Einstein versus FitzGerald, Lorentz, and Larmor Length Contraction
Einstein's Length Contraction is Also Consistent with Anisotropic One-Way Speed of Light

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Abstract

This paper discusses the similarities between Einstein's length contraction and the FitzGerald, Lorentz, and Larmor length contraction. The FitzGerald, Lorentz, and Larmor length contraction was originally derived for only the case of a frame moving relative to the ether frame, and not for two moving frames. When extending the FitzGerald, Lorentz, and Larmor length transformation to any two frames, we will clearly see that it is different than the Einstein length contraction. Under the FitzGerald, Lorentz, and Larmor length transformation we get both length contraction and length expansion, and non-reciprocality, while under Einstein's special relativity theory we have only length contraction and reciprocality. However, we show that there is a mathematical and logical link between the two methods of measuring length.

This paper shows that the Einstein length contraction can be derived from assuming an anisotropic one-way speed of light. Further, we show that that the reciprocality for length contraction under special relativity is an apparent reciprocality due to Einstein-Poincaré synchronization. The Einstein length contraction is real in the sense that the predictions are correct when measured with Einstein-Poincaré synchronized clocks. Still we will claim that there likely is a deeper and more fundamental reality that is better described with the extended FitzGerald, Lorentz, and Larmor framework, which, in the special case of using Einstein-Poincaré synchonized clocks gives Einstein's length contraction. The extended FitzGerald, Lorentz, and Larmor length contraction is also about length expansion, and it is not reciprocal between frames. Still, when using Einstein synchronized clocks the length contraction is apparently reciprocal. An enduring, open question concerns whether or not it is possible to measure the one-way speed of light without relying on Einstein-Poincaré synchronization or slow clock transportation synchronization, and if the one-way speed of light then is anisotropic or isotropic. Several experiments performed and published claim to have found an anisotropic one-way speed of light. These experiments have been ignored or ridiculed, but in our view they should be repeated and investigated further.

Key Words: length contraction, length expansion, special relativity theory, ether theories, atomism, reciprocality, anisotropic one-way speed of light.

1 Introduction

George FitzGerald was the first to suggest that the null result of the Michelson–Morley experiment could be that the length of any material object (including the Earth itself) contracts along the direction it moves through the ether, or, as explained in his own words:

> *I would suggest that almost the only hypothesis that can reconcile this opposition is that the length of the material bodies changes according as they are moving through the ether or across it, by an amount depending on the square of the ratio of their velocity to that of light.*
> – FitzGerald, May 1889

We should bear in mind that FitzGerald was a supporter of the ether theory and that here he is talking about the speed of light relative to the ether; in this context, the speed of the material bodies is the speed against the ether. Lorentz provides a 1892 mathematical formalization of length contraction. The Lorentz length contraction formula is given as

$$1-\frac{p^2}{2V^2} \tag{1}$$

where p is the velocity of the Earth relative to the ether, and V is the velocity of light relative to that of ether. The Lorentz formula is the first term of a series expansion of the more accurate formula

$$\sqrt{1-\frac{p^2}{V^2}} \approx 1-\frac{p^2}{2V^2} \tag{2}$$

that in todays notation would be written as $v = p$ and $c = V$; this gives

$$\sqrt{1-\frac{v^2}{c^2}} \approx 1-\frac{v^2}{2c^2} \tag{3}$$

Lorentz was "mainly" interested in experiments related to the motion of the Earth against the ether, and it was assumed the velocity of the Earth against the ether was close to the Earth orbital velocity around the sun, that is approximately 30 km/s. In this case the first term of the series expansion would be more than accurate enough.

What is often ignored in modern literature is that v in the FitzGerald and Lorentz length contraction originally was the velocity of the Earth against the ether and that c was the speed of the light against the ether. In the Einstein length contraction formula the velocity v is the relative speed against another frame as measured with Einstein-Poincaré synchronized clocks. Under Einstein's theory speeds against the ether are not even considered, as he assumed that the ether did not exist. Joseph Larmor (1900) combined the FitzGerald and Lorentz length contraction with time dilation and was the first to develop a consistent space and time transformation which, after rewritten in modern notation, at first sight looks identical to that of Lorentz and Einstein. However, it is actually different, as the velocity, v, in the Larmor transformation is clearly meant to be the velocity against the ether. In 1905, Einstein also published a length contraction formula that he related to the Lorentz transformation later on.

Larmor held on to the idea that the true one-way speed of light was anisotropic relative to a moving frame. Larmor proved that the null result in the Michelson–Morley experiment was fully consistent with the anisotropic one-way speed of light. Still, Larmor never came up with a suggestion of how to detect this true one-way speed of light, which is a way to detect motion against the ether (void). In 1905, Poincaré assumed that it was impossible to detect the Earth's motion against ether:

> *It seems that this impossibility of establishing experimentally the absolute motion of the Earth is a general law of nature: we are, of course, set towards admitting this law that we will call the Postulate of Relativity and to admit it without restrictions.* – Henri Poincaré

To measure the one-way speed of light, we need to synchronize two clocks, and to synchronize two clocks we need to know the one-way speed of light. This is why Poincaré assumed that we could never measure the one-way speed of light and therefore we could never detect motion against the ether.

After Poincaré (1905) simplified the Lorentz space and time transformation to a certain extent, Lorentz seems to have become heavily influenced by Poincaré's view that motion against the ether not could be detected. It is natural to believe that after 1905 Lorentz assumed the velocity, v, in his transformations was a relative velocity between frames rather than a velocity against the ether, but Lorentz himself does not seem clear on this point[1]. In this case the Lorentz transformation and the Lorentz length contraction would indeed be the same as under Einstein transformation, but with a somewhat different interpretation. In the Lorentz world, as influenced by Poincaré, the ether still existed, but motion against it could not be detected. In the Einstein world the ether simply did not exist or at least it was not necessary to take it into account if it was not possible to measure it. Larmor, on the other hand, seems to be holding on to the concept that the one-way speed of light is anisotropic (except in the ether frame) and that by using length contraction and time dilation, based on the velocity of the Earth against the ether, this was also fully consistent with the Michelson–Morley experiment. However, we will soon see that Larmor's theory was incomplete, as it only dealt with a moving frame against the ether frame and not two moving frames.

2 The Einstein Length Contraction

The Einstein length contraction is given by

[1] If anyone has found material where they feel he is clear on this, please point me towards it.

$$L_{e,1,2} = L_{e,2,2}\sqrt{1-\frac{v_e^2}{c^2}} \tag{4}$$

where v_e is the speed of the moving rod two relative to frame one as measured with Einstein-Poincaré synchronized clocks. I use the notation v_e rather than v, as we we will reserve v for the velocity of a rod (a reference frame) against the ether as measured from the ether. Further $L_{e,2,2}$ is the length of the a rod in frame two as measured from the same frame two. $L_{e,1,2}$ is the length of the rod at rest in frame two as measured from frame one. The speed of frame two as measured from frame one is v_e and the speed of frame one as observed from frame two using Einstein synchronized clocks is v_e. That is to say that the relative velocities between reference frames are reciprocal when using Einstein-Poincarè synchronized clocks, a point we fully agree on.

Further, we have

$$L_{e,2,1} = L_{e,1,1} = \sqrt{1-\frac{v_e^2}{c^2}} \tag{5}$$

This just shows that Einstein length contraction is reciprocal. Frame one can claim that the rod at rest in frame two is observed to be contracted and frame two will at the same time claim the rod at rest in frame one is contracted. That the length contraction is reciprocal is somewhat of an unsolved mystery, not mathematically, but conceptually. Exactly what is the reason for this? As we will see, the reciprocality has to do with the clock synchronization procedure.

3 Length Contraction: Is It for Real or Is It an "Illusion"?

"Surprisingly" there are still several different interpretations of the length contractions from special relativity among mainstream physicists. Physicists writing and lecturing about modern physics do not seem to be in complete agreement on whether length contraction is a real physical effect or if it is merely an illusionary effect, or actually some type of optical "illusion". For example, Lawden (1975) claims[2]

> The contraction is not to be thought of as the physical reaction of the rod to its motion and as belonging to the same category of physical effects as the contraction of a metal rod when cooled. It is due to changed relationship between the rod and the instruments measuring its length.

Patheria (1974) correctly points out that there is a major difference between FitzGerald-Lorentz transformation on one side and Einstein length transformation:

> It must be pointed out here that the contraction hypothesis, put forward by FitzGerald and Lorentz, was of entirely different character and must not be confused with the effect obtained here. That hypothesis did not refer to a mutually reciprocal effect; it is rather suggesting a contraction in the *absolute* sense, arising from the motion of an object with respect to the aether or, so to say, from 'absolute' motion. According to a relativistic standpoint, neither absolute motion nor any effect accruing therefrom has any physical meaning.[3]

In particular, this holds true for the FitzGerald length contraction and the original Lorentz contraction and also the way Larmor uses length contraction. For the Poincaré influenced Lorentz contraction one could argue that it is mathematically identical to the Einstein length contraction, as the velocity in the formula probably should be interpreted as relative velocity measured with Einstein-Poincaré synchronized clocks and not the velocity against the ether.

Shadowitz (1969) claims

> If the measurements are optical, then in order to avoid an incorrect result, the light photons must *leave* the two points at the same time as measured by the observers: they must leave simultaneously. It is clear that the process of length measurement is different from the process of seeing. Amazingly, this distinction was not noticed until 1959, when it was first pointed out by James Terrell.[4]

This point is also a rather important one.

Further Rindler (2001) writes

[2] Page 12.
[3] See Patheria (1974) page 41.
[4] See Shadowitz (1969) page 61.

> This *length contraction* is no illusion, no mere accident of measurement or convention. It is real in every sense. A moving rod is *really* short! It could really be pushed into a hole at rest in the lab into which it would not fit if it were moving and shrunk.[5]

Freedman and Young (2016) claims

> Length contraction is real! This is not an optical illusion! The ruler really is shorter in the reference frame S than it is in S‘.[6]

Further Harris (2008) writes

> It is a grave mistake to dismiss length contraction as an optical illusion caused by delays in light traveling to the observer from a moving object. This effect is real.[7]

University physics students are not always taught the same way though; for example, Krane (2012) seems to claim something different, at least in part:

> Length contraction suggests that the objects in motion are measured to have shorter length then they do at rest. The objects do not actually shrink; there is merely a difference in the length measured by different observers. For example, to observers on Earth a high-speed rocket ship would appear to be contracted along its direction of motion, but to an observer on the ship it is the passing Earth that appears to be contracted.[8]

As we see there are a number of opinions about length contraction and clearly not all seem to agree. They may agree on the formula, but not on the interpretation of it.

4 Back to the FitzGerald, Lorentz, and Larmor length contraction

The length contraction given by FitzGerald, Lorentz, and Larmor is

$$L_{0,1} = L_{1,1}\sqrt{1-\frac{v^2}{c^2}} \tag{6}$$

where $L_{1,1}$ is the length of the rod in the moving frame as observed from the same moving frame and $L_{0,1}$ is the length of the moving rod as observed from the ether frame. It is now of great importance to be aware that v here is the velocity of the rod against the ether as measured from the ether. Lorentz (1892) is clear that this is the speed against the ether. Larmor does not point out this directly, but it is obvious from reading his whole theory that this is intended to be the speed against the ether. This means that the length contraction formula above is not the same as the Einstein length contraction formula. Furthermore, the formula above only holds for the special case of a moving frame against the ether frame. Assume that we have two frames moving against the ether; the length contraction for each frame would be

$$L_{0,1} = L_{1,1}\sqrt{1-\frac{v_1^2}{c^2}} \tag{7}$$

And a rod at rest in the void frame $L_{0,0}$ will, from the frame moving at speed v_1 relative to the void frame, appear to be expanded relative to an identical rod at rest in frame one. That is to say,

$$L_{0,1} = \frac{L_{0,0}}{\sqrt{1-\frac{v_1^2}{c^2}}} \tag{8}$$

The subscript in $L_{0,1}$ indicates that the rod is at rest in frame zero (the ether or void frame) and is observed from frame one (the frame moving relative to the ether). The length of the rod at rest in frame two as observed from the void frame must be

$$L_{2,0} = L_{2,2}\sqrt{1-\frac{v_2^2}{c^2}} \tag{9}$$

and a rod at rest in the void frame $L_{0,0}$ will, from the frame moving at speed v_2 relative to the void frame, appear to be expanded. That is to say,

[5] See Rindler (2001) page 62.
[6] Freedman and Young (2016) page 1229.
[7] Harris (2008) page 11.
[8] Krane (2012) page 35.

$$L_{0,2} = \frac{L_{0,0}}{\sqrt{1-\frac{v_2^2}{c^2}}} \tag{10}$$

This also means that if we have a rod at rest in frame two, it must appear to have the following length as measured from frame one:

$$L_{2,1} = L_{2,2}\frac{\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} \tag{11}$$

A rod at rest in frame one, as observed from frame two, must be observed as having a length of

$$L_{1,2} = L_{1,1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}} \tag{12}$$

This is clearly different than the Einstein length contraction. The length contraction formula above is when using clocks with no clock synchronization error. In the ether frame, Larmor (and likely Lorentz and FitzGerald) assumed that the one-way speed of light is isotropic and the same as the round trip speed of light. In this frame we can actually synchronize clocks based on the Einstein and Poincaré procedure without introducing a clock synchronization error.

Further, we must have

$$L_{2,1} = L_{1,2}\frac{\left(1-\frac{v_2^2}{c^2}\right)}{\left(1-\frac{v_1^2}{c^2}\right)} \tag{13}$$

and

$$L_{1,2} = L_{2,1}\frac{\left(1-\frac{v_1^2}{c^2}\right)}{\left(1-\frac{v_2^2}{c^2}\right)} \tag{14}$$

In equation 13 and 14 we need to measure the length of a rod from a frame that itself is moving against the ether. In this case, using the Einstein and Poincaré method of synchronizing clocks will give a synchronization error. We will see that formula 13 and 14 actually are fully consistent with the Einstein length contraction formula. The difference is that my generalization of the Larmor length contraction relies on clocks without any synchronization error, while the Einstein length contraction contains an embedded clock synchronization error.

5 The One-Way Speed of Light

In general, we need to have two synchronized clocks to measure length contraction. To synchronize the two clocks we need to know the one-way speed of light. We will soon see that even if the true one-way speed of light is anisotropic and we derive the length contraction from this standpoint, we will get the same result as Einstein when using Einstein-Poincaré synchronized clocks.

If we have a static ether and light is moving through the ether, then the one-way speed of light relative to the ether must be isotropic, and we can call this speed c. Relative to a moving frame, the one-way speed of light as observed from the ether frame when the signal is sent in the same direction as the laboratory moves relative to the ether must be

$$c - v \tag{15}$$

The above one-way speeds of light are for a signal sent in the parallel direction to the moving frame. Larmor (1900) was the first to show that the one-way speed for light relative to a moving frame as observed from the ether frame must be

$$c_{ab} = \sqrt{c^2 - v^2\sin(\theta)^2} - v\cos(\theta) \tag{16}$$

where θ is the angle of the light signal relative to the direction of the moving frame relative to the ether. Prokhovnik (1967) gives the same result 67 years later without referring to Larmor, still Prokhovnik moves forward considerably in understanding ether theories and special relativity theory as well. Still, Prokhovnik (1967) theory is incomplete, but this is understandable, as a full understanding of a topic often takes many steps by many researchers over a long period of time. We will here concentrate on the length contraction in the parallel direction.

The one-way speed of light in ether theories is c and isotropic against the ether, and $c \pm v$ relative to a moving frame as observed from the ether frame. When the one-way speed of light is observed from the moving frame itself it is[9].

$$\hat{c}_{ab} = \frac{c - v}{1 - \frac{v^2}{c^2}} \tag{17}$$

and in the opposite direction we have

$$\hat{c}_{ba} = \frac{c + v}{1 - \frac{v^2}{c^2}} \tag{18}$$

What we will answer in this paper is what type of length contraction will we get if the ether does exist and the true one-way speed of light is anisotropic. This is an interesting question even if one assumes that one cannot detect the anisotropy in the one-way speed of light. In fact, we claim the anisotropy in the one-way speed of light already likely has been detected by several independent experiments, Marinov (1974), Torr and Kolen (1984), Cahill (2006b), Cahill (2006a), Cahill (2012) but this has been ignored and even ridiculed by main frame physicists.[10]. These experiments should be repeated and studied further instead of being ignored.

6 Clock Synchronization Error

If the one-way speed of light truly is isotropic against the ether as measured from the ether frame suggested by Joseph Larmor, then if one synchronizes clocks based on the Einstein-Poincaré method by assuming that the one-way speed of light is always isotropic, this will lead to a synchronization error in every moving frame except in the ether frame. Not only that, but the Einstein synchronization error will be different in each frame, something that will make it appear as though the one-way speed of light is isotropic in every frame, if measured from the same frame that the one-way light speed is measured against. This will lead to such things as apparent reciprocality for length contraction as long as we are measuring with Einstein-Poincaré synchronized clocks. Whether or not this error can be detected remains a partially open question, and the term "error" is possibly too strong at this point as if it is an 'error" or not depends on repetition on some experiments before we finally concluds.

Here we will be looking at the Einstein clock synchronization error for light signals sent parallel to the direction of the moving frame. If we use Einstein synchronization, we also assume that the one-way speed of light is isotropic in any frame. This means that we assume that the time it takes for light to travel from A to B or from B to A is the average of the one-way time of light. This is what Einstein assumed and what today's physicists still naively assume represents reality. The average of the one-way time of light in any frame as measured from that same frame the experiment is set up in is derived as

$$\hat{\tau} = \frac{1}{2}\left(\frac{L}{\hat{c}_{ab}} + \frac{L}{\hat{c}_{ba}}\right), \tag{19}$$

where $\hat{\tau}$ is the average one-way time of light that will only be identical to the true one-way time of light if the one-way speed of light is isotropic. Further, the true one-way time of light in the A to B direction must be

$$\hat{t}_{ab} = \frac{L}{\hat{c}_{ab}}, \tag{20}$$

and in the B to A direction it must be

$$\hat{t}_{ba} = \frac{L}{\hat{c}_{ba}}. \tag{21}$$

The Einstein synchronization error is the difference between the assumed one-way time we arrive at based on the incorrect Einstein assumption that the one-way speed of light is always isotropic and the true one-way time of light based on the true one-way speed of light. The Einstein synchronization error (or we could call it the Einstein time error) as observed from the moving frame for a light signal traveling

[9] See Haug (2014) for full derivation.

[10] Several other independent methods give indirectly the same speed of the solar system against the "ether", see Monstein and Wesley (1996) for an overview. It seems unlikely to be a coincidence that one-way speed of light experiments gives a speed close to this.

from A to B must be the real time it takes for the light to travel that distance minus the time that Einstein assumed the light would take; that is[11]:

$$
\begin{aligned}
\hat{\Delta}_{ab} &= \hat{t}_{ab} - \hat{\tau} \\
\hat{\Delta}_{ab} &= \frac{L}{\hat{c}_{ab}} - \frac{1}{2}\left(\frac{L}{\hat{c}_{ab}} + \frac{L}{\hat{c}_{ba}}\right) \\
\hat{\Delta}_{ab} &= \frac{1}{2}\left(\frac{L}{\hat{c}_{ab}} - \frac{L}{\hat{c}_{ba}}\right) \\
\hat{\Delta}_{ab} &= \frac{1}{2}\left(\frac{L}{\frac{c-v}{1-\frac{v^2}{c^2}}} - \frac{L}{\frac{c+v}{1-\frac{v^2}{c^2}}}\right) \\
\hat{\Delta}_{ab} &= \frac{1}{2}\left(\frac{L\left(1-\frac{v^2}{c^2}\right)}{c-v} - \frac{L\left(1-\frac{v^2}{c^2}\right)}{c+v}\right) \\
\hat{\Delta}_{ab} &= \frac{1}{2}\left(\frac{L\left(1-\frac{v^2}{c^2}\right)(c+v) - L\left(1-\frac{v^2}{c^2}\right)(c-v)}{(c-v)(c+v)}\right) \\
\hat{\Delta}_{ab} &= \frac{1}{2}\left(\frac{2Lv\left(1-\frac{v^2}{c^2}\right)}{c^2-v^2}\right) \\
\hat{\Delta}_{ab} &= \frac{Lv\left(1-\frac{v^2}{c^2}\right)}{c^2\left(1-\frac{v^2}{c^2}\right)} \\
\hat{\Delta}_{ab} &= \frac{Lv}{c^2}.
\end{aligned}
\tag{22}
$$

The assumed Einstein time is more than assumed; this is the one-way time we will actually observe when using Einstein-Poincaré synchronized clocks (also according to atomism). The point is that such clocks are synchronized based on the likely flawed assumption that the true one-way speed of light is isotropic, which it is not, or alternatively, that the true one-way speed of light can never be detected, as likely incorrectly assumed by Poincaré. The Einstein synchronization error for light that travels in the opposite direction of the direction of the moving frame must be

$$
\begin{aligned}
\hat{\Delta}_{ba} &= \hat{t}_{ba} - \hat{\tau} \\
\hat{\Delta}_{ba} &= \frac{L}{\hat{c}_{ba}} - \frac{1}{2}\left(\frac{L}{\hat{c}_{ab}} + \frac{L}{\hat{c}_{ba}}\right) \\
\hat{\Delta}_{ba} &= \frac{1}{2}\left(\frac{L}{\hat{c}_{ba}} - \frac{L}{\hat{c}_{ab}}\right) \\
\hat{\Delta}_{ba} &= \frac{1}{2}\left(\frac{L}{\frac{c+v}{1-\frac{v^2}{c^2}}} - \frac{L}{\frac{c-v}{1-\frac{v^2}{c^2}}}\right) \\
\hat{\Delta}_{ba} &= \frac{1}{2}\left(\frac{L\left(1-\frac{v^2}{c^2}\right)}{c+v} - \frac{L\left(1-\frac{v^2}{c^2}\right)}{c-v}\right) \\
\hat{\Delta}_{ba} &= \frac{1}{2}\left(\frac{L\left(1-\frac{v^2}{c^2}\right)(c-v) - L\left(1-\frac{v^2}{c^2}\right)(c+v)}{(c+v)(c-v)}\right) \\
\hat{\Delta}_{ba} &= \frac{1}{2}\left(\frac{-2Lv\left(1-\frac{v^2}{c^2}\right)}{c^2-v^2}\right) \\
\hat{\Delta}_{ba} &= \frac{-Lv\left(1-\frac{v^2}{c^2}\right)}{c^2\left(1-\frac{v^2}{c^2}\right)}
\end{aligned}
$$

[11] See also Lévy (2003) and Haug (2014).

$$\hat{\Delta}_{ba} = -\frac{Lv}{c^2}. \tag{23}$$

The synchronization error from B to A is exactly the opposite of the synchronization error from A to B. This is rather obvious if one thinks carefully about it. This symmetry, or more precisely, this perfect asymmetry (mirror symmetry) is extremely important. It means that when we measure the round-trip time of light, the Einstein synchronization errors in each direction will always perfectly cancel each other out to be zero,

$$\hat{\Delta}_{ab} + \hat{\Delta}_{ba} = \frac{Lv}{c^2} + \left(-\frac{Lv}{c^2}\right) = \frac{Lv}{c^2} - \frac{Lv}{c^2} = 0. \tag{24}$$

As long as we set up experiments that directly or indirectly measure the round-trip time of light, we will not be able to detect a synchronization error in most experimental set-ups, even if it is there.

7 Back to the Einstein Length Contraction

If we are using Einstein synchronized clocks, then we will actually also observe an apparent length contraction that differs from the true length contraction. Assume two frames moving against the ether. Frame one is the Earth moving at a velocity of v_1 against the ether as observed from the ether frame. Frame two is a train moving at a velocity of v_2 against the ether as observed from the ether frame. On the ground of Earth we have a rod with two lasers (placed perpendicular to the length of the rod), one on each end of the rod. The distance between the two lasers can be simply measured via a ruler in the same frame as the ruler.

Next we place this rod on board a train at rest relative to the Earth's ground. Then we accelerate the train up to a speed v_e relative the Earth as measured with Einstein-Poincaré synchronized clocks, and a speed of v_2 against the void as measured from the void. The speeds v_1 and v_2 have not been observed, as we assume we do not know how to detect and measure anisotropy in the one-way speed of light. And without finding the true one-way speed of light, we "must" synchronize clocks with Einstein-Poincaré synchronization or through slow clock transportation that leads to the same synchronization error, see Lévy (2003), Haug (2014). The easily observed velocity of the train against the ground using two Einstein synchronized clocks is linked to the Earth's velocity and the train's velocity against the ether in the following way (see Haug (2014) for the full derivation).

$$v_e = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}. \tag{25}$$

Bear in mind that this is an apparent velocity containing an Einstein speed error due to an embedded Einstein synchronization error. To be clear this is the velocity one will measure with Einstein-Poincaré synchronized clocks in real experiments, but the velocity is measure with clocks that are synchronized based on the assumption the one-way speed of light is isotropic. That is to say, it is based on the assumption that one cannot detect motion against the void. Bear in mind that the famous Michelson–Morley experiment is just a way to check that the round-trip speed of light is isotropic, after one takes into account length contraction and time dilation; it is not a test for the one-way speed of light.

To find the length of the rod in the train from the ground, we only need one clock with a laser receiver. This single clock connected to a laser receiver finds the time interval between the first and second laser beam moving over the detector. Let's call this time interval $\hat{t}_1$. This time interval must be

$$\begin{aligned}
\hat{t}_1 &= \frac{L_{2,1}}{\frac{v_2 - v_1}{1 - \frac{v_2^2}{c^2}}} \\
\hat{t}_1 &= \frac{L_{2,2}\frac{\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}}}{\frac{v_2 - v_1}{1 - \frac{v_1^2}{c^2}}} \\
\hat{t}_1 &= \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}}{\frac{v_2 - v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}} \\
\hat{t}_1 &= \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{v_2 - v_1}.
\end{aligned} \tag{26}$$

In the derivation above bear in mind that $L_{1,1} = L_{2,2}$ and that the time measurement is only done with one clock. So, this time measurement does not rely on any clock synchronization. However, to find the rod's length as observed from the other frame we also need to know the rod's velocity relative to the ground where we try to measure the length contraction. The velocity of the rod is equal to the velocity of the train. And to measure the velocity of the train, we have to use two Einstein-Poincaré synchronized clocks (assuming we did not know the true one-way speed of light). The length of the rod when at rest in frame two (the train) as measured by frame one (from the ground of the Earth) using Einstein-Poincaré synchronized clocks is therefore given by

$$\begin{aligned}
L_{e,2,1} &= \hat{t}_1 v_e \\
L_{e,2,1} &= \hat{t}_1 \frac{(v_2 - v_1)}{\left(1 - \frac{v_1 v_2}{c^2}\right)} \\
L_{e,2,1} &= \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{(v_2 - v_1)} \frac{(v_2 - v_1)}{\left(1 - \frac{v_1 v_2}{c^2}\right)} \\
L_{e,2,1} &= \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{\left(1 - \frac{v_1 v_2}{c^2}\right)}. \qquad (27)
\end{aligned}$$

The rod's length at rest in frame one as measured from frame two with Einstein-Poincaré synchronized clocks is

$$\begin{aligned}
L_{e,1,2} &= \hat{t}_2 v_e \\
L_{e,1,2} &= \hat{t}_2 \frac{(v_2 - v_1)}{\left(1 - \frac{v_1 v_2}{c^2}\right)} \\
L_{e,1,2} &= \frac{L_{2,2}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{(v_2 - v_1)} \frac{(v_2 - v_1)}{\left(1 - \frac{v_1 v_2}{c^2}\right)} \\
L_{e,1,2} &= \frac{L_{2,2}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{\left(1 - \frac{v_1 v_2}{c^2}\right)}. \qquad (28)
\end{aligned}$$

A rod as measured from the same reference frame in which it is at rest is always the same no matter what frame it is placed in. This is again because measuring sticks shrink/expand in the same way as the rod. This means, $L_{1,1} = L_{2,2}$, which also means

$$L_{e,1,2} = L_{e,2,1}. \qquad (29)$$

This is a rather important result as it means length contraction when using Einstein-Poincaré synchronized clocks will appear to be reciprocal. Each frame can claim it is the rods in the other frames that are contracted. This is, however, apparent length contraction and not true length contraction. Alternatively, if we had measured the train's speed via clocks without any clock synchronization error, we would have found the train (rod) speed to be

$$\hat{v}_{2,1} = \frac{v_2 - v_1}{1 - \frac{v_1^2}{c^2}}. \qquad (30)$$

The rod's true length contraction as measured without any clock synchronization error is given by

$$\begin{aligned}
L_{2,1} &= \hat{t}\hat{v}_{2,1} \\
L_{2,1} &= \hat{t}\frac{(v_2 - v_1)}{\left(1 - \frac{v_1^2}{c^2}\right)} \\
L_{2,1} &= \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{(v_2 - v_1)} \frac{(v_2 - v_1)}{\left(1 - \frac{v_1^2}{c^2}\right)}
\end{aligned}$$

$$
\begin{aligned}
L_{2,1} &= L_{1,1}\frac{\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} \\
L_{2,1} &= L_{2,2}\frac{\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}}. \qquad (31)
\end{aligned}
$$

Einstein's special relativity theory length contraction equation is given by

$$L_{2,1} = L_{2,2}\sqrt{1-\frac{v_e^2}{c^2}}.$$

When using Einstein synchronized clocks the relative speed is related to the absolute speeds (the speeds against the ether) in the following way

$$v_e = \frac{v_2 - v_1}{1-\frac{v_1 v_2}{c^2}}$$

Based on this we can rewrite the Einstein length contraction to

$$
\begin{aligned}
L_{e,2,1} &= L_{2,2}\sqrt{1-\frac{v_e^2}{c^2}} \\
L_{e,2,1} &= L_{2,2}\sqrt{1-\frac{\left(\frac{v_2-v_1}{1-\frac{v_1v_2}{c^2}}\right)^2}{c^2}} \\
L_{e,2,1} &= L_{2,2}\sqrt{1-\frac{(v_2-v_1)^2}{c^2\left(1-\frac{v_1v_2}{c^2}\right)^2}} \\
L_{e,2,1} &= L_{2,2}\frac{1}{\left(1-\frac{v_1v_2}{c^2}\right)}\sqrt{\left(1-\frac{v_1v_2}{c^2}\right)^2-\frac{(v_2-v_1)^2}{c^2}} \\
L_{e,2,1} &= L_{2,2}\frac{\sqrt{1-2\frac{v_1v_2}{c^2}+\frac{v_1^2v_2^2}{c^4}-\frac{v_2^2-2v_1v_2+v_1^2}{c^2}}}{\left(1-\frac{v_1v_2}{c^2}\right)} \\
L_{e,2,1} &= L_{2,2}\frac{\sqrt{1+\frac{v_1^2v_2^2}{c^4}-\frac{v_2^2}{c^2}-\frac{v_1^2}{c^2}}}{\left(1-\frac{v_1v_2}{c^2}\right)} \\
L_{e,2,1} &= \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{\left(1-\frac{v_1v_2}{c^2}\right)} \qquad (32) \\
L_{e,2,1} &= \frac{L_{2,2}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{\left(1-\frac{v_1v_2}{c^2}\right)}. \qquad (33)
\end{aligned}
$$

In other words, the Einstein length contraction equation is identical to apparent length contraction under the assumption of the existence of the ether (preferred void frame in atomism), equation 27, when using Einstein-Poincaré synchronized clocks. This of course also means that the Einstein length contraction equation, as well as observations with Einstein synchronized clocks, yields an apparent length contraction and not the true length contraction. The true length contraction is actually different than predicted by Einstein and as observed by Einstein synchronized clocks. Under ether theories, Einstein's length contraction equation is also consistent with Einstein-Poincaré synchronized clocks, but Einstein-Poincaré synchronized clocks contain an Einstein synchronization error that affects the measurement of length contraction. I call it an Einstein synchronization error rather than an Einstein-Poincaré synchronization error, as Poincaré likely would have claimed that the true one-way speed of light was anisotropic even if he claimed it not could be measured.

Furthermore, the Einstein length contraction is not identical to the length contraction initially suggested by FitzGerald and later by Lorentz and Larmor. The length contraction they introduced was, at least initially, the true length contraction related to motion against the ether (void). In Einstein's special relativity theory, the length contraction is due to relative velocities only and more importantly it contains an embedded Einstein synchronization error. However, FitzGerald (1889), Lorentz (1892), and Larmor (1900) are not very clear when they talk about the speed against the ether; they do not explain how this velocity should be measured exactly. Lorentz formalized length contraction for objects moving against the ether in 1892, where it is clear that the velocity in his length contraction formula was the velocity of the moving frame (in his examples Earth) against the ether (void). In his later work, Lorentz was heavily influenced by Poincaré, and it is no longer clear whether the velocity in Lorentz equations is just relative velocity as measured with Einstein-Poincaré synchronized clocks or actually the velocity against the ether.

7.1 Reciprocal Length Contraction?

Einstein and his adherents believe that length contraction is reciprocal, where each frame can claim it is the other frame that is moving and that a rod in the other frame will appear contracted relative to the frame we are observing it from. For example, assume we are making two identical rods. Next, one rod is placed on board a train, and the train then starts to move against the Earth's ground. Observers on the train and observers on the ground will now, according to Einstein's special relativity theory, be able to claim that it is the rod in the other frame that contracts. This seems impossible and against logic; the rod cannot shrink in both frames at the same time, can it? This is known as the length contraction paradox. Can length contraction really be reciprocal?

The correct answer is that the apparent length contraction we obtain when measured with Einstein-Poincaré synchronized clocks indeed is reciprocal. It is real in the sense that this is what we will measure with such clock synchronization, but again there is a deeper and more real reality behind this when removing the clock synchronization error. The true length contraction as measured with clocks without synchronization error is not reciprocal between frames. Assume one frame is moving at velocity v_1 and one frame is moving at velocity v_2 against the ether (void). Two identical rods are made in frame one, and then one of the rods is moved to frame two. The length of the rod at rest in frame one as measured from frame one is $L_{1,1}$, and the length of the rod at rest in frame two as measured from frame two is $L_{2,2}$. Furthermore, we must have $L_{1,1} = L_{2,2}$. The length of the rod in frame one as measured from frame two when using Einstein synchronized clocks must be

$$L_{e,1,2} = \frac{L_{1,1}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{1-\frac{v_1 v_2}{c^2}}, \tag{34}$$

and the length of the rod in frame two as measured from frame one when using Einstein synchronized clocks must be

$$L_{e,2,1} = \frac{L_{2,2}\sqrt{1-\frac{v_2^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}}}{1-\frac{v_1 v_2}{c^2}}. \tag{35}$$

This again means

$$L_{e,1,2} = L_{e,2,1}. \tag{36}$$

These are not the true rod lengths, but apparent length indirectly containing an Einstein synchronization error. Even if only one clock is used to measure the rod's length, we need to know the velocity of the rod relative to the clock. When measuring the velocity of the rod relative to the clock when using Einstein synchronized clocks, we will indirectly build an Einstein synchronization error into our length measurement.

The solution to the Einstein length contraction paradox is that not all the rods in the different frames are contracting; one of them is actually expanding relative to the other frame, but when using Einstein synchronized clocks we only observe apparent length contraction that indeed leads to reciprocal length contraction. The different sizes of the Einstein synchronization error in each frame makes it appear that the length contraction in the opposite frame is the same no matter if we observe the rod in frame two from frame one or the rod in frame one from frame two. Length contraction is indeed reciprocal when using Einstein synchronized clocks, but this is an apparent length contraction.

8 Conclusion

We have shown that Einstein length contraction not is the same as FitzGerald and Lorentz length contraction. The FitzGerald and Lorentz length contraction was developed for a moving frame and the ether frame. Here we have extended the FitzGerald and Lorentz length contraction to hold between any two reference frames. The equations are then clearly different than that of Einstein. Still, when deriving length contraction from Einstein-Poincaré synchronized clocks then even if the true one-way speed of light is anisotropic we get exactly the same formula as Einstein. However, we can see that the reciprocality is a kind of illusion due to clock synchronization "error.

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