

Conjecture on 3-Carmichael numbers of the form $(4h+3)(4j+1)(4k+3)$

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Abstract. In this paper I conjecture that for any 3-Carmichael number (absolute Fermat pseudoprime with three prime factors, see the sequence A087788 in OEIS) of the form $(4h+3)(4j+1)(4k+3)$ is true that $(k-h)$ and j must share a common factor (sometimes $(k-h)$ is a multiple of j). The conjecture is probably true even for the larger set of 3-Poulet numbers (Fermat pseudoprimes to base 2 with three prime factors, see the sequence 215672 in OEIS).

Conjecture:

For any 3-Carmichael number (absolute Fermat pseudoprime with three prime factors, see the sequence A087788 in OEIS) of the form $(4h+3)(4j+1)(4k+3)$ is true that $(k-h)$ and j must share a common factor (sometimes $(k-h)$ is a multiple of j). The conjecture is probably true even for the larger set of 3-Poulet numbers (Fermat pseudoprimes to base 2 with three prime factors, see the sequence 215672 in OEIS).

Note: The number of the 3-Carmichael numbers of the form $(4h+1)(4j+1)(4k+1)$ respectively $(4h+3)(4j+1)(4k+3)$ seems to be prevalent in front of those of the form $(4h+1)(4j+3)(4k+1)$ or $(4h+3)(4j+3)(4k+3)$.

Verifying the conjecture:

(for ten consecutive 3-Carmichael numbers of this form)

$$: \quad 7145178241130641 = 61987 \cdot 123973 \cdot 929791 = (4 \cdot 15496 + 3) \cdot (4 \cdot 30993 + 1) \cdot (4 \cdot 232447 + 3) \text{ and } 232447 - 15496 = 7 \cdot 30993;$$

$$: \quad 7161631253773501 = 46771 \cdot 654781 \cdot 233851 = (4 \cdot 11692 + 3) \cdot (4 \cdot 163695 + 1) \cdot (4 \cdot 58462 + 3) \text{ and } 58462 - 11692 \text{ share with } 163695 \text{ the factor } 54565;$$

$$: \quad 7162573122326497 = 64499 \cdot 36857 \cdot 3012979 = (4 \cdot 16124 + 3) \cdot (4 \cdot 9214 + 1) \cdot (4 \cdot 753244 + 3) \text{ and } 753244 - 16124 = 80 \cdot 9214;$$

- : $7181488986943501 = 29851 \cdot 1611901 \cdot 149251 = (4 \cdot 7462 + 3) \cdot (4 \cdot 402975 + 1) \cdot (4 \cdot 37312 + 3)$ and $37312 - 7462$ share with 402975 the factor 14925;
- : $7190899230439261 = 62119 \cdot 372709 \cdot 310591 = (4 \cdot 15529 + 3) \cdot (4 \cdot 93177 + 1) \cdot (4 \cdot 77647 + 3)$ and $77647 - 15529$ share with 93177 the factor 31059;
- : $7192327464624121 = 52711 \cdot 456821 \cdot 298691 = (4 \cdot 13177 + 3) \cdot (4 \cdot 114205 + 1) \cdot (4 \cdot 74672 + 3)$ and $74672 - 13177$ share with 114205 the factor 8785;
- : $7199906330209321 = 19927 \cdot 2590381 \cdot 139483 = (4 \cdot 4981 + 3) \cdot (4 \cdot 647595 + 1) \cdot (4 \cdot 34870 + 3)$ and $34870 - 4981$ share with 647595 the factor 9963;
- : $7202106745726369 = 27779 \cdot 444449 \cdot 583339 = (4 \cdot 6944 + 3) \cdot (4 \cdot 111112 + 1) \cdot (4 \cdot 145834 + 3)$ and $145834 - 6944$ share with 111112 the factor 27778;
- : $7206522548168833 = 9187 \cdot 146977 \cdot 5337067 = (4 \cdot 2296 + 3) \cdot (4 \cdot 36744 + 1) \cdot (4 \cdot 1334266 + 3)$ and $1334266 - 2296$ share with 36744 the factor 9186;
- : $7206889748909401 = 34211 \cdot 136841 \cdot 1539451 = (4 \cdot 8552 + 3) \cdot (4 \cdot 34210 + 1) \cdot (4 \cdot 384862 + 3)$ and $384862 - 8552 = 11 \cdot 34210$.