac{2\hbar\alpha_{01}}{bc_1} zQ \tag{12.28}$$

$$J = \frac{2\hbar\alpha_{01}}{bc_1} zQ \begin{bmatrix} 1 \\ \pm\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right) - 1\right)^{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$$

$$m_0 = m_0(x_1) = \pm(-\gamma_1)^{\frac{1}{2}} \cdot \frac{2\hbar\alpha_{01}}{bcc_1} Q$$

$$Q = Q(x_1)$$

if $m_0 \neq 0$. In these equations , either all of the ¨up¨ or all of the ¨down¨ signs apply.

We now check whether it can be $m_0 = 0$ in equation (12.18). For $m_0 = 0$ we ge

$$J_0^2 + J_1^2 = 0$$

$$J_1 = \pm iJ_0 . \tag{12.29}$$

We apply the $SV-T$ method, we deferentiate equation (12.29) with respect to $x_\nu, \nu = 0,1,2,3$, and, taking into account equation (7.43) we get

$$\frac{\partial J_1}{\partial x_\nu} = \pm i \frac{\partial J_0}{\partial x_\nu}$$

$$\frac{b}{\hbar} P_\nu J_1 + zQ\alpha_{\nu 1} = \pm i\left(\frac{b}{\hbar} P_\nu J_0 + zQ\alpha_{\nu 0}\right)$$

and with equation (12.29) we have

$$zQ\alpha_{\nu 1} = \pm izQ\alpha_{\nu 0}$$

and since $zQ \neq 0$ we get

$$\alpha_{\nu 1} = \pm i\alpha_{\nu 0}. \qquad (12.30)$$

For succesive $\nu = 0,1,2,3$ in equation (12.30) we get:

For $\nu = 0$,

$$\alpha_{01} = \pm i\alpha_{00} = \pm iT_0$$

and equivalently

$$T_0 = \mp i\alpha_{01}. \qquad (12.31)$$

For $\nu = 1$,

$$T_1 = \pm i\alpha_{10}$$

and equivalently

$$T_1 = \mp i\alpha_{01}. \qquad (12.32)$$

For $\nu = 2$,

$$\alpha_{21} = \pm i\alpha_{20}$$

which holds since

$$\alpha_{20} = \alpha_{21} = 0$$

for the symmetry T_{01}^{01}.

For $\nu = 3$,

$$\alpha_{31} = \pm i\alpha_{30}$$

which holds since

$$\alpha_{31} = \alpha_{30} = 0$$

for the symmetry T_{01}^{01}.

Equations (12.31) and (12.32) satisfy equation (12.7). Therefore, the rest mass m_0 of the USVI particle can be zero when equations (12.31) and (12.32) hold. Taking also into account equation (12.29) we get the case

$$\begin{aligned} &m_0 = 0 \\ &J_1 = \pm i J_0 \\ &T_0 = T_1 = \pm i\alpha_{01} \end{aligned} \quad . \tag{12.33}$$

for the T_{01}^{01} symmetry. Equations (12.33) resolve in the following four cases:

$$\begin{aligned} &c_0 \neq 0, c_1 = c_2 = c_3 = 0 \\ &J_0 = J_0\left(x_0\right) = \frac{i2\hbar}{bc_0} zQ\alpha_{01} \\ &J_1 = J_1\left(x_0\right) = -\frac{2\hbar}{bc_0} zQ\alpha_{01} \\ &J_2 = J_3 = 0 \\ &T_0 = T_1 = -i\alpha_{01} \\ &z = \exp\left(-\frac{bc_0}{2\hbar} x_0\right) \\ &Q = Q\left(x_0\right) \\ &m_0 = 0 \end{aligned} \qquad T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} -i & 1 & 0 & 0 \\ -1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12.34}$$

$$c_0 \neq 0, c_1 = c_2 = c_3 = 0$$
$$J_0 = J_0(x_0) = -\frac{i2\hbar}{bc_0} zQ\alpha_{01}$$
$$J_1 = J_1(x_0) = -\frac{2\hbar}{bc_0} zQ\alpha_{01}$$
$$J_2 = J_3 = 0$$
$$T_0 = T_1 = i\alpha_{01}$$
$$z = \exp\left(-\frac{bc_0}{2\hbar} x_0\right)$$
$$Q = Q(x_0)$$
$$m_0 = 0$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} i & 1 & 0 & 0 \\ -1 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12.35}$$

$$c_1 \neq 0, c_0 = c_2 = c_3 = 0$$
$$J_0 = J_0(x_1) = \frac{2\hbar}{bc_1} zQ\alpha_{01}$$
$$J_1 = J_1(x_1) = \frac{i2\hbar}{bc_1} zQ\alpha_{01}$$
$$J_2 = J_3 = 0$$
$$T_0 = T_1 = -i\alpha_{01}$$
$$z = \exp\left(-\frac{bc_1}{2\hbar} x_1\right)$$
$$Q = Q(x_1)$$
$$m_0 = 0$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} -i & 1 & 0 & 0 \\ -1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12.36}$$

$$c_1 \neq 0, c_0 = c_2 = c_3 = 0$$
$$J_0 = J_0(x_1) = \frac{2\hbar}{bc_1} zQ\alpha_{01}$$
$$J_1 = J_1(x_1) = -\frac{i2\hbar}{bc_1} zQ\alpha_{01}$$
$$J_2 = J_3 = 0$$
$$T_0 = T_1 = i\alpha_{01}$$
$$z = \exp\left(-\frac{bc_1}{2\hbar} x_1\right)$$
$$Q = Q(x_1)$$
$$m_0 = 0$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} i & 1 & 0 & 0 \\ -1 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12.37}$$

From the above it emerges that in the T_{01}^{01} symmetry there are six cases for the energy state of the generalized particle as given by relations (12.23), (12.24) and (12.34)-(12.37). In the first case, the rest mass of the USVI particle is non-zero, while in the other four it is equal to zero. In all cases the four-vector P is given by equation (3.5), $P = C - J$, and $P(Q) = -J(Q)$. From equation (3.6) and equations (12.14), (12.15) it emerges that

$$\left.\begin{matrix} c_0 = \pm iM_0c \neq 0 \\ c_1 = c_2 = c_3 = 0 \end{matrix}\right\} \vee \left\{\begin{matrix} c_1 = \pm iM_0c \neq 0 \\ c_0 = c_2 = c_3 = 0 \end{matrix}\right. . \qquad (12.38)$$

From equation (7.9), and taking into account the elements of matrix T_{01}^{01}, we get

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_{01} \\ 0 & 0 & \alpha_{01} & 0 \end{bmatrix} \qquad (12.39)$$

and from equation (7.23) we get

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_{01} \\ 0 & 0 & \alpha_{01} & 0 \end{bmatrix} \begin{bmatrix} j_0 \\ j_1 \\ j_2 \\ j_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and, equivalently,

$$\begin{matrix} -\alpha_{01} j_3 = 0 \\ \alpha_{01} j_2 = 0 \end{matrix}$$

and since $\alpha_{01} \neq 0$ we get

$$j_2 = j_3 = 0 \qquad (12.40)$$

$$j = \begin{bmatrix} j_0 \\ j_1 \\ 0 \\ 0 \end{bmatrix} \qquad (12.41)$$

for the current density four-vector j of the conserved physical quantities in the T_{01}^{01} symmetry.

We obtain a different expression of the four-vector j from equation (5.7). From equation (4.28) we get

$$M = \begin{bmatrix} 0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12.42}$$

for symmetry T_{01}^{01}. Combining equations (5.7) and (3.5) we get

$$j = -\frac{\sigma c^2 b}{\hbar} \Psi M \left((\lambda - \mu) J + \mu C \right)$$
$$j = -\frac{\sigma c^2 b}{\hbar} \Psi \left((\lambda - \mu) MJ + \mu MC \right)$$

and with equations (12.42) and (12.5), (12.14), (12.15) we get

$$j = -\frac{\sigma c^2 b \alpha_{01}}{\hbar} \Psi \left((\lambda - \mu) \begin{bmatrix} J_1 \\ -J_0 \\ 0 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -c_0 \\ 0 \\ 0 \end{bmatrix} \right) \tag{12.43}$$
$$c_0 \neq 0, c_1 = c_2 = c_3 = 0$$

$$j = -\frac{\sigma c^2 b \alpha_{01}}{\hbar} \Psi \left((\lambda - \mu) \begin{bmatrix} J_1 \\ -J_0 \\ 0 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} c_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \tag{12.44}$$
$$c_1 \neq 0, c_0 = c_2 = c_3 = 0$$

where λ, μ the degrees of freedom of the TSV.

For the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ it is $\lambda = \mu = -\frac{1}{2}$ and $\Psi = z$ and, substituting in equations (12.43) and (12.44), we get

$$
\begin{aligned}
&j = j(x_0) = -\frac{\sigma c^2 b \alpha_{01} z}{2\hbar}\begin{bmatrix} 0 \\ c_0 \\ 0 \\ 0 \end{bmatrix} \\
&c_0 \neq 0, c_1 = c_2 = c_3 = 0 \\
&z = \exp\left(-\frac{bc_0}{2\hbar} x_0\right)
\end{aligned}
\tag{12.45}
$$

$$
\begin{aligned}
&j = j(x_1) = \frac{\sigma c^2 b \alpha_{01} z}{2\hbar}\begin{bmatrix} c_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&c_1 \neq 0, c_0 = c_2 = c_3 = 0 \qquad . \\
&z = \exp\left(-\frac{bc_1}{2\hbar} x_1\right)
\end{aligned}
\tag{12.46}
$$

Using succesive $(k,i) = (0,1),(0,2),(0,3),(3,2),(1,3),(2,1)$ in equation (5.17) we get

$$
\begin{aligned}
&\sigma c^2 \alpha_{01}\left(\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2}\right) = \frac{\partial j_1}{\partial x_0} - \frac{\partial j_0}{\partial x_1} \\
&\frac{\partial j_0}{\partial x_2} = 0 \\
&\frac{\partial j_0}{\partial x_3} = 0 \\
&\frac{\partial j_1}{\partial x_3} = 0 \\
&\frac{\partial j_1}{\partial x_2} = 0
\end{aligned}
$$

and taking also into account equation (12.41) we get

$$
\begin{aligned}
&\sigma c^2 \alpha_{01}\left(\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2}\right) = \frac{\partial j_1}{\partial x_0} - \frac{\partial j_0}{\partial x_1} \\
&j_0 = j_0(x_0, x_1) \qquad . \\
&j_1 = j_1(x_0, x_1) \\
&j_2 = j_3 = 0
\end{aligned}
\tag{12.47}
$$

Equations (12.47) give the wave-equation of the TSV for the T_{01}^{01} symmetry.

In the case of the $(\boldsymbol{\alpha},\boldsymbol{\beta})$ field, we know the four-vector j in every symmetry of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$. For the T_{01}^{01} symmetry, the four-vector j of field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is given by equations (12.45) and (12.46). Equations (12.47) concern the case of equations (12.43) and (12.44).

From equations (4.19), (4.20), (12.41) and taking into account the elements of matrix T_{01}^{01} we obtain the USVI of the T_{01}^{01} symmetry:

$$
\begin{aligned}
&\frac{d}{dx_0}\left(\begin{bmatrix} J_0 \\ J_1 \end{bmatrix}\right) = \frac{dQ}{Qdx_0}\begin{bmatrix} J_0 \\ J_1 \end{bmatrix} - \frac{i}{c} zQ \begin{bmatrix} T_0 ic \\ T_1 u_1 \end{bmatrix} - \frac{i}{c} zQ\alpha_{01} \begin{bmatrix} -u_1 \\ ic \end{bmatrix} \\
&J_2 = J_3 = 0 \\
&\frac{d}{dx_0}\left(\begin{bmatrix} P_0 \\ P_1 \end{bmatrix}\right) = -\frac{dQ}{Qdx_0}\begin{bmatrix} J_0 \\ J_1 \end{bmatrix} + \frac{i}{c} zQ \begin{bmatrix} T_0 ic \\ T_1 u_1 \end{bmatrix} + \frac{i}{c} zQ\alpha_{01} \begin{bmatrix} -u_1 \\ ic \end{bmatrix} \\
&P_2 = c_2 \\
&P_3 = c_3
\end{aligned}
\qquad (12.48)
$$

We note that the term

$$\mp \frac{i}{c} zQ \begin{bmatrix} T_0 ic \\ T_1 u_1 \end{bmatrix}$$

on the right-hand side of equations (12.48) is different than zero, as follows from equation (12.8). The fact that this term is not zero in the USVI is a common characteristic of the symmetries of set Ω_1.

In the preceeding study we denoted J the four-vector $J(Q)$. Knowing the four-vector $J(Q)$ we can calculate the four-vectors J and P from equations (7.74). The calculation of the four-vectors J and P can also be performed through equations (11.6) and (11.7). The potential A is given by equations (11.16), (11.25) for $c_0 \neq 0 \wedge c_1 = c_2 = c_3 = 0$ and $c_1 \neq 0 \wedge c_0 = c_2 = c_3 = 0$, as emerges from equations (12.14), (12.15).

We complete the chapter with the observation that for the symmetries of the set Ω_1 theorem 7.3 applies. For symmetry T_{01}^{01} from equation (7.14) we obtain

$$\alpha^2 = \alpha_{01}^2 \neq 0 . \qquad (12.49)$$

From equation (12.42) we get

$$M^2 = -\alpha_{01}^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12.50}$$

From equations (7.15), (7.16) we obtain the pairs of eigenvalues and eigenvectors τ_1, ν_1 and τ_2, ν_2 of matrix M :

$$\tau_1 = i\alpha_{01}$$
$$\nu_1 = \begin{bmatrix} -i \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{12.51}$$

$$\tau_2 = -i\alpha_{01}$$
$$\nu_2 = \begin{bmatrix} i \\ 1 \\ 0 \\ 0 \end{bmatrix} . \tag{12.52}$$

From equation (12.39) we obtain

$$N^2 = -\alpha_{01}^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{12.53}$$

From equations (7.17), (7.18) we get

$$\tau_1 = i\alpha_{01}$$
$$n_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12.54}$$

$$\tau_2 = -i\alpha_{01}$$

$$n_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad . \tag{12.55}$$

Similar conclusions arise for the other symmetries of the set Ω_1.

13. THE SET Ω_4

13.1. Introduction

In this chapter we determine the basic characteristics of set Ω_4. We present the analytical study of the $T_{01033221}$ symmetry. The rest of the symmetries of set $\Omega_4 = \{T_{01023213}, T_{01033221}, T_{02031321}\}$ can be studied in a similar way.

In the symmetries of set Ω_4 the rest masses m_0, $\frac{E_0}{c^2}$ and M_0 are equal to zero, $m_0 = \frac{E_0}{c^2} = M_0 = 0$. The same is true for the four-vector j of the density of the conserved quantities of the generalized particle, which is $j = 0$. Additionally, the generalized particle is always in the form of the generalized photon. These three characteristics are also shared by the $T_{010203321321}$ symmetry of set Ω_6.

13.2. The symmetry $T_{01033221}$

The symmetry $T_{01033221}$ is given by equation (7.1) when $\alpha_{01}\alpha_{03}\alpha_{32}\alpha_{21} \neq 0$ and $\alpha_{02} = \alpha_{13} = 0$

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & T_2 & -\alpha_{32} \\ -\alpha_{03} & 0 & \alpha_{32} & T_3 \end{bmatrix}. \qquad (13.1)$$

$\alpha_{01}\alpha_{03}\alpha_{32}\alpha_{21} \neq 0$

From theorem 7.1 or, equivalently, from equations (7.4) we get $T_0 = T_1 = T_2 = T_3 = 0$ therefore, from equation (13.1) we get

$$T_{01033221} = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & 0 & \alpha_{32} & 0 \end{bmatrix} = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & \frac{\alpha_{03}}{\alpha_{01}} \\ -1 & 0 & -\frac{\alpha_{21}}{\alpha_{01}} & 0 \\ 0 & \frac{\alpha_{21}}{\alpha_{01}} & 0 & -\frac{\alpha_{32}}{\alpha_{01}} \\ -\frac{\alpha_{03}}{\alpha_{01}} & 0 & \frac{\alpha_{32}}{\alpha_{01}} & 0 \end{bmatrix}. \qquad (13.2)$$

We reiterate that, according to the notation we use, the elements $\alpha_{ki}, k \neq i, k, i \in \{0,1,2,3\}$ in the matrices of equation (7.47) have non-zero values.

From equation (2.13) or, equivalently, from equations (7.8) and taking into account the elements of matrix $T_{01033221}$ we get, after the calculations, equations

$$\begin{aligned} J_2 &= -\frac{\alpha_{01}}{\alpha_{21}} J_0 = \frac{\alpha_{03}}{\alpha_{32}} J_0 \\ J_3 &= -\frac{\alpha_{01}}{\alpha_{03}} J_1 = \frac{\alpha_{21}}{\alpha_{32}} J_1 \end{aligned} . \tag{13.3}$$

From equations (13.3) we get

$$\alpha_{01}\alpha_{32} + \alpha_{03}\alpha_{21} = 0 . \tag{13.4}$$

From the second of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get after the calculations

$$\begin{aligned} J_2 &= \frac{\alpha_{21}}{\alpha_{01}} J_0 = -\frac{\alpha_{32}}{\alpha_{03}} J_0 \\ J_3 &= \frac{\alpha_{03}}{\alpha_{01}} J_1 = -\frac{\alpha_{32}}{\alpha_{21}} J_1 \end{aligned} . \tag{13.5}$$

Comparing equations (13.3) and (13.5) we get

$$\begin{aligned} &\alpha_{01}^2 + \alpha_{21}^2 = 0 \\ &\alpha_{01}\alpha_{03} = \alpha_{32}\alpha_{21} \\ &\alpha_{01}^2 + \alpha_{03}^2 = 0 \\ &\alpha_{03}\alpha_{32} = \alpha_{01}\alpha_{21} \\ &\alpha_{03}^2 + \alpha_{32}^2 = 0 \\ &\alpha_{32}^2 + \alpha_{21}^2 = 0 \end{aligned} . \tag{13.6}$$

We solve the system of equations (13.4), (13.6) and get the following four cases for the relation of the physical quantities $\alpha_{01}, \alpha_{03}, \alpha_{32}, \alpha_{21}$:

$$\begin{aligned} (\alpha_{03},\alpha_{32},\alpha_{21}) &= (i\alpha_{01},\alpha_{01},i\alpha_{01}) \\ (\alpha_{03},\alpha_{32},\alpha_{21}) &= (i\alpha_{01},-\alpha_{01},-i\alpha_{01}) \\ (\alpha_{03},\alpha_{32},\alpha_{21}) &= (-i\alpha_{01},\alpha_{01},-i\alpha_{01}) \\ (\alpha_{03},\alpha_{32},\alpha_{21}) &= (-i\alpha_{01},-\alpha_{01},i\alpha_{01}) \end{aligned} . \tag{13.7}$$

One way to solve the system of equations (13.4), (13.6) is to write equation (13.4) in the form

$$\frac{\alpha_{03}}{\alpha_{01}} = -\frac{\alpha_{32}}{\alpha_{21}} = s\,.$$

We then make the replacement

$$\alpha_{03} = s\alpha_{01}$$
$$\alpha_{32} = -s\alpha_{21}$$

in equations (13.6), from where it emerges that $s = \pm i$, and we finally get equations (13.7).

From equations (13.3) and (13.5) we get

$$J = \begin{bmatrix} J_0 \\ J_1 \\ \frac{\alpha_{21}}{\alpha_{01}} J_0 \\ \frac{\alpha_{03}}{\alpha_{01}} J_1 \end{bmatrix}. \tag{13.8}$$

From the first of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get after the calculations

$$C = \begin{bmatrix} c_0 \\ c_1 \\ \frac{\alpha_{21}}{\alpha_{01}} c_0 \\ \frac{\alpha_{03}}{\alpha_{01}} c_1 \end{bmatrix}. \tag{13.9}$$

In the symmetry $T_{01033221}$ it is $\alpha_{01} \neq 0$, therefore, from equations (3.4) and (4.4) we get

$$zQ\alpha_{01} = \lambda_{01} = \frac{b}{2\hbar}\left(c_1 J_0 - c_0 J_1\right) \neq 0$$

$$c_0 J_1 \neq c_1 J_0\,. \tag{13.10}$$

Because of relation (13.10) the four-vectors J and C, as given by equations (13.8) and (13.9) cannot be parallel.

From equation (13.8) we get

$$P = P(Q) = -J(Q) = -\begin{bmatrix} J_0 \\ J_1 \\ \dfrac{\alpha_{21}}{\alpha_{01}} J_0 \\ \dfrac{\alpha_{03}}{\alpha_{01}} J_1 \end{bmatrix}. \tag{13.11}$$

From equations (2.7) and (13.8) we get

$$J_0^2 + J_1^2 + \frac{\alpha_{21}^2}{\alpha_{01}^2} J_0 + \frac{\alpha_{03}^2}{\alpha_{01}^2} J + m_0^2 c^2 = 0$$

$$\frac{J_0^2}{\alpha_{01}^2}\left(\alpha_{01}^2 + \alpha_{21}^2\right) + \frac{J_1^2}{\alpha_{01}^2}\left(\alpha_{01}^2 + \alpha_{03}^2\right) + m_0^2 c^2 = 0$$

and with equations (13.6) we get

$$0 + m_0^2 c^2 = 0$$

$$m_0 = 0\,. \tag{13.12}$$

Similarly, from equations (2.8), (13.11) and (3.6), (13.9) we get

$$E_0 = 0 \tag{13.13}$$

$$M_0 = 0\,. \tag{13.14}$$

Notice that while the rest masses m_0, $\frac{E_0}{c^2}$ and M_0 vanish, the Selfvariations continue to exist due to equation (7.77)

$$\begin{aligned} &\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki}, k, i = 0,1,2,3 \\ &J + P = C \\ &TJ = 0 \end{aligned}\,.$$

The Selfvariations exist in every case where $J \neq 0$.

From equations (13.7) and (13.2), (13.8), (13.9) emerge four sets of matrices $T_{01033221}$ and corresponding four-vectors J and C, as given by equations (13.15), (13.16), (13.17), (3.18):

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(i\alpha_{01},\alpha_{01},i\alpha_{01}\right)$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_0 \\ c_1 \\ ic_0 \\ ic_1 \end{bmatrix} \quad J = \begin{bmatrix} J_0 \\ J_1 \\ iJ_0 \\ iJ_1 \end{bmatrix} \tag{13.15}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(i\alpha_{01},-\alpha_{01},-i\alpha_{01}\right)$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_0 \\ c_1 \\ -ic_0 \\ ic_1 \end{bmatrix} \quad J = \begin{bmatrix} J_0 \\ J_1 \\ -iJ_0 \\ iJ_1 \end{bmatrix} \tag{13.16}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(-i\alpha_{01},\alpha_{01},-i\alpha_{01}\right)$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & -1 \\ i & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_0 \\ c_1 \\ -ic_0 \\ -ic_1 \end{bmatrix} \quad J = \begin{bmatrix} J_0 \\ J_1 \\ -iJ_0 \\ -iJ_1 \end{bmatrix} \tag{13.17}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(-i\alpha_{01},-\alpha_{01},i\alpha_{01}\right)$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_0 \\ c_1 \\ ic_0 \\ -ic_1 \end{bmatrix} \quad J = \begin{bmatrix} J_0 \\ J_1 \\ iJ_0 \\ -iJ_1 \end{bmatrix} \tag{13.18}$$

The condition (15.25), $c_0 J_1 \neq c_1 J_0$, is not necessarily valid for $c_0 c_1 \neq 0$ and $J_0 J_1 \neq 0$. It is easy to demonstrate that the condition (13.19), and hence equations (13.15)-(13.18) are valid for $c_0 \neq 0, J_1 \neq 0, c_1 = 0, J_0 = 0$ and also for $c_1 \neq 0, J_0 \neq 0, c_0 = 0, J_1 = 0$. These conditions complete the study of the symmetry $T_{01033221}$, and must be considered when studying the interaction $T_{01033221}$. In these cases frome the equations (13.15)-(13.18) we get

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(i\alpha_{01},\alpha_{01},i\alpha_{01}\right),c_0 \neq 0, J_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix} \quad C = c_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \quad J = J_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \tag{13.19}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(i\alpha_{01},\alpha_{01},i\alpha_{01}\right),c_1 \neq 0, J_0 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix} \quad C = c_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \quad J = J_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \tag{13.20}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(i\alpha_{01},-\alpha_{01},-i\alpha_{01}\right),c_0 \neq 0, J_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{bmatrix} \quad C = c_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \quad J = J_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \tag{13.21}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(i\alpha_{01},-\alpha_{01},-i\alpha_{01}\right),c_1\neq 0,J_0\neq 0$$

$$T=zQ\alpha_{01}\begin{bmatrix}0&1&0&i\\-1&0&i&0\\0&-i&0&1\\-i&0&-1&0\end{bmatrix}\quad C=c_1\begin{bmatrix}0\\1\\0\\i\end{bmatrix}\quad J=J_0\begin{bmatrix}1\\0\\i\\0\end{bmatrix}\tag{13.22}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(-i\alpha_{01},\alpha_{01},-i\alpha_{01}\right),c_0\neq 0,J_1\neq 0$$

$$T=zQ\alpha_{01}\begin{bmatrix}0&1&0&-i\\-1&0&i&0\\0&-i&0&-1\\i&0&1&0\end{bmatrix}\quad C=c_0\begin{bmatrix}1\\0\\i\\0\end{bmatrix}\quad J=J_1\begin{bmatrix}0\\1\\0\\i\end{bmatrix}\tag{13.23}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(-i\alpha_{01},\alpha_{01},-i\alpha_{01}\right),c_1\neq 0,J_0\neq 0$$

$$T=zQ\alpha_{01}\begin{bmatrix}0&1&0&-i\\-1&0&i&0\\0&-i&0&-1\\i&0&1&0\end{bmatrix}\quad C=c_1\begin{bmatrix}0\\1\\0\\i\end{bmatrix}\quad J=J_0\begin{bmatrix}1\\0\\i\\0\end{bmatrix}\tag{13.24}$$

$$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(-i\alpha_{01},-\alpha_{01},i\alpha_{01}\right),c_0\neq 0,J_1\neq 0$$

$$T=zQ\alpha_{01}\begin{bmatrix}0&1&0&-i\\-1&0&-i&0\\0&i&0&1\\i&0&-1&0\end{bmatrix}\quad C=c_0\begin{bmatrix}1\\0\\i\\0\end{bmatrix}\quad J=J_1\begin{bmatrix}0\\1\\0\\i\end{bmatrix}\tag{13.25}$$

$\left(\alpha_{03},\alpha_{32},\alpha_{21}\right)=\left(-i\alpha_{01},-\alpha_{01},i\alpha_{01}\right),c_1\neq 0,J_0\neq 0$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & -1 & 0 \end{bmatrix} \quad C = c_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \quad J = J_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \tag{13.26}$$

In the symmetry $T_{01033221}$ it is $T_0 = T_1 = T_2 = T_3 = 0$ therefore, from equation (4.11) we get $\Lambda = 0$, and from relation (7.38) of theorem 7.5 we get

$$j = -\frac{\sigma bc^2}{\hbar}\mu\Psi MC \tag{13.27}$$

for the current density four-vector j of the conserved physical quantities of the generalized particle. From equation (4.28), and considering the elements of symmetry $T_{01033221}$, we get

$$M = \begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & 0 & \alpha_{32} & 0 \end{bmatrix}. \tag{13.28}$$

From equations (13.18) and (13.9), (13.27) we get

$$j = -\frac{\sigma bc^2}{\hbar}\mu\Psi\begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & 0 & \alpha_{32} & 0 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ \dfrac{\alpha_{21}}{\alpha_{01}}c_0 \\ \dfrac{\alpha_{03}}{\alpha_{01}}c_1 \end{bmatrix}$$

$$j = -\frac{\sigma bc^2}{\hbar}\mu\Psi\begin{bmatrix} \alpha_{01}c_1 + \dfrac{\alpha_{03}^2}{\alpha_{01}}c_1 \\ -\alpha_{01}c_0 - \dfrac{\alpha_{21}^2}{\alpha_{01}}c_0 \\ \alpha_{21}c_1 - \dfrac{\alpha_{32}\alpha_{03}}{\alpha_{01}}c_1 \\ -\alpha_{03}c_0 + \dfrac{\alpha_{32}\alpha_{21}}{\alpha_{01}}c_0 \end{bmatrix}$$

$$j=-\frac{\sigma bc^2}{\hbar\alpha_{01}}\mu\Psi\begin{bmatrix}\left(\alpha_{01}^2+\alpha_{03}^2\right)c_1\\-\left(\alpha_{01}^2+\alpha_{21}^2\right)c_0\\\left(\alpha_{01}\alpha_{21}-\alpha_{32}\alpha_{03}\right)c_1\\\left(-\alpha_{01}\alpha_{03}+\alpha_{32}\alpha_{21}\right)c_0\end{bmatrix}$$

and with equations (13.6) we get

$$j=0. \tag{13.29}$$

Consequently, there is no flow of conserved physical quantities of the generalized perticle into the part of spacetime it occupies.

From the combination of equations (5.17) and (13.29) we get the wave equation

$$\nabla^2\Psi+\frac{\partial^2\Psi}{\partial x_0^2}=0 \tag{13.30}$$

for the $T_{01033221}$ symmetry. Additionally, for $j=0$ equation (5.24) of theorem 5.2 is fulfilled, so from equation (5.25) we get

$$\begin{aligned}&\nabla^2\boldsymbol{\xi}-\frac{\partial^2\boldsymbol{\xi}}{c^2\partial t^2}=0\\&\nabla^2\boldsymbol{\omega}-\frac{\partial^2\boldsymbol{\omega}}{c^2\partial t^2}=0\end{aligned} \tag{13.31}$$

for the field $(\boldsymbol{\xi},\boldsymbol{\omega})$. Consequently, in the symmetries of set Ω_4, the generalized particle is always in the form of the generalized photon.

In the preceeding study we denoted J the four-vector $J(Q)$. Knowing the four-vector $J(Q)$ we can calculate the four-vectors J and P from equations (7.74). The $T_{01033221}$ symmetry has twelve versions as defined by equations (13.19)-(13.26). Consequently, from the cobination of equations (13.15)-(13.18) and (13.19)-(13.26) twelve versions of the four-vectors J and P emerge. The four-vectors J and P can also be calculated through equations (11.6), (11.7) and (11.16). We note that in the $T_{01033221}$ symmetry, equation (4.33) is applied since it can be $c_1c_2\neq 0$ in the equations (13.15)-(13.18), and since is $\left(c_0c_2=ic_0^2\neq 0\right)\vee\left(c_1c_3=ic_1^2\neq 0\right)$ in the equations (13.19)-(13.26). Moreover, from corollaries 11.1 and 11.2 it is straightforward to show that the gauge function is $f = 0$ for the symmetries (13.15)-(13.18), and it may be $f \neq 0$ for the symmetries (13.19)-(13.26).

For every case of equations (13.7) we can calculate the USVI from equations (4.19) and (4.20). We note that the term

$$\frac{i}{c} zQ\Lambda u$$

of the USVI vanishes

$$\frac{i}{c} zQ\Lambda u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

since in the $T_{01033221}$ symmetry it is $\Lambda = 0$.

14. THE EMPTY SET Ω_5 AND THE DIMENSIONALITY OF SPACETIME. THE EQUATIONS OF THE TSV IN $N \in \mathbb{N}$ DIMENSIONS

14.1. Introduction

In this chapter we prove that Ω_5 is the empty set, $\Omega_5 = \varnothing$. We come to this conclusion by applying the $SV-T$ method. We present the detailed study for the $T_{0103321321}$ symmetry. The proof for the remaining symmetries of set $\Omega_5 = \{T_{0102033213}, T_{0102033221}, T_{0102023213}, T_{0102321321}, T_{0103321321}, T_{0203321321}\}$ is similar.

We present the analytical proof process for the rejection of the symmetries of set Ω_5 by the $SV-T$ method. The analytical application of the method shows that the rejection of the set Ω_5 is due to the number $\mathrm{N}=4$ of spacetime dimensions in which we have formulated the equations of the TSV.

The T matrices of the external symmetry depend on the number $\mathrm{N} \in \mathbb{N}$ of spacetime dimensions. We present the equations and the way in which the TSV is formulated in N dimensions for every $\mathrm{N} \in \mathbb{N}$.

14.2. The $T_{0103321321}$ symmetry. The equations of the TSV in $\mathrm{N} \in \mathbb{N}$ dimensions

The $T_{0103321321}$ symmetry is given by equation (7.1) for $\alpha_{01}\alpha_{03}\alpha_{32}\alpha_{13}\alpha_{21} \neq 0$ and $\alpha_{02} = 0$

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & T_2 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & T_3 \end{bmatrix}. \tag{14.1}$$

$\alpha_{01}\alpha_{03}\alpha_{32}\alpha_{13}\alpha_{21} \neq 0$

From theorem 7.1 or, equivalently, from equations (7.4) we get $T_0 = T_1 = T_2 = T_3 = 0$, therefore, from equation (14.1) we get

$$T_{0103321321} = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix}. \tag{14.2}$$

We note again that, according to the notation we follow, the elements $\alpha_{ki}, k \neq i, k, i \in \{0,1,2,3\}$ in the matrices of the sets of equation (7.47) are non-zero.

From equation (2.13) or, equivalently, from equations (7.8) and taking into account the elements of matrix $T_{0103321321}$ we get, after the calculations, equations

$$J_2 = \frac{\alpha_{03}}{\alpha_{32}} J_0 + \frac{\alpha_{13}}{\alpha_{32}} J_1$$
$$J_3 = \frac{\alpha_{21}}{\alpha_{32}} J_1 = -\frac{\alpha_{01}}{\alpha_{03}} J_1 .$$

(14.3)

From the second of equations (4.6) for $(i, \nu, k) = (0,1,2), (0,1,3), (0,2,3), (1,2,3)$ we get after the calculations

$$J_2 = \frac{\alpha_{21}}{\alpha_{01}} J_0 = -\frac{\alpha_{32}}{\alpha_{03}} J_0$$
$$J_3 = -\frac{\alpha_{13}}{\alpha_{01}} J_0 + \frac{\alpha_{03}}{\alpha_{01}} J_1 . \qquad (14.4)$$

The application of the $SV - T$ method, whether to equations (14.3) or to equations (14.4), rejects the $T_{0103321321}$ symmetry. We apply the method to equations (14.3).

From the second of equations (14.3) we get

$$\alpha_{32} J_3 = \alpha_{21} J_1 \qquad (14.5)$$

and differentiating with respect to $x_\nu, \nu = 0,1,2,3$, and taking into account equation (7.43) we get

$$\alpha_{32} \left(\frac{b}{\hbar} P_\nu J_3 + zQ\alpha_{\nu 3} \right) = \alpha_{21} \left(\frac{b}{\hbar} P_\nu J_1 + zQ\alpha_{\nu 1} \right)$$

and with equation (14.5) we get

$$\alpha_{32} zQ\alpha_{\nu 3} = \alpha_{21} zQ\alpha_{\nu 1}$$

and since $zQ \neq 0$ we get

$$\alpha_{32}\alpha_{\nu 3} = \alpha_{21}\alpha_{\nu 1} . \qquad (14.6)$$

Setting successively $\nu = 0,1,2,3$ in equation (14.6) we get:

For $\nu = 0$,

$$\alpha_{32}\alpha_{03} = \alpha_{21}\alpha_{01}.$$

For $\nu = 1$,

$$\alpha_{32}\alpha_{13} = \alpha_{21}\alpha_{11}$$
$$\alpha_{32}\alpha_{13} = \alpha_{21}T_1 = 0$$

$$\alpha_{32}\alpha_{13} = 0. \qquad (14.7)$$

Equality (14.7) is impossible in the $T_{0103321321}$ symmetry.

For $\nu = 2$,

$$\alpha_{32}\alpha_{23} = \alpha_{21}\alpha_{21}.$$

For $\nu = 3$,

$$\alpha_{32}\alpha_{33} = \alpha_{21}\alpha_{31}$$
$$\alpha_{32}T_3 = -\alpha_{21}\alpha_{13}$$

and we finally get

$$\alpha_{21}\alpha_{13} = 0 \qquad (14.8)$$

since $T_3 = 0$. Equality (14.8) is impossible in the $T_{0103321321}$ symmetry.

Fro the second of equations (14.3) we get

$$a_{03}J_3 = -a_{01}J_1$$

and, differentiating with respect to $x_\nu, \nu = 0,1,2,3$, we get

$$\alpha_{03}\alpha_{\nu 3} = -\alpha_{01}\alpha_{\nu 1} \qquad (14.9)$$

after the calculations.

Setting successively $\nu = 0,1,2,3$ in equation (14.9) we get:

For $\nu = 0$,

$\alpha_{03}\alpha_{03} = -\alpha_{01}\alpha_{01}$.

For $\nu = 1$,

$$\alpha_{03}\alpha_{13} = -\alpha_{01}\alpha_{11}$$
$$\alpha_{03}\alpha_{13} = -\alpha_{01}T_1 = 0$$

and we finally get

$$\alpha_{03}\alpha_{13} = 0\,. \qquad (14.10)$$

Equality (14.10) is impossible in the $T_{0103321321}$ symmetry.

For $\nu = 2$,

$$\alpha_{03}\alpha_{23} = -\alpha_{01}\alpha_{21}\,.$$

For $\nu = 3$,

$$\alpha_{03}\alpha_{33} = -\alpha_{01}\alpha_{31}$$
$$\alpha_{03}T_3 = a_{01}a_{13}$$

and we finally get

$$\alpha_{21}\alpha_{13} = 0 \qquad (14.11)$$

since $T_3 = 0$. Equality (14.11) is impossible in the $T_{0103321321}$ symmetry.

From the first of equations (14.3) we get

$$\alpha_{32}J_2 = \alpha_{03}J_0 + \alpha_{13}J_1$$

and, differentiating with respect to $x_\nu, \nu = 0,1,2,3$, we get

$$\alpha_{32}\alpha_{\nu 2} = \alpha_{03}\alpha_{\nu 0} + \alpha_{13}\alpha_{\nu 1} \qquad (14.12)$$

after the calculations.

Setting successively $\nu = 0,1,2,3$ in equation (14.12) we get:

For $\nu = 0$,

$$\alpha_{32}\alpha_{02} = \alpha_{03}\alpha_{20} + \alpha_{13}\alpha_{01}$$
$$0 = 0 + \alpha_{13}\alpha_{01}$$

and we finally get

$$\alpha_{13}\alpha_{01} = 0\,. \tag{14.13}$$

Equality (14.13) is impossible in the $T_{0103321321}$ symmetry.

For $\nu = 1$,

$$\alpha_{32}\alpha_{12} = \alpha_{03}\alpha_{10} + \alpha_{13}\alpha_{11}$$
$$-\alpha_{32}\alpha_{21} = -\alpha_{03}\alpha_{01} + \alpha_{13}T_1\,.$$
$$\alpha_{32}\alpha_{21} = \alpha_{03}\alpha_{01}$$

For $\nu = 2$,

$$\alpha_{32}\alpha_{22} = \alpha_{03}\alpha_{20} + \alpha_{13}\alpha_{21}$$
$$\alpha_{32}T_2 = -\alpha_{03}\alpha_{02} + \alpha_{13}\alpha_{21}$$

and we finally get

$$\alpha_{13}\alpha_{21} = 0 \tag{14.14}$$

since $\alpha_{02} = 0$ and $T_2 = 0$ in the $T_{0103321321}$ symmetry. Equality (14.14) is impossible in the $T_{0103321321}$ symmetry.

For $\nu = 3$,

$$\alpha_{32}\alpha_{32} = \alpha_{03}\alpha_{30} + \alpha_{13}\alpha_{31}\,.$$

The $T_{0103321321}$ symmetry is rejected since it is not compatible with equations (14.7), (14.8), (14.10), (14.11), (14.13), (14.14), which emerge from the $SV-T$ method. It is obvious that one of these equations suffices to reject the $T_{0103321321}$ symmetry. However, there is a very substantial reason for which we completed the proof, arriving at all these equations and not just the first one.

The analytical application of the $SV-T$ method, which we presented, showed that the matrices T of the external symmetry depend on the number $\mathrm{N} \in \mathbb{N}$ of spacetime dimensions. The only way to not have a new equation emerge during the differentiations with respect to $x_\nu, \nu \in \{0,1,2,3\}$, during the application of the $SV-T$ method, is for the four-vector J to be independent of $x_\nu, \nu \in \{0,1,2,3\}$. Therefore, in the cases where ν does not attain all values $\nu = 0,1,2,3$, the $SV-T$ method gives the $x_\nu, \nu \in \{0,1,2,3\}$ that the symmetry does not depend on (therefore also the $x_k k \in \{0,1,2,3\}$ on which it depends). In equations (14.7), (14.8), (14.10), (14.11), (14.13), (14.14), all values $\nu = 0,1,2,3$ appear. This is the real reason why the set Ω_5 is rejected ($\Omega_5 = \varnothing$). The set Ω_5 is rejected *in the 4-dimensional spacetime*. If we could increase the spacetime dimensions and, similarly, the number $\mathrm{N} \in \mathbb{N}$ of dimensions, the set Ω_5 would not be rejected ($\Omega_5 \neq \varnothing$).

In spacetimes with more than four dimensions, set $\Omega_{\mathrm{O}} = \{T = 0\}$ of the internal symmetry remains the same, but the number of sets $\Omega_0, \Omega_1, \Omega_2, \Omega_3, ..., \Omega_{\left(\frac{\mathrm{N}(\mathrm{N}-1)}{2}\right)}$ of equations (7.47), as well as the symmetries-elements of the sets, are altered. The number $\mathrm{N} \in \mathbb{N}$ of spacetime dimensions defines the symmetries contained in each of the sets

$$\Omega_{\mathrm{O}}, \Omega_0, \Omega_1, \Omega_2, ..., \Omega_{\left(\frac{\mathrm{N}(\mathrm{N}-1)}{2}\right)},$$

as well as which of the sets

$$\Omega_0, \Omega_1, \Omega_2, ..., \Omega_{\left(\frac{\mathrm{N}(\mathrm{N}-1)}{2}\right)}$$

are rejected.

Assigning one component of momentum to each dimension of a spacetime of $\mathrm{N} \in \mathbb{N}$ dimensions, equations (2.7) (2.8), and (3.6) are written in the form

$$\begin{aligned} & J_0^2 + J_1^2 + J_2^2 + ... + J_{\mathrm{N}-1}^2 + m_0^2 c^2 = 0 \\ & P_0^2 + P_1^2 + P_2^2 + ... + P_{\mathrm{N}-1}^2 + \frac{E_0^2}{c^2} = 0 \\ & c_0^2 + c_1^2 + c_2^2 + ... + c_{\mathrm{N}-1}^2 + M_0^2 c^2 = 0 \\ & \mathrm{N} \in \mathbb{N} \end{aligned} \quad . \qquad (14.15)$$

Combining equations (14.15) with equation (7.77)

$$\begin{aligned}&\frac{\partial J_i}{\partial x_k}=\frac{b}{\hbar}P_kJ_i+zQ\alpha_{ki},k,i=0,1,2,...,\mathrm{N}-1\\&J+P=C\\&TJ=0\end{aligned}\tag{14.16}$$

we get the TSV in $\mathrm{N}\in\mathbb{N}$ dimensions. Matrix T is $\mathrm{N}\times\mathrm{N}$, the N -vectors J , P and C are column matrices, $\mathrm{N}\times 1$, and function z is

$$z=\exp\left(-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+...+c_{\mathrm{N}-1}x_{\mathrm{N}-1}\right)\right).\tag{14.17}$$

Similarly, function Φ is

$$\Phi=K\exp\left(-\frac{b}{\hbar}\left(c_0x_0+c_1x_1+...+c_{\mathrm{N}-1}x_{\mathrm{N}-1}\right)\right).\tag{14.18}$$

Following the same process we used for $\mathrm{N}=4$, it is proven that all theorems of the TSV we have formulated are valid for every $\mathrm{N}\in\mathbb{N}$. The only difference is that in the place of $\mathrm{N}=4$, we have $\mathrm{N}\in\mathbb{N}$. Indicatively we mention equations (3.4) and (4.10) concerning the external symmetry, which become

$$\begin{aligned}&\lambda_{ki}=zQ\alpha_{ki}=\frac{b}{2\hbar}\left(J_kP_i-J_iP_k\right)=\frac{b}{2\hbar}\left(c_iJ_k-c_kJ_i\right)=\frac{b}{2\hbar}\left(c_kP_i-c_iP_k\right)\\&k\neq i,k,i=0,1,2,...,\mathrm{N}-1\end{aligned}\tag{14.19}$$

and

$$T_k=\alpha_{kk}=\frac{\lambda_{kk}}{zQ},k=0,1,2,...,\mathrm{N}-1\tag{14.20}$$

and the USVI where equations (4.19), (4.20) becomes

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J - \frac{izQ}{c} \Lambda u - \frac{izQ}{c} \begin{bmatrix} \sum\limits_{\substack{k=0 \\ k\neq 0}}^{\mathrm{N}-1} u_k \alpha_{k0} \\ \sum\limits_{\substack{k=0 \\ k\neq 1}}^{\mathrm{N}-1} u_k \alpha_{k1} \\ \bullet \\ \bullet \\ \bullet \\ \sum\limits_{\substack{k=0 \\ k\neq j}}^{\mathrm{N}-1} u_k \alpha_{kj} \\ \bullet \\ \bullet \\ \bullet \\ \sum\limits_{\substack{k=0 \\ k\neq \mathrm{N}-1}}^{\mathrm{N}-1} u_k \alpha_{k(\mathrm{N}-1)} \end{bmatrix} \tag{14.21}$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Qdx_0} J + \frac{izQ}{c} \Lambda u + \frac{izQ}{c} \begin{bmatrix} \sum\limits_{\substack{k=0 \\ k\neq 0}}^{\mathrm{N}-1} u_k \alpha_{k0} \\ \sum\limits_{\substack{k=0 \\ k\neq 1}}^{\mathrm{N}-1} u_k \alpha_{k1} \\ \bullet \\ \bullet \\ \bullet \\ \sum\limits_{\substack{k=0 \\ k\neq j}}^{\mathrm{N}-1} u_k \alpha_{kj} \\ \bullet \\ \bullet \\ \bullet \\ \sum\limits_{\substack{k=0 \\ k\neq \mathrm{N}-1}}^{\mathrm{N}-1} u_k \alpha_{k(\mathrm{N}-1)} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} T_0 & 0 & 0 & 0 & \bullet & \bullet & 0 \\ 0 & T_1 & 0 & 0 & \bullet & \bullet & 0 \\ 0 & 0 & T_2 & 0 & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & 0 & T_{N-2} & 0 \\ 0 & 0 & \bullet & \bullet & \bullet & 0 & T_{N-1} \end{bmatrix}$$

$$u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ \bullet \\ \bullet \\ \bullet \\ u_{N-1} \end{bmatrix} = \begin{bmatrix} \frac{dx_0}{dt} \\ \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \bullet \\ \bullet \\ \bullet \\ \frac{dx_{N-1}}{dt} \end{bmatrix} = \begin{bmatrix} ic \\ \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \bullet \\ \bullet \\ \bullet \\ \frac{dx_{N-1}}{dt} \end{bmatrix}.$$

Taking into account equation (4.28) we can write equations (14.21) in the form

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J - \frac{izQ}{c} \Lambda u + \frac{izQ}{c} Mu$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Qdx_0} J + \frac{izQ}{c} \Lambda u - \frac{izQ}{c} Mu \tag{14.22}$$

$$M = \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} & \bullet & \bullet & \alpha_{0(N-1)} \\ \alpha_{10} & 0 & \alpha_{12} & \alpha_{13} & \bullet & \bullet & \alpha_{1(N-1)} \\ \alpha_{20} & \alpha_{21} & 0 & \alpha_{23} & \bullet & \bullet & \alpha_{2(N-1)} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \alpha_{(N-2)0} & \alpha_{(N-2)1} & \bullet & \bullet & \alpha_{(N-2)(N-3)} & 0 & \alpha_{(N-2)(N-1)} \\ \alpha_{(N-1)0} & \alpha_{(N-1)1} & \bullet & \bullet & \bullet & \alpha_{(N-1)(N-2)} & 0 \end{bmatrix}.$$

Looking at equations (14.7), (14.8), (14.10), (14.11), (14.13) and (14.14) we find that they all contain the physical quantity α_{13} . Therefore, for $\alpha_{13}=0$ the disagreement with the $SV-T$ method is lifted. Setting $\alpha_{13}=0$ in equation (14.2), symmetry $T_{01033221}$ emerges. From equations (7.47) we see that this symmetry indeed exists in set Ω_4 . It is the symmetry which we studied in the previous chapter.

From the application of the $SV-T$ method to symmetry $T_{0103321321}$, which we presented above, we obtained a set of equations which were not numbered. These equations are

$$\alpha_{32}\alpha_{03}=\alpha_{21}\alpha_{01}$$

$$\alpha_{32}\alpha_{23}=\alpha_{21}\alpha_{21}$$

$$\alpha_{03}\alpha_{03}=-\alpha_{01}\alpha_{01}$$

$$\alpha_{03}\alpha_{23}=-\alpha_{01}\alpha_{21}$$

$$\alpha_{32}\alpha_{21}=\alpha_{03}\alpha_{01}$$

$$\alpha_{32}\alpha_{32}=\alpha_{03}\alpha_{30}+\alpha_{13}\alpha_{31}$$

from which for $\alpha_{13}=0$, and because of $\alpha_{ik}=-\alpha_{ki}\forall i\neq k,i,k=0,1,2,3$, we have

$$\begin{aligned}&\alpha_{32}\alpha_{03}=\alpha_{21}\alpha_{01}\\&\alpha_{32}^2+\alpha_{21}^2=0\\&\alpha_{03}^2+\alpha_{01}^2=0\\&\alpha_{32}\alpha_{21}=\alpha_{03}\alpha_{01}\\&\alpha_{32}^2+\alpha_{03}^2=0\end{aligned}\quad. \qquad (14.23)$$

It is easy proven that the equations (14.23) are equivalent with equations (13.6) of the previous chapter.

The equations of the TSV apply when, in equation (14.15), at least one component of the momentum J is non-zero. Therefore, the TSV is formulated for every $\mathrm{N}\in\mathbb{N}$. By induction we can conclude that the equations of the TSV are also valid for $\mathrm{N}\to\infty$.

It is easily proven that symmetry $T_{0103321321}$, as well as the rest of the symmetries of the set

$$\Omega_5 = \{T_{0102033213}, T_{0102033221}, T_{0102023213}, T_{0102321321}, T_{0103321321}, T_{0203321321}\} \text{ for } \mathrm{N} = 4,$$

are also rejected for every $\mathrm{N} > 4, \mathrm{N} \in \mathbb{N}$;

$$\left(\mathrm{N} > 4, \mathrm{N} \in \mathbb{N}, \{T_{0102033213}, T_{0102033221}, T_{0102023213}, T_{0102321321}, T_{0103321321}, T_{0203321321}\} = \varnothing\right).$$

The set Ω_5 is not rejected for $\mathrm{N} > 4, \mathrm{N} \in \mathbb{N}$, that is $\Omega_5 \neq \varnothing$, since the set Ω_5 for $\mathrm{N} > 4, \mathrm{N} \in \mathbb{N}$ includes more symmetries than the set

$$\Omega_5 = \{T_{0102033213}, T_{0102033221}, T_{0102023213}, T_{0102321321}, T_{0103321321}, T_{0203321321}\} \text{ for } \mathrm{N} = 4.$$

This conclusion is general and applies to all external symmetries as we increase the dimensions $\mathrm{N} \in \mathbb{N}$ of the spacetime.

15. THE SET Ω_6

15.1. Introduction

In this chapter we determine the basic characteristics of the $T_{010203321321}$ symmetry of set Ω_6. It is the only symmetry in which all physical quantities $\alpha_{ki}, k \neq i, k, i = 0,1,2,3$ are non-zero.

As is the case in the symmetries of set Ω_4, the rest masses m_0, $\frac{E_0}{c^2}$ and M_0 vanish, $m_0 = \frac{E_0}{c^2} = M_0 = 0$ in four-dimensional space time. The same is true for the four-vector j of the density of the conserved quantities of the generalized particle, which is $j = 0$. Additionally, the generalized particle is always in the form of the generalized photon. On top of this, the $T_{010203321321}$ symmetry has two properties not shared by any other symmetry of the set $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4$ in four-dimensional spacetime. The first is that the vectors $\mathbf{n}$ and $\boldsymbol{\tau}$ of equations (7.25), (7.24) are equal or opposite, $\boldsymbol{\tau} = \pm\mathbf{n} \neq \mathbf{0}$. The second property, which is a consequence of the first, is that the fundamental matrices M and N are equal or opposite, $M = \pm N$.

In some of the equations we present, the symbol $\pm$ appears. In these equations, either all of the ¨up¨ or all of the ¨down¨ signs apply. For $\boldsymbol{\tau} = +\mathbf{n}$ the up signs apply, while for $\boldsymbol{\tau} = -\mathbf{n}$ the down.

15.2. The symmetry $T_{010203321321}$

The $T_{010203321321}$ symmetry is given by equation (7.1) when $a_{ki} \neq 0, \forall k \neq i, k, i = 0,1,2,3$

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & T_2 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & T_3 \end{bmatrix}. \qquad (15.1)$$

$\alpha_{01}\alpha_{02}\alpha_{03}\alpha_{32}\alpha_{13}\alpha_{21} \neq 0$

From theorem 7.1 or, equivalently, from equations (7.4) we get $T_0 = T_1 = T_2 = T_3 = 0$, therefore, from equation (15.1) we get

$$T_{010203321321} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix}. \tag{15.2}$$

We reiterate that, according to the notation we follow, the elements $\alpha_{ki}, k \neq i, k, i \in \{0,1,2,3\}$ in the matrices of the sets of equation (7.47) are non-zero.

From equation (2.13) or, equivalently, from equations (7.8) and taking into account the elements of matrix $T_{010203321321}$ we get

$$\begin{aligned} &J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \\ &-J_0\alpha_{01} - J_2\alpha_{21} + J_3\alpha_{13} = 0 \\ &-J_0\alpha_{02} + J_1\alpha_{21} - J_3\alpha_{32} = 0 \\ &-J_0\alpha_{03} - J_1\alpha_{13} + J_2\alpha_{32} = 0 \end{aligned}. \tag{15.3}$$

We apply the $SV-T$ method to equations (15.3). We differentiate the first

$$J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \tag{15.4}$$

with respect to $x_\nu, \nu = 0,1,2,3$ and, taking into account equation (7.43) we get

$$\alpha_{01}\left(\frac{b}{\hbar}P_\nu J_1 + zQ\alpha_{\nu 1}\right) + \alpha_{02}\left(\frac{b}{\hbar}P_\nu J_2 + zQ\alpha_{\nu 2}\right) + \alpha_{03}\left(\frac{b}{\hbar}P_\nu J_3 + zQ\alpha_{\nu 3}\right) = 0$$

$$\frac{b}{\hbar}P_\nu\left(J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03}\right) + zQ\left(\alpha_{01}\alpha_{\nu 1} + \alpha_{02}\alpha_{\nu 2} + \alpha_{03}\alpha_{\nu 3}\right) = 0$$

and with equation (15.4) we get

$$zQ\left(\alpha_{01}\alpha_{\nu 1} + \alpha_{02}\alpha_{\nu 2} + \alpha_{03}\alpha_{\nu 3}\right) = 0$$

and because $zQ \neq 0$ we get

$$\alpha_{01}\alpha_{\nu 1} + \alpha_{02}\alpha_{\nu 2} + \alpha_{03}\alpha_{\nu 3} = 0. \tag{15.5}$$

Setting successively $\nu = 0,1,2,3$ into equation (15.5) we get:

For $\nu = 0$,

$$a_{01}^2 + a_{02}^2 + a_{03}^2 = 0. \tag{15.6}$$

For $\nu = 1$,

$$\alpha_{01}\alpha_{11} + \alpha_{02}\alpha_{12} + \alpha_{03}\alpha_{13} = 0$$
$$\alpha_{01}T_1 - \alpha_{02}\alpha_{21} + \alpha_{03}\alpha_{13} = 0$$
$$0 - a_{02}a_{21} + a_{03}a_{13} = 0$$
$$a_{03}a_{13} = a_{02}a_{21}$$

$$\frac{a_{13}}{a_{02}} = \frac{a_{21}}{a_{03}}. \tag{15.7}$$

For $\nu = 2$,

$$a_{01}a_{21} + a_{02}a_{22} + a_{03}a_{23} = 0$$
$$a_{01}a_{21} + a_{02}T_2 - a_{03}a_{32} = 0$$
$$\alpha_{01}\alpha_{21} + 0 - \alpha_{03}\alpha_{32} = 0$$
$$\alpha_{01}\alpha_{21} = \alpha_{03}\alpha_{32}$$

$$\frac{\alpha_{32}}{\alpha_{01}} = \frac{\alpha_{21}}{\alpha_{03}}. \tag{15.8}$$

For $\nu = 3$,

$$\alpha_{01}\alpha_{31} + \alpha_{02}\alpha_{32} + \alpha_{03}\alpha_{33} = 0$$
$$-\alpha_{01}\alpha_{13} + \alpha_{02}\alpha_{32} + \alpha_{03}T_3 = 0$$
$$-\alpha_{01}\alpha_{13} + \alpha_{02}\alpha_{32} + 0 = 0$$
$$-\alpha_{01}\alpha_{13} + \alpha_{02}\alpha_{32} = 0$$
$$\alpha_{02}\alpha_{32} = \alpha_{01}\alpha_{13}$$

$$\frac{\alpha_{32}}{\alpha_{01}} = \frac{\alpha_{13}}{\alpha_{02}}. \tag{15.9}$$

From equations (15.7), (15.8), (15.9) we get

$$\frac{\alpha_{32}}{\alpha_{01}} = \frac{\alpha_{13}}{\alpha_{02}} = \frac{\alpha_{21}}{\alpha_{03}} = \lambda. \tag{15.10}$$

From the second of equations (15.3) we get

$$J_3\alpha_{13} = J_0\alpha_{01} + J_2\alpha_{21} \tag{15.11}$$

and differentiating with respect to $x_\nu, \nu = 0,1,2,3$ and, taking into account equation (7.43) we get

$$\alpha_{13}\left(\frac{b}{\hbar}P_\nu J_3 + zQ\alpha_{\nu 3}\right) = \alpha_{01}\left(\frac{b}{\hbar}P_\nu J_0 + zQ\alpha_{\nu 0}\right) + \alpha_{21}\left(\frac{b}{\hbar}P_\nu J_2 + zQ\alpha_{\nu 2}\right)$$

$$\frac{b}{\hbar}P_\nu J_3\alpha_{13} + zQ\alpha_{13}\alpha_{\nu 3} = \frac{b}{\hbar}P_\nu\left(J_0\alpha_{01} + J_2\alpha_{21}\right) + zQ\left(\alpha_{01}\alpha_{\nu 0} + \alpha_{21}\alpha_{\nu 2}\right)$$

and with equation (15.11) we get

$$zQ\alpha_{13}\alpha_{\nu 3} = zQ\left(\alpha_{01}\alpha_{\nu 0} + \alpha_{21}\alpha_{\nu 2}\right)$$

and because $zQ \neq 0$ we get

$$\alpha_{13}\alpha_{\nu 3} = \alpha_{01}\alpha_{\nu 0} + \alpha_{21}\alpha_{\nu 2}\,. \tag{15.12}$$

Setting successively $\nu = 0,1,2,3$ in equation (15.12) we get:

For $\nu = 0$,

$$\begin{aligned}
\alpha_{13}\alpha_{03} &= \alpha_{01}\alpha_{00} + \alpha_{21}\alpha_{02} \\
\alpha_{13}\alpha_{03} &= \alpha_{01}T_0 + \alpha_{21}\alpha_{02} \\
\alpha_{13}\alpha_{03} &= 0 + \alpha_{21}\alpha_{02} \\
\alpha_{13}\alpha_{03} &= \alpha_{21}\alpha_{02}
\end{aligned}$$

$$\frac{a_{13}}{a_{02}} = \frac{a_{21}}{a_{03}}$$

which is equation (15.7).

For $\nu = 1$,

$$\begin{aligned}
\alpha_{13}\alpha_{13} &= \alpha_{01}\alpha_{10} + \alpha_{21}\alpha_{12} \\
\alpha_{13}^2 &= -\alpha_{01}^2 - \alpha_{21}^2 \\
\alpha_{13}^2 + \alpha_{21}^2 &= -\alpha_{01}^2
\end{aligned}$$

$$\alpha_{01}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0\,. \tag{15.13}$$

For $\nu = 2$,

$$\begin{aligned}
\alpha_{13}\alpha_{23} &= \alpha_{01}\alpha_{20} + \alpha_{21}\alpha_{22} \\
\alpha_{13}\alpha_{23} &= \alpha_{01}\alpha_{20} + \alpha_{21}T_2 \\
\alpha_{13}\alpha_{23} &= \alpha_{01}\alpha_{20}
\end{aligned}$$

$$\alpha_{32}\alpha_{13} = \alpha_{01}\alpha_{02}\,. \tag{15.14}$$

For $\nu = 3$,

$$\begin{aligned}
&\alpha_{13}\alpha_{33} = \alpha_{01}\alpha_{30} + \alpha_{21}\alpha_{32} \\
&\alpha_{13}T_3 = -\alpha_{01}\alpha_{03} + \alpha_{32}\alpha_{21} \\
&0 = -\alpha_{01}\alpha_{03} + \alpha_{32}\alpha_{21}
\end{aligned}$$

$$\alpha_{01}\alpha_{03} = \alpha_{32}\alpha_{21}\,. \tag{15.15}$$

From equations (15.6), (15.10), (15.13), (15.14) and (15.15) we get

$$\begin{aligned}
&\frac{\alpha_{32}}{\alpha_{01}} = \frac{\alpha_{13}}{\alpha_{02}} = \frac{\alpha_{21}}{\alpha_{03}} = \pm 1 \\
&\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0
\end{aligned}\,. \tag{15.16}$$

Similarly, the application of the $SV-T$ method to the third and fourth of equations (15.3), always gives either one of the first or one of the second of the equations (15.16).

From equations (15.16) and (7.24), (7.25) we get

$$\begin{aligned}
&\boldsymbol{\tau} = \pm\mathbf{n} \neq \mathbf{0} \\
&\begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} = \pm \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix}
\end{aligned}\,. \tag{15.17}$$

From equations (15.2) and (15.16) we get

$$T_{010203321321} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & \mp\alpha_{03} & \pm\alpha_{02} \\ -\alpha_{02} & \pm\alpha_{03} & 0 & \mp\alpha_{01} \\ -\alpha_{03} & \mp\alpha_{02} & \pm\alpha_{01} & 0 \end{bmatrix}. \tag{15.18}$$

Equation (15.18) gives the two mathematical expressions of matrix $T_{010203321321}$.

From the second of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get

$$\begin{aligned}
&J_0\alpha_{12} + J_2\alpha_{01} + J_1\alpha_{20} = 0 \\
&J_0\alpha_{13} + J_3\alpha_{01} + J_1\alpha_{30} = 0 \\
&J_0\alpha_{23} + J_3\alpha_{02} + J_2\alpha_{30} = 0 \\
&J_1\alpha_{23} + J_3\alpha_{12} + J_2\alpha_{31} = 0
\end{aligned}$$

$$\begin{aligned}
&-J_0\alpha_{21} + J_2\alpha_{01} - J_1\alpha_{02} = 0 \\
&J_0\alpha_{13} + J_3\alpha_{01} - J_1\alpha_{03} = 0 \\
&-J_0\alpha_{32} + J_3\alpha_{02} - J_2\alpha_{03} = 0 \\
&-J_1\alpha_{32} - J_3\alpha_{21} - J_2\alpha_{13} = 0
\end{aligned}. \tag{15.19}$$

It is easy to prove that equations (15.19) and (15.3) are equivalent if we consider the first of equations (15.16). Consequently, for the calculation of the momenta J_0, J_1, J_2, J_3 we have at our disposal four independent equations.

Equations (15.19) constitute a 4×4 linear homogeneous system with the momenta J_0, J_1, J_2, J_3 as unknowns. The determinant, as well as all the 3×3 subdeterminants of the system are equal to zero, while the 2×2 subdeterminants are non-zero. Consequently, there are two independent variables in the sytem. If we consider as independent variables the momenta J_0, J_1 we get the solution

$$\begin{aligned}
&J_2 = \frac{\alpha_{03}}{\alpha_{32}} J_0 + \frac{\alpha_{13}}{\alpha_{32}} J_1 \\
&J_3 = -\frac{\alpha_{02}}{\alpha_{32}} J_0 + \frac{\alpha_{21}}{\alpha_{32}} J_1
\end{aligned} \tag{15.20}$$

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{03}}{\alpha_{32}} \\ -\dfrac{\alpha_{02}}{\alpha_{32}} \end{bmatrix} + J_1 \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{13}}{\alpha_{32}} \\ \dfrac{\alpha_{21}}{\alpha_{32}} \end{bmatrix}. \tag{15.21}$$

From the first of equations (4.6) for $(i,\nu,k) = (0,1,2),(0,1,3),(0,2,3),(1,2,3)$ we get

$$\begin{aligned}
&c_0\alpha_{12} + c_2\alpha_{01} + c_1\alpha_{20} = 0 \\
&c_0\alpha_{13} + c_3\alpha_{01} + c_1\alpha_{30} = 0 \\
&c_0\alpha_{23} + c_3\alpha_{02} + c_2\alpha_{30} = 0 \\
&c_1\alpha_{23} + c_3\alpha_{12} + c_2\alpha_{31} = 0
\end{aligned}$$

$$\begin{aligned}
&-c_0\alpha_{21} + c_2\alpha_{01} - c_1\alpha_{02} = 0 \\
&c_0\alpha_{13} + c_3\alpha_{01} - c_1\alpha_{03} = 0 \\
&-c_0\alpha_{32} + c_3\alpha_{02} - c_2\alpha_{03} = 0 \\
&-c_1\alpha_{32} - c_3\alpha_{21} - c_2\alpha_{13} = 0
\end{aligned}. \tag{15.22}$$

Equations (15.22) constitute a 4×4 linear homogegeous system with the constants c_0, c_1, c_2, c_3 as unknowns. System (15.22) has the constants c_0, c_1, c_2, c_3 in place of the momenta J_0, J_1, J_2, J_3. Thus, it gives the solution corresponding to equation (15.19)

$$C = c_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{03}}{\alpha_{32}} \\ -\dfrac{\alpha_{02}}{\alpha_{32}} \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{13}}{\alpha_{32}} \\ \dfrac{\alpha_{21}}{\alpha_{32}} \end{bmatrix} \tag{15.23}$$

where c_0, c_1, are independent parameters.

In the $T_{010203321321}$ symmetry it is $a_{ki} \neq 0, \forall k \neq i, k, i = 0,1,2,3$, therefore, from equations (3.4) and (4.4) we get

$$c_k J_i \neq c_i J_k, \forall k \neq i, k, i = 0,1,2,3. \tag{15.24}$$

From relation (15.24) for $(k,i) = (0,1)$ we get

$$c_0 J_1 \neq c_1 J_0. \tag{15.25}$$

From relation (15.25) emerges that the four-vectors J and C of equations (15.21) and (15.23) cannot be parallel.

The four-vectors J and C have additional mathematical expressions other than (15.21) and (15.23). In total there are

$$\binom{4}{2} = 6$$

mathematical formulations arising from pairwise combinations of the parameters c_0, c_1, c_2, c_3 of the systems (15.19) and (15.22). The four-vector $P(Q)$ is calculated from equation

$$P = P(Q) = -J \tag{15.26}$$

and equation (15.21).

The condition (15.25), $c_0 J_1 \neq c_1 J_0$, is not necessarily valid for $c_0 c_1 \neq 0$ and $J_0 J_1 \neq 0$. It is easy to demonstrate that the condition (15.25), and hence equations (15.21) and (115.23) are valid for $c_0 \neq 0, J_1 \neq 0, c_1 = 0, J_0 = 0$ and also for $c_1 \neq 0, J_0 \neq 0, c_0 = 0, J_1 = 0$. These conditions complete the study of the symmetry $T_{010203321321}$, and must be considered when studying the interaction $T_{010203321321}$. In these cases frome the equations (15.21) and (15.23) we get

$$c_0 \neq 0, J_1 \neq 0$$

$$C = c_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{03}}{\alpha_{32}} \\ -\dfrac{\alpha_{02}}{\alpha_{32}} \end{bmatrix} \qquad J = J_1 \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{13}}{\alpha_{32}} \\ \dfrac{\alpha_{21}}{\alpha_{32}} \end{bmatrix} \tag{15.27}$$

$c_1 \neq 0, J_0 \neq 0$

$$C = c_1 \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{13}}{\alpha_{32}} \\ \dfrac{\alpha_{21}}{\alpha_{32}} \end{bmatrix} \tag{15.28}$$

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{03}}{\alpha_{32}} \\ -\dfrac{\alpha_{02}}{\alpha_{32}} \end{bmatrix}$$

From equation (2.7) and equation (15.21) we get

$$J_0^2 + J_1^2 + \left(\frac{\alpha_{03}}{\alpha_{32}} J_0 + \frac{\alpha_{13}}{\alpha_{32}} J_1\right)^2 + \left(-\frac{\alpha_{02}}{\alpha_{32}} J_0 + \frac{\alpha_{21}}{\alpha_{32}} J_1\right)^2 + m_0^2 c^2 = 0$$

$$J_0^2 + J_1^2 + \frac{\alpha_{03}^2}{\alpha_{32}^2} J_0^2 + \frac{\alpha_{13}^2}{\alpha_{32}^2} J_1^2 + \frac{2\alpha_{03}\alpha_{13}}{\alpha_{32}^2} J_0 J_1$$
$$+ \frac{\alpha_{02}^2}{\alpha_{32}^2} J_0^2 + \frac{\alpha_{21}^2}{\alpha_{32}^2} J_1^2 - \frac{2\alpha_{02}\alpha_{21}}{\alpha_{32}^2} J_0 J_1 + m_0^2 c^2 = 0$$

and since, as follows from equation (15.16), is $\alpha_{03}\alpha_{13} = \alpha_{02}\alpha_{21}$, we get

$$J_0^2 + J_1^2 + \frac{\alpha_{03}^2}{\alpha_{32}^2} J_0^2 + \frac{\alpha_{13}^2}{\alpha_{32}^2} J_1^2 + \frac{\alpha_{02}^2}{\alpha_{32}^2} J_0^2 + \frac{\alpha_{21}^2}{\alpha_{32}^2} J_1^2 + m_0^2 c^2 = 0$$

$$\frac{1}{\alpha_{32}^2}\left(J_0^2\left(\alpha_{32}^2 + \alpha_{03}^2 + \alpha_{02}^2\right) + J_1^2\left(\alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2\right)\right) + m_0^2 c^2 = 0$$

and since, as follows from equation (15.16), is $\alpha_{32} = \pm\alpha_{01}$, we get

$$\frac{1}{\alpha_{32}^2}\left(J_0^2\left(\alpha_{01}^2 + \alpha_{03}^2 + \alpha_{02}^2\right) + J_1^2\left(\alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2\right)\right) + m_0^2 c^2 = 0$$

and with equations (15.16) we get

$$0 + m_0^2 c^2 = 0$$

$$m_0 = 0 \,. \tag{15.29}$$

Similarly, from equations (2.8) and (15.26) we get

$$E_0 = 0 \tag{15.30}$$

and from (3.6) and (15.23) we get

$$M_0 = 0 \,. \tag{15.31}$$

In the preceding study we denoted J the four-vector $J(Q)$. Knowing four-vector $J(Q)$ we can calculate the four-vectors J and P from equations (7.74). The calculation of the four-vectors J and P can also be made through equations (11.6), (11.7) and (11.2). Also, from corollary 11.2 it is straightforward to show that for the symmetry $T_{010203321321}$ the gauge function is $f = 0$ ($f_k = 0, \forall k \in \{0,1,2,3\} \wedge c_k \neq 0$).

In the symmetry $T_{010203321321}$ it is $T_0 = T_1 = T_2 = T_3 = 0$, therefore, from equation (4.11) we get $\Lambda = 0$, and from relation (7.38) of theorem 7.5 we get

$$j = -\frac{\sigma b c^2}{\hbar} \mu \Psi M C \,. \tag{15.32}$$

Taking into account equations (15.16) we get from equations (4.28) and (7.9) equation

$$N = \pm M \tag{15.33}$$

for the fundamental matrices M and N in the $T_{010203321321}$ symmetry. From equations (15.32) and (15.33) we get

$$j = \pm \frac{\sigma b c^2}{\hbar} \mu \Psi N C$$

and with the first of equations (7.10) we get

$$j = 0 \tag{15.34}$$

for the current density four-vector j of the conserved physical quantities of the generalized particle. Equation (15.34) satisfies equation (5.24) of theorem 5.2, therefore, equations (5.23) and (5.25) of the theorem hold:

$$\nabla^2\Psi+\frac{\partial^2\Psi}{\partial x_0^2}=0 \tag{15.35}$$

$$\begin{aligned}&\nabla^2\boldsymbol{\xi}-\frac{\partial^2\boldsymbol{\xi}}{c^2\partial t^2}=0\\&\nabla^2\boldsymbol{\omega}-\frac{\partial^2\boldsymbol{\omega}}{c^2\partial t^2}=0\end{aligned} \tag{15.36}$$

for the $T_{010203321321}$ symmetry. Consequently, in the symmetry of set Ω_6, the generalized particle is always in the form of the generalized photon.

From equation (5.29) of corollary 5.3 and equation (15.34) we get

$$M\left[\frac{\partial\Psi}{\partial x}\right]=0$$

and with equations (4.28) and (5.28) we get

$$\begin{bmatrix}0&\alpha_{01}&\alpha_{02}&\alpha_{03}\\-\alpha_{01}&0&-\alpha_{21}&\alpha_{13}\\-\alpha_{02}&\alpha_{21}&0&-\alpha_{32}\\-\alpha_{03}&-\alpha_{13}&\alpha_{32}&0\end{bmatrix}\begin{bmatrix}\dfrac{\partial\Psi}{\partial x_0}\\\dfrac{\partial\Psi}{\partial x_1}\\\dfrac{\partial\Psi}{\partial x_2}\\\dfrac{\partial\Psi}{\partial x_3}\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{aligned}&\alpha_{01}\frac{\partial\Psi}{\partial x_1}+\alpha_{02}\frac{\partial\Psi}{\partial x_2}+\alpha_{03}\frac{\partial\Psi}{\partial x_3}=0\\&-\alpha_{01}\frac{\partial\Psi}{\partial x_0}-\alpha_{21}\frac{\partial\Psi}{\partial x_2}+\alpha_{13}\frac{\partial\Psi}{\partial x_3}=0\\&-\alpha_{02}\frac{\partial\Psi}{\partial x_0}+\alpha_{21}\frac{\partial\Psi}{\partial x_1}-\alpha_{32}\frac{\partial\Psi}{\partial x_3}=0\\&-\alpha_{03}\frac{\partial\Psi}{\partial x_0}-\alpha_{13}\frac{\partial\Psi}{\partial x_1}+\alpha_{32}\frac{\partial\Psi}{\partial x_2}=0\end{aligned}. \tag{15.37}$$

Equations (15.37) constitute a 4×4 linear homogeneous system with unknowns $\frac{\partial\Psi}{\partial x_k}, k=0,1,2,3$. The determinant as well as all of the 3×3 subdeterminants of the system are zero. The 2×2 subdeterminants of the system are non-zero. Therefore, there are two independent variables in the system. If we set

$$\frac{\partial\Psi}{\partial x_0},\frac{\partial\Psi}{\partial x_1}$$

as independent variables, we get the solution

$$\begin{aligned}\frac{\partial\Psi}{\partial x_2}&=\frac{\alpha_{03}}{\alpha_{32}}\frac{\partial\Psi}{\partial x_0}+\frac{\alpha_{13}}{\alpha_{32}}\frac{\partial\Psi}{\partial x_1}\\ \frac{\partial\Psi}{\partial x_3}&=-\frac{\alpha_{02}}{\alpha_{32}}\frac{\partial\Psi}{\partial x_0}+\frac{\alpha_{21}}{\alpha_{32}}\frac{\partial\Psi}{\partial x_1}\end{aligned}$$

and with equations (15.16) we get

$$\begin{aligned}\frac{\partial\Psi}{\partial x_2}&=\pm\frac{\alpha_{03}}{\alpha_{01}}\frac{\partial\Psi}{\partial x_0}+\frac{\alpha_{02}}{\alpha_{01}}\frac{\partial\Psi}{\partial x_1}\\ \frac{\partial\Psi}{\partial x_3}&=\mp\frac{\alpha_{02}}{\alpha_{01}}\frac{\partial\Psi}{\partial x_0}+\frac{\alpha_{03}}{\alpha_{01}}\frac{\partial\Psi}{\partial x_1}\end{aligned}. \tag{15.38}$$

Now the calculation of

$$\frac{\partial\Psi}{\partial x_0},\frac{\partial\Psi}{\partial x_1}$$

is performed through equation (5.3)

$$\begin{aligned}\frac{\partial\Psi}{\partial x_0}&=\frac{b}{\hbar}\left(\lambda J_0+\mu P_0\right)\Psi=\frac{b}{\hbar}\left(\left(\lambda-\mu\right)J_0+\mu c_0\right)\Psi\\ \frac{\partial\Psi}{\partial x_1}&=\frac{b}{\hbar}\left(\lambda J_1+\mu P_1\right)\Psi=\frac{b}{\hbar}\left(\left(\lambda-\mu\right)J_1+\mu c_1\right)\Psi\end{aligned} \tag{15.39}$$

where we necessarily introduce the degrees of freedom λ,μ of the TSV.

From equations (7.27), (7.28) and (15.17) we get

$$\begin{aligned}\boldsymbol{\xi}&=\pm ic\boldsymbol{\omega}\\ \boldsymbol{\omega}&=\mp\frac{i}{c}\boldsymbol{\xi}\end{aligned} \tag{15.40}$$

for the symetry $T_{010203321321}$. Field $\boldsymbol{\omega}$ emerges from field $\boldsymbol{\xi}$ through rottation by angle $\mp\frac{\pi}{2}$, considering vectors $\boldsymbol{\xi}$ and $\boldsymbol{\omega}$ on the complex plane. The sign of angle $\frac{\pi}{2}$ is opposite from the one in equation (15.17), according to the way in which we use the $\pm$ symbol.

Field $\boldsymbol{\alpha}$ is given by equation (4.14)

$$\boldsymbol{\alpha} = icz\begin{pmatrix}\alpha_{01}\\ \alpha_{02}\\ \alpha_{03}\end{pmatrix}$$

while for field $\boldsymbol{\beta}$ we get

$$\boldsymbol{\beta} = \pm z\begin{pmatrix}\alpha_{01}\\ \alpha_{02}\\ \alpha_{03}\end{pmatrix}$$

from the combination of equations (4.15) and (15.17). Taking into account that $\Lambda = 0$ we get from equation (4.19)

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0}J + \frac{izQ}{c}\begin{bmatrix}0 & \alpha_{01} & \alpha_{02} & \alpha_{03}\\ -\alpha_{01} & 0 & \mp\alpha_{03} & \pm\alpha_{02}\\ -\alpha_{02} & \pm\alpha_{03} & 0 & \mp\alpha_{01}\\ -\alpha_{03} & \mp\alpha_{02} & \pm\alpha_{01} & 0\end{bmatrix}\begin{bmatrix}ic\\ u_1\\ u_2\\ u_3\end{bmatrix} \tag{15.41}$$

for the USVI of the $T_{010203321321}$ symmetry.

We have studied one symmetry from every set $\Omega_0,\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_5,\Omega_6$. We did not dedicate a special chapter to the study of set Ω_2 since we have studied the T^1_{0121} symmetry as an example in chapter 7. The study of the symmetries we presented constitutes a prototype for the study of the rest of the symmetries of sets $\Omega_0,\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_5,\Omega_6$.

16. THE COSMOLOGICAL DATA AS A CONSEQUENCE OF THE THEOREM OF INTERNAL SYMMETRY

16.1. Introduction

The theorem 3.3., that is the theorem of internal symmetry, predicts and justifies the cosmological data. We present the relevant study in this chapter.

The emission of the electromagnetic spectrum of the far-distant astronomical objects we observe today has taken place a long time interval ago. At the moment of the emission the rest mass and the electric charge of the material particles had smaller values than the corresponding ones measured in the laboratory, “now”, on Earth, due to the manifestation of the Selfvariations. The consequences resulting from this difference are recorded in the cosmological data. The cosmological data have a microscopic and not a macroscopic cause.

Due to the Selfvariations of the rest masses of the material particles the gravitational interaction cannot play the role attributed to it by the Standard Cosmological Model (SCM). The gravitational interaction cannot cause neither the collapse, nor the expansion of the universe, since it decreases on a cosmological scale according to the factor $\frac{1}{1+z_\gamma}$. By z_γ we denote the redshift of the photons emitted from the cores of the material particles. The gravitational interaction exercised on our galaxy by a far-distant astronomical object with redshift $z_\gamma = 9$ is only the $\frac{1}{10}$ of the expected one. The Universe is static and flat, according to the law of Selfvariations.

16.2. The cosmological data as a consequence of the theorem of internal symmetry

For a non- moving particle, that is for $J_1 = J_2 = J_3 = 0$, from equation (3.12) we get that $c_1 = c_2 = c_3 = 0$ and from equation (3.9) we obtain

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0\right)$$
$$b, K \in \mathbb{R}$$

and since $x_0 = ict$, we have

$$\Phi = K \exp\left(-\frac{bicc_0}{\hbar} t\right)$$

and from equation (3.10) we obtain

$$m_0 = m_0(t) = \pm \frac{M_0}{1 + K \exp\left(-\frac{bicc_0}{\hbar} t\right)}. \tag{16.1}$$

The rest mass m_0 of the material particle is a function of time t .

We now denote by k the constant

$$k = -\frac{bicc_0}{\hbar}$$

and from equation (3.5) we have

$$k = -\frac{bicc_0}{\hbar} = \frac{b(W+E)}{\hbar} . \tag{16.2}$$

We also denote by A the time-dependent function

$$A = A(t) = -K \exp(kt) = -\Phi . \tag{16.3}$$

Following this notation, equation (16.1) is written as

$$m_0 = m_0(t) = \pm \frac{M_0}{1-A} . \tag{16.4}$$

From equation (16.3) we have

$$\frac{dA}{dt} = \dot{A} = kA . \tag{16.5}$$

for the expression of the parameter $A = A(t)$. Similarly, using the above notation equation (3.11) is written as

$$E_0 = E_0(t) = \mp \frac{M_0 c^2 A}{1-A}. \tag{16.6}$$

We consider an astronomical object at distance r from Earth. The emission of the electromagnetic spectrum of the far-distant astronomical objects we observe “now”on Earth has taken place before a time interval $\delta t = t - \frac{r}{c}$. From equation (16.3) we have that the parameter A obtained the value

$$A = A(r) = A(t)exp\left(-k\frac{r}{c}\right)$$

and from equation (16.4) we have

$$m_0(r) = \pm\frac{M_0}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \qquad (16.7)$$

Similarry from equation (16.6) we have

$$E_0(r) = \mp\frac{M_0 c^2 A\exp\left(-k\frac{r}{c}\right)}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \qquad (16.8)$$

From equations (16.4) and (16.7) we have

$$m_0(r) = m_0\frac{1 - A}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \qquad (16.9)$$

We can prove that for the electric charge q of the material particles an equation analogous to equation (16.7) is valid. From equation (4.2) we derive an equation corresponding to equation (16.9), which is the following equation

$$q(r) = q\frac{1 - B}{1 - B\exp\left(-k_1\frac{r}{c}\right)} \quad . \qquad (16.10)$$

The fine structure constant α is defined as

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 c\hbar} \qquad (16.11)$$

and using equation (16.10) we obtain

$$\alpha(r) = \alpha\left(\frac{1 - B}{1 - B\exp\left(-k_1\frac{r}{c}\right)}\right)^2 \quad . \qquad (16.12)$$

The wave length λ of the linear spectrum is inversely proportional to the factor $m_0 q^4$, where m_0 is the rest mass and q is the electric charge of the electron. If we denote by λ_0 the wavelength of a photon emitted by an atom “now”on Earth, and by λ the same wavelength of the same atom received “now” on Earth from the far-distant astronomical object, the following relation holds:

$$\frac{\lambda}{\lambda_0} = \frac{m_0 q^4}{m_0(r) q^4(r)}$$

and from equations (16.9) and (16.10) we obtain

$$\frac{\lambda}{\lambda_0} = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1-A}\left(\frac{1 - B\exp\left(-k_1\frac{r}{c}\right)}{1-B}\right)^4 . \qquad (16.13)$$

From equation (16.13) we have for the redshift

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1$$

of the astronomical object that

$$z = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1-A}\left(\frac{1 - B\exp\left(-k_1\frac{r}{c}\right)}{1-B}\right)^4 - 1 . \qquad (16.14)$$

Equation (16.14) can also be written as

$$z = \frac{1 - Aexp\left(-k\frac{r}{c}\right)}{1-A}\left(\frac{\alpha}{\alpha(r)}\right)^2 - 1 \qquad (16.15)$$

after considering equation (16.12).

From the cosmological data and from measurements conducted on Earth, we know that the variation of the fine structure constant is extremely small. Therefore, from equation (16.15), we obtain with extremely accurate approximation

$$z = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1 - A} - 1$$

$$z = \frac{A}{1 - A}\left(1 - e^{-\frac{kr}{c}}\right). \tag{16.16}$$

Equation (16.16) holds with great accuracy. The variation of the fine structure constant is so small, so that any contribution t redshift is overlapped by the same contributions from the far-distant astronomical objects, due to Doppler's effect.

For small distances r, we obtain from equation (16.16)

$$z = \frac{A}{1 - A}\left(1 - 1 + \frac{kr}{c}\right)$$

$$z = \frac{kA}{c(1 - A)}r$$

and comparing this with Hubble's law

$$cz = Hr$$

we get

$$\frac{kA}{1 - A} = H \tag{16.17}$$

where H is Hubble's parameter.

From equation (16.17) we have

$$\frac{dH}{dt} = \dot{H} = \frac{k\dot{A}(1 - A) + kA\dot{A}}{(1 - A)^2}$$

$$\dot{H} = \frac{k\dot{A}}{(1 - A)^2}$$

and with equation (16.5) we obtain

$$\dot{H} = \frac{k^2 A}{(1 - A)^2}$$

and from equation (16.17) we have

$$\dot{H} = \frac{H}{A} \quad . \tag{16.18}$$

For $\frac{m_0 c^2}{M_0 c^2} > 0 \wedge \frac{E_0}{M_0 c^2} < 0$ or $\frac{m_0 c^2}{M_0 c^2} < 0 \wedge \frac{E_0}{M_0 c^2} > 0$ we have

$$\frac{m_0 c^2}{M_0 c^2} \frac{E_0}{M_0 c^2} < 0$$

and with equations (3.10) and (3.11) we have

$$\frac{\Phi}{\left(1+\Phi\right)^2} < 0$$

$$\Phi < 0$$

and with equation (16.3) we finally get

$$\begin{aligned} &\Phi < 0 \\ &A = -\Phi > 0 \end{aligned} \quad . \tag{16.19}$$

From equation (16.17) we have

$$\frac{kA}{1-A} > 0$$

and considering relation (16.19) we get two combinations for the constant k and the parameter A:

$$\begin{aligned} &0 < A < 1 \wedge k > 0 \Leftrightarrow 0 < 1+\Phi < 1 \wedge k > 0 \\ &A > 1 \wedge k < 0 \Leftrightarrow 1+\Phi < 0 \wedge k < 0 \end{aligned} \quad . \tag{16.20}$$

From equations (16.7) and (16.8) it follows that the sign change of the constant k is equivalent with the interchange of the roles of the rest masses m_0 and $\frac{E_0}{c^2}$. Hence it suffices to present the conclusions resulting from the first case of (16.20).

For $k > 0$ for equation (16.16) we have

$$\lim_{r \to \infty} z = \frac{A}{1-A} \quad . \tag{16.21}$$

The redshift has an upper limit which depends on the value of the parameter A, even in the case that the Universe extends to infinity. In the case the Universe has finite extension, let $r_{\max} = R$ and from equation (16.16) we have

$$z_{\max} = \frac{A}{1-A}\left(1-e^{-\frac{kR}{c}}\right). \tag{16.22}$$

Thus redshift has a maximum value. The upper redshift limit of equation (16.21) and $z_{\max}$ of equation (16.22) are almost equal. Hence in the following we will use equation (16.21).

From equations (16.16) and (16.5) we get after the calculations

$$\frac{dz}{dt} = \dot{z} = \frac{kA}{(1-A)^2}\left(1-e^{-\frac{kr}{c}}\right)$$

and with equation (16.17) we have

$$\dot{z} = \frac{H}{1-A}\left(1-e^{-\frac{kr}{c}}\right) = \frac{H}{A}z > 0. \tag{16.23}$$

Thus the redshift of far distant astronomical objects increases slightly with the passage of time.

According to equation (16.21) it is

$$z < \frac{A}{1-A}$$

and because of $1-A>0$ we have

$$\frac{z}{1+z} < A$$

and because of $A<1$ we have

$$\frac{z}{1+z} < A < 1. \tag{16.24}$$

From the inequality (16.24) it follows that

$$A \to 1^- . \tag{16.25}$$

We prove now that as $A \to 1^-$ the equation (16.16) tends to Hubble's law $cz = Hr$. Let $x = \frac{1-A}{A}$ then $x \to 0^+$ for $A \to 1^-$, while from equation (16.17) we get $k = xH$ and equation (16.16) may be written as

$$z = \frac{1}{x}\left[1 - \exp\left(-x\frac{Hr}{c}\right)\right].$$

Hence we get

$$\lim_{A \to 1^-} z = \lim_{x \to 0^+} \frac{1}{x}\left[1 - \exp\left(-x\frac{Hr}{c}\right)\right] = \frac{Hr}{c}.$$

From relation (16.5) follows the conclusion that

$$\frac{dA}{dt} = kA > 0. \qquad (16.26)$$

Thus the parameter A increases with the passage of time. Hence according to the forementioned proof, the equation (16.16) tends to the Hubble law with the passage of time.

Combining equations (16.9) and (16.16) we have

$$m_0(z) = \frac{m_0}{1+z} . \qquad (16.27)$$

The equation (16.27) has multiple consequences on cosmological scale.

According to equation (16.27) the gravitational interaction between two astronomical objects is smaller than expected by the factor $\frac{1}{1+z}$. The redshift z depends on their distance r as given in equation (16.16). This is the redshift that an observer on one object would measure by observing the other object.

For the solar system or for the structure of a galaxy or a galaxy cluster, equation (16.27) has no consequences. On this distance scale we practically have $z = 0$. However we can seek consequences on this scale from another equation. From equation (16.4) we have

$$\frac{dm_0}{dt} = \dot{m}_0 = \pm \frac{M_o \dot{A}}{(1-A)^2}$$

and from equation (16.4) we have

$$\dot{m}_0 = m_0 \frac{\dot{A}}{1-A}$$

and with equation (16.5) we get

$$\frac{\dot{m}_0}{m_0} = \frac{kA}{1-A}$$

and with equation (16.17) we get

$$\frac{\dot{m}_0}{m_0} = H \tag{16.28}$$

for the rest mass of the electron.

Equation (16.28) concerns the mass $m_0 = m_0(t)$. Therefore its consequences can be found in our galaxy or even in the solar system. In any case the experimental verification of equation (16.28) requires measurements with sensitive instruments of observation.

Equation (16.27) has important consequences on cosmological scale distances. For such distances the gravitational interaction diminishes quickly and beyond some distance it practically vanishes. It has however played an important role for the creation of all large structures in the universe.

As we will see further down, the very early Universe differed only slightly from vacuum. The gravitational interaction strengthens with the passage of time, as the rest masses of material particles increase. Moreover, its strength depends on distance as predicted by the law of universal gravitation, but also for cosmological distances, as predicted by equation (16.27). Both these factors played an important role for the creation of all large structures in the universe and have not been both accounted for in the interpretation of the cosmological data via the SCM.

From equations $E = mc^2$ and (16.27) we have

$$E(z) = \frac{E}{1+z} \quad . \tag{16.29}$$

In every case of transformation of mass m to energy E. The production of energy in the universe is mainly achieved via hydrogen fusion and nuclear reactions. Therefore the energy produced in the past in the far distant astronomical objects was smaller than the corresponding energy produced today in our galaxy through the same mechanism. This fact has two immediate consequences.

The first is that equation (16.16) is valid for the redshift z_a of the radiation which stems from accelerated / decelerated electrons

$$z_a = \frac{A}{1-A}\left(1-e^{-\frac{kr}{c}}\right) . \tag{16.30}$$

And hence for the continuous spectrum. Similar mechanisms which accelerate electrons in our galaxy and in far distant astronomical objects do not give the same amount of energy to the electrons. According to equation (16.29) the energy which is supplied to the electrons in far distant astronomical objects is less than the corresponding energy in our galaxy.

The second consequence concers the luminosity distance D of far distant astronomical objects. The overall decrease of the energy produced in the past, due to equation (16.29) has as consequence the overall decrease of luminositites of distant astronomical objects. From the definition of the luminosity distance D it follows easy that

$$D = r\sqrt{1+z} \quad . \tag{16.31}$$

Between the distance r of the astronomical object and the distance D measured from its luminosity. The luminosity distance D is measured always larger than the real distance of the astronomical object. The real distance r of the distant astronomical object is given by equation

$$r = \frac{c}{k}\ln\left(\frac{A}{1-z(1-A)}\right) \tag{16.32}$$

which follows from equation (16.16). The distance measurement from equation (16.32) can be made if we know the constant k and the parameter A. Generally, due to equation (16.17) it suffices to know two of the parameters k, A, H.

The ionization energy as well as the excitation energy X_n of atoms is proportional to the factor m_0q^4, where m_0 is the rest mass and q the electric charge of the electron. Hence we get

$$\frac{X_n(r)}{X_n}=\frac{m_0(r)}{m_0}\left(\frac{q(r)}{q}\right)^4$$

$$\frac{X_n(r)}{X_n}=\frac{m_0(r)}{m_0}\left(\frac{\alpha(r)}{\alpha}\right)^2$$

and because of

$$\frac{\alpha(r)}{\alpha}\approx 1$$

we have

$$\frac{X_n(r)}{X_n}=\frac{m_0(r)}{m_0}$$

and with equation (16.27) we have

$$\frac{X_n(r)}{X_n}=\frac{X_n(z)}{X_n}=\frac{1}{1+z}$$

$$X_n(r)=X_n(z)=\frac{X_n}{1+z}\ . \tag{16.33}$$

From equation (16.33) we conclude tha the ionization and excitation energies of atoms decrease with inceasing redshift. This fact has consequences on the degree of ionization of atoms in the distant astronomical objects

The number of excited atoms in a gas in a state of thermodynamic equilibrium is given by Boltzmann's equation

$$\frac{N_n}{N_1}=\frac{g_n}{g_1}\exp\left(-\frac{X_n}{KT}\right) \tag{16.34}$$

where N_n is the number of atoms at energy level n, X_n the excitation energy from the the 1st to the n^{th} energy level, $K=1.38\times10^{-23}JK^{-1}$ Boltzmann's constant, T the temperature in degrees Kelvin, and g_n the multiplicity of level n, i.e. the number of levels into which level n is split apart inside a magnetic field.

Combining equations (16.33) and (16.34) we get

$$\frac{N_n}{N_1}=\frac{g_n}{g_1}\exp\left(-\frac{X_n}{KT(1+z)}\right). \tag{16.35}$$

For the hydrogen atom for $n=2, X_2=10.5eV=16.4\times10^{-19}J, g_1=2, g_2=8$ and at the surface of the Sun where $T\sim6000K$ equation (16.34) implies that just one in 10^8 atoms is at state $n=2$. Correspondingly from equation (16.35) and for $z=1$ we have $\frac{N_2}{N_1}=2.2\times10^{-4}$, for $z=2$ we have $\frac{N_2}{N_1}=5.8\times10^{-3}$, and for $z=5$ we have $\frac{N_2}{N_1}=0.15$.

Considering equation (16.21) we get from equation (16.33)

$$X_n(r\to\infty)=X_n(1-A). \tag{16.36}$$

Considering relations (16.24) and (16.25) we conclude that the ionization and excitation energies of atoms tend to zero in the very early universe. The universe went through an ionization phase in its initial phase of evolution.

The laboratory value of the Thomson scattering coefficient is given by equation

$$\sigma_{\mathrm{T}}=\frac{8\pi}{3}\frac{q^4}{m_0^2c^4} \tag{16.37}$$

where m_0 the rest mass and q the electric charge of the electron. Thus we have

$$\frac{\sigma_T(z)}{\sigma_T}=\left(\frac{m_0}{m_0(z)}\right)\left(\frac{\alpha}{\alpha(z)}\right)^2$$

and because of $\alpha(z)\sim\alpha$ we get

$$\frac{\sigma_T(z)}{\sigma_T}=\left(\frac{m_0}{m_0(z)}\right)^2$$

and with equation (16.27) we have

$$\frac{\sigma_T(z)}{\sigma_T} = (1+z)^2 . \qquad (16.38)$$

The Thomson coefficient concerns the scattering of photons with low energy E. For photons with high energy E the photon scattering is determined from the Klein-Nishina coefficient :

$$\sigma = \frac{3}{8}\sigma_T \frac{m_0}{E}\left[\ln\left(\frac{2E}{m_0 c^2}\right)+\frac{1}{2}\right] \qquad (16.39)$$

in the laboratory and

$$\sigma(z) = \frac{3}{8}\sigma_T(z)\frac{m_0(z)c^2}{E(z)}\left[\ln\left(\frac{2E(z)}{m_0(z)c^2}\right)+\frac{1}{2}\right] \qquad (16.40)$$

in astronomical objects with redshift z. From equations (16.27) and (16.29) we have

$$\frac{m_0(z)}{E(z)} = \frac{m_0}{E}$$

hence from equation (16.40) we get

$$\sigma(z) = \frac{3}{8}\sigma_T(z)\frac{m_0 c^2}{E}\left[\ln\left(\frac{2E}{m_0 c^2}\right)+\frac{1}{2}\right]$$

and with equation (16.38) we have

$$\frac{\sigma(z)}{\sigma} = \frac{\sigma_T(z)}{\sigma_T} = (1+z)^2 . \qquad (16.41)$$

From equation (16.41) we conclude that the Thomson and Klein-Nishina scattering coefficients increase with redshift and indeed in the same manner. Considering equation (16.21) we have

$$\frac{\sigma(r\to\infty)}{\sigma} = \frac{\sigma_T(r\to\infty)}{\sigma_T} = \frac{1}{(1-A)^2} . \qquad (16.42)$$

Considering equations (16.24) and (16.25) we conclude that the Thomson and Klein-Nishina scattering coefficients had enormous values in the vary early universe. In its initial phase the universe was totally opaque. From this initial phase stems the cosmic microwave background radiation (CMBR) we observe today.

The internal symmetry theorem (3.3) predicts that the initial Universe was at a 'vacuum state' with temperature $T = 0K$. Due to the Selfvariations the universe evolved to the state we observe today. This evolution agrees with the fact that the CMRB corresponds to a black body radiation with temperature $T \sim 2.73K$.

Combining equations (3.11) and (16.3) we have in the laboratory

$$J_i = \frac{c_i}{1 - A(t)}, i = 0,1,2,3$$

and

$$J_i(r) = \frac{c_i}{1 - A\left(t - \frac{r}{c}\right)} = \frac{c_i}{1 - A\exp\left(-\frac{kr}{c}\right)}, i = 0,1,2,3$$

for an astronomical object at distance r, and combining these two equations with equation (16.9) we get

$$\frac{J_i(r)}{J_i} = \frac{m_0(r)}{m_0}$$

and with equation (16.27) we have

$$\frac{J_i(z)}{J_i} = \frac{1}{1+z}$$

$$J_i(z) = \frac{J_i}{1+z}, i = 0,1,2,3. \tag{16.43}$$

From the Heisenberg uncertainty principle for the axis $x_1 = x$ we have

$$J_1 \Delta x \sim \hbar$$

in the lab, and

$$J_1(z) \Delta x(z) \sim \hbar$$

for the astronomical object, and combining these two relations we get

$$J_1(z)\Delta x(z) = J_1\Delta x$$

and with equation (16.43) we have

$$\Delta x(z) = (1+z)\Delta x. \tag{16.44}$$

From equation (16.44) we conclude that the uncertainty $\Delta x(z)$ of position of a material particle increases with redshift. Moreover as the universe evolved towards the state we observe today, the uncertainty of position of material particles was decreasing.

From equations (16.44) and (16.21) we have

$$\Delta x(r \to \infty) = \frac{\Delta x}{1-A}. \tag{16.45}$$

Considering relations (16.24) and (16.25) we conclude that in the very early universe there existed great uncertainty of position of material particles. The same conclusions arise for the Bohr radius R_{Bohr},

$$R_{Bohr}(z) = (1+z)R_{Bohr}$$

$$R_{Bohr}(r \to \infty) = \frac{R_{Bohr}}{1-A}.$$

The TSV is agrees with the uncertainty principle. In the next chapter we will see that the uncertainty of position of a material particle is one more consequence of theorem 3.3.

From equation (16.33) it follows that as the universe evolved to the state we observe today the ionization energy increased. This prediction is generally valid for any kind of negative dynamical energies which bind together material particles to produce more complex particles. From equation (16.27) we have

$$\Delta m_0(z)c^2 = \frac{\Delta m_0 c^2}{1+z} \tag{16.46}$$

for the energy $\Delta m_0 c^2$, the mass deficiency, which ties together the particles which constitute the nuclei of the elements. According to equation (16.46) the energy $\Delta m_0 c^2$, like the ionization energies, increased as the universe evolved towards its present state.

Particle like the electron, which today are considered fundamental may in fact be composed of other particles. Our inability to break them apart could be due to the strengthening of the binding energies of the constituent particles. The mass M_0 in equation (3.10) has many chances to the only realy fundamental rest mass, from which the masses of all other particles are composed.

From equations (16.27) and (16.21) we have

$$m_0(r \to \infty) = m_0(1 - A) \neq 0 . \tag{16.47}$$

Considering the relations (16.24) and (16.25) we conclude that, towards the initial state of the universe, the rest masses of material particles tend to zero:

$$m_0(r \to \infty) = m_0(1 - A) \to 0 . \tag{16.48}$$

From equation (16.8) we have

$$E_0(r \to \infty) = 0 . \tag{16.49}$$

According to the relations (16.48) and (16.49) the initial state of the universe slightly differed from vacuum. The same conclusion arises in the case the universe is finite, taking $r_{\max} = R < \infty$ instead of the condition $r \to \infty$ we have used.

We have studied the case of $J_1 = J_2 = J_3 = 0$ in equation (3.12) in order to bypass the consequences on the redshift produced by the proper motion of the electron. Thus, from equation (3.6) we obtain

$$M_0 = \pm \frac{ic_0}{c} .$$

From equation (16.2) we also have

$$M_0 = \pm \frac{\hbar k}{bc^2} . \tag{16.50}$$

From equation (16.17) we obtain that the constant k obtains an extremely small value. Therefore, the same holds and for the rest mass M_0, as a result of equation (16.50).

From equation (16.5) we conclude that the parameter A varies only very slightly with the passage of time. The age of the Universe is correlated at a greater degree with the value of the parameter A we measure today, and less with Hubble's parameter H . In any case the two parameters A and H are correlated via equation (16.17).

With the exception of equations (16.16) and (16.32), the TSV equations for cosmology do not depend on the values of the parameters k , A and H . They solely depend on z , which is accurately measured. Equation (16.27) allows us to express all the fundamental astrophysical equations as a function of z . For measurements with higher accuracy, and whenever allowed by the observation instruments, we have to consider equations (16.10) and (16.12).

The redshift in equation (16.14) comes from the selfvariations of the rest mass and the electric charge of the electron. The redshift in the main volume of the linear spectrum we observe from distant astronomical objects, is actually caused by this effect. Today, however, we have the capability [16] to perform high sensitivity measurements of the effects of the Selfvariations. The structure of matter predicted by TSV must be taken into account in these measurements. The fundamental rest mass M_0 of equation (16.50) is by far smaller than the neutrino mass. Neutrinos, not to speak of other particles, have internal structure. This structure could influence the sum $W+E$ in the right part of equation (16.2). In such a case, we will obtain a different value for the constants k and c_0 for different material particles.

Writing equation (16.2) in the form

$$k_p = -\frac{bic_{0,p}}{\hbar} = \frac{b\left(W_p + E_p\right)}{\hbar} \qquad (16.51)$$

we can introduce an index " p " in the equations of this chapter. Every index " p " corresponds to a specific particle when the right part of equation (16.2) is not constant.

The measurements we perform on a cosmological scale depend on the physical quantity

$$\frac{k_p A_p}{1-A_p} = H_p \,. \qquad (16.52)$$

The main volume of the linear spectrum we get from distant cosmological objects comes from the process of atomic excitation/relaxation, thus the Hubble parameter H as given by equation (16.17)

$$\frac{kA}{1-A}=H$$

expresses the consequences of the selfvariation of the electron rest mass. In equations (16.14), (16.16), (16.22), (16.32), (16.33), (16.35), (16.41) and (16.42) the rest mass of the electron comes into play. Therefore, these equations are unaffected by equation (16.51).

The energy of the γ radiation that comes from nuclear reactions, and not from accelerated/decelerated electrons, depends on the particles that take part in the reaction. Consequently, their energy depends on equation (16.51). In this case equation (16.9) takes the form

$$m_0\left(r\right)=m_0\frac{1-A_p}{1-A_p\exp\left(-k_p\frac{r}{c}\right)} \tag{16.53}$$

and considering that

$$\frac{\lambda_\gamma\left(r\right)}{\lambda_\gamma}=\frac{\Delta m_0c^2}{\Delta m_0\left(r\right)c^2}$$

we get

$$\frac{\lambda_\gamma\left(r\right)}{\lambda_\gamma}=\frac{1-A_p\exp\left(-k_p\frac{r}{c}\right)}{1-A_p}$$

and we finally get

$$z_\gamma=\frac{A_p}{1-A_p}\left(1-\exp\left(-k_p\frac{r}{c}\right)\right). \tag{16.54}$$

For relatively small distances, from equation (16.54) we get

$$z_\gamma=\frac{k_pA_p}{1-A_p}\frac{r}{c}=H_p\frac{r}{c}. \tag{16.55}$$

Correlating a source of γ radiation from nuclear reactions with a galaxy, we can compare the z and z_γ redshifts. From this comparison we can draw important conclusions about equation (16.51) as well as about the predictions of the TSV. We note that the SCM, in explaining the redshift through the hypothesis of universal expansion, does not predict any difference between the z and z_γ redshifts.

Equation (16.28)

$$\frac{\dot{m}_0}{m_0} = H$$

holds for the rest mass of the electron. For other particles " p " it is written in the form

$$\frac{\dot{m}_0}{m_0} = \frac{k_p A_p}{1 - A_p} = H_p \,. \tag{16.56}$$

The mass of the electron represents a small part of the mass of the atom. Therefore, in measurements based on the gravitational interaction, the consequences of the mass selfvariation are governed by equation (16.56).

The rest mass $m_{0,H}$ of the hydrogen atom is

$$m_{0,H} = m_{0,p} + m_{0,e} \tag{16.57}$$

where $m_{0,p}$ and $m_{0,e}$ the rest mass of the proton and the electron respectively. From equation (16.57) we have

$$\dot{m}_{0,H} = \dot{m}_{0,p} + \dot{m}_{0,e}$$

and with equations (16.56) for the proton and (16.28) for the electron we have

$$\dot{m}_{0,H} = H_p m_{0,p} + H m_{0,e} \,. \tag{16.58}$$

From equations (16.57) and (16.58) we have

$$\frac{\dot{m}_{0,H}}{m_{0,H}} = \frac{H_p m_{0,p} + H m_{0,e}}{m_{0,p} + m_{0,e}}$$

and considering that today it is

$$m_{0,e} = 5.4 \times 10^{-4} m_{0,p}$$

we have

$$\frac{\dot{m}_{0,H}}{m_{0,H}}=\frac{H_p m_{0,p}+H\times 5.4\times 10^{-4}m_{0,p}}{m_{0,p}+5.4\times 10^{-4}m_{0,p}}$$

$$\frac{\dot{m}_{0,H}}{m_{0,H}}=\frac{m_{0,p}\left(H_p+5.4\times 10^{-4}H\right)}{m_{0,p}\left(1+5.4\times 10^{-4}\right)}$$

$$\frac{\dot{m}_{0,H}}{m_{0,H}}=\frac{H_p+5.4\times 10^{-4}H}{1+5.4\times 10^{-4}}=H_p+5.4\times 10^{-4}H\ . \tag{16.59}$$

From equation (16.59) we conclude that the ratio

$$\frac{\dot{m}_{0,H}}{m_{0,H}}$$

of the hydrogen atom depends on the relation of the parameter H_p for the proton with the Hubble parameter H. Similarly we obtain

$$\frac{\dot{m}_{0,a}}{m_{0,a}}=\frac{\mathrm{Z}}{\mathrm{A}}H_p+\left(1-\frac{\mathrm{Z}}{\mathrm{A}}\right)H_n+\frac{\mathrm{Z}}{\mathrm{A}}\times 5.4\times 10^{-4}H \tag{16.60}$$

for any atom, where Z is the atomic number and A is the nucleon number of the atom, and H_n the parameter of the neutron.

Equations (16.59) and (16.60) is valid for relatively small distances, up to a few hundred kpc. For larger distances we have to repeat the procedure of the proof using equations (16.9) and (16.53) instead of (16.28) and (16.56), from which we get

$$\begin{aligned}\frac{dm_0(r)}{dt}&=\dot{m}_0(r)=m_0\frac{kA\exp\left(-\frac{kr}{c}\right)}{1-A\exp\left(-\frac{kr}{c}\right)}\\ \frac{dm_0(z)}{dt}&=\dot{m}_0(z)=m_0H\frac{A-(1-A)z}{A(1+z)}\end{aligned} \tag{16.61}$$

and

$$\frac{dm_{0,p}(r)}{dt} = \dot{m}_{0,p}(r) = m_{0,p}\frac{k_p A_p \exp\left(-\frac{k_p r}{c}\right)}{1 - A_p \exp\left(-\frac{k_p r}{c}\right)} \tag{16.62}$$

$$\frac{dm_{0,p}(z_\gamma)}{dt} = \dot{m}_{0,p}(z_\gamma) = m_{0,p}H_p\frac{A_p - (1 - A_p)z_\gamma}{A_p(1 + z_\gamma)}$$

after the calculations.

The measurement of parameters H_p and H_n can be made by matching some sources of γ radiation from nuclear reactions to the galaxy they are coming from. By comparing the redshifts z and z_γ we can find the relation of H_p and H_n with the Hubble parameter H. The cosmological models that attribute the redshift to the expansion of the Universe predict equal z and z_γ redshifts. On the contrary, the TSV predicts that $z_\gamma = z$ if $H_p = H$ for every particle " p ".

Equation (16.51) affects equations (16.27), (16.29), (16.30) and (16.31), which are written in the form

$$m_0(z_\gamma) = \frac{m_0}{1 + z_\gamma} \tag{16.63}$$

$$E(z_\gamma) = \frac{E}{1 + z_\gamma} \tag{16.64}$$

$$z_a \sim z_\gamma \tag{16.65}$$

$$D = r\sqrt{1 + z_\gamma}\ . \tag{16.66}$$

The energy produced in the past at distant astronomical objects was smaller than the corresponding energy produced today in our galaxy. The production of energy in the Universe is mainly through hydrogen fusion and nuclear reactions. Therefore, equation (16.64) is of greater accuracy than equation (16.29). Nevertheless, the selfvariation of the electron's rest mass played a defining role in the energy produced in the past at distant cosmological objects. This is due to the fact that the fundamental astrophysical parameters depend on the rest mass of the electron. These parameters, therefore, depend on the redshift z, and not on the z_γ, according to equations (16.33), (16.35), (16.36), (16.41) and (16.42).

The most characteristic example concerns type I_a supernovae. The value of the rest mass of the electron, given as a function of the redshift z from equation (16.27), plays a defining role at all phases of evolution of a star undergoing type I_a supernovae. In any case it is obvious which of the performed measurements are affected by redshift z and which by z_γ.

It is very likely that there exists a small set of elementary particles with rest masses

$$M_0 = \frac{\hbar k_p}{bc^2}$$

and not just one elementary particle of rest mass

$$M_0 = \frac{\hbar k}{bc^2}.$$

It seems improbable that the sum $W + E$ on the right side of equation (16.2) is not affected by the internal structure of the generalized particles.

All of the presented consequences of theorem 3.3 are recorded within the cosmological data [17-29]. For the confirmation of the predictions of the theorem for the initial state of the Universe the improvement of our observational instruments is demanded. We also recommend evaluating the data recorded in CMRB, based on the equations of the TSV.

In the observations conducted for distances of cosmological scales, we observe the Universe as it was in the past. That is, we observe directly the consequences of the Selfvariations. We do not possess this possibility for the distances of smaller scales. The cosmological data are the result of the immediate observation of the Selfvariations and their consequences.

17. OTHER CONSEQUENCES OF THE THEOREM OF THE INTERNAL SYMMETRY

17.1. Introduction

The consequences of the theorem of the internal symmetry cover a wider spectrum, than the one already stated for the cosmological data. In these, the consequences of the dependence of the function Φ on time $x_0 = ict$ are recorded. The function Φ, according to equation (3.9), is a function of the set of the coordinates x_0, x_1, x_2, x_3 and also of the constants c_0, c_1, c_2, c_3. Therefore function Φ may change as a consequence of the change of the constants c_0, c_1, c_2, c_3. In this chapter we study the conditions under which such a change can be. We come to the conclusion that the alteration of the constants c_0, c_1, c_2, c_3 implies an uncertainty of position of the material particle.

The uncertainty of the material particle is predicted from the equations of the TSV for other reasons as well. When the constant b of the law of Selfvariations is not a real number, when $b \in \mathbb{C} - \mathbb{R}$, the equations of the TSV do not determine the position of the material particle in spacetime.

17.2. Other consequences of the theorem of the internal symmetry

The function Φ, according to equation (3.9), is a function of the set of the coordinates and also of the constants $x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3$ and given as

$$\Phi = \Phi\left(x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3\right) = K \exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]. \tag{17.1}$$

As in the previous chapter, we refrain our study in the case of $\Phi \neq -1 \wedge \Phi \neq 0$, as included in theorem 3.3. This case is equivalent with the relations $C \neq 0$ and $P \neq 0$ respectively.

Equations (3.10) and (3.11) express the rest mass m_0 of the material particle and the rest energy E_0 of the STEM as a function of Φ

$$m_0 = \pm \frac{M_0}{1+\Phi} \tag{17.2}$$

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi} \tag{17.3}$$

$$E_0 + m_0 c^2 = \pm M_0 c^2 . \tag{17.4}$$

In these equations the only constant is the rest mass M_0 of the generalized particle. Additionally the rest masses m_0 and E_0 depend on the constants c_0, c_1, c_2, c_3, according to equations (17.2), (17.3) and (3.9), in the following sense: For a constant rest mass M_0 of a generalized particle there are infinite values of the constants c_0, c_1, c_2, c_3, i.e. infinite states of the 4-vector C, for which equation (3.6) is valid:

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2 c^2 . \tag{17.5}$$

According to equation (3.12) the different states of the 4-vector C are equivalent with the ability of the material particle to have different momentums at the same point $A(x_0, x_1, x_2, x_3)$. Therefore the evolution of the generalized particle depends on all physical quantities $x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3$. We now deduce corollary 17.1 of theorem 3.3.

Corollary 17.1. ′′ The only constant physical quantity for a material particle is its total rest mass M_0. The evolution of the Universe, or of a system of particles, or of one particle, does not depend only on time. Its evolution is determined by the Selfvariations, as this manifestation is expressed through the function Φ .′′

Proof. Corollary 17.1 is an immediate consequence of theorem 3.3.□

According to corollary 17.1, each material particle is uniquely defined from the rest mass M_0 of equations (17.2) and (17.3).

From equations (2.4), (2.5) and (3.5) , and since it holds that $x_0 = ict$, we can write the function Φ in the form

$$\Phi = \Phi(t, x_1, x_2, x_3) = K \exp\left(-\frac{b}{\hbar}\left[-(W+E)t + c_1 x_1 + c_2 x_2 + c_3 x_3\right]\right) \tag{17.6}$$

with the sum $W + E = -icc_0$ being constant. This equation gives Φ as a function of time t , instead of the variable $x_0 = ict$.

In the afterword we present the reasons, according to which the TSV strenghtens at an important degree the Theory of Special Relativity [30-31]. In contrast, the theorem of internal symmerty highlights a fundamental difference between the TSV and the Theory of General

Relativity. According to equations (17.1) and (17.2), the physical quantity, which is being introduced into the equations of the TSV and remains invariant with repsect to all systems of reference, is the quantity given by

$$\delta = \frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right) \in \mathbb{C} \ . \tag{17.7}$$

Therefore, the TSV studies the physical quantity δ , and not, the also invariant with respect to all systems of reference, physical quantity of the four-dimensional arc length

$$dS^2 = \left(dx_0\right)^2 + \left(dx_1\right)^2 + \left(dx_2\right)^2 + \left(dx_3\right)^2 \ . \tag{17.8}$$

This arc length is studied by the Theory of General Relativity. The study of dS^2 can be interpreted in the manner that the Theory of General Relativity is a macroscopic theory. On the contrary, in the TSV a differentiation between the levels of the macrocosm and the microcosm does not exist. In equations (3.12), and for the energy and the momentum of the material particle,

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3$$

the concept of velocity does not exist. With the exception of equations (4.19) and (4.20), within the totality of the equations of the TSV we already presented, the concept of velocity does not enter. The difference among these two theories is highlighted in a concrete manner by the comparison of equations (17.7) and (17.8). In the first, spacetime appers together with the four-vector C. The second equation refers only to spacetime.

We present an example which highlights the diffrences among these two theories. It is the famous Twin Paradox. We consider that the reader is familiar with this thought experiment, as well as the result of the Theory of General Relativity [32]. The Theory of General Relativity predicts correctly the time difference in the time duration counted by the two twins. On the other hand, according to corollary 17.1, this time difference does not suffice for providing a difference in the evolution of the twins. The twins have the same generalized particles, which acquire the same rest masses M_0, at the time they meet together. At the beginning and at the end of the travel the two twins are identical. Einstein drives the wrong conclusion, not because the Theory of General Relativity is wrong, but because he regards that this time difference implies a different evolution of the twins. But, this is not a characteristic of the Theory of General Relativity. This is a common characteristic of all the physical theories preceding the TSV.

At this point let me commentate. Einstein refers to this thought experiment as the "Twin Paradox", and not as a consequence of the Theory of General Relativity. According to my opinion, Einstein understood that something was missing from the Theory of General Relativity. To this point advocates also his peristance for determining the cause of the quantum phenomena.

General relativity has been experimentaly verified from a large number of experiments. Moreover on a distance scale of a few hundred kpc its predictions are not affected by equations (16.27) and (16.63). We expect that the combination of the two theories on this distance scale will give important results for the physical reality.

We consider a generalized particle with rest mass M_0. The material particle of the generalized particle (together with STEM) can be at the spacetime point $A(x_0, x_1, x_2, x_3)$ with its energy-momentum having any value. According to equations (3.9) and (3.12) this can happen only with the variation of the 4-vector C. For a generalized particle the rest mass M_0 is constant, which means that through the variation of the 4-vector C, equation (17.5) remains valid.

From equation (3.1) we have

$$\frac{\partial J_i}{\partial c_k} + \frac{\partial P_i}{\partial c_k} = \frac{\partial c_i}{\partial c_k}, k, i = 0,1,2,3 \tag{17.9}$$

and from equation (3.9) we have

$$\frac{\partial \Phi}{\partial c_k} = -\frac{b}{\hbar}\Phi\left(x_k + \sum_{\substack{j=0 \\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right), k = 0,1,2,3. \tag{17.10}$$

Equations (17.9) and (17.10) remain valid for any variation of the 4-vector C. We will know prove the following corollary of theorem 3.3:

Corollary 17.2 ′′The variation of the 4-vector C of a generalized particle with rest mass M_0,

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2 c^2 \tag{17.11}$$

implies the variation of the 4-vectors J and P according to equations

$$\frac{\partial J_i}{\partial c_k} = \frac{1}{1+\Phi}\left(\frac{\partial c_i}{\partial c_k} + \frac{b}{\hbar}\frac{c_i}{1+\Phi}\Phi\left(x_k + \sum_{\substack{j=0 \\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right)\right), k, i = 0,1,2,3 \tag{17.12}$$

$$\frac{\partial P_i}{\partial c_k} = \frac{\Phi}{1+\Phi}\left(\frac{\partial c_i}{\partial c_k} - \frac{b}{\hbar}\frac{c_i}{1+\Phi}\left(x_k + \sum_{\substack{j=0 \\ j\neq k}}^{3} x_j \frac{\partial c_j}{\partial c_k}\right)\right), k, i = 0,1,2,3 \text{ .′′} \tag{17.13}$$

Proof: From equation (17.11) it follows that any change of one of the constants $c_k, k = 0,1,2,3$, induces a change to the others. Taking the derivatives with respect to $c_k \neq 0, k = 0,1,2,3$ we have

$$c_k = -\sum_{\substack{j=0 \\ j\neq k}}^{3} c_j \frac{\partial c_j}{\partial c_k}, c_k \neq 0, k = 0,1,2,3. \tag{17.14}$$

The corollary is then implied from the combination of the equations (3.12), (17.10) and (17.9). □

According to the relation (16.19) we have $\Phi < 0$, while according to the relations (16.20) the physical quantity $1+\Phi$ can be a positive or negative number. Hence from equations (17.12) and (17.13) we may determine the consequences for the material particle depending on whether the rates of change of

$$\frac{\partial J_i}{\partial c_k}, \frac{\partial P_i}{\partial c_k}, k, i = 0,1,2,3$$

are negative, positive or zero.

As the 4-vector C variates there arises an uncertainty for the position of the material particle. Corollary 17.2 predicts many cases for this uncertainty. We will restrict ourselves to one of them. For

$$\frac{\partial J_i}{\partial c_k} > 0 \text{ and } 1+\Phi < 0$$

from equation (17.12) we have

$$\frac{\partial c_i}{\partial c_k}+\frac{b}{\hbar}\frac{c_i}{1+\Phi}\Phi\left(x_k+\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\right)<0, k,i=0,1,2,3$$

and from equation (3.12) we have

$$\frac{\partial c_i}{\partial c_k}+\frac{b}{\hbar}J_i\Phi\left(x_k+\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\right)<0, k,i=0,1,2,3 \ . \tag{17.15}$$

From inequality (17.15) for $i=k, k,i=0,1,2,3$ we have

$$1+\frac{b}{\hbar}J_k\Phi\left(x_k+\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\right)<0, k=0,1,2,3$$

and because $\Phi<0$ we have

$$J_\kappa\left(x_k+\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\right)>-\frac{\hbar}{b\Phi}, k=0,1,2,3 \, . \tag{17.16}$$

In the case where

$$\sum_{\substack{j=0\\j\neq k}}^{3}x_j\frac{\partial c_j}{\partial c_k}\le 0 \tag{17.17}$$

from inequality (17.16) we have

$$\begin{gathered}J_kx_k>-\frac{\hbar}{b\Phi}, k=0,1,2,3\\(\Phi<-1, b\in\mathbb{R})\end{gathered} \ . \tag{17.18}$$

From equations (3.12) and (3.13) it follows that $P_k=\Phi J_k, k=0,1,2,3$, and considering that $\Phi<0$ we get from inequality (17.18) that

$$\begin{gathered}P_kx_k<-\frac{\hbar}{b}, k=0,1,2,3\\(b\in\mathbb{R})\end{gathered} \ . \tag{17.19}$$

The inequalities (17.16), (17.17), (17.18) and (17.19) reverse direction for $1+\Phi>0$. The inequalities (17.18) and (17.19) correspond to Heisenberg's uncertainty principle [33]. Corollary 17.2 gives rise to restrictions in the position of the material particle and of the STEM, in the spacetime area occupied by the generalized particle. These restrictions concern the position-momentum product.

The USVI gives the variation of the 4-vectors J and P in spacetime. However the internal symmetry theorem brings about a 'hidden' parameter of the interactions: The 4-vectors J and P may variate according to the variation of the 4-vector C. One of the consequences of the variation of the 4-vector C is the intense uncertainty of position-momentum showing up in the laboratory.

The theorem of internal symmetry, as well as the two degrees of freedom appearing in equations (5.3) and (5.7), foundain on a novel basis the manipulation of quantum information. The generalized particle is a sustained state, with constant total rest mass M_0. We may however interfere with the internal structure of the generalized particle changing the momentum of the material particle. According to equation (3.12) the variation of the momentum of the material particle can be effected either with the change of position $A(x_0, x_1, x_2, x_3)$ of the material particle, or with the change of the 4-vector C. The first variation is determined by the USVI and the second from corollary 17.2. With a periodic variation of either the energy of the material particle or the 4-vector C we can achieve the redistribution of the physical quatities m_0, E_0, J, P, j in the spacetime area occupied by the generalized particle. Through the variation of the physical quantities m_0, E_0, J, P, j we can transmit information in spacetime. Until now the transmission of information was achieved only with the first approach. Moreover we did not know the origin or structure of STEM. Corollary 17.2 permits us to study the possibility of information transmission through the variation of the 4-vector C. The two degrees of freedom in equations (5.3) and (5.7) refer to the function Ψ, which has a fundamental role for the transmission of information in either way. This role is clearly visible in equation (5.7) for the 4-vector j of the current density of the preserved quantities of the generalized particle.

In corollary 17.2 we assumed that the change of the 4-vector C does not change the total rest mass M_0 of the generalized particle. It is easy to show that the relations (17.12) and (17.13) are also valid in the case where the change of the 4-vector C causes a change of the total rest mass M_0 of the generalized particle. In such a case there happens a transition to another generalized particle since the rest mass M_0 characterizes the generalized particle. In this case we have a change of equation (17.14) which becomes

$$c_k = -\sum_{\substack{j=0 \\ j\neq k}}^{3} c_j \frac{\partial c_j}{\partial c_k} - M_0\, \mathrm{c}^2 \frac{\partial M_0}{\partial c_k}, c_k \neq 0, k = 0,1,2,3. \tag{17.20}$$

Equation (17.20) is obtained from differentiation with respect to $c_k \neq 0, k = 0,1,2,3$ of equation (17.11).

If there is only one fundamental particle, with rest mass

$$M_0 = \frac{\hbar k}{bc^2} \ ,$$

equation (17.14) necessarily applies. The equation (17.20) can only be applied if there are more than one fundamental material particles, with rest masses

$$M_0 = \frac{\hbar k_p}{bc^2} .$$

Moreover, if there are more than one fundamental material particles, the constants $c_k, k = 0,1,2,3$ can vary independently, i.e. they can vary in a way such that

$$\frac{\partial c_j}{\partial c_k} = 0, j \neq k, k \in \{0,1,2,3\}, j = 0,1,2,3 \tag{17.21}$$

during the variation of the 4-vector C. In this case from corollary 17.2 there follow the inequalities (17.18) and (17.19) for $1+\Phi<0$, and in opposite direction for $1+\Phi>0$. The condition (17.21) can be realized when rest masses M_0 are *more than one.*

The Selfvariations illuminate a property of the Universe that cannot be illuminated by the theories of the last century. It is the «internality of the universe to the process of measurement». That is , the fact that the Selfvariations also affect the physical quantities we use as units for the measurement of other similar physical quantities.

Comparing the rest masses of two similar material particles, two electrons for example, we will always find them equal. This equality stems from the fact that a certain material particle, like the electron, expresses a certain state within set Ω;

$$\Omega = \Omega_O \cup \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6$$
$$\Omega_O = \{T = 0\}$$

a state in which the Selfvariations are evolving in the exact same maner.

Two different material particles express two different states of set Ω and, therefore, the selfvariations could evolve at different rates. The ratio of the rest masses of two different material particles could vary with time. In such a case, this variation will occur at an extremely slow rate, as emerges from the analysis of the cosmological data.

The electric charge of material particles can be either positive or negative. It also increases at a clearly slower rate than that of the rest mass of the electron. These are two fundamental differences we already know between the rest mass and the electric charge. We can make analogous hypotheses regarding the differences of the ′′selfvariating charge Q′′ and the rest mass. We have not presented these hypotheses in the current article, since it is about the basic study of the Selfvariations.

The observational instruments we have at our disposal, and the techniques that have been devised [16, 19, 20, 21, 25, 26, 28, 29] have brought us on the cusp of a direct detection of the consequences of the Selfvariations. Together with the measurements we perform, it is essential to analyse the microwave background radiation (CMBR) according to the predictions of the TSV. During the phase of evolution of the universe when the CMBR was produced, the rest masses of material particles were clearly smaller than the corresponding laboratory ones. A similar thing is true about the electric charge of the electron. We speculate that this fact is in some way recorded on the CMBR.

The continuous emmision of STEM in the surrounding spacetime of the material particles entails the continuous strengthening of negative potential energies of every kind. The consequences of this strengthening are multiple: from the shortening of the Bohr radius, to the strengthening of the cohesion of the material particles with time. The analysis of the cosmological data allows us to define the order of magnitude of the rest mass M_0. A conclusion that emerges is that particles we considered fundamental, like the electron and the neutrino, constitute composites of simpler particles. Our inability to break up the electron in the lab is a consequence of the strengthening of its cohesion, for the afforementioned reason.

There are a number of ways in which we can search for material particles with elementary rest masses

$$M_0 = \frac{\hbar k}{bc^2}$$

and

$$M_0 = \frac{\hbar k_p}{bc^2}$$

predicted by the TSV. But they all require the improvement of our observational instruments.

One way is to give material particles extremely large energies in the laboratory. The energies at which we collide particles today are far too small to achieve such a disintegration. Another way is to perform measurements at extremely large distances. In the very early universe the cohesion of material particles, even of the neutrino, tends to zero. This phase of the evolution of the universe is predicted over and above the limit set by the standard cosmological model for the size of the universe.

The TSV predicts M_0 as the minimum value of the rest mass of material particles. Since neutrinos have the smallest rest mass of all material particles we know off today, it is quite likely that they have the simplest structure of all known material particles. We, therefore, propose the intensification of the experimental study of neutrinos and their properties.

From equation (17.1), and taking into account that $x_0 = ict$, we derive that when the constant b of the law of the Selfvariations is a real number and $K \in \mathbb{R}$ then $\Phi \in \mathbb{R}$:

$$b, K \in \mathbb{R} \Rightarrow \Phi \in \mathbb{R}. \tag{17.22}$$

In that case, the rest mass m_0 of the material particle is uniquely defined by the point $A(x_0, x_1, x_2, x_3)$ of spacetime at which the material particle is located. This case was examined in chapter 16.

Similarly, from equation (17.1) we derive that when the constant b of the law of Selfvariations is not a real number, it holds that

$$\left.\begin{array}{l} b \in \mathbb{C} - \mathbb{R} \\ K \in \mathbb{R} \end{array}\right\} \Rightarrow \Phi \in \mathbb{C} - \mathbb{R}. \tag{17.23}$$

In that case, function Φ, and through equation (17.1) also function $m_0 = m_0\left(x_0, x_1, x_2, x_3\right)$, are periodic. There is an infinite number of spacetime points at which function $m_0 = m_0\left(x_0, x_1, x_2, x_3\right)$ obtains the same value. Simultaneously, however, the quotient $\frac{m_0}{M_0}$ is not a real number,

$$\frac{m_0}{M_0} = \frac{1}{1+\Phi} \in \mathbb{C} - \mathbb{R} \tag{17.24}$$

as it is derived from equations (17.1) and (17.23).

The inequalities (17.15)-(17.19) are meaningful only if the function Φ acquires real values, $\Phi \in \mathbb{R}$. This is always the case when $b, K \in \mathbb{R}$, as follows from relation (17.22). In case where $b \in \mathbb{C} - \mathbb{R}, K \in \mathbb{R}$ the function Φ generally acquires values in the set $\mathbb{C}$ of complex numbers, as follows from relation (17.23). In this case the inequalities (17.15)-(17.19) are valid only for values of x_0, x_1, x_2, x_3 such that $\Phi = \Phi\left(x_0, x_1, x_2, x_3\right) \in \mathbb{R}$.

18. THE SET Ω

18.1. Introduction

The four-vector C determines the total rest mass M_0 of the generalized particle through equation (3.6). In the previous chapter we examined the variation of the four-vector C in the internal symmetry. In this chapter, we examine the variation of the four-vector C in the external symmetry.

18.2. The set Ω

We present a physical procedure where the 4-vector C varies in the external symmetry. Initially we use the notation Ω_{O} for the set which has as unique element the internal symmetry matrix. The internal symmetry matrix is the 4×4 zero matrix $T=0$, hence

$$\Omega_{\mathrm{O}} = \{T=0\}. \tag{18.1}$$

The sets $\Omega_{\mathrm{O}},\Omega_0,\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_6$ contain all symmetries of the TSV in *four-dimensional* spacetime ($\mathrm{N}=4$). Moreover the sets $\Omega_{\mathrm{O}},\Omega_0,\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_6$ do not have common elements, that is

$$\begin{gathered}\Omega = \Omega_{\mathrm{O}} \cup \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_6 \\ \Omega_i \cap \Omega_j = \varnothing, i \neq j, i, j \in \{\mathrm{O},0,1,2,3,4,6\}\end{gathered}. \tag{18.2}$$

Every generalized particle corresponds to a particular matrix of the set Ω. Therefore the set Ω all possible states of matter, all possible states of physical reality predicted by the TSV (in four-dimensional spacetime).

Inside the set Ω any transition can happen, with the generalized particle going from one symmetry to another. This transition can happen either inside one set $\Omega_k, k \in \{\mathrm{O},0,1,2,3,4,6\}$ or between two different sets $\Omega_k,\Omega_i,k,i \in \{\mathrm{O},0,1,2,3,4,6\}$. An example of a transition from one symmetry to another is presented at the end of chapter 14, where we demonstrated the transition from symmetry $T_{0103321321}$ to symmetry $T_{01033221}$.

In four-dimensional spacetime ($\mathrm{N}=4$) the set Ω contains $N_{\mathrm{O}}+N_0+N_1+N_2+(N_3-1)+N_4+N_6 = 1+14+6+24+11+3+1=60$ matrices. The transitions inside the set Ω can happen to both directions, therefore there are in total

$$N = 2\binom{60}{2} = 3540 \tag{18.3}$$

possible transitions inside the set Ω. In 3524 of the 3540 possible transitions the 4-vector C changes, and only in 16 cases it remains unchanged. In the 3524 cases where the 4-vector C changes the corollary 17.2 is valid. Of particular interest are the

$$2\binom{7}{2} = 42 \tag{18.4}$$

transitions between sets Ω_k and Ω_i with $k \neq i, k, i \in \{\mathrm{O}, 0, 1, 2, 3, 4, 6\}$.

In contrast with the variation of the 4-vector C, in all transitions there is either a constant rest mass

$$M_0 = \frac{\hbar k}{bc^2}$$

or a *limited*, possibly small number of rest masses

$$M_0 = \frac{\hbar k_p}{bc^2}$$

as a result of the study of the previous chapter.

In the 4 symmetries of the set $\Omega_4 \cup \Omega_6$ the matrix T of the symmetry, as well as the 4-vector C, have more than one mathematical expressions. In these symmetries the 4-vector C can vary inside the symmetry. These transitions are not included in the 3540 possible transitions of equation (18.3). There are symmetries in which the 4-vector C has more than one mathematical expressions, with $M_0 = 0$ ($m_0 = \frac{E_0}{c^2} = M_0 = 0$). An example of such a symmetry is $T_{01033221}$ of the set $\Omega_4 \subset \Omega_4 \cup \Omega_6$ (see chapter 13). The matrix $T_{01033221}$ has four different mathematical expressions

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix}$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{bmatrix}$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & -1 \\ i & 0 & 1 & 0 \end{bmatrix}$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & -1 & 0 \end{bmatrix}$$

and correspondingly twelve different 4-vectors C

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} c_0 \\ c_1 \\ ic_0 \\ ic_1 \end{bmatrix} \quad c_0c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} c_0 \\ c_1 \\ -ic_0 \\ ic_1 \end{bmatrix} \quad c_0c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & -1 \\ i & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_0 \\ c_1 \\ -ic_0 \\ -ic_1 \end{bmatrix} \quad c_0c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_0 \\ c_1 \\ ic_0 \\ -ic_1 \end{bmatrix} \quad c_0c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix} \quad C = c_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \quad c_0 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & -1 \\ -i & 0 & 1 & 0 \end{bmatrix} \quad C = c_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \quad c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{bmatrix} \quad C = c_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \quad c_0 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{bmatrix} \quad C = c_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \quad c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & -1 \\ i & 0 & 1 & 0 \end{bmatrix} \quad C = c_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \quad c_0 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & -1 \\ i & 0 & 1 & 0 \end{bmatrix} \quad C = c_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \quad c_1 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & -1 & 0 \end{bmatrix} \quad C = c_0\begin{bmatrix} 1 \\ 0 \\ i \\ 0 \end{bmatrix} \quad c_0 \neq 0$$

$$T = zQ\alpha_{01}\begin{bmatrix} 0 & 1 & 0 & -i \\ -1 & 0 & -i & 0 \\ 0 & i & 0 & 1 \\ i & 0 & -1 & 0 \end{bmatrix} \quad C = c_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ i \end{bmatrix} \quad c_1 \neq 0 .$$

The generalized particle has rest mass $M_0 = 0$ ($m_0 = \frac{E_0}{c^2} = M_0 = 0$) .

There are symmetries in which the 4-vector C has more than one mathematical expressions, with $M_0 \neq 0$. In these symmetries the 4-vector C can also vary inside the symmetry. As an example we mention the symmetry T_{01}^{01} of the set Ω_1 (see chapter 12), which has two mathematical expressions two the 4-vector C

$$C = \begin{bmatrix} c_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$c_0 \neq 0$

$$C = \begin{bmatrix} 0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}$$

$$c_1 \neq 0$$

six mathematical expressions for the matrix T_{01}^{01}

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} \pm\left(\gamma_0 \exp\left(\frac{bc_0x_0}{\hbar}\right)-1\right)^{-\frac{1}{2}} & 1 & 0 & 0 \\ -1 & \mp\left(\gamma_0 \exp\left(\frac{bc_0x_0}{\hbar}\right)-1\right)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma_0 \in \mathbb{C}, \gamma_0 c_0 \neq 0, c_1 = c_2 = c_3 = 0$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} \pm\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right)-1\right)^{-\frac{1}{2}} & 1 & 0 & 0 \\ -1 & \mp\left(\gamma_1 \exp\left(\frac{bc_1x_1}{\hbar}\right)-1\right)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma_1 \in \mathbb{C}, \gamma_1 c_1 \neq 0, c_0 = c_2 = c_3 = 0$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} i & 1 & 0 & 0 \\ -1 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T_{01}^{01} = zQ\alpha_{01} \begin{bmatrix} -i & 1 & 0 & 0 \\ -1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the rest mass $M_0 \neq 0$ ($M_0 = \pm\frac{ic_0}{c} \neq 0 \vee M_0 = \pm\frac{ic_1}{c} \neq 0$).

The uncertainty of the material particle is predicted from the equations of the TSV for other reasons as well. When the constant b of the law of Selfvariations is not a real number, when $b \in \mathbb{C} - \mathbb{R}$, the equatons of the TSV do not determine the position of the material particle in spacetime. For example we show the case where $b = i$ for symmetry T^{12} . From equation (8.18) it follows that the charge Q has the same value at the different points $A(x_0, x_1, x_2, x_3)$ and $B(x_0 + icT, x_1, x_2, x_3 + X_3)$ of spacetime when

$$\begin{gathered} c_3 X_3 - \| c_0 \| cT = 2\pi n\hbar, n = 0, \pm 1, \pm 2, \pm 3, ... \\ \| c_0 \| = -ic_0 = \frac{W + E}{c} \end{gathered} \quad . \qquad (18.5)$$

For $b = i$ there are infinite points in spacetime where the charge Q has the same value. Analogous conclusions are predicted for all external symmetries when $b \in \mathbb{C} - \mathbb{R}$.

19. AFTERWORD

We offer as an epilogue some comments on the TSV as a whole. Having concluded our study, it has become clear that the network of equations of the TSV arises from the combination of the axiom of the Selfvariations, as given by equation (4.2), and the principle of conservation of the momentum 4-vector and equation (2.7). The conservation principle of the 4-dimentional momentum has emerged empirically, from the experimental data. The TSV lays the axiomatic foundations of theoretical physics with just three axioms. Indeed, it is very likely that equation (2.7) emerges from the other two axioms. As far as we know, no other science is axiomatically founded with such a small number of axioms. Equation (2.7) comes from special relativity. We, therefore, start our comments with the relation of the TSV with special relativity.

Special relativity imposes constraints on the mathematical formulation of physical laws. All mathematical equations of physical laws have to be invariant under Lorentz-Einstein transformations. The TSV comes to impose an even greater constraint on these mathematical formulations. If we denote L the set of equations that are Lorentz-Einstein-invariant and S the set of equations compatible with the law of Selfvariations, it is $S \neq L$ with $S \subset L$.

A classic example are the Lienard-Wiechert electromagnetic potentials. These were proposed by Lienard and Wiechert in 1899 and give the correct electromagnetic field and electromagnetic radiation for a randomely moving electric charge. After the formulation of special relativity by Einstein in 1905, the Lienard-Wiechert potentials proved to also be invariant under Lorentz-Einstein transformations. On the contrary, with the formulation of the TSV they prove to be incompatible with the Selfvariations. The TSV replaces the Lienard-Wiechert electromagnetic potentials with the macroscopic potentials of the TSV, which give exactly the same field with the Lienard-Wiechert potentials. The macroscopic potentials of the TSV are compatible with the Selfvariations as well as with the Lorentz-Einstein transformations ($S \subset L$). An additional characteristic of the macroscopic potentials of the TSV is this: Whether we consider that the Selfvariations happen, or whether we consider the electric charge constant, exactly the same field emerges. It is an expression of the “internality of the universe to the process of measurement”.

To get the Lorentz-Einstein transformations we consider two observers exchanging signals with velocity c. If the observers are moving with the same velocity with respect to each other, the Lorentz-Einstein transformations emerge. If the observers exchange signals with a velocity different from c, for example sound signals, other transformations emerge which are incorrect. As is natural, Einstein was asked about this issue. His answer essentially was that we chose the exchange of signals with light speed because of the result: the transformations that emerge in this way are the correct ones.

On this point the TSV reinforces special relativity to a superlative degree. There is a constant exchange of STEM between material particles which, in the macrocosm, and when spacetime is flat, occurs with velocity c. According to the TSV the exchange of signals with velocity c is not just a hypothesis we can make in order to obtain the Lorentz-Einstein transformations, but constitutes a continuous physical reality.

The Selfvariations of the rest massess occur if and only if they are counterbalanced by a corresponding emmission of negative energy in the surrounding spacetime of the material particle, so that the energy-momentum conservation holds. This energy-content of the spacetime is expressed by the 4-vector P of equation (2.5). Macroscopicaly this energy is expressed by equation (16.8), which has emerged from equation (3.11), i.e. the internal symmetry theorem. This is expected since the internal symmetry expresses specifically the spontaneous realization of the Selfvariations. Something analogous holds for the electric charge and for any selfvariating charge Q. The spontaneous emmision of negative energy in spacetime has two fundamental consequences.

The continuous exchange of STEM implies a continuous exchange of information between material particles. If the universe is finite, with a finite age, there are still parts of it that have not exchanged information through the STEM, as a consequence of the finite speed of the STEM. This, however, will occur in the future. With the passage of time, every part of the universe interacts with an ever larger part of the rest of the universe. According to equation (16.45), going back in time the uncertainty of the position of material particles tends to become infinite. Regions of the universe which will interact through STEM at a future time, have already interacted through material particles at a past time. The above hold even in the case where the universe is infinite and of infinite age. The only difference is that all of its parts have also interacted through STEM. We thus come to the following fundamental conclusion of the TSV: The Universe behaves as one object.

The second consequence is the indirect dynamic interaction of the material particles (USVI). When I was differentiating for the calculation of the rate of change of the momentum of the material particle, there where specific conditions: the law of Selfvariations predicts a unified mechanism for all interactions. Consequently, the Lorentz force should emerge after the differentiation and also, in some way, the relation of the USVI with the curvature of spacetime according to Einstein's work on gravity. We now know that equations (4.19), (4.20) and (14.22) contains both of these terms.

I was often asked why we set as an axiom the Selfvariations of the rest masses and the electric charge, and not the Selfvariation of some other physical constant. An axiom is judged exclusively by the conclusions to whch it leads. Nevertheless, there is always a specific ′′logic′′ for the introduction of an axiom in a scientific field. This is also the case for the axiom of the Selfvariations. Taking into account the energy-momentum conservation principle, the Selfvariation of the rest mass of the material particle can only take place with the simultaneous emmision of energy-momentum into the surrounding spacetime of the particle. The combination of the Selfvariations with the conservation of energy-momentum has as a consequence the presence of energy-momentum in the surrounding spacetime of the material particle. The introduction of the axiom of the rest mass Selfvariation was made with the expectation that this energy-momentum in spacetime could provide a cause for the interaction of material particles. In retrospect, this expectation was confirmed. The fundamental physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ that emerge from this combination lead to the USVI, and are at the heart of the TSV. Being aware of the existence of the gravitational interaction we set as an axiom the selfvariation of the rest mass. Similarly, due to the existence of the electromagnetic interaction we set as an axiom the selfvariation of the electric charge. Following the same ′′logic′′ we introduce in the TSV the ′′selfvariating charge Q′′ through equation (4.2).

The internal symmetry theorem and the set Ω_O express the isotropic emission of STEM in the flat spacetime of special relativity. The theorems of external symmetry and the set $\Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_6$ express the anisotropic emmision of STEM in an anisotropic four-dimensional spacetime. Every generalized particle corresponds to a matrix-element of the set $\Omega = \Omega_O \cup \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_6$. Every element of set Ω contains an extremely large amount of data and information about the physical condition it describes. The set Ω contains all the information about the physical reality predicted by the TSV, in four-dimensional spacetime ($\mathrm{N} = 4$).

In the internal symmetry, the distribution of the total rest mass M_0 of the generalized particle, between the material particle and the STEM, is given by equations (3.10) and (3.11). In the external symmetry, for every selfvariating charge Q there are $N_T = 59$ external symmetry matrices in four-dimensional spacetime ($\mathrm{N} = 4$). There are, therefore, $N_T = 59$ ways to distribute the rest mass M_0 in the external symmetry. In chapter 8 we proved that in the $N_0 = 14$ symmetries $T = zQ\Lambda$, it can be $m_0 \neq 0$ for the rest mass of the USVI particle. Similarly, for the $N_1 = 6$ symmetries of the set Ω_1 it can be $m_0 \neq 0$. In the remaining $59-14-6=39$ external symmetries, it is $m_0 = 0$ for the rest mass of the USVI particle.

We left for last some comments on equations (2.10), (2.13) and (4.6). Equation (2.10) cannot arise without the axiom of the Selfvariations. The fundamental physical quantities $\lambda_{ki}, k, i = 0.1, 2, 3$ cannot arise from the physical theories of the last century. That is, they cannot arise in any way other than the hypothesis of the Selfvariations. From equations (2.10) flows the entire network of equations of the TSV, including equations (2.13) and (4.6).

Equations (2.13) and (4.6) predict the USVI and, additionally, correlate the corpuscular with the wave behaviour of matter. The properties of the wave-function Ψ as well as the 4-vector j of the conserved physical quantities, express exactly these equations. The laws of Maxwell are four precisely because the first of equations (4.6) expands into four distinct equations. The theorems of chapter 7, which define the corpuscular structure of matter, are nothing more than the consequences of equation (2.10). The same holds for the internal symmetry theorem and its consequences, which emerge from equation (2.10).

Both the sets $\Omega_k, k = 0,1,2,...,\frac{\mathrm{N}(\mathrm{N}-1)}{2}$, as well as the properties of their elements, i.e. the properties of external symmetries, depend on the number $\mathrm{N} \in \mathbb{N}$ of spacetime dimensions. That is, from the number of coordinates $x_k, k = 0,1,2,...,\mathrm{N}-1$ required to express the $\mathrm{N}-\mathrm{dim}$ vectors J and P. For example the symmetries of the set Ω_1 are 1-dimensional. In contrast, for the symmetries of the set Ω_5 at least 5 ($\mathrm{N} \geq 5, \mathrm{N} \in \mathbb{N}$) spacetime dimensions are required, while the symmetry $T_{010203321321}$ exist for every N-dimensional spacetime, $\mathrm{N} \geq 4, \mathrm{N} \in \mathbb{N}$. The relation of the sets $\Omega_k, k = 0,1,2,...,\frac{\mathrm{N}(\mathrm{N}-1)}{2}$ with the number of spacetime dimensions $\mathrm{N} \in \mathbb{N}$ is determined by the entirety of the theorems of the TSV. The $SV-T$ method provides the simplest way to determine this relation. A detailed application of this method is presented in chapter 14.

The matrix of the internal symmetry is the zero matrix, $\Omega_{O} = \{T = 0\}$ independently of the number $N \in \mathbb{N}$ of spacetime dimensions. Consequently, the study presented in chapter 16 for the cosmological data is independent of the number $N \in \mathbb{N}$ of spacetime dimensions.

Using the $SV - T$ method we can verify the self-consistency of the network of equations we provide. The TSV is a closed and self-consistent theory.

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