Fine Structure Constant - a mystery resolved

Jose P Koshy E.mail: josepkoshy@gmail.com

The Fine Structure Constant (FSC), a dimensionless constant, has been known since 1916. As Richard Feynman has stated, it's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding. Attempts to find a mathematical basis for it have been unsuccessfull so far. Here I demonstrate that a dimensionless constant resembling FSC, both in the equation-form and value, can be obtained from a 'symmetrical spherical-packing' model. Thus in the hundredth year of its discovery, the so-called magic number has a rational explanation: it is packing constant, a length- mass ratio (both expressed in number of entities) when particles get packed into spherical structures. The finding reveals a possible relation between spherical-packing and integration of particles: composite particles may be containing the minimum number of entities required to give the best possible spherical shape.

Symmetrical packing of uniform entities into spherical structures is a mathematical problem. To obtain the best possible spherical shape with the least number of entities, the diameter of the structure should be 7 entities, because the smallest whole number ratio of π is 22/7. To construct the structure, first the entities are linearly arranged, such linear structures are then closely packed to form a layer, and such layers are then stacked. For symmetry, the distance between layers should be the same as the distance between lines. The entities thus remain at exact lattice-points, which can be clearly identified. Taking any entity as the centre, and removing the ones beyond a certain distance, we get the structure. The number of layers in it will be equal to the number of lines in its central layer.

Here we consider a two-stage packing. In the first stage, as described above, a structutre is built using spheres of unit radius, taking the limiting distance as 6 units from the centre. It contains 155 spheres (refer annexure-1). The central layer is shown here (fig.1). The spheres in it remain in 7 lines and the diameter contains 7 spheres. Layers are built by placing spheres in the troughs in each line, taking any set of parallel lines. Thus the distance between layers and the distance between lines are equal to $\sqrt{3}$ units. What we get is a 7-layer hexagonal structure with blunt corners. It is similarly layerwise in the perpendicular plane also, and so its breadth and thickness are equal. The effective radius of its central layer (refer annexure-2) is approximately $42.141592/2\pi$ units.

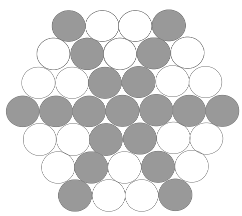


Fig.1: Central layer – first-stage packing

In the second stage, such hexagonal structures 'taken in pairs' are packed. To identify the lattice-points, we will take these as perfectly hexagonal (with pointed corners). So if length is taken as one unit, breadth and thickness are equal to $\sqrt{3}$ units. As there will be an odd number of entities in a centre-filled symmetrical structure, the centre has to be empty in the case of pairs, and so the diameter has to be 7.5 pairs, that is, 15 entities. A single layer having the required diameter is shown here (fig.2). Here, the nearest position in the eighth line (marked X) is at a distance of √192 units and remains outside, but positions at the corners (shown darkened) are at a distance of √196 units, and these remain inside. So we can take these as the limits in the two directions. The thickness of the layer being √3 units, the packing can be completed by stacking 15 such layers exactly one over the other, and removing the central entity and the entities beyond the limit. The structure contains 1838 entities, and all lattice points within the limit (except the central point) remain filled.

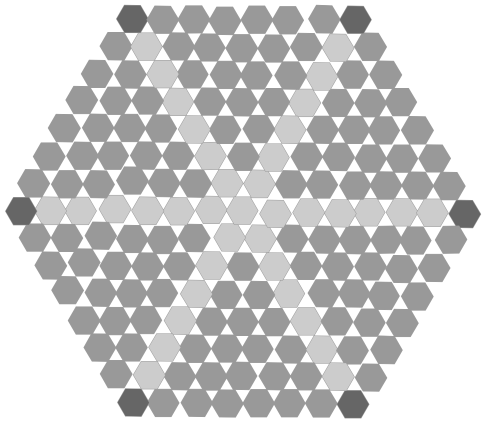


Fig. 2: A layer having diameter 15 units

Thus we have spheres of radius one unit as the first- level entity; the second-level entity is 155 times heavier than the first, and its radius is $42.141592/2\pi$ units; and the third-level entity is 1838 times heavier than the second. Using their radii and masses, a number of dimensionless constants can be obtained. Consider one such constant, $2(r_2 / r_1) / (m_3 / m_2) = \alpha_0$, where r_1, and r_2 are the radii of the first and second level entities, and m_2 and m_3, the masses of second and third level entities. Substituting,

$$\alpha_0 = 2(r_2 / r_1) / (m_3 / m_2) \quad \text{..............................(i)}$$
$$= 2(42.141592 / 2\pi) / 1838$$
$$= 0.00729819$$

This is very close to the Codata value of FSC ($\alpha = 0.00729735$). The equation for FSC (refer annexure-3) can be written as, $\alpha = 2(r_e / r) / (m_n / m_e)$, where r =. λ_n / π. The equation is similar to the equation for α_0. Thus the dimensionless constant obtained from the spherical-packing model is simlar to FSC, both in the equation-form and value.

Spherical-packing was studied as a possible way to explain the formation of composite particles and not as part of arriving at FSC. Neutron is 1838.6 times heavier than electron. Being neutral, it can be made up of electron-positron pairs, and if it has perfect symmetry and an empty centre, there will be no residual charge. Symmetry implies that the outermost positions will be occupied by either electron or positron, and neutron will have magnetic moment. A subtle difference between electron and positron can lead to a small (but hiherto unnoticed) mass difference, making 1838 a magic number when viewed in that perspective. A 15-layer structure made up of uniform spheres was considered. This immediately gave a positive result: the number of spheres in it can vary between 1804 and 1862. So different types of packing were studied expecting that there would exist a packing which allows exactly 1838 positions. A two-stage packing explained earlier provided the answer.

In short, the search was for a structure containing 1838 entities, and it was successful. But even a blind-fold search based on π, symmetry and perfection will lead to 1838 as the most probable number. Above all, the surprise finding was that the structure has a built-in FSC: it contains 1838 entities and the perimeter of its central layer (fig.2) is 42 entities, and $42/1838\pi = 0.007274 \approx \alpha$, the FSC. FSC thus appeared to be a diameter- mass ratio related to packing. Incorporating the first stage packing also, a more accurate value and an equation identical to that of FSC were obtained. Considering all these, it was concluded that FSC is a packing constant.

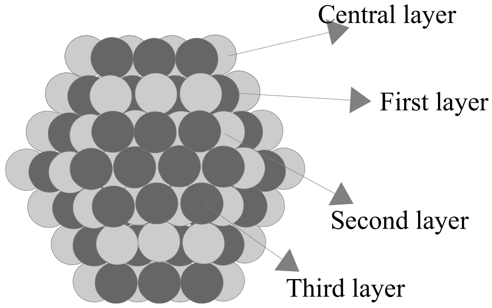
Ref:

1. Richard P. Feynman (1985). QED: The Strange Theory of Light and Matter. Princeton University Press. p.129.ISBN0-691-08388-6.

Annexure 1: Number of entities in the structures formed

In symmetrical spherical packing, the entities remain in layers, and in each layer, in a set of parallel lines. The distance between entities in a line is 2 units, and distance between lines/layers is √3 units. The lattice-points in such packing can be idenified using x, y, z cordinates with respect to the centre of the structure to be formed. 'z' denotes the layer, 'y' denotes the line, and 'x' denotes the position in the line. $|z| = 0, \sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ for layers 0,1,2,3, ... , $|y| = 0, \sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ for lines 0,1,2,3, ... and $|x| = 0,2,4, \ldots$ in even lines and 1, 2, 3, ... in odd lines. The distance from the centre can be given by the relation, $d^2 = x^2+y^2+z^2$. The number of entites in the structure is equal to the number of lattice points within the given limit (distance from the centre).

Here, a two-stage packing is considered. In the first stage, a 7-layer structure is formed. Here the limit is $d \leq 6$ units. The figure (fig.3) shows the central and the three layers on one side stacked one over the other. The number of spheres in each layer can be counted easily. There are 155 lattice points within the limit and this can be easily verfied.

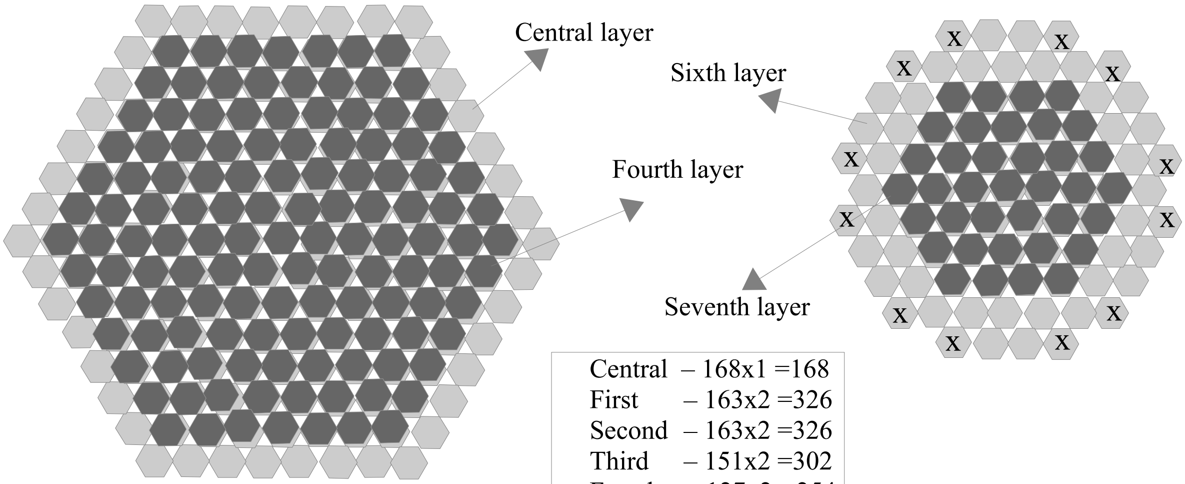


No. of spheres

Central	–	37x1 =37
1st	–	30x2 =60
2nd	–	19x2 =38
3rd	–	10x2 =20
Total	–	155

Fig. 3: First-stage packing – layers and number of spheres in each layer

In the second stage, a 15-layer structure is formed. The limit is $d \leq \sqrt{196}$ towards the corner and $d < \sqrt{192}$ towards the sides. The number of entities in each layer can be ascertained from the given figures (fig.4). The central layer has a vacant centre, the rest are



Central	– 168x1	=168
First	– 163x2	=326
Second	– 163x2	=326
Third	– 151x2	=302
Fourth	– 127x2	=254
Fifth	– 109x2	=218
Sixth	– 85x2	=170
Seventh	– 37x2	= 74
Total	–	1838

Fig.4: Layers in the second-stage packing

centre-filled. Central layer contains 169-1=168 entities. First and second layers are identical and contains 163 each (removing 1 from corners of central layer). Third layer contains 151 (removing 3 from corners of the central layer). Fourth layer contains 127. Fifth layer contains 109 (removing 3 from corners of the fourth layer). Sixth layer contains 85 and seventh layer, 37.

The structure contains 1838 entities. Out of this, 6 in the central layer (at the corners) remain at √196, and 24 in the sixth layers (points marked 'x') remain at √192 – this is close to the corner and so well within the limit. The rest are below √192. So no confusion arises in deciding whether a lattice point remain within the structure or not, and all lattice points within the limit (except the centre) remain filled. This can be easily verified because the structure is symmetric and only few positions need verification.

Annexure 2: Radius of the second-level structure

The approximate perimeter of the second-level structure can be calculated in two ways:

(i). by taking the central layer as a regular hexagon of side 7 units; this gives the perimeter as, 6x7 = 42 units. This is slightly less because a small portion of the spheres remains outside the line joining the corners (fig.5a).

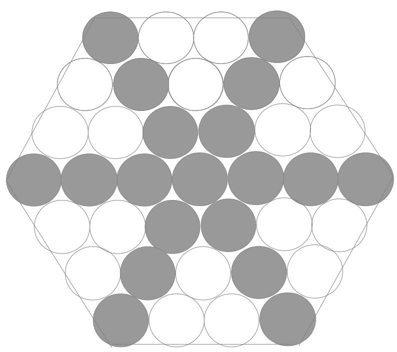


Fig.5a: Perimeter of the layer

(ii). by measuring the distance along the outermost points (that is, the corners rounded). The length of the arc at the corner is $2\pi/6$, because the angle substended by the arc is 60^0 and the radius is one unit (fig.5b). So perimeter = 6 x $(6+2\pi/6)$ = 42.283185 units. This is slightly greater because it includes the gaps between spheres.

The radius can be obtained by dividing the perimeter by 2π, and so its value lies between $42/2\pi$ and $42.283185/2\pi$ units. The average value, $42.141592/2\pi$ can be taken as the approximate radius of the structure.

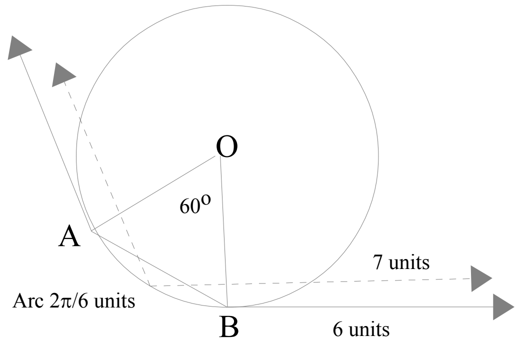


Fig.5b: The corner sphere

Annexure 3: Equation of fine structure constant

The equation for fine structure constant can be written in may ways using other physical constants. Generally the equation is given as, $\alpha = (1/\hbar c)(e^2/4\pi\varepsilon_0)$. We have the following relations also : $\hbar c = hc/2\pi$; $hc = m_n c^2 \lambda_n$; and the Classical radius of electron, $r_e = (e^2/4\pi\varepsilon_0)/m_e c^2$.

So, $(1/\hbar c) = 2\pi/hc = 2\pi/m_n c^2 \lambda_n$

and, $(e^2/4\pi\varepsilon_0) = r_e m_e c^2$

Substituting we get,

$$\alpha = (2\pi/m_n c^2 \lambda_n)(r_e m_e c^2) = (2\pi r_e m_e/m_n \lambda_n) = (2\pi r_e/\lambda_n)/(m_n/m_e)$$

If we put $\lambda_n/\pi = r$, the above equation can be written as,

$$\alpha = 2(r_e/r)/(m_n/m_e) .$$