Medical treatment options selection using extended TODIM method with single valued trapezoidal neutrosophic numbers

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Abstract: The selection process of medical treatment options is a multi-criteria decision-making (MCDM) one, and single valued trapezoidal neutrosophic numbers (SVTNNs) are useful in depicting information and fuzziness in selection processes. Some comparison methods for SVTNNs have been developed and applied in MCDM problems. However, some defects exist in these comparison methods. Furthermore, few studies have been focused on the distance measurements for SVTNNs. Moreover, the previous MCDM methods under SVTNN environments assume that decision makers are perfectly rational. Nevertheless, in practical problems like the selection of medical treatment options, decision makers' bounded rationality should be considered due to the complexity of human cognition. To address the above deficiencies, in this paper, an improved comparison method and several distance measurements for SVTNNs are defined. Furthermore, a novel MCDM method for medical treatment options selection is established based on an acronym in Portuguese of interactive and MCDM method (TODIM method) with SVTNNs to consider the risk preference of physicians. In addition, a numerical example of the selection of medical treatment options is provided in order to verify the proposed method and the influence of the parameter. Finally, a comparative analysis is conducted to demonstrate the feasibility of the proposed method.

Keywords: single valued trapezoidal neutrosophic number; comparison method; distance measurement; TODIM; selection of medical treatment options

1 Introduction

As a common but significant process in clinical medicine, the selection of medical treatment options is,

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indeed, a multi-criteria decision-making (MCDM) problem, and it can be summarized as a physician makes the decision which medical treatment option is most probable for a particular patient considering several factors like the survival rate and the probability of a recurrence. Due to the complexity of human cognition, a lot of imprecise, uncertain, incomplete information which cannot be depicted by crisp values may exist in the selection problems of medical treatment options [1]. For instance, a physician may not sure how much the survival rate of a treatment option is. To handle fuzziness and uncertainty, fuzzy logic and fuzzy sets (FSs) are introduced. FSs were proposed by Zadeh [2] in 1965. After that, many extensions of FSs have been developed. For example, Turksen [3] extended FSs by utilizing an interval value to depict the degree of membership, and defined the interval valued fuzzy sets (IVFSs). On the basis of FSs and IVFSs, Atanassov and Gargov [4, 5] introduced the notion of non-membership, and presented the intuitionistic fuzzy sets (IFSs) and the interval valued intuitionistic fuzzy sets (IVIFSs). In some situations, people may be hesitant in selecting single values to express their preferences regarding an individual object. To deal with these situations, Torra [6] utilized several single values to reflect the degree of membership, and developed the hesitant fuzzy sets (HFSs). Moreover, all these extensions of FSs have been applied in decision-making [7-9] with further extensions still being developed [10, 11] and applied [12-16].

Neutrosophic sets (NSs), which were defined by Smarandache [17, 18], are important extensions of FSs. Unlike FSs and IFSs, NSs make use of the degrees of truth, indeterminacy, and falsity to describe fuzzy information in decision-making processes. Moreover, the values of these three degrees lie in $]^{-}0,1^{+}[$. It is obvious that NSs are difficult to be applied in actual decision-making problems. Therefore, Wang and Smarandache [19] defined the single valued neutrosophic sets (SVNSs) whose degrees of truth, indeterminacy, and falsity are between 0 and 1. In addition, many other extensions of NSs have been proposed [20-22]. For instance, the simplified neutrosophic sets (SNSs) were developed by Ye [23], and the interval valued neutrosophic sets (IVNSs) were defined by Wang and Smarandache [24]. Furthermore, NSs have been effectively applied in many fields, such as decision-making [25-30] and image segmentation [31, 32].

Recently, the above extensions of NS have been applied in medical decision-making problems to depict fuzzy information. Such as, SNSs are utilized by Ye [33] and SVNSs are used by Ye and Fu [34] to deal with medical decision-making problems. In some practical medical decision-making problems like the selection of medical treatment options, truth, indeterminacy, and falsity may exist simultaneously in evaluation information provided by physicians. These three factors can be accurately and fully depicted by the degrees of truth, indeterminacy, and falsity in NSs. However, SNSs and SVNSs are crisp sets. That is to say, they utilize crisp values to denote the degrees of truth, indeterminacy, and falsity, which may result in a loss of information.

To overcome the above shortcoming, some researchers studied the extensions of NSs in the field of the continuous sets [35]. In particular, Deli and Subas [36] extended NSs into the domain of continuous sets, and defined the single valued trapezoidal neutrosophic numbers (SVTNNs). In SVTNNs, the degrees of truth, indeterminacy and falsity are trapezoidal fuzzy numbers rather than single values. Comparing to discrete sets, continuous sets like SVTNNs can depict more information and the fuzziness in decision-making processes. SVTNNs are more suitable to be utilized to represent the fuzzy information in medical decision-making problems like the selection of medical treatment options than the extensions of NSs in the domain of crisp sets like SNSs and SVNSs. Furthermore, the comparison method for SVTNNs has been defined in some studies. For instance, Ye [37] gave the definition of the score function, and presented a comparison method for SVTNNs. Other than the comparison method proposed by Ye [37], the comparison method for SVTNNs defined by Deli and Subas [36] utilized a different score function and introduced the notion of accuracy function.

Many kinds of continuous sets including SVTNNs have been widely used in solving decision-making method [38-43]. For example, Zhang and Jin [44] proposed a grey relational projection method for decision-making problems based on intuitionistic trapezoidal fuzzy number. Dong and Wan [45] developed a new method for decision-making problems in which the criteria values are triangular intuitionistic fuzzy numbers. SVTNNs are also an important kind of tools in constructing decision-making methods. As for decision-making methods with SVTNNs, Deli and Subas [36] presented a decision-making method based on the proposed aggregation operator with the parameter γ of SVTNNs. Similarly, Ye [37] also constructed decision-making methods utilizing aggregation operators of SVTNNs, but the aggregation operators in Ref. [37] are the weighted arithmetic averaging operator and the weighted geometric averaging operator for SVTNNs.

Distance measurements, which are important tools in decision-making, have been studied under various fuzzy environments. For instance, researchers have studied distance measurements of various FSs in the domain of crisp sets: distance measurements of FSs [46-49], distance measurements of IFSs [50-53], distance measurements of HFSs [54-57], distance measurements of NSs [58-60]. As for distance measurements of FSs in the domain of continuous sets, Önüt and Soner [61] and Chen [62] defined the Euclidean distance between two triangular fuzzy numbers. Fu [63] extended the distance defined by Önüt and Soner [61] and Chen [62] by introducing a parameter *p*, and developed a generalized distance measurement of triangular fuzzy numbers. Distance measurements of trapezoidal intuitionistic fuzzy numbers have also been defined. Wan [64] gave the definition of Hamming distance and Euclidean distance between two trapezoidal intuitionistic fuzzy numbers based on Hausdorff metric. Furthermore, distance measurements have been applied in plenty of extant fuzzy decision-making methods [57, 58, 60, 65, 66].

An acronym in Portuguese of interactive and decision-making method named Tomada de decisao interativa e multicritério (TODIM method) has also been used in decision-making problems to consider the risk preference of decision makers. TODIM method was proposed by Gomes and Lima [67, 68] on the basis of prospect theory [69]. However, the TODIM method proposed by Gomes and Lima [67, 68] can only deal with crisp values. To overcome this deficiency, Krohling and Souza [70] originally extended TODIM method into fuzzy environments. On the basis of the fuzzy TODIM method developed by Krohling and Souza [70], many researchers have studied TODIM methods for decision-making problems under various fuzzy environments. For example, Lourenzutti and Krohling [71] established a decision-making method based on the proposed intuitionistic fuzzy TODIM method. Moreover, Li *et al*. [72] further extended TODIM method into interval valued intuitionistic fuzzy environments, and constructed an extended TODIM method for decision-making problems. TODIM methods for decision-making problems have also been proposed under various other discrete fuzzy set environments, like hesitant fuzzy environments [73], and multi-valued neutrosophic environments [74].

TODIM methods under fuzzy continuous set environments have been studied and applied in decision-making. For instance, Tseng *et al*. [75] proposed a triangular fuzzy number TODIM method for decision-making problems and applied it to evaluate green supply chain practices. Similarly, Tosun and Akyüz [76] utilized the triangular fuzzy number TODIM method to solve decision-making problems in the selection of supplier. Unlike the TODIM method proposed by Tseng *et al*. [75], Gomes *et al*. [77] combined Choquet integral with TODIM methods under triangular fuzzy number environments, and presented a decision-making method for decision-making problems where criteria are interdependent and the decision makers are bounded rational. Furthermore, a TODIM method under trapezoidal intuitionistic fuzzy number environments has been developed by Krohling *et al*. [78] to tackle decision-making problems.

SVTNNs can comprehensively denote fuzzy, uncertain, and imprecise information in medical decision-making problems like the selection of medical treatment options. Fuzzy TODIM method takes into account the risk preference of decision makers, and it is a significant vehicle in decision-making problems. However, few studies have been focused on the distance measurement of SVTNNs or the fuzzy TODIM method under SVTNN environments. In addition, the aforementioned comparison methods for SVTNNs have some drawbacks, *i.e*. some results obtained by these comparison methods may be not consistent with reality in some cases (the details will be discussed in Section 2). To overcome these deficiencies, this paper proposed an improved comparison method and several distance measurements for SVTNNs. Furthermore, TODIM method was extended to SVTNN environments, and a decision-making method for medical treatment options selection was constructed based on the TODIM method to take into consideration the risk preference of physicians.

The structure of this paper is organized as follows. In Section 2, we introduce several relevant concepts of SVTNNs. In Section 3, an improved comparison method for SVTNNs is developed to overcome the deficiencies of the extant comparison method. Moreover, some distance measurements are presented in Section 3. In Section 4, a novel decision-making method for medical diagnosis problems with SVTNNs is constructed based on TODIM method. Furthermore, a numerical example for medical treatment options selection is given and the influence of the parameter is discussed in Section 5. Furthermore, a comparative analysis is provided in Section 5 to verify the feasibility of the proposed method. Finally, Section 6 concludes the paper.

2 Preliminaries

This section reviews some basic concepts of SVTNNs. And these concepts will be used in the reminder of this paper.

Definition 1 [19]. Let X be a space of points (objects), with a generic element in X denoted by x. An

SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard $[0,1]$. That is $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$. Then, a simplification of A is denoted by

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | \, x \in X\}.$$

It is a subclass of NSs. And the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies that $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$

Definition 2 [36, 37]. An SVTNN $a = \langle [a_1, a_2, a_3, a_4], T(a), I(a), F(a) \rangle$ is a special NS on the real number set *R*, whose truth-membership function $T_a(x)$, indeterminacy-membership function $I_a(x)$ and falsity-membership $F_a(x)$ are given as follows:

$$T_a(x) = \begin{cases} (x - a_1)T(a)/(a_2 - a_1) & (a_1 < x < a_2), \\ T(a) & (a_2 < x < a_3), \\ (a_4 - x)T(a)/(a_4 - a_3) & (a_3 < x < a_4), \\ 0 & \textit{otherwise.} \end{cases}$$

$$I_a(x) = \begin{cases} (a_2 - x + I(a)(x - a_1))/(a_2 - a_1) & (a_1 < x < a_2), \\ I(a) & (a_2 < x < a_3), \\ (x - a_3 + I(a)(a_4 - x))/(a_4 - a_3) & (a_3 < x < a_4), \\ 1 & \textit{otherwise.} \end{cases} \text{ and}$$

$$F_a(x) = \begin{cases} (a_2 - x + F(a)(x - a_1))/(a_2 - a_1) & (a_1 < x < a_2), \\ F(a) & (a_2 < x < a_3), \\ (x - a_3 + F(a)(a_4 - x))/(a_4 - a_3) & (a_3 < x < a_4), \\ 1 & \textit{otherwise.} \end{cases}$$

When $a_1 > 0$, $a = \langle [a_1, a_2, a_3, a_4], T(a), I(a), F(a) \rangle$ is called a positive SVTNN, denoted by $a > 0$. Similarly, when $a_4 \le 0$, $a = \langle [a_1, a_2, a_3, a_4], T(a), I(a), F(a) \rangle$ is called a negative SVTNN, denoted by $a < 0$. When $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $T(a), I(a), F(a) \in [0,1]$, $a = \langle [a_1, a_2, a_3, a_4], T(a), I(a), F(a) \rangle$ is called a normalized SVTNN, which is used for this paper. It should be noted that a_1, a_2, a_3 and a_4 are not zero simultaneously.

Definition 3 [36, 37]. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ and $b=\langle[b_1,b_2,b_3,b_4], T(b),I(b),F(b)\rangle$ be two SVTNNs and $\lambda\geq 0$, then

(1) $a+b=\langle[a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4],T(a)+T(b)-T(a)T(b),I(a)I(b),\mathrm{F}(a)F(b)\rangle$;

(2) $a\cdot b=\langle[a_1b_1,a_2b_2,a_3b_3,a_4b_4],T(a)T(b),I(a)+I(b)-I(a)I(b),\mathrm{F}(a)+F(b)-$

$\mathrm{F}(a)F(b)\rangle$;

(3) $\lambda a=\left\langle[\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4],1-(1-T(a))^{\lambda},(I(a))^{\lambda},(F(a))^{\lambda}\right\rangle$; and

(4) $a^{\lambda}=\left\langle[a_1^{\lambda},a_2^{\lambda},a_3^{\lambda},a_4^{\lambda}],(T(a))^{\lambda},1-(1-I(a))^{\lambda},1-(1-F(a))^{\lambda}\right\rangle$.

Furthermore, some comparison methods for SVTNNs are defined in order to compare two SVTNNs.

Definition 4 [37]. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ be an SVTNN. The score function of a is defined as:

$$s(a)=\frac{1}{12}[a_1+a_2+a_3+a_4]\times(2+T(a)-I(a)-F(a)).$$

Definition 5 [37]. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ and $b=\langle[b_1,b_2,b_3,b_4],T(b), I(b),F(b)\rangle$ be two SVTNNs. When $s(a)>s(b)$, $a\succ b$; when $s(a)=s(b)$, $a=b$.

However, a shortcoming exists in Definition 5, and it is illustrated in the following example.

Example 1. Let $a=\langle[0.3,0.4,0.6,0.7],0.2,0.4,0.6\rangle$ and $b=\langle[0.6,0.7,0.8,0.9],0.1,0.8,0.5\rangle$ be two SVTNNs, it is clear that $a\neq b$. However, according to Definitions 4 and 5, $s(a)=s(b)=0.2$, and $a=b$, which does not conform to our intuition.

Moreover, Deli and Subas [36] defined a novel score function and the accuracy function for SVTNNs, and they also presented a new comparison method for SVTNNs.

Definition 6 [36]. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ be an SVTNN. The score function of a is defined as:

$$s(a)=\frac{1}{16}[a_1+a_2+a_3+a_4]\times(2+T(a)-I(a)-F(a)),$$

and the accuracy function of a is defined as:

$$h(a)=\frac{1}{16}[a_1+a_2+a_3+a_4]\times(2+T(a)-I(a)+F(a)).$$

Definition 7 [36]. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ and $b=\langle[b_1,b_2,b_3,b_4],T(b),\ I(b),F(b)\rangle$ be two SVTNNs. The comparison method for a and b can be defined as:

1. When $s(a)<s(b)$, $a\prec b$;

2. When $s(a)=s(b)$ and $h(a)<h(b)$, $a\prec b$; and

3. When $s(a)=s(b)$ and $h(a)=h(b)$, $a=b$.

Example 2. Utilizing Definition 7 to compare a and b in Example 1, we can obtain that $s(a)=s(b)=0.15$, $h(a)=0.3$, $h(b)=0.3375$. Therefore, it is true that $a\prec b$.

This comparison method solves the problem in Example 1. Nevertheless, another drawback exists in this comparison method and the following example presents the drawback.

Example 3. Let $a=\langle[0.3,0.4,0.6,0.7],0.1,0.9,0\rangle$ and $b=\langle[0.1,0.4,0.5,0.6],0.1,0.6,0\rangle$ be two SVTNNs, it is clear that $a\neq b$. However, according to Definitions 6 and 7, $s(a)=s(b)=0.15$, $h(a)=h(b)=0.15$ and $a=b$, which is contradictory to our intuition.

Lemma 1 (Minkowski's inequality [56]) Let $(x_1,x_2,\cdots,x_n)$ and $(y_1,y_2,\cdots,y_n)$ be two sequences of real numbers, and $1\leq p\leq\infty$. Then

$$\left(\sum_{i=1}^{n}|x_i+y_i|^p\right)^{1/p}\leq\left(\sum_{i=1}^{n}|x_i|^p\right)^{1/p}+\left(\sum_{i=1}^{n}|y_i|^p\right)^{1/p}.$$

3 Comparison method and distance measurements for SVTNNs

In this section, we propose the novel score and accuracy functions for SVTNNs. Subsequently, an improved comparison method for SVTNNs is defined on the basis of the score and accuracy functions. Moreover, several distance measurements for SVTNNs are defined based on the prior distance measurements under various fuzzy environments.

Definition 8. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ be an SVTNN. A score function $s(a)$ of the SVTNN *a* can be defined as:

$$s(a)=\frac{a_1+a_2+a_3+a_4}{4}\times\left(\frac{1+T(a)-2I(a)-F(a)}{2}\right). \tag{1}$$

The bigger the value of $s(a)$ is, the better the SVTNN *a* will be.

Definition 9. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ be an SVTNN. An accuracy function $h(a)$ of the SVTNN *a* can be defined as:

$$h(N)=\frac{a_1+a_2+a_3+a_4}{4}\times\left(T(a)-I(a)(1-T(a))-F(a)(1-I(a))\right). \tag{2}$$

The bigger the value of $s(a)$ is, the better the SVTNN *a* will be.

According to the score and accuracy functions for SVTNNs, we can obtain the following comparison method.

Definition 10. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$ and $b=\langle[b_1,b_2,b_3,b_4],T(b),I(b),\ F(b)\rangle$ be two SVTNNs. The comparison method for *a* and *b* can be defined as:

(1) $a\le b$ if and only if $a_1\le b_1$, $a_2\le b_2$, $a_3\le b_3$, $a_4\le b_4$, $T(a)\le T(b)$, $I(a)\ge I(b)$, and $F(a)\ge F(b)$;

(2) $a=b$ if and only if $a\le b$ and $b\le a$ (*i.e.* $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$, $T(a)=T(b)$, $I(a)=I(b)$, and $F(a)=F(b)$);

(3) When $a\le b$ and $a\ne b$, $a<b$;

(4) When neither $a\le b$ nor $b\le a$ exists and $s(a)<s(b)$, $a\prec b$;

(5) When neither $a\le b$ nor $b\le a$ exists, $s(a)=s(b)$ and $h(a)<h(b)$, $a\prec b$; and

(6) When $a\ne b$, $s(a)=s(b)$ and $h(a)=h(b)$, $a\sim b$.

It is obvious that $a\prec b$ when $a<b$.

Example 4. Let $a=\langle[0.3,0.4,0.6,0.7],0.1,0.9,0\rangle$ and $b=\langle[0.1,0.4,0.5,0.6],0.1,0.6,0\rangle$ be two SVTNNs.

By applying the comparison method in Definition 10, since $0.3 \geq 0.1$, $0.4 \geq 0.4$, $0.6 \geq 0.5$, $0.7 \geq 0.6$, $0.1 \geq 0.1$, $0.9 \geq 0.6$ and $0 \geq 0$, neither $b \leq a$ nor $b \leq a$ exists. Furthermore, $s(a) = -0.175$ and $s(b) = -0.02$. Here $s(a) < s(b)$, thus $a \prec b$.

As can be seen from Example 4, the improved comparison method overcomes the defect of the comparison method in Definition 7.

In addition, we define several distance measurements for SVTNNs as follows.

Definition 11. Let $a = \langle [a_1, a_2, a_3, a_4], T(a), I(a), F(a) \rangle$ and $b = \langle [b_1, b_2, b_3, b_4], T(b), I(b), F(b) \rangle$ be two SVTNNs. The distances between *a* and *b* can be defined by the following equations:

(1) The Hamming distance:

$$\begin{aligned} d_1(a,b) = \frac{1}{24}\Big(& |a_1 - b_1| + 2|a_2 - b_2| + 2|a_3 - b_3| + |(a_4 - b_4)| + |a_1 T(a) - b_1 T(b)| \\ & + 2|a_2 T(a) - b_2 T(b)| + 2|a_3 T(a) - b_3 T(b)| + |a_4 T(a) - b_4 T(b)| \\ & + |a_1 I(a) - b_1 I(b)| + 2|a_2 I(a) - b_2 I(b)| + 2|a_3 I(a) - b_3 I(b)| \\ & + |a_4 I(a) - b_4 I(b)| + |a_1 F(a) - b_1 F(b)| + 2|a_2 F(a) - b_2 F(b)| \\ & + 2|a_3 F(a) - b_3 F(b)| + |a_4 F(a) - b_4 F(b)| \Big) \end{aligned} \tag{3}$$

(2) The Euclidean distance:

$$\begin{aligned} d_2(a,b) = \bigg(\frac{1}{24}\Big(& |a_1 - b_1|^2 + 2|a_2 - b_2|^2 + 2|a_3 - b_3|^2 + |(a_4 - b_4)|^2 + |a_1 T(a) - b_1 T(b)|^2 \\ & + 2|a_2 T(a) - b_2 T(b)|^2 + 2|a_3 T(a) - b_3 T(b)|^2 + |a_4 T(a) - b_4 T(b)|^2 \\ & + |a_1 I(a) - b_1 I(b)|^2 + 2|a_2 I(a) - b_2 I(b)|^2 + 2|a_3 I(a) - b_3 I(b)|^2 \\ & + |a_4 I(a) - b_4 I(b)|^2 + |a_1 F(a) - b_1 F(b)|^2 + 1|a_2 F(a) - b_2 F(b)|^2 \\ & + 2|a_3 F(a) - b_3 F(b)|^2 + |a_4 F(a) - b_4 F(b)|^2 \Big) \bigg)^{1/2} \end{aligned} \tag{4}$$

(3) The Chebyshev distance:

$$d_3(a,b)=\frac{1}{24}\max\Big(|a_1-b_1|,2|a_2-b_2|,2|a_3-b_3|,|(a_4-b_4)|,|a_1T(a)-b_1T(b)|,$$
$$2|a_2T(a)-b_2T(b)|,2|a_3T(a)-b_3T(b)|,|a_4T(a)-b_4T(b)|,$$
$$|a_1I(a)-b_1I(b)|,2|a_2I(a)-b_2I(b)|,2|a_3I(a)-b_3I(b)|, \tag{5}$$
$$|a_4I(a)-b_4I(b)|,|a_1F(a)-b_1F(b)|,2|a_2F(a)-b_2F(b)|,$$
$$2|a_3F(a)-b_3F(b)|,|a_4F(a)-b_4F(b)|\Big)$$

(4) The generalized distance:

$$d_4(a,b)=\Big(\frac{1}{24}\Big(|a_1-b_1|^p+2|a_2-b_2|^p+2|a_3-b_3|^p+|(a_4-b_4)|^p+|a_1T(a)-b_1T(b)|^p$$
$$+2|a_2T(a)-b_2T(b)|^p+2|a_3T(a)-b_3T(b)|^p+|a_4T(a)-b_4T(b)|^p$$
$$+|a_1I(a)-b_1I(b)|^p+2|a_2I(a)-b_2I(b)|^p+2|a_3I(a)-b_3I(b)|^p \tag{6}$$
$$+|a_4I(a)-b_4I(b)|^p+|a_1F(a)-b_1F(b)|^p+1|a_2F(a)-b_2F(b)|^p$$
$$+2|a_3F(a)-b_3F(b)|^p+|a_4F(a)-b_4F(b)|^p\Big)\Big)^{1/p}$$

where $p\geq 1$. Especially, when $p=1$, the generalized distance in Eq. (6) reduces to the Hamming distance in Eq. (3); when $p=2$, the generalized distance in Eq. (6) reduces to the Euclidean distance in Eq. (4); when $p=\infty$, the generalized distance in Eq. (6) reduces to the Chebyshev distance in Eq. (5).

Theorem 1. Let that *a*, *b* and *c* be three SVTNNs, then above distance measurements $d_k(a,b)$ $(k=1,2,3,4)$ satisfy the following properties:

(1) $0\leq d_k(a,b)\leq 1$;

(2) $d_k(a,b)=0$ if and only if $a=b$;

(3) $d_k(a,b)=d_k(b,a)$; and

(4) $d_k(a,c)\leq d_k(a,b)+d_k(b,c)$.

Proof. Let $a=\langle[a_1,a_2,a_3,a_4],T(a),I(a),F(a)\rangle$, $b=\langle[b_1,b_2,b_3,b_4],T(b),I(b),F(b)\rangle$ and $c=\langle[c_1,c_2,c_3,c_4],T(c),I(c),F(c)\rangle$. The proof of the generalized distance measurements $d_4(a,b)$ is shown as follows.

(1) According to Definition 2, $0\leq a_1\leq a_2\leq a_3\leq a_4\leq 1$, $0\leq b_1\leq b_2\leq b_3\leq b_4\leq 1$, $T(a)$, $I(a),F(a)\in[0,1]$, and $T(b),I(b),F(b)\in[0,1]$. Since $p\geq 1$, it holds that $|a_1-b_1|^p,|(a_4-b_4)|^p,|a_1T(a)-b_1T(b)|^p$,

$\left|a_4T(a)-b_4T(b)\right|^p, \left|a_1I(a)-b_1I(b)\right|^p, \left|a_4I(a)-b_4I(b)\right|^p, \left|a_1F(a)-b_1F(b)\right|^p, \left|a_4F(a)-b_4F(b)\right|^p \in [0,1]$, and $2\left|a_2-b_2\right|^p, 2\left|a_3-b_3\right|^p, 2\left|a_2T(a)-b_2T(b)\right|^p, 2\left|a_3T(a)-b_3T(b)\right|^p, 2\left|a_2I(a)-b_2I(b)\right|^p, 2\left|a_3I(a)-b_3I(b)\right|^p$, $2\left|a_2F(a)-b_2F(b)\right|^p, 2\left|a_3F(a)-b_3F(b)\right|^p \in [0,2]$. Therefore, $\Big(\left|a_1-b_1\right|^p+2\left|a_2-b_2\right|^p+2\left|a_3-b_3\right|^p+\left|a_4-b_4\right|+$ $\left|a_1T(a)-b_1T(b)\right|^p+2\left|a_2T(a)-b_2T(b)\right|^p+2\left|a_3T(a)-b_3T(b)\right|^p+\left|a_4T(a)-b_4T(b)\right|^p+\left|a_1I(a)-b_1I(b)\right|^p+$ $2\left|a_2I(a)-b_2I(b)\right|^p+2\left|a_3I(a)-b_3I(b)\right|^p+\left|a_4I(a)-b_4I(b)\right|^p+\left|a_1F(a)-b_1F(b)\right|^p+2\left|a_2F(a)-b_2F(b)\right|^p+$ $2\left|a_3F(a)-b_3F(b)\right|^p+\left|a_4F(a)-b_4F(b)\right|^p\Big) \in [0,24]$. Thus, according to Eq. (6), $d_4(a,b) \in [0,1]$, *i.e*. the property (1) holds.

(2) According to Definition 10, if $a=b$, we have that $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$, $T(a)=T(b)$, $I(a)=I(b)$, and $F(a)=F(b)$. Thus, $d_4(a,b)=0$.

If $d_4(a,b)=0$, we can obtain that $\left|a_1-b_1\right|=2\left|a_2-b_2\right|=2\left|a_3-b_3\right|=\left|a_4-b_4\right|=\left|a_1T(a)-b_1T(b)\right|=$ $2\left|a_2T(a)-b_2T(b)\right|=2\left|a_3T(a)-b_3T(b)\right|=\left|a_4T(a)-b_4T(b)\right|=\left|a_1I(a)-b_1I(b)\right|=2\left|a_2I(a)-b_2I(b)\right|=$ $2\left|a_3I(a)-b_3I(b)\right|=\left|a_4I(a)-b_4I(b)\right|=\left|a_1F(a)-b_1F(b)\right|=2\left|a_2F(a)-b_2F(b)\right|=2\left|a_3F(a)-b_3F(b)\right|=$ $\left|a_4F(a)-b_4F(b)\right|=0$. Hence, it is true that $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$. Moreover, according to Definition 2, a_1, a_2, a_3 and a_4 are not zero simultaneously, we have that $T(a)=T(b)$, $I(a)=I(b)$, and $F(a)=F(b)$. Therefore, according to Definition 10, $a=b$ holds.

(3) It is clear that $\left|a_1-b_1\right|=\left|b_1-a_1\right|$, $2\left|a_2-b_2\right|=2\left|b_2-a_2\right|$, $2\left|a_3-b_3\right|=2\left|b_3-a_3\right|$, $\left|a_4-b_4\right|=\left|b_4-a_4\right|$, $\left|a_1T(a)-b_1T(b)\right|=\left|b_1T(b)-a_1T(a)\right|$, $2\left|a_2T(a)-b_2T(b)\right|=2\left|b_2T(b)-a_2T(a)\right|$, $2\left|a_3T(a)-b_3T(b)\right|=$ $2\left|b_3T(b)-a_3T(a)\right|$, $\left|a_4T(a)-b_4T(b)\right|=\left|b_4T(b)-a_4T(a)\right|$, $\left|a_1I(a)-b_1I(b)\right|=\left|b_1I(b)-a_1I(a)\right|$, $2\left|a_2I(a)-b_2I(b)\right|=2\left|b_2I(b)-a_2I(a)\right|$, $2\left|a_3I(a)-b_3I(b)\right|=2\left|b_3I(b)-a_3I(a)\right|$, $\left|a_4I(a)-b_4I(b)\right|=$ $\left|b_4I(b)-a_4I(a)\right|$, $\left|a_1F(a)-b_1F(b)\right|=\left|b_1F(b)-a_1F(a)\right|$, $2\left|a_2F(a)-b_2F(b)\right|=2\left|b_2F(b)-a_2F(a)\right|$, $2\left|a_3F(a)-b_3F(b)\right|=2\left|b_3F(b)-a_3F(a)\right|$ and $\left|a_4F(a)-b_4F(b)\right|=\left|b_4F(b)-a_4F(a)\right|$. Therefore, according to Eq. (6), $d_4(a,b)=d_4(b,a)$.

(4) According to Eq. (6), we have that $d_4(a,c)=\left(\frac{1}{24}\left(|a_1-c_1|^p+2|a_2-b_2|^p+2|a_3-b_3|^p+|(a_4-c_4)|^p+\right.\right.$ $+|a_1T(a)-c_1T(c)|^p+2|a_2T(a)-b_2T(b)|^p+2|a_3T(a)-b_3T(b)|^p+|a_4T(a)-c_4T(c)|^p+|a_1I(a)-c_1I(c)|^p+$ $2|a_2I(a)-b_2I(b)|^p+2|a_3I(a)-b_3I(b)|^p+|a_4I(a)--c_4I(c))|^p+|a_1F(a)-c_1F(c)|^p+2|a_2F(a)-b_2F(b)|^p+$ $\left.\left.2|a_3F(a)-b_3F(b)|^p+|a_4F(a)-c_4I(c))|^p\right)\right)^{1/p}$. According to the Minkowski's inequality in Lemma 1, $d_4(a,c)=\left(\frac{1}{24}\left(|a_1-b_1+b_1-c_1|^p+2|a_2-b_2+b_2-c_2|+2|a_3-b_3+b_3-c_3|^p+|a_4-b_4+b_4-c_4|^p+|a_1T(a)-\right.\right.$ $b_1T(b)+b_1T(b)-c_1T(c)|^p+2|a_2T(a)-b_2T(b)+b_2T(b)-c_2T(c)|^p+2|a_3T(a)-b_3T(b)+b_3T(b)-c_3T(c)|^p+$ $|a_4T(a)-b_4T(b)+b_4T(b)-c_4T(c)|^p+|a_1I(a)-b_1I(b)+b_1I(b)-c_1I(c)|^p+2|a_2I(a)-b_2I(b)+b_2I(b)$ $-c_2I(c)|^p+2|a_3I(a)-b_3I(b)+b_3I(b)-c_3I(c)|+|a_4I(a)-b_4I(b)+b_4I(b)-c_4I(c)|^p+|a_1F(a)-b_1F(b)+$ $b_1F(b)-c_1F(c)|^p+2|a_2F(a)-b_2F(b)+b_2F(b)-c_2F(c)|^p+2|a_3F(a)-b_3F(b)+b_3F(b)-c_3F(c)|^p+$ $\left.\left.|a_4F(a)-b_4F(b)+b_4F(b)-c_4F(c)|^p\right)\right)^{1/p}\leq\left(\frac{1}{24}\left(|a_1-b_1|^p+2|a_2-b_2|^p+2|a_3-b_3|^p+|a_4-b_4|^p+|a_1T(a)-\right.\right.$ $b_1T(b)|^p+2|a_2T(a)-b_2T(b)|^p+2|a_3T(a)-b_3T(b)|^p+|a_4T(a)-b_4T(b)|^p+|a_1I(a)-b_1I(b)|^p$ $+2|a_2I(a)-b_2I(b)|^p+2|a_3I(a)-b_3I(b)|^p+|a_4I(a)-b_4I(b)|^p+|a_1F(a)-b_1F(b)|^p++2|a_2F(a)-b_2F(b)|^p$ $\left.\left.+2|a_3F(a)-b_3F(b)|^p+|a_4F(a)-b_4F(b)|^p\right)\right)^{1/p}+\left(\frac{1}{24}\left(|c_1-b_1|^p+2|c_2-b_2|^p+2|c_3-b_3|^p+|c_4-c_4|^p+\right.\right.$ $|c_1T(c)-b_1T(b)|^p+2|c_2T(c)-b_2T(b)|^p+2|c_3T(c)-b_3T(b)|^p+|c_4T(c)-b_4T(b)|^p+|c_1I(c)-b_1I(b)|^p+$ $2|c_2I(c)-b_2I(b)|^p+|2(c_3I(c)-b_3I(b))|^p+|c_4I(c)-b_4I(b)|^p+|c_1F(c)-b_1F(b)|^p+2|c_2F(c)-b_2F(b)|^p+$ $\left.\left.2|c_3F(c)-b_3F(b)|^p+|c_4F(c)-b_4F(b)|^p\right)\right)^{1/p}=d_4(a,c)+d_4(c,b)$. Thus, the generalized distance measurement satisfies the property that $d_4(a,c)\leq d_4(a,b)+d_4(b,c)$.

Furthermore, as mentioned in Definition 11, when $p=1$, the generalized distance in Eq. (6) reduces to the Hamming distance in Eq. (3); when $p=2$, the generalized distance in Eq. (6) reduces to the Euclidean distance in Eq. (4); when $p=\infty$, the generalized distance in Eq. (6) reduces to the Chebyshev distance in Eq.

(5). Since the generalized distance satisfies the properties (1)-(4), the Hamming distance $d_1(a,b)$, the Euclidean distance $d_2(a,b)$, and the Chebyshev distance $d_3(a,b)$ satisfy the properties (1)-(4).

Therefore, Theorem 1 holds.

4 The TODIM method with SVTNNs for medical treatment options selection

Here we present a TODIM method for medical treatment options selection with SVTNNs.

Assume there are m medical treatment options $A=\{A_1, A_2, \cdots, A_m\}$ which are evaluated by the physicians concerning n criteria $C=\{C_1, C_2, \cdots, C_n\}$. The evaluation values provided by the physicians are transformed into SVTNNs, and U_{ij} represents the evaluation value for the treatment options A_i $(i=1,2,\cdots,m)$ under the criterion C_j $(j=1,2,\cdots,n)$. The decision matrix, which is transformed from the evaluation values provided by the physicians, can be denoted as U:

$$U=\begin{pmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ U_{21} & U_{22} & \cdots & U_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ U_{m1} & U_{m2} & \cdots & U_{mn} \end{pmatrix}.$$

Since each criterion has distinct weight, the weight vector of criteria is $w=(w_1,w_2,\cdots,w_n)^T$, where $w_j \geq 0$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n} w_j = 1$.

In the following part, the TODIM method to rank the medical treatment options and select the best one is proposed based upon the proposed comparison method and distance measurements. The procedure of the method is as follows:

Step 1: Normalize the decision matrix.

Benefit criterion and cost criterion may exist in an individual MCDM problem simultaneously. To unify all criteria, it is necessary to normalize the evaluation value under the cost criterion. It should be noted that normalization is needless if all criteria are benefit ones. If the criterion C_j is a cost one, the evaluation value $U_{ij} = \left\langle \left[u_1^{ij}, u_2^{ij}, u_3^{ij}, u_4^{ij} \right], T\left(u^{ij}\right), \ I\left(u^{ij}\right), F\left(u^{ij}\right) \right\rangle$ of the treatment option A_i under the criterion C_j should be normalized utilizing the following equation:

$$N_{ij} = \left\langle \left[1 - U_4^{ij}, 1 - U_3^{ij}, 1 - U_2^{ij}, 1 - U_1^{ij} \right], T\left(u^{ij}\right), I\left(u^{ij}\right), F\left(u^{ij}\right) \right\rangle . \tag{7}$$

It is obvious that the normalized values N_{ij} are also SVTNNs.

Step 2: Obtain score values.

Utilizing the score function in Eq. (1), we can obtain the score value $s\left(N_{ij}\right)$ $\left(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\right)$ of the treatment option A_i concerning the criterion C_j.

Step 3: Obtain accuracy values.

Utilizing the accuracy function in Eq. (2), we can obtain the accuracy value $h\left(N_{ij}\right)$ $\left(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\right)$ of the treatment option A_i concerning the criterion C_j.

Step 4: Obtain distance matrices.

The generalized distance measurement is utilized here seeing that the Hamming, Euclidean, and Chebyshev distance measurements are special cases of the generalized distance measurement. Based on the generalized distance in Definition 11, we can obtain the distance d_{ir}^{j} $\left(i = 1, 2, \cdots, m; r = 1, 2, \cdots, m; j = 1, 2, \cdots, n\right)$ between two treatment options A_i and A_r concerning the criterion C_j.

Step 5: Obtain partial dominance matrices.

The distance matrix Φ^j under the criterion C_j is composed of partial dominance degrees Φ_{ir}^{j} $\left(i = 1, 2, \cdots, m; r = 1, 2, \cdots, m; j = 1, 2, \cdots, n\right)$ of the treatment option A_i over the treatment option A_r concerning the criterion C_j and Φ_{ir}^{j} can be obtained by the following formula:

$$\Phi_{ir}^{j}=\begin{cases}\sqrt{\dfrac{\omega_{ju}d_{ir}^{j}}{\sum_{j=1}^{n}\omega_{ju}}}, & N_{ij}\succ N_{rj}\,\text{or}\,N_{ij}>N_{rj}\\ 0, & N_{ij}\sim N_{rj}\,\text{or}\,N_{ij}=N_{rj}\\ \dfrac{-1}{t}\sqrt{\dfrac{\sum_{j=1}^{n}\omega_{ju}d_{ir}^{j}}{\omega_{ju}}}, & N_{ij}\prec N_{rj}\,\text{or}\,N_{ij}<N_{rj}\end{cases}, \tag{8}$$

where $\omega_{ju}=\dfrac{\omega_j}{\omega_u}$ and $\omega_u=\max\left(\omega_j\right)$ $\left(j=1,2,\cdots,n\right)$. The comparison relation between N_{ij} and N_{rj} can be obtained according to the comparison method in Definition 10 utilizing the score values and accuracy values. If $N_{ij}\succ N_{rj}$ or $N_{ij}>N_{rj}$, it represents a gain; if $N_{ij}=N_{rj}$, it is breakeven; if $N_{ij}\prec N_{rj}$ or $N_{ij}<N_{rj}$, it represents a loss. The parameter t in the situation where $N_{ij}\prec N_{rj}$ or $N_{ij}<N_{rj}$ is the decay factor of the loss and $t>0$.

Step 6: Obtain the final dominance matrix Φ.

The final dominance matrix Φ is composed of dominance degrees. The dominance degree Φ_{ir} $\left(i=1,2,\cdots,m;r=1,2,\cdots,m\right)$ denotes the degree that the treatment option A_i is better than the treatment option A_r and can be obtained by:

$$\Phi_{ir}=\sum_{j=1}^{n}\Phi_{ir}^{j}\,. \tag{9}$$

Step 7: Calculate the global value of each treatment option.

The global value ξ_i $\left(i=1,2,\cdots,m\right)$ of the treatment option A_i can be obtained by:

$$\xi_i=\frac{\sum_{r=1}^{m}I_{ir}-\min\limits_{1\le i\le m}\left(\sum_{r=1}^{m}I_{ir}\right)}{\max\limits_{1\le i\le m}\left(\sum_{r=1}^{m}I_{ir}\right)-\min\limits_{1\le i\le m}\left(\sum_{r=1}^{m}I_{ir}\right)}\,. \tag{10}$$

Step 8: Rank the treatment options.

The treatment options can be ranked based on the global value of each treatment option. The bigger the global value of an individual treatment option is, the better the option will be.

5 A numerical example for medical treatment options selection

In this section, a numerical example for medical treatment options selection with SVTNNs is used to demonstrate the applicability of the proposed method.

We utilize the proposed method to tackle a selection problem of medical treatment options. Assume that a physician wants to find a medical treatment option for a particular patient with verruca plantaris. There are five medical treatment options: (1) A_1 is carbon dioxide laser; (2) A_2 is high frequency therapeutic instrument; (3) A_3 is microwave therapeutic instrument; (4) A_4 is cryotherapy; and (5) A_5 is apoxesis. They will be evaluated by the physician from the three aspects: (1) C_1 is the probability of a cure; (2) C_2 is severity of the side effects; and (3) C_3 is cost. The weight vector of the three criteria is supposed as $(0.4, 0.3, 0.3)^T$. Furthermore, Table 1 shows the decision matrix U which is transformed from the evaluation values of the physician.

Table 1. The decision matrix transformed from the evaluation values of the experts

	C_1	C_2	C_3
A_1	$\langle[0.3,0.4,0.5,0.7],0.5,0.4,0.3\rangle$	$\langle[0.4,0.5,0.7,0.8],0.5,0.3,0.7\rangle$	$\langle[0.2,0.3,0.8,0.9],0.9,0.1,0.5\rangle$
A_2	$\langle[0.2,0.5,0.6,0.9],0.8,0.2,0.4\rangle$	$\langle[0.2,0.4,0.6,0.8],0.1,0.2,0.3\rangle$	$\langle[0.3,0.4,0.7,0.8],0.5,0.3,0.8\rangle$
A_3	$\langle[0.3,0.5,0.8,0.8],0.7,0.2,0.5\rangle$	$\langle[0.1,0.2,0.5,0.7],0.2,0.5,0.8\rangle$	$\langle[0.4,0.5,0.6,0.7],0.8,0.2,0.6\rangle$
A_4	$\langle[0.3,0.5,0.8,0.8],0.7,0.2,0.5\rangle$	$\langle[0.2,0.3,0.7,0.8],0.9,0.8,0.7\rangle$	$\langle[0.2,0.3,0.4,0.6],0.5,0.4,0.2\rangle$
A_5	$\langle[0.4,0.6,0.7,0.8],0.3,0.5,0.6\rangle$	$\langle[0.3,0.5,0.7,0.9],0.9,0.7,0.5\rangle$	$\langle[0.3,0.4,0.5,0.6],0.9,0.3,0.6\rangle$

5.1 The steps of the proposed method

Step 1: Normalize the decision matrix.

Since the criterion C_1 is a benefit one while the criteria C_2 and C_3 are cost ones, the decision matrix needs to be normalized utilizing Eq. (7). Table 2 lists the normalized decision matrix.

Table 2. The normalized decision matrix

	C_1	C_2	C_3
A_1	$\langle[0.3,0.4,0.5,0.7],0.5,0.4,0.3\rangle$	$\langle[0.2,0.3,0.5,0.6],0.5,0.3,0.7\rangle$	$\langle[0.1,0.2,0.7,0.8],0.9,0.1,0.5\rangle$
A_2	$\langle[0.2,0.5,0.6,0.9],0.8,0.2,0.4\rangle$	$\langle[0.2,0.4,0.6,0.8],0.1,0.2,0.3\rangle$	$\langle[0.2,0.3,0.6,0.7],0.5,0.3,0.8\rangle$
A_3	$\langle[0.3,0.5,0.8,0.8],0.7,0.2,0.5\rangle$	$\langle[0.3,0.5,0.8,0.9],0.2,0.5,0.8\rangle$	$\langle[0.3,0.4,0.5,0.6],0.8,0.2,0.6\rangle$
A_4	$\langle[0.3,0.5,0.8,0.8],0.7,0.2,0.5\rangle$	$\langle[0.2,0.3,0.7,0.8],0.9,0.8,0.7\rangle$	$\langle[0.4,0.6,0.7,0.8],0.5,0.4,0.2\rangle$
A_5	$\langle[0.4,0.6,0.7,0.8],0.3,0.5,0.6\rangle$	$\langle[0.1,0.3,0.5,0.7],0.9,0.7,0.5\rangle$	$\langle[0.4,0.5,0.6,0.7],0.9,0.3,0.6\rangle$

Step 2: Obtain score values.

Utilizing Eq. (1), we can obtain the score value of an individual treatment option concerning each criterion, and these score values are shown in Table 3.

Table 3. The score value of each treatment option concerning each criterion

	C_1	C_2	C_3
A_1	0.095	0.04	0.27
A_2	0.275	0.1	0.0225
A_3	0.24	-0.188	0.18
A_4	0.24	-0.1	0.1562
A_5	-0.0937	0	0.1925

Step 3: Obtain accuracy values.

Utilizing Eq. (2), we can obtain the accuracy value of each treatment option concerning each criterion.

Table 4. The accuracy value of each treatment option concerning each criterion

	C_1	C_2	C_3
A_1	0.057	-0.056	0.198
A_2	0.242	-0.16	-0.0945
A_3	0.144	-0.375	0.126
A_4	0.144	0.34	0.1125
A_5	-0.2188	0.272	0.2475

Step 4: Obtain distance matrices.

Here for ease of computation, we assume that $p=1$. Utilizing Eq. (6), the distance matrix D_j $(j=1,2,3)$ concerning the criterion C_j can be obtained as:

$$D_1=\begin{pmatrix} 0 & 0.1208 & 0.145 & 0.145 & 0.145 \\ 0.1208 & 0 & 0.0717 & 0.0717 & 0.1833 \\ 0.145 & 0.0717 & 0 & 0 & 0.1475 \\ 0.145 & 0.0717 & 0 & 0 & 0.1475 \\ 0.145 & 0.1833 & 0.1475 & 0.1475 & 0 \end{pmatrix},\quad D_2=\begin{pmatrix} 0 & 0.1 & 0.1825 & 0.175 & 0.1092 \\ 0.1 & 0 & 0.1958 & 0.2417 & 0.1608 \\ 0.1825 & 0.1958 & 0 & 0.1758 & 0.2058 \\ 0.175 & 0.2417 & 0.1758 & 0 & 0.115 \\ 0.1092 & 0.1608 & 0.2058 & 0.115 & 0 \end{pmatrix},$$

and $$D_3=\begin{pmatrix} 0 & 0.1271 & 0.1313 & 0.1796 & 0.1513 \\ 0.1271 & 0 & 0.0942 & 0.1567 & 0.1242 \\ 0.1313 & 0.0942 & 0 & 0.1333 & 0.0925 \\ 0.1796 & 0.1567 & 0.1333 & 0 & 0.1383 \\ 0.1513 & 0.1242 & 0.0925 & 0.1383 & 0 \end{pmatrix}.$$

Step 5: Obtain partial dominance matrices.

Utilizing Eq. (8) (Suppose $t=1$ [79]), the partial dominance matrix Φ^j $(j=1,2,3)$ concerning the criterion C_j can be obtained as:

$$\Phi^1=\begin{pmatrix} 0 & -0.5496 & -0.6021 & -0.6021 & 0.2408 \\ 0.2198 & 0 & 0.1693 & 0.2708 & 0.1732 \\ 0.2408 & -0.4233 & 0 & 0 & 0.2429 \\ 0.2408 & -0.4233 & 0 & 0 & 0.2429 \\ -0.6021 & -0.677 & -0.6702 & -0.6702 & 0 \end{pmatrix},$$

$$\Phi^2 = \begin{pmatrix} 0 & -0.5774 & 0.234 & 0.2291 & 0.181 \\ 0.1732 & 0 & 0.2424 & 0.2693 & 0.2197 \\ -0.78 & -0.8079 & 0 & -0.7656 & -0.8283 \\ -0.7638 & -0.8975 & 0.2297 & 0 & -0.6191 \\ -0.6032 & -0.7322 & 0.2485 & 0.1857 & 0 \end{pmatrix},$$ and

$$\Phi^3 = \begin{pmatrix} 0 & 0.1953 & 0.1984 & 0.2321 & 0.213 \\ -0.6509 & 0 & -0.5603 & -0.7226 & -0.6433 \\ -0.6614 & 0.1681 & 0 & 0.2 & -0.5553 \\ -0.7737 & 0.2168 & -0.6667 & 0 & -0.6791 \\ -0.71 & 0.193 & 0.1666 & 0.2037 & 0 \end{pmatrix}.$$

Step 6: Obtain the final dominance matrix.

Utilizing Eq. (9), the final dominance matrix Φ can be obtained as:

$$\Phi = \begin{pmatrix} 0 & -0.9317 & -0.1697 & -0.1408 & 0.6348 \\ -0.2578 & 0 & -0.1486 & -0.2841 & -0.1529 \\ -1.2006 & -1.0632 & 0 & -0.5656 & -1.1407 \\ -1.12966 & -1.104 & -0.437 & 0 & -1.0553 \\ -1.9154 & -1.2162 & -0.1922 & -0.2178 & 0 \end{pmatrix}.$$

Step 7: Calculate the global value of each treatment option.

The global value ξ_i of the treatment option A_i can be obtained as: $\xi_1 = 1$, $\xi_2 = 0.9298$, $\xi_3 = 0$, $\xi_4 = 0.0229$, and $\xi_5 = 0.1274$.

Step 8: Rank the treatment options.

Since $\xi_1 > \xi_2 > \xi_5 > \xi_4 > \xi_3$, the ranking order of the five treatment options is $A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$. Thus, the best treatment option is A_1.

5.2The influences of the parameter

In this subsection, we investigate and discuss the influences of the parameter t in Eq. (8) and p in the generalized distance measurement in Eq. (6) in detail.

Firstly, the influence of t on the shape of the prospect value function is verified and discussed. Fig. 1 depicts the prospect value functions using different values of t, *i.e.* $t = 1$ [79] and $t = 2.5$ [70]. It can be seen from

Section 5.1 that the score values of two SVTNNs in the decision matrix are same if and only if these two SVTNNs are same. That is to say, the disparity between any two SVTNNs in this numerical example can be reflected by the difference between their score values. Thus, in Fig. 1, the horizontal axis is the difference between any two score values concerning the same criterion and the vertical axis is the corresponding partial dominance degrees.

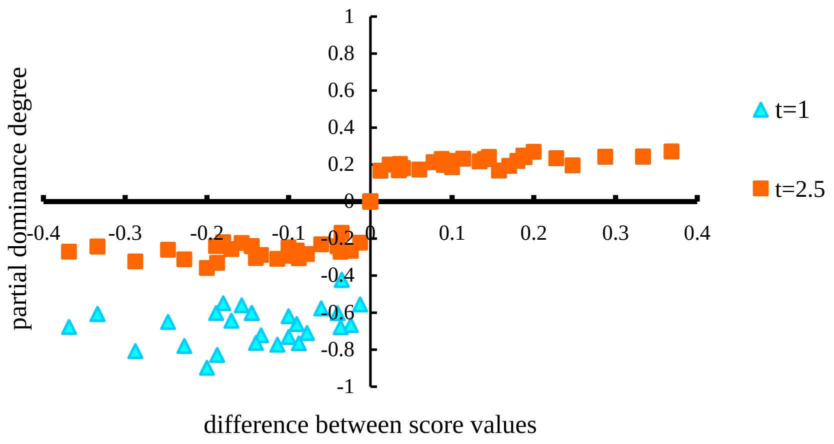

Fig. 1. Prospect value functions using $t=1$ and $t=2.5$

As presented in Fig. 1, the shape of the prospect value function in the third quadrant is influenced by the value of the parameter *t* while that in the first quadrant is immune to the value of *t*. Moreover, the partial dominance obtained by $t=2.5$ is bigger than the corresponding partial dominance obtained by $t=1$ when the difference of score values is negative. The reason for this phenomenon is given as follows. The value of *t* denotes the degree that the losses are attenuated when $t>1$. The greater the value of *t* is, the bigger the degree that the losses are attenuated will be. Thus, it is reasonable that the shape of the prospect value function when $t=1$ is deeper than that when $t=2.5$.

Secondly, we investigate the influence of the parameter *t* on the ranking order of the treatment options by obtaining the ranking orders with different values of *t*. As the value of *t* changes from 0.001 to 50, the corresponding ranking order of the treatment options can be obtained. Table 5 lists the value of *t*, the corresponding global values, and the ranking order of the treatment options.

Table 5. Ranking orders of the treatment options with different values of t

t	global value ξ_i	Ranking order
$t=0.001$	$\xi_1=1$, $\xi_2=0.9013$ $\xi_3=0.0005$, $\xi_4=0$, $\xi_5=0.114$	$A_1 \succ A_2 \succ A_5 \succ A_3 \succ A_4$
$t=0.01$	$\xi_1=1$, $\xi_2=0.9016$ $\xi_3=0.0002$, $\xi_4=0$, $\xi_5=0.1139$	$A_1 \succ A_2 \succ A_5 \succ A_3 \succ A_4$
$t=0.1$	$\xi_1=1$, $\xi_2=0.905$ $\xi_3=0$, $\xi_4=0.0025$, $\xi_5=0.1153$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$t=0.5$	$\xi_1=1$, $\xi_2=0.9177$ $\xi_3=0$, $\xi_4=0.0129$, $\xi_5=0.1215$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$t=1$	$\xi_1=1$, $\xi_2=0.9298$ $\xi_3=0$, $\xi_4=0.0229$, $\xi_5=0.1274$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$t=20$	$\xi_1=1$, $\xi_2=0.9977$ $\xi_3=0$, $\xi_4=0.0786$, $\xi_5=0.1604$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$t=24$	$\xi_1=1$, $\xi_2=0.9998$ $\xi_3=0$, $\xi_4=0.0803$, $\xi_5=0.1614$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$t=25$	$\xi_1=0.9998$, $\xi_2=1$ $\xi_3=0$, $\xi_4=0.0806$, $\xi_5=0.1616$	$A_2 \succ A_1 \succ A_5 \succ A_4 \succ A_3$
$t=50$	$\xi_1=0.9945$, $\xi_2=1$ $\xi_3=0$, $\xi_4=0.0845$, $\xi_5=0.1633$	$A_2 \succ A_1 \succ A_5 \succ A_4 \succ A_3$

As shown in Table 5, the ranking order of the treatment options may be different when the value of t

changes. When $t \le 0.01$ the ranking orders are same and the best treatment option is A_1 and the worst one is A_4. The ranking orders are same when t changes from 0.1 to 24. The best treatment option when $0.1 \le t \le 24$ is same to that when $t \le 0.01$ while the worst one when $0.1 \le t \le 24$ is different from that when $t \le 0.01$. And when $0.1 \le t \le 24$, A_3 is the worst treatment option. Moreover, the best treatment option becomes A_2 and the worst one remains A_3 when $t \ge 25$. The reason for these differences is provided as follows. From Eq. (8), we can see that the losses are amplified when $t < 1$ and they are attenuated when $t > 1$. When $t \le 0.01$, the losses are amplified, and eventually the global value of A_4 becomes smaller than that of A_3. When $0.1 \le t < 1$, the losses are amplified but the degree of amplification is smaller than the degree when $t \le 0.01$, which makes the global value of A_4 become bigger than that of A_3. When $1 < t \le 24$, the losses are attenuated, and the global value of A_4 remains bigger than that of A_3. When $t \ge 25$, the losses are attenuated and the degree of attenuation is bigger than the degree when $1 < t \le 24$, which makes the global value of A_2 becomes bigger than that of A_1. Thus, it is reasonable that the value of *t* influences the ranking order of the treatment options.

Thirdly, we explore the influence of the parameter *p* on the final ranking order of the treatment options with different values of *p*. Table 6 shows the value of *p*, the corresponding global values (suppose $t = 1$).

Table 6. Global values of treatment options with different values of *p*

p	global value ξ_i	Ranking order
$p = 1$	$\xi_1 = 1, \xi_2 = 0.905$ $\xi_3 = 0, \xi_4 = 0.0025, \xi_5 = 0.1153$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$p = 2$	$\xi_1 = 1, \xi_2 = 0.9260$ $\xi_3 = 0, \xi_4 = 0.0010, \xi_5 = 0.1134$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$

$p=3$	$\xi_1=1, \xi_2=0.9282$ $\xi_3=0, \xi_4=0.0048, \xi_5=0.1014$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$p=6$	$\xi_1=1, \xi_2=0.9393$ $\xi_3=0, \xi_4=0.0227, \xi_5=0.0686$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$
$p=\infty$	$\xi_1=1, \xi_2=0.9051$ $\xi_3=0, \xi_4=0.0284, \xi_5=0.1231$	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$

As presented in Table 6, the ranking order of the treatment options remains same when the value of *p* changes. In other words, the ranking order of the treatment options is unacted on the value of the parameter *p*. The best treatment option is A_1 and the worst one is A_3.

In generally, the parameter *t* in Eq. (8) influences the shape of the prospect function in the third quadrant. Furthermore, the parameter *t* may influence the ranking order obtained by the proposed method while the parameter *p* in the generalized distance measurement in Eq. (6) cannot. The proposed method can be deemed as a flexible one considering the influence of *t*.

5.3 Comparative analysis

We conduct a comparative analysis to verify and discuss the feasibility of the proposed method in this subsection. Two cases are included in this comparative analysis. In the first case, the proposed method is compared to the method that was outlined in Ref. [78] using trapezoidal intuitionistic fuzzy information. In the second case, the proposed method is compared to the methods that were proposed by Deli and Subas [36] under SVTNN environments.

Case 1: comparative analysis under trapezoidal intuitionistic fuzzy environments.

Krohling *et al*. [78] take into account the risk preference of decision makers, and developed a MCDM method for problems with trapezoidal intuitionistic fuzzy information based on fuzzy TODIM method. The proposed method and the method proposed by Krohling *et al*. [78] are utilized to solve MCDM problems in Ref. [78]. Table 7 lists the ranking orders of the proposed TODIM method and the method proposed by Krohling *et al*. [78] when $t=1$ and $t=2.5$.

Table 7. The ranking orders of different methods

Method	Ranking order	The best treatment option(s)	The worst treatment option(s)
Method 1 $(t=1)$ [78]	$A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 2 $(t=2.5)$ [78]	$A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
The proposed method $(t=1)$	$A_2 \succ A_1 \succ A_3 \succ A_4 \succ A_5$	A_2	A_5
The proposed method $(t=2.5)$	$A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$	A_2	A_5

From Table 7, the best treatment option of these four methods is A_2 while the worst ones are different. A_1 is the worst treatment option of the method proposed by Krohling *et al*. [78] when $t=1$ and $t=2.5$ while A_5 is the worst one of the proposed method when $t=1$ and $t=2.5$. Furthermore, the ranking orders of the method proposed by Krohling *et al*. [78] when $t=1$ and $t=2.5$ are same while the proposed method obtains two different ranking orders when $t=1$ and $t=2.5$. The reasons for these differences are illustrated as follows. the comparison methods and distance measurements utilized in the MCDM method proposed by Krohling *et al*. [78] and the proposed MCDM method are greatly distinct even though both of these two MCDM methods consider the risk preference of decision makers. Furthermore, the proposed method makes use of the intuitionistic fuzzy index $\pi = 1-\mu-\nu$ besides the degrees of membership μ and non-membership ν while the method proposed by Krohling *et al*. [78] only utilizes μ and ν. It is reasonable that these two MCDM methods may obtain different ranking order when the value of *t* is same.

The ranking order of the proposed method meets decision makers' actual preferences better compared to that of the method proposed by Krohling *et al*. [78]. Moreover, as presented in Section 5.2, the value of t may influence the ranking order of the proposed method. When $t \geq 1$, the losses are attenuated. However, the degrees of attenuation are different when $t = 1$ and $t = 2.5$. Therefore, the ranking orders of the proposed method are different when $t = 1$ and $t = 2.5$. The ranking orders of the method proposed by Krohling *et al*. [78] are same when $t = 1$ and $t = 2.5$. It is because that the difference between the degrees of attenuation when $t = 1$ and $t = 2.5$ is not big enough to make the ranking orders become distinct.

Overall, the proposed method can be effectively used to solve MCDM problems under trapezoidal intuitionistic fuzzy environments, and the ranking orders obtained by the proposed method are closer to decision makers' actual preferences than those obtained by the extant trapezoidal intuitionistic fuzzy methods like the method proposed by Krohling *et al*. [78].

Case 2: comparative analysis under SVTNN environments.

Deli and Subas [36] constructed a MCDM method by utilized a single valued trapezoidal neutrosophic weighted aggregation operator with the parameter γ and the comparison method in Definition 7. The proposed method and the method proposed by Deli and Subas [36] are utilized to solve the selection problem of medical treatment options (a MCDM problem) in Section 5.1. Table 8 lists the ranking orders of the proposed TODIM method $(t = 1)$ and the method proposed by Deli and Subas [36].

Table 8. The ranking orders of different methods under SVTNN environments

Method	Ranking order	The best treatment option(s)	The worst treatment option(s)
Method 3 $(\gamma = 1)$ [36]	$A_1 \succ A_4 \succ A_5 \succ A_2 \succ A_3$	A_1	A_3
Method 4 $(\gamma = 2)$ [36]	$A_4 \succ A_1 \succ A_5 \succ A_2 \succ A_3$	A_4	A_1

The proposed method	$A_1 \succ A_2 \succ A_5 \succ A_4 \succ A_3$	A_1	A_3

From Table 8, the best treatment option of the method proposed by Deli and Subas [36] when $\gamma = 2$ is A_4 while the worst one is A_1. A_1 is the best treatment option of the method proposed by Deli and Subas [36] when $\gamma = 1$ and the proposed TODIM method while A_3 is the worst treatment option of these two methods. Furthermore, different ranking orders of the other three treatment options (*i.e.* A_2, A_4, A_5) are obtained by these two methods. The reasons for the differences are that the aggregation operator calculates the weighted arithmetic average value of elements when $\gamma = 1$ while the aggregation operator obtains the weighted arithmetic average value of the square values of elements when $\gamma = 2$. It is clear that the aggregation operator with $\gamma = 2$ enlarges the differences between treatment options while the aggregation operator with $\gamma = 1$ does not. Thus, different ranking orders are obtained by these two methods. Moreover, in practice, decision makers are bounded rational, it is indispensable to consider their risk preference in decision-making process. The proposed TODIM method takes into account the decision makers' bounded rationality while the methods proposed by Deli and Subas [36] assume that the decision makers are perfectly rational. Therefore, the ranking orders of the three methods are different. In addition, it is true that the proposed method is effective in tackling the selection problems of medical treatment options with SVTNNs.

Generally speaking, the proposed method is applicable and feasible to tackle MCDM problems like the selection problems of medical treatment options under trapezoidal intuitionistic fuzzy environments and SVTNN environments. However, the extant trapezoidal intuitionistic fuzzy methods cannot solve MCDM problems under SVTNN environments. From this perspective, the proposed method is a flexible one. Furthermore, the proposed method overcomes the shortcomings exist in the prior MCDM methods with SVTNNs, and its ranking order better corresponds with decision makers' real preferences than those obtained by the previous MCDM methods.

6 Conclusion

SVTNNs can be used to reflect fuzzy information in the selection of medical treatment options. Moreover, TODIM methods under fuzzy environments can be utilized to construct MCDM methods to consider the risk preference of decision makers. In this paper, an improved comparison method for SVTNNs was proposed to overcome the deficiencies of the prior comparison methods. Furthermore, we developed several distance measurements for SVTNNs. Subsequently, a TODIM method for the selection problems of medical treatment options with SVTNNs was constructed on the basis of the proposed comparison method and distance measurements. In addition, a numerical example for medical treatment options selection was provided to illustrate the process of the proposed method and the influence of the parameter was discussed. Finally, a comparative analysis was conducted to verify and discuss the feasibility of the proposed method.

The prominent characteristics of this paper are that the proposed method for medical treatment options selection utilizes SVTNNs, which can depict much information and fuzziness in selection processes. Moreover, the comparison method utilized in the proposed method cover the defects of the extant comparison methods. Furthermore, the proposed method for medical treatment options selection extends TODIM method to SVTNN environments, and considers the risk attitude of physicians which makes the decision-making results closer to physicians' actual preferences than the previous methods.

There are two directions for future research: first, the proposed MCDM method can be applied to solve practical issues in other fields, such as purchasing decision-making, and tourism destination selection. Second, the proposed MCDM method does not take into consideration the interrelationships among criteria. It would be interesting to improve the proposed method by introducing independent aggregation operators.

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1.an improved comparison method and several distance measurements for SVTNNs are defined.

2.a novel MCDM method for medical treatment options selection is established based on TODIM method with SVTNNs.

3. a example of the selection of medical treatment options is provided in order to verify the proposed method and the influence of the parameter.