Simplified Neutrosophic Linguistic Multi-criteria Group Decision-Making Approach to Green Product Development

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Abstract For many companies, green product development has become a key strategic consideration due to regulatory requirements and market trends. In this paper, the life cycle assessment technique is used to develop an innovative multi-criteria group decision-making approach that incorporates power aggregation operators and a TOPSIS-based QUALIFLEX method in order to solve green product design selection problems using neutrosophic linguistic information. Differences in semantics as well as the risk preferences of decision-makers are considered in the proposed method. The practicality and effectiveness of the proposed approach are then demonstrated through an illustrative example, in which the proposed method is used to select the optimum green product design, followed by sensitivity and comparative analyses.

Keywords Multi-criteria group decision-making · Simplified neutrosophic linguistic set · Power aggregation operator · QUALIFLEX · Green product development

1 Introduction

Due to the increasing public awareness of environmental issues, businesses have begun to promote practices that alleviate or prevent negative environmental effects. As a result, green design, also known as eco-design or design for the environment, has become an important facet of new product development. The life cycle assessment (LCA) technique, a practical model, is used to analyze the environmental effects of a product based on the characteristics of its life cycle (Junnila 2008). This model

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is based on the theory that the decisions made during the design stage of a product could significantly impact the environment throughout the life cycle of that product (Frei and Züst 1997). Usually, life cycle of a product is comprised of six phases, including the material selection (l_1), manufacturing (l_2), distribution (l_3), installation (l_4), usage (l_5) and end-of-life (l_6) stages. However, the life cycles of some products may not include all six of these phases (Chan et al. 2013). For example, the installation phase is not included in the life cycle of a battery-driven electronic product. Numerous decision-making methods for green product development have been proposed. In this paper, a green product design selection method is developed based on the life cycle assessment technique.

1.1 Review of Green Product Development Decision-Making Methods

In recent years, an increasing number of studies concerning the application of LCA-based methods to green product development have been conducted. The existing decision-making methods used for green product development can be roughly categorized into two groups, including traditional LCA-based methods and extended LCA-based fuzzy multi-criteria decision-making (MCDM) methods.

Traditional LCA-based methods include the input-output LCA (IO-LCA) method (Junnila 2008), analytic hierarchy process (AHP) (Yang et al. 2010; Wang et al. 2015c), semi-quantitative screening eco-design method (Simanovska et al. 2012), and dynamic eco-strategy explorer model (Mansoux et al. 2014). In one study, Junnila (2008) investigated whether the IO-LCA method effectively reflects the life cycle effects of energy-using products. In another study, Wang et al. (2015c) developed a hierarchical model in order to assess eco-design options. Moreover, Yang et al. (2010) constructed an assessment framework for product development by combining the AHP and DELPHI methods. Simanovska et al. (2012) developed a semi-quantitative screening eco-design method in order to identify the health-related and environmental effects of the hazardous substances in various products. Furthermore, Mansoux et al. (2014) established a dynamic eco-design explorer model in order to assess the environmental effects of various products.

Extended LCA-based fuzzy multi-criteria decision-making (MCDM) methods have also been extensively applied to green product development (Ng and Chuah 2012; Chan et al. 2013, 2014; Wang and Chan 2013; Wang et al. 2014, 2015d). In one study, Ng and Chuah (2012) discussed the feasibility of the LCA-based integration of a fuzzy AHP and a fuzzy extension of the technique for order of preference by similarity to ideal solution (TOPSIS). In addition, Wang and Chan (2013) combined the fuzzy extent analysis and fuzzy hierarchical TOPSIS methods in order to evaluate product designs from a remanufacturing perspective. Furthermore, Chan et al. (2013) developed an LCA-based fuzzy AHP approach in order to assess the overall environmental performance of various product designs, and Wang et al. (2014) developed an LCA-based fuzzy analytic network process (ANP) method for the selection of environmentally sustainable product designs. Based on the multitier AHP framework proposed by Chan et al. (2013), Wang et al. (2015d) developed a fuzzy hierarchical TOPSIS technique in order to evaluate green product designs. Chan et al. (2014) also combined the concepts

of LCA and environmental management accounting (EMA) with the aforementioned multitier AHP framework in order to analyze the environmental and organizational performances of various product designs.

Although these methods can be effectively applied to green product development, they do have some limitations.

(1) In the aforementioned methods, crisp numbers (Junnila 2008; Yang et al. 2010; Simanovska et al. 2012; Mansoux et al. 2014; Wang et al. 2015c) and fuzzy sets (FSs) (Ng and Chuah 2012; Chan et al. 2013, 2014; Wang and Chan 2013; Wang et al. 2014, 2015d) are used to represent assessment information. Due to the complexity of green product development, real numbers cannot effectively describe evaluated objects. In addition, in FSs, a single membership function is used to determine the degree to which an element belongs to a reference set. As a result, FSs cannot accurately represent incomplete or inconsistent information.

(2) In MCDM methods (Ng and Chuah 2012; Chan et al. 2013, 2014; Wang and Chan 2013; Wang et al. 2014, 2015d), linguistic information is transformed into triangular fuzzy numbers (TFSs). Thus, since these methods do not account for differences in semantics, they can only be applied to fixed semantic situations.

(3) The risk preferences of decision-makers (DM) are not considered in the afore-mentioned extended LCA-based fuzzy (MCDM) methods (Ng and Chuah 2012; Chan et al. 2013, 2014; Wang and Chan 2013; Wang et al. 2014, 2015d). Due to the increasing complexity and competitiveness of markets, companies often seek assistance from experts in various fields in order to improve the efficiency of the product selection process. These experts have different risk preferences and back-grounds, which influence their selections. Therefore, green product development should be treated as a typical comprehensive multi-criteria group decision-making (MCGDM) problem, in which experts with different backgrounds and experiences are assimilated to evaluate a number of product designs with various criteria.

1.2 Goals and Innovations Presented in this Paper

Simplified neutrosophic linguistic sets (SNLSs) (Ye 2015; Tian et al. 2015a; Ma et al. 2016), an extension of FSs (Zadeh 1965) and intuitionistic fuzzy sets (IFSs) (Atanassov 1986), consist of linguistic term sets (Herrera et al. 1996; Herrera and Herrera-Viedma 2000) and neutrosophic sets (NS) (Ye 2014; Peng et al. 2016; Tian et al. 2015b; Zhang et al. 2015). Simplified neutrosophic linguistic numbers (SNLNs), which act as the elements in SNLSs, are used to effectively describe uncertain, incomplete, and inconsistent information. For example, when a paper is sent to a reviewer, he or she may state that the paper is "good", with probabilities of truth, falsity, and uncertainty of 60, 50, and 20 %, respectively. This information, which cannot be effectively managed with FSs and IFSs, can be expressed as $\langle s_5, (0.6, 0.2, 0.5) \rangle$ using SNLNs. Thus, SNLNs can be used to describe linguistic information.

The power averaging (PA) operator proposed by Yager (2001) is one of the most important information aggregation tools used in decision-making models (Wan 2013; Liu and Yu 2014; Peng et al. 2015; Cao et al. 2015; Liu and Teng 2015). In various methods (Wan 2013; Liu and Yu 2014; Peng et al. 2015), PA operators are used to

manage information provided by decision-makers and other criteria. PA operators account for information regarding the interrelationships among aggregated values and enable those values to reinforce one another as they are aggregated. Thus, the weight vectors in PA operators depend on the input arguments. However, according to the LCA concept, aggregating the evaluation information regarding a green product based on different criteria would not necessarily be beneficial since the attributes of a green product in the different phases of its life cycle are not interchangeable. Therefore, a PA operator could be a suitable tool to aggregate evaluation information of experts, but not criteria. In contrast, the qualitative flexible multiple criteria method (QUALIFLEX) originally proposed by Paelinck (1976, 1977, 1978), a valuable outranking tool, can be used to effectively manage large or complex MCDM problems involving numerous criteria and a limited number of alternatives (Chen et al. 2013; Chen 2014; Wang et al. 2015b; Zhang and Xu 2015). In a previous study, based on the extended QUALIFLEX and signed distance-based comparison methods (Chen et al. 2013; Zhang and Xu 2015), a comparison was conducted by constructing signed distances with only a positive ideal solution (PIS) or negative ideal solution (NIS). The results indicated that, although the proposed method correctly identified the differences between items, the final rankings were occasionally incorrect, especially when only the PIS or NIS was taken into account. In another study, inspired by TOPSIS (Chen and Hwang 1992), the PIS and NIS were considered simultaneously, and positive and negative values were used to identify the rankings of alternatives based on their signed distances (Yao and Wu 2000).

These analyses provided the motivation necessary to develop an LCA-based green product design selection method using SNLNs. In this study, a comprehensive approach was developed by integrating power aggregation operators and a TOPSIS-based QUALIFLEX method in order to solve fuzzy MCGDM problems. The primary contributions of this paper can be summarized as follows:

(1) Green product development was described as an MCGDM problem with SNLNs in order to compensate for the complexity of the selection process.
(2) Linguistic scale functions were used to transform qualitative data into quantitative data.
(3) A simplified neutrosophic linguistic power weighted averaging (SNLPWA) operator and simplified neutrosophic linguistic power weighted geometric (SNLPWG) operator were developed in order to aggregate neutrosophic linguistic evaluation information provided by DMs.
(4) A TOPSIS-based QUALIFLEX method was developed in order to rank alternatives of green product designs considering the risk preferences of DMs.

The remainder of this paper is organized as follows. In Sect. 2, concepts regarding linguistic term sets, neutrosophic sets (NSs), simplified neutrosophic sets (SNSs), SNLNSs, and SNLN operations are briefly reviewed. In Sect. 3, a family of distance measurements is defined, and a TOPSIS-based QULIFLEX method is developed. In Sect. 4, an SNLPWA operator and SNLPWG operator are developed. In Sect. 5, a simplified neutrosophic linguistic MCGDM approach is developed by integrating power aggregation operators and the proposed TOPSIS-based QUALIFLEX method. In Sect. 6, an illustrative example of a green product design selection problem is pro-

vided in order to demonstrate the feasibility and applicability of the proposed approach. In addition, sensitivity and comparative analyses are conducted. The conclusions of this paper are presented in Sect. 7.

2 Preliminaries

In this section, definitions and operations related to SNLSs, including linguistic term sets, linguistic scale functions, and SNLN operations, are briefly reviewed.

Let $S = \{s_\tau | \tau = 1, 2, \ldots, 2t + 1\}$ be a finite and totally ordered discrete term set, where t is a nonnegative integer. Then s_τ and s_υ ($s_\tau, s_\upsilon \in S$) is ordered $s_\tau < s_\upsilon$ if and only if $\tau < \upsilon$. If a negation operator exists, then $neg(s_\tau) = s_{(2t+2-\tau)}$ ($\tau, \upsilon = 1, 2, \ldots, 2t + 1$), in which the indices can be obtained using the subscript function $sub(s_\tau) = \tau$ (Herrera et al. 1996; Herrera and Herrera-Viedma 2000).

When aggregation information is utilized in the decision-making process, the aggregation results often do not correspond with the elements in the language assessment scale. In order to preserve information, Xu (2006) extended the discrete linguistic set $S = \{s_\tau | \tau = 1, 2, \ldots, 2t + 1\}$ into the continuous linguistic term set $\bar{S} = \{s_\tau | 1 \leq \tau \leq L\}$, in which $s_\tau < s_\upsilon$ if and only if $\tau < \upsilon$, and L ($L > 2t + 1$) is a sufficiently large positive integer. If $s_\tau \in S$, then s_τ is called the original linguistic term; otherwise s_τ is called the virtual linguistic term. Virtual linguistic terms, which have no practical meaning, are primarily used to rank alternatives (Mart et al. 2010). Usually, DMs use original linguistic terms to evaluate alternatives, and virtual linguistic terms are only used in operations to prevent information loss and enhance the decision-making process. In practice, calculation results are normalized using weights; thus, the subscripts of the linguistic terms do not exceed $2t + 1$.

Definition 1 (Xu 2006). Let $S = \{s_\tau | \tau = 1, 2, \ldots, 2t + 1\}$ be a linguistic term set and $s_\tau, s_\upsilon \in S$ be two arbitrary linguistic terms. Then the operations can be defined as:

(1) $\lambda s_\tau = s_{\lambda \times \upsilon}$;
(2) $s_\tau \oplus s_\upsilon = s_{\tau + \upsilon}$;
(3) $s_\tau \otimes s_\upsilon = s_{\tau \times \upsilon}$;
(4) $(s_\tau)^\lambda = s_{\tau^\lambda}$.

Example 1 Assume $S = \{s_1, s_2, \ldots, s_7\}$ =\{*extremely poor, very poor, poor, medium, good, very good, extremely good*\} is a linguistic term set and $s_5, s_6 \in S$ are two linguistic terms. According to the operation laws presented in Definition 1:

$$0.4s_5 \oplus 0.6s_6 = s_{0.4 \times 5} \oplus s_{0.6 \times 6} = s_{5.6}.$$

Therefore, since the obtained result is a virtual linguistic term that does not possess any semantics or proper linguistic syntax, the comprehensive result $s_{5.6}$ lies between s_5 ("good") and s_6 ("very good"), but is more approximate to s_6.

2.1 Linguistic Scale Functions

When fuzzy numbers are directly combined with linguistic terms, operations cannot be conducted in a normal fashion. Thus, in order to define SNLN operations, linguistic scale functions must be used for linguistic modeling. Linguistic scale functions can be used to assign different semantic values to linguistic terms in different situations in order to use qualitative data more efficiently and express semantics with flexibility (Wang et al. 2014b; Zhou et al. 2016). In linguistic evaluation scales with increasing linguistic subscripts, the absolute deviation between any two adjacent linguistic subscripts can increase or decrease. Therefore, linguistic scale functions are preferable in practice since they are flexible and can yield relatively deterministic results regardless of differences in semantics.

Definition 2 (Wang et al. 2014b). Let $s_\tau \in S$ be a linguistic term. If $\theta_\tau \in [0, 1]$ is a numerical value, then the linguistic scale function f that conducts the mapping from s_τ to θ_τ ($\tau = 1, 2, \ldots, 2t + 1$) can be defined as

$$f : s_\tau \to \theta_\tau \ (\tau = 1, 2, \ldots, 2t + 1), \tag{1}$$

where $0 \leq \theta_1 < \theta_2 < \cdots < \theta_{2t+1} \leq 1$.

Thus, the function f monotonically increases with respect to the subscript τ. In addition, θ_τ ($\tau = 1, 2, \ldots, 2t + 1$) reflects the preferences of DMs when using the linguistic term $s_\tau \in S$ ($\tau = 1, 2, \ldots, 2t + 1$). Therefore, the function and values reflect differences in semantics. The following functions can act as linguistic scale functions:

(1) The linguistic scale function based on the subscript function $sub(s_\tau) = \tau$.

$$f_1(s_x) = \theta_x = \frac{x - 1}{2t} \ (x = 1, 2, \ldots, 2t + 1). \tag{2}$$

The evaluation scale of the given linguistic information is averaged; this linguistic scale function is simple and commonly used (Liu and Wei 2011).
(2) The linguistic scale function based on the exponential scale.

$$f_2(s_y) = \theta_y = \begin{cases} \frac{\alpha^t - \alpha^{t-y+1}}{2\alpha^t - 2} \ (y = 1, 2, \ldots, t + 1) \\ \frac{\alpha^t + \alpha^{y-t-1} - 2}{2\alpha^t - 2} \ (y = t + 2, t + 3, \ldots, 2t + 1) \end{cases}. \tag{3}$$

The value of α can be determined using a subjective approach. Let A and B be two indicators. Assume that A is more significant than B, with an importance ratio of m. Then $\alpha^k = m$, where k represents the scale level, and $\alpha = \sqrt[k]{m}$. Currently, most researchers believe that the upper limit of the importance ratio is $m = 9$. If the scale level is 7, then $\alpha = \sqrt[7]{9} \approx 1.37$ (Bao et al. 2010). As the middle of the given linguistic term set increases on both sides, the absolute deviation between any two adjacent linguistic subscripts also increases.

(3) The linguistic scale function based on prospect theory.

$$f_3(s_z) = \theta_z = \begin{cases} \frac{t^\beta - (t-z+1)^\beta}{2t^\beta} & (z = 1, 2, \ldots, t+1) \\ \frac{t^\gamma + (z-t-1)^\gamma}{2t^\gamma} & (z = t+2, t+3, \ldots, 2t+1) \end{cases}. \tag{4}$$

The values $\beta, \gamma \in [0, 1]$ denote the curvature of the subjective value function for gains and losses, respectively. (Kahneman and Tversky 1979) experimentally determined that $\beta = \gamma = 0.88$, which corresponds with empirical data. Therefore, as the middle of the given linguistic term set increases on both ends, the absolute deviation between any two adjacent linguistic subscripts decreases.

In order to preserve the information produced in the calculation of evaluation data, the above linguistic scale functions f_1, f_2, and f_3 can be extended to $f^* : \bar{S} \to R^+$ ($R^+ = \{r \mid r \geq 0, r \in R\}$), which satisfies $f^*(s_\tau) = \theta_\tau$, a strictly monotonically increasing and continuous function. Therefore, the mapping from $\bar{S}$ to R^+ is one-to-one due to its monotonicity, and the inverse function of f^*, denoted as f^{*-1}, exists.

2.2 Neutrosophic Sets and Simplified Neutrosophic Sets

Definition 3 (Smarandache 1999). Let X be a space of points (objects) with a generic element in X, denoted by x. Then an NS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. In addition, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$; that is, $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$. Since the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ is unrestricted, $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Since NSs cannot easily be applied to practical problems, Ye (2014) reduced the NSs of nonstandard interval numbers into the SNSs of standard interval numbers.

Definition 4 (Rivieccio 2008; Ye 2014). Let X be a space of points (objects) with a generic element in X, denoted by x. Then an NS A in X is characterized by $T_A(x)$, $I_A(x)$, and $F_A(x)$, which are single subintervals or subsets in the real standard $[0, 1]$; that is, $T_A(x):X \to [0, 1]$, $I_A(x):X \to [0, 1]$, and $F_A(x):X \to [0, 1]$. In addition, the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Thus, A can be simplified as $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$, which is an SNS (a subclass of NSs).

For an SNS $\{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$, the ordered triple components $(T_A(x), I_A(x), F_A(x))$ are described as a simplified neutrosophic number (SNN), and each SNN can be expressed as $a = (T_a, I_a, F_a)$, where $T_a \in [0, 1]$, $I_a \in [0, 1]$, $F_a \in [0, 1]$, and $0 \leq T_a + I_a + F_a \leq 3$.

2.3 Simplified Neutrosophic Linguistic Sets and Operations for Simplified Neutrosophic Linguistic Numbers

Definition 5 (Ye 2015; Tian et al. 2015a). Let X be a space of points (objects) with a generic element in X denoted by x and $S = \{s_\tau \mid \tau = 1, 2, \ldots, 2t + 1\}$ be a finite and totally ordered discrete linguistic term set, where t is a nonnegative integer. Then an SNLS A in X is characterized as $A = \{\langle x, h_A(x), (T_A(x), I_A(x), F_A(x)) \rangle \mid x \in X\}$, where $h_A(x) \in S$, $T_A(x) \in [0, 1]$, $I_A(x) \in [0, 1]$, and $F_A(x) \in [0, 1]$ if $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for any $x \in X$. In addition, $T_A(x), I_A(x)$, and $F_A(x)$ represent the degree of truth-membership, indeterminacy-membership, and falsity-membership of the element x in X to the linguistic term $h_A(x)$, respectively.

For an SNLS $\{\langle h_A(x), (T_A(x), I_A(x), F_A(x)) \rangle \mid x \in X\}$, the ordered quadruple components $\langle h_A(x), (T_A(x), I_A(x), F_A(x)) \rangle$ are described as an SNLN, and each SNLN can be expressed as $a = \langle h_a, (T_a, I_a, F_a) \rangle$, where $h_a \in S$, $T_a \in [0, 1]$, $I_a \in [0, 1]$, $F_a \in [0, 1]$, and $0 \leq T_a + I_a + F_a \leq 3$. Therefore, when $T_a = 1$ and $I_a = F_a = 0$, the SNLN is degenerated into a linguistic term.

Definition 6 (Tian et al. 2015a). Let $a = \langle h_a, (T_a, I_a, F_a) \rangle$ and $b = \langle h_b, (T_b, I_b, F_b) \rangle$ be any two SNLNs, f^* be a linguistic scale function, and $\lambda \geq 0$. Then the following SNLN operations can be defined:

(1) $a \oplus b = \left\langle f^{*-1} \left(f^*(h_{\theta_a}) + f^*(h_{\theta_b}) \right), \left(\frac{f^*(h_{\theta_a})T_a + f^*(h_{\theta_b})T_b}{f^*(h_{\theta_a}) + f^*(h_{\theta_b})}, \frac{f^*(h_{\theta_a})I_a + f^*(h_{\theta_b})I_b}{f^*(h_{\theta_a}) + f^*(h_{\theta_b})}, \right. \right.$
$\left. \left. \frac{f^*(h_{\theta_a})F_a + f^*(h_{\theta_b})F_b}{f^*(h_{\theta_a}) + f^*(h_{\theta_b})} \right) \right\rangle$;

(2) $a \otimes b = \left\langle f^{*-1} \left(f^*(h_{\theta_a})f^*(h_{\theta_b}) \right), (T_a T_b, I_a + I_b - I_a I_b, F_a + F_b - F_a F_b) \right\rangle$;

(3) $\lambda a = \left\langle f^{*-1} \left(\lambda f^*(h_{\theta_a}) \right), (T_a, I_a, F_a) \right\rangle$;

(4) $a^\lambda = \left\langle f^{*-1} \left((f^*(h_{\theta_a}))^\lambda \right), (T_a^\lambda, 1 - (1 - I_a)^\lambda, 1 - (1 - F_a)^\lambda) \right\rangle$.

According to Definition 2, f^* is a mapping from the linguistic term s_τ to the numerical value θ_τ, and f^{*-1} is a mapping from θ_τ to s_τ. Therefore, in the calculation results, the linguistic parts are denoted as linguistic terms, and the membership parts are denoted as SNNs. The results obtained using Definition 6 are also SNLNs. These aforementioned operations were obtained using the linguistic scale function, which can yield different results when a different linguistic function f^* is employed. Thus, DMs can flexibly select a linguistic function f^* based on their personal preferences and different semantic situations.

3 TOPSIS-Based QUALIFLEX Method with Simplified Neutrosophic Linguistic Numbers

In this section, several SNLN distance measurements are developed using the above linguistic scale functions. Then, a TOPSIS-based QUALIFLEX method is developed based on the proposed distance measurements.

3.1 Distance Measurement Between Two Simplified Neutrosophic Linguistic Numbers

Distance measurements are widely used to measure the amount of deviation and degree of proximity between arguments. In recent years, numerous studies concerning the development of distance measurements have been conducted. These distance measurements include traditional distance measurements, such as the Hamming distance measurement, Euclidean distance measurement, and Hausdorff metric (Xu and Chen 2008; Zeng 2013; Liao and Xu 2015; Zhou et al. 2016), and weighted distance measurements, such as hybrid weighted distance measurements (Liao et al. 2015), directional distance measurements (Branda 2015; Wang et al. 2015a), and fuzzy ordered distance measurements (Xian and Sun 2014). All of these distance measurements have been extended into IFSs (Liao and Xu 2014; Szmidt 2014), SNSs (Zhang and Wu 2014; Peng et al. 2014), interval neutrosophic sets (INSs) (Zhang et al. 2016), MVNSs (Peng et al. 2015), HFSs (Xu and Xia 2011; Wang et al. 2014a; Zhang and Xu 2015), HFLSs (Wang et al. 2015c), HFLTSs (Liao et al. 2015; Wang et al. 2015a), and SNLSs (Ye 2015). The distance measurements proposed in this study were developed based on these previous studies.

Definition 7 Let $a = \langle h_a, (T_a, I_a, F_a) \rangle$ and $b = \langle h_b, (T_b, I_b, F_b) \rangle$ be any two SNLNs and f^* be a linguistic scale function. Then the generalized distance measure between a_i and a_j can be defined as

$$d(a, b) = \left(\frac{1}{3} \left(\left| f^*(h_a) T_a - f^*(h_b) T_b \right|^\lambda + \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|^\lambda \right. \right.$$
$$\left. \left. + \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^\lambda \right) \right)^{\frac{1}{\lambda}}. \tag{5}$$

When $\lambda = 1, 2$, Eq. (5) is reduced to the Hamming distance and Euclidean distance, respectively.

Theorem 1 *Let $a = \langle h_a, (T_a, I_a, F_a) \rangle$, $b = \langle h_b, (T_b, I_b, F_b) \rangle$, and $c = \langle h_c, (T_c, I_c, F_c) \rangle$ be any three SNLNs and f^* be a linguistic scale function. Then the distance measurement presented in Definition 7 satisfies the following properties:*

(1) $d(a, b) \geq 0$;
(2) $d(a, b) = d(b, a)$;
(3) If $h_a \leq h_b \leq h_c, T_a \leq T_b \leq T_c, I_a \geq I_b \geq I_c$, and $F_a \geq F_b \geq F_c$, then $d(a, b) \leq d(a, c)$, and $d(b, c) \leq d(a, c)$.

Proof As shown, $d(a, b)$ satisfies Properties (1) and (2). The proof of Property (3) is presented below.

Since $h_a \leq h_b \leq h_c, T_a \leq T_b \leq T_c, I_a \geq I_b \geq I_c, F_a \geq F_b \geq F_c$, and f^* is a strictly monotonically increasing and continuous function, $f^*(h_a) \leq f^*(h_b) \leq f^*(h_c)$. Thus, the following inequalities can be obtained:

$$f^*(h_a)T_a \leq f^*(h_b)T_b$$
$$\leq f^*(h_c)T_c \Rightarrow \left| f^*(h_a)T_a - f^*(h_b)T_b \right|^\lambda$$
$$\leq \left| f^*(h_a)T_a - f^*(h_c)T_c \right|^\lambda,$$
$$f^*(h_a)(1 - I_a) \leq f^*(h_b)(1 - I_b)$$
$$\leq f^*(h_c)(1 - I_c) \Rightarrow \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|^\lambda$$
$$\leq \left| f^*(h_a)(1 - I_a) - f^*(h_c)(1 - I_c) \right|^\lambda$$

In addition, if

$$f^*(h_a)(1 - F_a) \leq f^*(h_b)(1 - F_b)$$
$$\leq f^*(h_c)(1 - F_c) \Rightarrow \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^\lambda$$
$$\leq \left| f^*(h_a)(1 - F_a) - f^*(h_c)(1 - F_c) \right|^\lambda.$$

then

$$\left(\frac{1}{3} \left(\left| f^*(h_a)T_a - f^*(h_b)T_b \right|^\lambda + \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|^\lambda \right.\right.$$
$$\left.\left. + \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^\lambda \right) \right)^{\frac{1}{\lambda}}$$
$$\leq \left(\frac{1}{3} \left(\left| f^*(h_a)T_a - f^*(h_c)T_c \right|^\lambda + \left| f^*(h_a)(1 - I_a) - f^*(h_c)(1 - I_c) \right|^\lambda \right.\right.$$
$$\left.\left. + \left| f^*(h_a)(1 - F_a) - f^*(h_c)(1 - F_c) \right|^\lambda \right) \right)^{\frac{1}{\lambda}}.$$

Thus, $d(a, b) \leq d(a, c)$. The inequality $d(b, c) \leq d(a, c)$ can be proven in a similar manner. This concludes the proof of Theorem 1. $\qquad\square$

The Hausdorff distance measurement can also be integrated into SNLNs. For two SNLNs a and b, the Hausdorff distance measurement can be defined as

$$d_{haud}(a, b) = \max \left(\left| f^*(h_a)T_a - f^*(h_b)T_b \right|, \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|, \right.$$
$$\left. \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right| \right). \tag{6}$$

In addition, several hybrid distance measurements can be developed by combining the above distance measurements.

(1) The hybrid Hamming distance between a and b:

$$d_{hhd}(a, b)$$
$$= \frac{1}{2} \left(\frac{1}{3} \left(\left| f^*(h_a)T_a - f^*(h_b)T_b \right| + \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right| \right.\right.$$
$$\left.\left. + \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right| \right) \right)$$

$$+ \max \left(\left| f^*(h_a)T_a - f^*(h_b)T_b \right|, \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|, \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right| \right). \quad (7)$$

(2) The hybrid Euclidean distance between a and b:

$$
\begin{aligned}
d_{hed}(a, b) \\
= \Bigg(\frac{1}{2} \bigg(\frac{1}{3} \Big(&\left| f^*(h_a)T_a - f^*(h_b)T_b \right|^2 + \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|^2 \\
&+ \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^2 \Big) \\
&+ \max \Big(\left| f^*(h_a)T_a - f^*(h_b)T_b \right|^2, \left| f^*(h_a)(1 - I_a) \right. \\
&\left. - f^*(h_b)(1 - I_b) \right|^2, \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^2 \Big) \bigg) \Bigg)^{\frac{1}{2}}. \quad (8)
\end{aligned}
$$

(3) The generalized hybrid distance between a and b:

$$
\begin{aligned}
d_{ghd}(a, b) \\
= \Bigg(\frac{1}{2} \bigg(\frac{1}{3} \Big(&\left| f^*(h_a)T_a - f^*(h_b)T_b \right|^\lambda + \left| f^*(h_a)(1 - I_a) - f^*(h_b)(1 - I_b) \right|^\lambda \\
&+ \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^\lambda \Big) \\
&+ \max \Big(\left| f^*(h_a)T_a - f^*(h_b)T_b \right|^\lambda, \left| f^*(h_a)(1 - I_a) \right. \\
&\left. - f^*(h_b)(1 - I_b) \right|^\lambda, \left| f^*(h_a)(1 - F_a) - f^*(h_b)(1 - F_b) \right|^\lambda \Big) \bigg) \Bigg)^{\frac{1}{\lambda}}. \quad (9)
\end{aligned}
$$

When $\lambda = 1, 2$, Eq. (9) is reduced to Eqs. (7) and (8), respectively.
Similar to Eqs. (5), (6)–(9) satisfy the properties in Theorem 1.

Example 2 Assume that $a_1 = \langle s_4, (0.7, 0.4, 0.6) \rangle$ and $a_2 = \langle s_5, (0.8, 0.3, 0.5) \rangle$ are two SNLNs, and let $\lambda = 2, t = 3$, and $f^* = f_1^*$. Thus:

$$d_{gd}(a_1, a_2) = 0.1625, \quad d_{haud}(a_1, a_2) = 0.1833 \text{ and} d_{ghd}(a_1, a_2) = 0.1732.$$

Consider a decision matrix R that refers to m alternatives for n criteria. Then the simplified neutrosophic linguistic rating r_{ij} can be denoted by $R = [r_{ij}]_{m \times n} = [\langle h_{ij}, (T_{ij}, I_{ij}, F_{ij}) \rangle]_{m \times n}$, where r_{ij} represents the evaluation value of the alternative a_i ($i = 1, 2, \ldots, m$) for criterion c_j ($j = 1, 2, \ldots, n$).

According to Definition 5, since the ordered triple components (T_{ij}, I_{ij}, F_{ij}) are considered to be the degrees of true-membership, indeterminacy-membership and falsity-membership of the linguistic term h_{ij}, respectively, the SNLN$\langle h_{ij}, (T_{ij}, I_{ij}, F_{ij}) \rangle$ is reduced to a linguistic term if $T_{ij} = 1$ and $I_{ij} = F_{ij} = 0$. Moreover, since $h_{ij} \in S$, the smallest linguistic term is s_1, and the largest linguistic term is s_{2t+1}.

Thus, the simplified neutrosophic linguistic PIS and NIS can be expressed as $a^+ = \langle s_{2t+1}, (1, 0, 0)\rangle$ and $a^- = \langle s_1, (1, 0, 0)\rangle$, respectively (Ye 2015).

In order to select the optimum alternative, the distance between each alternative a_i $(i = 1, 2, \ldots, m)$ and the PIS a^+, denoted by $d(a_i, a^+)$, as well as the distance between each alternative a_i and the NIS a^-, denoted by $d(a_i, a^-)$, can be calculated. The closeness coefficient of TOPSIS for a given alternative a_i for criterion c_j $(j = 1, 2, \ldots, n)$ is defined as

$$\zeta(a_i) = \frac{(1 - \delta)d(a_i, a^-)}{\delta d(a_i, a^+) + (1 - \delta)d(a_i, a^-)}. \tag{10}$$

In this equation, δ denotes the risk preference of the DM, where $\delta \in [0, 0.5)$ indicates that the DM is risk-seeking, $\delta = 0.5$ indicates that the DM is risk-neutral, and $\delta \in (0.5, 1]$ indicates that the DM is risk-averse.

Since $\zeta(a_i) \in [0, 1]$ $(i = 1, 2, \ldots, m)$ for any $\delta \in [0, 1]$, high values of $\zeta(a_i)$ are associated with improved values of a_i.

Example 3 Let $\delta = 0.5$. According to the data presented in Example 2, since $\zeta(a_2) > \zeta(a_1), a_2 \succ a_1$, Eq. (9) can be used to obtain $\zeta(a_1) = 0.2971$ and $\zeta(a_2) = 0.4451$.

3.2 TOPSIS-Based QUALIFLEX Method

In this subsection, a novel TOPSIS-based QUALIFLEX approach is developed. In the proposed approach, the aforementioned SNLN closeness coefficient is used to identify the corresponding concordance/discordance index.

First, the concordance/discordance index is computed based on the successive permutations of all of the possible rankings of the alternatives. For a set A containing m alternatives, assume that $m!$ permutations of the ranking of alternatives exist. If P_l denotes the lth permutation, then

$$P_l = (\ldots, a_\alpha, \ldots, a_\beta, \ldots), \text{ for } l = 1, 2, \ldots, m!, \tag{11}$$

where $a_\alpha, a_\beta \in A$, and the ranking of a_α is higher than or equal to that of a_β.

If a_α and a_β are ranked in the same order in two preorders, then concordance exists. If a_α and a_β have the same ranking, then ex aequo exists. If a_α and a_β are counter-ranked, then discordance exists.

Therefore, the corresponding concordance/discordance index $\Upsilon_j^l(a_\alpha, a_\beta)$ of each pair of alternatives (a_α, a_β) $(a_\alpha, a_\beta \in A)$ at the preorder level under the n criteria in C as well as the ranking corresponding to permutation P_l can be defined using Eq. (10) as

$$\Upsilon_j^l(a_\alpha, a_\beta) = \zeta(a_\alpha) - \zeta(a_\beta) = \frac{(1 - \delta)d(a_\alpha, a^-)}{\delta d(a_\alpha, a^+) + (1 - \delta)d(a_\alpha, a^-)}$$
$$- \frac{(1 - \delta)d(a_\beta, a^-)}{\delta d(a_\beta, a^+) + (1 - \delta)d(a_\beta, a^-)}, \tag{12}$$

where $\Upsilon_j^l(a_\alpha, a_\beta) \in [-1, 1]$.

(1) If $\Upsilon_j^l(a_\alpha, a_\beta) > 0$, or $\zeta(a_\alpha) > \zeta(a_\beta)$, then a_α is ranked higher than a_β under c_j ($j = 1, 2, \ldots, n$). Thus, concordance exists between the closeness coefficient-based ranking and preorder of a_α and a_β under P_l.

(2) If $\Upsilon_j^l(a_\alpha, a_\beta) = 0$, or $\zeta(a_\alpha) = \zeta(a_\beta)$, then a_α and a_β have the same ranking under c_j ($j = 1, 2, \ldots, n$). Thus, ex aequo exists between the closeness coefficient-based ranking and preorder of a_α and a_β under P_l.

(3) If $\Upsilon_j^l(a_\alpha, a_\beta) < 0$, or $\zeta(a_\alpha) < \zeta(a_\beta)$, then a_β is ranked higher than a_α under c_j ($j = 1, 2, \ldots, n$). Thus, discordance exists between the closeness coefficient-based ranking and preorder of a_α and a_β under P_l.

For convenience, the concordance/discordance index $I_j^l(a_\alpha, a_\beta)$ can be rewritten as

$$\Upsilon_j^l(a_\alpha, a_\beta) = \begin{cases} \zeta(a_\alpha) - \zeta(a_\beta) > 0 \Leftrightarrow \text{concordance exists} \\ \zeta(a_\alpha) - \zeta(a_\beta) = 0 \Leftrightarrow \text{ex aequo exists} \\ \zeta(a_\alpha) - \zeta(a_\beta) < 0 \Leftrightarrow \text{discordance exists} \end{cases} . \qquad (13)$$

Assume that ω_j ($j = 1, 2, \ldots, n$) is the importance weight of c_j ($j = 1, 2, \ldots, n$). Then the weighted concordance/discordance index $\Upsilon^l(a_\alpha, a_\beta)$ of each pair of alternatives (a_α, a_β) at the preorder level with respect to n criteria in C as well as the ranking corresponding to permutation P_l can be derived using Eq. (12).

$$\Upsilon^l(a_\alpha, a_\beta) = \sum_{j=1}^{n} \Upsilon_j^l(a_\alpha, a_\beta)\omega_j = \sum_{j=1}^{n} \left(\zeta(a_\alpha) - \zeta(a_\beta) \right) \omega_j. \qquad (14)$$

The comprehensive concordance/discordance index I^l for permutation P_l can be obtained by substituting Eqs. (9) and (10) into Eq. (14).

$$\Upsilon^l = \sum_{a_\alpha, a_\beta \in A} \sum_{j=1}^{n} \Upsilon_j^l(a_\alpha, a_\beta)\omega_j = \sum_{a_\alpha, a_\beta \in A} \sum_{j=1}^{n} \left(\zeta(a_\alpha) - \zeta(a_\beta) \right) \omega_j. \qquad (15)$$

According to the closeness coefficient-based comparison method of SNLNs, as the value of Υ^l increases, the final ranking of the alternatives becomes more credible. Therefore, the optimum ranking of the alternatives P^* can be expressed as

$$\Upsilon* = \max_{l=1}^{m!} \left\{ \Upsilon^l \right\}. \qquad (16)$$

4 Power Aggregation Operators with Simplified Neutrosophic Linguistic Numbers

In this section, SNLPWA and SNLPWG operators are developed based on the traditional power average (PA) operator.

Definition 8 ([Yager 2001](#)). Let a_j $(j = 1, 2, \ldots, n)$ be a collection of values and Ω be the set of all given values. Then the PA operator is the mapping $PA : \Omega^n \rightarrow \Omega$, defined as

$$PA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \frac{1 + \Psi(a_j)}{\sum_{j=1}^{n} \left(1 + \Psi(a_j)\right)} a_j, \tag{17}$$

where $\Psi(a_j) = \sum_{i=1, i \neq j}^{n} Supp(a_j, a_i)$, and $Supp(a_j, a_i)$ is the support for a_j and a_i, which satisfies the following three properties:

(1) $Supp(a_i, a_j) \in [0, 1]$;
(2) $Supp(a_i, a_j) = Supp(a_j, a_i)$;
(3) If $d(a_i, a_j) < d(a_l, a_r)$, then $Supp(a_i, a_j) \geq Supp(a_l, a_r)$, where $d(a_i, a_j)$ is the distance between a_i and a_j.

Therefore, these two values increasingly support one another as they converge.

4.1 Power Weighted Averaging Operator with Simplified Neutrosophic Linguistic Numbers

Definition 9 Let $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$ be a collection of SNLNs, Ω be the set of all SNLNs, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be the weight vector of $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$, where $\omega_j \geq 0$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. Then the SNLPWA operator is the mapping $SNLPWA : \Omega^n \rightarrow \Omega$, defined as

$$SNLPWA(a_1, a_2, \ldots, a_n) = \bigoplus_{j=1}^{n} \frac{\omega_j \left(1 + \Psi(a_j)\right) a_j}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right)}, \tag{18}$$

where $\Psi(a_j) = \sum_{i=1, i \neq j}^{n} \omega_i Supp(a_j, a_i)$, and $Supp(a_j, a_i)$ is the support for a_j and a_i, which satisfies the following three properties:

(1) $Supp(a_i, a_j) \in [0, 1]$;
(2) $Supp(a_i, a_j) = Supp(a_j, a_i)$;
(3) If $d(a_i, a_j) < d(a_l, a_r)$, then $Supp(a_i, a_j) \geq Supp(a_l, a_r)$, where $d(a_i, a_j)$ is the distance measurement between a_i and a_j, defined in Sect. [3.1](#).

Theorem [2](#) can be derived based on the operations presented in Definition [6](#) and Eq. ([18](#)).

Theorem 2 *Let $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$ be a collection of SNLNs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be the weight vector of $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$, where $\omega_j \geq 0$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. Therefore, the aggregation result, which is also an SNLN, can be calculated using Eq. ([18](#)) as*

$SNLPWA(a_1, a_2, \ldots, a_n)$

$$
= \left\langle f^{*-1} \left(\frac{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right)} \right), \left(\frac{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)T_j}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)}, \right. \right.
$$

$$
\left. \left. \frac{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)I_j}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)}, \frac{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)F_j}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)} \right) \right\rangle, \tag{19}
$$

where $\Psi(a_j) = \sum_{i=1, i \neq j}^{n} \omega_i Supp(a_j, a_i)$, which satisfies the conditions presented in Definition 9.

Proof For convenience, let $\eta_j = \frac{\omega_j(1 + \Psi(a_j))}{\sum_{j=1}^{n} \omega_j(1 + \Psi(a_j))}$ for the purposes of this proof. Then Eq. (19) can be proven via the mathematical induction of n.

(1) When $n = 2$, the following equation can be calculated using the operations presented in Definition 6:

$$
SNLPWA(a_1, a_2) = \eta_1 a_1 \oplus \eta_2 a_2
$$

$$
= \left\langle f^{*-1} \left(\eta_1 f^*(h_1) + \eta_2 f^*(h_2) \right), \left(\frac{\eta_1 f^*(h_1)T_1 + \eta_2 f^*(h_2)T_2}{\eta_1 f^*(h_1) + \eta_2 f^*(h_2)}, \right. \right.
$$

$$
\left. \left. \frac{\eta_1 f^*(h_1)I_1 + \eta_2 f^*(h_2)I_2}{\eta_1 f^*(h_1) + \eta_2 f^*(h_2)}, \frac{\eta_1 f^*(h_1)F_1 + \eta_2 f^*(h_2)F_2}{\eta_1 f^*(h_1) + \eta_2 f^*(h_2)} \right) \right\rangle.
$$

$$
= \left\langle f^{*-1} \left(\frac{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1) + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)}{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right)} \right), \right.
$$

$$
\left(\frac{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1)T_1 + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)T_2}{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1) + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)}, \right.
$$

$$
\frac{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1)I_1 + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)I_2}{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1) + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)},
$$

$$
\left. \left. \frac{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1)F_1 + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)F_2}{\omega_1 \left(1 + \Psi(a_1)\right) f^*(h_1) + \omega_2 \left(1 + \Psi(a_2)\right) f^*(h_2)} \right) \right\rangle
$$

$$
= \left\langle f^{*-1} \left(\frac{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)}{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right)} \right), \left(\frac{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)T_j}{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)}, \right. \right.
$$

$$
\left. \left. \frac{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)I_j}{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)}, \frac{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)F_j}{\sum_{j=1}^{2} \omega_j \left(1 + \Psi(a_j)\right) f^*(h_j)} \right) \right\rangle.
$$

Therefore, when $n = 2$, Eq. (19) is true.

(2) Assume that when $n = k$, Eq. (19) is true. Thus,

$$
SNLPWA(a_1, a_2, \ldots, a_k) = \left\langle f^{*-1} \left(\sum_{j=1}^{k} \eta_j f^*(h_j) \right), \right.
$$

$$
\left. \left(\frac{\sum_{j=1}^{k} \eta_j f^*(h_j)T_j}{\sum_{j=1}^{k} \eta_j f^*(h_j)}, \frac{\sum_{j=1}^{k} \eta_j f^*(h_j)I_j}{\sum_{j=1}^{k} \eta_j f^*(h_j)}, \frac{\sum_{j=1}^{k} \eta_j f^*(h_j)F_j}{\sum_{j=1}^{k} \eta_j f^*(h_j)} \right) \right\rangle.
$$

$\square$

Therefore, when $n = k + 1$, the following results can be calculated:

$$SNLPWA(a_1, a_2, \ldots, a_k, a_{k+1}) = SNLPWA(a_1, a_2, \ldots, a_k) \oplus \eta_{k+1} a_{k+1}$$

$$= \left\langle f^{*-1} \left(\sum_{j=1}^{k} \eta_j f^*(h_j) \right), \left(\frac{\sum_{j=1}^{k} \eta_j f^*(h_j) T_j}{\sum_{j=1}^{k} \eta_j f^*(h_j)}, \frac{\sum_{j=1}^{k} \eta_j f^*(h_j) I_j}{\sum_{j=1}^{k} \eta_j f^*(h_j)}, \frac{\sum_{j=1}^{k} \eta_j f^*(h_j) F_j}{\sum_{j=1}^{k} \eta_j f^*(h_j)} \right) \right\rangle$$

$$\oplus \left\langle f^{*-1} \left(\eta_{k+1} f^*(h_{k+1}) \right), \left(T_{k+1}, I_{k+1}, F_{k+1} \right) \right\rangle$$

$$= \left\langle f^{*-1} \left(\sum_{j=1}^{k} \eta_j f^*(h_j) + \eta_{k+1} f^*(h_{k+1}) \right), \left(\frac{\sum_{j=1}^{k} \eta_j f^*(h_j) T_j + \eta_{k+1} f^*(h_{k+1}) T_{k+1}}{\sum_{j=1}^{k} \eta_j f^*(h_j) + \eta_{k+1} f^*(h_{k+1})}, \right.\right.$$

$$\left.\left. \frac{\sum_{j=1}^{k} \eta_j f^*(h_j) I_j + \eta_{k+1} f^*(h_{k+1}) I_{k+1}}{\sum_{j=1}^{k} \eta_j f^*(h_j) + \eta_{k+1} f^*(h_{k+1})}, \frac{\sum_{j=1}^{k} \eta_j f^*(h_j) F_j + \eta_{k+1} f^*(h_{k+1}) F_{k+1}}{\sum_{j=1}^{k} \eta_j f^*(h_j) + \eta_{k+1} f^*(h_{k+1})} \right) \right\rangle$$

$$= \left\langle f^{*-1} \left(\sum_{j=1}^{k+1} \eta_j f^*(h_j) \right), \left(\frac{\sum_{j=1}^{k+1} \eta_j f^*(h_j) T_j}{\sum_{j=1}^{k+1} \eta_j f^*(h_j)}, \frac{\sum_{j=1}^{k+1} \eta_j f^*(h_j) I_j}{\sum_{j=1}^{k+1} \eta_j f^*(h_j)}, \frac{\sum_{j=1}^{k+1} \eta_j f^*(h_j) F_j}{\sum_{j=1}^{k+1} \eta_j f^*(h_j)} \right) \right\rangle$$

$$= \left\langle f^{*-1} \left(\frac{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j)}{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right)} \right), \left(\frac{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j) T_j}{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j)}, \right.\right.$$

$$\left.\left. \frac{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j) I_j}{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j)}, \frac{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j) F_j}{\sum_{j=1}^{k+1} \omega_j \left(1 + \Psi(a_j) \right) f^*(h_j)} \right) \right\rangle.$$

Thus, since Eq. (19) holds for $n = k + 1$, Eq. (19) holds for all values of n.

The SNLPWA operator satisfies the following properties.

Theorem 3 (Idempotency). *Let* $a_j = a$ $(j = 1, 2, \ldots, n)$. *Then* $SNLPWA(a_1, a_2, \ldots, a_n) = a$.

Proof Since $a_j = a$ for all j, the following equation can be obtained:

$$SNLPWA(a_1, a_2, \ldots, a_n) = \bigoplus_{j=1}^{n} \frac{\omega_j \left(1 + \Psi(a_j) \right) a_j}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j) \right)}$$

$$= \bigoplus_{j=1}^{n} \frac{\omega_j \left(1 + \Psi(a_j) \right) a}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j) \right)}$$

$$= a \sum_{j=1}^{n} \frac{\omega_j \left(1 + \Psi(a_j) \right)}{\sum_{j=1}^{n} \omega_j \left(1 + \Psi(a_j) \right)} = a.$$

$\square$

Theorem 4 (Boundedness). *Let* $a_j = \left\langle h_j, (T_j, I_j, F_j) \right\rangle$ $(j = 1, 2, \ldots, n)$ *be a collection of SNLNs,* $a = \langle \min_j \{h_j\}, (\min_j \{T_j\}, \max_j \{I_j\}, \max_j \{F_j\}) \rangle$, *and* $b = \langle \max_j \{h_j\}, (\max_j \{T_j\}, \min_j \{I_j\}, \min_j \{F_j\}) \rangle$. *Then* $a \leq SNLPWA(a_1, a_2, \ldots, a_n) \leq b$.

Proof Since $h_j \geq \min_j \{h_j\}$, $T_j \geq \min_j \{T_j\}$, $I_j \leq \max_j \{I_j\}$, and $F_j \leq \max_j \{F_j\}$, according to Theorem 3, the following inequality can be obtained:

$$a = SNLPWA(a, a, \ldots, a) \leq SNLPWA(a_1, a_2, \ldots, a_n).$$

The following inequality can be obtained similarly:

$$SNLPWA(a_1, a_2, \ldots, a_n) \leq SNLPWA(b, b, \ldots, b) = b.$$

Thus, $a \leq SNLPWA(a_1, a_2, \ldots, a_n) \leq b$. □

4.2 Power Weighted Geometric Operator with Simplified Neutrosophic Linguistic Numbers

Definition 10 Let $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$ be a collection of SNLNs, Ω be the set of all SNLNs, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be the weight vector of $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$, where $\omega_j \geq 0$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. Then the SNLPWG operator is the mapping $SNLPWG : \Omega^n \to \Omega$, defined as

$$SNLPWG(a_1, a_2, \ldots, a_n) = \prod_{j=1}^{n} (a_j)^{\frac{\omega_j (1 + \Psi(a_j))}{\sum_{j=1}^{n} \omega_j (1 + \Psi(a_j))}}, \tag{20}$$

where $\Psi(a_j) = \sum_{i=1, i \neq j}^{n} \omega_i \, Supp(a_j, a_i)$, and $Supp(a_j, a_i)$ is the support for a_j and a_i, which satisfies the following three properties:

(1) $Supp(a_i, a_j) \in [0, 1]$;
(2) $Supp(a_i, a_j) = Supp(a_j, a_i)$;
(3) If $d(a_i, a_j) < d(a_l, a_r)$, then $Supp(a_i, a_j) \geq Supp(a_l, a_r)$, where $d(a_i, a_j)$ is the distance measurement between a_i and a_j, as defined in Sect. 3.1.

Theorem 5 can be derived using the operations presented in Definition 6 and Eq. (20).

Theorem 5 Let $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$ be a collection of SNLNs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be the weight vector of $a_j = \langle h_j, (T_j, I_j, F_j) \rangle$ $(j = 1, 2, \ldots, n)$, where $\omega_j \geq 0$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. Then the aggregation result, which is also an SNLN, can be obtained using Eq. (20) as

$$SNLPWG(a_1, a_2, \ldots, a_n)$$
$$= \left\langle f^{*-1} \left(\prod_{j=1}^{n} \left(f^*(h_{\theta_j}) \right)^{\frac{\omega_j (1 + \Psi(a_j))}{\sum_{j=1}^{n} \omega_j (1 + \Psi(a_j))}} \right), \left(\prod_{j=1}^{n} (T_j)^{\frac{\omega_j (1 + \Psi(a_j))}{\sum_{j=1}^{n} \omega_j (1 + \Psi(a_j))}}, \right.$$
$$\left. 1 - \prod_{j=1}^{n} (1 - I_j)^{\frac{\omega_j (1 + \Psi(a_j))}{\sum_{j=1}^{n} \omega_j (1 + \Psi(a_j))}}, 1 - \prod_{j=1}^{n} (1 - F_j)^{\frac{\omega_j (1 + \Psi(a_j))}{\sum_{j=1}^{n} \omega_j (1 + \Psi(a_j))}} \right) \right\rangle, \tag{21}$$

where $\Psi(a_j) = \sum_{i=1, i \neq j}^{n} \omega_i Supp(a_j, a_i)$, *which satisfies the conditions presented in Definition* 10.

Proof Theorem 5 can be proven via mathematical induction. This process is omitted herein. Similarly, the SNLPWG operator satisfies the properties of idempotency and boundedness. □

5 A Multi-criteria Group Decision-Making Approach Based on QUALIFLEX and Power Aggregation Operators with Simplified Neutrosophic Linguistic Numbers

In this section, the TOPSIS-based QUALIFLEX method and power aggregation operators are combined in order to effectively manage SNLNs. Furthermore, a simplified neutrosophic linguistic MCGDM approach is developed.

For a group decision-making problem with a finite set of m alternatives, let $A = \{a_1, a_2, \ldots, a_m\}$ be a set of alternatives, $C = \{c_1, c_2, \ldots, c_n\}$ be a set of criteria, and $E = \{e_1, e_2, \ldots, e_q\}$ be a set of DMs. Assume that the weight vector of the criteria is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, where $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, and the weight vector of the DMs is $w = (w_1, w_2, \ldots, w_q)$, where $w_k \in [0, 1]$ and $\sum_{k=1}^{q} w_k = 1$. Then for a DM e_k $(k = 1, 2, \ldots, q)$, the assessment information of a_i $(i = 1, 2, \ldots, m)$ with respect to c_j $(j = 1, 2, \ldots, n)$ is denoted by $D^k = [d_{ij}^k]_{m \times n} = \left[\langle h_{ij}, (T_{ij}, I_{ij}, F_{ij}) \rangle \right]_{m \times n}$, where $d_{ij}^k = \langle h_{ij}, (T_{ij}, I_{ij}, F_{ij}) \rangle$ is expressed in the form of SNLNs.

According to the above analysis, the simplified neutrosophic linguistic MCGDM approach with the TOPSIS-based QUALIFLEX method and power aggregation operators can be summarized by the following steps:

Step 1: Normalize the decision matrix.

Two types of criteria are included in decision matrices, including maximizing and minimizing criteria. In order to uniform criteria, the minimizing criteria must be transformed into maximizing criteria. Assume that there exists a standardized matrix expressed as $R^k = [r_{ij}^k]_{m \times n}$ $(k = 1, 2, \ldots, q)$. The original decision matrix D^k can be transformed into R^k using the primary transformation principle proposed by Ye (2015), where

$$r_{ij}^k = \begin{cases} \langle h_{ij}, (T_{ij}, I_{ij}, F_{ij}) \rangle, & \text{for maximizing criterion } c_j \\ \langle s_{(2t+2-sub(h_{ij}))}, (T_{ij}, I_{ij}, F_{ij}) \rangle, & \text{for minimizing criterion } c_j \end{cases}, \quad (22)$$

For convenience, the normalized values of a_i $(i = 1, 2, \ldots, m)$ with respect to c_j $(j = 1, 2, \ldots, n)$ are also expressed as $\langle h_{ij}, (T_{ij}, I_{ij}, F_{ij}) \rangle$.

Step 2: Calculate the supports.

The supports can be obtained using the formula:

$$Supp\left(r_{ij}^{k_1}, r_{ij}^{k_2}\right) = 1 - d\left(r_{ij}^{k_1}, r_{ij}^{k_2}\right)$$
$$(i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n; \ k_1, k_2 = 1, 2, \ldots, q), \quad (23)$$

where $d(r_{ij}^{k_1}, r_{ij}^{k_2})$ is the distance between $r_{ij}^{k_1}$ and $r_{ij}^{k_2}$, as defined in Definition 7.

Step 3: Calculate the weights associated with $r_{ij}^{k_1}$ $(k_1 = 1, 2, \ldots, q)$.

$$\eta_{ij}^{k_1} = \frac{w_{k_1}\left(1 + \Psi(a_{k_1})\right)}{\sum_{k_1=1}^{q} w_{k_1}\left(1 + \Psi(a_{k_1})\right)}, \tag{24}$$

where $\Psi(a_{k_1}) = \sum_{k_2=1, k_2 \neq k_1}^{q} w_{k_2} Supp(a_{k_1}, a_{k_2})$, and w_{k_2} is the weight of DM e_{k_2}.

Step 4: Obtain the comprehensive evaluation information.

Using Eqs. (19) or (21), the normalized evaluation information provided by experts can be aggregated, and the integrated decision matrix $R = \left[r_{ij}\right]_{m \times n}$ can be obtained.

Step 5: List all of the possible permutations of the alternatives.

List all of the possible permutations $m!$ of the m alternatives that are to be tested. Let P_l $(l = 1, 2, \ldots, m!)$ denote the lth permutation.

Step 6: Calculate the comprehensive concordance/discordance indices.

Using Eq. (12), the concordance/discordance index $\Upsilon_j^l(a_\alpha, a_\beta)$ $(j = 1, 2, \ldots, n)$ of each pair of alternatives (a_α, a_β) with respect to the criterion c_j can be calculated. Then the comprehensive concordance/discordance index Υ_l $(l = 1, 2, \ldots, m!)$ can be derived using Eq. (15).

Step 7: Determine the ranking of the alternatives.

Using Eq. (16), the final ranking results can be obtained, and the optimum ranking(s) can be selected.

6 Illustrative Example of the Green Product Design Selection Process

In this section, the proposed simplified neutrosophic linguistic MCGDM approach is applied to the selection of an environmentally sustainable personal electronic product design. In addition, sensitivity and comparative analyses are conducted.

ABC TECHNOLOGIES Co., Ltd. is a technological company in China that is primarily involved in the development and production of wireless electronic products. This company would like to incorporate environmentally friendly features into the product design stage since any decisions made during that stage could significantly impact the environment. First, an LCA was conducted on one of the electronic products produced by ABC TECHNOOGIES. Since this product is electronic, it does not undergo the installation and maintenance phases of production. Therefore, only five phases, including the material selection (l_1), manufacturing (l_2), distribution (l_3), usage (l_4) and end-of-life (l_5) phases were considered in this example. After a thorough investigation, four different design options were identified, denoted by $\{a_1, a_2, a_3, a_4\}$.

The company invited three experts (DMs), denoted by e_1, e_2, and e_3, to evaluate the four designs based on their re-usability, recycling, and material and/or energy recovery characteristics throughout the five stages of their life cycles l_j $(j = 1, 2, 3, 4, 5)$. The experts included an engineer from the R&D department, a production manager, and an operational management specialist. The evaluations provided by the experts were all weighed equally. The four designs a_i $(i = 1, 2, 3, 4)$ were evaluated

Table 1 Evaluation information of e_1

	l_1	l_2	l_3	l_4	l_5
a_1	$\langle s_5, (0.8, 0.2, 0.3)\rangle$	$\langle s_5, (0.8, 0.2, 0.3)\rangle$	$\langle s_6, (0.7, 0.2, 0.2)\rangle$	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_5, (0.7, 0.2, 0.1)\rangle$
a_2	$\langle s_6, (0.6, 0.1, 0.2)\rangle$	$\langle s_6, (0.6, 0.1, 0.2)\rangle$	$\langle s_5, (0.5, 0.3, 0.2)\rangle$	$\langle s_5, (0.7, 0.1, 0.2)\rangle$	$\langle s_5, (0.5, 0.2, 0.2)\rangle$
a_3	$\langle s_5, (0.3, 0.2, 0.3)\rangle$	$\langle s_4, (0.5, 0.2, 0.3)\rangle$	$\langle s_6, (0.7, 0.3, 0.1)\rangle$	$\langle s_5, (0.6, 0.2, 0.3)\rangle$	$\langle s_4, (0.7, 0.3, 0.1)\rangle$
a_4	$\langle s_5, (0.7, 0.2, 0.1)\rangle$	$\langle s_6, (0.6, 0.3, 0.2)\rangle$	$\langle s_4, (0.6, 0.2, 0.2)\rangle$	$\langle s_4, (0.6, 0.1, 0.2)\rangle$	$\langle s_5, (0.6, 0.2, 0.2)\rangle$

Table 2 Evaluation information of e_2

	l_1	l_2	l_3	l_4	l_5
a_1	$\langle s_5, (0.4, 0.3, 0.4)\rangle$	$\langle s_6, (0.5, 0.1, 0.2)\rangle$	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_6, (0.6, 0.1, 0.2)\rangle$	$\langle s_5, (0.7, 0.1, 0.3)\rangle$
a_2	$\langle s_6, (0.7, 0.2, 0.3)\rangle$	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_6, (0.6, 0.3, 0.2)\rangle$	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_6, (0.6, 0.2, 0.2)\rangle$
a_3	$\langle s_6, (0.4, 0.2, 0.4)\rangle$	$\langle s_6, (0.6, 0.3, 0.4)\rangle$	$\langle s_5, (0.6, 0.1, 0.3)\rangle$	$\langle s_5, (0.6, 0.3, 0.4)\rangle$	$\langle s_5, (0.6, 0.1, 0.3)\rangle$
a_4	$\langle s_5, (0.8, 0.1, 0.2)\rangle$	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_6, (0.6, 0.2, 0.2)\rangle$	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_5, (0.4, 0.2, 0.2)\rangle$

Table 3 Evaluation information of e_3

	l_1	l_2	l_3	l_4	l_5
a_1	$\langle s_4, (0.5, 0.2, 0.3)\rangle$	$\langle s_6, (0.6, 0.2, 0.4)\rangle$	$\langle s_5, (0.5, 0.1, 0.6)\rangle$	$\langle s_6, (0.6, 0.2, 0.4)\rangle$	$\langle s_4, (0.5, 0.1, 0.4)\rangle$
a_2	$\langle s_5, (0.7, 0.2, 0.3)\rangle$	$\langle s_6, (0.7, 0.2, 0.2)\rangle$	$\langle s_6, (0.8, 0.1, 0.1)\rangle$	$\langle s_5, (0.7, 0.2, 0.2)\rangle$	$\langle s_6, (0.7, 0.2, 0.2)\rangle$
a_3	$\langle s_6, (0.7, 0.1, 0.3)\rangle$	$\langle s_5, (0.6, 0.1, 0.3)\rangle$	$\langle s_5, (0.6, 0.2, 0.2)\rangle$	$\langle s_4, (0.6, 0.1, 0.3)\rangle$	$\langle s_6, (0.6, 0.2, 0.2)\rangle$
a_4	$\langle s_4, (0.6, 0.1, 0.2)\rangle$	$\langle s_4, (0.5, 0.2, 0.2)\rangle$	$\langle s_5, (0.8, 0.1, 0.1)\rangle$	$\langle s_6, (0.6, 0.2, 0.2)\rangle$	$\langle s_5, (0.8, 0.1, 0.1)\rangle$

based on the simplified neutrosophic linguistic information provided by the DMs. The weight vector of the life cycle stages was $\omega = (0.381, 0.278, 0.104, 0.093, 0.144)$, and the weight vector of the DMs was $w = (1/3, 1/3, 1/3)$ (Chan et al. 2013; Wang et al. 2015d). In addition, the linguistic term set $S = \{s_1, s_2, \ldots, s_7\} = \{extremely\ poor\ ,\ very\ poor,\ poor,\ medium,\ good,\ very\ good,\ extremely\ good\}$ was used herein. The evaluation information provided by the experts is displayed in the form of SNLNs in Tables 1, 2, and 3.

6.1 Illustration of the Proposed Approach

The procedures used to obtain the optimum ranking of the alternatives can be summarized by the following steps:

Step 1: Normalize the decision matrix.

Since all of the criteria consist of maximizing criteria, the decision information did not require normalization; therefore, $R_k = D_k$ ($k = 1, 2, 3$).

Step 2: Calculate the supports.

Equation (9) was substituted into Eq. (23) in order to obtain the supports, assuming that $f^* = f_1^*(s_\tau) = (\tau - 1)/6$ and $\lambda = 2$.

$$Supp\left(r_{ij}^1, r_{ij}^2\right) = Supp\left(r_{ij}^2, r_{ij}^1\right) = \begin{bmatrix} 0.7789 & 0.7994 & 0.8214 & 0.8047 & 0.8878 \\ 0.9167 & 0.8047 & 0.8459 & 0.9391 & 0.8437 \\ 0.8775 & 0.7741 & 0.7568 & 0.9391 & 0.7947 \\ 0.9333 & 0.8349 & 0.7432 & 0.8571 & 0.8911 \end{bmatrix},$$

$$Supp\left(r_{ij}^1, r_{ij}^3\right) = Supp\left(r_{ij}^3, r_{ij}^1\right) = \begin{bmatrix} 0.7576 & 0.8894 & 0.6567 & 0.8894 & 0.7374 \\ 0.8047 & 0.9239 & 0.6913 & 0.9456 & 0.7818 \\ 0.6713 & 0.8192 & 0.8068 & 0.8909 & 0.7200 \\ 0.8199 & 0.7482 & 0.7772 & 0.7512 & 0.8845 \end{bmatrix} \text{ and}$$

$$Supp\left(r_{ij}^2, r_{ij}^3\right) = Supp\left(r_{ij}^3, r_{ij}^2\right) = \begin{bmatrix} 0.9415 & 0.8557 & 0.8257 & 0.8597 & 0.8008 \\ 0.8720 & 0.8214 & 0.8441 & 0.9456 & 0.9320 \\ 0.7903 & 0.9169 & 0.9391 & 0.9156 & 0.8295 \\ 0.7926 & 0.8129 & 0.9376 & 0.8273 & 0.7789 \end{bmatrix}.$$

Step 3: Calculate the weights associated with $r_{ij}^{k_1}$ ($k_1 = 1, 2, 3$).

Eq. (24) was used to obtain the weights associated with $r_{ij}^{k_1}$ ($k_1 = 1, 2, 3$) based on the results presented in Step 2.

$$\eta_{ij}^1 = \begin{bmatrix} 0.3251 & 0.3328 & 0.3291 & 0.3327 & 0.3339 \\ 0.3328 & 0.3354 & 0.3297 & 0.3332 & 0.3277 \\ 0.3326 & 0.3276 & 0.3258 & 0.3333 & 0.3298 \\ 0.3373 & 0.3323 & 0.3248 & 0.3322 & 0.3385 \end{bmatrix},$$

$$\eta_{ij}^2 = \begin{bmatrix} 0.3382 & 0.3304 & 0.3415 & 0.3306 & 0.3385 \\ 0.3375 & 0.3281 & 0.3408 & 0.3332 & 0.3383 \\ 0.3413 & 0.3346 & 0.3353 & 0.3350 & 0.3378 \\ 0.3354 & 0.3370 & 0.3364 & 0.3377 & 0.3310 \end{bmatrix} \text{ and}$$

$$\eta_{ij}^3 = \begin{bmatrix} 0.3367 & 0.3368 & 0.3294 & 0.3366 & 0.3276 \\ 0.3297 & 0.3365 & 0.3295 & 0.3336 & 0.3340 \\ 0.3262 & 0.3378 & 0.3389 & 0.3317 & 0.3324 \\ 0.3273 & 0.3307 & 0.3388 & 0.3301 & 0.3305 \end{bmatrix}.$$

Step 4: Obtain the comprehensive evaluation information.

Equation (19) (i.e., the SNLPWA operator) was used to aggregate the evaluation information provided by the experts. The comprehensive evaluation information was also obtained, as shown in Table 4.

Step 5: List all of the possible permutations of the alternatives.

The 24 (= 4!) permutations were listed, where P_l ($l = 1, 2, \ldots, 24$) represents the lth permutation.

$$P_1 = (a_1, a_2, a_3, a_4), \quad P_2 = (a_1, a_2, a_4, a_3), \quad P_3 = (a_1, a_3, a_2, a_4),$$
$$P_4 = (a_1, a_3, a_4, a_2), \quad P_5 = (a_1, a_4, a_2, a_3),$$
$$P_6 = (a_1, a_4, a_3, a_2), \quad P_7 = (a_2, a_1, a_3, a_4), \quad P_8 = (a_2, a_1, a_4, a_3),$$
$$P_9 = (a_2, a_3, a_1, a_4), \quad P_{10} = (a_2, a_3, a_4, a_1),$$
$$P_{11} = (a_2, a_4, a_1, a_3), \quad P_{12} = (a_2, a_4, a_3, a_1), \quad P_{13} = (a_3, a_1, a_2, a_4),$$

Table 4 Comprehensive evaluation information

	l_1	l_2	l_3
a_1	$\langle s_{4.6630}, (0.5695, 0.2369, 0.3369)\rangle$	$\langle s_{5.6674}, (0.6216, 0.1646, 0.3007)\rangle$	$\langle s_{5.3290}, (0.6391, 0.1696, 0.3533)\rangle$
a_2	$\langle s_{5.6704}, (0.6644, 0.1644, 0.2644)\rangle$	$\langle s_{5.6722}, (0.6641, 0.1641, 0.2281)\rangle$	$\langle s_{5.6704}, (0.6423, 0.2294, 0.1647)\rangle$
a_3	$\langle s_{5.6674}, (0.4763, 0.1651, 0.3366)\rangle$	$\langle s_{5.0068}, (0.5755, 0.2080, 0.3418)\rangle$	$\langle s_{5.3260}, (0.6377, 0.2067, 0.1933)\rangle$
a_4	$\langle s_{4.6726}, (0.7098, 0.1367, 0.1633)\rangle$	$\langle s_{5.0014}, (0.6089, 0.2415, 0.2337)\rangle$	$\langle s_{5.0116}, (0.6676, 0.1662, 0.1662)\rangle$

	l_4	l_5
a_1	$\langle s_{5.6674}, (0.6285, 0.1646, 0.3006)\rangle$	$\langle s_{4.6726}, (0.6465, 0.1364, 0.2540)\rangle$
a_2	$\langle s_{5.0002}, (0.7000, 0.1667, 0.2333)\rangle$	$\langle s_{5.6722}, (0.6077, 0.2000, 0.2000)\rangle$
a_3	$\langle s_{4.6684}, (0.6000, 0.2094, 0.3365)\rangle$	$\langle s_{5.0026}, (0.6247, 0.1910, 0.2090)\rangle$
a_4	$\langle s_{4.9978}, (0.6338, 0.1751, 0.2238)\rangle$	$\langle s_{5.0002}, (0.5999, 0.1669, 0.1669)\rangle$

Table 5 The concordance/discordance indices

	l_1	l_2	l_3	l_4	l_5
$\Upsilon_j^l(a_1, a_2)$	−0.1638	−0.0229	−0.0538	0.0492	−0.0907
$\Upsilon_j^l(a_1, a_3)$	−0.0969	0.1000	−0.0081	0.1329	−0.0229
$\Upsilon_j^l(a_1, a_4)$	−0.0746	0.0896	0.0087	0.0693	−0.0265
$\Upsilon_j^l(a_2, a_1)$	0.1638	0.0229	0.0538	−0.0492	0.0907
$\Upsilon_j^l(a_2, a_3)$	0.0670	0.1229	0.0457	0.0837	0.0678
$\Upsilon_j^l(a_2, a_4)$	0.0892	0.1125	0.0625	0.0201	0.0642
$\Upsilon_j^l(a_3, a_1)$	0.0969	−0.1000	0.0081	−0.1329	0.0229
$\Upsilon_j^l(a_3, a_2)$	−0.0670	−0.1229	−0.0457	−0.0837	−0.0678
$\Upsilon_j^l(a_3, a_4)$	0.0222	−0.0105	0.0168	−0.0636	−0.0036
$\Upsilon_j^l(a_4, a_1)$	0.0746	−0.0896	−0.0087	−0.0693	0.0265
$\Upsilon_j^l(a_4, a_2)$	−0.0892	−0.1125	−0.0625	−0.0201	−0.0642
$\Upsilon_j^l(a_4, a_3)$	−0.0222	0.0105	−0.0168	0.0636	0.0036

$$P_{14} = (a_3, a_1, a_4, a_2), \quad P_{15} = (a_3, a_2, a_1, a_4),$$
$$P_{16} = (a_3, a_2, a_4, a_1), \quad P_{17} = (a_3, a_4, a_1, a_2), \quad P_{18} = (a_3, a_4, a_2, a_1),$$
$$P_{19} = (a_4, a_1, a_2, a_3), \quad P_{20} = (a_4, a_1, a_3, a_2),$$

$P_{21} = (a_4, a_2, a_1, a_3), \quad P_{22} = (a_4, a_2, a_3, a_1), \quad P_{23} = (a_4, a_3, a_1, a_2)$ and $P_{24} = (a_4, a_3, a_2, a_1)$.

Step 6: Calculate the comprehensive concordance/discordance indices (let $\delta = 0.5$).

Equation (12) was used to calculate the concordance/discordance indices. The results are shown in Table 5.

The comprehensive concordance/discordance indices were derived using Eq. (15). For example, for Υ_1 and the permutation $P_1 = (a_1, a_2, a_3, a_4)$:

$$
\begin{aligned}
\Upsilon_1 &= \sum_{a_\alpha, a_\beta \in A} \sum_{j=1}^{5} \Upsilon_j^1(a_\alpha, a_\beta)\omega_j \\
&= (-0.1638 - 0.0969 - 0.0746 + 0.0670 + 0.0892 + 0.0222) \times 0.381 \\
&\quad + (-0.0229 + 0.1000 + 0.0896 + 0.1229 + 0.1125 - 0.0105) \times 0.278 \\
&\quad + (-0.0538 - 0.0081 + 0.0087 + 0.0457 + 0.0625 + 0.0168) \times 0.104 \\
&\quad + (0.0492 + 0.1329 + 0.0693 + 0.0837 + 0.0201 - 0.0636) \times 0.093 \\
&\quad + (-0.0907 - 0.0229 - 0.0265 + 0.0678 + 0.0642 - 0.0036) \times 0.144 \\
&= 0.0819.
\end{aligned}
$$

The other comprehensive concordance/discordance indices were obtained similarly.

$$\Upsilon_2 = 0.0802, \quad \Upsilon_3 = -0.0820, \quad \Upsilon_4 = -0.2477, \quad \Upsilon_5 = -0.0855,$$

$\Upsilon_6 = -0.2495, \quad \Upsilon_7 = 0.2477, \quad \Upsilon_8 = 0.2459, \quad \Upsilon_9 = 0.2494,$

$\Upsilon_{10} = 0.2495, \quad \Upsilon_{11} = 0.2460, \quad \Upsilon_{12} = 0.2477, \quad \Upsilon_{13} = -0.0803,$

$\Upsilon_{14} = -0.2460, \quad \Upsilon_{15} = 0.0855, \quad \Upsilon_{16} = 0.0855, \quad \Upsilon_{17} = -0.2459,$

$\Upsilon_{18} = -0.0802, \quad \Upsilon_{19} = -0.0855, \quad \Upsilon_{20} = -0.2494, \quad \Upsilon_{21} = 0.0803,$

$\Upsilon_{22} = 0.0820, \quad \Upsilon_{23} = -0.2477$ and $\Upsilon_{24} = -0.0819.$

Step 7: Determine the ranking of all of the alternatives.

Equation (16) was used to rank all Υ_l $(l = 1, 2, \ldots, 24)$.

$$\Upsilon* = \max_{l=1}^{24} \left\{ \Upsilon^l \right\} = \Upsilon_{10} \quad \text{and} \quad P^* = P_{10} = (a_2, a_3, a_4, a_1).$$

Thus, alternatives were ranked $a_2 \succ a_3 \succ a_4 \succ a_1$, with an optimum design of a_2.

Moreover, for the same process, the SNLPWG operator yielded different ranking result $a_2 \succ a_1 \succ a_4 \succ a_3$.

6.2 Sensitivity Analysis

In order to investigate the effects of the risk preferences of the DMs, semantics, and distance parameters on the ranking results, different values of δ, f^*, and λ were taken into consideration. Let $\alpha = 1.37$ and $\beta = \gamma = 0.88$. The results are shown in Table 6 and Table 7. In the tables, "$a_i \succ a_j \succ a_k \succ a_l$" is denoted by "$a_i, a_j, a_k, a_l$" due to limited space.

As shown, the ranking results varied with the risk preferences of the DMs and distance parameters under different semantic environments. However, a_2 was consistently identified as the optimum design. In the first semantic situation, when the hybrid Euclidean distance and SNPWA operator were used to aggregate the evaluation information, the possible alternatives were ranked as $a_2 \succ a_4 \succ a_3 \succ a_1$ when $0.1 \leq \delta \leq 0.2$, $a_2 \succ a_3 \succ a_4 \succ a_1$ when $0.4 \leq \delta \leq 0.5$, and $a_2 \succ a_1 \succ a_3 \succ a_4$ when $0.6 \leq \delta \leq 0.9$. The effects of the risk preferences of the DMs and distance parameters on the ranking results were similar when the SNLPWG operator was employed. The ranking results also varied when the risk preferences of the DMs and distance parameters remained the same under different semantic situations.

These results indicate that the risk preferences of the DMs, distance measurements, and semantics all influenced the decision-making process. In addition, the multiple experts from various backgrounds involved in the green product design process must be considered. Moreover, the inherent characteristics of the SNLPWA and SNLPWG operators must also be considered since they emphasize the effects of the overall criterion values and a single item, respectively.

6.3 Comparative Analysis and Discussion

In this subsection, a comparative study is conducted by applying different methods to the same MCGDM problem in order to validate the feasibility and effectiveness of the proposed approach.

Table 6 Ranking results for different values of δ and $f^*(\lambda = 2)$

δ	f_1^*		f_2^*		f_3^*	
	SNLPWA	SNLPWG	SNLPWA	SNLPWG	SNLPWA	SNLPWG
0.1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1
0.2	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1
0.3	a_2, a_3, a_4, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_1, a_3
0.4	a_2, a_3, a_4, a_1	a_2, a_4, a_1, a_3	a_2, a_3, a_4, a_1	a_2, a_4, a_3, a_1	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3
0.5	a_2, a_3, a_4, a_1	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
0.6	a_2, a_1, a_3, a_4	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
0.7	a_2, a_1, a_3, a_4	a_2, a_1, a_4, a_3	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
0.8	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
0.9	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3

Table 7 Ranking results for different values of λ and f^* ($\delta = 0.5$)

λ	f_1^*		f_2^*		f_3^*	
	SNLPWA	SNLPWG	SNLPWA	SNLPWG	SNLPWA	SNLPWG
1	a_2, a_3, a_4, a_1	a_2, a_1, a_4, a_3	a_2, a_4, a_3, a_1	a_2, a_4, a_1, a_3	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3
2	a_2, a_3, a_4, a_1	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
3	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
4	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_4, a_1, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
5	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_3, a_4, a_1	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
6	a_2, a_3, a_1, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_4, a_1	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
7	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
8	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3	a_2, a_1, a_4, a_3
9	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_1, a_4	a_2, a_1, a_4, a_3	a_2, a_1, a_3, a_4	a_2, a_1, a_4, a_3
10	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_3, a_1, a_4	a_2, a_1, a_3, a_4	a_2, a_1, a_3, a_4	a_2, a_1, a_4, a_3

Table 8 Ranking results obtained using different methods

Methods	Ranking results
Ye (2015)'s method	$a_4 \succ a_3 \succ a_2 \succ a_1$
Tian et al. (2015a)'s method	
Method with f_1^* and $p = q = 1$	$a_2 \succ a_4 \succ a_1 \succ a_3$
Method with f_2^* and $p = q = 1$	$a_2 \succ a_4 \succ a_3 \succ a_1$
Method with f_3^* and $p = q = 1$	$a_2 \succ a_4 \succ a_1 \succ a_3$
The proposed approach based on SNLPWA with f_1^*, $\lambda = 2$ and $\delta = 0.5$	$a_2 \succ a_3 \succ a_4 \succ a_1$

(1) The method developed by Ye (2015) consists of two main steps. First, the comprehensive evaluation information is aggregated using SNLN operations. Then, an extended TOPSIS method is used to rank all of the alternatives. According to Ye (2015), the relative closeness coefficients are $S_1 = 0.8996$, $S_2 = 0.8960$, $S_3 = 0.8956$, and $S_4 = 0.8913$.

(2) In the method developed by Tian et al. (2015a), an SNLNWBM operator based on the linguistic scale function is used to aggregate the evaluation information. Then, the alternatives are ranked using the score function. Adjustments to this method were implemented since it was originally developed to solve MCDM problems involving a single DM.

The ranking results obtained using different methods are summarized in Table 8.

As shown, the different methods yielded different results. Other than the method proposed by Ye (2015), all of the methods identified a_2 as the optimum design. This inconsistency was likely because, in the method developed by Ye (2015), qualitative information is transformed into quantitative information via the labels of linguistic terms. In addition, in this method, linguistic information and SNNs are calculated separately in the SNLN operations. These factors may lead to significant information distortion and/or loss and, thereby, inaccurate results (Tian et al. 2015a).

Although both utilize the same linguistic scale functions and SNLN operations, the method developed by Tian et al. (2015a) and the approach proposed herein yielded differing results for the worst design using the first and second linguistic scale functions. This discrepancy could have been caused by the distinct inherent characteristics of the aggregation operators and ranking rules utilized in the two methods. In the SNLNWBM operator, the interrelationships among the input arguments are represented using the parameters p and q. In contrast, in the SNLPWA operator, the input arguments can support one another, and the weight vectors depend on the arguments. The ranking results obtained using the method developed by Tian et al. (2015a) verified the effectiveness of the linguistic scale functions under different semantic situations.

Since the method of developed by Ye (2015) has some limitations, the results identifying a_4 as the optimum design are unconvincing. Although they utilize different algorithms, the results obtained using both the proposed approach and the approach developed by Tian et al. (2015a) indicated that a_2 is the optimum design. Since these methods employ linguistic scale functions and possess intrinsic features that are capa-

ble of managing simplified neutrosophic linguistic MCDM problems, their results were considered to be relatively convincing.

According to the results of the comparative analysis, the characteristics of the proposed approach can be summarized as follows:

(1) Linguistic scale functions were utilized in the proposed approach to conduct the transformation between qualitative and quantitative data. As a result, the semantics of the original linguistic evaluation information were maintained and sufficiently reflected in the final ranking results.

(2) In the proposed method, power aggregation operators and the TOPSIS-based QUALIFLEX method were combined, providing an innovative, robust approach to simplified neutrosophic linguistic MCGDM problems. The PA operators allowed for the inter-reinforcement of aggregated evaluation values as well as the nonlinear aggregation of data sets clustered around a common value. In the TOPSIS-based QUALIFLEX method, the closeness coefficient of TOPSIS was incorporated into QUALIFLEX. In addition, both the PIS and NIS were considered simultaneously, improving the practicality and validity of the proposed approach. The increasing number of criteria did not significantly influence the computational complexity, but a large number of alternatives did affect the calculations. Although tedious and intricate, the involved procedures yielded reliable results, and the computational burden could be significantly reduce using proven properties and powerful software.

(3) The risk preferences of DMs, distance measurements, and semantics involved in complex group decision-making problems were considered in the proposed approach. However, it may be difficult for DMs to select appropriate parameters due to the limited knowledge and self-cognition. Thus, in a relatively typical decision-making environment, DMs could select the linguistic scale function f_1^* for simplicity. Similarly, a default hybrid Euclidean distance measurement of $\lambda = 2$ and risk-neutral of $\delta = 0.5$ could be employed by default.

7 Conclusions

An increasing number of companies have begun to incorporate green product development into their business models in order to compete in the marketplace. Selecting an optimum product design is a typical MCGDM problem involving SNLNs. In this paper, two simplified neutrosophic linguistic power aggregation operators were developed to address the DMs' evaluation information in the form of SNLNs. In addition, a TOPSIS-based QUALIFLEX method was developed to rank various product designs.

In the proposed approach, the advantages of linguistic term sets and SNS were combined using SNLNs. These SNLNs were used to describe qualitative data involving uncertain, incomplete, and inconsistent information. The risk preferences of DMs, distance measurements, and semantic situations were also considered in the proposed approach. In future studies, a multi-object approach to green product design program selection will be developed using neutrosophic linguistic information.

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