Using metaballs to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes

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March 20, 2017

Abstract

In this paper, Blinn's metaballs are used to model the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes. An analytical solution is provided.

1 Metaballs

Metaballs have been used in computer graphics ever since their discovery by Jim Blinn. They were originally used to visualize electron density [1]. In this paper we will model the pre-ringdown phase of the merger of $n = 2$

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Schwarzschild black holes using the simple mathematics of metaballs [2]. Analogously, we will be visualizing graviton density.

Where $G = c = \hbar = k = 1$, there is a metaball-based solution for the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes travelling directly toward each other:

$$f(l) = \sum_{i=1}^{n=2} \frac{2M_i}{r_i} = \frac{2M_1}{r_1} + \frac{2M_2}{r_2}. \tag{1}$$

Here M_i is the mass of the ith black hole (metaball), and

$$r_i = \sqrt{(l.x - v_i.x)^2 + (l.y - v_i.y)^2 + (l.z - v_i.z)^2}, \tag{2}$$

where l is a sample location, and v_i is the centre of the ith black hole.[1] The event horizon (the isosurface) is given by $f(l) = 1$.

Included are figures of the pre-ringdown phase of the merger of $n = 2$ Schwarzschild black holes. It is shown that the event horizon resembles the Cassini ovals (or some generalization thereof), which makes for an analytical solution.[2]

The C++/OpenGL code for this paper can be found at [3].

[1]Here the '.' operator is the 'member of' operator, like in the C++ programming language – that is, both l and v_i are instantiations that each encapsulate three variables: x, y, and z.

[2]Here we simplify by disregarding the black hole velocity and acceleration, as well as Hawking and gravitational radiation. We also simplify by assuming that change in the gravitational field propagates at a speed of infinite, like in Newtonian gravitation. Getting rid of these simplifications is left for a future paper.

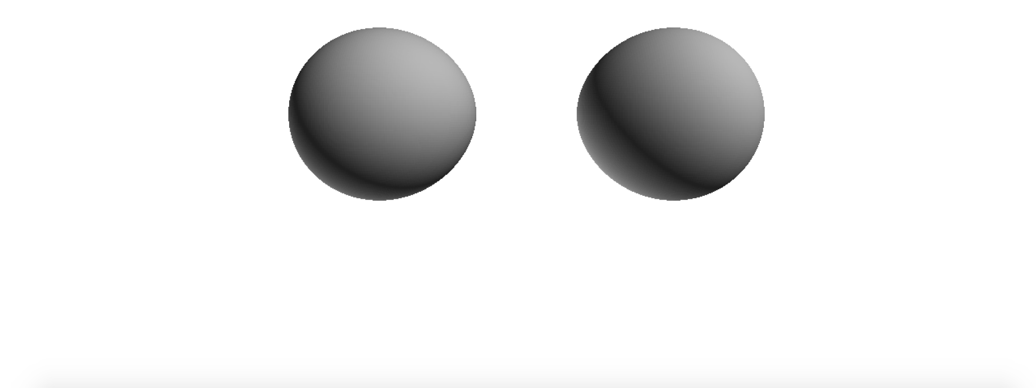

Figure 1: Two black holes of unit mass each, at a distance of 9. Here the unit mass is the Planck mass $M_P = \sqrt{\hbar c/G} = 1$, and the unit distance is the Planck length $\ell_P = \sqrt{\hbar G/c^3} = 1$. Note that the event horizon resembles Cassini ovals.

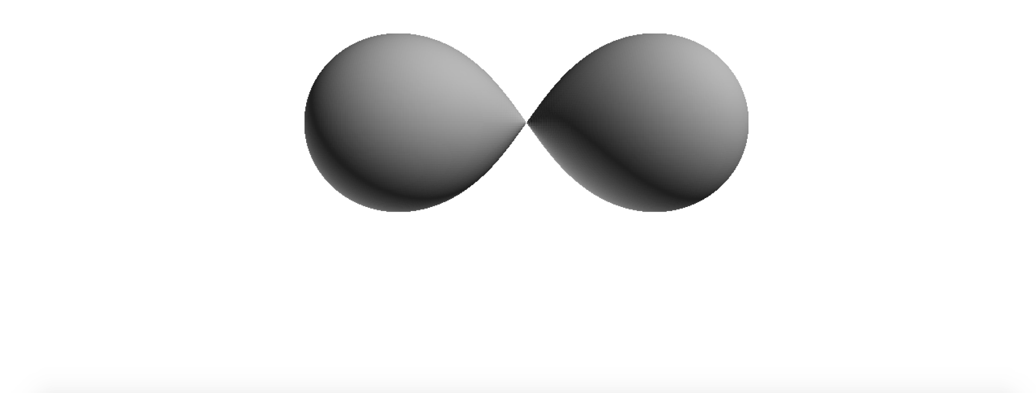

Figure 2: Two black holes of unit mass each, at a distance of 8. Note that the event horizon resembles the lemniscate of Bernoulli, which is a special case of Cassini ovals.

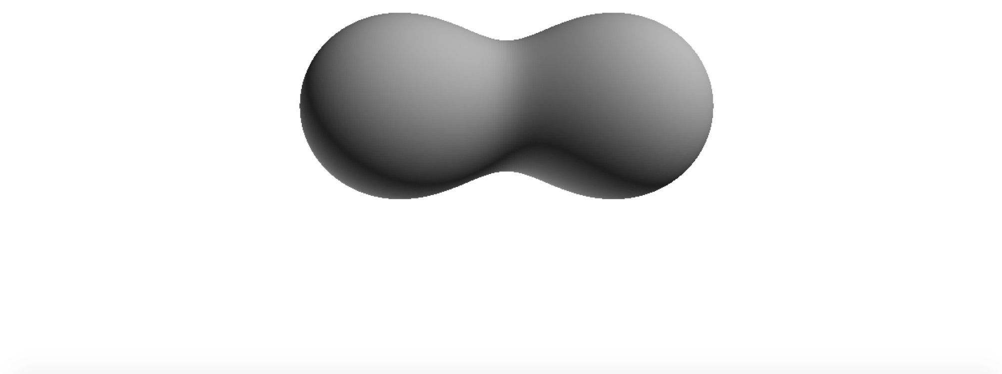

Figure 3: Two black holes of unit mass each, at a distance of 7.

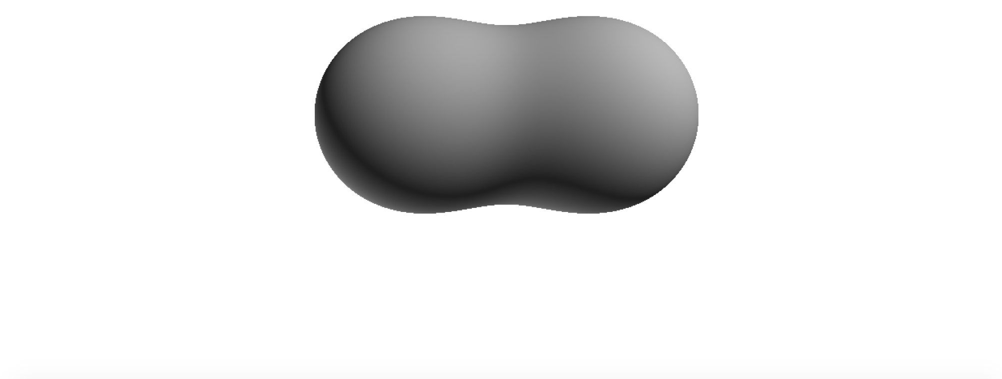

Figure 4: Two black holes of unit mass each, at a distance of 6.

Figure 5: Two black holes of unit mass each, at a distance of 5.

Figure 6: Two black holes of unit mass each, at a distance of 4.

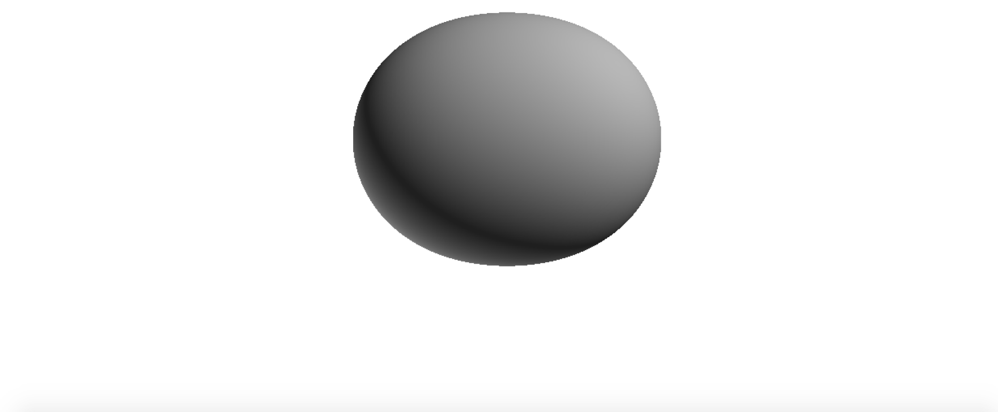

Figure 7: Two black holes of unit mass each, at a distance of 3.

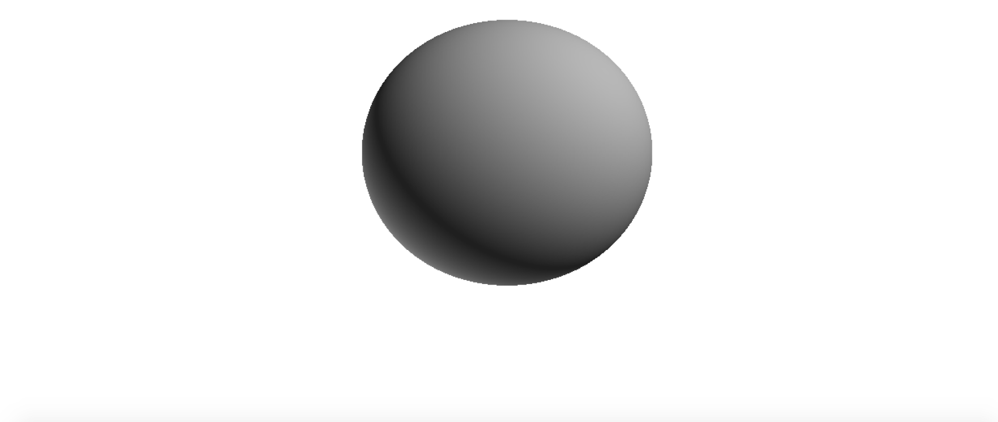

Figure 8: Two black holes of unit mass each, at a distance of 2.

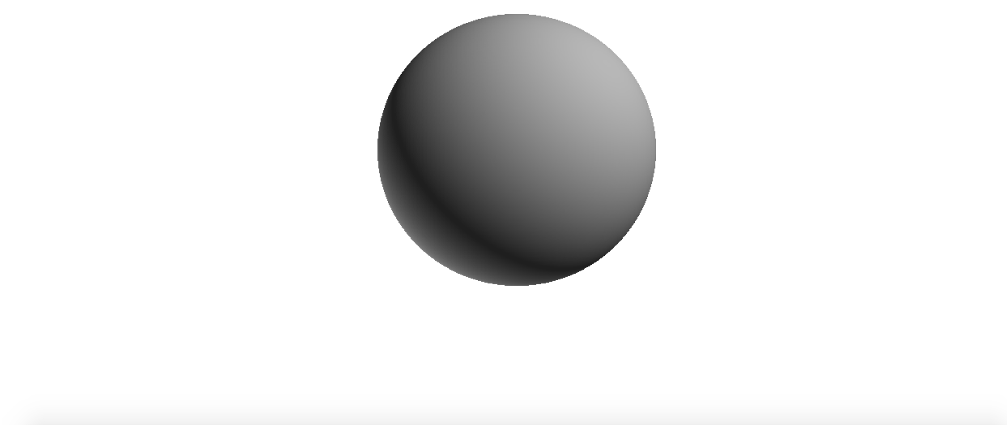

Figure 9: Two black holes of unit mass each, at a distance of 1.

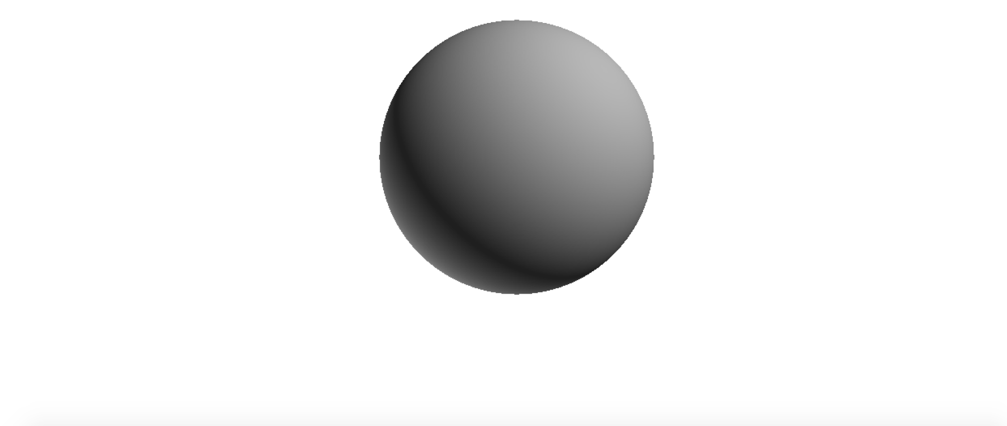

Figure 10: *One* black hole of mass = 2.

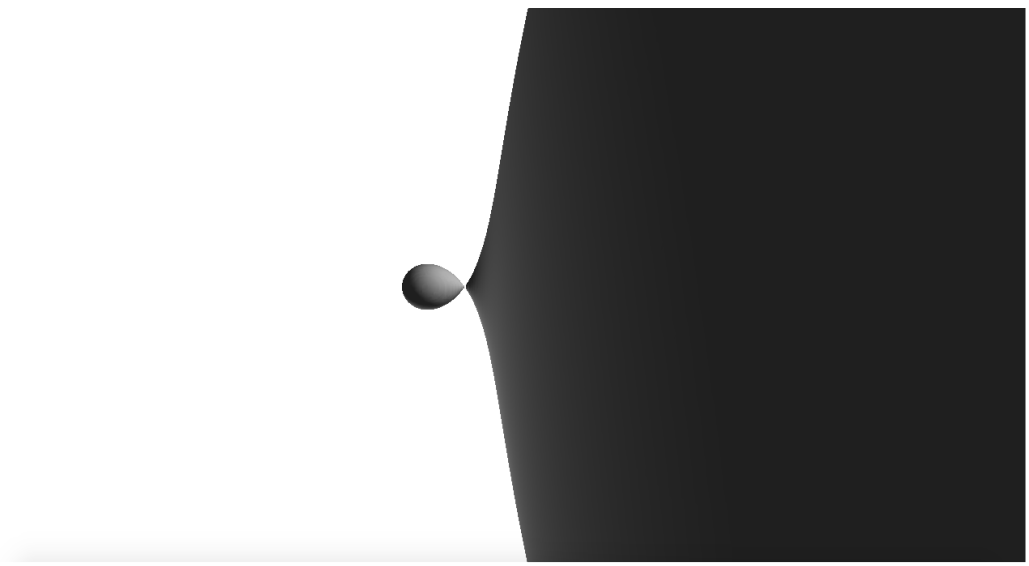

Figure 11: A unit mass black hole merging with a black hole of mass = 1000. Note that the event horizon is likely to be given by some generalization of Cassini ovals, where the torus in question is lopsided like a ring cyclide (Dupin cyclide). This ring cyclide is used instead of the elliptic functions used in [4].

References

[1] Blinn J. *A generalization of algebraic surface drawing* ACM Transactions on Graphics, Vol. 1, No. 3

[2] Daoust M. http://physics.stackexchange.com/questions/18769/black-hole-collision-and-the-event-horizn

[3] Halayka S. *Code* https://github.com/sjhalayka/bhmerger

[4] Emparan R, Martinez M. *Exact event horizon of a black hole merger* Classical and Quantum Gravity, Vol. 33, No. 15