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PID Tuning with Neutrosophic Similarity Measure

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Abstract In this paper, a method for adjusting the proportional-integral-derivative (PID) coefficients based on the neutrosophic similarity measure is proposed. First, rough PID coefficients were determined by the Ziegler–Nichols method, and the upper and lower limit values for the search range of the PID coefficients were determined. At each step of the search range, we applied a unit step function to the system and obtained the transient response characteristics. The obtained values were converted into a neutrosophic set (real set) by using defined membership functions. Then, the optimal PID coefficients were obtained using the similarity ratio between the real and ideal (target) neutrosophic sets. In calculating the similarity ratio, the Hamming, Euclidean, Set-theoretic, Jaccard, and Dice approaches were applied, and the results were compared. Finally, the proposed method was tested on two transfer functions, and it was demonstrated that the proposed method can be used to adjust PID coefficients.

Keywords Decision making · Neutrosophic logic · Neutrosophic similarity measure · PID tuning

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1 Introduction

A wide range of controller design methods including fuzzy logic controllers, lead-lag compensators, sliding mode controllers, and proportional-integral-derivative (PID) controllers have been proposed for various applications. The PID controller is preferred in process control applications, because of its robustness, ease of design, zero steady-state error, low oscillation rate, fast system response, and high stability [1, 2]. The PID controller has three basic parameters called K_p, K_i, and K_d. The setting of these parameters, in a process called PID tuning, is very important in PID controller design. PID tuning methods are divided into two main classes: closed-loop and open-loop methods. Ziegler–Nichols is an example of a closed-loop method, and Cohen–Coon is an example of an open-loop method.

Software-based approaches for adjusting the PID coefficients are available. Genetic algorithms (GA), particle swarm optimization (PSO), and Fuzzy Logic approaches are among the most widely studied topics in the literature on software-based controller design [3–5]. An efficient and quick PID tuning method based on the modified genetic algorithm (MGA) has been proposed [3]. In [3], an optimization method based on integrating a classical genetic algorithm structure with a systematic neighborhood structure was used. In an alternative approach, the results obtained from a PSO-based PID tuning method were compared with results from a Ziegler–Nichols PID tuning method [4]. In another software-based PID tuning study, a method using fuzzy logic for optimal PID controller design was proposed [5]. In this study, the researchers used the fuzzy set point weighting method, and also investigated the effects of different membership functions.

Since Zadeh first proposed fuzzy logic in 1965 [6], researchers have introduced a range of innovations to the concept, including L-fuzzy sets [7], interval-valued fuzzy sets

[8–10], four valued logic [11], intuitionistic fuzzy sets [12], interval-valued intuitionistic fuzzy sets [13], vague sets [14].

Smarandache proposed the concepts of neutrosophy and neutrosophic sets [15–17]. These are a special case of fuzzy logic. Unlike classical fuzzy logic, they have True (*T*), Indeterminate (*I*), and False (*F*) membership values. Neutrosophy resembles the logic of human thought by including uncertainty. Today, neutrosophic sets and neutrosophic logic approaches are applied in image processing, robot control, and computer science [18–20]. Following the introduction of neutrosophic sets, neutrosophic similarity measures were proposed in [21]. This method identifies the percentage similarity between two or more neutrosophic sets, and is used mainly in decision-making processes.

In control applications, reach of a system to the reference value in a short time and with minimal errors is aimed. Also, minimum overshoot ratio must be obtained. The process of setting the PID coefficients is intended to meet as many as possible of these characteristics. The process of determining the optimal values of the PID coefficients can in fact be considered as the decision-making process. In this process, setting the PID coefficients according to the unit step response of a system can lead to "good," "bad," or "uncertain/indeterminate" ("neither good nor bad") outcomes. These can be considered as linguistic expressions in fuzzy logic. When designing the fuzzy logic controller for a temperature control application, the designer may use terms like "a little cold," or "a little hot," and the same is true when reviewing the unit step responses of a system. For example, in the assessment of a direct current motor step response graph, a designer commenting on the rise time (or any other unit step parameter) can judge the response "good," "medium," or "bad," according to the unit step response he/she wants from the system. Here "good" can be considered as the *T* value, "medium" ("neither good nor bad") can be considered as the *I* value, and "bad" can be considered as the *F* value in the neutrosophic set concept. By using this approach, designers are able to provide the appropriate PID coefficients to the membership values, without resorting to numerical values obtained from intensive mathematical operations. The interpretation can be made by looking at how close to 1 the *T* membership value of each unit step response characteristic (rising time, settling time, peak time, etc.) is, and how close to 0 the *I* and *F* membership values are.

In this study, a method based on the neutrosophic similarity measure is proposed for adjusting PID coefficients, using some of the measures currently available in the literature. Our suggested method differs from currently used PID tuning methods. Many PID tuning methods use performance indices, and such methods are called minimum error criteria methods. Frequently used performance indices are the integral of the absolute value of the error (IAE), the integral of the square value of the error (ISE), the integral of the time weighted absolute value of the error (ITAE), and the integral of the time weighted square of the error (ITSE) [1]. Some PID tuning methods apply transfer function models such as Internal Model Control (IMC) [1] and the pole placement-zero cancelation method [22]. The method used in this study does not take into account performance indexes and it does not need to know the transfer function. Based on neutrosophic similarity measures, it contains time-domain step-response characteristics (rising time, settling time, overshoot ratio, undershoot ratio, peak time, and steady-state error) and is a decision-making process. The proposed method is intended to stay at the design values of all the unit step response characteristics. The method requires no complex mathematical calculations.

Our proposed approach was applied to two sample transfer functions.

2 Preliminaries

In this section some of the preliminaries that form the basis of the proposed study are described.

2.1 PID Control

PID control is a feedback method commonly used in control applications. The PID controller takes into account the error rate, total error, and the derivative of the error. The error rate, sum of errors, and derivative of errors are separately multiplied by coefficients called K_p, K_i, and K_d respectively. The goal of the process is to reduce the error to zero. PID tuning is a process to achieve zero error with the desired steady-state characteristics by finding the optimal K_p, K_i, and K_d values. In some applications, one or any two of the K_p, K_i, K_d values can take the zero value, and in such cases the PID controller takes on P, PI, and PD controller form. System block diagrams for a PID controlled feedback system, in the time domain and *s* domain respectively, are given in Figs. 1 and 2.

From Fig. 1,

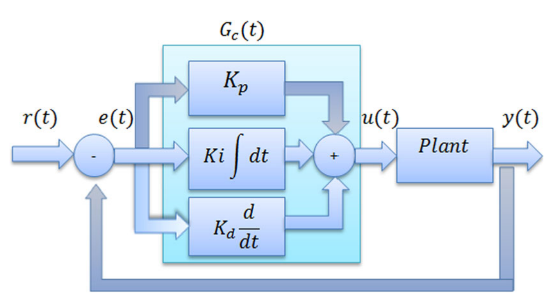

Fig. 1 PID controlled feedback system in the time domain

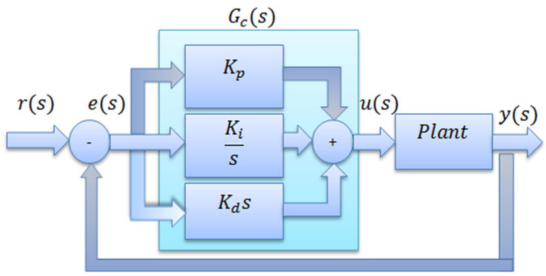

Fig. 2 PID controlled feedback system in the S domain

$$e(t) = r(t) - y(t) \tag{1}$$

$$u(t) = K_p e(t) + K_i \int e(t)\mathrm{d}t + K_d \frac{\mathrm{d}}{\mathrm{d}t} e(t) \tag{2}$$

and from Fig. 2,

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \tag{3}$$

if $T_i = \frac{K_p}{K_i}$ and $T_d = \frac{K_d}{K_p}$ changed;

$$G_c(s) = K_p \left(\frac{T_d s^2 + s + \frac{1}{T_i}}{s} \right) \tag{4}$$

Here $r(t)$ is the reference value, $e(t)$ is the error value, $u(t)$ is the control signal, and $y(t)$ is the output signal. $G_c(s)$ shows the transfer function of the PID controller in s domain. The s domain representation of the control signal can also be expressed as follows:

$$U(s) = E(s)\left(K_p + \frac{K_i}{s} + K_d s \right) \tag{5}$$

Equations 2, 3, and 5 show that the PID controller includes the K_p, K_i, and K_d coefficients. These coefficients can also take the K_p, T_i, and T_d shape, as shown in Eq. 4. The PID tuning procedure searches for the optimal K_p, K_i, and K_d or T_i and T_d values.

2.2 Neutrosophic Logic, Neutrosophic Sets, and Neutrosophic Similarity Measure

In contrast with classical fuzzy logic, the neutrosophic set contains true, indeterminate, and false membership values. In neutrosophic logic or the neutrosophic set approach, the value of a phenomenon is illustrated by the T, F, and I membership values. For example, in an $x(0.7, 0.4, 0.2)$ illustration, the T membership value of the x phenomenon is 0.7, the I value is 0.4, and the F value is 0.2. In some cases, and especially in engineering problems, these values may be assigned to a 0–1 range. There is no limitation on the sum of $T + I + F$ [15]. In neutrosophic sets, there is no limitation on the range of T, F, and I membership values. They have real values and can be discrete, continuous, single-valued, finite (countable or uncountable), or infinite, or a union or intersection of subsets of various sets [16].

Wang et al. proposed the interval neutrosophic set (INS) and single-valued neutrosophic set (SVNS) for solving engineering problems [23, 24]. Ye later introduced the simplified neutrosophic set (SNS) [25].

The similarity measure is a method used to determine the degree of similarity between two or more sets which is often applied to decision-making problems. Distance-based, probability-based, fuzzy set theory-based, and graph theory-based approaches are available for the similarity measure. The neutrosophic similarity measure is used to determine the degree of similarity between two or more neutrosophic sets, and is also widely used in decision making. A wide range of approaches are available for solving such decision-making problems [25–32].

2.3 Some Definitions and Theorems

This section reviews the definitions and theorems which have been suggested by different researchers concerning neutrosophic sets. We review neutrosophic set descriptions and some features of neutrosophic sets and the single-valued neutrosophic set concept [24, 33], soft sets, and neutrosophic soft set (NSS) concepts [34–36]. We discuss the definitions and theorems of both neutrosophic Hamming, Euclidean, and Set-theoretic similarity measures [28] and neutrosophic Jaccard, Dice, and Cosine similarity measures [29].

Definition 1 [24] X be a universe of discourse. An element in X denoted by x. For a neutrosophic set A in X;

$T_A(x)$: Truth-membership function
$I_A(x)$: Indeterminacy-membership function.
$F_A(x)$: Falsity-membership function
The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$.

$T_A(x)$: $X \rightarrow]0^-, 1^+[$
$I_A(x)$: $X \rightarrow]0^-, 1^+[$
$F_A(x)$: $X \rightarrow]0^-, 1^+[$
There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup\ T_A(x) + \sup\ I_A(x) + \sup\ F_A(x) \leq 3^+$.

Definition 2 [24] The complement of a neutrosophic set A is denoted by A^c

$T_A^c(x) = \{1^+\} - T_A(x)$
$I_A^c(x) = \{1^+\} - I_A(x)$
$F_A^c(x) = \{1^+\} - F_A(x)$ for all $x \in X$.

Definition 3 [24] A neutrosophic set A and other neutrosophic set B, $A \subseteq B$, if and only if;

$\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$
$\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$
$\inf F_A(x) \ge \inf F_B(x)$, $\sup F_A(x) \ge \sup F_B(x)$
for all $x \in X$.

Definition 4 [24] A single-valued neutrosophic set (SVNS) A in X. $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle: x \in X\}$.

$T_A(x): X \to [0,1]$
$I_A(x): X \to [0,1]$
$F_A(x): X \to [0,1]$
$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

Definition 5 [24] The complement of an SVNS A is denoted by A^c and for all $x \in X$;

$T_A^c(x) = F_A(x)$
$I_A^c(x) = 1 - I_A(x)$
$F_A^c(x) = T_A(x)$
That is, $A^c = \{\langle x, F_A(x), 1-I_A(x), T_A(x)\rangle: x \in X\}$.

Definition 6 [24] A and B are SVNS, $A \subseteq B$, if and only if;

$T_A(x) \le T_B(x)$
$I_A(x) \ge I_B(x)$
$F_A(x) \ge F_B(x)$
for all $x \in X$.

Definition 7 [24] SVNS A and SVNS B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

Definition 8 [34, 35] U is the initial universe and E is a set of parameters. $P(U)$ denotes the power set of U and $A \subseteq E$. (F,A) is called a soft set over U, and F is a mapping given by $F: A \to P(U)$.

Definition 9 [36] U is the universe set, E the set of parameters, $A \subseteq E$ and $NS(U)$ the set of all neutrosophic sets of U. The collection (F,A) is the NSS over U, F is a mapping given by $F: A \to NS(U)$.

Definition 10 [28] $U = \{x_1, x_2, x_3,\ldots, x_n\}$ is an initial universe and $E = \{e_1, e_2, e_3,\ldots, e_n\}$ is a set of parameters. $NS(U)$ denotes the set of all neutrosophic sets over the U. A and B are NSS over the U, A and B are mappings given by A,B: $E \to NS(U)$. Hamming, Normalized Hamming, Euclidean, and Normalized Euclidean distances between A and B sets are given in Eqs. 6, 7, 8, 9 respectively.

$$L_H(A,B) = \frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\begin{array}{l}\left|T_{A(x_i)(e_j)} - T_{B(x_i)(e_j)}\right| \\ + \left|I_{A(x_i)(e_j)} - I_{B(x_i)(e_j)}\right| \\ + \left|F_{A(x_i)(e_j)} - F_{B(x_i)(e_j)}\right|\end{array}\right\} \tag{6}$$

$$L_{NH}(A,B) = \frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\begin{array}{l}\left|T_{A(x_i)(e_j)} - T_{B(x_i)(e_j)}\right| \\ + \left|I_{A(x_i)(e_j)} - I_{B(x_i)(e_j)}\right| \\ + \left|F_{A(x_i)(e_j)} - F_{B(x_i)(e_j)}\right|\end{array}\right\} \tag{7}$$

$$L_E(A,B) = \sqrt{\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\begin{array}{l}\left(T_{A(x_i)(e_j)} - T_{B(x_i)(e_j)}\right)^2 \\ + \left(I_{A(x_i)(e_j)} - I_{B(x_i)(e_j)}\right)^2 \\ + \left(F_{A(x_i)(e_j)} - F_{B(x_i)(e_j)}\right)^2\end{array}\right)} \tag{8}$$

$$L_{NE}(A,B) = \sqrt{\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\begin{array}{l}\left(T_{A(x_i)(e_j)} - T_{B(x_i)(e_j)}\right)^2 \\ + \left(I_{A(x_i)(e_j)} - I_{B(x_i)(e_j)}\right)^2 \\ + \left(F_{A(x_i)(e_j)} - F_{B(x_i)(e_j)}\right)^2\end{array}\right)} \tag{9}$$

Definition 11 [28] U is a universe, E the set of parameters and A, B NSS over U. Based on the distances similarity measure (SM) between A and B is given below:

$$SM(A,B) = \frac{1}{1 + L(A,B)} \tag{10}$$

Another similarity measure of A and B is given below:

$$SM(A,B) = e^{-\alpha L(A,B)} \tag{11}$$

$L(A,B)$ is the distance between the interval-valued NSS A and B and α is steepness measure and it is a positive real number.

Definition 12 [28] $S = \{x_1, x_2, x_3,\ldots, x_n\}$ is the universe and $E = \{e_1, e_2, e_3,\ldots, e_n\}$ is a set of parameters. $NS(U)$ denotes the set of all Neutrosophic Subsets of S. A and B are NSS over S. A and B are mappings given by $A, B : E \to NS(U)$. Similarity Measure $SM(A,B)$ between A and B based on Set-theoretic approach is given below:

$$SM(A,B) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\begin{array}{l}\left(T_{A(x_i)(e_j)} \wedge T_{B(x_i)(e_j)}\right) \\ + \left(I_{A(x_i)(e_j)} \wedge I_{B(x_i)(e_j)}\right) \\ + \left(F_{A(x_i)(e_j)} \wedge F_{B(x_i)(e_j)}\right)\end{array}\right]}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\begin{array}{l}\left(T_{A(x_i)(e_j)} \vee T_{B(x_i)(e_j)}\right) \\ + \left(I_{A(x_i)(e_j)} \vee I_{B(x_i)(e_j)}\right) \\ + \left(F_{A(x_i)(e_j)} \vee F_{B(x_i)(e_j)}\right)\end{array}\right]} \tag{12}$$

Theorem 1 [28] *SM(A,B)* similarity measure between *A* and *B* sets then:

(i) $SM(A,B) = SM(B,A)$
(ii) $0 \le SM(A,B) \le 1$
(iii) $SM(A,B) = 1$ if and only if $A = B$

Definition 13 [29] S_J, S_D, S_C represents to Jaccard, Dice, and Cosine vector similarity measures, respectively. The three vector similarity measures of SVNSs:

$$S_J(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\left(\begin{array}{l}(T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) \\ + (T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)) \\ - (T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))\end{array}\right)} \quad (13)$$

$$S_D(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\left(\begin{array}{l}(T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) \\ + (T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i))\end{array}\right)} \quad (14)$$

$$S_C(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)}{\sqrt{T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)}\sqrt{T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)}} \quad (15)$$

The cosine measure is undefined if the *A* and/or *B* membership values are equal to zero, and the Jaccard and Dice measures are undefined if all *A* and *B* membership values are equal to zero.

Theorem 2 [29] According to the Jaccard, Dice, and Cosine similarity measures [30–32], $S_k(A, B)$ similarity measure ($k = J, D, C$) implements the properties given below:

(i) $0 \le S_k(A,B) \le 1$;
(ii) $S_k(A,B) = S_k(B,A)$;
(iii) $S_k(A,B) = 1$ if $A = B$, i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ for all $x_i \in X$.

2.4 Neutrosophic Similarity Measure Between Two Neutrosophic Sets

In this section, an example is given for calculating the similarity measure between two neutrosophic sets. In this example *A* represents the ideal neutrosophic set, *B* represents the real neutrosophic set, and *E* represents the parameter set. e_1, e_2, e_3, and e_4 represent the neutrosophic components of the *E* parameter set. If *x* has *E* parameters, it can represent an object, a status, or a person. Consider the problem of selecting a manager for a company when three suitable candidates are available. In this example, *i* represents the ideal characteristics, *x*, *y*, and *z* represent the managerial candidates, and *E* represents the required characteristics of a manager. In this case, the e_1, e_2, e_3 and e_4 values represent talent, experience, communication skills, and practical intelligence. Assume that these values have been measured in a screening test given to the candidates. *A* is the ideal set and *B* is the real set from the screening tests.

The values in Tables 1, 2, 3, and 4 were fed into Eqs. 6, 8, 10, 12, 13, 14, and 15, and the similarity measure for each candidate is given in Table 5.

Table 5 shows that the Hamming, Euclidean, and Set-theoretic approaches produced similar results, while the Jaccard approach produced higher results than the other three approaches. Dice and Cosine reached the maximum similarity measure ratio. The results suggest that *y* is the most suitable candidate based on the expected criteria, as the measures of candidate *y* showed the largest similarity ratio in all approaches.

Table 1 Ideal neutrosophic set

A	e_1	e_2	e_3	e_4
i	(0.8,0.3,0.2)	(0.7,0.3,0.3)	(0.8,0.2,0.1)	(0.6,0.5,0.4)

Table 2 Neutrosophic set of candidate *x*

B	e_1	e_2	e_3	e_4
x	(0.2,0.4,0.8)	(0.7,0.8,0.7)	(0.7,0.3,0.3)	(0.5,0.9,0.5)

Table 3 Neutrosophic set of candidate *y*

B	e_1	e_2	e_3	e_4
y	(0.6,0.4,0.3)	(0.7,0.5,0.5)	(0.7,0.2,0.4)	(0.8,0.2,0.3)

Table 4 Neutrosophic set of candidate *z*

B	e_1	e_2	e_3	e_4
z	(0.6,0.4,0.5)	(0.6,0.2,0.2)	(0.2,0.5,0.5)	(0.5,0.2,0.7)

Table 5 Neutrosophic SM of candidate *x, y, z* according to different SM criteria

Candidates	SM(A,B)					
	Hamming	Euclidean	Set-theoretic	Jaccard	Dice	Cosine
x	0.65	0.67	0.57	0.70	0.80	0.83
y	0.76	0.79	0.71	0.88	0.94	0.93
z	0.67	0.71	0.56	0.72	0.82	0.81

Table 6 Rule table of first Ziegler–Nichols method

Controller type	K_p	T_i	T_d
P	T/L	∞	0
PI	0.9 (T/L)	$L/3$	0
PID	1.2 (T/L)	$2L$	$0.5L$

Table 7 Rule table of second Ziegler–Nichols method

Controller type	K_p	T_i	T_d
P	$0.5\ K_u$	∞	0
PI	$0.45\ K_u$	$\frac{1}{1.2}P_u$	0
PID	$0.6\ K_u$	$0.5\ P_u$	$0.125\ P_u$

2.5 Ziegler–Nichols PID Tuning Method

The Ziegler–Nichols method is a well-known closed-loop PID tuning method, first proposed in the early 1940s. Using this method, it is possible to obtain rough K_p, K_i, and K_d values based on values of the time domain transient response characteristics. Two Ziegler–Nichols tuning methods are available [37].

2.6 First Ziegler–Nichols Method

In this method, the unit step signal is applied to the system. Then, the values obtained from the unit step response curve of the system are processed in the formulas given in Table 6, to determine the PID coefficients. The unit step response curve example of a system is given in Fig. 3.

2.7 Second Ziegler–Nichols Method

In this method, primarily T_i and T_d values are set up to zero. Then, K_p value is increased until the system output reach oscillation. K_p value at which output of the system oscillates with a constant amplitude is K_u and the period of the oscillation is P_u. So, PID coefficients are determined according to Table 7, where K_u is called critical gain and the P_u is called critical period.

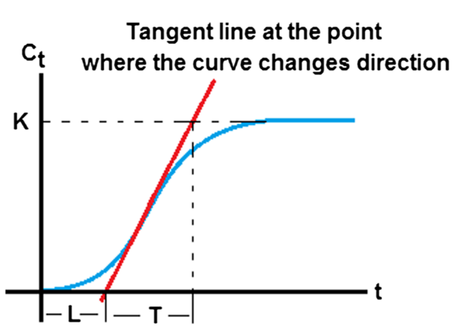

Fig. 3 Unit step response curve of a system

3 The Proposed Method

In this study, a multi-criteria decision-making application was used to determine the appropriate PID coefficients by the similarity measure. Determining PID coefficients can be considered as a multi-criteria decision-making process, because it must take account of a variety of criteria such as rise time, overshoot ratio, settling time, and steady-state error when controlling a system. The use of these criteria is made possible by choosing the appropriate PID coefficients. In this study, neutrosophic Hamming, Euclidean, Set-theoretic, Jaccard, and Dice similarity measures were used to determine the appropriate PID coefficients. The unit step response properties obtained from the system were passed from the membership functions, and neutrosophic values were calculated. In some cases the membership functions had zero value and the cosine similarity measures were therefore undefined. The cosine similarity measure was therefore not used in this study. The proposed method was tested by two example transfer functions having second and third degree open-loop characteristic equations. A flow chart is given Fig. 4 to summarize the methodology.

First, rough K_p, K_i, and K_d values were determined using the Ziegler–Nichols method. Next, a similarity rate measurement algorithm was run in MATLAB. In this algorithm, the K_p, K_i, and K_d values were increased one by one around the K_p, K_i, and K_d Ziegler–Nichols values, from a given lower value to an upper value, in step-up nested loops. At the end of each increment, the transient response values of the system were measured. These values were then passed to the neutrosophic membership functions (Neutrosophication) and a neutrosophic set called a real set derived. The

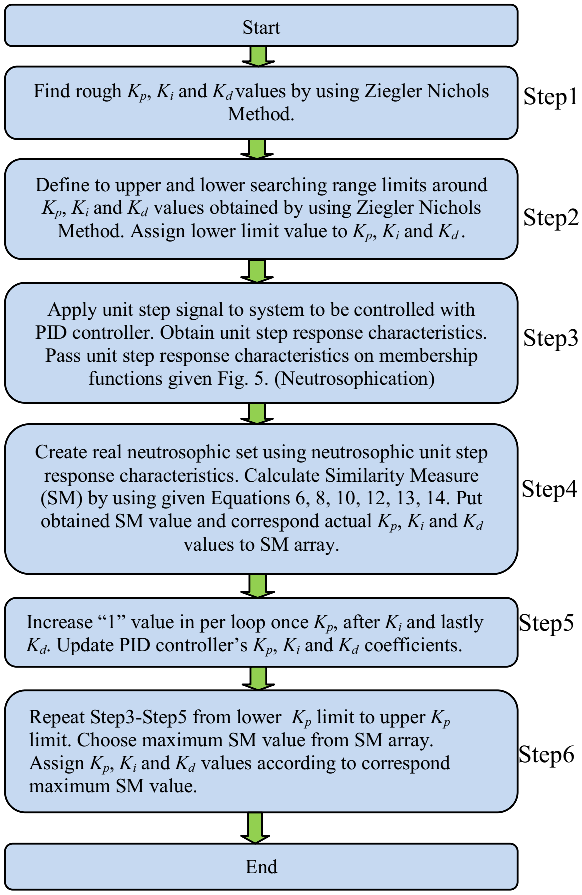

Fig. 4 The flow chart of the methodology

similarity measure between the actual (real) set and a previously prepared ideal neutrosophic set was calculated, and the resulting similarity measure recorded in an SM array. Finally, it was determined whether the maximum value in the SM array satisfied the appropriate PID values.

3.1 Neutrosophic Membership Functions and Neutrosophication

Neutrosophication is similar to fuzzification, though fuzzification methods such as largest and smallest membership degree (min, max) are not used. Instead, the membership value of the phenomenon is directly transferred to the *T*, *I*, and *F* subsets. In this study, triangle and trapezoid membership functions were used. The range of *T*, *I*, and *F* values in the membership function, and type of membership function used, may vary from application to application. In the neutrosophication process, the rising time, settling time, % overshoot ratio, % undershoot ratio, peak time, and steady-state error values obtained from the transient response characteristics of the system are passed through the membership functions in Fig. 5, and the real neutrosophic set *B* is obtained.

In Fig. 5, *T* corresponds to the true membership function, *I* corresponds to the indeterminate membership function, and *F* corresponds to the false membership function. In Fig. 5a, a range of 0–3 s is defined as the good (*T*) value for the rise time, a range of 2.5–3.5 s is neither good nor bad, and a range of (*I*), greater than 3 s is bad (*F*). Now, consider the rise time value of 1.7227 given in Table 8. This value was passed through the membership function in Fig. 5a. A 0.851 value was obtained from the true membership function, and a 0 value was obtained from the *I* and *F* membership functions. Thus, the neutrosophic membership value of the rise time was $(T, I, F) = (0.851, 0, 0)$. As another example, consider the 4.8543 valued peak time. When this value was passed through *T, I,* and *F* membership functions, it took values of 0, 0.14, and 0.42, respectively. Thus, the neutrosophic membership value was $(T, I, F) = (0, 0.14, 0.42)$ for the peak time. Examples of step response characteristics and the neutrosophic provisions obtained by neutrosophication are shown in Table 8.

As in fuzzy logic design, the type and range of the membership functions are determined by the known characteristics of the system to be controlled and the experience of the designer. For example, the selection of a membership function representing a fast rise time is generally not suitable in temperature control applications. Conversely, the selection of a membership function representing a slow rise time is also not suitable in servo motor control applications. The type and range of a membership functions are also a function of the designer's experience and expertise. The example membership functions used in this study were selected after considering the desirable system responses in general control applications. For example, an overshoot ratios of less than 5 % is in the goal in most applications, and undershoot and steady-state errors are undesirable in all applications. In many control applications, rise times of 4–5 s or longer are not preferred.

In the neutrosophication process, selection of rise time, settling time, % overshoot ratio, % undershoot ratio, peak time, and steady-state error values from transient state characteristics of the system have a large impact on performance, because these values represent a large proportion of the transient state characteristics of the system. In this study, the target unit step values were compared with the values obtained from the system to determine the appropriate PID coefficients. A sufficient number of features must be used in determining the similarity ratio. Using too few unit steps adversely affects the estimates of the rate of similarity. For example, consider only the rise time and the settling time. In some cases, two different step responses with similar rise times and settling times may have different % overshoot rates and different steady-state errors. One of these systems may even undershoot. Therefore, six transient state features of the system (rise time, settling

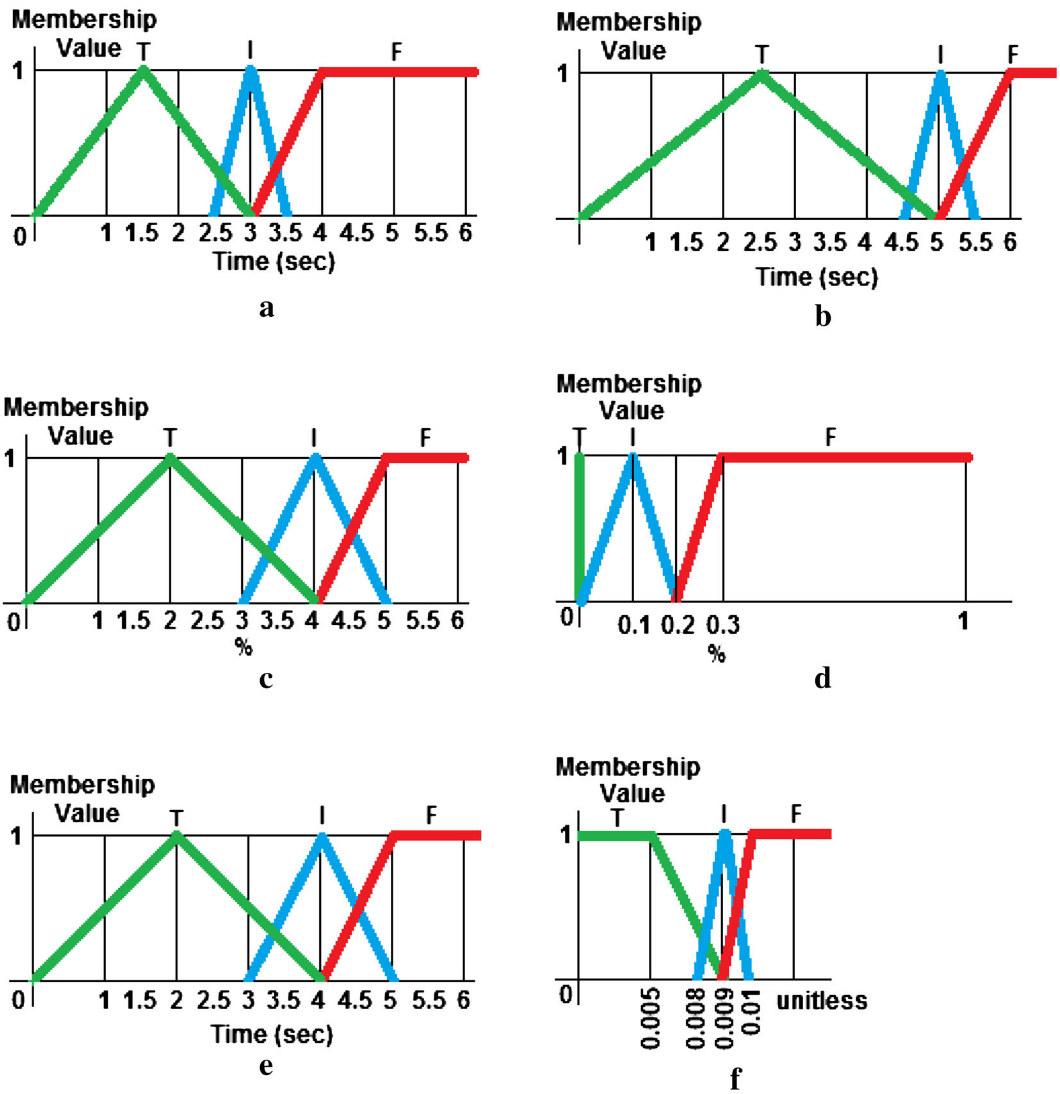

Fig. 5 Membership functions used for the unit step response characteristics. **a** Rising time **b** Settling time **c** % Overshoot ratio **d** % Undershoot ratio **e** Peak time. **f** Steady-state error (absolute values of negative values are taken)

Table 8 An example for step response characteristics and neutrosophic provisions

Step response characteristics	Step response value	Neutrosophic provision
Rising time	1.7227	(0.851,0,0)
Settling time	35.6180	(0,0,1)
% Overshoot ratio	50.5434	(0,0,1)
% Undershoot ratio	0	(1,0,0)
Peak time	4.8543	(0,0.14,0.42)
Steady-state error	0	(1,0,0)

time, % overshoot rate, % undershoot rate, peak time, and steady-state error) were used. These features are members of the E parameter set in the real neutrosophic set B, where e_1–e_6 refer to the rise time, settling time, % overshoot ratio, % undershoot ratio, peak time, and steady-state error values, respectively.

3.2 Application Example 1

In this application, the proposed method was tested on a transfer function which was a quadratic open-loop characteristic equation. Consider the following transfer function:

$$G(s) = \frac{1}{(s+1)(s+4)} \quad (16)$$

The first Ziegler–Nichols method was performed on the $G(s)$ transfer function and rough PID values were obtained as $K_p = 7$, $K_i = 4$, and $K_d = 2$. Then, the three-loop similarity ratio measurement algorithm was run in MATLAB. The lower and upper limits were selected, based on the $K_p = 7$, $K_i = 4$, and $K_d = 2$ values.

$1 \leq K_p \leq +20$ (for minimum $K_p > 1$)
$-20 \leq K_i \leq +20$
$-20 \leq K_d \leq +20$

In the algorithm, K_p, K_i, and K_d values were changed respective to per unit increase, and the coefficients of the PID controller were reworked using these new values. Then, a unit step signal was applied to the G(s) transfer function using MATLAB's "*feedback*($G_c(s)$*$G(s)$,*1*)" command at the end of each step. The unit step response characteristics of the system were obtained using MATLAB's *"stepinfo"* function, and these characteristics were subjected to the neutrosophication process through the membership functions shown in Fig. 5. The values from the neutrosophication process were arranged to form a neutrosophic set and the real *B* neutrosophic set was obtained. The similarity ratio between the *A* ideal neutrosophic set given in Table 9 and the *B* real set was calculated using the 6, 8, 10, 12, 13, and 14 equations to generate the SM values. The results from this calculation were transferred to an *SM*(*A*,*B*) array. When the three nested loop search algorithm completed, it was decided that the K_p, K_i, K_d values corresponding to the maximum SM value within the *SM*(*A*,*B*) array were appropriate PID coefficients. In this example, the SM value reached the maximum value at the 12771-th order in the *SM*(*A*,*B*) array in calculations made using the Hamming, Euclidean, Set-theoretic, Jaccard, and Dice approaches.

For application 1, the algorithm used is given below. The same algorithm was used for application 2. In this algorithm, MF is used as an abbreviation of membership function.

```
for P = 1:1:Max_ Kp _Value
  for I = Min_ Ki _Value:1: Max_ Ki _Value
    for D = Min_ Kd _Value:1: Max_ Kd _Value
      Kp = P;Ki = I;Kd = D;
      Gc = (Kp + Ki*1/s + Kd*s);
      Gcl = feedback(G*Gc,1);
      SS = stepinfo(Gcl,1);
      Pass Rise Time on MF for obtain T,I,F (Fig. 5a)
      Pass Settling Time on MF for obtain T,I,F (Fig. 5b)
      Pass Overshoot on MF for obtain T,I,F (Fig. 5c)
      Pass Undershoot on MF for obtain T,I,F (Fig. 5d)
      Pass Peak Time on MF for obtain T,I,F (Fig. 5e)
      Pass Steady-State Error on MF for obtain T,I,F (Fig. 5f)
      Calculate SMs using Eqs. 6, 8, 10, 12, 13, 14
      Put calculated SM values and correspond P,I,D values to SM array
    end
  end
end
Choose maximum SM value and correspond P,I,D values from SM array. And assign these P,I,D values as Kp, Ki, Kd coefficients of PID controller.
```

The e_1–e_6 values given in Table 9 were determined by the peak value of the *T* membership function in Fig. 5. Here, while the *T* membership value was 1, the *I* and *F* values were 0. This shows the desired values.

The similarity measuring algorithm required a total of 45,387 steps. The highest value in the *SM*(*A, B*) sequence order of this value and the corresponding K_p, K_i, and K_d values are shown in Table 10.

The change in the similarity ratio against the number of steps is shown in Fig. 6.

In Fig. 6a, the initial (lowest) value of the Hamming and Euclidean similarity measures was approximately 0.5, which is a negative result in the Hamming and Euclidean approaches. A 0.5 minimum initial value represents a 50 % similarity ratio, which may cause errors. In Fig. 6b, c, the initial value of the similarity measure of the set-theoretic Jaccard and Dice approaches were approximately 0.2.

In Fig. 7, the step response curves are drawn for the K_p, K_i, and K_d values in each 10,000 steps of the similarity measuring algorithm. Graphics 1-4 represent the step responses to the K_p, K_i, and K_d values at the 10,000–40000-th steps. Graphic 5 represents the step response to the K_p, K_i, and K_d values obtained by the proposed method at the 12771-th step.

The unit step response curves following the proposed method ($K_p = 8$, $K_i = 8$, and $K_d = 1$) and the Ziegler–

Table 9 The *A* ideal neutrosophic set

A	e_1	e_2	e_3	e_4	e_5	e_6
i	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)

Table 10 Obtained maximum value from *SM(A,B)* array and correspond PID values

Method	*SM(A,B)*	Array order	K_p	K_i	K_d
Hamming	0.9183	12771	8	8	1
Euclidean	0.8719	12771	8	8	1
Set-theoretic	0.911	12771	8	8	1
Jaccard	0.9729	12771	8	8	1
Dice	0.9855	12771	8	8	1

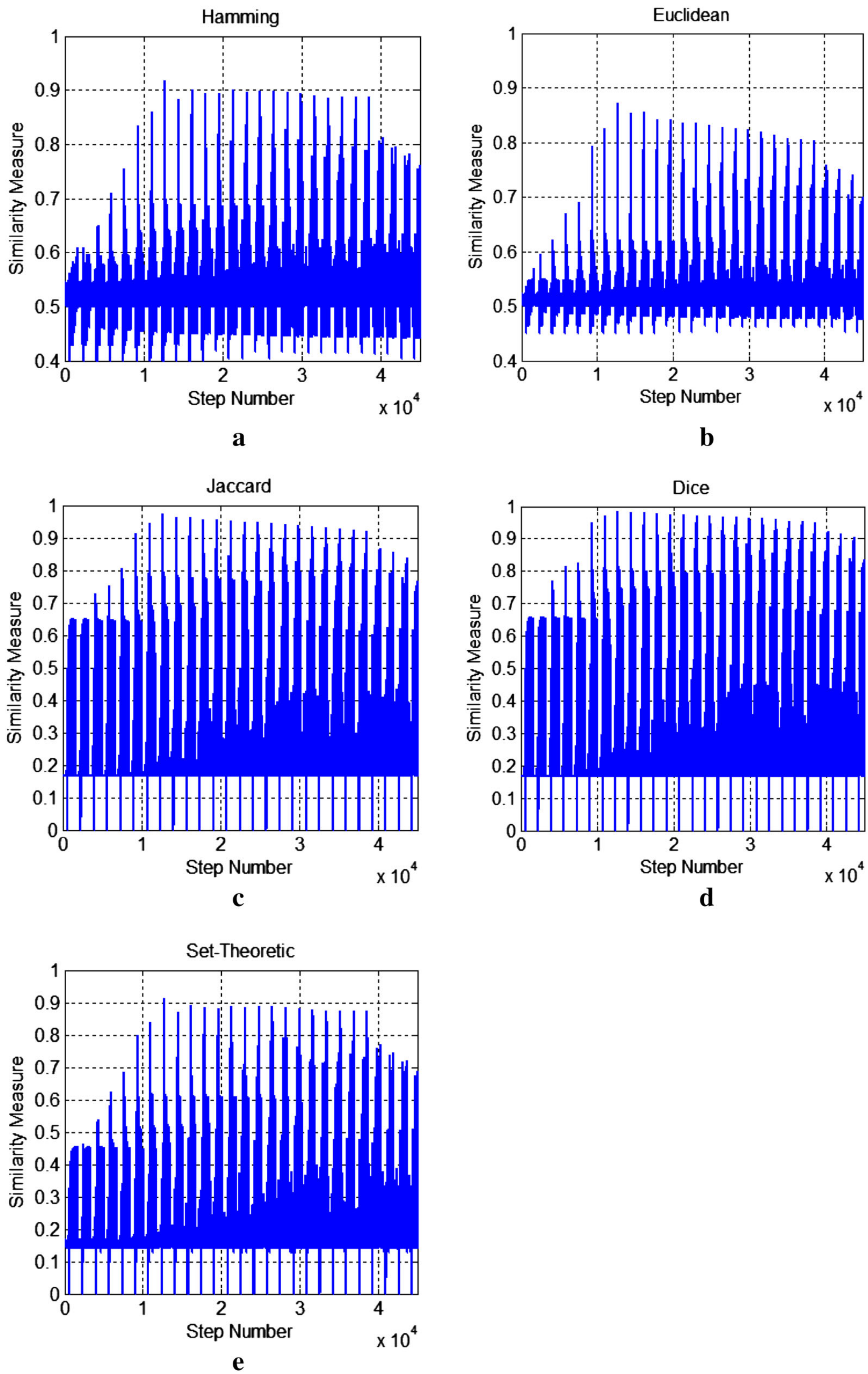

Fig. 6 Similarity ratio change graph obtained from the similarity ratio measuring algorithm

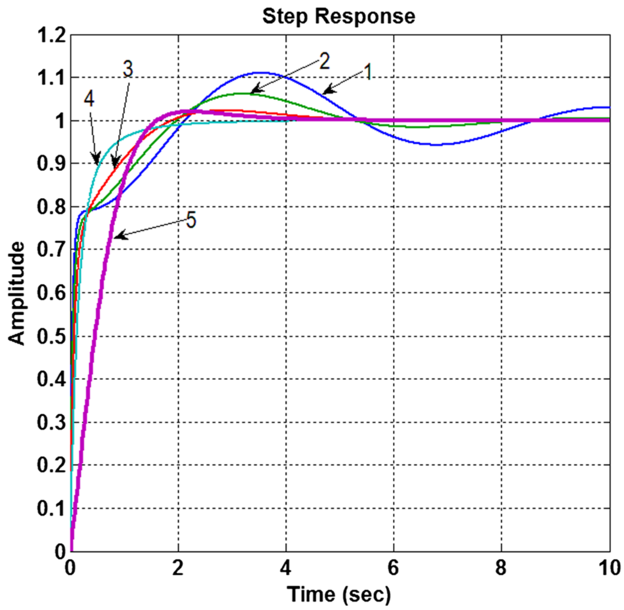

Fig. 7 Step response curves obtained from the *G(s)* transfer function according to K_p, K_i and K_d values in each 10,000 step in the similarity measuring algorithm

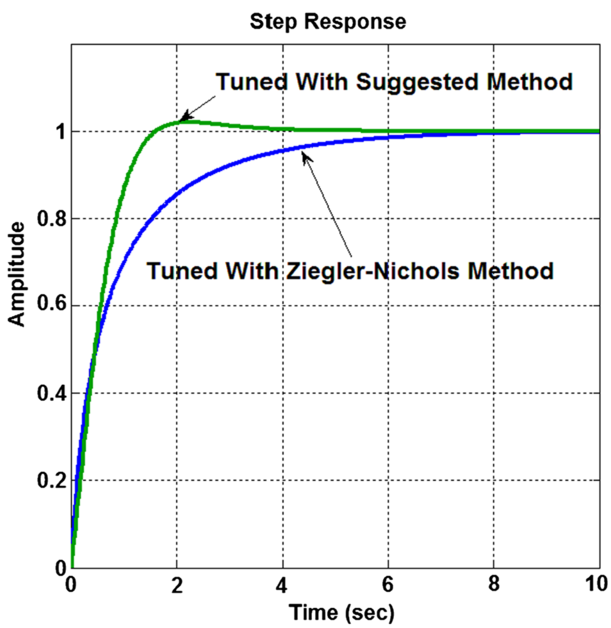

Fig. 8 Step response of the system according to proposed method and Ziegler–Nichols method

Nichols method ($K_p = 7$, $K_i = 4$, and $K_d = 2$) are presented in Fig. 8.

The real neutrosophic set for $K_p = 8$, $K_i = 8$, and $K_d = 1$ is given in Table 11.

Table 11 shows that the *B* real set is close to the *A* ideal set Table 9. In this example, Table 10 shows that the SM values were close to each other and close to 1 on all similarity measures. This indicates that the desired step response has been obtained.Table 12 shows the unit step response characteristics according to K_p, K_i, K_d values obtained by using different similarity measures.

Table 11 *B* real neutrosophic set for application example 1

B	e_1	e_2	e_3	e_4	e_5	e_6
r	(0.66,0,0)	(0.92,0,0)	(0.99,0,0)	(1,0,0)	(0.88,0,0)	(1,0,0)

Table 12 Transient response characteristics of *G(s)* according to K_p, K_i, K_d values obtained Set-theoretic, Hamming, euclidean, Jaccard, Dice methods

Transient response characteristics	Set-theoretic, Hamming, Euclidean, Jaccard, Dice
Rising time	1.0027
Settling time	2.3000
% Over shoot	2.0152
% Undershoot	0
Peak time	2.2295
Steady-state error	0

3.3 Application Example 2

The proposed method was then tested on an open-loop transfer function with a third degree characteristic equation, using the same methods and the same membership functions as in the first example. Consider the following transfer function:

$$G(s) = \frac{1}{s(s+1)(s+5)} \tag{17}$$

In this example, the first Ziegler–Nichols method was used as in the first example and values of $K_p = 18$, $K_i = 13$, and $K_d = 6$ were obtained. Same as Example 1, lower and upper limits were used for K_p, K_i, and K_d search range. The search range is given below.

$1 \leq K_p \leq +20$ (for minimum $K_p > 1$)
$-20 \leq K_i \leq +20$
$-20 \leq K_d \leq +20$

Table 13 Obtained maximum value from *SM(A,B)* array and correspond PID values

Method	*SM(A,B)*	Array order	K_p	K_i	K_d
Hamming	0.8904	10394	7	0	6
Euclidean	0.8586	12076	8	0	7
Set-theoretic	0.8769	10394	7	0	6
Jaccard	0.9668	12076	8	0	7
Dice	0.9827	12076	8	0	7

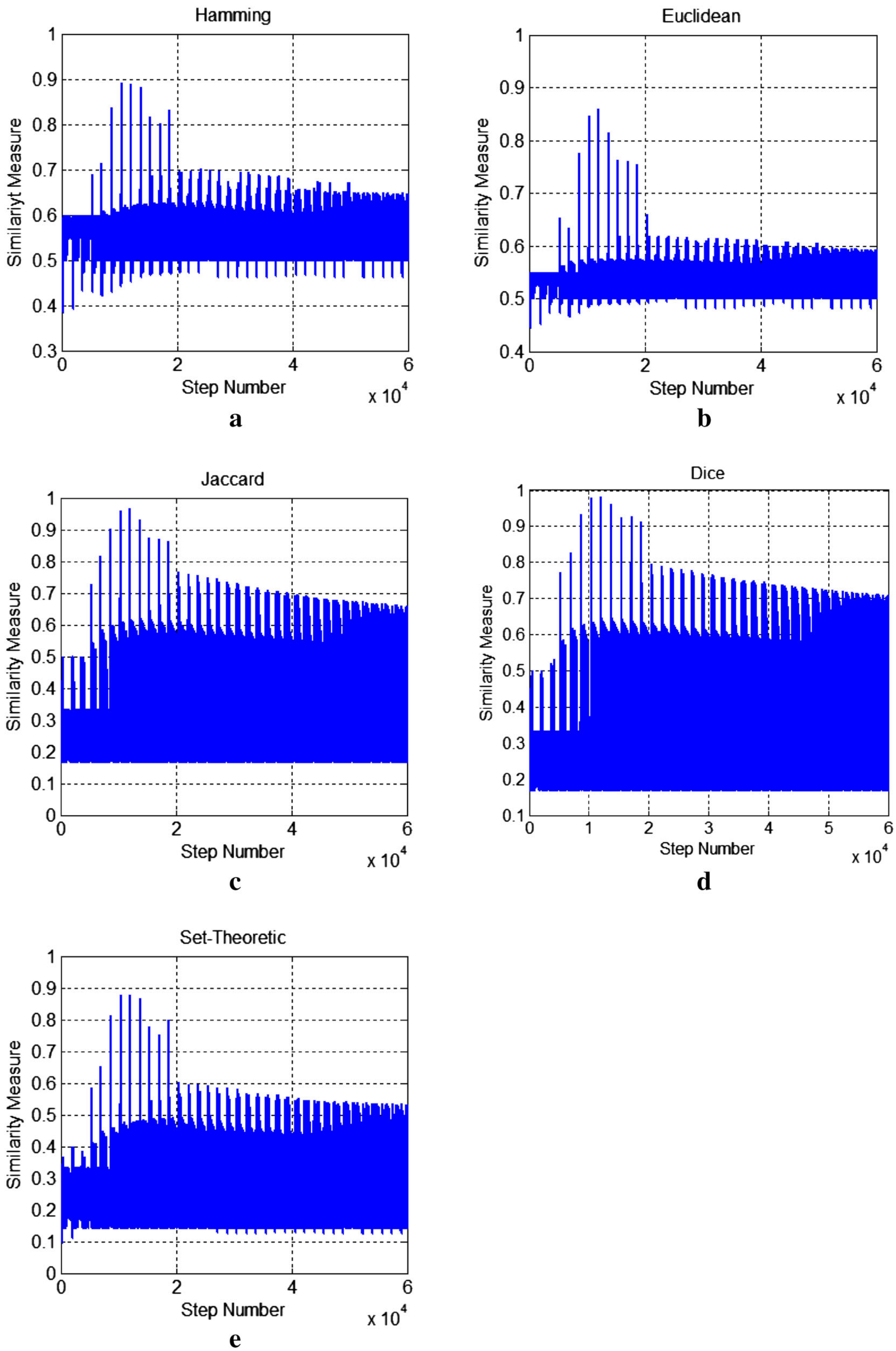

Fig. 9 Similarity ratio change graph obtained from the similarity ratio measuring algorithm

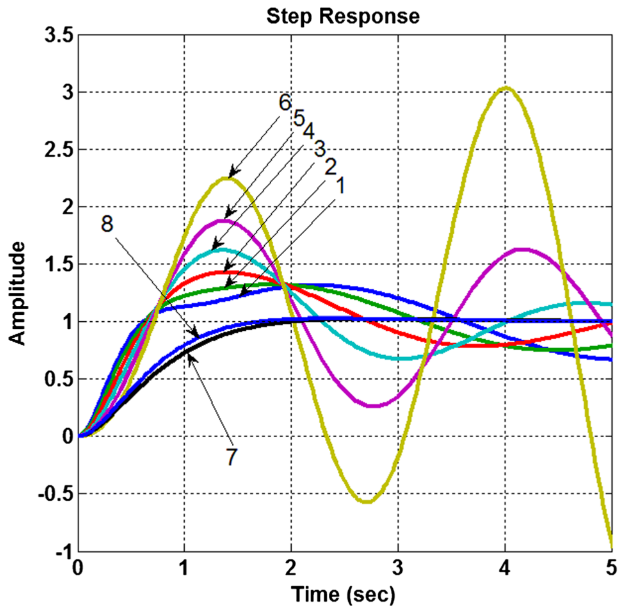

Fig. 10 Step response curves obtained from the *G(s)* transfer function according to K_p, K_i, and K_d values in each 10,000 step in the similarity measuring algorithm

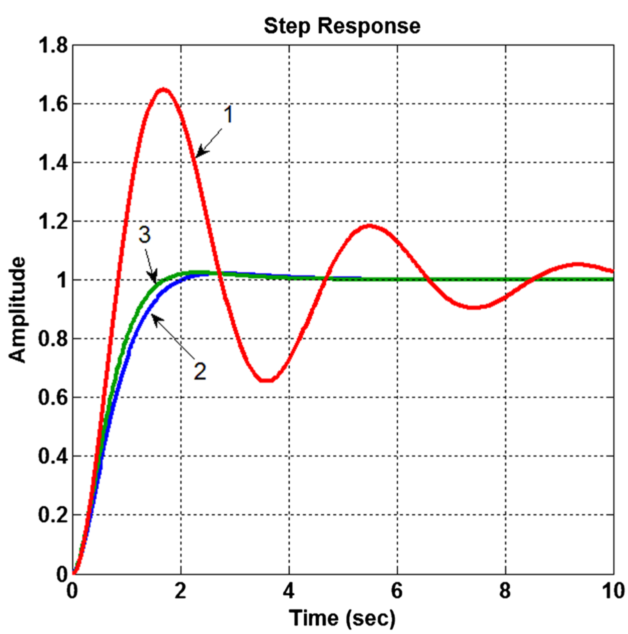

Fig. 11 Step response of the system according to proposed method and Ziegler–Nichols method. *1* Ziegler–Nichols, *2* Set-theoretic and Hamming, *3* Euclidean, Jaccard, and Dice

The similarity ratio measuring algorithm took a total of 63,878 steps. The maximum value at the end of the algorithm run from the *SM*(*A*,*B*) array is shown in Table 13.

The similarity ratio change is related to the step number graph in Fig. 9.

In Fig. 9a, the results are consistent with the results of the other examples. The initial (lowest) value of the Hamming and Euclidean similarity measures was approximately 0.5; this may be considered negative aspect of the Hamming and Euclidean approaches. In Fig. 9b, c, the initial values of the similarity measure under the set-theoretic, Jaccard, and Dice approaches were approximately 0.2.

Figure 10 shows the step response curves drawn according to the K_p, K_i, and K_d values in each 10,000 steps of the similarity measuring algorithm, where 1–6 represents the 10000–60000-th step response and 7–8 represents the 10394-th and 12076-th step response under the proposed method.

Figure 11 compares the step responses of the $G(s)$ transfer function using the K_p, K_i, and K_d values obtained by the Ziegler–Nichols method and the proposed method.

In Tables 14 and 15, the real neutrosophic set found by the proposed method is similar to a target (ideal) neutrosophic set. There are very small differences between the Set-theoretic, Hamming, and Euclidean approaches, and the Jaccard and Dice approaches. Curves 2 and 3 in Fig. 11 and the values in Table 16 demonstrate that the unit step responses of the system under the Set-theoretic, Hamming, Euclidean, Jaccard, and Dice approaches were very similar.

Table 14 Obtained *B* real neutrosophic set according to Set-theoretic and Hamming similarity measurement

B	e_1	e_2	e_3	e_4	e_5	e_6
r	(0.80,0,0)	(0.84,0,0)	(0.98,0,0)	(1,0,0)	(0.62,0,0)	(1,0,0)

Table 15 Obtained *B* real neutrosophic set according to Euclidean, Jaccard, and Dice

B	e_1	e_2	e_3	e_4	e_5	e_6
r	(0.69,0,0)	(0.90,0,0)	(0.82,0,0)	(1,0,0)	(0.83,0,0)	(1,0,0)

Table 16 Transient response characteristics according to Set-theoretic, Hamming, Euclidean, Jaccard, Dice SM

Transient response characteristic	Set-theoretic, Hamming	Euclidean, Jaccard, Dice
Rising time	1.2063	1.0373
Settling time	2.8848	2.7353
% Overshoot	2.0334	2.3509
% Undershoot	0	0
Peak time	2.7442	2.3344
Steady-state error	0	0

4 Conclusions

We have shown through two transfer function examples that any of the Hamming, Euclidean, Set-theoretic, Jaccard, and Dice approaches to the neutrosophic similarity measure can be used in the PID tuning procedure. Since in some cases the denominator would be undefined, the cosine similarity measure was not tested. In tests carried out on two different second and third order transfer functions, the values obtained from the different neutrosophic SM approaches were found to be approximately the same. The most important advantage of the proposed method is that it does not require any complex calculations. In both example applications, the same membership functions were used for the neutrosophication process. Depending on the type of application being addressed, the membership functions may be revised.

Another advantage of the proposed method is that it is based on time domain values, and does not require the transfer function of the system to be known. This makes it suitable for application in real systems with unknown transfer functions. The Ziegler–Nichols method was used in the study to narrow down the search range for the appropriate PID coefficients. The method can also be used alone, without the Ziegler–Nichols method. In future studies, the proposed method will be tested in real system platforms, and will be compared with other PID tuning methods from the literature. In addition, a graphical user interface (GUI) running in MATLAB is planned, to improve its practicality.

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