A Note on the Gravitational Equations analogous to Maxwell's Electromagnetic Equations.

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Abstract.

Ever since Oliver Heaviside's suggestion of the possible existence of a set of equations, analogous to Maxwell's equations for the electromagnetic field, to describe the gravitational field, others have considered and built on the original notion. However, if such equations do exist and really are analogous to Maxwell's electromagnetic equations, new problems could arise related to presently accepted notions concerning special relativity. This note, as well as offering a translation of a highly relevant paper by Carstoiu, addresses these concerns in the same manner as similar concerns regarding Maxwell's equations were.

Introduction.

Maxwell's electromagnetic equations are among the best known and most widely used equations in physics. Possibly for this reason, there has been little or no critical examination of their range of validity until relatively recent years[1]. With all symbols having their usual meanings, for a non-conducting medium at rest when no charge is present these equations are

$$\boldsymbol{\nabla}.\boldsymbol{E} = 0\,, \qquad \boldsymbol{\nabla}\mathbf{x}\boldsymbol{E} = -\frac{\mu}{c}\frac{\partial \boldsymbol{H}}{\partial t}\,,$$

$$\boldsymbol{\nabla}.\boldsymbol{H} = 0\,, \qquad \boldsymbol{\nabla}\mathbf{x}\boldsymbol{H} = \frac{\varepsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t}\,.$$

Here $\boldsymbol{D} = \varepsilon\boldsymbol{E}$, $\boldsymbol{B} = \mu\boldsymbol{H}$ and ε, μ are assumed constant in time. The first two equations are seen to lead to

$$\nabla^2\boldsymbol{E} = \frac{\varepsilon\mu}{c^2}\frac{\partial^2\boldsymbol{E}}{\partial t^2}$$

and the latter two to

$$\nabla^2\boldsymbol{H} = \frac{\varepsilon\mu}{c^2}\frac{\partial^2\boldsymbol{H}}{\partial t^2}\,,$$

that is, in this special case, *provided* the medium is at rest, both $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy the well-known wave equation. However, it has been shown by Thornhill[2] that, if the mean flow is steady and uniform and, therefore, both homentropic and irrotational, the system of equations governing small-amplitude homentropic irrotational wave motion in such a flow reduces to the equation

$$\nabla^2\varphi = \frac{1}{c^2}\frac{D^2\varphi}{Dt^2}$$

where ${}^{D}/_{Dt} = {}^{\partial}/_{\partial t} + \boldsymbol{u}.\boldsymbol{\nabla}$ and this final equation is sometimes referred to as the convected, or progressive, wave equation. It has been shown subsequently[1], using an approach initially considered by Abraham & Becker[3] that, for a medium not at rest, Maxwell's electromagnetic equation should be modified with the operator ${}^{\partial}/_{\partial t}$ replaced by ${}^{D}/_{Dt}$ just as Thornhill has shown for the case of the familiar wave equation mentioned above.

In the more recent examination of Maxwell's equations[1], it was noted that the modified equations agreed in form with all similar equations of continuum mechanics and were invariant under Galilean transformation. This raises the question that, if Maxwell had himself derived his equations for a moving medium, would a need for special relativity, as we presently know it, ever have arisen?

Maxwell-Type Equations for the Gravitational Field.

The notion that a set of equations, similar to Maxwell's electromagnetic equations, exists for the gravitational field has been around at least since the time of Heaviside[4]. Since his time, the idea has been considered by Carstoiu[5], Brillouin[6] and Jefimenko[7], to name but three. Each has considered such a set of equations and contemplated the meaning of the possible second field necessary to enable the set of equations to be exactly analogous to the electromagnetic ones which relate to two fields - the electric field and the magnetic field. Hence, since the required invariance of Maxwell's electromagnetic equations helped lead to the development of special relativity, if similar equations do exist for the gravitational field, this question concerning a moving medium must be addressed again. First, though, as an introduction to the topic, there follows a translation of the paper due to Carstoiu[5]:

GRAVITATION – The two fields of gravitation and the propagation of gravitational waves.

Note by M. John Carstoiu[1], presented by M. Henri Villat.

1. The theory of general gravity (which is the correct name for general relativity) is based on the supposition that the gravitational field $\boldsymbol{G}$ propagates in space at the speed of light. This was suggested by Einstein when he tried to unify the theories of gravity and electromagnetism, but even up until the present day, nobody has been able to prove this, and as such the effect has never been verified. Considerable work is currently ongoing in this area. M Fock[2] observed that it is not reasonable to keep the theory quite so general as Einstein had presented it and that simple rules led to over simplification of the mathematics. M. Brillouin[3,4] noted that while Fock's idea was certainly interesting, he was not sure that it was the only possible solution. M Weber's[5] precise experiences are not excluded from the existence of such an effect; yet it is too small to be observed currently. M. Forward[6] has made important contributions on this subject; using Einstein's equations, he has indicated the existence of a second gravitational field, corresponding to that of magnetic induction $\boldsymbol{B}$. However, he has recognised that his analogy is not perfect, because it leads him to the conclusion that gravitational waves propagate at a speed which is half that of the speed of light. This is what Einstein's equations have shown.

New ideas on the subject are presented within this article; Einstein's equations are not used, and we imagine ourselves in a flat universe – without curves. I will show, by means of appropriate comparison, the existence of a second gravitational field, which I am calling the gravitational vortex $\boldsymbol{\Omega}$. The two fields $\boldsymbol{G}$ and $\boldsymbol{\Omega}$ are subject to a system of equations presented within this article. As a result it appears that these two fields propagate through space at a speed which we will call c. Therefore, the assumptions made here, replace the Einstein condition.

Possible applications for this new theory appear numerous. I will point to just a few within this article. In the first instance, take Newton's law which cannot explain the rotation of the planets; this second gravitational field offers a solution. Secondly, it has recently been suggested[7] that some strange phenomena in the atmosphere could be put down to the existence of a second gravitational field, which is also considered here. Finally, we observe – along with M Brillouin – that the gravitational waves with the speed c, produced within this article, could perhaps replace the traditional radio and light waves. On the other hand, these new waves could eventually interrupt our electro-magnetic communications. Consequently, the debate contained within this article opens the gateway to new research and very important technical applications.

2. We begin with the static fields. The laws of Newton and Coulomb are expressed by the formulas:

(1) $$\boldsymbol{f}_N = -\gamma \frac{MM\prime}{r^2}\boldsymbol{r}^0$$

(2) $$\boldsymbol{f}_c = \frac{QQ\prime}{4\pi\varepsilon r^2}\boldsymbol{r}^0$$

(3) $$\boldsymbol{f}_{c_m} = \frac{\mu Q_m Q'_m}{4\pi r^2}\boldsymbol{r}^0$$

where, furthermore, in the recognised notation $\boldsymbol{Q}$ is the magnetic charge. As we have already shown[8], the formulas (1), (2), and (3) are identical, provided we take:

(4) $$\frac{k^2}{\varepsilon} = \mu k_m^2 = -4\pi\gamma$$

from whence

(5) $$\frac{k_m^2}{k^2} = c^2,$$

where k and k_m are two absolute constants. Following MM Brillouin and Lucas[9] take $k = 1$. It follows that $k_m = c$. Consequently:

(6) $$\varepsilon = -\frac{1}{4\pi\gamma}$$

(7) $$\mu = -\frac{4\pi\gamma}{c^2}$$

Therefore, not only do we obtain an isomorphism between the electrical field, $\boldsymbol{E} = [Q/4\pi\varepsilon r^2]\boldsymbol{r}^0$ and the gravitational field, $\boldsymbol{G} = -[\gamma M/r^2]\boldsymbol{r}^0$ but a second gravitational field, $\boldsymbol{\Omega}$, appears which corresponds to the magnetic induction $\boldsymbol{B} = [\mu Q_m/4\pi r^2]\boldsymbol{r}^0$; we have

(8) $$\boldsymbol{\Omega} = -\frac{\gamma}{c}\frac{M}{r^2}\boldsymbol{r}^0$$

It is clear that $\boldsymbol{\Omega} = \boldsymbol{G}/c$ and has dimensions T^{-1}. This will be termed the "Gravitational Vortex."

3. Assuming that our analogy can be extended to variable fields over time, and using the equations of Maxwell, it follows that

(9) $$\boldsymbol{\nabla}\mathbf{x}\boldsymbol{G} = -\frac{\partial\Omega}{\partial t}$$

(10) $$\boldsymbol{\nabla}\mathbf{x}\boldsymbol{\Omega} = \frac{1}{c^2}\frac{\partial \boldsymbol{G}}{\partial t} - \frac{4\pi\gamma}{c^2}\boldsymbol{J_g}$$

(11) $$\boldsymbol{\nabla}.\boldsymbol{G} = -4\pi\gamma\rho$$

(12) $$\boldsymbol{\nabla}.\boldsymbol{\Omega} = 0$$

where $\boldsymbol{J_g}$ is called the current gravitational density. For $\rho = \boldsymbol{J_g} = O$, these equations show that the two fields $\boldsymbol{G}$ and $\boldsymbol{\Omega}$ propagate in all directions with the speed c.

4. Our analogy introduces the gravitational tensor as follows

(13) $$\left(T_g\right)_{ij} = -\frac{1}{4\pi\gamma}\left(G_iG_j + c^2\Omega_i\Omega_j\right) - W_g\delta_{ij}$$

where

(14) $$W_g = -\frac{1}{8\pi\gamma}(G^2 + c^2\Omega^2)$$

which represents the gravitational energy per unit volume. This last term is a negative quantity and corresponds to a *negative mass distributed in a field*. [See – on this subject – a very interesting article by MM Brillouin and Lucas[9].] The generated force, per unit volume is

(15) $$\boldsymbol{k} = \rho\boldsymbol{G} + \boldsymbol{J_g}\mathbf{x}\boldsymbol{\Omega} + \frac{1}{4\pi\gamma}\frac{\partial}{\partial t}(\boldsymbol{\Omega}\mathbf{x}\boldsymbol{G}).$$

This is the analogue of the pondermotive force of magnetohydrodynamics.

5. It is clear that the magnetohydrodynamic waves do not admit a gravitational analogue; because the speed of M Alfven $A_0 = B_0(\mu\rho)^{-1/2}$ would correspond to the quantity $\Omega_0 c(-4\pi\gamma\rho)^{-1/2}$ which is imaginary. At the end of the day, the propagation of the magnetohydrodynamic waves is governed by partial differential equations of the hyperbolic type, while our transposition gives an elliptical system.The study of the latter goes beyond the purpose of this article. We observe here, that the quantity $\Omega_0^2 c^2/(4\pi\gamma\rho)$ (the square of a speed) can reach enormous values; yet this one cannot exceed *c*. Hence, the condition:

(16) $$\Omega_0^2 \leq 4\pi\gamma\rho$$

that is to say, a rotation of $\boldsymbol{\Omega_0}$ of external origin applied to a fluid mass of density must obey condition (16); otherwise the mass can disintegrate. It is curious to note that we find a similar condition in the *Theory of Equilibrium Figures of a Fluid Mass*, where Poincaré[10] has given $\Omega_0^2 < 2\pi\gamma\rho$. Note that an increase in the angular velocity of the Earth may well violate Poincaré's condition or that of (16) which is more generous. Since this increase is transmitted to us at the speed of light, it follows that the words, which conclude the famous work of Heaviside[11], have a clear meaning:
The destruction of this wicked world may come at any moment without any warning. There is no possibility of foretelling this calamity (or blessing possibly), because the cause thereof cannot give us any information until it arrives, when it will be too late to take precautions against destruction.

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Comments.

Here a set of equations is presented, deliberately chosen to be very similar to Maxwell's electromagnetic equations, said to represent the gravitational field. Like Maxwell's equations, this set has only one current associated with the gravitational field, that appearing in equation (10). It might be considered appropriate to include a similar term in equation (9) also, representing a second current associated with the gravitational field; that is, one current associated with each component of the field. However, it is the intention here to consider quite specifically the form of the equations most closely linked with those of Maxwell and so attention will be restricted to the form exhibited in equations (9) and (10) above.

For this set of equations, if ρ and $\boldsymbol{J_g}$ are both zero, the resulting four equations may be combined in pairs to produce two wave equations – one each for the variables $\boldsymbol{G}$ and Ω. However, such a set of equations could conceivably lead eventually to the same problems which surfaced concerning the original electromagnetic equations of Maxwell. As mentioned earlier, that particular set of equations was found not to be invariant under Galilean transformation and that, in turn, led finally to the establishment of special relativity. What had been forgotten though was that Maxwell's equations had been derived for a medium at rest and, in all other areas of continuum mechanics, when the wave equation for a moving medium was desired, it was found that the partial derivative with respect to time, $\partial/\partial t$, had to be replaced by the total time derivative $D/Dt = \partial/\partial t + \boldsymbol{v}.\nabla$ and, subsequently, it was found that, if Maxwell's equations were derived for a moving medium, this replacing of the partial time derivation by the total time derivative occurred quite naturally[1]. Hence, it must follow that these gravitational equations proposed by, amongst others, Carstoiu are based on the involvement of a stationary medium. In accordance with what has preceded this discussion, it is suggested, therefore, that the Maxwell-like equations for describing a gravitational field as shown in equations (9) – (12) above should be replaced by the more general equations

$$\boldsymbol{\nabla}\mathbf{x}\boldsymbol{G} = -\frac{D\boldsymbol{\Omega}}{Dt}$$

$$\boldsymbol{\nabla}\mathbf{x}\boldsymbol{\Omega} = \frac{1}{c^2}\frac{D\boldsymbol{G}}{Dt} - \frac{4\pi\gamma}{c^2}\boldsymbol{J_g}$$

$$\boldsymbol{\nabla}.\boldsymbol{G} = -4\pi\gamma\rho$$

$$\boldsymbol{\nabla}.\boldsymbol{\Omega} = 0$$

where $D/Dt = \partial/\partial t + \boldsymbol{v}.\nabla$

These equations, for zero valued ρ and $\boldsymbol{J_g}$, would be invariant under Galilean transformation and so would not cause problems similar to those experienced with the original electromagnetic equations of Maxwell. Again, it might be noted that these equations are exactly analogous to the similar electromagnetic equations[1].

Other approaches.

As mentioned earlier, the fascination with describing the gravitation field via equations similar to those derived by Maxwell for the electromagnetic field has persisted for many years. However, another approach to the problem which does not actually rely on such a set of equations is that outlined by Wesley[8], who concentrates on the formal similarity between Newton's universal law of gravitation and Coulomb's law. He notes that both have the same form but with the $-GMM'$ of Newton being replaced by the QQ' of Coulomb. He then postulates that gravitational theory may be derived from electrodynamics by altering the form of the constant and changing the sign of the force from plus to minus. He then notes that only two potentials are sufficient to characterise the electrodynamic field, defines two analogous gravitational potentials and proceeds to develop his theory after first postulating a finite velocity of action c, as in electrodynamics. This alternative approach is certainly different but, crucially, is shown to lead to the production of a number of well-known results and hence lend some credence to this notion of it being reasonable to treat the gravitational field as being in many ways analogous to the electromagnetic field. How far this alleged analogy may be taken is, of course, another matter, but one worthy of further serious consideration.

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