Int. J. Mach. Learn. & Cyber.
DOI 10.1007/s13042-016-0505-3

ORIGINAL ARTICLE

A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems

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Received: 2 June 2015 / Accepted: 25 January 2016
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Abstract The concept of a single valued neutrosophic number (SVN-number) is of importance for quantifying an ill-known quantity and the ranking of SVN-numbers is a very difficult problem in multi-attribute decision making problems. The aim of this paper is to present a methodology for solving multi-attribute decision making problems with SVN-numbers. Therefore, we firstly defined the concepts of cut sets of SVN-numbers and then applied to single valued trapezoidal neutrosophic numbers (SVTN-numbers) and triangular neutrosophic numbers (SVTrN-numbers). Then, we proposed the values and ambiguities of the truth-membership function, indeterminacy-membership function and falsity-membership function for a SVN-numbers and studied some desired properties. Also, we developed a ranking method by using the concept of values and ambiguities, and applied to multi-attribute decision making problems in which the ratings of alternatives on attributes are expressed with SVTN-numbers.

Keywords Neutrosophic set · Single valued neutrosophic numbers · Trapezoidal neutrosophic numbers · Triangular neutrosophic numbers · Decision making

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1 Introduction

Smarandache [32] proposed concept of neutrosophic set which is generalization of classical set, fuzzy set [50], intuitionistic fuzzy set [3], and so on. In the neutrosophic set, for an element x of the universe, the membership functions independently indicates the truth-membership degree, indeterminacy-membership degree, and false-membership degree of the element x belonging to the neutrosophic set. Also, fuzzy, intuitionistic and neutrosophic models have been studied by many authors (e.g. [1, 2, 5, 7–9, 14, 15, 19, 31, 33, 34, 36, 41, 42]).

Multi-attribute decision making (MADM) which is an important part of decision science is to find an optimal alternative, which are characterized in terms of multiple attributes, from alternative sets. In some practical applications, the decision makers may be not able to evaluate exactly the values of the MADM problems due to uncertain and asymmetric information between decision makers. As a result, values of the MADM problems are not measured by accurate numbers. It is feasible for some sets which contain uncertainty such as; a fuzzy set, intuitionistic set and neutrosophic set to represent an uncertainty of values of the MADM problems. Intuitionistic fuzzy numbers, intuitionistic triangular fuzzy numbers and intuitionistic trapezoidal fuzzy numbers is introduced by Mahapatra and Roy [27], Liang [26] and Jianqiang [20], respectively. Li [23] gave a ranking method of intuitionistic fuzzy numbers and application to multiattribute decision-making problems in which the attribute ratings are expressed with intuitionistic fuzzy numbers in management. Therefore, he defined the notation of cut sets of intuitionistic fuzzy numbers and their values and ambiguities of membership and nonmembership functions. Also, the notions of intuitionistic fuzzy numbers were studied in [4, 6, 10, 12, 17,

Published online: 19 February 2016

18, 28, 35, 39, 40, 49] and applied to multi-attribute decision making problems in [6, 24, 37, 38, 40, 43, 46, 48].

Ye [45] presented the notations of simplified neutrosophic sets and gave the operational laws of simplified neutrosophic sets. Then, he introduced some aggregation operators are called simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator. Also, he developed ranking method by using cosine similarity measure between an alternative and the ideal alternative. Peng et al. [30] introduced the concept of multi-valued neutrosophic set with the operations. Then, they gave two multi-valued neutrosophic power aggregation operators and applied to multi-criteria group decision-making problems. A novel concept of expected values of fuzzy variables, which is essentially a type of Choquet integral and coincides with that of random variables is introduced by Liu et al. [22]. Li [25] developed a new methodology for ranking TIFNs and cut sets of intuitionistic trapezoidal fuzzy numbers as well as arithmetical operations are developed. Then, he gave the values and ambiguities of the membership function and the non-membership function for a intuitionistic trapezoidal fuzzy number and a new ranking method with applications. Kumar and Kaur [21] and Nehi [29] presented a new ranking approach by modifying an existing ranking approach for comparing intuitionistic fuzzy numbers. Zhang [51] proposed a methodology for intuitionistic trapezoidal fuzzy multiple and give a numerical example by using similarity measure. Zeng et al. [52] and De and Das [11] developed a method for ranking trapezoidal intuitionistic fuzzy numbers and gave cut sets over intuitionistic triangular fuzzy numbers. Then, they presented the values and ambiguities of the membership degree and the nonmembership degree for intuitionistic triangular fuzzy numbers and developed a method. Ye [44] proposed the expected values for intuitionistic trapezoidal fuzzy numbers and presented a handling method for intuitionistic trapezoidal fuzzy multicriteria decision-making problems, in which the preference values of an alternative on criteria and the weight values of criteria take the form of intuitionistic trapezoidal fuzzy numbers.

In a multiple-attribute decision-making problem the decision makers need to rank the given alternatives and the ranking of alternatives with neutrosophic numbers is many difficult because neutrosophic numbers are not ranked by ordinary methods as real numbers. However it is possible with score functions [16], aggregation operators[16], distance measures[13], and so on. Therefore; in this study we extend the ranking method as well as applications of fuzzy numbers by given [53] and intuitionistic fuzzy numbers by given [23, 25, 28] to SVN-numbers for solving MAGDM problems in which the ratings of alternatives with respect to each attribute are represented by SVN-numbers. To do so, the rest of this paper is organized as: In the next section, we will present some basic definitions and operations of SVN-numbers. In Sect. 3, we introduce the concepts of cut sets of N-numbers and applied to single valued trapezoidal neutrosophic numbers (SVTN-numbers) and triangular neutrosophic numbers (SVTrN-numbers). Meanwhile, we also describe the values and ambiguities of the truth-membership function, indeterminacy-membership function and falsity-membership function for a SVN-numbers and studied some desired properties. In Sect. 4, we develop a novel ranking method is called SVTrN-multi-attribute decision-making method. Afterwards, we present a decision algorithm and applied to multi-attribute decision making problems in which the ratings of alternatives on attributes are expressed with SVTrN-numbers. In Sect. 5, the method is compared with the other methods that were outlined in Refs. [13], [16] and [47] using SVTrN-numbers. In last section, a short conclusion are given. The present expository paper is a condensation of part of the dissertation [13].

2 Preliminary

In this section, we recall some basic notions of fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, single valued neutrosophic sets and single valued neutrosophic numbers.

From now on we use $I_n = \{1, 2, \ldots, n\}$ and $I_m = \{1, 2, \ldots, m\}$ as an index set for $n \in \mathbb{N}$ and $m \in \mathbb{N}$, respectively.

Definition 2.1 [3] Let E be a universe. An intuitionistic fuzzy set K over E is defined by

$$K = \{ <x, \mu_K(x), \gamma_K(x)> \ : \ x \in E\}$$

where $\mu_K : E \to [0,1]$ and $\gamma_K : E \to [0,1]$ such that $0 \le \mu_K(x) + \gamma_K(x) \le 1$ for any $x \in E$. For each $x \in E$, the values $\mu_K(x)$ and $\gamma_K(x)$ are the degree of membership and degree of non-membership of x, respectively.

Definition 2.2 [32] Let E be a universe. A neutrosophic sets A over E is defined by

$$A = \{ <x, (T_A(x), I_A(x), F_A(x))> \ : x \in E\}.$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by

$$T_A : E \to]^-0, 1^+[, \quad I_A : E \to]^-0, 1^+[, \quad F_A : E \to]^-0, 1^+[$$

such that $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.3 [36] Let E be a universe. An single valued neutrosophic set (SVN-set) over E is a neutrosophic set over E, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$$T_A : E \to [0,1], \quad I_A : E \to [0,1], \quad F_A : E \to [0,1]$$

such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.4 [13] Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0,1]$ be any real numbers, $a_i, b_i, c_i, d_i \in \mathbb{R}$ and $a_i \leq b_i \leq c_i \leq d_i$ (i = 1, 2, 3). Then a single valued neutrosophic number (SVN-number)

$$\tilde{a} = \langle ((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}}) \rangle$$

is a special neutrosophic set on the set of real numbers $\mathbb{R}$, whose truth-membership function $\mu_{\tilde{a}}$, indeterminacy-membership function $v_{\tilde{a}}$ and falsity-membership function $\lambda_{\tilde{a}}$ are respectively defined by

$$\mu_{\tilde{a}} : \mathbb{R} \to [0, w_{\tilde{a}}], \quad \mu_{\tilde{a}}(x) = \begin{cases} f_\mu^l(x), & a_1 \leq x < b_1 \\ w_{\tilde{a}}, & b_1 \leq x < c_1 \\ f_\mu^r(x), & c_1 \leq x \leq d_1 \\ 0, & otherwise \end{cases}$$

$$v_{\tilde{a}} : \mathbb{R} \to [u_{\tilde{a}}, 1], \quad v_{\tilde{a}}(x) = \begin{cases} f_v^l(x), & a_2 \leq x < b_2 \\ u_{\tilde{a}}, & b_2 \leq x < c_2 \\ f_v^r(x), & c_2 \leq x \leq d_2 \\ 1, & otherwise \end{cases}$$

$$\lambda_{\tilde{a}} : \mathbb{R} \to [y_{\tilde{a}}, 1], \quad \lambda_{\tilde{a}}(x) = \begin{cases} f_\lambda^l(x), & (a_3 \leq x < b_3) \\ y_{\tilde{a}}, & (b_3 \leq x < c_3) \\ f_\lambda^r(x), & (c_3 \leq x \leq d_3) \\ 1, & otherwise \end{cases}$$

where the functions $f_\mu^l : [a_1, b_1] \to [0, w_{\tilde{a}}]$, $f_v^r : [c_2, d_2] \to [u_{\tilde{a}}, 1]$ $f_\lambda^r : [c_3, d_3] \to [y_{\tilde{a}}, 1]$ are continuous and nondecreasing, and satisfy the conditions: $f_\mu^l(a_1) = 0, f_\mu^l(b_1) = w_{\tilde{a}}, f_v^r(c_2) = u_{\tilde{a}}$, $f_v^r(d_2) = 1$, $f_\lambda^r(c_3) = y_{\tilde{a}}$, and $f_\lambda^r(d_3) = 1$; the functions $f_\mu^r : [c_1, d_1] \to [0, w_{\tilde{a}}]$, $f_v^l : [a_2, b_2] \to [u_{\tilde{a}}, 1]$ and $f_\lambda^l : [a_3, b_3] \to [y_{\tilde{a}}, 1]$ are continuous and nonincreasing, and satisfy the conditions: $f_\mu^r(c_1) = w_{\tilde{a}}$, $f_\mu^r(d_1) = 0$, $f_v^l(a_2) = 1$, $f_v^l(b_2) = u_{\tilde{a}}$, $f_\lambda^l(a_3) = 1$ and $f_\lambda^l(b_3) = y_{\tilde{a}}$. $[b_1, c_1]$, a_1 and d_1 are called the mean interval and the lower and upper limits of the general neutrosophic number $\tilde{a}$ for the truth-membership function, respectively. $[b_2, c_2]$, a_2 and d_2 are called the mean interval and the lower and upper limits of the general neutrosophic number $\tilde{a}$ for the indeterminacy-membership function, respectively. $[b_3, c_3]$, a_3 and d_3 are called the mean interval and the lower and upper limits of the general neutrosophic number $\tilde{a}$ for the falsity-membership function, respectively. $w_{\tilde{a}}$, $u_{\tilde{a}}$ and $y_{\tilde{a}}$ are called the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree, respectively.

Example 2.5 Assume that $\tilde{a} = \langle ((1,3,5,7), 0.9), ((0,1,7,9), 0.1), ((0,3,5,9), 0.3) \rangle$ be a SVN-number. Then,

The meanings of $\tilde{a}$ is interpreted as follows: For example; the truth-membership degree of the element $3 \in \mathbb{R}$ belonging to $\tilde{a}$ is 0.9 whereas the indeterminacy-membership degree is 0.1 and falsity-membership degree is 0.3 i.e., $\mu_{\tilde{a}}(3) = 0.9$, $v_{\tilde{a}}(3) = 0.1$, $\lambda_{\tilde{a}}(3) = 0.3$.

Definition 2.6 [13] A single valued trapezoidal neutrosophic number (SVTN-number)

$$\tilde{a} = \langle (a, b, c, d); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$$

is a special neutrosophic set on the real number set $\mathbb{R}$, whose truth-membership, indeterminacy-membership, and a falsity-membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a)w_{\tilde{a}}/(b-a) & (a \leq x < b) \\ w_{\tilde{a}} & (b \leq x \leq c) \\ (d-x)w_{\tilde{a}}/(d-c) & (c < x \leq d) \\ 0 & otherwise \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} (b-x+u_{\tilde{a}}(x-a))/(b-a) & (a \leq x < b) \\ u_{\tilde{a}} & (b \leq x \leq c) \\ (x-c+u_{\tilde{a}}(d-x))/(d-c) & (c < x \leq d) \\ 0 & otherwise \end{cases}$$

and

$$\lambda_{\tilde{a}}(x) = \begin{cases} b-x+y_{\tilde{a}}(x-a))/(b-a) & (a \leq x < b) \\ y_{\tilde{a}} & (b \leq x \leq c) \\ (x-c+y_{\tilde{a}}(d-x))/(d-c) & (c < x \leq d) \\ 0 & otherwise \end{cases}$$

respectively.

Definition 2.7 [13] A single valued triangular neutrosophic number (SVTrN-number)

$$\tilde{a} = \langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$$

is a special neutrosophic set on the set of real numbers $\mathbb{R}$, whose truth-membership, indeterminacy-membership and falsity-membership functions are respectively defined by

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a)w_{\tilde{a}}/(b-a), & (a \leq x < b) \\ (c-x)w_{\tilde{a}}/(c-b), & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} (b-x+u_{\tilde{a}}(x-a))/(b-a) & (a \leq x < b), \\ (x-b+u_{\tilde{a}}(c-x))/(c-b), & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b-x+y_{\tilde{a}}(x-a))/(b-a), & (a \leq x < b) \\ (x-b+y_{\tilde{a}}(c-x))/(c-b), & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

If $a \geq 0$ and at least $c > 0$ then $\tilde{a} = \langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a positive SVTrN-number, denoted by $\tilde{a} > 0$.

Likewise, if $c \le 0$ and at least $a < 0$, then $\tilde{a} = \langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a negative SVTrN-number, denoted by $\tilde{a} < 0$. A SVTrN-number $\tilde{a} = \langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ may express an ill-known quantity about a, which is approximately equal to a.

Definition 2.8 [13] Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ be two SVTN-numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

2. $$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (a_1 d_2, b_1 c_2, c_1 b_2, d_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1 d_2, c_1 c_2, b_1 b_2, a_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$$

3. $$\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$$

Definition 2.9 [13] Let $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ be two SVTrN-numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

2. $$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (a_1 c_2, b_1 b_2, c_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1 c_2, b_1 b_2, a_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$$

3. $$\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$$

3 Concepts of values and ambiguities for SVN-numbers

In this section, we first define the concept of cut (or level) sets, values, ambiguities, weighted values and weighted ambiguities of SVN-numbers and give some desired properties. Also we developed a ranking method of SVN-numbers. In the following, some definitions and operations on intuitionistic sets defined in [23–25, 37, 48], we extend these definitions and operations to single valued neutrosophic sets [36].

Definition 3.1 Let $\tilde{a} = \langle ((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}}) \rangle$ be a SVN-number. Then, $\langle \alpha, \beta, \gamma \rangle$-cut set of the SVN-number $\tilde{a}$, denoted by $\tilde{a}_{\langle \alpha, \beta, \gamma \rangle}$, is defined as;

$$\tilde{a}_{\langle \alpha, \beta, \gamma \rangle} = \{x \mid \mu_{\tilde{a}}(x) \ge \alpha,\ \nu_{\tilde{a}}(x) \le \beta,\ \lambda_{\tilde{a}}(x) \le \gamma,\ x \in R\}.$$

which satisfies the conditions as follows: $0 \le \alpha \le w_{\tilde{a}}$, $u_{\tilde{a}} \le \beta \le 1$, $y_{\tilde{a}} \le \gamma \le 1$ and $0 \le \alpha + \beta + \gamma \le 3$.

Clearly, any $\langle \alpha, \beta, \gamma \rangle$-cut set $\tilde{a}_{\langle \alpha, \beta, \gamma \rangle}$ of a SVN-number $\tilde{a}$ is a crisp subset of the real number set R.

Definition 3.2 Let $\tilde{a} = \langle ((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}}) \rangle$ be a SVN-number. Then, α-cut set of the SVN-number $\tilde{a}$, denoted by $\tilde{a}_\alpha$, is defined as;

$$\tilde{a}_\alpha = \{x \mid \mu_{\tilde{a}}(x) \ge \alpha,\ x \in R\}.$$

where $\alpha \in [0, w_{\tilde{a}}]$.

Clearly, any α-cut set of a SVN-number $\tilde{a}$ is a crisp subset of the real number set R.

In here, any α-cut set of a SVN-number $\tilde{a}$ for truth-membership function is a closed interval, denoted by $\tilde{a}_\alpha = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)]$.

Definition 3.3 Let $\tilde{a} = \langle ((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}}) \rangle$ be a SVN-number. Then, β-cut set of the SVN-number $\tilde{a}$, denoted by $\tilde{a}^\beta$, is defined as;

$$\tilde{a}^\beta = \{x \mid \nu_{\tilde{a}}(x) \le \beta,\ x \in R\}$$

where $\beta \in [u_{\tilde{a}}, 1]$.

Clearly, any β-cut set of a SVN-number $\tilde{a}$ is a crisp subset of the real number set R.

In here, any β-cut set of a SVN-number $\tilde{a}$ for indeterminacy-membership function is a closed interval, denoted by $\tilde{a}^\beta = [L'_{\tilde{a}}(\beta), R'_{\tilde{a}}(\beta)]$.

Definition 3.4 Let $\tilde{a} = \langle ((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}}) \rangle$ be a SVN-number. Then, γ-cut set of the SVN-number $\tilde{a}$, denoted by ${}^\gamma\tilde{a}$, is defined as;

$${}^\gamma\tilde{a} = \{x \mid \lambda_{\tilde{a}}(x) \le \gamma,\ x \in R\}$$

where $\gamma \in [y_{\tilde{a}}, 1]$.

Clearly, any γ-cut set of a SVN-number $\tilde{a}$ is a crisp subset of the real number set R.

In here, any γ-cut set of a SVN-number $\tilde{a}$ for falsity-membership function is a closed interval, denoted by ${}^\gamma\tilde{a} = [L''_{\tilde{a}}(\gamma), R''_{\tilde{a}}(\gamma)]$.

Theorem 3.5 *Let* $\tilde{a} = \langle ((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}}) \rangle$ *be a SVN-number. Then,* $\tilde{a}_{\langle \alpha, \beta, \gamma \rangle} = \tilde{a}_\alpha \cap \tilde{a}^\beta \cap {}^\gamma\tilde{a}$ *is hold.*

Proof Its trivial.

Definition 3.6 Let $\tilde{a} = \langle((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}})\rangle$ be a SVN-number and $\tilde{a}_\alpha = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)]$, $\tilde{a}^\beta = [L'_{\tilde{a}}(\beta), R'_{\tilde{a}}(\beta)]$ and ${}^\gamma\tilde{a} = [L''_{\tilde{a}}(\gamma), R''_{\tilde{a}}(\gamma)]$, be any α-cut set, β-cut set and γ-cut set of the SVN-number $\tilde{a}$, respectively. Then,

1. The values of the SVN-number $\tilde{a}$ for α-cut set, denoted by $V_\mu(\tilde{a})$, is defined as;
$$V_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} (L_{\tilde{a}}(\alpha) + R_{\tilde{a}}(\alpha)) f(\alpha) d\alpha \tag{1}$$
where $f(\alpha) \in [0, 1]$ $(\alpha \in [0, w_{\tilde{a}}])$, $f(0) = 0$ and $f(\alpha)$ is monotonic and nondecreasing of $\alpha \in [0, w_{\tilde{a}}]$.
2. The values of the SVN-number $\tilde{a}$ for β-cut set, denoted by $V_\nu(\tilde{a})$, is defined as;
$$V_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^1 (L'_{\tilde{a}}(\beta) + R'_{\tilde{a}}(\beta)) g(\beta) d\beta \tag{2}$$
where $g(\beta) \in [0, 1]$ $(\beta \in [u_{\tilde{a}}, 1])$, $g(1) = 0$ and $g(\beta)$ is monotonic and nonincreasing of $\beta \in [u_{\tilde{a}}, 1]$.
3. The values of the SVN-number $\tilde{a}$ for γ-cut set, denoted by $V_\lambda(\tilde{a})$, is defined as;
$$V_\lambda(\tilde{a}) = \int_{y_{\tilde{a}}}^1 (L''_{\tilde{a}}(\gamma) + R''_{\tilde{a}}(\gamma)) h(\gamma) d\gamma \tag{3}$$
where $h(\gamma) \in [0, 1]$ $(\gamma \in [y_{\tilde{a}}, 1])$, $h(1) = 0$ and $h(\gamma)$ is monotonic and nonincreasing of $\gamma \in [y_{\tilde{a}}, 1]$.

From now on, without loss of generality, we use $f(\alpha) = \alpha$ $(\alpha \in [0, w_{\tilde{a}}])$, $g(\beta) = 1 - \beta$ $(\beta \in [u_{\tilde{a}}, 1])$ and $h(\gamma) = 1 - \gamma$ $(\gamma \in [y_{\tilde{a}}, 1])$ for f, g and h, respectively.

Definition 3.7 Let $\tilde{a} = \langle((a_1, b_1, c_1, d_1), w_{\tilde{a}}), ((a_2, b_2, c_2, d_2), u_{\tilde{a}}), ((a_3, b_3, c_3, d_3), y_{\tilde{a}})\rangle$ be a SVN-number and $\tilde{a}_\alpha = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)]$, $\tilde{a}^\beta = [L'_{\tilde{a}}(\beta), R'_{\tilde{a}}(\beta)]$ and ${}^\gamma\tilde{a} = [L''_{\tilde{a}}(\gamma), R''_{\tilde{a}}(\gamma)]$, be any α-cut set, β-cut set and γ-cut set of the SVN-number $\tilde{a}$, respectively. Then,

1. The ambiguities of the SVN-number $\tilde{a}$ for α-cut set, denoted by $A_\mu(\tilde{a})$, is defined as;
$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} (R_{\tilde{a}}(\alpha) - L_{\tilde{a}}(\alpha)) f(\alpha) d\alpha$$
where $f(\alpha) \in [0, 1]$ $(\alpha \in [0, w_{\tilde{a}}])$.
2. The ambiguities of the SVN-number $\tilde{a}$ for β-cut set, denoted by $A_\nu(\tilde{a})$, is defined as;
$$A_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^1 (R'_{\tilde{a}}(\beta) - L'_{\tilde{a}}(\beta)) g(\beta) d\beta,$$
where $g(\beta) \in [0, 1]$ $(\beta \in [u_{\tilde{a}}, 1])$, $g(1) = 0$ and $g(\beta)$ is monotonic and nonincreasing of $\beta \in [u_{\tilde{a}}, 1]$.
3. The ambiguities of the SVN-number $\tilde{a}$ for γ-cut set, denoted by $A_\lambda(\tilde{a})$, is defined as;
$$A_\lambda(\tilde{a}) = \int_{y_{\tilde{a}}}^1 (R''_{\tilde{a}}(\gamma) - L''_{\tilde{a}}(\gamma)) h(\gamma) d\gamma,$$
where $h(\beta) \in [0, 1]$ $(\gamma \in [y_{\tilde{a}}, 1])$, $h(1) = 0$ and $h(\gamma)$ is monotonic and nonincreasing of $\gamma \in [y_{\tilde{a}}, 1]$.

In here, $A_\mu(\tilde{a})$, $A_\nu(\tilde{a})$ and $A_\lambda(\tilde{a})$ basically measure how much there is vagueness in the SVN-number $\tilde{a}$.

Corollary 3.8 *Let $\tilde{a} = \langle(a, b, c, d); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is an arbitrary SVTN-number. Then,*

1. *α-cut set of the SVTN-number $\tilde{a}$ for truth-membership is calculated as;*
$$\tilde{a}_\alpha = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)] = \left[\frac{(w_{\tilde{a}} - \alpha)a + \alpha b}{w_{\tilde{a}}}, \frac{(w_{\tilde{a}} - \alpha)d + \alpha c}{w_{\tilde{a}}}\right]$$
where $\alpha \in [0, w_{\tilde{a}}]$. If $f(\alpha) = \alpha$, we can obtain the value and ambiguity of the SVTN-number $\tilde{a}$, respectively, as;
$$\begin{aligned} V_\mu(\tilde{a}) &= \int_0^{w_{\tilde{a}}} \left[(a + d) + \frac{(b + c - a - d)\alpha}{w_{\tilde{a}}}\right] \alpha d\alpha \\ &= \left[\frac{(a + d)\alpha^2}{2} + \frac{(b + c - a - d)\alpha^3}{3w_{\tilde{a}}}\right]\Bigg|_0^{w_{\tilde{a}}} \\ &= \frac{(a + 2b + 2c + d)w_{\tilde{a}}^2}{6} \end{aligned}$$
and
$$\begin{aligned} rlA_\mu(\tilde{a}) &= \int_0^{w_{\tilde{a}}} \left[(d - a) - \frac{(d - a + b - c)\alpha}{w_{\tilde{a}}}\right] \alpha d\alpha \\ &= \left[\frac{(d - a)\alpha^2}{2} - \frac{(d - a + b - c)\alpha^3}{3w_{\tilde{a}}}\right]\Bigg|_0^{w_{\tilde{a}}} \\ &= \frac{(d - a + 2c - 2b)w_{\tilde{a}}^2}{6} \end{aligned}$$
2. *β-cut set of the SVTN-number $\tilde{a}$ for indeterminacy-membership is calculated as;*
$$\begin{aligned} \tilde{a}^\beta &= [L'_{\tilde{a}}(\beta), R'_{\tilde{a}}(\beta)] \\ &= \left[\frac{(1 - \beta)b + (\beta - u_{\tilde{a}})a}{1 - u_{\tilde{a}}}, \frac{(1 - \beta)c + (\beta - u_{\tilde{a}})d}{1 - u_{\tilde{a}}}\right]. \end{aligned}$$
where $\beta \in [u_{\tilde{a}}, 1]$. If $g(\beta) = 1 - \beta$, we can obtain the value and ambiguity of the SVTN-number $\tilde{a}$, respectively, as;
$$\begin{aligned} V_\nu(\tilde{a}) &= \int_{u_{\tilde{a}}}^1 \left[(a + d) + \frac{(b + c - a - d)(1 - \beta)}{1 - u_{\tilde{a}}}\right] (1 - \beta) d\beta \\ &= \left[-\frac{(a + d)(1 - \beta)^2}{2} - \frac{(b + c - a - d)(1 - \beta)^3}{3(1 - u_{\tilde{a}})}\right]\Bigg|_{u_{\tilde{a}}}^1 \\ &= \frac{(a + 2b + 2c + d)(1 - u_{\tilde{a}})^2}{6} \end{aligned}$$

and

$$A_\nu(\tilde a)=\int_{u_{\tilde a}}^{1}\left[(d-a)-\frac{(d-a+b-c)(1-\beta)}{1-u_{\tilde a}}\right](1-\beta)d\beta$$
$$=\left[-\frac{(d-a)(1-\beta)^2}{2}-\frac{(d-a+b-c)(1-\beta)^3}{3(1-u_{\tilde a})}\right]\Bigg|_{u_{\tilde a}}^{1}$$
$$=\frac{(d-a+2c-2b)(1-u_{\tilde a})^2}{6}$$

3. *γ-cut set of the SVTN-number $\tilde a$ for falsity-membership is calculated as;*

$$^{\gamma}\tilde a=[L''_{\tilde a}(\gamma),R''_{\tilde a}(\gamma)]$$
$$=\left[\frac{(1-\gamma)b+(\gamma-y_{\tilde a})a}{1-y_{\tilde a}},\frac{(1-\gamma)c+(\gamma-y_{\tilde a})d}{1-y_{\tilde a}}\right]$$

where $\gamma\in[y_{\tilde a},1]$. If $h(\gamma)=1-\gamma$, we can obtain the value and ambiguity of the SVTN-number $\tilde a$, respectively, as;

$$V_\lambda(\tilde a)=\int_{y_{\tilde a}}^{1}\left[(a+d)+\frac{(b+c-a-d)(1-\gamma)}{1-y_{\tilde a}}\right](1-\gamma)d\gamma$$
$$=\left[-\frac{(a+d)(1-\gamma)^2}{2}-\frac{(b+c-a-d)(1-\gamma)^3}{3(1-y_{\tilde a})}\right]\Bigg|_{y_{\tilde a}}^{1}$$
$$=\frac{(a+2b+2c+d)(1-y_{\tilde a})^2}{6}$$

and

$$A_\lambda(\tilde a)=\int_{y_{\tilde a}}^{1}\left[(d-a)-\frac{(d-a+b-c)(1-\gamma)}{1-u_{\tilde a}}\right](1-\gamma)d\gamma$$
$$=\left[-\frac{(d-a)(1-\gamma)^2}{2}-\frac{(d-a+b-c)(1-\gamma)^3}{3(1-y_{\tilde a})}\right]\Bigg|_{y_{\tilde a}}^{1}$$
$$=\frac{(d-a+2c-2b)(1-y_{\tilde a})^2}{6}$$

Corollary 3.9 *Let $\tilde a=\langle(a,b,c);w_{\tilde a},u_{\tilde a},y_{\tilde a}\rangle$ is an arbitrary SVTrN-number. Then,*

1. *α-cut set of the SVTrN-number $\tilde a$ for truth-membership is calculated as;*

$$[L_{\tilde a}(\alpha),R_{\tilde a}(\alpha)]=\left[\frac{(w_{\tilde a}-\alpha)a+\alpha b}{w_{\tilde a}},\frac{(w_{\tilde a}-\alpha)c+\alpha b}{w_{\tilde a}}\right]$$

If $f(\alpha)=\alpha$, we can obtain the value and ambiguity of the SVTrN-number $\tilde a$, respectively, as;

$$V_\mu(\tilde a)=\int_0^{w_{\tilde a}}\left[(a+c)+\frac{(2b-a-c)\alpha}{w_{\tilde a}}\right]\alpha d\alpha$$
$$=\left[\frac{(a+c)\alpha^2}{2}+\frac{(2b-a-c)\alpha^3}{3w_{\tilde a}}\right]\Bigg|_0^{w_{\tilde a}}$$
$$=\frac{(a+4b+c)w_{\tilde a}^2}{6}$$

and

$$A_\mu(\tilde a)=\int_0^{w_{\tilde a}}\left[(c-a)-\frac{(c-a)\alpha}{w_{\tilde a}}\right]\alpha d\alpha$$
$$=\left[\frac{(c-a)\alpha^2}{2}-\frac{(c-a)\alpha^3}{3w_{\tilde a}}\right]\Bigg|_0^{w_{\tilde a}}$$
$$=\frac{(c-a)w_{\tilde a}^2}{6}$$

2. *β-cut set of the SVTrN-number $\tilde a$ for indeterminacy-membership is calculated as;*

$$[L'_{\tilde a}(\beta),R'_{\tilde a}(\beta)]$$
$$=\left[\frac{(1-\beta)b+(\beta-u_{\tilde a})a}{1-u_{\tilde a}},\frac{(1-\beta)b+(\beta-u_{\tilde a})c}{1-u_{\tilde a}}\right]$$

If $g(\beta)=1-\beta$, we can obtain the value and ambiguity of the SVTrN-number $\tilde a$, respectively, as;

$$V_\nu(\tilde a)=\int_{u_{\tilde a}}^{1}\left[(a+c)+\frac{(2b-a-c)(1-\beta)}{1-u_{\tilde a}}\right](1-\beta)d\beta$$
$$=\left[-\frac{(a+c)(1-\beta)^2}{2}+\frac{(2b-a-c)(1-\beta)^3}{3(1-u_{\tilde a})}\right]\Bigg|_{u_{\tilde a}}^{1}$$
$$=\frac{(a+4b+c)(1-u_{\tilde a})^2}{6}$$

and

$$A_\nu(\tilde a)=\int_{u_{\tilde a}}^{1}\left[(c-a)-\frac{(c-a)(1-\beta)}{1-u_{\tilde a}}\right](1-\beta)d\beta$$
$$=\left[-\frac{(c-a)(1-\beta)^2}{2}+\frac{(c-a)(1-\beta)^3}{3(1-u_{\tilde a})}\right]\Bigg|_{u_{\tilde a}}^{1}$$
$$=\frac{(c-a)(1-u_{\tilde a})^2}{6}$$

3. *γ-cut set of the SVTrN-number $\tilde a$ for falsity-membership is calculated as;*

$$[L''_{\tilde a}(\gamma),R''_{\tilde a}(\gamma)]$$
$$=\left[\frac{(1-\gamma)b+(\gamma-y_{\tilde a})a}{1-y_{\tilde a}},\frac{(1-\gamma)b+(\gamma-y_{\tilde a})c}{1-y_{\tilde a}}\right].$$

where $\gamma\in[y_{\tilde a},1]$. If $h(\gamma)=1-\gamma$, we can obtain the value and ambiguity of the SVTrN-number $\tilde a$, respectively, as;

$$V_\lambda(\tilde a)=\int_{y_{\tilde a}}^{1}\left[(a+c)+\frac{(2b-a-c)(1-\gamma)}{1-y_{\tilde a}}\right](1-\gamma)d\gamma$$
$$=\left[-\frac{(a+c)(1-\gamma)^2}{2}-\frac{(2b-a-c)(1-\gamma)^3}{3(1-y_{\tilde a})}\right]\Bigg|_{y_{\tilde a}}^{1}$$
$$=\frac{(a+4a+c)(1-y_{\tilde a})^2}{6}$$

and

$$\begin{aligned}A_\lambda(\tilde{a}) &= \int_{y_{\tilde{a}}}^{1}\left[(c-a)-\frac{(c-a)(1-\gamma)}{1-y_{\tilde{a}}}\right](1-\gamma)d\gamma\\ &= \left[-\frac{(c-a)(1-\gamma)^2}{2}-\frac{(c-a)(1-\gamma)^3}{3(1-y_{\tilde{a}})}\right]\Bigg|_{y_{\tilde{a}}}^{1}\\ &= \frac{(c-a)(1-y_{\tilde{a}})^2}{6}\end{aligned}$$

Definition 3.10 Let $\tilde{a}=\langle(a,b,c);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ be a SVN-number. Then, for $\theta\in[0,1]$,

1. The θ-weighted value of the SVN-number $\tilde{a}$ are defined as;

$$V_\theta(\tilde{a})=\theta V_\mu(\tilde{a})+(1-\theta)V_\nu(\tilde{a})+(1-\theta)V_\lambda(\tilde{a})$$

2. The θ-weighted ambiguity of the SVN-number $\tilde{a}$ are defined as;

$$A_\theta(\tilde{a})=\theta A_\mu(\tilde{a})+(1-\theta)A_\nu(\tilde{a})+(1-\theta)A_\lambda(\tilde{a})$$

Corollary 3.11 *Let $\tilde{a}=\langle(a,b,c,d);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is an arbitrary SVTN-number. Then,*

1. *The weighted value $V_\theta(\tilde{a})$ is calculated as;*

$$\begin{aligned}V_\theta(\tilde{a})=&\frac{a+2b+2c+d}{6}\\ &\times\left[\theta w_{\tilde{a}}^2+(1-\theta)(1-u_{\tilde{a}})^2+(1-\theta)(1-y_{\tilde{a}})^2\right]\end{aligned}$$

2. *The weighted ambiguity $A_\theta(\tilde{a})$ is calculated as;*

$$\begin{aligned}A_\theta(\tilde{a})=&\frac{d-a+2c-2b}{6}\\ &\times\left[\theta w_{\tilde{a}}^2+(1-\theta)(1-u_{\tilde{a}})^2+(1-\theta)(1-y_{\tilde{a}})^2\right]\end{aligned}$$

Corollary 3.12 *Let $\tilde{a}=\langle(a,b,c);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is an arbitrary SVTrN-number. Then,*

1. *The weighted value $V_\theta(\tilde{a})$ is calculated as;*

$$\begin{aligned}V_\theta(\tilde{a})=&\frac{a+4b+c}{6}\\ &\times\left[\theta w_{\tilde{a}}^2+(1-\theta)(1-u_{\tilde{a}})^2+(1-\theta)(1-y_{\tilde{a}})^2\right]\end{aligned}$$

2. *The weighted ambiguity $A_\theta(\tilde{a})$ is calculated as;*

$$\begin{aligned}A_\theta(\tilde{a})=\frac{c-a}{6}\Big[&\theta w_{\tilde{a}}^2+(1-\theta)(1-u_{\tilde{a}})^2\\ &+(1-\theta)(1-y_{\tilde{a}})^2\Big]\end{aligned}$$

No we give a ranking method of SVN-numbers based on the weighted value and ambiguity can be developed as follows;

Definition 3.13 Let $\tilde{a}$ and $\tilde{b}$ be two SVN-number and $\theta\in[0,1]$. For weighted values and ambiguities of the SVN-numbers $\tilde{a}$ and $\tilde{b}$, the ranking order of $\tilde{a}$ and $\tilde{b}$ is defined as;

1. If $V_\theta(\tilde{a})>V_\theta(\tilde{b})$, then $\tilde{a}$ is bigger than $\tilde{b}$, denoted by $\tilde{a}\tilde{<}\tilde{b}$;
2. If $V_\theta(\tilde{a})>V_\theta(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}\tilde{<}\tilde{b}$;
3. If $V_\theta(\tilde{a})=V_\theta(\tilde{b})$, then
   (a) If $A_\theta(\tilde{a})=A_\theta(\tilde{b})$, then $\tilde{a}$ is equal to $\tilde{b}$, denoted by $\tilde{a}\tilde{=}\tilde{b}$;
   (b) If $A_\theta(\tilde{a})>A_\theta(\tilde{b})$, then $\tilde{a}$ is bigger than $\tilde{b}$, denoted by $\tilde{a}\tilde{>}\tilde{b}$ and
   (c) If $A_\theta(\tilde{a})<A_\theta(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}\tilde{<}\tilde{b}$.

Example 3.14 Let $\tilde{a}=\langle(2,4,6);0.8,0.7,0.6\rangle$ and $\tilde{b}=\langle(2,6,8);0.4,0.6,0.5\rangle$ be two SVTrN-numbers. Then, we can compare the SVTrN-numbers $\tilde{a}$ and $\tilde{b}$ as;

Solving We firstly can obtain the weighted value and ambiguity of the SVTrN-number $\tilde{a}$ respectively, as;

$$\begin{aligned}V_\theta(\tilde{a})&=\frac{2+4\times4+6}{6}\Big[\theta(0.8)^2+(1-\theta)(1-0.7)^2\\ &\quad+(1-\theta)(1-0.6)^2\Big]\\ &=4(0.25-0.39\theta)\\ &=1-1.560\theta\end{aligned}$$

and

$$\begin{aligned}A_\theta(\tilde{a})&=\frac{6-2}{6}\Big[\theta(0.8)^2+(1-\theta)(1-0.7)^2\\ &\quad+(1-\theta)(1-0.6)^2\Big]\\ &=\frac{2}{3}(0.25-0.39\theta)\\ &=0.166-0.26\theta\end{aligned}$$

Similarly, we can obtain the weighted value and ambiguity of the SVTrN-number $\tilde{b}$ respectively, as;

$$\begin{aligned}V_\theta(\tilde{b})&=\frac{2+4\times6+8}{6}\Big[\theta(0.4)^2+(1-\theta)(1-0.6)^2\\ &\quad+(1-\theta)(1-0.5)^2\Big]\\ &=\frac{34}{6}(0.09-0.25\theta)\\ &=0.51-1.416\theta\end{aligned}$$

$$A_\theta(\tilde{b}) = \frac{8-2}{6} \times \left[\theta(0.4)^2 + (1-\theta)(1-0.6)^2 + (1-\theta)(1-0.5)^2\right] = 0.09 - 0.25\theta$$

Then, we have

$$V_\theta(\tilde{a}) - V_\theta(\tilde{b}) = (1 - 1.560\theta) - (0.51 - 1.416\theta) = 0.49 - 0.144\theta,$$

Therefore, it is clearly that $0.346 \leq 0.49 - 0.144\theta \leq 0.49$, (for $0 \leq \theta \leq 1$) which directly implies that $V_\theta(\tilde{a}) > V_\theta(\tilde{b})$ for any $\theta \in [0, 1]$. Hence, it easily the ranking order of the SVTrN-numbers $\tilde{a}$ and $\tilde{b}$ is $\tilde{a} \tilde{>} \tilde{b}$.

Theorem 3.15 *Suppose that* $\tilde{a} = \langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$, $\tilde{b} = \langle(a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}}\rangle$ *are two SVTrN-numbers with* $w_{\tilde{a}} = w_{\tilde{b}}$, $u_{\tilde{a}} = u_{\tilde{b}}$ *and* $y_{\tilde{a}} = y_{\tilde{b}}$. *If* $a_1 > c_2$, *then* $\tilde{a} \tilde{>} \tilde{b}$.

Proof It is easily derived from Eq. (1) that

$$V_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} (L_{\tilde{a}}(\alpha) + R_{\tilde{a}}(\alpha))f(\alpha)d\alpha \geq \int_0^{w_{\tilde{a}}} 2a_1 f(\alpha)d\alpha = 2a_1 \int_0^{w_{\tilde{a}}} f(\alpha)d\alpha$$

and

$$V_\mu(\tilde{b}) = \int_0^{w_{\tilde{b}}} (L_{\tilde{b}}(\alpha) + R_{\tilde{b}}(\alpha))f(\alpha)d\alpha \leq \int_0^{w_{\tilde{b}}} 2c_2 f(\alpha)d\alpha = 2c_2 \int_0^{w_{\tilde{b}}} f(\alpha)d\alpha$$

Noticing that the assumption condition: $w_{\tilde{a}} = w_{\tilde{b}}$, we have

$$\int_0^{w_{\tilde{a}}} f(\alpha)d\alpha = \int_0^{w_{\tilde{b}}} f(\alpha)d\alpha$$

Combining with the assumption condition: $a_1 > c_2$, we can prove that $V_\mu(\tilde{a}) > V_\mu(\tilde{b})$.

It easily follows from Eq. (2) that

$$V_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^1 (L'_{\tilde{a}}(\beta) + R'_{\tilde{a}}(\beta))g(\beta)d\beta, \geq \int_{u_{\tilde{a}}}^1 2a_1 g(\beta)d\beta = 2a_1 \int_{u_{\tilde{a}}}^1 g(\beta)d\beta$$

and

$$V_\nu(\tilde{b}) = \int_{u_{\tilde{b}}}^1 (L'_{\tilde{b}}(\beta) + R'_{\tilde{b}}(\beta))g(\beta)d\beta, \leq \int_{u_{\tilde{b}}}^1 2c_2 g(\beta)d\beta = 2c_2 \int_{u_{\tilde{b}}}^1 g(\beta)d\beta$$

Due to the assumption condition: $u_{\tilde{a}} = u_{\tilde{b}}$, it directly follows that

$$\int_{u_{\tilde{a}}}^1 g(\beta)d\beta = \int_{u_{\tilde{b}}}^1 g(\beta)d\beta$$

Combining with the assumption condition: $a_1 > c_2$, we have: $V_\nu(\tilde{a}) > V_\nu(\tilde{b})$.

Likewise, it easily follows from Eq. (3) that

$$V_\lambda(\tilde{a}) = \int_{y_{\tilde{a}}}^1 (L''_{\tilde{a}}(\gamma) + R''_{\tilde{a}}(\gamma))h(\gamma)d\gamma, \geq \int_{y_{\tilde{a}}}^1 2a_1 h(\gamma)d\gamma = 2a_1 \int_{y_{\tilde{a}}}^1 h(\gamma)d\gamma$$

and

$$V_\lambda(\tilde{b}) = \int_{y_{\tilde{b}}}^1 (L''_{\tilde{b}}(\gamma) + R''_{\tilde{b}}(\gamma))h(\gamma)d\gamma, \leq \int_{y_{\tilde{b}}}^1 2c_2 h(\gamma)d\gamma = 2c_2 \int_{y_{\tilde{b}}}^1 h(\gamma)d\gamma$$

Due to the assumption condition: $y_{\tilde{a}} = y_{\tilde{b}}$, it directly follows that

$$\int_{y_{\tilde{a}}}^1 h(\gamma)d\gamma = \int_{y_{\tilde{b}}}^1 h(\gamma)d\gamma$$

Combining with the assumption condition: $a_1 > c_2$, we have: $V_\lambda(\tilde{a}) > V_\lambda(\tilde{b})$.

According to Definition 3.10, for any $\theta \in [0, 1]$, we have

$$\theta V_\mu(\tilde{a}) + (1-\theta)V_\nu(\tilde{a}) + (1-\theta)V_\lambda(\tilde{a}) > \theta V_\mu(\tilde{b}) + (1-\theta)V_\nu(\tilde{b}) + (1-\theta)V_\lambda(\tilde{b})$$

i.e. $V_\theta(\tilde{a}) > V_\theta(\tilde{b})$. Therefore, it directly follows from the case (1) of Definition 3.13 that $\tilde{a} \tilde{>} \tilde{b}$.

Theorem 3.16 *Suppose that* $\tilde{a}$, $\tilde{b}$ *and* $\tilde{c}$ *are any SVN-numbers, where* $w_{\tilde{a}} = w_{\tilde{b}}$, $u_{\tilde{a}} = u_{\tilde{b}}$ *and* $y_{\tilde{a}} = y_{\tilde{b}}$. *If* $\tilde{a} \tilde{>} \tilde{b}$, *then* $\tilde{a} + \tilde{c} \tilde{>} \tilde{b} + \tilde{c}$.

Proof According to Eq. (1), we have

$$
\begin{aligned}
&V_{\mu}(\tilde{a}+\tilde{c})\\
&=\int_0^{w_{\tilde{a}}\wedge w_{\tilde{c}}}\Big[\big(L_{\tilde{a}}(\alpha)+R_{\tilde{a}}(\alpha)\big)+\big(L_{\tilde{c}}(\alpha)+R_{\tilde{c}}(\alpha)\big)\Big]f(\alpha)d\alpha\\
&=\int_0^{w_{\tilde{a}}\wedge w_{\tilde{c}}}\big(L_{\tilde{a}}(\alpha)+R_{\tilde{a}}(\alpha)\big)f(\alpha)d\alpha\\
&\quad+\int_0^{w_{\tilde{a}}\wedge w_{\tilde{c}}}\big(L_{\tilde{c}}(\alpha)+R_{\tilde{c}}(\alpha)\big)f(\alpha)d\alpha
\end{aligned}
$$

and

$$
\begin{aligned}
&V_{\mu}(\tilde{b}+\tilde{c})\\
&=\int_0^{w_{\tilde{b}}\wedge w_{\tilde{c}}}\Big[\big(L_{\tilde{b}}(\alpha)+R_{\tilde{b}}(\alpha)\big)+\big(L_{\tilde{c}}(\alpha)+R_{\tilde{c}}(\alpha)\big)\Big]f(\alpha)d\alpha\\
&=\int_0^{w_{\tilde{b}}\wedge w_{\tilde{c}}}\big(L_{\tilde{b}}(\alpha)+R_{\tilde{b}}(\alpha)\big)f(\alpha)d\alpha\\
&\quad+\int_0^{w_{\tilde{b}}\wedge w_{\tilde{c}}}\big(L_{\tilde{c}}(\alpha)+R_{\tilde{c}}(\alpha)\big)f(\alpha)d\alpha
\end{aligned}
$$

where $w_{\tilde{c}}$ is the truth-membership of the SVN-number $\tilde{c}$. Noticing that the assumption conditions: $\tilde{a} \gtrsim \tilde{b}$ and $w_{\tilde{a}} = w_{\tilde{b}}$, we have

$$
\int_0^{w_{\tilde{a}}\wedge w_{\tilde{c}}}\big(L_{\tilde{a}}(\alpha)+R_{\tilde{a}}(\alpha)\big)f(\alpha)d\alpha >
\int_0^{w_{\tilde{b}}\wedge w_{\tilde{c}}}\big(L_{\tilde{b}}(\alpha)+R_{\tilde{b}}(\alpha)\big)f(\alpha)d\alpha
$$

we have

$$
V_{\mu}(\tilde{a}+\tilde{c}) > V_{\mu}(\tilde{b}+\tilde{c}) \tag{4}
$$

Likewise, it is derived from Eq. (2) that

$$
\begin{aligned}
&V_{\nu}(\tilde{a}+\tilde{c})\\
&=\int_{u_{\tilde{a}}\vee u_{\tilde{c}}}^{1}\Big[\big(L'_{\tilde{a}}(\beta)+R'_{\tilde{a}}(\beta)\big)+\big(L'_{\tilde{c}}(\beta)+R'_{\tilde{c}}(\beta)\big)\Big]g(\beta)d\beta,\\
&=\int_{u_{\tilde{a}}\vee u_{\tilde{c}}}^{1}\big(L'_{\tilde{a}}(\beta)+R'_{\tilde{a}}(\beta)\big)g(\beta)d\beta\\
&\quad+\int_{u_{\tilde{a}}\vee u_{\tilde{c}}}^{1}\big(L'_{\tilde{c}}(\beta)+R'_{\tilde{c}}(\beta)\big)g(\beta)d\beta
\end{aligned}
$$

and

$$
\begin{aligned}
&V_{\nu}(\tilde{b}+\tilde{c})\\
&=\int_{u_{\tilde{b}}\vee u_{\tilde{c}}}^{1}\Big[\big(L'_{\tilde{b}}(\beta)+R'_{\tilde{b}}(\beta)\big)+\big(L'_{\tilde{c}}(\beta)+R'_{\tilde{c}}(\beta)\big)\Big]g(\beta)d\beta,\\
&=\int_{u_{\tilde{b}}\vee u_{\tilde{c}}}^{1}\big(L'_{\tilde{b}}(\beta)+R'_{\tilde{b}}(\beta)\big)g(\beta)d\beta\\
&\quad+\int_{u_{\tilde{b}}\vee u_{\tilde{c}}}^{1}\big(L'_{\tilde{c}}(\beta)+R'_{\tilde{c}}(\beta)\big)g(\beta)d\beta
\end{aligned}
$$

where $u_{\tilde{c}}$ is the indeterminacy-membership of the SVN-number $\tilde{c}$. Noticing that the assumption conditions: $\tilde{a} \gtrsim \tilde{b}$ and $u_{\tilde{a}} = u_{\tilde{b}}$, we have

$$
\int_{u_{\tilde{a}}\vee u_{\tilde{c}}}^{1}\big(L'_{\tilde{a}}(\beta)+R'_{\tilde{a}}(\beta)\big)g(\beta)d\beta >
\int_{u_{\tilde{b}}\vee u_{\tilde{c}}}^{1}\big(L'_{\tilde{b}}(\beta)+R'_{\tilde{b}}(\beta)\big)g(\beta)d\beta
$$

Hereby, we have

$$
V_{\nu}(\tilde{a}+\tilde{c}) > V_{\nu}(\tilde{b}+\tilde{c}) \tag{5}
$$

Therefore, we have

$$
V_{\lambda}(\tilde{a}+\tilde{c}) > V_{\lambda}(\tilde{b}+\tilde{c})
$$

Likewise, it is derived from Eq. (3) that

$$
\begin{aligned}
&V_{\lambda}(\tilde{a}+\tilde{c})\\
&=\int_{y_{\tilde{a}}\vee y_{\tilde{c}}}^{1}\Big[\big(L''_{\tilde{a}}(\gamma)+R''_{\tilde{a}}(\gamma)\big)+\big(L''_{\tilde{c}}(\gamma)+R''_{\tilde{c}}(\gamma)\big)\Big]h(\gamma)d\gamma,\\
&=\int_{y_{\tilde{a}}\vee y_{\tilde{c}}}^{1}\big(L''_{\tilde{a}}(\gamma)+R''_{\tilde{a}}(\gamma)\big)h(\gamma)d\gamma\\
&\quad+\int_{y_{\tilde{a}}\vee y_{\tilde{c}}}^{1}\big(L''_{\tilde{c}}(\gamma)+R''_{\tilde{c}}(\gamma)\big)h(\gamma)d\gamma
\end{aligned}
$$

and

$$
\begin{aligned}
&V_{\lambda}(\tilde{b}+\tilde{c})\\
&=\int_{y_{\tilde{b}}\vee y_{\tilde{c}}}^{1}\Big[\big(L''_{\tilde{b}}(\gamma)+R''_{\tilde{b}}(\gamma)\big)+\big(L''_{\tilde{c}}(\gamma)+R''_{\tilde{c}}(\gamma)\big)\Big]h(\gamma)d\gamma,\\
&=\int_{y_{\tilde{b}}\vee y_{\tilde{c}}}^{1}\big(L''_{\tilde{b}}(\gamma)+R''_{\tilde{b}}(\gamma)\big)h(\gamma)d\gamma\\
&\quad+\int_{y_{\tilde{b}}\vee y_{\tilde{c}}}^{1}\big(L''_{\tilde{c}}(\gamma)+R''_{\tilde{c}}(\gamma)\big)h(\gamma)d\gamma
\end{aligned}
$$

where $y_{\tilde{c}}$ is the falsity-membership of the SVN-number $\tilde{c}$. Noticing that the assumption conditions: $\tilde{a} \gtrsim \tilde{b}$ and $y_{\tilde{a}} = y_{\tilde{b}}$, we have

$$
\int_{y_{\tilde{a}}\vee y_{\tilde{c}}}^{1}\big(L''_{\tilde{a}}(\gamma)+R''_{\tilde{a}}(\gamma)\big)h(\gamma)d\gamma >
\int_{y_{\tilde{b}}\vee y_{\tilde{c}}}^{1}\big(L''_{\tilde{b}}(\gamma)+R''_{\tilde{b}}(\gamma)\big)h(\gamma)d\gamma
$$

Hereby, we have

$$
V_{\lambda}(\tilde{a}+\tilde{c}) > V_{\lambda}(\tilde{b}+\tilde{c}) \tag{6}
$$

According to Definition 3.10, and combining with Eqs. (4), (5) and (6) the following inequality is always valid for any $\theta \in [0, 1]$:

$$\theta V_{\mu}(\tilde{a}+\tilde{c})+(1-\theta)V_{\nu}(\tilde{a}+\tilde{c})+(1-\theta)V_{\lambda}(\tilde{a}+\tilde{c})>\theta V_{\mu}(\tilde{b}+\tilde{c}) \\ +(1-\theta)V_{\nu}(\tilde{b}+\tilde{c})+(1-\theta)V_{\lambda}(\tilde{b}+\tilde{c})$$

i.e

$$V_{\theta}(\tilde{a}+\tilde{c})>V_{\theta}(\tilde{b}+\tilde{c})$$

Therefore, it is easy to see from the case (1) of Definition 3.13 that $\tilde{a}+\tilde{c}>\tilde{b}+\tilde{c}$.

4 A multi-attribute decision-making method with SVTrN-numbers

In this section, we define a multi-attribute decision making method, so called SVTrN-multi-attribute decision-making method. Its adopted from [23–25].

From now on we use the weight of each attribute u_i $(i \in I_m)$ is ω_i, which should satisfy the normalized conditions: $\omega_i \in [0,1]$ $(i \in I_m)$ and $\sum_{i=1}^{m}\omega_i = 1$.

Definition 4.1 Let $X=(x_1,x_2,\ldots,x_n)$ be a set of alternatives, $U=(u_1,u_2,\ldots,u_m)$ be the set of attributes and $[\tilde{A}_{ij}]=\langle(a_{ij},b_{ij},c_{ij});w_{\tilde{a}_{ij}},u_{\tilde{a}_{ij}},y_{\tilde{a}_{ij}}\rangle$(for $i\in I_m; j\in I_n$) be a SVTrN-numbers. Then,

$$[\tilde{A}_{ij}]_{m\times n}=\begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ u_m \end{array}\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \cdots & \tilde{A}_{1n} \\ \tilde{A}_{21} & \tilde{A}_{22} & \cdots & \tilde{A}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{A}_{m1} & \tilde{A}_{m2} & \cdots & \tilde{A}_{mn} \end{pmatrix}\end{array}$$

s called an SVTrN-multi-attribute decision making matrix of the decision maker.

Now, we can give an algorithm of the SVTrN multi-attribute decision-making method as follows;

Algorithm:

Step 1. Construct the decision-making matrix $A=(\tilde{A}_{ij})_{m\times n}$; for decision;

Step 2. Compute the normalized decision-making matrix $\mathbf{R}=(\tilde{r}_{ij})_{m\times n}$ of $\mathbf{A}$where

$$\tilde{r}_{ij}=\left\langle\left(\frac{a_{ij}}{\bar{a}^{+}},\frac{b_{ij}}{\bar{a}^{+}},\frac{c_{ij}}{\bar{a}^{+}}\right);w_{\tilde{a}_{ij}},u_{\tilde{a}_{ij}},y_{\tilde{a}_{ij}}\right\rangle \\ \times(j\in I_n;\ \ i\in I_m)$$

such that $\bar{a}^{+}=max\{c_{ij}\,|\,j\in I_n, i\in I_m\}$.

Step 3. Compute the $\mathbf{U}=(\tilde{\boldsymbol{u}}_{ij})_{m\times n}$ of $\mathbf{R}$,where $\tilde{\mathbf{u}}_{ij}=\omega_i\tilde{r}_{ij}$ $(i\in I_m;\ j\in I_n)$

Step 4. Calculate the comprehensive values $\tilde{S}_j$ as;

$$\tilde{S}_j=\sum_{i=1}^{m}\tilde{u}_{ij}\ (j\in I_n)$$

Step 5. Determine the nonincreasing order of $\tilde{S}_j(j\in I_n)$

Step 6. Rank the alternatives x_j according to $\tilde{S}_j(j\in I_n)$ and select the best alternative.

Since humans might feel more comfortable using words by means of linguistic labels or terms to articulate their preferences, the ratings of each alternative with respect to each attribute are given as linguistic variables characterized by SVTrN-numbers in the evaluation process.(see Table 1, also the values can be replace by experts).

Example 4.2 (Its adopted from [23, 24]) Suppose that a software company desires to hire a system analyst. After preliminary screening, three candidates (i.e., alternatives) x_1, x_2 and x_3 remain for further evaluation. The panel (or decision making committee) assesses the three candidates according to the five attributes (or criteria, factors), which are emotional steadiness u_1, oral communication skill u_2, personality u_3, past experience u_4 and self-confidence u_5, respectively. Also, the weight vector of the five attributes may be given as $\omega=(0.15,0.25,0.20,0.25,0.15)^T$.

The four possible candidates (or alternatives) are to be evaluated under the above five attributes by corresponding to linguistic values of SVTrN-numbers for linguistic terms (adapted from [13]), as shown in Table 1.

Step 1. Construct the decision-making matrix $A=(\tilde{A}_{ij})_{m\times n}$; for decision as;

$$\begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} \langle(4.6,5.5,8.6);0.4,0.7,0.2\rangle & \langle(6.2,7.6,8.2);0.4,0.1,0.3\rangle & \langle(5.5,6.2,7.3);0.8,0.1,0.2\rangle \\ \langle(5.8,6.9,8.5);0.6,0.2,0.3\rangle & \langle(7.1,7.7,8.3);0.5,0.2,0.4\rangle & \langle(4.7,6.9,8.5);0.7,0.2,0.6\rangle \\ \langle(5.3,6.7,9.9);0.3,0.5,0.2\rangle & \langle(6.2,8.9,9.1);0.6,0.3,0.5\rangle & \langle(7.1,8.5,8.9);0.5,0.2,0.7\rangle \\ \langle(4.4,5.9,7.2);0.7,0.2,0.3\rangle & \langle(6.3,7.5,8.9);0.7,0.4,0.6\rangle & \langle(6.6,8.8,10);0.6,0.2,0.2\rangle \\ \langle(6.5,6.9,8.5);0.6,0.8,0.1\rangle & \langle(7.5,7.9,8.5);0.8,0.5,0.4\rangle & \langle(5.3,7.3,8.7);0.7,0.2,0.8\rangle \end{pmatrix}\end{array}$$

Step 2. Compute the normalized decision-making matrix $\mathbf{R} = (\tilde{r}_{ij})_{m\times n}$ of $\mathbf{A}$ as;

$$
\begin{array}{c}
 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5
\end{array}
\begin{array}{c}
\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\
\begin{pmatrix}
\langle(0.46,0.55,0.86);0.4,0.7,0.2\rangle & \langle(0.62,0.76,0.82);0.4,0.1,0.3\rangle & \langle(0.55,0.62,0.73);0.8,0.1,0.2\rangle \\
\langle(0.58,0.69,0.85);0.6,0.2,0.3\rangle & \langle(0.71,0.77,0.83);0.5,0.2,0.4\rangle & \langle(0.47,0.69,0.85);0.7,0.2,0.6\rangle \\
\langle(0.53,0.67,0.99);0.3,0.5,0.2\rangle & \langle(0.62,0.89,0.91);0.6,0.3,0.5\rangle & \langle(0.71,0.85,0.89);0.5,0.2,0.7\rangle \\
\langle(0.44,0.59,0.72);0.7,0.2,0.3\rangle & \langle(0.63,0.75,0.89);0.7,0.4,0.6\rangle & \langle(0.66,0.88,1.00);0.6,0.2,0.2\rangle \\
\langle(0.65,0.69,0.85);0.6,0.8,0.1\rangle & \langle(0.67,0.88,0.96);0.8,0.5,0.4\rangle & \langle(0.53,0.73,0.87);0.7,0.2,0.8\rangle
\end{pmatrix}
\end{array}
$$

Step 3. Compute the $\mathbf{U} = (\tilde{\boldsymbol{u}}_{ij})_{m\times n}$ of $\mathbf{R}$,where $\tilde{\mathbf{u}}_{ij} = \omega_i\tilde{r}_{ij}\ \ i \in I_m;\ j \in I_n)$ as;

$$
\begin{array}{c}
 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5
\end{array}
\begin{array}{c}
\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\
\begin{pmatrix}
\langle(0.069,0.083,0.129);0.4,0.7,0.2\rangle & \langle(0.093,0.114,0.123);0.4,0.1,0.3\rangle & \langle(0.083,0.093,0.110);0.8,0.1,0.2\rangle \\
\langle(0.145,0.173,0.213);0.6,0.2,0.3\rangle & \langle(0.178,0.193,0.208);0.5,0.2,0.4\rangle & \langle(0.118,0.173,0.213);0.7,0.2,0.6\rangle \\
\langle(0.106,0.134,0.198);0.3,0.5,0.2\rangle & \langle(0.124,0.178,0.182);0.6,0.3,0.5\rangle & \langle(0.142,0.170,0.178);0.5,0.2,0.7\rangle \\
\langle(0.110,0.148,0.180);0.7,0.2,0.3\rangle & \langle(0.158,0.188,0.223);0.7,0.4,0.6\rangle & \langle(0.165,0.220,0.250);0.6,0.2,0.2\rangle \\
\langle(0.098,0.104,0.128);0.6,0.8,0.1\rangle & \langle(0.101,0.132,0.144);0.8,0.5,0.4\rangle & \langle(0.080,0.110,0.131);0.7,0.2,0.8\rangle
\end{pmatrix}
\end{array}
$$

Step 4. Calculate the comprehensive values $\tilde{S}_j$ for $j = 1, 2, 3$ as;

$$\tilde{S}_1 = \langle(0.528, 0.640, 0.847); 0.3, 0.8, 0.3\rangle$$

$$\tilde{S}_2 = \langle(0.653, 0.804, 0.879); 0.4, 0.5, 0.6\rangle$$

and

$$\tilde{S}_3 = \langle(0.587, 0.765, 0.881); 0.5, 0.2, 0.8\rangle$$

respectively.

Step 5. Determine the nonincreasing order of $\tilde{S}_j (j \in I_n)$ The values of the $\tilde{S}_1, \tilde{S}_2$ *and* $\tilde{S}_3$ respectively, as follows:

$$V_\mu(\tilde{S_1}) = 0.656 \times 0.3^2 = 0.059,\ V_v(\tilde{S_1}) = 0.656 \times (1-0.8)^2 = 0.026,\ V_\lambda(\tilde{S_1}) = 0.656 \times (1-0.3)^2 = 0.321$$

$$V_\mu(\tilde{S_2}) = 0.791 \times 0.4^2 = 0.127,$$
$$V_v(\tilde{S_2}) = 0.791 \times (1-0.5)^2 = 0.198,$$
$$V_\lambda(\tilde{S_2}) = 0.791 \times (1-0.6)^2 = 0.127$$

and

$$V_\mu(\tilde{S_3}) = 0.755 \times 0.5^2 = 0.189,$$
$$V_v(\tilde{S_3}) = 0.755 \times (1-0.2)^2 = 0.483,$$
$$V_\lambda(\tilde{S_3}) = 0.755 \times (1-0.8)^2 = 0.030$$

Then we have the weighted values of the SVTrN-numbers $\tilde{S_1}$, $\tilde{S_2}$ and $\tilde{S_3}$ respectively, as follows:

$$V_\theta(\tilde{S_1}) = 0.059\theta + 0.026(1-\theta) + 0.321(1-\theta)$$

$$V_\theta(\tilde{S_2}) = 0.127\theta + 0.198(1-\theta) + 0.127(1-\theta)$$

and

$$V_\theta(\tilde{S_3}) = 0.189\theta + 0.483(1-\theta) + 0.030(1-\theta)$$

It is depicted as in Fig. 1

It is easy to see from Fig. 1 that the weighted values of the SVTrN-numbers $\tilde{S_1}$ and $\tilde{S_2}$ are identical if $\theta = 0.268$

Also, we have the weighted ambiguities of $\tilde{S_1}$ and $\tilde{S_2}$ can be calculated, respectively, as follows:

$$A_{0.268}(\tilde{S_1}) = \frac{0.847-0.528}{6}\left[0.268 \times 0.3^2 + (1-0.268)(1-0.8)^2 + (1-0.268)(1-0.3)^2\right] = 0.022$$

and

$$A_{0.268}(\tilde{S_2}) = \frac{0.878-0.587}{6}\left[0.268 \times 0.5^2 + (1-0.268)(1-0.2)^2 + (1-0.268)(1-0.8)^2\right] = 0.001$$

Therefore, the ranking order of $\tilde{S}_1$ and $\tilde{S}_2$ is $\tilde{S}_1 \gtrsim \tilde{S}_2$.

Further, it is easy to see from Fig. 1 that for any given weight $\theta \in [0, 0.268)$, we have

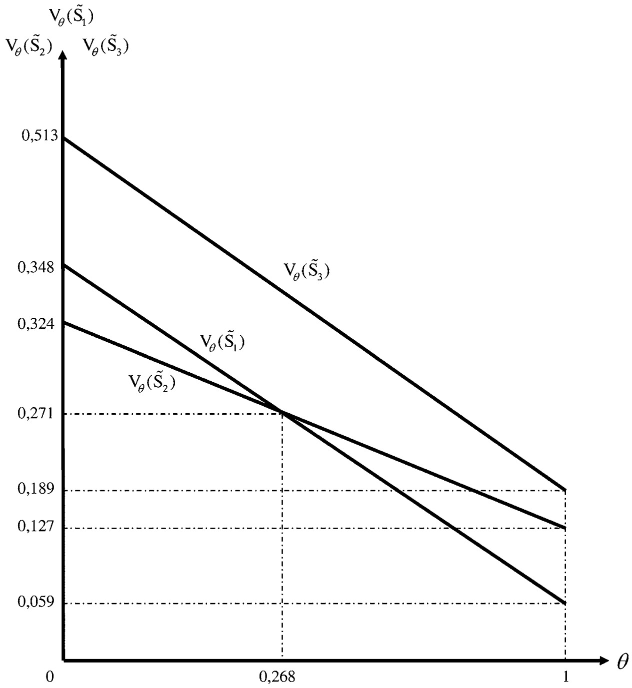

Fig. 1 The weighted values of the $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$

Table 1 Linguistic values of SVTrN-numbers

Linguistic terms	Linguistic values
Low	$\langle(4.6, 5.5, 8.6); 0.4, 0.7, 0.2\rangle$
Not low	$\langle(4.7, 6.9, 8.5); 0.7, 0.2, 0.6\rangle$
Very low	$\langle(6.2, 7.6, 8.2); 0.4, 0.1, 0.3\rangle$
Completely low	$\langle(7.1, 7.7, 8.3); 0.5, 0.2, 0.4\rangle$
More or less low	$\langle(5.8, 6.9, 8.5); 0.6, 0.2, 0.3\rangle$
Fairly low	$\langle(5.5, 6.2, 7.3); 0.8, 0.1, 0.2\rangle$
Essentially low	$\langle(5.3, 6.7, 9.9); 0.3, 0.5, 0.2\rangle$
Neither low nor high	$\langle(6.2, 8.9, 9.1); 0.6, 0.3, 0.5\rangle$
High	$\langle(6.2, 8.9, 9.1); 0.6, 0.3, 0.5\rangle$
Not high	$\langle(4.4, 5.9, 7.2); 0.7, 0.2, 0.3\rangle$
Very high	$\langle(6.6, 8.8, 10); 0.6, 0.2, 0.2\rangle$
Completely high	$\langle(6.3, 7.5, 8.9); 0.7, 0.4, 0.6\rangle$
More or less high	$\langle(5.3, 7.3, 8.7); 0.7, 0.2, 0.8\rangle$
Fairly high	$\langle(6.5, 6.9, 8.5); 0.6, 0.8, 0.1\rangle$
Essentially high	$\langle(7.5, 7.9, 8.5); 0.8, 0.5, 0.4\rangle$

$$V_\theta(\tilde{S}_3) > V_\theta(\tilde{S}_1) > V_\theta(\tilde{S}_2)$$

Step 6. Rank the alternatives x_j according to $\tilde{S}_j (j \in I_n)$ as;

$$x_3 \succ x_1 \succ x_2$$

and the best candidate is x_3. However, for any given $\theta = (0.268, 1]$, we have

Table 2 The results of the methods

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method 1	$x_3 > x_2 > x_1$	x_3	x_1
Method 2	$x_3 > x_2 > x_1$	x_3	x_1
Method 3	$x_3 > x_2 > x_1$	x_3	x_1
Method 4	$x_3 > x_2 > x_1$	x_3	x_1
The proposed method	$x_3 > x_2 > x_1$	x_3	x_1

$$V_\theta(\tilde{S}_3) > V_\theta(\tilde{S}_2) > V_\theta(\tilde{S}_1)$$

which infers that the ranking order of the three candidates is $x_3 \succ x_2 \succ x_1$ and the best candidate is x_3.

5 Comparison analysis and discussion

In order to show the feasibility of the introduced method, a comparative study with other methods was conducted. The proposed method is compared to the methods that were outlined in Refs. [13, 16] and [47] using SVTrN-numbers. With regard to the method in Ref. [13], the score values were firstly found and used to determine the final ranking order of all the alternatives, and then arithmetic and geometric aggregation operators were developed in order to aggregate the SVTrN-numbers which is used in Method 1 and Method 2, respectively. Also, a SV-trapezoidal neutrosophic number is a SVTrN-numbers (In Definition, 2.8 if $b_1 = c_1$ then the SV-trapezoidal neutrosophic number is a SVTrN-number). Therefore, the method in Ref. [16] and [47] were used to determine the final ranking order of all the alternatives Method 3 and Method 4, respectively. The results from the different methods used to resolve the MCDM problem in Example 4.2 are shown in Table 2.

From the results presented in Table 2, the best alternatives is x_3 and the worst one is x_1 in all methods. Firstly, method 1–4 use distance measure, score function, and aggregation operator and it is very difficult for decision makers to confirm their judgments when using operators and measures that have similar characteristics. Secondly, the proposed method in this paper pays more attention to the impact that uncertainty has on the alternatives and also takes into θ-weighted value of the SVN-numbers by using the concepts of cut sets of SVN-numbers. By comparison, the proposed method in this paper focuses on the θ-weighted value of the SVN-numbers, the ranking of the proposed method is the same as that of the other results. Therefore, the proposed method is effective and feasible.

6 Conclusion

The ranking method which developed by Li [23, 24] for intuitionistic numbers plays an important role in solving multiple-attribute decision-making problems and successfully applied in many fields. Therefore, developing a method, by using the methods by given Li [23, 24], in order to solve SVN-numbers is seen as a valuable research topic. This paper gives two characteristics of a SVN-number is called the value and ambiguity. Then, a ratio ranking method is developed for the ordering of SVN-numbers and applied to solve multi-attribute decision making problem with SVN-numbers. It is easily seen that the proposed ratio ranking method can be extended to rank more general SVN-numbers in a straightforward manner. Due to the fact that a SVN-numbers is a generalization of a fuzzy number and intuitionistic fuzzy number, the other existing methods of ranking fuzzy numbers and intuitionistic fuzzy number may be extended to SVN-numbers. More effective ranking methods of SVN-numbers will be investigated in the near future and applied this concepts to game theory, algebraic structure, optimization and so on.

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