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NEUTROSOPHIC MULTI-OBJECTIVE LINEAR PROGRAMMING

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ABSTRACT
For modeling imprecise and indeterminate data for multi-objective decision making, two different methods: neutrosophic multi-objective linear/non-linear programming, neutrosophic goal programming, which have been very recently proposed in the literatuire. In many economic problems, the well-known probabilities or fuzzy solutions procedures are not suitable because they cannot deal the situation when indeterminacy inherently involves in the problem. In this case we propose a new concept in optimization problem under uncertainty and indeterminacy. It is an extension of fuzzy and intuitionistic fuzzy optimization in which the degrees of indeterminacy and falsity (rejection) of objectives and constraints are simultaneously considered together with the degrees of truth membership (satisfaction/acceptance). The drawbacks of the existing neutrosophic optimization models have been presented and new framework of multi-objective optimization in neutrosophic environment has been proposed. The essence of the proposed approach is that it is capable of dealing with indeterminacy and falsity simultaneously.

1. INTRODUCTION
Multi-objective programming has evolved in the last six decades into a recognized specially of operations research. Its development has occurred primarily in three disciplines, namely, operations research, economics and psychology. In 1955, Gass and Saaty [1] studied the first approach applicable to multi-objective programming problem.

Multi-objective programming and planning is concerned with decision making problem having several conflicting objectives. It is one of the popular methods to deal with decision problems. Multi-objective programming problem may be characterized by an attempt to optimize a set of potentially conflicting objectives as completely as possible in an environment comprised of a set of finite resources, conflicting interest and a set of constraints. When the objective functions and constraints are linear, the multi-objective programming problem is a linear. If any objective function and/ or constraints are nonlinear, then the problem is called as a nonlinear multi-objective programming prolem. Several computational methods have been proposed in the literature for characterizing Pareto optimal solutions depending on the different approaches to scalarize the multi-objective programming problems. The details of multi-objective programming problem can be found in the books authored by Hwang, and Masud [2], M. Zeleny [3], R.E. Steur [4], Chankong and Haimes [5], M. Sakawa [6], Lai, and Hwang [7], K. Miettinen [8], For constructing of a multi-objective linear programming (MOLP) problem, various factors related to the problem should be reflected in the description of the objective functions and the constraints. The objectives functions and constraints may be characterized by many parameters. It is, naturally, recognized that the possible values of these parameters are often imprecisely or ambiguously known to the domain experts. To deal with this uncertainty, researchers employed fuzzy set due to L.A. Zadeh [9]. In 1970, Bellman and Zadeh [10] introduced three basic concepts, namely, fuzzy goal, fuzzy constraint, and fuzzy decision and explored the application of these concepts to decision making processes under fuzziness. In 1978, H. –J. Zimmermann [11] extended his fuzzy linear programming [12] to MOLP. In 1981, H. Leberling [13] studied special nonlinear functions and showed that the resulting nonlinear programming problem can be equivalently transformed into a conventional linear programming problem. In 1981, E.L. Hannan [14] adopted piecewise linear membership function to represent the fuzzy goal of the decision maker and converted the multi-objective programming problem into the ordinary linear programming problem. In 1983, M. Sakawa [6] proposed interactive fuzzy multi-objective programming problem using five types of membership functions, namely, linear, exponential, hyperbolic, hyperbolic inverse, and piecewise linear functions. In the same study, M. Sakawa [6] presented linear fuzzy multi-objective programming problem by combining the use of the bisection method and the linear programming method [15]. K. Miettinen [16] presented an overview of the interactive methods for solving non-linear multi-objective programming problem and introduced the basic features of several methods by providing some theoretical results. Various methods have been proposed in the literature for MOLP to derive a satisfactory solution for decision maker based on their subjective value judgment and preference. Some popular types of such methods are namely, goal programming [17-35], fuzzy goal programming [36-61], interactive programming [2], fuzzy programming [62-66] and interactive fuzzy programming [6].

Many optimization approaches/methods and techniques for modeling and solving fuzzy MOLP problems have been proposed in the literature to deal with decision making situations, which involve fuzzy values in objective functions, parameters, constraints, or goals. However, in the fuzzy model, the degree of non-membership (falsity/ rejection) as independent component due to K. Atanassov [67, 68] has not been incorporated. The principles of fuzzy optimization problems have been critically studied by P. Angellov and proposed intuitionistic fuzzy programming [69, 70] by considering truth membership (acceptance) and falsity (non-membership/rejection) simultaneously. P. Angellov [71] also presented multi-objective optimization in air–conditioning systems based on intuitionistic fuzzy programming method. Thereafter few studies [72-90] have been reported in the literature. In 2005, Pramanik and Roy [91] presented intuitionistic fuzzy goal programming by extending fuzzy goal programming model of Pramanik and Roy [47] in intuitionistic fuzzy environment. Pramanik and Roy [92, 93] also presented intuitionistic fuzzy goal programming to transportation problem and quality control problem respectively.

Multi objective programming in crisp and fuzzy environment have been well developed in order to deal realistic problems. Multi-objective programming in intuitionistic fuzzy environment is still in its infancy. MOLP in fuzzy and intuitionistic fuzzy environment are not capable of dealing with indeterminacy which exists in realistic multi-objective programming problem. So to deal MOLP involving indeterminacy, neutrosophic set studied by F. Smarandache [94, 95, 96, 97] and single valued neutrosophic set [98] are suitable tools. In 2015, Roy and Das [99] proposed neutrosophic optimization approach to solve multi-objective linear programming problem that can be considered as an extension of fuzzy programming [11] and intuitionistic fuzzy optimization [70]. In 2015, Das and Roy [100] proposed multi-objective non-linear programming problem based on neutrosophic optimization technique. Hezam et al. [101] presented Taylor series approximation to solve neutrosophic multi-objective programming problem. Kar et al. [102] applied single valued neutrosophic set theory to generalized assignment problem. Kar et al. [103] also presented neutrosophicmulti-criteria assignment problem. In 2015, Kour and Basu [104] presented neutrosophicreal life transportation problem. In 2016, Thamaraiselvi and Santhi [105] presented neutrosophgic transportation model. In the optimum solution, Thamaraiselvi and Santhi [105] considered that the degrees of indeterminacy and falsity are the same in the optimum level. In 2016, Abdel-Baset et al. [106] presented two models of neutrosophic goal programming. Roy and Das [107] applied neutrosophic goal programming model of Abdel-Baset et al. [106] to bank investment problem. In 2016, S. Pramanik [108] critically studied the results of neutrosophic optimization models presented in [99, 100,101, 106] and presented new direction of research and proposed new framework of neutrosophic linear goal programming. In Pramanik's model [107] falsity membership function and indeterminacy membership functions are minimized while truth membership functions are maximized.

Neutrosophic optimization is an open field for research work. Very little research work has been reported on neutrosophic optimization in the literature. Since in the studies [99, 100], the indeterminacy membership function has been maximized and that is not realistic goal of an organization, new methodology is urgently needed to address the issue. In this paper, new approach to neutrosophic multi-objective programming has been presented by extending Zimmermann's approach [11] in neutrosophic environment. New insight in neutrosophic multi-objective programming has been also introduced by providing the concept of minimizing the indeterminacy membership function in multi-objective linear programming problem.

Remainder of the paper has been organized in the following way. Section 2 presents some basic definitions of neutrosophic sets. Section 3 is devoted to present the proposed framework of neutrosophic multi-objective programming problems. Section 4 presents the conclusion and future direction of research.

2. PRELIMINARIES

We recall some basic definitions related to neutrosophic sets which are important to develop the paper.

2.1 ***Definition: Neutrosophic set*** [94]

Let *Y* be a space of points (objects) with a generic element y $\in Y$. A neutrosophic set S in *Y* is characterized by a truth membership function $T_S(y)$, an indeterminacy membership function $I_S(y)$, and a falsity membership function $F_S(y)$ and is denoted by

$S=\{y,\langle T_S(y), I_S(y), F_S(y)\rangle \mid y \in Y\}$

Here $T_S(y)$, $I_S(y)$ and $F_S(y)$ can be defined as follows:

$T_S : Y \to]^-0, 1^+[$

$I_S : Y \to]^-0, 1^+[$

$F_S : Y \to]^-0, 1^+[$

Here, $T_S(y)$, $I_S(y)$ and $F_S(y)$ are the real standard and non-standard subset of $]^-0, 1^+[$. In general, there is no restriction on $T_S(y)$, $I_S(y)$, and $F_S(y)$. Therefore,

$^-0 \le \operatorname{Inf} T_S(y) + \inf I_S(y) + \inf F_S(y)) \le \operatorname{Sup} T_S(y) + \operatorname{Sup} I_S(y) + \operatorname{Sup} F_S(y) \le 3^+$

2.2*Definition: Single valued neutrosophic set* [98]

Let *Y* be a space of points with generic element $y \in Y$. A single valued neutrosophic set S in *Y* is characterized by a truth-membership function $T_S(y)$, an indeterminacy-membership function $I_S(y)$ and a falsity-membership function $F_S(y)$, for each point y in *Y*, $T_S(y)$, $I_S(y)$, $F_S(y) \in [0, 1]$. When *Y* is continuous, then single-valued neutrosophic set S can be written as

$S = \int_Y < T_S(y), I_S(y), F_S(y) > / y, y \in Y.$

When *Y* is discrete, single-valued neutrosophic set S can be written as follows:

$S = \sum_{i=1}^{n} < T_S(y_i), I_S(y_i), F_S(y_i) > \mid y_i, y_i \in Y$

2.3 Definition: Complement of a single valued neutrosophic set [98]

The complement of a single valued neutrosophic set S is denoted by S^c and is defined by

$T_{S^c}(y) = F_S(y)$; $I_{S^c}(y) = 1 - I_S(y)$; $F_{S^c}(y) = T_S(y)$

2.4 Definition: Equality of two single valued neutrosophic sets [98]

Equality of two single valued neutrosophic sets M and N are equal, written as M= N, if and only if $M \subseteq N$ and $M \supseteq N$.

2.5. Definition: Union of two single valued neutrosophic sets [109]

The union of two single valued neutrosophic sets M and N is a single valued neutrosophic set P, written as $P = M \cup N$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of M and N by $T_P(y) = \max(T_M(y), T_N(y))$; $I_P(y) = \min(I_M(y), I_N(y))$; $F_P(y) = \min(F_M(y), F_N(y))$ for all y in *Y*.

2.6. Definition: Intersection of two single valued neutrosophic sets [109]

The intersection of two single valued neutrosophic sets M and N is a neutrosophic set P written as $P = M \cap N$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of M and N by $T_P(y) = \min(T_M(y), T_N(y))$; $I_p(y) = \max(I_M(y), I_N(y))$; $F_P(y) = \max(F_M(y), F_N(y))$ for all y in *Y*.

Definition 2.7 [109]: Assume that $\{M_t : t \in T\}$ be an arbitrary family of single valued neutrosophic sets in *Y*, then

i) $\bigcup_{t\in T} M_t$ can be defined as follows:

$$\underset{t\in T}{\cup} M_t = \left\langle y, \underset{t\in T}{\vee} T_{M_t}(y), \underset{t\in T}{\wedge} I_{M_t}(y), \underset{t\in T}{\wedge} F_{M_t}(t) \right\rangle$$

(ii) $\underset{t\in T}{\cap} M_t$ can be defined as follows:

$$\underset{t\in T}{\cap} M_t = \left\langle y, \underset{t\in T}{\wedge} T_{M_t}(y), \underset{t\in T}{\vee} I_{M_t}(y), \underset{t\in T}{\vee} F_{M_t}(t) \right\rangle$$

3. FORMULATION OF NEUTROSOPHIC MULTI-OBJECTIVE LINEAR PROGRAMMING

To formulate neutrosophic programming, we start from multi-objective programing problem in crisp environment.
Consider an optimization problem of the form in crisp environment:

$$\text{Max } \xi_j(\bar{y}),\ j = 1, 2, \ldots, q_1 \tag{1}$$

Subject to

$$\zeta_j(\bar{y}) \le 0,\ j = q_1+1, \ \ldots, q$$

$$\bar{y} \ge \bar{0}$$

where $\xi_j(\bar{y})$ represents the i-th objective function, $\bar{y}$ is the vector of n decision variables ($y_1, y_2, \ldots, y_n$), $\zeta_j(\bar{y})$ denotes i-th constraint, q_1 denotes the number of objective functions and q- q_1 denotes the number of constraints.

3.1 *Analogous fuzzy optimization problem*

In general, fuzzy optimization problem comprises of a set of objectives and constraints. The objectives and or constraints or parameters and relations can be expressed by fuzzy sets, which explain the degree of satisfaction of the respective conditions and expressed by their membership functions [11].
Consider the analogous fuzzy optimization problem:

$$\widetilde{\text{Max}}\ \xi_j(\bar{y}),\ j = 1, 2, \ldots, q_1 \tag{2}$$

Subject to

$$\zeta_j(\bar{y}) \ \tilde{\le}\ 0,\ j = q_1+1, \ \ldots, q$$

$$\bar{y} \ge \bar{0}$$

$\widetilde{\text{Max}}$ denotes fuzzy maximization and $\tilde{\le}$ denotes the fuzzy inequality.
To maximize the degree of membership of the objectives and constraints to the respective fuzzy sets, we can write:

$$\text{Max } \mu_j(\bar{y}),\ \bar{y} \in \Re^n,\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q \tag{3}$$

Subject to

$$0 \le \mu_i(\bar{y}) \le 1,\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q.$$

Here $\mu_j(\bar{y})$ denotes the membership function of the j-th objective function $\xi_j(\bar{y})$ (j = 1, 2, …q_1) and $\mu_j(\bar{y})$ denotes the membership function of the j-th membership function of the constraint $\zeta_j(\bar{y})$ (j = q_1+1, …, q).
Minimum operator of Bellman and Zadeh [10] can be applied to the optimization problem (3).

$$\mu_D(\bar{y}) = \overset{q}{\underset{j=1}{\wedge}} \mu_j(\bar{y}),\ \bar{y} \ge \bar{0},\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q \tag{4}$$

Therefore, $\mu_D(\bar{y}) \le \mu_j(\bar{y})$, j = 1, 2, …, q_1, q_1+1, …, q. (5)

According to Zimmermann [11], the problem can be solved as follows:

$$\mu_D(\bar{v}^o) = \text{Max } (\min (\mu_1(\bar{y}), \mu_2(\bar{y}), \ldots, \mu_{q_1}(\bar{y}), \mu_{q_1+1}(\bar{y}), \ldots, \mu_q(\bar{y})) \tag{6}$$

Subject to

$0 \leq \mu_j(\bar{y}) \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\bar{y} \geq \bar{0}$.
The problem (6) is equivalent to the following problem:
Max γ
Subject to (7)
$\gamma \leq \mu_j(\bar{y})$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\bar{y} \geq \bar{0}$.

3.2 ***An analogous intuitionistic fuzzy optimization (IFO) problem***

An analogous intuitionistic fuzzy optimization problem can be represented as follows:
To maximize the degree of acceptance of intuitionistic fuzzy objective functions and constraints, and to minimize the degree of rejection of intuitionistic fuzzy objective functions and constraints we can write:

$$\text{Max } \mu_j(\bar{y}),\ \bar{y} \in \Re^n,\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q \tag{8}$$

$$\text{Min } \nu_j(\bar{y}),\ \bar{y} \in \Re^n,\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q \tag{9}$$

Subject to
$\mu_j(\bar{y}) + \nu_j(\bar{y}) \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$0 \leq \mu_j(\bar{y}) \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$0 \leq \nu_j(\bar{y}) \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\bar{y} \geq \bar{0}$.

Here, $\mu_j(\bar{y})$ denotes the degree of membership of j-th objective function $\xi_j(\bar{y})$ $(j = 1, 2, \ldots q_1)$ and $\mu_j(\bar{y})$ denotes the degree of j-th membership function of constraint $\zeta_j(\bar{y})$ $(j = q_1+1, \ldots, q)$.

Here, $\nu_j(\bar{y})$ denotes the degree of non-membership of j-th objective function $\xi_j(\bar{y})$ $(j = 1, 2, \ldots q_1)$ and $\nu_j(\bar{y})$ denotes the degree of j-th non-membership function of constraint $\zeta_j(\bar{y})$ $(j = q_1+1, \ldots, q)$.

Conjunction [68] of two intuitionistic fuzzy sets A and B can be defined as follows:

$$A \cap B = \{\langle \bar{y}, \mu_A(\bar{y}) \wedge \mu_B(\bar{y}), \nu_A(\bar{y}) \vee \nu_B(\bar{y}) \rangle | \bar{y} \in \Re^n\}, \tag{10}$$

where A represents an intuitionistic fuzzy objectives and B represents intuitionistic fuzzy constraints. This conjunction operator can be easily generalized and applied to the IFO problem.

$$\text{Here, } D = \{\langle \bar{y}, \mu_D(\bar{y})), \nu_D(\bar{y}) \rangle | \bar{y} \in \Re^n\},\ \mu_D(\bar{y}) = \bigwedge_{j=1}^{q} \mu_j(\bar{y}), \nu_D(\bar{y}) = \bigvee_{j=1}^{q} \nu_j(\bar{y}) \tag{11}$$

Here D represents an intuitionistic fuzzy set based representation of the decision.
Min-operator can be used for conjunction and max-operator for disjunction [70].

$$\mu_D(\bar{y}) = \bigwedge_{j=1}^{q} \mu_j(\bar{y}),\ \bar{y} \in \Re^n, j = 1, 2, \ldots, q_1, q_1+1, \ldots, q, \tag{12}$$

$$\nu_D(\bar{y}) = \bigvee_{j=1}^{q} \nu_j(\bar{y}),\ \bar{y} \in \Re^n, j = 1, 2, \ldots, q_1, q_1+1, \ldots, q, \tag{13}$$

$$\text{Therefore, } \mu_D(\bar{y}) \leq \mu_j(\bar{y}), \nu_D(\bar{y}) \geq \nu_j(\bar{y}),\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q. \tag{14}$$

Therefore, for optimal decision we can write
Max γ,
Min δ,
Subject to
$\mu_j(\bar{y}) \geq \gamma$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\nu_j(\bar{y}) \leq \delta$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$0 \leq \gamma \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$0 \le \delta \le 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\gamma + \delta \le 1$,
$\bar{y} \ge \bar{0}$.

Here γ represents minimal acceptable degree of objectives and constraints and δ represents the maximal degree of rejection of objectives and constraints.

Now the IFO problem (7) can be transformed into the following crisp (non-fuzzy) optimization problem:

$$\text{Max } (\gamma - \delta) \quad (15)$$

Subject to

$\mu_j(\bar{y}) \ge \gamma$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\nu_j(\bar{y}) \le \delta$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$0 \le \gamma \le 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$
$0 \le \delta \le 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$
$\gamma + \delta \le 1$,
$\bar{y} \ge \bar{0}$.

3. 3. Formulation of neutrosophic multi objective linear programming

Neutrosophic optimization problem can be represented as follows:
To maximize the degree of acceptance (truth) of neutrosophic objectives and constraints, to minimize the degree of indeterminacy and to minimize the degree of rejection (falsity) of neutrosophic objectives and constraints:

$$\text{Max } \mu_j(\bar{y}),\ \bar{y} \in \Re^n,\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q, \quad (16)$$

$$\text{Min } \omega_i(\bar{y}) \in [0, 1],\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q, \quad (17)$$

$$\text{Min } \nu_j(\bar{y}),\ \bar{y} \in \Re^n,\ j = 1, 2, \ldots, q_1, q_1+1, \ldots, q, \quad (18)$$

Subject to

$0 \le \mu_j(\bar{y}) \le 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$0 \le \omega_j(\bar{y}) \le 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$0 \le \nu_j(\bar{y}) \le 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\mu_j(\bar{y}) + \omega_i(\bar{v}) + \nu_j(\bar{y}) \le 3$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,
$\bar{y} \ge \bar{0}$.

Here $\mu_j(\bar{y})$ denotes the degree of truth membership of $\bar{y}$ to the j-th SVNS, $\omega_j(\bar{y})$ denotes the degree of indeterminacy and $\nu_j(\bar{y})$ denotes the degree of falsity (rejection) of functions $\bar{y}$ from the j-th SVNS.
Conjunction [109] of SVNSs is defined by

$$G \cap C = \{\langle \bar{y}, \mu_G(\bar{y}) \wedge \mu_C(\bar{y}), \omega_G(\bar{y}) \vee \omega_C(\bar{y}), \nu_G(\bar{y}) \vee \nu_C(\bar{y})\rangle | \bar{y} \in \Re^n\} \quad (19)$$

Here G represents a neutrosophic objective function and C represents neutrosophic constraint. This conjunction operator can be easily generalized and applied to the neutrosophic optimization problem:

$$D = \{\langle \bar{y}, \mu_D(\bar{y})), \nu_D(\bar{y})\rangle | \bar{v} \in \Re^n\},\ \mu_D(\bar{y}) = \bigwedge_{j=1}^{q} \mu_j(\bar{y}), \omega_D(\bar{y}) = \bigvee_{j=1}^{q} \omega_j(\bar{y}),\ \nu_D(\bar{y}) = \bigvee_{j=1}^{q} \nu_j(\bar{y}) \quad (20)$$

Here D represents a single valued neutrosophic set based representation of the decision.
Min-operator has been used for conjunction and max-operator for disjunction:

$$\mu_D(\bar{y}) = \bigwedge_{j=1}^{q} \mu_j(\bar{y}),\ \bar{y} \in \Re^n,\quad \omega_D(\bar{y}) = \bigvee_{j=1}^{q} \omega_j(\bar{y}),\ \bar{y} \in \Re^n,\ \nu_D(\bar{y}) = \bigvee_{j=1}^{q} \nu_j(\bar{y}),\ \bar{y} \in \Re^n. \quad (21)$$

Therefore, $\mu_D(\bar{y}) \le \mu_j(\bar{y}), \omega_D(\bar{y}) \ge \omega_j(\bar{y}),\ \nu_D(\bar{y}) \ge \nu_j(\bar{y}),$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$. (22)

Here $\mu_j(\bar{y})$ denotes the degree of truth membership of $\bar{y}$ to the j-th SVNS, $\omega_j(\bar{y})$ denotes the degree of indeterminacy membership, and $\nu_i(\bar{y})$ denotes the degree of falsity (rejection) of functions $\bar{y}$ from the j-th SVNS.

Neutrosophic multi-objective linear programming can be presented as follows:

$$\text{Max } (\gamma - \lambda - \delta) \tag{23}$$

Subject to

$\mu_j(\bar{y}) \geq \gamma$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$\omega_j(\bar{y}) \leq \lambda$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$\nu_j(\bar{y}) \leq \delta$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$0 \leq \gamma \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$0 \leq \lambda \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$0 \leq \delta \leq 1$, $j = 1, 2, \ldots, q_1, q_1+1, \ldots, q$,

$\gamma + \lambda + \delta \leq 3$,

$\bar{y} \geq \bar{0}$.

To solve this problem, indeterminacy membership functions and falsity membership functions should be suitably constructed in the decision making context.

CONCLUSION

This paper deals with the framework of neutrosophic multi-objective linear programming problem. The essence of the proposed neutrosophic multi-objective linear programming problem is that it is capable of dealing with indeterminacy and falsity simultaneously. Roy and Das [99] and Das and Roy [100] presented neutrosophic multi-objective linear programming and neutrosophic multi-objective non-linear programming respectively. However, in their approaches they maximize indeterminacy membership function that is not realistic in decision making context. In this paper the definition of intersection of two single valued neutrosophic sets due to Salama and Alblowi [109] has been utilized and minimization of falsity membership function and indeterminacy membership functions have been simultaneously considered. The proposed framework of neutrosophic multi-objective programming problem reflects the new direction of research in neutrosophic environment. Optimization problem in neutrosophic environment is a promising field of study. Neutrosophic optimization problem does, however, need a broader philosophy and new methods of dealing with problems in more versatile ways. To draw attentions, neutrosophic optimization technique must open its eyes to fresh possibilities dealing with, clearly defined indeterminacy function and falsity membership function simultaneously in realistic way.

The author hopes that the proposed framework of neutrosophic multi-objective linear programming will accelerate the study of optimization problem in neutrosopohic environment.

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