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Neutrosophic Combination Rules Based on Dempster-Shafer Theory and Dezert-Smarandache Theory

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Abstract: Several neutrosophic combination rules based on the Dempster-Shafer theory (DST) and Dezert-Smarandache theory (DSmT) are presented in this study. The new information fusing approaches proposed the neutrosophic belief assignment to represent the evidences and connect the DST and DSmT with neutrosophic theory. Neutrosophic theory used the components truth, indeterminacy and falsity to represent any idea, which provides a better approach to express the real world. The DST-based and DSmT-based combination rules can fuse information more effectively. The new combination rules utilized the advantages of DST, DSmT and neutrosophic theory. They are more adapted to describing the human mind and helping to make correct decisions. To fuse the information expressed in natural language, the qualitative neutrosophic combination rules of information have been proposed in this study too. Some application examples have been given to support the proposed combination rules. Compared with the original DST-based and DSmT-based combination rules, the neutrosophic combination rules can provide more useful information and have wider application field.

Key words: Information fusion, combination rule, DST, DSmT, neutrosophic theory

INTRODUCTION

Nowadays, many fusion theories have been studied for the combination of information and some information fusing technologies have been applied in many fields successfully, such as defense, economy and medicine. In real application, the information provided by multiple sources is often fuzzy, imprecise and even conflicting. Belief function can be used to represent the uncertain information effectively. As a result, the belief function theory is more adapted to the real and complex fusion of information.

Shafer (1976) proposed the Dempster-Shafer Theory (DST). Some improvement of DST has been presented in the past years (Bauer, 1997; Lucas and Araabi, 1999; Murphy, 2000). As a mathematical theory of evidence, the DST provides a classical information fusing method and has been applied widely (Beynon *et al*., 2001; Foucher *et al*., 2002; Rombaut and Zhu, 2002). Belief function and Shafer's model are the foundations of DST. In the Shafer's model, all elements of the frame are exhaustive and fully exclusive and the bodies of evidence are associated with basic belief assignment (bba). On the other hand, the Dempster's rule is used in DST to combine independent sources of evidence. The DST is suitable for combining uncertain information with low conflict. However, when the conflict between sources of information becomes very high, the combination result is unreliable.

To overcome the weakness of DST, Smarandache and Dezert (2004) presented the Dezert-Smarandache theory (DSmT). The DSmT, which can be regarded as an extension of DST, proposed some new useful combination rules of information and can resolve the complex fusion problems. It also represents the information in terms of belief function just as DST. However, it can work in the Shafer's model, the free DSm model and the hybrid DSm model. In the free DSm model, the elements of the frame can overlap and change with time. The hybrid DSm model is between free DSm model and Shfter's model and takes account of some real exclusivity constraints. The main combination rules in DSmT include the classic DSm rule (DSmC), the hybrid DSm rule (DSmH) and the proportional conflict redistribution rule (PCR) (Smarandache and Dezert, 2006). The DSmH and PCR can work in any models.

Smarandache (2003) presented neutrosphy to study the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosphy is a new branch of philosophy and considers every idea to be represented by three components truth, indeterminacy and falsity. It provides a better definition

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approach to the real world. The neutrosophic logic, the neutrosophic set, the neutrosophic probability and the neutrosophic statistics are all its derivatives and can be applied in the information processing.

Smarandache and Dezert (2006) have used the N-norms and N-conorms to connect the conjunctive rule and disjunctive rule with neutrosophic logic. In this research, we connect the DST-based combination rules and the DSmT-based combination rules with neutrosophic logic and propose the neutrosophic combination rules based on DST and DSmT. We also present the neutrosophic set based on linguistic label and apply the new combination rules in the fusion of qualitative neutrosophic beliefs.

THE RELATED THEORIES

Neutrosophic theory: The non-standard real unit interval $]^-0,1^+[$ is an important definition in the neutrosophic theory (Smarandache, 2003). It arose from the non-standard analysis. Let $\varepsilon>0$ be a infinitesimal number. Then, the non-standard numbers $^-0 = 0-\varepsilon$ and $1^+ = 1+\varepsilon$. Therefore, 0 and 1 belong to the non-standard unit interval. In general, the left and the right borders of a non-standard interval are vague and imprecise. In the neutrosophic theory, as the neutrosophic components, T, I, F represent the truth, the indeterminacy and the falsity respectively. They are all the standard or non-standard real subsets of $]^-0,1^+[$.

Neutrosophic logic is a generalization of the fuzzy logic and estimates each proposition to be t% true, i% indeterminate and f% false (t% $\in$T, i% $\in$I, f% $\in$F). Compared with fuzzy logic, the neutrosophic logic considers not only the truth and the falsity but also the indeterminacy. Neutrosophic set is based on neutrosphy and generalizes the fuzzy set. If M is a neutrosophic set and the element x(T, I, F) belongs to M, x is t% true, i% indeterminate and f% false in the set M. Neutrosophic probability extends the classical probability and imprecise probability. In the neutrosophic probability, the occurring chance of one event is t% true, i% indeterminate and f% false. Therefore, the neutrosophic theory is more adapted to the vagueness, the imprecision and the uncertainty in real world and can represent the real proposition better.

DST and DSmT: The DST (Shafer, 1976) based on the Shafer's model. $\Theta = \{\theta_1, \theta_2,\ldots, \theta_n\}$ is considered as the frame of discernment of the fusion problem. In the Shafer's model, the n elements θ_i in Θ are fully exclusive. All subsets of Θ are made up of the power set 2^Θ. If $\Theta = \{\theta_1, \theta_2\}$, then $2^\Theta = \{\phi, \theta_1, \theta_2, \theta_1\cup\theta_2\}$. The DST associates each evidence with a basic belief assignment (bba) and maps the power set 2^Θ onto [0,1] to define the bba:

$$m(\phi) = 0 \text{ and } \sum_{X\in 2^\Theta} m(X) = 1 \quad (1)$$

The definitions of the belief and plausibility functions are both based on the bba:

$$\mathrm{Bel}(Y) = \sum_{X\in 2^\Theta, X\subseteq Y} m(X) \text{ and } \mathrm{Pl}(Y) = \sum_{X\in 2^\Theta, X\cap Y\neq\phi} m(X) \quad (2)$$

To get the combined global belief function of two independent and equally reliable sources (m_1 and m_2), we can firstly combine the bba of the two sources of information through the Dempster's combination rule:

$$\begin{cases} m_{DS}(\phi) = 0 \\ m_{DS}(Y) = \left(\frac{1}{1-k_{12}}\right) \sum_{\substack{X_1,X_2\in 2^\Theta \\ X_1\cap X_2 = Y}} m_1(X_1)m_2(X_2) \quad \forall Y\in 2^\Theta\setminus\{\phi\} \end{cases} \quad (3)$$

$$\text{and } k_{12} = \sum_{\substack{X_1,X_2\in 2^\Theta \\ X_1\cap X_2 = \phi}} m_1(X_1)m_2(X_2)$$

In the equation, k_{12} is the degree of the conflict between the two sources. The DST is fit for the combination of uncertain information, but it also has some limitations. When the conflict is absolute ($k_{12} = 1$), we cannot get the result through the Dempster's rule.

The DSmT (Smarandache and Dezert, 2004, 2006) extends DST and overcomes the limitations of DST. The DSmT can work in not only the Shafer's model but also the free DSm model and the hybrid DSm model. In the free DSm model, the n elements θ_i in Θ are only exhaustive and there is no other assumption. In the hybrid DSm model, the existing constraints have been considered. In fact, the Shafer's model and the free DSm model can be considered as the specific models of the hybrid DSm model. In the DSmT, all subsets of Θ are made up of the hyper-power set D^Θ. If $\Theta = \{\theta_1, \theta_2\}$, then $D^\Theta = \{\phi, \theta_1\cap\theta_2, \theta_1, \theta_2, \theta_1\cup\theta_2\}$. Just as DST, the DSmT associates each evidence with a generalized basic belief assignment (gbba) and maps the hyper-power set D^Θ onto [0,1] to define the gbba:

$$m(\phi) = 0 \text{ and } \sum_{X\in D^\Theta} m(X) = 1 \quad (4)$$

The generalized belief and plausibility functions in DSmT are also based on the gbba.

The main combination rules in DSmT include DSmC, DSmH and PCR5. Corresponding to the free DSm model, the DSmC rule is:

$$m_{DSmC}(Y) = \sum_{\substack{X_1,X_2\in D^\Theta \\ X_1\cap X_2 = Y}} m_1(X_1)m_2(X_2) \quad \forall Y\in D^\Theta \quad (5)$$

Corresponding to the hybrid DSm model, the DSmH rule is:

$$\begin{aligned} m_{DSmH}(Y) &= \phi(Y)\left[S_1(Y)+S_2(Y)+S_3(Y)\right] \quad \forall Y \in G, \\ S_1(Y) &= \sum_{\substack{X_1,X_2 \in D^{\Theta} \\ X_1 \cap X_2 = Y}} m_1(X_1)m_2(X_2), \\ S_2(Y) &= \sum_{\substack{X_1,X_2 \in \phi \\ [U=Y]\vee[(U\in\phi)\wedge(Y=I_t)]}} m_1(X_1)m_2(X_2) \text{ and} \\ S_3(Y) &= \sum_{\substack{X_1,X_2 \in D^{\Theta} \\ X_1 \cup X_2 = Y \\ X_1 \cap X_2 \in \phi}} m_1(X_1)m_2(X_2) \end{aligned} \tag{6}$$

In the equation, G is the power set 2^{Θ} or the hyper-power set D^{Θ}, $I_t = \theta_1 \cup \theta_2 \cup \ldots \cup\theta_n$. If $Y \in \phi$, then $\phi(Y) = 0$, otherwise, $\phi(Y) = 1$. If u(X) is the union of all singletons θ_i that compose X, then $U = u(X_1) \cup u(X_2)$.

Different from DSmH, the PCR transfers the conflicting masses to the sets involved in the conflicts proportionally. Smarandache and Dezert (2006) have proposed five PCR rules and the PCR5 is the most effective.

$$\begin{cases} m_{PCR5}(\phi) = 0 \\ m_{PCR5}(Y) = m_{12}(Y) + \sum\limits_{\substack{Z \in G\setminus\{Y\} \\ Y \cap Z = \phi}} \left[\dfrac{m_1(Y)^2 m_2(Z)}{m_1(Y)+m_2(Z)} + \dfrac{m_2(Y)^2 m_1(Z)}{m_2(Y)+m_1(Z)}\right] \quad \forall Y \in G\setminus\{\phi\} \end{cases}$$
$$\text{and } m_{12}(Y) = \sum_{\substack{X_1,X_2 \in G \\ X_1 \cap X_2 = Y}} m_1(X_1)m_2(X_2) \tag{7}$$

Compared with DST, the DSmT is more adapted to combining the highly conflicting information.

THE NEUTROSOPHIC COMBINATION RULES OF INFORMATION

The information acquired in real application is always vague and uncertain. In the DST and DSmT, only the degree of truth (as bba and gbba) is considered. However, in the neutrosophic logic, not only the truth and the falsity but also the indeterminacy is considered. Therefore, the neutrosophic logic is more adapted to representing the real proposition. In the following, we connect the DST and DSmT with neutrosophic theory and present the neutrosophic combination rules based on DST and DSmT.

Fusion of quantitative information: T, I and F are supposed to be the standard subsets of [0, 1]. We define the neutrosophic basic belief assignment (nbba):

$$\begin{aligned} &m_{neu}(\cdot): G \to [0,1]^3,\ m_{neu}(X) = (t_X, i_X, f_X) \text{ and} \\ &m_{neu}(\phi) = (0,0,1),\ \sum_{X \in G} t_X = 1 \end{aligned} \tag{8}$$

G represents the power set 2^{Θ} or the hyper-power set D^{Θ}. $t_X \in T$, $i_X \in I$, $f_X \in F$. Smarandache (2003) also provided the extension of bba on neutrosophic sets and the defined nbba is required to normalize all the truth, the indeterminacy and the falsity values:

$$\sum_{X \in G} (t_X + i_X + f_X) = 1 \tag{9}$$

The admissibility condition is different from that of our definition. In the DST and DSmT, the bba and gbba are needed to be normalized. In fact, the bba and gbba only represent the truth. Therefore, to get the coincident results, we only normalize all the truth values. We think that our definition of nbba can contain bba and gbba better.

Let $X_1, X_2, \ldots, X_n$ be the n elements of G. $m_{neu}(X_i)$ is the nbba of X_i and is provided by experts. In general, the sum of truth value, indeterminacy value and falsity value of X_i equals one, but the sum of truth values of all X_i $(i = 1,2,\ldots,n)$ is not one probably. Therefore, all the nbba need to be normalized firstly. We define the normalization as follows:

$$\begin{aligned} &t_{X_i} = t_{X_i-ori} \Big/ \sum_{j=1}^{n} t_{X_j-ori},\ i_{X_i} = i_{X_i-ori} \cdot t_{X_i} / t_{X_i-ori}, \\ &f_{X_i} = f_{X_i-ori} \cdot t_{X_i} / t_{X_i-ori}\ (i = 1,2,\ldots,n) \end{aligned} \tag{10}$$

The operation not only normalizes the truth values but also keeps the ratio between the truth value and the indeterminacy value or the falsity value.

Let $m_{neu1}(.) = (t_1, i_1, f_1)$ and $m_{neu2}(.) = (t_2, i_2, f_2)$ be the nbba of two sources of information. In this research, we used the followed operations to compute the combination of two nbba:

$$m_{neu1}(\cdot) + m_{neu2}(\cdot) = (t_1 + t_2, i_1 + i_2, f_1 + f_2) \tag{11}$$

$$m_{neu1}(\cdot) - m_{neu2}(\cdot) = (t_1 - t_2, i_1 - i_2, f_1 - f_2) \tag{12}$$

$$m_{neu1}(\cdot) m_{neu2}(\cdot) = (t_1 \cdot t_2, i_1 \cdot i_2, f_1 \cdot f_2) \tag{13}$$

$$k \cdot m_{neu1}(\cdot) = (k \cdot t_1, k \cdot i_1, k \cdot f_1) \tag{14}$$

$$m_{neu1}(\cdot) / m_{neu2}(\cdot) = (t_1/t_2, i_1/i_2, f_1/f_2) \tag{15}$$

Then, the neutrosophic Dempster's combination rule is:

$$\begin{cases} m_{neu-DS}(\phi) = (0,0,1) \\ m_{neu-DS}(Y) = \left(\dfrac{I_{neu-DS}}{I_{neu-DS} - K_{neu12}}\right) \sum\limits_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = Y}} m_{neu1}(X_1) m_{neu2}(X_2) \quad \forall Y \in 2^\Theta \setminus \{\phi\} \end{cases}$$
$$\text{and } I_{neu-DS} = \sum_{X_1, X_2 \in 2^\Theta} m_{neu1}(X_1) m_{neu2}(X_2),\ K_{neu12} = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \phi}} m_{neu1}(X_1) m_{neu2}(X_2) \tag{16}$$

In the equation, $I_{neu\text{-}DS}$-K_{neu12} is used to do the normalization which eliminates the conflicts between two sources. When $K_{neu12} = I_{neu\text{-}DS}$, the combined nbba does not exist. The neutrosophic DSmC combination rule is:

$$m_{neu-DSmC}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cap X_2 = Y}} m_{neu1}(X_1) m_{neu2}(X_2) \quad \forall Y \in D^\Theta \tag{17}$$

The neutrosophic DSmH combination rule is:

$$\begin{cases} m_{neu-DSmH}(\phi) = (0,0,1) \\ m_{neu-DSmH}(Y) = S_{neu1}(Y) + S_{neu2}(Y) + S_{neu3}(Y) \quad \forall Y \in G \setminus \{\phi\} \end{cases}$$
$$S_{neu1}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cap X_2 = Y}} m_{neu1}(X_1) m_{neu2}(X_2),$$
$$S_{neu2}(Y) = \sum_{\substack{X_1, X_2 \in \phi \\ [U=Y] \vee [(U \in \phi) \wedge (Y = I_t)]}} m_{neu1}(X_1) m_{neu2}(X_2), \tag{18}$$
$$\text{and} \quad S_{neu3}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cup X_2 = Y \\ X_1 \cap X_2 \in \phi}} m_{neu1}(X_1) m_{neu2}(X_2)$$

The meanings of U and I_t are the same as the original DSmH. The neutrosophic PCR5 combination rule is:

$$\begin{cases} m_{neu-PCR5}(\phi) = (0,0,1) \\ m_{neu-PCR5}(Y) = m_{neu12}(Y) + \sum\limits_{\substack{Z \in G \setminus \{Y\} \\ Y \cap Z = \phi}} \left[\dfrac{m_{neu1}(Y)^2 m_{neu2}(Z)}{m_{neu1}(Y) + m_{neu2}(Z)} + \dfrac{m_{neu2}(Y)^2 m_{neu1}(Z)}{m_{neu2}(Y) + m_{neu1}(Z)}\right] \quad \forall Y \in G \setminus \{\phi\} \end{cases}$$
$$\text{and } m_{neu12}(Y) = \sum_{\substack{X_1, X_2 \in G \\ X_1 \cap X_2 = Y}} m_{neu1}(X_1) m_{neu2}(X_2) \tag{19}$$

Although the above rules are for the combination of two sources, they can be easily extended to combine N sources of information.

Fusion of qualitative information: The classical belief function theories all focus on the fusion of quantitative information. However, in real world, much information, especially provided by human sources, is the qualitative information. Therefore, with the development of artificial intelligence, the qualitative fusion methods of information have been received increasingly more attention.

Many researches about qualitative analysis have been done in the past few years (Brewka *et al*., 2004; Bryson and Mobolurin, 1998; Parsons, 1998; Zadeh, 2008). In the DSmT, Smarandache and Dezert (2006) have also introduced a new method to combine qualitative information directly and proposed the qualitative DSmT.

$L' = \{L_1, L_2, \ldots, L_n\}$ is a finite set of linguistic labels and $L_1 < L_2 < \ldots < L_n$. L_0 (the minimal qualitative value) and L_{n+1} (the maximal qualitative value) are used to extend L'. Then $L = \{L_0, L_1, L_2, \ldots, L_n, L_{n+1}\}$ and $L_0 < L_1 < L_2 < \ldots < L_n < L_{n+1}$. L can be mapped onto [0,1], L_0 corresponds to the number 0 and L_{n+1} corresponds to the number 1. The qualitative addition, subtraction and multiplication defined in the qualitative DSmT are shown as follows (Smarandache and Dezert, 2006):

$$L_i + L_j = \begin{cases} L_{i+j} & i + j \le n + 1 \\ L_{n+1} & i + j > n + 1 \end{cases} \tag{20}$$

$$L_i - L_j = \begin{cases} L_{i-j} & i \ge j \\ L_0 & i < j \end{cases} \tag{21}$$

$$L_i \times L_j = L_{\min\{i,j\}} \tag{22}$$

Dezert and Smarandache (2006) also have defined the qualitative belief assignment (qba), the qualitative DSmC rule and the qualitative DSmH rule:

$$qm_{DSmC}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cap X_2 = Y}} qm_1(X_1) qm_2(X_2) \quad \forall Y \in D^\Theta \tag{23}$$

$$qm_{DSmH}(Y) = \phi(Y)\left[qS_1(Y) + qS_2(Y) + qS_3(Y)\right] \quad \forall Y \in G \tag{24}$$

However, they did not propose the division operators of linguistic labels. Therefore, they did not define the mathematical expression of qualitative PCR rule.

In the neutrosophic theory, the truth T, the indeterminacy I and the falsity F are all shown by numerical values. For example, we use 80% truth, 0% indeterminacy and 20% falsity to represent a proposition. However, the received information sometimes is qualitative. For example, the truth of a proposition is probable, the indeterminacy is no and the falsity is impossible. Therefore, we propose the T, I and F based on linguistic labels. T, I and F are supposed to be the subsets of L and $L = \{L_0, L_1, L_2, \ldots, L_n, L_{n+1}\}$. We also define the qualitative neutrosophic belief assignment (qnba):

$$qm_{neu}(\cdot): G \to L^3,\ qm_{neu}(X) = (t_X, i_X, f_X)$$
$$\text{and } qm_{neu}(\phi) = (L_0, L_0, L_{n+1}),\ \sum_{X \in G} t_X = L_{n+1} \tag{25}$$

G represents the power set 2^Θ or the hyper-power set D^Θ. $t_X \in T$, $i_X \in I$, $f_X \in F$. According to Eq. 11, 13, 17 and 18, we can get the qualitative neutrosophic DSmC rule and the qualitative neutrosophic DSmH rule:

$$qm_{neu-DSmC}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cap X_2 = Y}} qm_{neu1}(X_1) qm_{neu2}(X_2) \quad \forall Y \in D^\Theta, \tag{26}$$

$$\begin{cases} qm_{neu-DSmH}(\phi) = (L_0, L_0, L_{n+1}) \\ qm_{neu-DSmH}(Y) = qS_{neu1}(Y) + qS_{neu2}(Y) + qS_{neu3}(Y) \quad \forall Y \in G \setminus \{\phi\} \end{cases},$$

$$qS_{neu1}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cap X_2 = Y}} qm_{neu1}(X_1)\, qm_{neu2}(X_2),$$
$$qS_{neu2}(Y) = \sum_{\substack{X_1, X_2 \in \phi \\ [U=Y] \vee [(U \in \phi) \wedge (Y = I_t)]}} qm_{neu1}(X_1)\, qm_{neu2}(X_2), \tag{27}$$
$$\text{and } qS_{neu3}(Y) = \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cup X_2 = Y \\ X_1 \cap X_2 \in \phi}} qm_{neu1}(X_1)\, qm_{neu2}(X_2)$$

It is very difficult to divide or normalize the linguistic labels and there are still no feasible definitions of the division and normalization operation of linguistic labels. Therefore, we do not define the qualitative neutrosophic Dempster's combination rule and qualitative neutrosophic PCR rule.

If the linguistic labels are equidistant, we can map L onto {0, 1/(n+1), 2/(n+1), ..., n/(n+1), 1}. Then, we can use the quantitative combination rules to fuse the qualitative information represented by qnba and transfer the fusion results into linguistic labels. In fact, the results of this method are more precise than those of the above combination rules. If the linguistic labels are not equidistant, we can also apply the method, but the results are imprecise.

THE IMPLEMENTATION OF THE NEW COMBINATION RULES

Example 1: Let us consider $\Theta = \{\theta_1, \theta_2\}$. The initial nbba $m_{neu1}'(.)$ and $m_{neu2}'(.)$ are provided as follows:

$m_{neu1}'(\theta_1) = (0.6, 0.2, 0.2)$, $m_{neu1}'(\theta_2) = (0.4, 0.0, 0.6)$,
$m_{neu1}'(\theta_1 \cup \theta_2) = (0.0, 0.2, 0.8)$,
$m_{neu2}'(\theta_1) = (0.4, 0.4, 0.2)$, $m_{neu2}'(\theta_2) = (0.6, 0.2, 0.2)$,
$m_{neu2}'(\theta_1 \cup \theta_2) = (0.2, 0.2, 0.6)$.

We can extract the truth values of nbba as the initial bba $m_1'(.)$ and $m_2'(.)$:
$m_1'(\theta_1) = 0.6$, $m_1'(\theta_2) = 0.4$, $m_1'(\theta_1 \cup \theta_2) = 0.0$,
$m_2'(\theta_1) = 0.4$, $m_2'(\theta_2) = 0.6$, $m_2'(\theta_1 \cup \theta_2) = 0.2$.

Before computing the combination results, we need to normalize the initial nbba and bba firstly. According to Eq. 10, we can get the normalized nbba. The normalized nbba ($m_{neu1}(.)$, $m_{neu2}(.)$) and the normalized bba ($m_1(.)$, $m_2(.)$) are all shown in Table 1. If we hold the free DSm model, then according to Eq. 5 and 17, we can get $m_{DSmC}(.)$ and $m_{neu\text{-}DSmC}(.)$:

$m_{DSmC}(\theta_1) = 0.300$, $m_{DSmC}(\theta_2) = 0.267$, $m_{DSmC}(\theta_1 \cup \theta_2) = 0$, $m_{DSmC}(\theta_1 \cap \theta_2) = 0.433$,
$m_{neu\text{-}DSmC}(\theta_1) = (0.300, 0.167, 0.267)$, $m_{neu\text{-}DSmC}(\theta_2) = (0.267, 0.033, 0.534)$,
$m_{neu\text{-}DSmC}(\theta_1 \cup \theta_2) = (0, 0.033, 0.400)$, $m_{neu\text{-}DSmC}(\theta_1 \cap \theta_2) = (0.433, 0.033, 0.133)$.

If we hold the Shafer's model, then according to Eq. 3, 6, 7, 16, 18 and 19, we can get $m_{DS}(.)$, $m_{DSmH}(.)$, $m_{PCR5}(.)$, $m_{neu\text{-}DS}(.)$, $m_{neu\text{-}DSmH}(.)$ and $m_{neu\text{-}PCR5}(.)$. The combination results are shown in Table 1.

From the data, we can see the combination results of Dempster's rule, DSmC rule, DSmH rule and PCR5 rule are the same as the truth values of combination results of the corresponding neutrosophic rules. Therefore, the neutrosophic combination rules based on DST and DSmT can be considered as the generalization of the original DST-based and DSmT-based rules and provide more information.

Example 2: Let us consider $\Theta = \{\theta_1, \theta_2\}$ and hold the Shafer's model. θ_1 represents that a patient has contracted disease A. θ_2 represents that the patient has contracted disease B. $\theta_1 \cup \theta_2$ represents that the patient has contracted both A and B diseases at the same time. $m_{neu1}'(.)$ and $m_{neu2}'(.)$ are the diagnosis provided by two doctors respectively:

$m_{neu1}'(\theta_1) = (0.6, 0.3, 0.1)$, $m_{neu1}'(\theta_2) = (0.4, 0.1, 0.5)$,
$m_{neu1}'(\theta_1 \cup \theta_2) = (0.1, 0.2, 0.7)$,
$m_{neu2}'(\theta_1) = (0.4, 0.4, 0.2)$, $m_{neu2}'(\theta_2) = (0.6, 0.2, 0.2)$,
$m_{neu2}'(\theta_1 \cup \theta_2) = (0.2, 0.2, 0.6)$.

We extract the truth values of $m_{neu1}'(.)$ and $m_{neu2}'(.)$ as the initial bba $m_1'(.)$ and $m_2'(.)$:

Table 1: The combination results of different rules in the example 1

Belief assignment	θ_1	θ_2	$\theta_1 \cup \theta_2$
$m_1(.)$	0.6	0.4	0
$m_2(.)$	0.333	0.500	0.167
$m_{DS}(.)$	0.529	0.471	0
$m_{DSmH}(.)$	0.300	0.267	0.433
$m_{PCR5}(.)$	0.524	0.476	0
$m_{neu1}(.)$	(0.6, 0.2, 0.2)	(0.4, 0, 0.6)	(0, 0.2, 0.8)
$m_{neu2}(.)$	(0.333, 0.333, 0.167)	(0.500, 0.167, 0.167)	(0.167, 0.167, 0.500)
$m_{neu\text{-}DS}(.)$	(0.529, 0.191, 0.296)	(0.471, 0.038, 0.594)	(0, 0.038, 0.444)
$m_{neu\text{-}DSmH}(.)$	(0.300, 0.167, 0.267)	(0.267, 0.033, 0.534)	(0.433, 0.066, 0.533)
$m_{neu\text{-}PCR5}(.)$	(0.524, 0.185, 0.307)	(0.476, 0.048, 0.627)	(0, 0.033, 0.400)

Table 2: The combination results of different rules in the example 2

Belief assignment	θ_1	θ_2	$\theta_1\cup\theta_2$
$m_1(.)$	0.545	0.364	0.091
$m_2(.)$	0.333	0.500	0.167
$m_{DS}(.)$	0.498	0.477	0.025
$m_{DSmH}(.)$	0.302	0.289	0.409
$m_{PCR5}(.)$	0.502	0.483	0.015
$m_{neu1}(.)$	(0.545, 0.273, 0.091)	(0.364, 0.091, 0.455)	(0.091, 0.182, 0.636)
$m_{neu2}(.)$	(0.333, 0.333, 0.167)	(0.500, 0.167, 0.167)	(0.167, 0.167, 0.500)
$m_{neu\text{-}DS}(.)$	(0.498, 0.250, 0.184)	(0.477, 0.076, 0.452)	(0.025, 0.038, 0.350)
$m_{neu\text{-}DSmH}(.)$	(0.302, 0.198, 0.167)	(0.289, 0.060, 0.410)	(0.409, 0.106, 0.409)
$m_{neu\text{-}PCR5}(.)$	(0.502, 0.251, 0.192)	(0.483, 0.083, 0.476)	(0.015, 0.030, 0.318)

$m_1'(\theta_1) = 0.6$, $m_1'(\theta_2) = 0.4$, $m_1'(\theta_1\cup\theta_2) = 0.1$,
$m_2'(\theta_1) = 0.4$, $m_2'(\theta_2) = 0.6$, $m_2'(\theta_1\cup\theta_2) = 0.2$.

$m_{neu1}(.)$, $m_{neu2}(.)$, $m_1(.)$ and $m_2(.)$ are the normalizations of $m_{neu1}'(.)$, $m_{neu2}'(.)$, $m_1'(.)$ and $m_2'(.)$ respectively. Then, according to the above rules mentioned, we can fuse the information provided by doctors. As shown in Table 2, the combined bba of θ_1 is larger than and very close to that of θ_2. We can diagnose the patient's illness as disease A. However, because the difference of the combined bba of θ_1 and θ_2 is small, this diagnosis is uncertain probably. The neutrosophic combination results provide more information. The falsity value of combined nbba of θ_1 is less than that of θ_2 obviously, which can support the above diagnosis.

Example 3: The linguistic labels $L = \{L_0, L_1, L_2, L_3, L_4, L_5\}$. $L_0, L_1, L_2, L_3, L_4, L_5$ represent no, impossible, improbable, possible, probable and certain, respectively. $\Theta = \{\theta_1, \theta_2\}$ and the qnba and qba are as follows:

$qm_{neu1}(\theta_1) = (L_3, L_1, L_1)$, $qm_{neu1}(\theta_2) = (L_2, L_0, L_3)$,
$qm_{neu1}(\theta_1\cup\theta_2) = (L_0, L_1, L_4)$,
$qm_{neu2}(\theta_1) = (L_2, L_2, L_1)$, $qm_{neu2}(\theta_2) = (L_2, L_1, L_2)$,
$qm_{neu2}(\theta_1\cup\theta_2) = (L_1, L_1, L_3)$.
$qm_1(\theta_1) = L_3$, $qm_1(\theta_2) = L_2$, $qm_1(\theta_1\cup\theta_2) = L_0$,
$qm_2(\theta_1) = L_2$, $qm_2(\theta_2) = L_2$, $qm_2(\theta_1\cup\theta_2) = L_1$.

If we hold the free DSm model, then according to Eq. 23 and 26, we can get $qm_{DSmC}(.)$ and $qm_{neu\text{-}DSmC}(.)$:

$qm_{DSmC}(\theta_1) = L_3$, $qm_{DSmC}(\theta_2) = L_3$, $qm_{DSmC}(\theta_1\cup\theta_2) = L_0$,
$qm_{DSmC}(\theta_1\cap\theta_2) = L_4$,
$qm_{neu\text{-}DSmC}(\theta_1) = (L_3, L_3, L_3)$, $qm_{neu\text{-}DSmC}(\theta_2) = (L_3, L_1, L_5)$,
$qm_{neu\text{-}DSmC}(\theta_1\cup\theta_2) = (L_0, L_1, L_3)$, $qm_{neu\text{-}DSmC}(\theta_1\cap\theta_2) = (L_4, L_1, L_2)$.

If we hold the Shafer's model, then according to Eq. 24 and 27, we can get $qm_{DSmH}(.)$ and $qm_{neu\text{-}DSmH}(.)$:

$qm_{DSmH}(\theta_1) = L_3$, $qm_{DSmH}(\theta_2) = L_3$, $qm_{DSmH}(\theta_1\cup\theta_2) = L_4$,
$qm_{neu\text{-}DSmH}(\theta_1) = (L_3, L_3, L_3)$, $qm_{neu\text{-}DSmH}(\theta_2) = (L_3, L_1, L_5)$,
$qm_{neu\text{-}DSmH}(\theta_1\cup\theta_2) = (L_4, L_2, L_5)$.

From the above results, we can see that the qualitative neutrosophic combination rules based on DSmT generalizes the original qualitative rules based on DSmT. Because the limitation of operators of linguistic labels, we cannot normalize the combination results and the results are imprecise.

Example 4: Let us consider the previous example of disease diagnosis. However, the diagnosis provided by two doctors is qualitative. $qm_{neu1}'(.)$ and $qm_{neu2}'(.)$ are:

$qm_{neu1}'(\theta_1) = (L_3, L_1, L_1)$, $qm_{neu1}'(\theta_2) = (L_2, L_0, L_3)$,
$qm_{neu1}'(\theta_1\cup\theta_2) = (L_0, L_1, L_4)$,
$qm_{neu2}'(\theta_1) = (L_2, L_2, L_1)$, $qm_{neu2}'(\theta_2) = (L_3, L_1, L_1)$,
$qm_{neu2}'(\theta_1\cup\theta_2) = (L_1, L_1, L_3)$.
We extract the truth values of $qm_{neu1}'(.)$ and $qm_{neu2}'(.)$ as the initial qba $qm_1'(.)$ and $qm_2'(.)$:
$qm_1'(\theta_1) = L_3$, $qm_1'(\theta_2) = L_2$, $qm_1'(\theta_1\cup\theta_2) = L_0$,
$qm_2'(\theta_1) = L_2$, $qm_2'(\theta_2) = L_3$, $qm_2'(\theta_1\cup\theta_2) = L_1$.

The meanings of $L_0, L_1, L_2, L_3, L_4, L_5$ are the same as those in example 3. We suppose the linguistic labels are equidistant and use $\{0, 0.2, 0.4, 0.6, 0.8, 1\}$ to map $\{L_0, L_1, L_2, L_3, L_4, L_5\}$. Then, we use the quantitative combination rules to fuse the qualitative information and transfer the combination results into linguistic labels. The corresponding numerical values of qnba and qba are the same as the values of nbba and bba in example 1. Therefore, we can utilize the results in example 1 directly and transfer the quantitative results into the closest linguistic labels.

Table 3 shows the qualitative combination results. We can diagnose the patient's illness as disease A through the results. Compared with the combination results in example 3, the results are normalized and more precise. However, this method needs to suppose the linguistic labels to be equidistant.

Table 3: The combination results of different qualitative rules in the example 4

Belief assignment	θ_1	θ_2	$\theta_1 \cup \theta_2$
$qm_1(.)$	L_3	L_2	L_0
$qm_2(.)$	L_2	L_3	L_1
$qm_{DS}(.)$	L_3	L_2	L_0
$qm_{DSmH}(.)$	L_2	L_1	L_2
$qm_{PCR5}(.)$	L_3	L_2	L_0
$qm_{neu1}(.)$	(L_3, L_1, L_1)	(L_2, L_0, L_3)	(L_0, L_1, L_4)
$qm_{neu2}(.)$	(L_2, L_2, L_1)	(L_3, L_1, L_1)	(L_1, L_1, L_3)
$qm_{neu\text{-}DS}(.)$	(L_3, L_1, L_1)	(L_2, L_0, L_3)	(L_0, L_0, L_2)
$qm_{neu\text{-}DSmH}(.)$	(L_2, L_1, L_1)	(L_1, L_0, L_3)	(L_2, L_0, L_3)
$qm_{neu\text{-}PCR5}(.)$	(L_3, L_1, L_2)	(L_2, L_0, L_3)	(L_0, L_0, L_2)

CONCLUSIONS

In this study, we have developed the neutrosophic combination rules based on DST and DSmT, which are the generalizations of the original DST-based and DSmT-based combination rules. In real world, the acquired information is always vague, uncertain and imprecise. It is incomplete that only the degree of truth is considered. The three members including truth, indeterminacy and falsity make the neutrosophic theory be adapted to representing the real proposition. Therefore, we propose the neutrosophic belief assignment, which can represent the human mind more effectively and connect the DST-based combination rules and the DSmT-based combination rules with neutrosophic logic. The information provided by human sources is expressed in natural language sometimes. To fuse not only the quantitative information but also the qualitative information, we have presented both quantitative and qualitative neutrosophic combination rules. Because of the limitations of qualitative operators, the qualitative combination rules need more developments. We also have provided some application examples. Compared with the original DST-based and DSmT-based combination rules, the neutrosophic combination rules based on DST and DSmT can provide more important information and help to make correct decisions. Certainly, the new neutrosophic fusing approaches also need more computation.

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