Some Algebraic Properties of Picture Fuzzy t-Norms and Picture Fuzzy t – Conorms on Standard Neutrosophic Sets

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Abstract

In 2013 we introduced a new notion of picture fuzzy sets (PFS), which are direct extensions of the fuzzy sets and the intuitonistic fuzzy sets. Then some operations on PFS with some properties are considered in [9,10]. In [15] these sets are considered as standard neutrosophic sets. Some basic operators of fuzzy logic as negation, t-norms ,t-conorms for picture fuzzy sets firstly are defined and studied in [13,14]. This paper is devoted to some algebraic properties of picture fuzzy t-norms and picture fuzzy t-conorms on standard neutrosophic sets.

Key words: Picture fuzzy sets,, Picture fuzzy t-norms, Picture fuzzy t-conorm , Algebraic property

1. Introduction

Recently, Bui Cong Cuong and Kreinovich (2013) first defined "picture fuzzy sets" [9,10], which are a generalization of the Zadeh' fuzzy sets [1, 2] and the Antanassov's intuitionistic fuzzy sets [3,4]. This concept is particularly effective in approaching the practical problems in relation to the synthesis of ideas, make decisions, such as voting, financial forecasting, estimation of risks in business. The new concept are supporting to new algorithms in computational intelligence problems [18].

In this paper we study some algebraic properties of the picture fuzzy t-norms and the picture fuzzy t-conorms on the standard neutrosophic sets , which are basic operators of the neutrosophic logics. Some classifications of the representable picture fuzzy t-norms and the representable picture fuzzy t-conorms will be presented.

We first recall some basic notions of the picture fuzzy sets.

Definition 1.1. [9] *A picture fuzzy set A on a universe X is an object of the form*

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\},$$

where $\mu_A(x), \eta_A(x), \nu_A(x)$ *are respectively called the degree of positive membership, the degree of neutral membership, the degree of negative membership of x in A, and the following conditions are satisfied:*

$$0 \le \mu_A(x), \eta_A(x), \nu_A(x) \le 1 \quad and \quad \mu_A(x) + \eta_A(x) + \nu_A(x) \le 1, \forall x \in X.$$

Then, $\forall x \in X:\ 1-(\mu_A(x)+\eta_A(x)+\nu_A(x))$ is called the degree of refusal membership of x in A.

Consider the set $D^* = \left\{x = (x_1, x_2, x_3) \mid x \in [0,1]^3, x_1 + x_2 + x_3 \le 1\right\}$. From now on, we will assume that if $x \in D^*$: , then x_1, x_2 and x_3 denote, respectively, the first, the second and the third component of x, i.e. , $x = (x_1, x_2, x_3)$.

We have a lattice $(D^*, \le_1)$, where $\le_1$ defined by

$$\forall x, y \in D^* : (x \le_1 y). \Leftrightarrow \left(x_1 < y_1, x_3 \ge y_3\right) \vee \left(x_1 = y_1, x_3 > y_3\right) \vee \left(\left\{x_1 = y_1, x_3 = y_3, x_2 \le y_2\right\}\right),$$

$$x = y \Leftrightarrow \left\{x_1 = y_1, x_3 = y_3, x_2 = y_2\right\}.$$

We define the first, second and third projection mapping pr_1 , then pr_2 and pr_3 on D^* , defined as $pr_1(x) = x_1$ and $pr_2(x) = x_2$ and $pr_3(x) = x_3$, on all $x \in D^*$.

Note that, if for $x, y \in D^*$ that neither $x \le_1 y$ nor $y \le_1 x$, then x and y are incomparable w.r.t $\le_1$, denoted as $x \|_{\le_1} y$.

From now on , we denote $u \wedge v = \min(u, v), u \vee v = \max(u, v)$ for all $u, v \in R^1$.

For each $x, y \in D^*$, we define

$$\inf(x, y) = \begin{cases} \min(x, y), & if \quad x \le_1 y \ or\ y \le_1 x \\ (x_1 \wedge y_1, 1 - x_1 \wedge y_1 - x_3 \vee y_3, x_3 \vee y_3), & else \end{cases}$$

$$\sup(x, y) = \begin{cases} \max(x, y), & if \quad x \le_1 y \ or\ y \le_1 x \\ (x_1 \vee y_1, 0, x_3 \wedge y_3), & else \end{cases}$$

Proposition 1.2. With these operators $(D^*, \le_1)$ is a complete lattice.

Proof. The units of these lattice are $1_{D^*} = (1, 0, 0)$ and $0_{D^*} = (0, 0, 1)$.

For each nonempty $A \subseteq D^*$, we have

$\inf A = (\inf pr_1 A, \inf pr_2 A, \inf pr_3 A)$, where

$\inf pr_1 A = \inf \{ x_1 \in [0,1] | \exists x = (x_1, x_2, x_3) \in A \}$, ,

$\inf pr_3 A = \sup \{ x_3 \in [0,1] | \exists x = (x_1, x_2, x_3) \in A \}$,

Denote $B_2 = \{ x_2 : (\inf pr_1 A, x_2, \inf pr_3 A) \in A \}$,

$$\inf pr_2 A = \begin{cases} \inf B_2, & if \quad B_2 \neq \emptyset \\ 1 - \inf pr_1 A - \inf pr_3 A, & else \end{cases}$$

and $\sup A = (\sup pr_1 A, \sup pr_2 A, \sup pr_3 A)$, , where

$\sup pr_1 A = \sup \{ x_1 \in [0,1] | \exists x = (x_1, x_2, x_3) \in A \}$, ,

$\sup pr_3 A = inf \{ x_3 \in [0,1] | \exists x = (x_1, x_2, x_3) \in A \}$,

Denote $B_1 = \{ x_2 \in [0,1] : (\sup pr_1 A, x_2, \sup pr_3 A) \in A \}$,

$$\sup pr_2 A = \begin{cases} \sup B_1, & if \quad B_1 \neq \emptyset \\ 0, & else \end{cases} .$$

Using this lattice, we easily see that every picture fuzzy set $A = \{ (x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X \}$ corresponds an D^* – fuzzy set [12] mapping, i.e., we have a mapping

$A : X \to D^* : x \to \{ (x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X \}$.

2. Picture fuzzy t-norms and picture fuzzy t-conorms

Now we consider some basic fuzzy operators of the Picture Fuzzy Logics.

Picture fuzzy negations form an extension of the fuzzy negations [5] and the intuitionistic fuzzy negations [4]. They are defined as follows.

Definition 2.1. A mapping $N : D^* \to D^*$ satisfying conditions $N(0_{D^*}) = 1_{D^*}$ and $N(1_{D^*}) = 0_{D^*}$ and N is nonincreasing is called a picture fuzzy negation.

If $N(N(x)) = x$ for all $x \in D^*$, then N is called an *involutive* negation .

Let $x=(x_1,x_2,x_3)\in D^*$. The mapping N_0 defined by $N_0(x)=(x_3,0,x_1)$, for each $x\in D^*$, is a picture fuzzy negation.

Denote $x_4=1-(x_1+x_2+x_3)$.

The mapping N_S defined by $N_S(x)=(x_3,x_4,x_1)$, for each $x\in D^*$, is an involutive negation and is called the *picture fuzzy standard negation*.

Some picture fuzzy negations were given and studied in [13, 14].

Fuzzy t-norms on [0,1] and fuzzy t-conorms on [0,1] were defined and considered in [5,6].

In 2004 , G.Deschrijver et al.[11] introduced the notion of intuitionistic fuzzy t-norms and t-conorms and investigated under which conditions a similar representation theorem could be obtained.

For further usage, we define $L^*=\{x\in D^* \,|\, x_2=0\}$.

We can considerthe set L^* defined by $L^*=\{u=(u_1,u_3) \,|\, u\in[0,1]^2, u_1+u_3\le 1\}$.

Consider the order relation $u\le v$ on L^*, defined by
$u\le v \Leftrightarrow ((u_1\le v_1)\wedge(u_3\ge v_3))$, for all $u,v\in L^*$.

We define the first, and second projection mapping pr_1 and pr_3 on L^* , defined as $pr_1(u)=u_1$ and $pr_3(u)=u_3$,on all $u\in L^*$. The units of L^* are $1_{L^*}=(1,0)$ and $0_{L^*}=(0,1)$.

Definition 2.2. [12]. An intuitionistic fuzzy t-norm is a commutative, associative, increasing

$(L^*)^2\to L^*$ mapping T satisfying $T(1_{L^*},u)=u$, for all $u\in L^*$.

Definition 2.3. [12]. An intuitionistic fuzzy t-conorm is a commutative, associative, increasing

$(L^*)^2\to L^*$ mapping S satisfying $S(v,0_{L^*})=v$, for all $v\in L^*$.

Definition 2.4.[12]. An intuitionistic fuzzy t-norm T is called t-representable iff there exist a fuzzy t-norm t_1 on [0,1] and a fuzzy t-conorm s_3 on [0,1] satisfying for all $u,v\in L^*$,

$$T(u,v)=(t_1(u_1,v_1),s_3(u_3,v_3)) .$$

Definition 2.5. [12]. An intuitionistic fuzzy t-conorm S is called t-representable iff there exist a fuzzy t-norm t_1 on [0,1] and a fuzzy t-conorm s_3 on [0,1] satisfying for all $u,v\in L^*$,

$S(u,v)=(s_3(u_1,v_1),t_1(u_3,v_3))$.

Now we define picture fuzzy t-norms and picture fuzzy t-conorms . It means that we will give some classes of conjunction operators and and some classes of disjunction operators ,which are basic operators of the neutrosophic logics .

Picture fuzzy t-norms are direct extensions of the fuzzy t-norms in [2, 5, 6] and of the intuitionistic fuzzy t-norms in [4].

Let $x=(x_1,x_2,x_3)\in D^*$. Denote $I(x)=\left\{y\in D^*: y=(x_1,y_2,x_3), 0\leq_1 y_2\leq_1 x_2\right\}$.

Definition 2.6. A mapping $T:D^*\times \mathrm{D}^*\to \mathrm{D}^*$ is a *picture fuzzy t-norm* if the mapping T satisfies the following conditions:

1. $T(x,y)=T(y,x), \quad \forall x,y\in \mathrm{D}^*$ (commutative),
2. $T(x,T(y,z))=T(T(x,y),z), \quad \forall x,y,z\in \mathrm{D}^*$ (associativity)
3. $T(x,y)\leq_1 T(x,z), \quad \forall x,y,z\in \mathrm{D}^*, y\leq_1 z$ (monotonicity)
4. $T(1_{D^*},x)\in I(x), \quad \forall x\in \mathrm{D}^*$ (boundary condition).

Fisrt we give a special picture fuzzy t-norm on picture fuzzy sets

For all $x,y\in D^*$:

$$T_{\inf}(x,y)=\inf\{x,y\}=\begin{cases}(x_1\wedge y_1, 1-x_1\wedge y_1-x_3\vee y_3, x_3\vee y_3) & \text{if } x\,\|_{\leq_1}\, y\\ \min\{x,y\}, & else\end{cases}$$

Let $T_1:D^*\times D^*\to D^*$, $T_2:D^*\times D^*\to D^*$

Definition 2.7.. We say that T_1 *is weaker than* T_2 *if* $T_1(x,y)\leq_1 T_2(x,y), \quad \forall x,y\in D^*$

then we write $T_1\leq T_2$, We write $T_1<T_2$, if $T_1\leq T_2$, and $T_1\neq T_2$

Proposition 2.8.. For any picture fuzzy t-norm $T(x,y)\leq_1 T_{\inf}(x,y), \quad \forall x,y\in D^*$

It means that for any picture fuzzy t-norm we have $T\leq T_{\inf}$

Proof. Let T be a picture fuzzy t-norm, we have $T(x,y)\leq_1 T(x,1_{D^*})\leq_1 x, \quad \forall x,y\in D^*$

and $T(x,y) \le_1 T(y, 1_{D^*}) \le_1 y, \quad \forall x, y \in D^*$

It implies for any picture fuzzy t-norm T we have $T(x,y) \le_1 T(x, 1_{D^*}) \le \inf(x,y), \forall x, y \in D^*$

It means $T \le T_{\inf}$

Definition 2.9. A picture fuzzy t-norm T is called *representable* iff there exist two fuzzy t-norms t_1, t_2 on [0,1] and a fuzzy t-conorm s_3 on [0,1] satisfy:

$$T(x,y) = \left(t_1(x_1,y_1), t_2(x_2,y_2), s_3(x_3,y_3)\right), \forall x, y \in D^*.$$

We give some representable picture fuzzy t-norms, for all $x, y \in D^*$:

1. $T_{\min}(x,y) = \left(\min(x_1,y_1), \min(x_2,y_2), \max(x_3,y_3)\right)$.
2. $T_{02}(x,y) = \left(\min(x_1,y_1), x_2y_2, \max(x_3,y_3)\right)$.
3. $T_{03}(x,y) = \left(x_1y_1, x_2y_2, \max(x_3,y_3)\right)$.
4. $T_{04}(x,y) = \left(x_1y_1, x_2y_2, x_3 + y_3 - x_3y_3\right)$.
5. $T_{05}(x,y) = \left(\begin{cases} x_1 \wedge y_1 & if \ x_1 \vee y_1 = 1 \\ 0 & if \ x_1 \vee y_1 < 1 \end{cases}, \begin{cases} x_2 \wedge y_2 & if \ x_2 \vee y_2 = 1 \\ 0 & if \ x_2 \vee y_2 < 1 \end{cases}, \begin{cases} x_3 \vee y_3 & if \ x_3 \wedge y_3 = 0 \\ 1 & if \ x_3 \wedge y_3 \ne 0 \end{cases}\right)$.
6. $T_{06}(x,y) = \left(\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), \min(1, x_3 + y_3)\right)$.
7. $T_{07}(x,y) = \left(\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), x_3 + y_3 - x_3y_3\right)$.
8. $T_{08}(x,y) = \left(\max\left\{\frac{1}{2}(x_1 + y_1 - 1 + x_1y_1), 0\right\}, \max\left\{\frac{1}{2}(x_2 + y_2 - 1 + x_2y_2), 0\right\}, x_3 + y_3 - x_3y_3\right)$.
9. $T_{09}(x,y) = \left(x_1y_1, \max(0, x_2 + y_2 - 1), x_3 + y_3 - x_3y_3\right)$.
10. $T_{010}(x,y) = \left(\max(0, x_1 + y_1 - 1), x_2y_2, x_3 + y_3 - x_3y_3\right)$.

Definition 2.10. A mapping $S : D^* \times D^* \to D^*$ is a *picture fuzzy t-conorm* if S satisfies all folowing conditions*:*

1. $S(x,y) = S(y,x), \forall x, y \in D^*$.
2. $S(x, S(y,z)) = S(S(x,y), z), \forall x, y, z \in D^*$.
3. $S(x,y) \le_1 S(x,z), \forall x, y, z \in D^*, y \le_1 z.$
4. $S(x, 0_{D^*}) \in I(x), \forall x \in D^*$.

Now we give a new special picture fuzzy t-conorm. For all $x, y \in D^*$

$$S_{\sup}(x,y) = \sup\{x,y\} = \begin{cases} (x_1 \vee y_1, 0, x_3 \wedge y_3), & if\ x \|_{\leq_1} y \\ \max\{x,y\}, & else \end{cases}.$$

Proposition 2.11 . For any picture fuzzy t-conorm $S(x,y) \geq_1 S_{\sup}(x,y), \quad \forall x, y \in D^*$

It means that for any picture fuzzy t-conorm S we have $S \geq S_{\sup}$

Proof. Let S be a picture fuzzy t-conorm, we have $S(x,y) \geq_1 S(x, 0_{D^*}) \geq_1 x, \quad \forall x, y \in D^*$

and $S(x,y) \geq_1 S(y, 0_{D^*}) \geq_1 y, \quad \forall x, y \in D^*$

It implies for any S be a picture fuzzy t-norm, $S(x,y) \geq_1 S(x, 1_{D^*}) \geq_1 \sup(x,y), \forall x, y \in D^*$

It means $S \geq S_{\sup}$

Definition 2.12. A picture fuzzy t-conorm S is called *representable* iff there exist two fuzzy t-norms t_1, t_2 on [0,1] and a fuzzy t-conorm s_3 on [0,1] satisfy:

$$S(x,y) = \left(s_3(x_1,y_1), t_2(x_2,y_2), t_1(x_3,y_3)\right), \forall x, y \in D^*.$$

Some examples of representable picture fuzzy t-conorms, for all $x, y \in D^*$:

1. $S_{\max}(x,y) = \left(\max(x_1,y_1), \min(x_2,y_2), \min(x_3,y_3)\right).$
2. $S_{02}(x,y) = \left(\max(x_1,y_1), x_2 y_2, \min(x_3,y_3)\right).$
3. $S_{03}(x,y) = \left(\max(x_1,y_1), x_2 y_2, x_3 y_3\right).$
4. $S_{04}(x,y) = \left(x_1 + y_1 - x_1 y_1, x_2 y_2, x_3 y_3\right).$
5. $S_{05}(x,y) = \left(x_1 \vee y_1, \begin{cases} x_2 \wedge y_2 & if\ x_2 \vee y_2 = 1 \\ 0 & if\ x_2 \vee y_2 < 1 \end{cases}, x_3 \wedge y_3 \right).$
6. $S_{06}(x,y) = \left(\begin{cases} x_1 \vee y_1 & if\ x_1 \wedge y_1 = 0 \\ 1 & if\ x_1 \wedge y_1 \neq 0 \end{cases}, x_2 \wedge y_2, \begin{cases} x_3 \wedge y_3 & if\ x_3 \vee y_3 = 1 \\ 0 & if\ x_3 \vee y_3 < 1 \end{cases} \right).$

Proposition 2.13. For any representable picture fuzzy t-norm T we have:

$$T_{05}(x,y) \leq_1 T(x,y) \leq_1 T_{\min}(x,y), \ \forall x, y \in D^*.$$

Proposition 2.14. For any representable picture fuzzy t-conorm S we have:

$$S_{05}(x,y) \leq_1 S(x,y) \leq_1 S_{06}(x,y), \forall x, y \in D^*.$$

Proposition 2.15. Assume $T(u,v)$ is a t- representable intuitionistic fuzzy t-norm:

$$T(u,v) = \left(t_1(u_1,v_1), s_3(u_3,v_3)\right), \forall u = (u_1,u_3), v = (v_1,v_3) \in L^*$$

where, t_1 is a fuzzy t-norm on [0,1], s_3 is a fuzzy t-conorm on [0,1]. Assume t_2 is a t-norm on [0,1] satisfies: $0 \leq t_1(x_1,y_1) + t_2(x_2,y_2) + s_3(x_3,y_3) \leq 1, \forall x, y \in D^*$ then:

$$T(x,y) = \left(t_1(x_1,y_1), t_2(x_2,y_2), s_3(x_3,y_3)\right), \forall x, y \in D^*$$

is a representable picture fuzzy t-norm.

Proposition 2.16. Assume $S(u,v)$ is a t-representable intuitionistic fuzzy t-conorm:

$$S(u,v) = \left(s_3(u_1,v_1), t_1(u_3,v_3)\right), \quad \forall u = (u_1,u_3), v = (v_1,v_3) \in L^*$$

where, t_1 is a fuzzy t-norm on [0,1], s_3 is a fuzzy t-conorm on [0,1]. Assume t_2 is a t-norm on [0,1] satisfies: $0 \leq t_1(x_1,y_1) + t_2(x_2,y_2) + s_3(x_3,y_3) \leq 1, \forall x, y \in D^*$ then:

$$S(x,y) = \left(s_3(x_1,y_1), t_2(x_2,y_2), t_1(x_3,y_3)\right), \forall x, y \in D^*$$

is a representable picture fuzzy t-conorm.

Now we define some new concepts for the neutrosophic logics.

Definition 2.17. A picture fuzzy t-norm T is called *Achimerdean* iff:

$$\forall x \in D^* \setminus \left\{0_{D^*}, 1_{D^*}\right\}, T(x,x) <_1 x.$$

Definition 2.18. A picture fuzzy t-norm T is called:

- *nilpotent iff:* $\exists x, y \in D^* \setminus \left\{0_{D^*}\right\}, T(x,y) = 0_{D^*}.$
- *strict iff:* $\forall x, y \in D^* \setminus \left\{0_{D^*}\right\}, T(x,y) \neq 0_{D^*}.$

With these defitions we have the following proposition:

Proposition 2.19. Let

$T^* = \left\{nilpotent\ \ picture\ \ fuzzy\ t - norms\right\},$

$$T^{**} = \{strict\ \ picture\ t - norms\}.\quad Then\quad T^{*} \cap T^{**} = \varnothing$$

Definition 2.20. A picture fuzzy t-conorm S is called *Achimerdean* iff:

$$\forall x \in D^{*} \setminus \{0_{D^{*}}, 1_{D^{*}}\}, S(x,x) >_{1} x.$$

Definition 2.21. A picture fuzzy t-conorm S is called:

- *nilpotent* iff: $\exists x, y \in D^{*} \setminus \{1_{D^{*}}\}, S(x,y) = 1_{D^{*}}$.
- *strict* iff: $\forall x, y \in D^{*} \setminus \{1_{D^{*}}\}, S(x,y) \neq 1_{D^{*}}$.

Proposition 2.22. Let

$$S^{*} = \{nilpotent\ \ picture\ \ fuzzy\ t - conorms\},$$

$$S^{**} = \{strict\ \ picture\ t - conorms\}.\quad Then\ S^{*} \cap S^{**} = \varnothing$$

Proposition 2.23. Assume T is a representable picture fuzzy t-norm:

$$T(x,y) = \left(t_1(x_1,y_1), t_2(x_2,y_2), s_3(x_3,y_3)\right), \forall x, y \in D^{*},$$

and $t_1, t_2,\ s_3$ are Archimedean on [0,1] [5], then T is Archimedean.

Proof. For all $x \in D^{*} \setminus \{0_{D^{*}}, 1_{D^{*}}\}$, we have:

$$T(x,x) = \left(t_1(x_1,x_1), t_2(x_2,x_2), s_3(x_3,x_3)\right).$$

Since $t_1, t_2,\ s_3$ are Archimedean on [0,1] . It follows that $t_1(x_1,x_1) < x_1$, $s_3(x_3,x_3) > x_3$, so $T(x,x) <_1 x$. Thus T is Archimedean.

Proposition 2.24. Assume S is a representable picture fuzzy t-conorm:

$$S(x,y) = \left(s_3(x_1,y_1), t_2(x_2,y_2), t_1(x_3,y_3)\right), \forall x, y \in D^{*}.$$

and $t_1, t_2,\ s_3$ are Archimedean on [0,1], then S is Archimedean.

3. A classification of representable picture fuzzy t-norms

We can give a classification of representable picture fuzzy t-norms to subclasses as follows:

3.1. Strict-strict-strict t-norms subclass, denoted by Δ_{sss}

Definition 3.1. A picture fuzzy t-norm T is called *strict-strict-strict* iff

$$T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$$

where t_1, t_2 are strict fuzzy t-norms on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Example 3.1. $T_1(x,y)=(x_1y_1,x_2y_2,x_3+y_3-x_3y_3)$,

$$T_2(x,y)=\left(\frac{x_1y_1}{\lambda_1+(1-\lambda_1)(x_1+y_1-x_1y_1)},\frac{x_2y_2}{\lambda_2+(1-\lambda_2)(x_2+y_2-x_2y_2)},\right.$$

$$\left.(x_3^a+y_3^a-x_3^ay_3^a)^{\frac{1}{a}}\right),\quad \lambda_1,\lambda_2,a\in[1,+\infty),$$

3.2.Nipoltent-nipoltent-nipoltent t-norms subclass, denoted by Δ_{nnn} **:**

Definition 3.2. A picture fuzzy t-norm T is called *nipoltent-nipoltent-nipoltent* iff

$$T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$$

where t_1, t_2 are nipoltent fuzzy t-norms on [0,1] and s_3 is a nipoltent fuzzy t-conorm on [0,1].

Example 3.2.

$$T_3(x,y)=(0\vee(x_1+y_1-1),0\vee(x_2+y_2-1),1\wedge(x_3+y_3)),$$

$$T_4(x,y)=(((x_1+y_1-1)(1+\lambda_1)-\lambda_1x_1y_1)\vee 0,((x_2+y_2-1)(1+\lambda_2)-\lambda_2x_2y_2)\vee 0,$$

$$1\wedge(x_3^a+y_3^a)^{\frac{1}{a}}),\quad \lambda_1,\lambda_2\in[0,+\infty),a\geq 1,$$

$$T_5(x,y)=((0\vee(x_1^a+y_1^a-1))^{\frac{1}{a}},(0\vee(x_2^b+y_2^b-1))^{\frac{1}{b}},1\wedge(x_3^c+y_3^c)^{\frac{1}{c}}),\quad a,b,c\geq 1,$$

$$T_6(x,y)=((\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0),(\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0),$$

$$1\wedge(x_3^c+y_3^c)^{\frac{1}{c}}),\quad a,b\in(0,1];c\geq 1,$$

$$T_7(x,y)=((\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0),((x_2+y_2-1)(1+\lambda)-\lambda x_2y_2)\vee 0,$$

$$1\wedge(x_3^b+y_3^b)^{\frac{1}{b}}),\quad a\in(0,1],\lambda\geq 0,b\geq 1,$$

$$T_8(x,y)=(((x_1+y_1-1)(1+\lambda)-\lambda x_1y_1)\vee 0,(\frac{1}{a}(x_2+y_2-1+(a-1)x_2y_2)\vee 0),$$

$$1\wedge(x_3^b+y_3^b)^{\frac{1}{b}}),\quad a\in(0,1],b\geq 1,\lambda\geq 0,$$

$$T_9(x,y)=((\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0),0\vee(x_2^b+y_2^b-1)^{\frac{1}{b}},$$

$$1\wedge(x_3^c+y_3^c)^{\frac{1}{c}}),\quad a\in(0,1],b,c\geq 1,$$

$$T_{10}(x,y)=(0\vee(x_1^a+y_1^a-1)^{\frac{1}{a}},(\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0),$$

$$1\wedge(x_3^c+y_3^c)^{\frac{1}{c}}),\quad b\in(0,1],a,c\geq 1.$$

$$T_{11}(x,y)=((((x_1+y_1-1)(1+\lambda)-\lambda x_1y_1)\vee 0,0\vee(x_2^a+y_2^a-1)^{\frac{1}{a}},$$

$$1\wedge(x_3^b+y_3^b)^{\frac{1}{b}}),\quad \lambda\geq 0,a,b\geq 1,$$

$$T_{12}(x,y)=(0\vee(x_1^a+y_1^a-1)^{\frac{1}{a}},((x_2+y_2-1)(1+\lambda)-\lambda x_2y_2)\vee 0,$$

$$1\wedge(x_3^b+y_3^b)^{\frac{1}{b}}),\quad \lambda\geq 0,a,b\geq 1.$$

3.3. Nipoltent-nipoltent-strict t-norms subclass, denoted by Δ_{nns}

Definition 3.3. A picture fuzzy t-norm T is called *nipoltent-nipoltent-strict* iff

$$T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$$

where t_1, t_2 are nipoltent fuzzy t-norms on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Examples 3.3

$$T_{13}(x,y)=(0\vee(x_1+y_1-1),0\vee(x_2+y_2-1),x_3+y_3-x_3y_3).$$

$$T_{14}(x,y)=(\frac{1}{2}(x_1+y_1-1+x_1y_1)\vee 0,\frac{1}{2}(x_2+y_2-1+x_2y_2)\vee 0,(x_3^a+y_3^a-x_3^ay_3^a)^{\frac{1}{a}}),a\geq 1,$$

$$T_{15}(x,y)=((((x_1+y_1-1)(1+\lambda_1)-\lambda_1x_1y_1)\vee 0,((x_2+y_2-1)(1+\lambda_2)-\lambda_2x_2y_2)\vee 0,$$

$$(x_3^a+y_3^a-x_3^ay_3^a)^{\frac{1}{a}}),\quad \lambda_1,\lambda_2\in[0,+\infty),a\geq 1,$$

$$T_{16}(x,y)=(0\vee(x_1^a+y_1^a-1)^{\frac{1}{a}},(x_2^b+y_2^b-1)^{\frac{1}{b}}\vee 0,(x_3^c+y_3^c-x_3^cy_3^c)^{\frac{1}{c}}),\quad a,b,c\geq 1,$$

$$T_{17}(x,y)=(\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0,0\vee(x_2^b+y_2^b-1)^{\frac{1}{b}},$$

$$(x_3^c+y_3^c-x_3^cy_3^c)^{\frac{1}{c}}),\quad a\in(0,1];b,c\geq 1,$$

$$T_{18}(x,y)=(\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0,\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$

$$(x_3^c+y_3^c-x_3^cy_3^c)^{\frac{1}{c}}),\quad a,b\in(0,1];c\geq 1,$$

$$T_{19}(x,y)=(\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0,((x_2+y_2-1)(1+b)-bx_2y_2)\vee 0,$$

$$(x_3^c+y_3^c-x_3^cy_3^c)^{\frac{1}{c}}),\quad a\in(0,1];b\geq 0;c\geq 1,$$

$$T_{20}(x,y)=(0\vee(x_1^a+y_1^a-1)^{\frac{1}{a}},\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$

$$(x_3^c+y_3^c-x_3^cy_3^c)^{\frac{1}{c}}),\quad b\in(0,1];a,c\geq 1,$$

$$T_{21}(x,y)=(((x_1+y_1-1)(1+a)-ax_1y_1)\vee 0,\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$

$$(x_3^c+y_3^c-x_3^cy_3^c)^{\frac{1}{c}}),\quad a\geq 0;b\in(0,1];c\geq 1,$$

$$T_{22}(x,y)=(((x_1+y_1-1)(1+\lambda)-\lambda x_1y_1)\vee 0,0\vee(x_2^a+y_2^a-1)^{\frac{1}{a}},$$

$$(x_3^b+y_3^b-x_3^by_3^b)^{\frac{1}{b}}),\quad \lambda\geq 0,a,b\geq 1,$$

$$T_{23}(x,y)=(0\vee(x_1^a+y_1^a-1)^{\frac{1}{a}},((x_2+y_2-1)(1+\lambda)-\lambda x_2y_2)\vee 0,$$

$$(x_3^b+y_3^b-x_3^by_3^b)^{\frac{1}{b}}),\quad \lambda\geq 0,a,b\geq 1.$$

3.4. Strict-nipoltent-strict t-norms subclass, denoted by Δ_{sns}

Definition 3.4. A picture fuzzy t-norm T is called *strict -nipoltent-strict* iff

$$T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$$

where t_1 is a strict fuzzy t-norm on [0,1], t_2 is a nipoltent fuzzy t-norm on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Example 3.4.

$$T_{24}(x,y)=(x_1y_1,0\vee(x_2+y_2-1),x_3+y_3-x_3y_3).$$

$$T_{25}(x,y)=(\frac{x_1y_1}{\lambda_1+(1-\lambda_1)(x_1+y_1-x_1y_1)},((x_2+y_2-1)(1+\lambda_2)-\lambda_2x_2y_2)\vee 0,$$

$$(x_3^a+y_3^a-x_3^ay_3^a)^{\frac{1}{a}}),\quad \lambda_1\ge 1,\lambda_2\ge 0,a\ge 1,$$

$$T_{26}(x,y)=(\frac{x_1y_1}{\lambda_1+(1-\lambda_1)(x_1+y_1-x_1y_1)},0\vee(x_2^a+y_2^a-1)^{\frac{1}{a}},$$

$$(x_3^b+y_3^b-x_3^by_3^b)^{\frac{1}{b}}),\quad \lambda_1\ge 1,a,b\ge 1,$$

$$T_{27}(x,y)=(\frac{x_1y_1}{\lambda_1+(1-\lambda_1)(x_1+y_1-x_1y_1)},\frac{1}{a}(x_2+y_2-1+(a-1)x_2y_2)\vee 0,$$

$$(x_3^b+y_3^b-x_3^by_3^b)^{\frac{1}{b}}),\quad \lambda_1,b\ge 1,a\in(0,1].$$

3.5. Nipoltent-strict-strict t-norms subclass, denoted by Δ_{nss}

Definition 3.5. A picture fuzzy t-norm T is called *nipoltent-strict-strict* iff

$$T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$$

where t_1 is a nipoltent fuzzy t-norm on [0,1], t_2 is a strict fuzzy t-norm on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Example 3.5.

$$T_{28}(x,y)=(0\vee(x_1+y_1-1),x_2y_2,x_3+y_3-x_3y_3),$$

$$T_{29}(x,y)=(\frac{1}{a}(x_1+y_1-1+(a-1)x_1y_1)\vee 0,\frac{x_2y_2}{\lambda+(1-\lambda)(x_2+y_2-x_2y_2)},$$

$$(x_3^b+y_3^b-x_3^by_3^b)^{\frac{1}{b}}),\quad a\in(0,1];b,\lambda\ge 1,$$

$$T_{30}(x,y)=(0\vee(x_1^a+y_1^a-1)^{\frac{1}{a}},\frac{x_2y_2}{\lambda+(1-\lambda)(x_2+y_2-x_2y_2)},$$

$$(x_3^b+y_3^b-x_3^by_3^b)^{\frac{1}{b}}),\quad a,b,\lambda\ge 1,$$

$$T_{31}(x,y)=(((x_1+y_1-1)(1+\lambda_1)-\lambda_1x_1y_1,\frac{x_2y_2}{\lambda_2+(1-\lambda_2)(x_2+y_2-x_2y_2)},$$

$$(x_3^a+y_3^a-x_3^ay_3^a)^{\frac{1}{a}}),\quad a,\lambda_2\ge 1,\lambda_1\in(0,1].$$

Proposition 3.6. There doesn't exist t-representable picture fuzzy t-norm T :

$$T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$$

where t_1 or t_2 is a strict fuzzy t-norm on [0,1], and s_3 is a nipoltent fuzzy t-conorm on [0,1].

Proof. Assume $T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right),\forall x,y\in D^*,$ with t_1 is a strict t-norm and exist $x_3,y_3\in(0,1)$ such that $S_3(x_3,y_3)=1.$ Let $x_1,x_2\neq 0\,|\,x_1+x_2+x_3\leq 1; y_1,y_2\neq 0\,|\,y_1+y_2+y_3\leq 1,$ and since t_1 is strict t-norm then: $t_1(x_1,y_1)>0.$

Let $x=(x_1,x_2,x_3),y=(y_1,y_2,y_3),$ we have a contradition: $t_1(x_1,y_1)+t_2(x_2,y_2)+s_3(x_3,y_3)>1.$

Similarly, if t_2 is strict t-norm and s_3 is nipoltent t-conorm then we have a contraddition.

Proposition 3.7. If T belongs to one of four classes Δ_{sss}, Δ_{nns}, Δ_{sns}, Δ_{nss} then T is strict.

Proof. Assume for all $x,y\in D^*$, s_3 is a strict fuzzy t-conorm on [0,1], T is representable picture fuzzy t-norm: $T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right)$ and T is nipoltent.

Then $\exists x,y\in D^*\setminus\{0_{D^*}\},T(x,y)=0_{D^*},$ and it implies $t_1(x_1,y_1)=0,$ $t_2(x_2,y_2)=0,$ $s_3(x_3,y_3)=1.$ Since s_3 is a strict fuzzy t-conorm on [0,1],then $x_3=1$ or $y_3=1$, which is a contradition.

Proposition 3.8. If T belongs to the class Δ_{nnn} then T is a nipoltent picture fuzzy t-norm.

Proof. Assume $T\in\Delta_{nnn},\forall x,y\in D^*:T(x,y)=\left(t_1(x_1,y_1),t_2(x_2,y_2),s_3(x_3,y_3)\right)$

Since t_1,t_2 are nipoltent fuzzy t-norms on [0,1], we have

$\exists x_1,y_1,x_2,y_2\,|\,t_1(x_1,y_1)=0,t_2(x_2,y_2)=0.$ Since t_1,t_2 are not decreasing, so:
$\forall x_1'\leq x_1,y_1'\leq y_1;x_2'\leq x_2,y_2'\leq y_2\,|\,t_1(x_1',y_1')=0,t_2(x_2',y_2')=0.$ Since s is a nipoltent fuzzy t-conorm on [0,1] so: $\exists x_3,y_3\neq 1\,|\,s_3(x_3,y_3)=1.$ Let $x=(x_1',x_2',x_3),y=(y_1',y_2',y_3)\in D^*.$ Then:

$T(x,y)=\left(t_1(x_1',y_1'),t_2(x_2',y_2'),s_3(x_3,y_3)\right)=0_{D^*}.$ T is a nipoltent picture fuzzy t-norm.

4. A classification of representable picture fuzzy t-conorms

Similar to the section 3, we can give some subclasses of represntable picture fuzzy t-conorms as follows:

4.1. *Strict-strict-strict t-conorms subclass,* denoted by ∇_{sss}

Definition 4.1. A picture fuzzy t-conorm S is called *strict-strict-strict* iff

$$S(x,y)=\left(s_3(x_1,y_1),t_2(x_2,y_2),t_1(x_3,y_3)\right),\forall x,y\in D^*.$$

where t_1, t_2 are strict fuzzy t-norms on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Example 4.1.

$$S_1(x,y)=\left(x_1+y_1-x_1y_1,x_2y_2,x_3y_3\right),$$

$$S_2(x,y)=((x_1^a+y_1^a-x_1^ay_1^a)^{\frac{1}{a}},\frac{x_2y_2}{\lambda_1+(1-\lambda_1)(x_2+y_2-x_2y_2)},$$
$$\frac{x_3y_3}{\lambda_2+(1-\lambda_2)(x_3+y_3-x_3y_3)}),\quad \lambda_1,\lambda_2,a\in[1,+\infty).$$

4.2.*Nipoltent-nipoltent-nipoltent t-conorms subclass*, denoted by ∇_{nnn} :

Definition 4.2. A picture fuzzy t-conorm S is called *nipoltent-nipoltent-nipoltent* iff

$$S(x,y)=\left(s_3(x_1,y_1),t_2(x_2,y_2),t_1(x_3,y_3)\right),\forall x,y\in D^*.$$

where t_1, t_2 are nipoltent fuzzy t-norms on [0,1] and s_3 is a nipoltent fuzzy t-conorm on [0,1].

Example 4.2.

$$S_3(x,y)=(1\wedge(x_1+y_1),\ 0\vee(x_2+y_2-1),\ 0\vee(x_3+y_3-1)),$$

$$S_4(x,y)=(1\wedge(x_1^a+y_1^a)^{\frac{1}{a}},\ ((x_2+y_2-1)(1+\lambda_1)-\lambda_1x_2y_2)\vee 0,$$
$$((x_3+y_3-1)(1+\lambda_2)-\lambda_2x_3y_3)\vee 0),\quad \lambda_1,\lambda_2\in[0,+\infty),a\geq 1,$$

$$S_5(x,y)=(1\wedge(x_1^a+y_1^a)^{\frac{1}{a}},(0\vee(x_2^b+y_2^b-1))^{\frac{1}{b}},$$
$$0\vee(x_3^c+y_3^c-1)^{\frac{1}{c}}),\quad a,b,c\geq 1,$$

$$S_6(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, (\frac{1}{b}(x_2 + y_2 - 1 + (b-1)x_2y_2) \vee 0),$$
$$(\frac{1}{c}(x_3 + y_3 - 1 + (c-1)x_3y_3) \vee 0)), \quad a \ge 1; b, c \in (0,1],$$

$$S_7(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, (\frac{1}{b}(x_2 + y_2 - 1 + (b-1)x_2y_2) \vee 0),$$
$$((x_3 + y_3 - 1)(1+\lambda) - \lambda x_3y_3) \vee 0), \quad a \ge 1, b \in (0,1], \lambda \ge 0,$$

$$S_8(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, ((x_2 + y_2 - 1)(1+\lambda) - \lambda x_2y_2) \vee 0,$$
$$(\frac{1}{b}(x_3 + y_3 - 1 + (b-1)x_3y_3) \vee 0)), \quad a \ge 1, b \in (0,1], \lambda \ge 0,$$

$$S_9(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, (\frac{1}{b}(x_2 + y_2 - 1 + (b-1)x_2y_2) \vee 0),$$
$$0 \vee (x_3^c + y_3^c - 1)^{\frac{1}{c}}), \quad b \in (0,1], a, c \ge 1,$$

$$S_{10}(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, 0 \vee (x_2^b + y_2^b - 1)^{\frac{1}{b}},$$
$$(\frac{1}{c}(x_3 + y_3 - 1 + (c-1)x_3y_3) \vee 0)), \quad c \in (0,1], a, b \ge 1,$$

$$S_{11}(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, ((x_2 + y_2 - 1)(1+\lambda) - \lambda x_2y_2) \vee 0,$$
$$0 \vee (x_3^b + y_3^b - 1)^{\frac{1}{b}}), \quad \lambda \ge 0, a, b \ge 1,$$

$$S_{12}(x,y) = (1 \wedge (x_1^a + y_1^a)^{\frac{1}{a}}, 0 \vee (x_2^b + y_2^b - 1)^{\frac{1}{b}},$$
$$((x_3 + y_3 - 1)(1+\lambda) - \lambda x_3y_3) \vee 0), \quad \lambda \ge 0, a, b \ge 1.$$

4.3.Strict-nipoltent-nipoltent t-conorms subclass, denoted by ∇_{snn}

Definition 4.3. A picture fuzzy t-conorm S is called *strict-nipoltent-nipoltent* iff

$$S(x,y) = \left(s_3(x_1,y_1), t_2(x_2,y_2), t_1(x_3,y_3)\right), \forall x, y \in D^*.$$

where t_1, t_2 are nipoltent fuzzy t-norms on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Examples 4.3.

$$S_{13}(x,y) = (x_1 + y_1 - x_1y_1, 0 \vee (x_2 + y_2 - 1), 0 \vee (x_3 + y_3 - 1)),$$

$$S_{14}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},\frac{1}{2}(x_2+y_2-1+x_2y_2)\vee 0,$$
$$\frac{1}{2}(x_3+y_3-1+x_3y_3)\vee 0),\quad a\geq 1,$$

$$S_{15}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},((x_2+y_2-1)(1+\lambda_1)-\lambda_1 x_2y_2)\vee 0,$$
$$((x_3+y_3-1)(1+\lambda_2)-\lambda_2 x_3y_3)\vee 0),\quad \lambda_1,\lambda_2\in[0,+\infty),a\geq 1,$$

$$S_{16}(x,y)=((x_1^c+y_1^c-x_1^c y_1^c)^{\frac{1}{c}},0\vee(x_2^a+y_2^a-1)^{\frac{1}{a}},0\vee(x_3^b+y_3^b-1)^{\frac{1}{b}}),\quad a,b,c\geq 1,$$

$$S_{17}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$
$$0\vee(x_3^c+y_3^c-1)^{\frac{1}{c}}),\quad b\in(0,1];a,c\geq 1,$$

$$S_{18}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$
$$\frac{1}{c}(x_3+y_3-1+(c-1)x_3y_3)\vee 0),\quad b,c\in(0,1];a\geq 1,$$

$$S_{19}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$
$$((x_3+y_3-1)(1+c)-cx_3y_3)\vee 0),\quad a\geq 1,b\in(0,1];c\geq 0,$$

$$S_{20}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},(x_2^b+y_2^b-1)^{\frac{1}{b}}\vee 0,$$
$$\frac{1}{c}(x_3+y_3-1+(c-1)x_3y_3)\vee 0),\quad c\in(0,1];a,b\geq 1,$$

$$S_{21}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},((x_2+y_2-1)(1+b)-bx_2y_2)\vee 0,$$
$$\frac{1}{c}(x_3+y_3-1+(c-1)x_3y_3)\vee 0),\quad a\geq 1;b\geq 0;c\in(0,1],$$

$$S_{22}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},((x_2+y_2-1)(1+\lambda)-\lambda x_2y_2)\vee 0,$$
$$0\vee(x_3^b+y_3^b-1)^{\frac{1}{b}}),\quad \lambda\geq 0,a,b\geq 1,$$

$$S_{23}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}},0\vee(x_2^b+y_2^b-1)^{\frac{1}{b}},$$
$$((x_3+y_3-1)(1+\lambda)-\lambda x_3y_3)\vee 0),\quad \lambda\geq 0,a,b\geq 1.$$

4.4.Strict-nipoltent-strict t-conorms subclass, denoted by ∇_{sns}

Definition 4.4. A picture fuzzy t-conorm S is called *strict-nipoltent-strict* iff

$$S(x,y)=\left(s_3(x_1,y_1),t_2(x_2,y_2),t_1(x_3,y_3)\right),\forall x,y\in D^*.$$

where t_1 is a strict fuzzy t-norm on [0,1], t_2 is a nipoltent fuzzy t-norm on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Example 4.4.

$$S_{24}(x,y)=(x_1+y_1-x_1y_1,\, 0\vee(x_2+y_2-1),\, x_3y_3),$$

$$S_{25}(x,y)=((x_1^a+y_1^a-x_1^ay_1^a)^{\frac{1}{a}},((x_2+y_2-1)(1+\lambda_1)-\lambda_1x_2y_2)\vee 0,$$
$$\frac{x_3y_3}{\lambda_2+(1-\lambda_2)(x_3+y_3-x_3y_3)}),\quad \lambda_1\geq 0;\lambda_2,a\geq 1,$$

$$S_{26}(x,y)=((x_1^a+y_1^a-x_1^ay_1^a)^{\frac{1}{a}},\, 0\vee(x_2^b+y_2^b-1)^{\frac{1}{b}},$$
$$\frac{x_3y_3}{\lambda_1+(1-\lambda_1)(x_3+y_3-x_3y_3)}),\quad \lambda_1\geq 1;a,b\geq 1,$$

$$S_{27}(x,y)=((x_1^a+y_1^a-x_1^ay_1^a)^{\frac{1}{a}},\frac{1}{b}(x_2+y_2-1+(b-1)x_2y_2)\vee 0,$$
$$\frac{x_3y_3}{\lambda_1+(1-\lambda_1)(x_3+y_3-x_3y_3)}),\quad a,\lambda_1\geq 1;b\in(0,1].$$

4.5.Strict-strict-nipoltent t-conorms subclass, denoted by ∇_{ssn}

Definition 4.5. A picture fuzzy t-conorm S is called *strict-strict-nipoltent* iff

$$S(x,y)=\left(s_3(x_1,y_1),t_2(x_2,y_2),t_1(x_3,y_3)\right),\forall x,y\in D^*.$$

where t_1 is a nipoltent fuzzy t-norm on [0,1], t_2 is a strict fuzzy t-norm on [0,1] and s_3 is a strict fuzzy t-conorm on [0,1].

Example4.5.

$$S_{28}(x,y)=(x_1+y_1-x_1y_1,\, x_2y_2,\, 0\vee(x_3+y_3-1)),$$

$$S_{29}(x,y)=((x_1^a+y_1^a-x_1^ay_1^a)^{\frac{1}{a}},\frac{x_2y_2}{\lambda+(1-\lambda)(x_2+y_2-x_2y_2)},$$
$$\frac{1}{b}(x_3+y_3-1+(b-1)x_3y_3)\vee 0),\quad a,\lambda\geq 1;b\in(0,1],$$

$$S_{30}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}}, \frac{x_2 y_2}{\lambda+(1-\lambda)(x_2+y_2-x_2 y_2)},$$
$$0\vee(x_3^b+y_3^b-1)^{\frac{1}{b}}), \quad a,b,\lambda\ge 1,$$
$$S_{31}(x,y)=((x_1^a+y_1^a-x_1^a y_1^a)^{\frac{1}{a}}, \frac{x_2 y_2}{\lambda+(1-\lambda)(x_2+y_2-x_2 y_2)},$$
$$((x_2+y_2-1)(1+b)-bx_2 y_2)\vee 0), \quad a,\lambda\ge 1; b\ge 0.$$

Proposition 4.6. There doesn't exist representable picture fuzzy t-conorm S :

$$S(x,y)=\left(s_3(x_1,y_1),t_2(x_2,y_2),t_1(x_3,y_3)\right),\forall x,y\in D^*.$$

where t_1 or t_2 is strict fuzzy t-norm on [0,1] and s_3 is a nipoltent fuzzy t-conorm on [0,1].

Proposition 4.7. If S belongs to one of four classes ∇_{sss}, ∇_{snn}, ∇_{sns}, ∇_{ssn} then S is strict.

Proposition 4.8. If S belongs to the class ∇_{nnn} then S is nipoltent.

5. Conclusion

t-norms and t-conorms are basic operators of the fuzzy logics [5,6]. Picture fuzzy t-norms and picture fuzzy t-conorms firstly defined and studied in 2015 [13]. In this paper we give some algebraic properties of the picture fuzzy t-norms and the picture fuzzy t-conrms on standard neutrsophic sets , including some classifications of the class of representable picture fuzzy t-norms and and of the class of representable picture fuzzy t-conorms. Another important operators of picture fuzzy logics should be considered in the future papers.

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