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TOPSIS APPROACH TO CHANCE CONSTRAINED MULTI - OBJECTIVE MULTI-LEVEL QUADRATIC PROGRAMMING PROBLEM

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ABSTRACT
This paper presents TOPSIS approach to solve chance constrained multi – objective multi – level quadratic programming problem. The proposed approach actually combines TOPSIS and fuzzy goal programming. In the TOPSIS approach, most appropriate alternative is to be finding out among all possible alternatives based on both the shortest distance from positive ideal solution (PIS) and furthest distance from the negative ideal solution (NIS). PIS and NIS for all objective functions of each level have been determined in the solution process. Distance functions which measure distances from PIS and NIS have been formulated for each level. The membership functions of the distance functions have been constructed and linearized in order to approximate nonlinear membership functions into equivalent linear membership functions. Stanojevic's normalization technique for normalization has been employed in the proposed approach. For avoiding decision deadlock, each level decision maker provides relaxation on the upper and lower bounds of the decision variables. Two FGP models have been developed in the proposed approach. Euclidean distance function has been utilized to identify the optimal compromise solution. An illustrative example has been solved to demonstrate the proposed approach.

INTRODUCTION
Multilevel programming (MLP) is very useful technique to solve hierarchical decision making problem with multiple decision makers (DMs) in a hierarchical system. In MLP, each level DM independently controls some variables and tries to optimize his own objectives. For the successful running of a multilevel system, DMs try to find out a way/solution so that each level DM is satisfied at reasonable level. There are many approaches to solve MLP problem (MLPP) in the literature. Ananndalingam [1] proposed mathematical programming model of decentralized multi-level systems in crisp environment.

Lai [2] at first developed an effective fuzzy approach by using the concept of tolerance membership functions of fuzzy set theory [3] for solving MLPPs in 1996. Shih et al. [4] extended Lai's concept by employing non-compensatory max-min aggregation operator for solving MLPPs. Shih and Lee [5] further extended Lai's concept by introducing the compensatory fuzzy operator for obtaining satisfactory solution for MLPP. Sakawa et al. [6] developed interactive fuzzy programming to solve MLPP. Sinha [7, 8] further established an alternative fuzzy mathematical programming for MLPP. Pramanik and Roy [9] established fuzzy goal programming (FGP) approach to solve MLPP and presented sensitive analysis on relaxation provided by the upper level decision maker.. Linear plus linear fractional multilevel multi objective programming problem had been studied by Pramanik et al [10]. Han et al. [11] presented a case study on production inventory planning using reference based uncooperative multi follower tri level decision problem based on K – th best algorithm.

Pramanik [12] developed bilevel programming problem (BLPP) with fuzzy parameters using FGP. Pramanik and Dey [13] extended Pramanik's concept [12] to multi objective BLPP with fuzzy parameters. Pramanik et al. [14] further discussed decentralized multi objective BLPP with fuzzy parameters. In 2015, Pramanik [15] studied MLPP with fuzzy parameters by extending Pramanik's concept [12] and developed three FGP models. Sakawa et al. [16] established interactive fuzzy programming for MLPP with fuzzy parameters.

Pramanik and Banerjee [17] developed chance constrained BLPP with quadratic objective functions. In their study [17] they converted chance constraints into deterministic constraints and solved the problem using FGP models. Pramanik et al. [18] studied linear plus linear fractional chance constrained BLPP. Pramanik et al. [19] also studied multilevel linear programming problem with chance constraints based on FGP.

TOPSIS stands for technique for order preference by similarity to ideal solution. In TOPSIS approach, most appropriate alternative is to be selected among all alternatives based on the shortest distance from positive ideal solution (PIS) and furthest distance from negative ideal solution (NIS). TOPSIS approach reduces multiple numbers of conflicting objectives to two objectives; one is the minimum of distance function which measures distance from PIS and another is the maximum of distance function which measures distance from NIS. Hwang and Yoon [20] introduced the TOPSIS approach to solve multi – attribute decision making problem. Lai et al. [21] established TOPSIS method for solving multi – objective decision making (MODM) problem. Chen [22] developed TOPSIS to solve multi criteria decision making. Jahanshahloo et al. [23] extended TOPSIS for decision making problem with fuzzy data. They used triangular fuzzy numbers for rating of each alternatives and weight of each criterion using α - cut method for normalization.

In neutrosophic environment Biswas et al. [24] proposed TOPSIS method for multi attribute decision making. In the evaluation process, Biswas et al. [24] employed linguistic variables to present the ratings of each alternative with respect to each attribute characterized by single-valued neutrosophic number. Biswas et al. [24] employed neutrosophic aggregation operator to aggregate all the opinions of decision makers. Dey et al. [25] studied generalized neutrosophic soft multi attribute group decision making based on TOPSIS. Pramanik et al. [26] established TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. Dey et al. [27] studied TOPSIS for solving multi-attribute decision making problem under bi-polar neutrosophic environment.

Wang and Lee [28] developed fuzzy TOPSIS approach based on subjective weights and objective weights. Subjective weights are normalized into comparable scale. They adopted Shannon's entropy [29] theory and defined closeness coefficient. Baky and Abo-Sinna [30] studied non-linear MODM problem using TOPSIS approach. Baky [31] developed interactive TOPSIS algorithms for solving multi – level non – linear multi – objective decision making problem. Dey et al. [32] studied TOPSIS approach to linear fractional MODM problem based on FGP. In their study they compared obtained results with Baky and Abo-Sinna's [33] method and obtained better satisfactory results in terms of distance function.

In the present paper, we have presented TOPSIS approach to solve chance constrained multi-level multi objective quadratic programming problem. In the proposed approach, firstly, we have transformed chance constraints into equivalent deterministic constraints using known means and variances and confidence levels. PIS and NIS have been calculated for each objective function of each level. We have employed first order Taylor series for each nonlinear membership function to convert them into linear membership function. Then we have normalized them using Stanojevic's normalization technique [34]. The above process has been done for each level and using FGP model to find optimal solution for each level separately. Each level DM provides his choice on the upper and lower bounds of the decision variables under his control. Two FGP models have been developed to get the optimal solution. Euclidean distance function has been used to find out the most appropriate optimal solution. The proposed TOPSIS method has been illustrated by solving a CCMLMOQPP.

The rest of the paper is designed as follow: In the section 2, CCMLMOQPP has been formulated. In Section 3 we have presented the conversion of chance constraints into equivalent deterministic constraints. In the Section 4, TOPSIS approach with normalization method for all levels have been described. Selection of preference bounds has been presented in the section 5. Two FGP models have been formulated in the next section. Section 7 presents Zeleny's distance function [35] which helps to select optimal compromise solution. A numerical example of CCMLMOQPP has been presented in the section 8. In Section 9, we have presented conclusion and future scope of research.

PROBLEM FORMULATION

Consider the following CCMLMOQPP.

$$\underset{x_1}{\text{Max}}\ Z_1(\bar{x}) = \underset{x_1}{\text{Max}}\ Z_1\ (x_1, x_2, ..., x_p) = \underset{x_1}{\text{Max}}\ (Z_{11}(\bar{x}), Z_{12}(\bar{x}), ..., Z_{1m_1}(\bar{x}))\ [\text{FLDM}] \quad (1)$$

$$\underset{x_2}{\text{Max}}\ Z_2(\bar{x}) = \underset{x_2}{\text{Max}}\ Z_2\ (x_1, x_2, ..., x_p) = \underset{x_2}{\text{Max}}\ (Z_{21}(\bar{x}), Z_{22}(\bar{x}), ..., Z_{2m_2}(\bar{x}))\ [\text{SLDM}] \quad (2)$$

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$$\underset{x_p}{\text{Max}}\ Z_p(\bar{x}) = \underset{x_p}{\text{Max}}\ Z_p\ (x_1, x_2, ..., x_p) = \underset{x_p}{\text{Max}}\ (Z_{p1}(\bar{x}), Z_{p2}(\bar{x}), ..., Z_{pm_p}(\bar{x}))\ [\text{LLDM}] \quad (3)$$

Subject to

$$\bar{x} \in S = (\bar{x} = (x_1, x_2, ..., x_p) \in R^n : \Pr(\bar{A}\bar{x} \overset{\leq}{\underset{\geq}{}} \bar{B}) \geq \bar{I} - \bar{\beta} \text{ and } \bar{x} \geq 0). \quad (4)$$

Where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots & a_{1p} \\ a_{21} & a_{22} & a_{23} \dots & a_{2p} \\ \dots & \dots & \dots & \dots \\ a_{s1} & a_{s2} & a_{s3} \dots & a_{sp} \end{bmatrix}_{s \times p}$

$B = (b_1, b_2, ..., b_s)_{s\times 1}$

$\bar{I}, \bar{\beta}$ are vectors of order $s \times 1$ and $\bar{I}$ is a vector having s numbers of 1, $x_i = (x_{i1}, x_{i2}, ..., x_{in_i})$, $\bar{\beta} = (\beta_1, \beta_2, ..., \beta_s)$, $i = 1, 2, \dots, p$; $n = n_1 + n_2 + \dots + n_p$

$$Z_{ij}(\bar{x}) = \bar{c}_{ij}\bar{x} + \frac{1}{2}\bar{x}^t \bar{d}_{ij}\bar{x},\ (i = 1, 2, ..., p;\ j = 1, 2, ..., m_i) \quad (5)$$

Where $\bar{c}_{ij}$ is a vector of order $1 \times n$ and $\bar{d}_{ij}$ is a vector of order $n \times n$.

CONVERSION OF CHANCE CONSTRAINTS INTO EQUIVALENT DETERMINISTIC CONSTRAINTS

Consider the chance constraints of the form $\Pr(\sum_{j=1}^{p} a_{ij}x_j \leq b_i) \geq 1 - \beta_i$, $\quad i = 1, 2, ..., s_1$

Using [17], the constraints can be written as follows:

$$\sum_{j=1}^{p} a_{ij}x_j \leq E(b_i) + \phi^{-1}(\beta_i)\sqrt{\text{var}(b_i)} \quad i = 1, 2, ..., s_1 \quad (6)$$

Now, consider the chance constraints of the form

$$\Pr(\sum_{j=1}^{p} a_{ij}x_j \geq b_i) \geq 1 - \beta_i, \quad i = s_1 + 1, ..., s$$

Using [17], the constraint can be written as follows:

$$\sum_{j=1}^{p} a_{ij}x_j \geq E(b_i) - \phi^{-1}(\beta_i)\sqrt{\text{var}(b_i)}, \quad i = s_1 + 1, ..., s \quad (7)$$

$$\bar{x} > 0 \quad (8)$$

where $E(b_i)$ and $\text{var}(b_i)$ are the expectation and variance of the random variable b_i and $\phi(.), \phi^{-1}(.)$ represent the distribution and inverse distribution functions of standard normal variable respectively. We denote the equivalent deterministic system constraints (6), (7) and (8) by S'. Here, S' and S are equivalent set of constraints.

TOPSIS APPROACH

TOPSIS model for the FLDM

Consider the first level problem as

$$\underset{x_1}{Max}\ Z_1(\bar{x}) = \underset{x_1}{Max}\ Z_1(x_1, x_2, ..., x_p) = \underset{x_1}{Max}\ (Z_{11}(\bar{x}), Z_{12}(\bar{x}), ..., Z_{1m_1}(\bar{x}))\ [FLDM]$$

Subject to $\bar{x} \in S'$

Let $\underset{\bar{x}\in S'}{Max} Z_{1j}(\bar{x}) = Z_{1j}^{+}$ and $\underset{\bar{x}\in S'}{Min} Z_{1j}(\bar{x}) = Z_{1j}^{-}$ (j = 1, 2, …, m$_1$) be the PIS and NIS for the FLDM. The distance functions measuring the distances from the PIS and NIS can be defined as follows:

$$d_q^{PIS(F1)}(\bar{x}) = \left[\sum_{j=1}^{m_1} \alpha_j^q \left(\frac{Z_{1j}^{+} - Z_{1j}(\bar{x})}{Z_{1j}^{+} - Z_{1j}^{-}}\right)^q\right]^{1/q} \text{ and } d_q^{NIS(F1)}(\bar{x}) = \left[\sum_{j=1}^{m_1} \alpha_j^q \left(\frac{Z_{1j}(\bar{x}) - Z_{1j}^{-}}{Z_{1j}^{+} - Z_{1j}^{-}}\right)^q\right]^{1/q} \quad (9)$$

Thus the problem becomes

$$Min\ d_q^{PIS(F1)}(\bar{x})$$

$$Max\ d_q^{NIS(F1)}(\bar{x})$$

Subject to $\bar{x} \in S'$.

Construction of membership function for $d_q^{PIS(F1)}(\bar{x}), d_q^{NIS(F1)}(\bar{x})$

Let $\underset{\bar{x}\in S'}{Min}\ d_q^{PIS(F1)}(\bar{x}) = (d_q^{PIS(F1)}(\bar{x}))^{+}$ and $\underset{\bar{x}\in S'}{Max}\ d_q^{PIS(F1)}(\bar{x}) = (d_q^{PIS(F1)}(\bar{x}))^{-}$

The membership function for $d_q^{PIS(F1)}(\bar{x})$ can be constructed as follows:

$$\mu_{d_q^{PIS(F1)}}(\bar{x}) = \left\langle \begin{array}{ll} 0, & (d_q^{PIS(F1)}(\bar{x}))^{-} \le d_q^{PIS(F1)}(\bar{x}) \\ \dfrac{(d_q^{PIS(F1)}(\bar{x}))^{-} - d_q^{PIS(F1)}(\bar{x})}{(d_q^{PIS(F1)}(\bar{x}))^{-} - (d_q^{PIS(F1)}(\bar{x}))^{+}} & (d_q^{PIS(F1)}(\bar{x}))^{+} \le d_q^{PIS(F1)}(\bar{x}) \le (d_q^{PIS(F1)}(\bar{x}))^{-} \\ 1 & (d_q^{PIS(F1)}(\bar{x}))^{+} \ge d_q^{PIS(F1)}(\bar{x}) \end{array} \right\rangle \quad (10)$$

Let $\underset{\bar{x}\in S'}{Min}\ d_q^{NIS(F1)}(\bar{x}) = (d_q^{NIS(F1)}(\bar{x}))^{-}$ and $\underset{\bar{x}\in S'}{Max}\ d_q^{NIS(F1)}(\bar{x}) = (d_q^{NIS(F1)}(\bar{x}))^{+}$

The membership function for $d_q^{NIS(F1)}(\bar{x})$ can be constructed as follows:

$$\mu_{d_q^{NIS(F1)}}(\bar{x}) = \left\langle \begin{array}{ll} 0, & d_q^{NIS(F1)}(\bar{x}) \le (d_q^{|NIS(F1)}(\bar{x}))^{-} \\ \dfrac{d_q^{NIS(F1)}(\bar{x}) - (d_q^{NIS(F1)}(\bar{x}))^{-}}{(d_q^{NIS(F1)}(\bar{x}))^{+} - (d_q^{NIS(F1)}(\bar{x}))^{-}} & (d_q^{NIS(F1)}(\bar{x}))^{-} \le d_q^{NIS(F1)}(\bar{x}) \le (d_q^{NIS(F1)}(\bar{x}))^{+} \\ 1 & (d_q^{NIS(F1)}(\bar{x}))^{+} \le d_q^{NIS(F1)}(\bar{x}) \end{array} \right\rangle \quad (11)$$

Conversion of non – linear membership function into linear membership function

Let $\underset{\bar{x}\in S'}{Max}\ \mu_{d_q^{PIS(F1)}}(\bar{x}) = \mu_{d_q^{PIS(F1)}}(\bar{x}^{PIS(F1)^*})$ and $\bar{x}^{PIS(F1)^*} = (x_1^{PIS(F1)^*}, x_2^{PIS(F1)^*}, ..., x_p^{PIS(F1)^*})$

Applying first order Taylor's series we have obtained the linear membership function

$$\mu_{d_q^{PIS(F1)}}(\bar{x}) \approx \mu_{d_q^{PIS(F1)}}(\bar{x}^{PIS(F1)^*}) + \sum_{i=1}^{p}\sum_{j=1}^{n_1}(x_{ij} - x_{ij}^{PIS(F1)^*})\left(\frac{\partial \mu_{d_q^{PIS(F1)}}(\bar{x})}{\partial x_{ij}}\right)_{\bar{x}=\bar{x}^{PIS(F1)^*}} = \hat{\mu}_{d_q^{PIS(F1)}}(\bar{x}) \quad (12)$$

Let $\underset{\bar{x}\in S'}{Max}\ \mu_{d_q^{NIS(F1)}}(\bar{x}) = \mu_{d_q^{NIS(F1)}}(\bar{x}^{NIS(F1)^*})$ and $\bar{x}^{NIS(F1)^*} = (x_1^{NIS(F1)^*}, x_2^{NIS(F1)^*}, ..., x_p^{NIS(F1)^*})$

Applying first order Taylor's series we have obtained the linear membership function

$$\mu_{d_q^{NIS(F1)}}(\bar{x}) \approx \mu_{d_q^{NIS(F1)}}(\bar{x}^{NIS(F1)^*}) + \sum_{i=1}^{p}\sum_{j=1}^{n_1}(x_{ij} - x_{ij}^{NIS(F1)^*})\left(\frac{\partial \mu_{d_q^{NIS(F1)}}(\bar{x})}{\partial x_{ij}}\right)_{\bar{x}=\bar{x}^{NIS(F1)^*}} = \hat{\mu}_{d_q^{NIS(F1)}}(\bar{x}) \quad (13)$$

Stanojevic's normalization technique

Adopting Stanojevic's normalization technique [34], the linear membership functions can be normalized as follows:

$$\bar{\mu}_{d_q^{PIS(F1)}}(\bar{x}) = \frac{\hat{\mu}_{d_q^{PIS(F1)}}(\bar{x}) - a^{PIS(F1)}}{b^{PIS(F1)} - a^{PIS(F1)}} \text{ where } a^{PIS(F1)} = \underset{\bar{x}\in S'}{Min}\, \hat{\mu}_{d_q^{PIS(F1)}}(\bar{x})\ \ b^{PIS(F1)} = \underset{\bar{x}\in S'}{Max}\, \hat{\mu}_{d_q^{PIS(F1)}}(\bar{x}) \quad (14)$$

$$\bar{\mu}_{d_q^{NIS(F1)}}(\bar{x}) = \frac{\hat{\mu}_{d_q^{NIS(F1)}}(\bar{x}) - a^{NIS(F1)}}{b^{NIS(F1)} - a^{NIS(F1)}} \text{ where } a^{NIS(F1)} = \underset{\bar{x}\in S'}{Min}\, \hat{\mu}_{d_q^{NIS(F1)}}(\bar{x})\ \ b^{NIS(F1)} = \underset{\bar{x}\in S'}{Max}\, \hat{\mu}_{d_q^{NIS(F1)}}(\bar{x}) \quad (15)$$

FGP model to obtain satisfactory solution for FLDM

Using the FGP model of Pramanik and Banerjee [17] in order to obtain the satisfactory solution for the FLDM, the FGP model appears as:

$$\underset{\bar{x}\in S'}{Min}\ \lambda_1$$

(16)

$$\bar{\mu}_{d_q^{PIS(F1)}}(\bar{x}) + d^-_{PIS(F1)} = 1,$$

$$\bar{\mu}_{d_q^{NIS(F1)}}(\bar{x}) + d^-_{NIS(F1)} = 1,$$

$$\lambda_1 \geq d^-_{PIS(F1)},$$

$$\lambda_1 \geq d^-_{NIS(F1)},$$

$$0 \leq d^-_{PIS(F1)} \leq 1,$$

$$0 \leq d^-_{NIS(F1)} \leq 1,$$

Subject to $\bar{x} \in S'$

Solving the above model, the satisfactory solution for the FLDM has been obtained as

$$\bar{x}^{F1^*} = (x_1^{F1^*}, x_2^{F1^*}, \ldots, x_p^{F1^*})$$

TOPSIS model for the SLDM

Let $\underset{\bar{x}\in S'}{Max}\, Z_{2j}(\bar{x}) = Z^+_{2j}$ and $\underset{\bar{x}\in S'}{Min}\, Z_{2j}(\bar{x}) = Z^-_{2j}$ (j = 1, 2, …, m_2) be the PIS and NIS for the SLDM. The distance functions measuring the distances from the PIS and NIS can be defined as

$$d_q^{PIS(F2)}(\bar{x}) = \left[\sum_{j=1}^{m_2} \alpha_j^q \left(\frac{Z^+_{2j} - Z_{2j}(\bar{x})}{Z^+_{2j} - Z^-_{2j}}\right)^q\right]^{1/q} \text{ and } d_q^{NIS(F2)}(\bar{x}) = \left[\sum_{j=1}^{m_2} \alpha_j^q \left(\frac{Z_{2j}(\bar{x}) - Z^-_{2j}}{Z^+_{2j} - Z^-_{2j}}\right)^q\right]^{1/q} \quad (17)$$

Thus the problem becomes

$$Min\ d_q^{PIS(F2)}(\bar{x})$$

$$Max\ d_q^{NIS(F2)}(\bar{x})$$

Subject to $\bar{x} \in S'$

Construction of membership function for $d_q^{PIS(F2)}(\bar{x}), d_q^{NIS(F2)}(\bar{x})$

Let $\underset{\bar{x}\in S'}{Min}\, d_q^{PIS(F2)}(\bar{x}) = (d_q^{PIS(F2)}(\bar{x}))^+$ and $\underset{\bar{x}\in S'}{Max}\, d_q^{PIS(F2)}(\bar{x}) = (d_q^{PIS(F2)}(\bar{x}))^-$

The membership function for $d_q^{PIS(F2)}(\bar{x})$ can be constructed as follows:

$$\mu_{d_q^{PIS(F2)}}(\overline{x}) = \begin{cases} 0, & (d_q^{PIS(F2)}(\overline{x}))^- \le d_q^{PIS(F2)}(\overline{x}) \\ \dfrac{(d_q^{PIS(F2)}(\overline{x}))^- - d_q^{PIS(F2)}(\overline{x})}{(d_q^{PIS(F2)}(\overline{x}))^- - (d_q^{PIS(F2)}(\overline{x}))^+} & (d_q^{PIS(F2)}(\overline{x}))^+ \le d_q^{PIS(F2)}(\overline{x}) \le (d_q^{PIS(F2)}(\overline{x}))^- \\ 1 & (d_q^{PIS(F2)}(\overline{x}))^+ \ge d_q^{PIS(F2)}(\overline{x}) \end{cases} \quad (18)$$

Let $\underset{\overline{x}\in S'}{Min}\, d_q^{NIS(F2)}(\overline{x}) = (d_q^{NIS(F2)}(\overline{x}))^-$ and $\underset{\overline{x}\in S'}{Max}\, d_q^{NIS(F2)}(\overline{x}) = (d_q^{NIS(F2)}(\overline{x}))^+$

The membership function for $d_q^{NIS(F2)}(\overline{x})$ can be constructed as follows:

$$\mu_{d_q^{NIS(F2)}}(\overline{x}) = \begin{cases} 0, & d_q^{NIS(F2)}(\overline{x}) \le (d_q^{|NIS(F2)}(\overline{x}))^- \\ \dfrac{d_q^{NIS(F2)}(\overline{x}) - (d_q^{NIS(F2)}(\overline{x}))^-}{(d_q^{NIS(F2)}(\overline{x}))^+ - (d_q^{NIS(F2)}(\overline{x}))^-} & (d_q^{NIS(F2)}(\overline{x}))^- \le d_q^{NIS(F2)}(\overline{x}) \le (d_q^{NIS(F2)}(\overline{x}))^+ \\ 1 & (d_q^{NIS(F2)}(\overline{x}))^+ \le d_q^{NIS(F2)}(\overline{x}) \end{cases} \quad (19)$$

Conversion of non – linear membership function into linear membership function

Let $\underset{\overline{x}\in S'}{Max}\, \mu_{d_q^{PIS(F2)}}(\overline{x}) = \mu_{d_q^{PIS(F2)}}(\overline{x}^{PIS(F2)^*})$ and $\overline{x}^{PIS(F2)^*} = (x_1^{PIS(F2)^*}, x_2^{PIS(F2)^*}, \ldots, x_p^{PIS(F2)^*})$

Applying first order Taylor's series we have obtained the linear membership function

$$\mu_{d_q^{PIS(F2)}}(\overline{x}) \approx \mu_{d_q^{PIS(F2)}}(\overline{x}^{PIS(F2)^*}) + \sum_{i=1}^{p}\sum_{j=1}^{n2}(x_{ij} - x_{ij}^{PIS(F2)^*})\left(\frac{\partial \mu_{d_q^{PIS(F2)}}(\overline{x})}{\partial x_{ij}}\right)_{\overline{x}=\overline{x}^{PIS(F2)^*}} = \hat{\mu}_{d_q^{PIS(F2)}}(\overline{x}) \quad (20)$$

Let $\underset{\overline{x}\in S'}{Max}\, \mu_{d_q^{NIS(F2)}}(\overline{x}) = \mu_{d_q^{NIS(F2)}}(\overline{x}^{NIS(F2)^*})$ and $\overline{x}^{NIS(F2)^*} = (x_1^{NIS(F2)^*}, x_2^{NIS(F2)^*}, \ldots, x_p^{NIS(F2)^*})$

Applying first order Taylor's series we have obtained the linear membership function

$$\mu_{d_q^{NIS(F2)}}(\overline{x}) \approx \mu_{d_q^{NIS(F2)}}(\overline{x}^{NIS(F2)^*}) + \sum_{i=1}^{p}\sum_{j=1}^{n2}(x_{ij} - x_{ij}^{NIS(F2)^*})\left(\frac{\partial \mu_{d_q^{NIS(F2)}}(\overline{x})}{\partial x_{ij}}\right)_{\overline{x}=\overline{x}^{NIS(F2)^*}} = \hat{\mu}_{d_q^{NIS(F2)}}(\overline{x}) \quad (21)$$

Stanojevic normalization technique

Adopting Stanojevic's normalization technique [34] the linear membership functions have been normalized as follows:

$$\overline{\mu}_{d_q^{PIS(F2)}}(\overline{x}) = \frac{\hat{\mu}_{d_q^{PIS(F2)}}(\overline{x}) - a^{PIS(F2)}}{b^{PIS(F2)} - a^{PIS(F2)}} \text{ where } a^{PIS(F2)} = \underset{\overline{x}\in S'}{Min}\, \hat{\mu}_{d_q^{PIS(F2)}}(\overline{x}),\ b^{PIS(F2)} = \underset{\overline{x}\in S'}{Max}\, \hat{\mu}_{d_q^{PIS(F2)}}(\overline{x}) \quad (22)$$

$$\overline{\mu}_{d_q^{NIS(F2)}}(\overline{x}) = \frac{\hat{\mu}_{d_q^{NIS(F2)}}(\overline{x}) - a^{NIS(F2)}}{b^{NIS(F2)} - a^{NIS(F2)}} \text{ where } a^{NIS(F2)} = \underset{\overline{x}\in S'}{Min}\, \hat{\mu}_{d_q^{NIS(F2)}}(\overline{x}),\ b^{NIS(F2)} = \underset{\overline{x}\in S'}{Max}\, \hat{\mu}_{d_q^{NIS(F2)}}(\overline{x}) \quad (23)$$

FGP model to obtain satisfactory solution for SLDM

Using the model of Pramanik and Banerjee [17] in order to obtain the satisfactory solution for the SLDM, the FGP model can be written as:

$$\underset{\overline{x}\in S'}{Min}\, \lambda_2 \quad (24)$$

$\overline{\mu}_{d_q^{PIS(F2)}}(\overline{x}) + d^-_{PIS(F2)} = 1$,

$\overline{\mu}_{d_q^{NIS(F2)}}(\overline{x}) + d^-_{NIS(F2)} = 1$,

$\lambda_2 \ge d^-_{PIS(F2)}$,

$\lambda_2 \ge d^-_{NIS(F2)}$,

$0 \le d^-_{PIS(F2)} \le 1$,

$0 \le d^{-}_{NIS(F2)} \le 1,$

Subject to $\bar{x} \in S'$.

Solving the above model, the obtained satisfactory solution for the SLDM has been denoted by $\bar{x}^{F2^*} = (x_1^{F2^*}, x_2^{F2^*}, \ldots, x_p^{F2^*})$.

TOPSIS model for the LLDM

Let $\underset{\bar{x}\in S'}{\text{Max}}\, Z_{pj}(\bar{x}) = Z^{+}_{pj}$ and $\underset{\bar{x}\in S'}{\text{Min}}\, Z_{pj}(\bar{x}) = Z^{-}_{pj}$ (j = 1, 2, …, m_p) be the PIS and NIS for the LLDM. The distance functions measuring the distances from the PIS and NIS can be defined as follows:

$$d_q^{PIS(Fp)}(\bar{x}) = \left[\sum_{j=1}^{m_p} \alpha_j^q \left(\frac{Z^{+}_{pj} - Z_{pj}(\bar{x})}{Z^{+}_{pj} - Z^{-}_{pj}}\right)^q\right]^{1/q} \text{ and } d_q^{NIS(Fp)}(\bar{x}) = \left[\sum_{j=1}^{m_p} \alpha_j^q \left(\frac{Z_{pj}(\bar{x}) - Z^{-}_{pj}}{Z^{+}_{pj} - Z^{-}_{pj}}\right)^q\right]^{1/q} \tag{25}$$

Thus the problem becomes

$\text{Min } d_q^{PIS(Fp)}(\bar{x})$

$\text{Max } d_q^{NIS(Fp)}(\bar{x})$

Subject to $\bar{x} \in S'$

Construction of membership function for $d_q^{PIS(Fp)}(\bar{x}), d_q^{NIS(Fp)}(\bar{x})$

Let $\underset{\bar{x}\in S'}{\text{Min}}\, d_q^{PIS(Fp)}(\bar{x}) = (d_q^{PIS(Fp)}(\bar{x}))^{+}$ and $\underset{\bar{x}\in S'}{\text{Max}}\, d_q^{PIS(Fp)}(\bar{x}) = (d_q^{PIS(Fp)}(\bar{x}))^{-}$

The membership function for $d_q^{PIS(Fp)}(\bar{x})$ can be constructed as follows:

$$\mu_{d_q^{PIS(Fp)}}(\bar{x}) = \begin{cases} 0, & (d_q^{PIS(Fp)}(\bar{x}))^{-} \le d_q^{PIS(Fp)}(\bar{x}) \\ \dfrac{(d_q^{PIS(Fp)}(\bar{x}))^{-} - d_q^{PIS(Fp)}(\bar{x})}{(d_q^{PIS(Fp)}(\bar{x}))^{-} - (d_q^{PIS(Fp)}(\bar{x}))^{+}} & (d_q^{PIS(Fp)}(\bar{x}))^{+} \le d_q^{PIS(Fp)}(\bar{x}) \le (d_q^{PIS(Fp)}(\bar{x}))^{-} \\ 1 & (d_q^{PIS(Fp)}(\bar{x}))^{+} \ge d_q^{PIS(Fp)}(\bar{x}) \end{cases} \tag{26}$$

Let $\underset{\bar{x}\in S'}{\text{Min}}\, d_q^{NIS(Fp)}(\bar{x}) = (d_q^{NIS(Fp)}(\bar{x}))^{-}$ and $\underset{\bar{x}\in S'}{\text{Max}}\, d_q^{NIS(Fp)}(\bar{x}) = (d_q^{NIS(Fp)}(\bar{x}))^{+}$

The membership function for $d_q^{NIS(Fp)}(\bar{x})$ can be constructed as follows:

$$\mu_{d_q^{NIS(Fp)}}(\bar{x}) = \begin{cases} 0, & d_q^{NIS(Fp)}(\bar{x}) \le (d_q^{|NIS(Fp)}(\bar{x}))^{-} \\ \dfrac{d_q^{NIS(Fp)}(\bar{x}) - (d_q^{NIS(Fp)}(\bar{x}))^{-}}{(d_q^{NIS(Fp)}(\bar{x}))^{+} - (d_q^{NIS(Fp)}(\bar{x}))^{-}} & (d_q^{NIS(Fp)}(\bar{x}))^{-} \le d_q^{NIS(Fp)}(\bar{x}) \le (d_q^{NIS(Fp)}(\bar{x}))^{+} \\ 1 & (d_q^{NIS(Fp)}(\bar{x}))^{+} \le d_q^{NIS(Fp)}(\bar{x}) \end{cases} \tag{27}$$

Conversion of non – linear membership function into linear membership function

Let $\underset{\bar{x}\in S'}{\text{Max}}\, \mu_{d_q^{PIS(Fp)}}(\bar{x}) = \mu_{d_q^{PIS(Fp)}}(\bar{x}^{PIS(Fp)^*})$ and $\bar{x}^{PIS(Fp)^*} = (x_1^{PIS(Fp)^*}, x_2^{PIS(Fp)^*}, \ldots, x_p^{PIS(Fp)^*})$

Applying first order Taylor's series we have obtained the linear membership function

$$\mu_{d_q^{PIS(Fp)}}(\bar{x}) \approx \mu_{d_q^{PIS(Fp)}}(\bar{x}^{PIS(Fp)^*}) + \sum_{i=1}^{p}\sum_{j=1}^{np}(x_{ij} - x_{ij}^{PIS(Fp)^*})\left(\frac{\partial \mu_{d_q^{PIS(Fp)}}(\bar{x})}{\partial x_{ij}}\right)_{\bar{x}=\bar{x}^{PIS(Fp)^*}} = \hat{\mu}_{d_q^{PIS(Fp)}}(\bar{x}) \tag{28}$$

Let $\underset{\bar{x}\in S'}{\text{Max}}\, \mu_{d_q^{NIS(Fp)}}(\bar{x}) = \mu_{d_q^{NIS(Fp)}}(\bar{x}^{NIS(Fp)^*})$ and $\bar{x}^{NIS(Fp)^*} = (x_1^{NIS(Fp)^*}, x_2^{NIS(Fp)^*}, \ldots, x_p^{NIS(Fp)^*})$

Applying first order Taylor's series we have obtained the linear membership function

$$\mu_{d_q^{NIS(Fp)}}(\bar{x}) \approx \mu_{d_q^{NIS(Fp)}}(\bar{x}^{NIS(Fp)^*}) + \sum_{i=1}^{p}\sum_{j=1}^{n_p}(x_{ij} - x_{ij}^{NIS(Fp)^*})\left(\frac{\partial \mu_{d_q^{NIS(Fp)}}(\bar{x})}{\partial x_{ij}}\right)_{\bar{x}=\bar{x}^{NIS(Fp)^*}} = \hat{\mu}_{d_q^{NIS(Fp)}}(\bar{x}) \qquad (29)$$

Stanojevic's normalization technique

Adopting Stanojevic's normalization technique [34] the linear membership functions can be normalized as follows:

$$\bar{\mu}_{d_q^{PIS(Fp)}}(\bar{x}) = \frac{\hat{\mu}_{d_q^{PIS(Fp)}}(\bar{x}) - a^{PIS(Fp)}}{b^{PIS(Fp)} - a^{PIS(Fp)}} \text{ where } a^{PIS(Fp)} = \underset{\bar{x}\in S'}{Min}\, \hat{\mu}_{d_q^{PIS(Fp)}}(\bar{x}),\ b^{PIS(Fp)} = \underset{\bar{x}\in S'}{Max}\, \hat{\mu}_{d_q^{PIS(Fp)}}(\bar{x}) \qquad (30)$$

$$\bar{\mu}_{d_q^{NIS(Fp)}}(\bar{x}) = \frac{\hat{\mu}_{d_q^{NIS(Fp)}}(\bar{x}) - a^{NIS(Fp)}}{b^{NIS(Fp)} - a^{NIS(Fp)}}$$

where $a^{NIS(Fp)} = \underset{\bar{x}\in S'}{Min}\, \hat{\mu}_{d_q^{NIS(Fp)}}(\bar{x})$, $b^{NIS(Fp)} = \underset{\bar{x}\in S'}{Max}\, \hat{\mu}_{d_q^{NIS(Fp)}}(\bar{x})$ (31)

FGP model to obtain satisfactory solution for LLDM

Using the model of Pramanik and Banerjee [17] in order to obtain the satisfactory solution for the LLDM, the FGP model can be written as:

$$\underset{\bar{x}\in S'}{Min}\,\lambda_p$$

(32)

$$\bar{\mu}_{d_q^{PIS(p)}}(\bar{x}) + d^{-}_{PIS(Fp)} = 1,$$

$$\bar{\mu}_{d_q^{NIS(Fp)}}(\bar{x}) + d^{-}_{NIS(Fp)} = 1,$$

$$\lambda_p \ge d^{-}_{PIS(Fp)};\ \lambda_p \ge d^{-}_{NIS(Fp)},$$

$$0 \le d^{-}_{PIS(Fp)} \le 1; 0 \le d^{-}_{NIS(Fp)} \le 1,$$

Subject to $\bar{x} \in S'$.

Solving the above model, the obtained satisfactory solution for the SLDM has been denoted by $\bar{x}^{Fp^*} = (x_1^{Fp^*}, x_2^{Fp^*}, \ldots, x_p^{Fp^*})$

SELECTION OF PREFERENCE BOUNDS

In the multi – level decision making problem, the goals of all levels are generally conflicting. To execute the decision making in the real situation, cooperation between DMs is needed. For the overall satisfaction, each level DM provides some relaxation on their decision variables. So, the i-th level provides the upper and lower bounds on the decision variable x_i. Let $t_i^{L(Fi)}$ and $t_i^{R(Fi)}$ be the lower and upper bounds on the decision variables (i = 1, 2, …, p). Then the bounds can be written as follows:

$$x_i^{Fi^*} - t_i^{L(Fi)} \le x_i \le x_i^{Fi^*} + t_i^{R(Fi)} \qquad (33)$$

Where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in_i})$, $x_i^{Fi^*} = (x_{i1}^{Fi^*}, x_{i2}^{Fi^*}, \ldots, x_{in_i}^{Fi^*})$, $t_i^{L(Fi)} = (t_{i1}^{L(Fi)}, t_{i2}^{L(Fi)}, \ldots, t_{in_i}^{L(Fi)})$, $t_i^{R(Fi)} = (t_{i1}^{R(Fi)}, t_{i2}^{R(Fi)}, \ldots, t_{in_i}^{R(Fi)})$

FGP MODELS

Using FGP model of Pramanik and Banerjee [17], the two FGP models have been presented as follows:

Model – 1

Min ν (34)

Subject to

$$\bar{\mu}_{d_q^{PIS(Fi)}}(\bar{x}) + d^{-}_{PIS(Fi)} = 1,$$

$\bar{\mu}_{d_q^{NIS(Fi)}}(\bar{x}) + d^-_{NIS(Fi)} = 1,$

$\nu \geq d^-_{PIS(Fi)},$

$\nu \geq d^-_{NIS(Fi)},$

$0 \leq d^-_{PIS(Fi)} \leq 1,$

$0 \leq d^-_{NIS(Fi)} \leq 1,$

$x_i^{Fi^*} - t_i^{L(Fi)} \leq x_i \leq x_i^{Fi^*} + t_i^{R(Fi)}, \text{ for } i = 1, 2, \ldots, p$

$\bar{x} \in S'.$

Model – 2

$$\text{Min } \eta = \sum_{i=1}^{p} (w_i^{PIS} d^-_{PIS(Fi)} + w_i^{NIS} d^-_{NIS(Fi)}) \quad (35)$$

subject to

$\bar{\mu}_{d_q^{PIS(Fi)}}(\bar{x}) + d^-_{PIS(Fi)} = 1,$

$\bar{\mu}_{d_q^{NIS(Fi)}}(\bar{x}) + d^-_{NIS(Fi)} = 1,$

$0 \leq d^-_{PIS(Fi)} \leq 1,$

$0 \leq d^-_{NIS(Fi)} \leq 1,$

$\sum_{i=1}^{p} (w_i^{PIS} + w_i^{NIS}) = 1,$

$x_i^{Fi^*} - t_i^{L(Fi)} \leq x_i \leq x_i^{Fi^*} + t_i^{R(Fi)}, \text{ for } i = 1, 2, \ldots, p$

$\bar{x} \in S'.$

SELECTION OF OPTIMAL SOLUTION

Zeleny's distance function [35] can be defined as follows:

$$L_r(\tau, K) = [\sum_{k=1}^{K} \tau_k^r (1 - \omega_k)^r]^{1/r} \quad (36)$$

Here, τ_k means attribute level and $\sum_{k=1}^{K} \tau_k = 1$ $r (0 \leq r \leq \infty)$ denotes the distance parameter and ω_k represents the degree of closeness between compromise solution and individual best solution of the k – th objective function.

In this paper, we consider r = 2, then distance function becomes $L_2(\tau, K) = [\sum_{k=1}^{K} \tau_k^2 (1 - \omega_k)^2]^{1/2}$ (37)

For the maximization problem, ω_k is the ratio of the compromise solution and individual best solution of the k – th objective function. For the minimization type, the ratio would be reversed. Minimum L_2 reflects the best optimal compromise solution.

NUMERICAL EXAMPLE

Consider the following numerical example to illustrate the proposed approach.

$\max_{x_1} (z_{11} = (x_1 + 2)(x_2 + 3) + (x_3 + 4); z_{12} = x_1^2 + x_2^2 + x_3)$[FLDM]

$\max_{x_2} (z_{21} = (x_1 + x_2 x_3; z_{22} = x_1 x_2 + x_3)$ [SLDM]

$\max_{x_3} (z_{31} = (x_2 + 7) + (x_1 + 1)(x_3 + 5); z_{32} = 2x_1^2 + 3x_2 x_3)$[LLDM]

Subject to

$\Pr(x_1 + x_2 + x_3 \leq b_1) \geq 1 - \beta_1,$

$\Pr(-2x_1 + 5x_2 + 3x_3 \le b_2) \ge 1 - \beta_2$,
$\Pr(3x_1 - 4x_2 + 2x_3 \le b_3) \ge 1 - \beta_3$,
$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$.
The means, variances and the confidence levels are given below:
$E(b_1) = 3$, $var(b_1) = 2$, $\beta_1 = 0.03$
$E(b_2) = 12$, $var(b_2) = 8$, $\beta_2 = 0.01$
$E(b_3) = 10$, $var(b_3) = 18$, $\beta_3 = 0.05$
Using (6), (7) the chance constraints involved in the proposed problem have been transformed into equivalent deterministic constraints as:
$x_1 + x_2 + x_3 \le 5.666$,
$-2x_1 + 5x_2 + 3x_3 \le 18.576$,
$3x_1 - 4x_2 + 2x_3 \le 3.021$,

First level MODM problem

$$\max_{x_1} (z_{11} = (x_1 + 2)(x_2 + 3) + (x_3 + 4); z_{12} = x_1^2 + x_2^2 + x_3)[\text{FLDM}]$$

Subject to
$x_1 + x_2 + x_3 \le 5.666$,
$-2x_1 + 5x_2 + 3x_3 \le 18.576$,
$3x_1 - 4x_2 + 2x_3 \le 3.021$,
$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$.
The individual best solutions (PISs) and the individual worst solutions (NISs) of the objective functions considered by the FLDM subject to the constraints have been respectively shown in the Table 1 and Table 2.

Table 1: The individual best solutions of FLDM in Numerical Example

Maximal value of the objective function of FLDM	z_{11}^{+}	z_{12}^{+}
$\max_{x \in S'} z_{1j}(x)$, (j = 1,2)	32.33 at (3.669, 1.997, 0)	32.103 at (5.666, 0, 0)

Table 2: The individual worst solutions of FLDM in Numerical Example

Minimal value of the objective function of FLDM	z_{11}^{-}	z_{12}^{-}
$\min_{x \in S'} z_{1j}(x)$, (j = 1,2)	11.51 at (0, 0, 1.51)	0.948 at (0.75, 0, 0.38)

Considering $\alpha_1 = \alpha_2 = 0.5$, and q = 2, we have obtained

$$d_2^{PIS(F1)}(x) = \left\{ (0.5)^2 \left[\frac{32.328 - \dfrac{(x_1+2)*(x_2+3)+(x_3+4)}{2x_1 - x_2 + 3}}{32.328 - 11.51} \right]^2 + (0.5)^2 \left[\frac{32.103 - (x_1^2 + x_2^2 + x_3)}{32.103 - 0.948} \right]^2 \right\}^{1/2},$$

$$d_2^{NIS(F1)}(x) = \left\{ (0.5)^2 \left[\frac{\dfrac{(x_1+2)*(x_2+3)+(x_3+4)}{2x_1 - x_2 + 3} - 11.51}{32.328 - 11.51} \right]^2 + (0.5)^2 \left[\frac{(x_1^2 + x_2^2 + x_3) - 0.948}{32.103 - 0.948} \right]^2 \right\}^{1/2}.$$

Now, $\left(d_2^{PIS(F1)}(x)\right)^+ = \underset{x\in S}{Min}\ d_2^{PIS(F1)}(x) = 0.109$ at (5.338, 0.328, 0); $\left(d_2^{PIS(F1)}(x)\right)^- = \underset{x\in S}{Max}\ d_2^{PIS(F1)}(x) = 0.7007$ at (0, 0, 1.51); $\left(d_2^{NIS(F1)}(x)\right)^+ = \underset{x\in S}{Max}\ d_2^{NIS(F1)}(x) = 0.599$ at (0, 0, 5.666); $\left(d_2^{NIS(F1)}(x)\right)^- = \underset{x\in S}{Min}\ d_2^{NIS(F1)}(x) = 0.0205$ at (0.459, 0, 0.822).

The membership functions of $d_2^{PIS(F1)}(x)$ and $d_2^{NIS(F1)}(x)$ can be formulated as follows:

$$\mu_{d_2^{PIS(F1)}}(x) = \begin{cases} 0, & \text{if } 0.701 \le d_2^{PIS(F1)}(x) \\ \dfrac{0.701 - d_2^{PIS(F1)}(x)}{0.701 - 0.109}, & \text{if } 0.109 \le d_2^{PIS(F1)}(x) \le 0.701, \\ 1, & \text{if } d_2^{PIS(F1)}(x) \le 0.109 \end{cases}$$

$$\mu_{d_2^{NIS(F1)}}(x) = \begin{cases} 0, & \text{if } \left(d_2^{NIS(F1)}(x)\right) \le 0.02 \\ \dfrac{d_2^{NIS(F1)}(x) - 0.02}{0.599 - 0.02}, & \text{if } 0.02 \le d_2^{NIS(F1)}(x) \le 0.599 \\ 1, & \text{if } d_2^{NIS(F1)}(x) \ge 0.599 \end{cases}$$

Applying the first order Taylor's series the non-linear membership functions $\mu_{d_2^{PIS(F1)}}(x)$ and $\mu_{d_2^{NIS(F1)}}(x)$ have been transformed into equivalent linear membership functions $\hat{\mu}_{d_2^{PIS(F1)}}(x)$ and $\hat{\mu}_{d_2^{NIS(F1)}}(x)$ respectively as follows:

$$\mu_{d_2^{PIS(F1)}}(x) \tilde{=} \mu_{d_2^{PIS(F1)}}(5.34, 0.33, 0) + (x_1 - 5.34)\left(\frac{\partial \mu_{d_2^{PIS(F1)}}(x)}{\partial x_1}\right)_{\text{at } x=(5.34, 0.33, 0)} + (x_2 - 0.33)\left(\frac{\partial \mu_{d_2^{PIS(F1)}}(x)}{\partial x_2}\right)_{\text{at } x=(5.34, 0.33, 0)} + (x_3 - 0)\left(\frac{\partial \mu_{d_2^{PIS(F1)}}(x)}{\partial x_1}\right)_{\text{at } x=(5.34, 0.33, 0)}$$

$= \hat{\mu}_{d_2^{PIS(F1)}}(x)$

$= 1 + 3.895149((x_1 - 5.34)0.06814 + (x_2 - 0.33)0.0682 + (x_3 - 0)0.01255)$

$$\mu_{d_2^{NIS(F1)}}(x) \tilde{=} \mu_{d_2^{NIS(F1)}}(5.666, 0, 0) + (x_1 - 5.666)\left(\frac{\partial \mu_{d_2^{NIS(F1)}}(x)}{\partial x_1}\right)_{\text{at } x=(5.666, 0, 0)} + (x_2 - 0)\left(\frac{\partial \mu_{d_2^{NIS(F1)}}(x)}{\partial x_2}\right)_{\text{at } x=(5.666, 0, 0)} + (x_3 - 0)\left(\frac{\partial \mu_{d_2^{NIS(F1)}}(x)}{\partial x_3}\right)_{\text{at } x=(5.666, 0, 0)}$$

$= \hat{\mu}_{d_2^{NIS(F1)}}(x)$

$= 1 + 3.3128((x_1 - 5.666) \times 0.4696 + (x_2 - 0) \times 0.2734 + (x_3 - 0)0.0677)$,

Normalizing $\hat{\mu}_{d_2^{PIS(F1)}}(x)$ and $\hat{\mu}_{d_2^{NIS(F1)}}(x)$, we have obtained the following,

$\bar{\mu}_{d_2^{PIS(F1)}}(x) = \dfrac{\hat{\mu}_{d_2^{PIS(F1)}}(x) - a^{PIS(F1)}}{b^{PIS(F1)} - a^{PIS(F1)}}$, where $b^{PIS(F1)} = \underset{x\in S}{Max}\ \hat{\mu}_{d_2^{PIS(F1)}}(x) = 0.999$ and $a^{PIS(F1)} = \underset{x\in S}{Min}\ \hat{\mu}_{d_2^{PIS(F1)}}(x) = -0.431$;

$\bar{\mu}_{d_2^{NIS(F1)}}(x) = \dfrac{\hat{\mu}_{d_2^{NIS(F1)}}(x) - a^{NIS(F1)}}{b^{NIS(F1)} - a^{NIS(F1)}}$, where $b^{NIS(F1)} = \underset{x\in S}{Max}\ \hat{\mu}_{d_2^{NIS(F1)}}(x) = 1$ and $a^{NIS(F1)} = \underset{x\in S}{Min}\ \hat{\mu}_{d_2^{NIS(F1)}}(x) = -7.476$.

Solve the model (16) in order to get the satisfactory solution of FLDM:

Min λ_1

$\bar{\mu}_{d_q^{PIS(F_1)}}(\bar{x}) + d^-_{PIS(F_1)} = 1,$

$\bar{\mu}_{d_q^{NIS(F_1)}}(\bar{x}) + d^-_{NIS(F_1)} = 1,$

$\lambda_1 \geq d^-_{PIS(F_1)},$

$\lambda_1 \geq d^-_{NIS(F_1)},.$

$0 \leq d^-_{PIS(F_1)} \leq 1,$

$0 \leq d^-_{NIS(F_1)} \leq 1,$

$x_1 + x_2 + x_3 \leq 5.666,$

$-2x_1 + 5x_2 + 3x_3 \leq 18.576,$

$3x_1 - 4x_2 + 2x_3 \leq 3.021,$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

The satisfactory solution of FLDM has been provided in the Table 3.

Table 3: The satisfactory solution of FLDM

$x_1^{F1^*}$	$x_2^{F1^*}$	$x_3^{F1^*}$
5.664737	0.001263	0

Suppose that the FLDM decides $x_1^{F1^*} = 5.66$ with upper tolerance $t_1^{R(F1)} = 0.34$ and lower tolerance $t_1^{L(F1)} = 3.66$ such that $5.66 - 3.66 \leq x_1 \leq 5.66 + 0.34$.

8.2 Second level MODM problem

$\max_{x_2} (z_{21} = (x_1 + x_2 x_3; z_{22} = x_1 x_2 + x_3)$[SLDM]

Subject to

$x_1 + x_2 + x_3 \leq 5.666,$

$-2x_1 + 5x_2 + 3x_3 \leq 18.576,$

$3x_1 - 4x_2 + 2x_3 \leq 3.021,$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

,he individual best solutions (PISs) and the individual worst solutions (NISs) of the objective functions considered by the SLDM subject to the constraints have been respectively shown in the Table 4 and Table 5.

Table 4: The individual best solutions of SLDM in Numerical Example

Maximal value of the objective function of SLDM	z_{21}^{+}	z_{22}^{+}
$\max_{x \in S'} z_{2j}(x)$, (j = 1,2)	5.941871 at (0.26, 1.43, 3.98)	7.326 at (3.67, 1.997, 0)

Table 5: The individual worst solutions of SLDM in Numerical Example

Minimal value of the objective function of SLDM	z_{21}^{-}	z_{22}^{-}
$\min_{x \in S'} z_{2j}(x)$, (j = 1,2)	0 at (0, 0, 1.51)	0 at (1.007, 0, 0)

Considering $\alpha_1 = \alpha_2 = 0.5$, and q = 2, we have obtained

$$d_2^{PIS(F2)}(x) = \left\{ (0.5)^2 \left[\frac{5.942 - (x_1 + x_2 x_3)}{5.942 - 0} \right]^2 + (0.5)^2 \left[\frac{7.326 - (x_1 x_2 + x_3)}{7.326 - 0} \right]^2 \right\}^{1/2},$$

$$d_2^{NIS(F2)}(x) = \left\{ (0.5)^2 \left[\frac{x_1 + x_2 x_3 - 0}{5.942 - 0} \right]^2 + (0.5)^2 \left[\frac{(x_1 x_2 + x_3) - 0}{7.326 - 0} \right]^2 \right\}^{1/2}.$$

Now, $\left(d_2^{PIS(F2)}(x)\right)^+ = \underset{x\in S}{\text{Min}}\ d_2^{PIS(F2)}(x) = 0.129$ at (2.59, 1.817, 1.259); $\left(d_2^{PIS(F2)}(x)\right)^- = \underset{x\in S}{\text{Max}}\ d_2^{PIS(F2)}(x) = 0.6499$ at (1.007,0, 0); $\left(d_2^{NIS(F2)}(x)\right)^+ = \underset{x\in S}{\text{Max}}\ d_2^{NIS(F2)}(x) = 0.5877$ at (3.669, 1.997, 0); $\left(d_2^{NIS(F2)}(x)\right)^- = \underset{x\in S}{\text{Min}}\ d_2^{NIS(F2)}(x) = 0.065$ at (0.6, 0, 0.609).

The membership functions of $d_2^{PIS(F2)}(x)$ and $d_2^{NIS(F2)}(x)$ can be formulated as follows:

$$\mu_{d_2^{PIS(F2)}}(x) = \begin{cases} 0, & \text{if } 0.6499 \le d_2^{PIS(F2)}(x) \\ \dfrac{0.6499 - d_2^{PIS(F2)}(x)}{0.6499-0.129}, & \text{if } 0.129 \le d_2^{PIS(F2)}(x) \le 0.6499, \\ 1, & \text{if } d_2^{PIS(F2)}(x) \le 0.129 \end{cases}$$

$$\mu_{d_2^{NIS(F2)}}(x) = \begin{cases} 0, & \text{if } \left(d_2^{NIS(F2)}(x)\right) \le 0.065 \\ \dfrac{d_2^{NIS(F2)}(x) - 0.065}{0.5877-0.065}, & \text{if } 0.065 \le d_2^{NIS(F2)}(x) \le 0.5877 \\ 1, & \text{if } d_2^{NIS(F2)}(x) \ge 0.5877 \end{cases}$$

Applying the first order Taylor's series the non-linear membership functions $\mu_{d_2^{PIS(F2)}}(x)$ and $\mu_{d_2^{NIS(F2)}}(x)$ can be transformed into equivalent linear membership functions $\hat{\mu}_{d_2^{PIS(F2)}}(x)$ and $\hat{\mu}_{d_2^{NIS(F2)}}(x)$ respectively as follows:

$$\mu_{d_2^{PIS(F2)}}(x) \tilde{=} \hat{\mu}_{d_2^{PIS(F2)}}(x)$$

$= 1 + (x_1 - 2.59)0.2835 + (x_2 - 1.817)0.3854 + (x_3 - 1.259)0.2980$

$$\mu_{d_2^{NIS(F2)}}(x) \tilde{=} \hat{\mu}_{d_2^{NIS(F2)}}(x)$$

$= 1 + (x_1 - 0.6)(-0.1245) + (x_2 - 0)(-0.1257) + (x_3 - 0.609)(-0.083)$,

Normalizing $\hat{\mu}_{d_2^{PIS(F2)}}(x)$ and $\hat{\mu}_{d_2^{NIS(F2)}}(x)$, we have obtained

$\bar{\mu}_{d_2^{PIS(F2)}}(x) = \dfrac{\hat{\mu}_{d_2^{PIS(F2)}}(x) - a^{PIS(F2)}}{b^{PIS(F2)} - a^{PIS(F2)}}$, where $b^{PIS(F2)} = \underset{x\in S}{\text{Max}}\ \hat{\mu}_{d_2^{PIS(F2)}}(x) = 0.999$ and $a^{PIS(F2)} = \underset{x\in S}{\text{Min}}\ \hat{\mu}_{d_2^{PIS(F2)}}(x) = -0.5243$;

$\bar{\mu}_{d_2^{NIS(F2)}}(x) = \dfrac{\hat{\mu}_{d_2^{NIS(F2)}}(x) - a^{NIS(F2)}}{b^{NIS(F2)} - a^{NIS(F2)}}$, where $b^{NIS(F2)} = \underset{x\in S}{\text{Max}}\ \hat{\mu}_{d_2^{NIS(F2)}}(x) = 1$ and $a^{NIS(F2)} = \underset{x\in S}{\text{Min}}\ \hat{\mu}_{d_2^{NIS(F2)}}(x) = 0.4176$.

Solve the model (24) in order to get the satisfactory solution of SLDM:

Min λ_2

$\bar{\mu}_{d_q^{PIS(F2)}}(\bar{x}) + d^-_{PIS(F2)} = 1,$

$\bar{\mu}_{d_q^{NIS(F2)}}(\bar{x}) + d^-_{NIS(F2)} = 1,$

$\lambda_2 \ge d^-_{PIS(F2)}$,

$\lambda_2 \ge d^-_{NIS(F2)}$,

$0 \le d^-_{PIS(F2)} \le 1$,

$0 \le d^-_{NIS(F2)} \le 1$,

$x_1 + x_2 + x_3 \le 5.666$,

$-2x_1 + 5x_2 + 3x_3 \le 18.576$,

$3x_1 - 4x_2 + 2x_3 \le 3.021$,

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$

The satisfactory solution of SLDM has been provided in the Table 6.

Table 6: The satisfactory solution of SLDM

$x_1^{F2^*}$	$x_2^{F2^*}$	$x_3^{F2^*}$
3.669308	1.996692	0

Assume that the SLDM decides $x_2^{F2^*}$ = 1.997 with upper tolerance $t_2^{R(F2)}$ = 0.003 and lower tolerance $t_2^{L(F2)}$ =1.997 such that 1.997 – 1.997 $\leq x_2 \leq$ 1.997 + 0.003.

Last level MODM problem

$$\max_{x_3} (z_{31} = (x_2 + 7) + (x_1 + 1)(x_3 + 5); z_{32} = 2x_1^2 + 3x_2x_3)[\text{LLDM}]$$

Subject to

$x_1 + x_2 + x_3 \leq 5.666$,

$-2x_1 + 5x_2 + 3x_3 \leq 18.576$,

$3x_1 - 4x_2 + 2x_3 \leq 3.021$,

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

The individual best solutions (PISs) and the individual worst solutions (NISs) of the objective functions of the LLDM subject to the constraints have been respectively shown in the Table 7 and Table 8.

Table 7: The individual best solutions of LLDM in Numerical Example

Maximal value of the objective function of LLDM	z_{31}^{+}	z_{32}^{+}
$\underset{x \in S'}{\text{Max}}\ z_{3j}(x)$, (j = 1,2)	41.024 at (4.833, 0, 0.833)	64.21 at (5.666, 0, 0)

Table 8: The individual worst solutions of LLDM in Numerical Example

Minimal value of the objective function of LLDM	z_{31}^{-}	z_{32}^{-}
$\underset{x \in S'}{\text{Min}}\ z_{3j}(x)$, (j = 1,2)	13.51 at (0, 0, 1.51)	0 at (0, 0, 2.546)

Let us assume that $\alpha_1 = \alpha_2$ = 0.5, and q = 2.

$$d_2^{PIS(F3)}(x) = \left\{ (0.5)^2 \left[\frac{41.024 - (x_2 + 7 + (x_1 + 1)(x_3 + 5))}{41.024 - 13.51} \right]^2 + (0.5)^2 \left[\frac{64.21 - (2x_1^2 + 3x_2x_3)}{64.21 - 0} \right]^2 \right\}^{1/2},$$

$$d_2^{NIS(F3)}(x) = \left\{ (0.5)^2 \left[\frac{(x_2 + 7 + (x_1 + 1)(x_3 + 5)) - 13.51}{41.024 - 13.51} \right]^2 + (0.5)^2 \left[\frac{(2x_1^2 + 3x_2x_3) - 0}{64.21 - 0} \right]^2 \right\}^{1/2}.$$

Now, $\left(d_2^{PIS(F3)}(x)\right)^+ = \underset{x \in S}{\text{Min}}\ d_2^{PIS(F3)}(x)$ = 0.012 at (5.65, 0, 0.011); $\left(d_2^{PIS(F3)}(x)\right)^- = \underset{x \in S}{\text{Max}}\ d_2^{PIS(F3)}(x)$ = 0.707 at (0, 0, 1.51); $\left(d_2^{NIS(F3)}(x)\right)^+ = \underset{x \in S}{\text{Max}}\ d_2^{NIS(F3)}(x)$ = 0.698 at (5.67, 0, 0); $\left(d_2^{NIS(F3)}(x)\right)^- = \underset{x \in S}{\text{Min}}\ d_2^{NIS(F3)}(x)$ = 0 at (0, 0, 1.51).

The membership functions of $d_2^{PIS(F3)}(x)$ and $d_2^{NIS(F3)}(x)$ can be formulated as follows:

$$\mu_{d_2^{PIS(F3)}}(x) = \begin{cases} 0, & \text{if } 0.71 \leq d_2^{PIS(F3)}(x) \\ \dfrac{0.71 - d_2^{PIS(F3)}(x)}{0.71 - 0.01}, & \text{if } 0.01 \leq d_2^{PIS(F3)}(x) \leq 0.71, \\ 1, & \text{if } d_2^{PIS(3)}(x) \leq 0.01 \end{cases}$$

$$\mu_{d_2^{NIS(F3)}}(x)=\begin{cases}0, & \text{if } d_2^{NIS(F3)}(x)\le 0\\ \dfrac{d_2^{NIS(F3)}(x)-0}{0.698-0}, & \text{if } 0\le d_2^{NIS(F3)}(x)\le 0.698\\ 1, & \text{if } d_2^{NIS(F3)}(x)\ge 0.698\end{cases}$$

Applying the first order Taylor's series the non-linear membership functions $\mu_{d_2^{PIS(F3)}}(x)$ and $\mu_{d_2^{NIS(F3)}}(x)$ can be transformed into equivalent linear membership functions $\hat{\mu}_{d_2^{PIS(F3)}}(x)$ and $\hat{\mu}_{d_2^{NIS(F3)}}(x)$ respectively as follows:

$\mu_{d_2^{PIS(F3)}}(x) \stackrel{\sim}{=} \hat{\mu}_{d_2^{PIS(F3)}}(x)$

$= 1 + 0.0093((x_1 - 5.65)0.00333 + (x_2 - 0)*0.00309 + (x_3 - 0.01)0.00047)$

$\mu_{d_2^{NIS(F3)}}(x) \stackrel{\sim}{=} \hat{\mu}_{d_2^{NIS(F3)}}(x)$

$= 1 + 1.02474((x_1 - 5.67)\ 0.72871 + (x_2 - 0)\ 0.18832 + (x_3 - 0)0.48636)$,

Now, Normalize $\hat{\mu}_{d_2^{PIS(F3)}}(x)$ and $\hat{\mu}_{d_2^{NIS(F3)}}(x)$ we get the following,

$\bar{\mu}_{d_2^{PIS(F3)}}(x)=\dfrac{\hat{\mu}_{d_2^{PIS(F3)}}(x)-a^{PIS(F3)}}{b^{PIS(F3)}-a^{PIS(F3)}}$, where $b^{PIS(F3)}=\underset{x\in S}{Max}\ \hat{\mu}_{d_2^{PIS(F3)}}(x) = 1$ and $a^{PIS(F3)}=\underset{x\in S}{Min}\ \hat{\mu}_{d_2^{PIS(F3)}}(x) = 0.9998$;

$\bar{\mu}_{d_2^{NIS(F3)}}(x)=\dfrac{\hat{\mu}_{d_2^{NIS(F3)}}(x)-a^{NIS(F3)}}{b^{NIS(F3)}-a^{NIS(F3)}}$, where $b^{NIS(F3)}=\underset{x\in S}{Max}\ \hat{\mu}_{d_2^{NIS(F3)}}(x) = 0.99701$ and $a^{NIS(F3)}=\underset{x\in S}{Min}\ \hat{\mu}_{d_2^{NIS(F3)}}(x) = -2.48204$.

Solve the model (32) in order to get the satisfactory solution of LLDM.

$\text{Min } \lambda_3$

$\bar{\mu}_{d_q^{PIS(F3)}}(\bar{x}) + d^-_{PIS(F3)} = 1,$

$\bar{\mu}_{d_q^{NIS(F3)}}(\bar{x}) + d^-_{NIS(F3)} = 1,$

$\lambda_3 \ge d^-_{PIS(F3)},$

$\lambda_3 \ge d^-_{NIS(F3)},$

$0 \le d^-_{PIS(F3)} \le 1,$

$0 \le d^-_{NIS(F3)} \le 1,$

$x_1 + x_2 + x_3 \le 5.666,$

$-2x_1 + 5x_2 + 3x_3 \le 18.576,$

$3x_1 - 4x_2 + 2x_3 \le 3.021,$

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$

The satisfactory solution of LLDM has been provided in the Table 9.

Table 9: The satisfactory solution of FLDM

$x_1^{F3^*}$	$x_2^{F3^*}$	$x_3^{F3^*}$
5.666	0	0

Assume that the LLDM decides $x_3^{F3^*} = 0$ with upper tolerance $t_3^{R(F3)} = 1$ and lower tolerance $t_3^{L(F3)} = 0$ such that $0-0 \le x_3 \le 0 + 1$.

Using Model 1(35) and Model 2 (36) for p= 3, obtained solutions have been shown in the Table 10 and Table 11 respectively.

Table 10: The optimal solution of the FGP model 1

Methods	Decision variables x_1, x_2, x_3	Objective values of FLDM z_{11}, z_{12}	Objective values of SLDM Z_{21}, z_{22}	Objective values of LLDM Z_{31}, z_{32}	Membership values $\mu_{z_{11}}$, $\mu_{z_{12}}$, $\mu_{z_{21}}$, $\mu_{z_{22}}$., $\mu_{z_{31}}$, $\mu_{z_{32}}$
FGP Model 1	$x_1 = 2$, $x_2 = 1.24$ $x_3 = 1$	$z_{11} = 21.979$, $z_{12} = 6.549$.	z_{21}=3.245, z_{22}=3.245.	Z_{31} =26.979, z_{22} =11.734.	$\mu_{z_{11}} = 0.374$, $\mu_{z_{12}} = 0.427$, $\mu_{z_{21}} = 0.558$, $\mu_{z_{22}} = 0.548$, $\mu_{z_{31}} = 0.326$, $\mu_{z_{32}} = 0.374$,

The optimal solution of the FGP Model 2 is shown in the Table 11.

Table 11: The optimal solution of the FGP model 2

Methods	Decision variables x_1, x_2, x_3	Objective values of FLDM z_{11}, z_{12}	Objective values of SLDM Z_{21}, z_{22}	Objective values of LLDM Z_{31}, z_{32}	Membership values $\mu_{z_{11}}$, $\mu_{z_{12}}$, $\mu_{z_{21}}$, $\mu_{z_{22}}$., $\mu_{z_{31}}$, $\mu_{z_{32}}$
FGP Model 2	$x_1 = 2$, $x_2 = 1.24$ $x_3 = 1$	$z_{11} = 21.979$, $z_{12} = 6.549$.	z_{21}=3.245, z_{22}=3.245.	Z_{31} =26.979, z_{22} =11.734.	$\mu_{z_{11}} = 0.374$, $\mu_{z_{12}} = 0.427$, $\mu_{z_{21}} = 0.558$, $\mu_{z_{22}} = 0.548$, $\mu_{z_{31}} = 0.326$, $\mu_{z_{32}} = 0.374$,

Note1. Two tables show the two FGP Models provide the same results

CONCLUSION

In the paper we have developed TOPSIS approach to solve chance constrained multi-level multi objective quadratic programming problem. In the proposed approach two FGP models have been developed and solved. Further, the proposed approach can be extended to solve multi-level multi objective quadratic programming problem with fuzzy coefficient of objective functions. If the coefficients of constraints are taken as random variables, the proposed concept can also be adopted.

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