Solving The Mystery of the Fine Structure Constant

Nikola Perkovic

Department of Physics and Mathematics, Faculty of Sciences, Novi Sad, 21000, Serbia
email: personal perce90gm@gmail.com faculty nikola.perkovic@uns.ac.rs

Abstract The paper will make new claims regarding the fine structure constant. The specific value of the electromagnetic coupling constant, that is the fine structure constant, will be explained as a consequence of mass energy equivalence. Special Relativity and Quantum Electrodynamics will be used to attain the mass energy equivalence equation and after which a new, quantized equation of mass energy equivalence will be postulated and tested. A new way will be presented to determine the mass of neutrons by using the strong nuclear coupling constant and protons by using the fine structure constant.

Keywords Fine structure constant, Strong nuclear constant, Mass Energy Equivalence

1. Introduction

The very specific value of fine structure constant is still a mystery, even though it is used in a variety of aspects. However, determining the specific value of α is a truly painstaking process. The anomalous magnetic dipole moment is used for this purpose

$$a_e = \sum_{n=1}^{\infty} \left({}^{\alpha}/_{2\pi}\right)^n a_e^{(2n)} \qquad (1)$$

where α is the fine structure constant and a_e is the anomalous magnetic dipole moment. To obtain the value of the fine structure constant α experts in Quantum Electrodynamics, both theoretical and experimental, have to perform rather difficult tasks such as measuring $a_e = A_1 {}^{\alpha}/_{\pi} + A_2({}^{\alpha}/_{\pi})^2 + A_3({}^{\alpha}/_{\pi})^3 + \cdots + a\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}, \ldots\right)$ due to the lack of a theoretical prediction established that the amplitude of an electron in order to absorb or emit a photon $e = 0.08542455$ [1] is directly connected with the constant α. One should not confuse the amplitude of an electron with the elementary charge also symbolized with the letter e.

Usually the equation $\delta F_1(q^2) \rightarrow \delta F_1(q^2) - \delta F_1(0)$ where $F_1(0) = 1$ and δF_1 is the first order correction to F_1 is used. From this we can define for electrons $F_1(q^2) = 1 + \frac{\alpha}{\pi}\int_0^1 dx\, dy\, dz\, \delta(x+y+z+1)\left[\log\left(\frac{m^2(1-z)^2}{m^2(1-z)^2 - q^2xy}\right) + \left(\frac{m^2(1-4z+z^2)+q^2(1-x)(1-y)}{m^2(1-z)^2 - q^2xy + \mu^2 z}\right) - \left(\frac{m^2(1-4z+z^2)}{m^2(1-z)^2 + \mu^2 z}\right)\right] + \mathcal{O}\{\alpha^2\}$

which is calculated to be $F_2(q^2) = \frac{\alpha}{2\pi}\int_0^1 x\, dy\, dz\, \delta(x+y+z-1)\left[\frac{2m^2(1-z)}{m^2(1-z)^2 - q^2xy}\right] + \mathcal{O}\{\alpha^2\}$ and therefore $F_2(q^2 = 0) = \frac{\alpha}{2\pi}\int_0^1 x\, dy\, dz\, \delta(x+y+z-1)\frac{2m^2(1-z)}{m^2(1-z)^2} = \frac{\alpha}{\pi}\int_0^1 dz \int_0^{1-z} dy \frac{z}{1-z} = \frac{\alpha}{2\pi}$ hence obtaining us the anomalous magnetic dipole moment $a_e = \frac{\alpha}{2\pi} = 0.001161409695$.

The value of alpha has been experimentally measured to be $\alpha^{-1} = 137.035\,999\,084\,(51)$ [2] in latest experiments. The theoretical value is in near agreement but the process of determining the value of α is truly painstaking. The current theoretical value is $\alpha_{theory}^{-1} = 137.035\,999\,174\,(35)$ [3].

The way to explain the specific value of the α constant is to use the mass energy equivalence principle which will be performed based on momentum conservation.

2. Derivation of mass energy equivalence based on momentum conservation

In an inertial frame, we have a body at rest and it emits two photons $\boldsymbol{a}, \boldsymbol{b}$ that have equal energy but move in opposite directions. The total energy as measured in $\boldsymbol{S}$ is $\boldsymbol{E}$. Due to the conservation of momentum, the total momentum of the body is:

$$\boldsymbol{P}'_i = -\boldsymbol{m}'_i\, \boldsymbol{v}\, \boldsymbol{e}_x \qquad (2)$$

Where $\boldsymbol{m}_i$ is the rest mass of the body before the emission process. After the emission of two photons, the total momentum will remain conserved:

$$-\boldsymbol{m}'_i \cdot \boldsymbol{v} \cdot \boldsymbol{e}_x = -\boldsymbol{m}'_f \cdot \boldsymbol{v} \cdot \boldsymbol{e}_x + \boldsymbol{P}'_a + \boldsymbol{P}'_b = -\boldsymbol{m}'_f \cdot \boldsymbol{v} \cdot \boldsymbol{e}_x + (\boldsymbol{P}'_{a_x} + \boldsymbol{P}'_{b_x})\boldsymbol{e}_x + \left(\boldsymbol{P}'_{a_y} + \boldsymbol{P}'_{b_y}\right)\boldsymbol{e}_y \qquad (3)$$

Where $\boldsymbol{m}'_f$ is the mass of the body after the emission process. $\boldsymbol{P}'_a, \boldsymbol{P}'_b$ are the moments of the two photons, respectively, in $\boldsymbol{S}'$. Equating the x components from the equation above, we attain:

$$-\left(\frac{\boldsymbol{\mu_0 e^2}}{\boldsymbol{4\pi d}}/\boldsymbol{\alpha}\right)\boldsymbol{v}\boldsymbol{e}_x = -(\boldsymbol{m}'_i - \boldsymbol{m}'_f)\boldsymbol{v}\, \boldsymbol{e}_x = \boldsymbol{P}'_{a_x} + \boldsymbol{P}'_{b_x} \quad (4)$$

The energies of photons $\boldsymbol{a}$ and $\boldsymbol{b}$ in $\boldsymbol{S}'$ are:

$$\boldsymbol{E}'_a = \frac{\boldsymbol{\gamma E}}{\boldsymbol{2c}}(\boldsymbol{1} - \boldsymbol{\beta} \cos \boldsymbol{\varphi}) \qquad (5)$$

and:

$$\boldsymbol{E}'_b = \frac{\boldsymbol{\gamma E}}{\boldsymbol{2c}}(\boldsymbol{1} + \boldsymbol{\beta} \cos \boldsymbol{\varphi}) \qquad (6)$$

Therefore $\boldsymbol{E}'_a + \boldsymbol{E}'_b = \boldsymbol{\gamma E} = \boldsymbol{E}'$. The x components of the photon momenta are:

$$P'_{a_x} = \frac{E'_a}{c} \cos \varphi'_a \qquad (7)$$

and:

$$P'_{b_x} = \frac{E'_b}{c} \cos \varphi'_b \qquad (8)$$

The cosines are $\cos \varphi_a = \frac{u_{a_x}}{c}$ and $\cos \varphi_b = -\frac{u_{b_x}}{c}$ where $\cos \varphi_b = -\cos \varphi_a = -\cos \varphi$. The velocity transformation relation for the x component of the said velocity is changed to:

$$u'_x = \frac{u_x - v}{1 - u_x \frac{v}{\frac{1}{\varepsilon_0 \mu_0}}} \qquad (9)$$

Using the equations $7, 8$ we attain $\cos \varphi'_a = \frac{u'_{a_x}}{c} = \frac{\cos \varphi_a - a}{1 - a\ \cos \varphi_a} = \frac{\cos \varphi - a}{1 - a\ \cos \varphi}$ and $\cos \varphi'_b = \frac{u'_{b_x}}{c} = \frac{\cos \varphi_b - a}{1 - a\ \cos \varphi_b} = \frac{\cos \varphi + a}{1 + a\ \cos \varphi}$ after which we evaluate:

$$P'_{a_x} = \left[\frac{\gamma E}{2c}(1 - \beta \cos \varphi)\right]\left[\frac{\cos \varphi - \beta}{1 - \beta \cos \varphi}\right] = \frac{\gamma E}{2c}(1 - \beta \cos \varphi) \quad (10)$$

$$P'_{b_x} = \left[\frac{\gamma E}{2c}(1 + \beta \cos \varphi)\right]\left[-\frac{\cos \varphi + \beta}{1 + \beta \cos \varphi}\right] = -\frac{\gamma E}{2c}(1 + \beta \cos \varphi) \quad (11)$$

Therefore:

$$P'_{a_x} + P'_{b_x} = \frac{aE}{2c}(\cos \varphi - a) - \frac{aE}{2c}(\cos \varphi + a) = \frac{aEv}{\frac{1}{\varepsilon_0 \mu_0}} \quad (12)$$

Thus:

$$-\left(\frac{\mu_0 e^2}{4\pi d}/\alpha\right) v = P'_{a_x} + P'_{b_x} = \frac{\gamma E}{2c}(\cos \varphi - \beta) - \frac{\gamma E}{2c}(\cos \varphi + \beta) = -\frac{E' v}{\frac{1}{\varepsilon_0 \mu_0}} \qquad (13)$$

Finally we conclude:

$$E' = \left(\frac{\mu_0 e^2}{4\pi d}/\alpha\right) \cdot \frac{1}{\varepsilon_0 \mu_0} \qquad (14)$$

Where e is the elementary charge, α is the fine structure constant and d is the angular wavelength.

It is obvious that $\frac{1}{\varepsilon_0 \mu_0} = c^2$ but the claim that $\frac{\mu_0 e^2}{4\pi d}/\alpha = \Delta m'$ leads to the equation 14 to change into:

$$E' = \Delta m' \cdot c^2 \qquad (15)$$

attaining us the equation for mass energy equivalence. From equation 14 it follows that:

$$E = \left(\frac{\mu_0 e^2}{4\pi d}/\alpha\right) \cdot \frac{1}{\varepsilon_0 \mu_0} = \frac{e^2}{4\pi \varepsilon_0 d}/\alpha \qquad (16)$$

This means that the fine structure constant can be calculated experimentally by knowing the wavelength and either mass or energy from the relation. Therefore, a new equation for mass energy equivalence is:

$$\boldsymbol{\alpha} = \frac{\boldsymbol{\mu_0 e^2}}{\boldsymbol{4\pi d}}/\Delta \boldsymbol{m}' = \frac{\boldsymbol{e^2}}{\boldsymbol{4\pi \varepsilon_0 d}}/\boldsymbol{E}' \qquad (17)$$

where $\Delta \mathbf{m}'$ is the inertial or relativistic mass [4, 5] and $\boldsymbol{E}'$ is the relativistic energy. We constitute that the same can also be applied for the rest mass and rest energy.

The equation $\mathbf{17}$ explains how the specific value of the fine structure constant emerges from mass energy equivalence. This is the new, quantized equation for mass energy equivalence. The equation now has to be proven.

3. Testing the equation

In order for the equation to be tested adequately both sides of it have to be tested, mass and energy respectively. Electrons will be used in both cases since the electromagnetic interaction between electrons and protons forms the atom. In the next chapter we will also form different equations for the proton and additionally, using the strong nuclear coupling constant, for the neutron as well.

3.1. Testing the equation for mass

We proceed to test the equation:

$$\alpha = \frac{\mu_0 e^2}{4\pi d}/m_e \qquad (18)$$

where m_e is the electron mass. From experiments we know that:

$$m_e = \frac{2R_\infty h}{c\alpha^2} \qquad (19)$$

Where $R_\infty = \alpha^2/2\lambda_e$ is the Rydberg constant and $d = \lambda_e/2\pi$ is the angular wavelength therefore:

$$\alpha = \frac{\mu_0 e^2}{4\pi d}/\frac{2R_\infty h}{c\alpha^2} = \frac{\mu_0 e^2 c\alpha^2}{4\pi d 2hR_\infty} = \frac{\mu_0 e^2 c\alpha^2}{4\pi d 2h\frac{\alpha^2}{4\pi d}} \qquad (20)$$

Finally we conclude:

$$\alpha = \frac{\mu_0 e^2 c}{2h} = 0.00729735256 \qquad (21)$$

The result is in agreement with the experimental results [2, 6] referenced in this paper.

3.2. Testing the equation for energy

We test the other side of the equation for the energy of the electron:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 d}/E_e \qquad (22)$$

Knowing that $E = hc/\lambda_e$ we solve the equation:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 d}/\frac{hc}{\lambda_e} = \frac{e^2}{4\pi\varepsilon_0 d}\cdot\frac{2\pi d}{hc} = \frac{e^2}{4\pi\varepsilon_0 d}\cdot\frac{d}{\hbar c} \qquad (23)$$

Finally:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = 0.00729735256 \qquad (24)$$

which is in agreement with the experimental results [2, 6].

4. Protons and Neutrons

Protons can be explained with the quantized equation for mass energy equivalence just like electrons, but unlike electrons they are composite particles. This is not strange since protons, even though they are hadrons, participate in the electromagnetic interaction with electrons to form the atom. However it is strange that neutrons that clearly do not participate in the electromagnetic interaction, can still be explained by the quantized mass energy equation that is clearly based on the electromagnetic coupling constant, that is the fine structure constant. This mystery might have a solution in the nature of QCD binding energy.

4.1. Protons

For protons we use the equation:

$$E_0^p = \frac{hc}{\lambda_p} \qquad (25)$$

Where the proton's wavelength corresponds to:

$$\lambda_p = \sum \lambda_q \times 10^{-И} \qquad (26)$$

where $И$ is the proton loop number, a dimensionless value that equals:

$$И = 2 - \left(\frac{\alpha}{n_q\,\pi}\right) = 2 - \left(\frac{\alpha}{3\,\pi}\right) \qquad (27)$$

where $n_q = 3$ is the number of quarks in the proton and it has the value $И = 1.9992257268457278$. Further on we have the quark wavelengths that correspond to:

$$\sum \lambda_q = \frac{hc}{\sum E_0^q} \qquad (28)$$

Where $\sum E_0^q$ is the sum of rest energies of the three quarks. Therefore the equation 25 becomes:

$$E_0^p = \frac{hc}{\frac{hc}{\sum E_0^q}\times 10^{-И}} = \sum E_0^q \times 10^{И} \approx 938.3\ MeV \qquad (29)$$

which means that the mass is $m_p \approx 938.3\ MeV/c^2$. A proton consists of two up quarks and one down quark meaning that $\sum E_0^q \approx 9.4\ MeV$ [7]. The accuracy of this method depends on the measures of quarks and the measure of α. In this paper we used the value $\alpha = 0.00729735256$. Higher accuracy can be attained with more expensive research.

4.2. Neutrons

Although neutrons are irrelevant to this paper, we will use the same type of equations as for protons with the exception of the fine structure constant. Instead we use the strong nuclear coupling constant for the distance of $\mathbf{1\,fm}$ at which the strong nuclear coupling constant equals $\boldsymbol{\alpha_s \approx 1}$. We claim that:

$$\boldsymbol{E_0^n = \frac{hc}{\lambda_n}} \tag{30}$$

the same as with protons we have:

$$\boldsymbol{\lambda_n = \sum \lambda_q \times 10^{-\text{Ш}}} \tag{31}$$

where Ш is the neutron loop number, also a dimensionless value that equals:

$$\text{Ш} = \boldsymbol{2 - \left(\frac{\alpha_s}{n_q\,\pi}\right) = 2 - \left(\frac{\alpha_s}{3\,\pi}\right)} \tag{32}$$

and it has the value Ш $= \mathbf{1.8974087236829643}$. Here the $\boldsymbol{\alpha_s = 0.9669}$ for the distance of approximately $\mathbf{1\,fm}$.

A neutron is formed out of two down quarks and one up quark therefore $\boldsymbol{\sum E_0^q \approx 11.9\,MeV}$[7] so the equation $\mathbf{28}$ will be identical but with a different result.

$$\boldsymbol{E_0^n = \frac{hc}{\frac{hc}{\sum E_0^q} \times 10^{-\text{Ш}}} = \sum E_0^q \times 10^{\text{Ш}} \approx 939.6\,MeV} \tag{33}$$

This new method is as effective as the methodology in Quantum Chromodynamics [8] if the accurate measurements of quarks and the strong nuclear coupling constant where used. The QCD binding energy is the explanation to why the mass of hadrons is much larger than the collective masses of the quarks that form them [9] which is explained by using gluons, gluon fields and the interactions of quarks and gluons.

5. Conclusion

The conclusion is that the new, quantized equation for mass energy equivalence:

$$\alpha = \frac{\mu_0 e^2}{4\pi d}/m = \frac{e^2}{4\pi\varepsilon_0 d}/E \tag{34}$$

explains the specific value of the fine structure constant.

The relationship between mass and energy, in the equation also the relationship between the constants $\boldsymbol{\mu_0}$ and $\boldsymbol{\varepsilon_0}$ which are vacuum permeability [10] and vacuum permittivity [11] respectfully, brings about the specific value of the electromagnetic coupling constant. This makes formation of atoms possible and thus even life can be, which is why the electromagnetic coupling constant has the nickname “fine structure constant”.

This means that the “Finely tuned Universe” hypothesis is wrong since the value of α, on which this hypothesis is built, arises from natural means that is of laws of nature or, to be specific: the value of α arises from mass energy equivalence.

The relationship of mass and energy has to be as such that the equation $E = mc^2$ applies and therefore the equation 34 will apply as well since it is a quantized version of the famous equation mentioned above.

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