Fault diagnoses of hydraulic turbine using the dimension root similarity measure of single-valued neutrosophic sets

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Abstract

This paper proposes a dimension root distance and its similarity measure of single-valued neutrosophic sets (SVNSs), and then develops the fault diagnosis method of hydraulic turbine by using the dimension root similarity measure of SVNSs. By the similarity measure between the fault diagnosis patterns and a testing sample with single-valued neutrosophic information and the relation indices, we can determine the fault type and rank faults. Then, the vibration fault diagnosis of hydraulic turbine is presented to demonstrate the effectiveness and rationality of the proposed diagnosis method by the comparative analysis. The fault diagnosis results of hydraulic turbine show that the proposed diagnosis method not only gives the main fault types of hydraulic turbine but also provides useful information for multi-fault analyses and future fault trends. The developed method is effective and reasonable in the fault diagnosis of hydraulic turbine under single-valued neutrosophic environment.

Keywords**:** Single-valued neutrosophic set; Dimension root distance; Similarity measure; Hydraulic turbine; Fault diagnosis

1. Introduction

Due to the complex structure of turbine-generator sets, if there is a fault of the equipment, it will produce a chain reaction and cause the fault of other parts or equipments, and then will seriously impact the reliability of power generation. Therefore, one is convinced of the importance of fault diagnoses. Various diagnosis methods has been developed and applied in hydraulic turbine-generator sets. For example, the vibration fault diagnosis of hydro-turbine generating unit was presented based on wavelet packet analysis and support vector machine [1, 2]. The neural network was applied to the vibration fault diagnosis of hydraulic turbine by particle swarm optimization [3]. The fault diagnosis expert system was built by using fuzzy synthesized evaluation combined with practical experience of experts [4]. The data stream mining method of associative rule classification was applied to the fault diagnosis of hydro-turbine generating unit [5]. Furthermore, fault diagnosis methods were investigated in wind turbines [6, 7]. The fault diagnosis of steam turbine generator unit was introduced based on support vector machine [8].

However, the above mentioned diagnosis methods imply some disadvantages. For instance, the

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fault diagnoses based on neural networks and support vector machine are only suitable for unique fault diagnosis and require learning process for updating fault knowledge. While the fault diagnosis expert systems indicate the diagnosis complexity and need the practical experience of experts. Therefore, the above motioned diagnosis methods are difficult to handle multiple fault diagnoses and fault prediction. In many real situations, the diagnosis data cannot provide deterministic values because the fault testing data obtained by experts are usually imprecise or uncertain due to a lack of data, time pressure, or the experts' limited attention and knowledge. Therefore, for expressing imprecise and incomplete information in real problems, Zadeh firstly proposed fuzzy sets [9]. Then, fuzzy sets have been extended to intuitionistic fuzzy sets (IFSs) [10], vague sets [11] and interval-valued intuitionistic fuzzy sets (IVIFSs) [12] and so on. Therefore, the intelligent fault diagnosis frameworks have been developed based on fuzzy integral [13]. Some researchers have developed the fault diagnosis methods of steam turbine based on the similarity measures of vague sets [14-16] and based on the cross entropy of vague sets [17]. However, existing diagnosis methods cannot deal with fault diagnosis problems with incomplete, indeterminate and inconsistent information comprehensively, which exist in real world. Hence, a neutrosophic set proposed by Smarandache [18] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It extend the theory of fuzzy sets, vague sets, IFSs and IVIFSs, then the neutrosophic set is characterized by a truth-membership degree, an indeterminacy-membership degree and a falsity-membership degree independently, which lie within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Specially, the indeterminacy presented in the neutrosophic set is independent on the truth and falsity values and can include inconsistent information. While vague sets, IFSs and IVIFSs are only characterized by a truth-membership degree and a falsity-membership degree, and then they can include the incomplete and uncertain information (hesitant degree) which is dependent on the truth and falsity values, but not include indeterminate and inconsistent information. Hence, the neutrosophic set is more suitable for expressing incomplete, indeterminate and inconsistent information comprehensively. However, the neutrosophic set is difficult to apply directly in engineering fields because the range of the truth-membership, indeterminacy-membership and falsity-membership degrees is within the nonstandard unit interval $]^{-}0, 1^{+}[$. Therefore, their range can be restrained within the real standard unit interval [0, 1] to apply easily in engineering problems. For this purpose, the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) were introduced as the subclasses of the neutrosophic set [19, 20]. In a fault diagnosis problem, various symptoms usually imply a lot of incomplete, uncertainty and inconsistent information for a fault, which characterizes a relation between symptoms and a fault. Thus we work with the uncertainties and inconsistencies to lead us to proper fault diagnosis. Recently, Ye [21] proposed cotangent similarity measures between SVNSs based on cotangent function and successfully applied them to the fault diagnosis of steam turbine under single-valued neutrosophic environment. Till now neutrosophic theory has been not applied to fault diagnoses of hydraulic turbine. However, existing fault diagnosis methods scarcely handle the fault diagnosis problems of turbine-generator sets with neutrosophic information. To extend existing fault diagnosis methods, the main purposes of this paper are to propose a new similarity measure based on a dimension root distance of SVNSs and its fault diagnosis method for the vibration fault diagnosis of hydraulic turbine with single-valued neutrosophic information.

The remainder of this paper is organized as follows. Section 2 briefly describes some basic concepts of SVNSs. Section 3 presents a dimension root distance of SVNSs and its similarity measure of SVNSs (called the dimension root similarity measure of SVNSs) and investigates their properties. Based on the dimension root similarity measure of SVNSs, we establish a fault diagnosis method for the vibration fault diagnosis of hydraulic turbine under single-valued neutrosophic environment and demonstrate the effectiveness and nationality of the developed method in Section 4. Section 5 gives conclusions and future research.

2. Some basic concepts of single-valued neutrosophic sets

Smarandache [18] firstly introduced the concept of the neutrosophic set from philosophical point of view. As mentioned above, it is difficult to apply the neutrosophic set to real problems. Therefore, Wang et al. [20] introduced the concept of SVNS, which is a subclass of the neutrosophic set, and gave the following definition.

Definition 1 [20]. Let X be a universal set. A SVNS N in X is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$. Then, a SVNS N can be denoted by the following form:

$$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\},$$

where $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$ for each point x in X. Obviously, the sum of $T_N(x)$, $I_N(x)$ and $F_N(x)$ satisfies the condition $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$.

Let $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\}$ and $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle \mid x \in X\}$ be two SVNSs. Then there are the following relations [20]:

(1) Complement: $N^c = \{\langle x, F_N(x), 1 - I_N(x), T_N(x)\rangle \mid x \in X\}$;

(2) Inclusion: $N \subseteq M$ if and only if $T_N(x) \le T_M(x)$, $I_N(x) \ge I_M(x)$ and $F_N(x) \ge F_M(x)$ for any x in X;

(3) Equality: $N = M$ if and only if $N \subseteq M$ and $M \subseteq N$.

3. Dimension root distance and its similarity measure of SVNSs

In this section, we propose a dimension root distance of SVNSs and its similarity measure between SVNSs.

Definition 2. Let two SVNSs N and M in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ be $N = \{\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j)\rangle \mid x_j \in X\}$ and $M = \{\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j)\rangle \mid x_j \in X\}$.

Then, a dimension root distance between SVNSs N and M is defined as

$$D_R(N, M) = \frac{1}{n}\sum_{j=1}^{n}\left\{\frac{1}{3}\left(\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2\right)\right\}^{1/3}, \quad (1)$$

Proposition 1. The dimension root distance measure $D_R(N, M)$ satisfies the following properties (D1-D4):

(D1) $0 \le D_R(N, M) \le 1$;

(D2) $D_R(N, M) = 0$ if and only if $N = M$;

(D3) $D_R(N, M) = D_R(M, N)$;

(D4) If P is a SVNS in X and $N \subseteq M \subseteq P$, then $D_R(N, P) \ge D_R(N, M)$ and $D_R(N, P) \ge D_R(M, P)$.

Proof:

It is obvious that $D_R(N, M)$ satisfies the properties (D1-D3). Hence, we only prove the property

(D4). Since $N \subseteq M \subseteq P$, this implies $T_N(x_j) \le T_M(x_j) \le T_P(x_j)$, $I_N(x_j) \ge I_M(x_j) \ge I_P(x_j)$ and $F_N(x_j) \ge F_M(x_j) \ge F_P(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Then, we have

$\left|T_N(x_j) - T_M(x_j)\right| \le \left|T_N(x_j) - T_P(x_j)\right|$, $\left|T_M(x_j) - T_P(x_j)\right| \le \left|T_N(x_j) - T_P(x_j)\right|$, $\left|I_N(x_j) - I_M(x_j)\right| \le \left|I_N(x_j) - I_P(x_j)\right|$, $\left|I_M(x_j) - I_P(x_j)\right| \le \left|I_N(x_j) - I_P(x_j)\right|$, $\left|F_N(x_j) - F_M(x_j)\right| \le \left|F_N(x_j) - F_P(x_j)\right|$, and $\left|F_M(x_j) - F_P(x_j)\right| \le \left|F_N(x_j) - F_P(x_j)\right|$.

Hence, there are $\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2 \le \left|T_N(x_j) - T_P(x_j)\right|^2 + \left|I_N(x_j) - I_P(x_j)\right|^2 + \left|F_N(x_j) - F_P(x_j)\right|^2$ and $\left|T_M(x_j) - T_P(x_j)\right|^2 + \left|I_M(x_j) - I_P(x_j)\right|^2 + \left|F_M(x_j) - F_P(x_j)\right|^2 \le \left|T_N(x_j) - T_P(x_j)\right|^2 + \left|I_N(x_j) - I_P(x_j)\right|^2 + \left|F_N(x_j) - F_P(x_j)\right|^2$.

Combining the above inequalities with the defined distance formula (1), we can obtain that $D_R(N, P) \ge D_R(N, M)$ and $D_R(N, P) \ge D_R(M, P)$.

Thus, we complete the proof of the properties. □

Considering the importance of elements in the universe, one needs to give the weight w_j of the element x_j (j = 1, 2,…, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_i = 1$. Then, the weighted dimension root distance measure between SVNSs N and M can be defined as

$$W_R(N, M) = \sum_{j=1}^{n} w_j \left\{ \frac{1}{3} \left(\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2 \right) \right\}^{1/3}, \quad (2)$$

Especially, when $w_j = 1/n$ (j = 1, 2,…, n), Eq. (2) reduces to Eq. (1). Obviously, the weighted dimension root distance measure also satisfies the above properties (D1-D4).

Based on the complementary relationship between the distance measure and the similarity measure, we can define the similarity measure based on the weighted dimension root distance.

Definition 3. Let two SVNSs N and M in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ be $N = \{\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j)\rangle \mid x_j \in X\}$ and $M = \{\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j)\rangle \mid x_j \in X\}$. Then, the similarity measure based on the weighted dimension root distance between SVNSs N and M is defined as follows:

$$S_R(N, M) = 1 - \sum_{j=1}^{n} w_j \left\{ \frac{1}{3} \left(\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2 \right) \right\}^{1/3}, \quad (3)$$

which is call the dimension root similarity measure of SVNSs.

Proposition 2. The dimension root similarity measure $S_R(N, M)$ satisfies the following properties (S1-S4):

(S1) $0 \le S_R(N, M) \le 1$;

(S2) $S_R(N, M) = 1$ if and only if $N = M$;

(S3) $S_R(N, M) = S_R(M, N)$;

(S4) If P is a SNS in X and $N \subseteq M \subseteq P$, then $S_R(N, P) \leq S_R(N, M)$ and $S_R(N, P) \leq S_R(M, P)$.

According to the above distance properties and the complementary relationship between the distance and the similarity measure, we can easily prove that the dimension root similarity measure should satisfy the above properties (S1-S4).

4. Fault diagnosis method of hydraulic turbine based on the dimension root similarity measure

For a volume of fault feature information obtained from modem measurement technologies, the fault information contains a lot of incomplete, uncertain and inconsistent information. In some practical situations, there is the possibility of each feature including truth and falsity information and indeterminacy information, by which a SVNS is expressed. Hence, the dimension root similarity measure of SVNSs is a suitable tool to deal with fault diagnosis problems with single-valued neutrosophic information. This section applies the proposed similarity measure of SVNSs to the fault diagnosis of hydraulic turbine.

4.1 ***Fault diagnosis method***

For a fault diagnosis problem of hydraulic turbine, the fault diagnosis of hydraulic turbine realized by the frequency features extracted from the vibration signals of hydraulic turbine is a simple and effective method [2, 5]. The spectrum analysis of the vibration signals measured by the sensor is carried out, and the different frequency components in the spectrum are composed of different fault characteristic frequencies, which reflect the different fault reasons. The frequency components in the spectrum of the fault vibration data are used as the fault feature vectors. The type of faults can be determined by these fault feature vectors (see [2, 5] in detail).

Assume that a set of m fault diagnosis patterns (fault diagnosis knowledge) is $D = \{D_1, D_2, \ldots, D_m\}$ and a set of n frequency features is $S = \{s_1, s_2, \ldots, s_n\}$. Then the knowledge of a fault diagnosis pattern D_k ($k = 1, 2, \ldots, m$) with respect to a frequency feature s_i ($i = 1, 2, \ldots, n$) is represented by a SVNS D_k ($k = 1, 2, \ldots, m$):

$$D_k = \left\{ \left\langle s_j, T_{D_k}(s_j), I_{D_k}(s_j), F_{D_k}(s_j) \right\rangle \mid s_j \in S \right\}.$$

Then, the information of testing samples is represented by a SVNS R_t ($t = 1, 2, \ldots, q$):

$$R_t = \left\{ \left\langle s_j, T_{R_t}(s_j), I_{R_t}(s_j), F_{R_t}(s_j) \right\rangle \mid s_j \in S \right\}.$$

The measure value δ_k ($k = 1, 2, \ldots, m$) can be obtained by the following similarity measure between D_K and R_t:

$$\begin{aligned} \delta_k &= S_R(D_k, R_t) \\ &= 1 - \sum_{j=1}^{n} w_j \left\{ \frac{1}{3} \left(\left| T_{D_k}(s_j) - T_{R_t}(s_j) \right|^2 + \left| I_{D_k}(s_j) - I_{R_t}(s_j) \right|^2 + \left| F_{D_k}(s_j) - F_{R_t}(s_j) \right|^2 \right) \right\}^{1/3} . \end{aligned} \quad (3)$$

For easy fault diagnosis analysis, the measure values of δ_k ($k = 1, 2, \ldots, m$) are normalized into the values of relation indices by the following formula:

$$\rho_k = \frac{2\delta_k - \delta_{\min} - \delta_{\max}}{\delta_{\max} - \delta_{\min}}, \ k=1, 2, \ldots, m, \quad (4)$$

where ρ_k lies within the interval [–1, 1], $\delta_{\min} = \min_{1 \leq k \leq m} \{\delta_k\}$ and $\delta_{\max} = \max_{1 \leq k \leq m} \{\delta_k\}$.

According to the relation indices, we can rank faults and determine the fault type or predict possible fault trend for the tested equipment.

If the maximum value of the relation indices is $\rho_k = 1$, then we can determine that the testing sample R_t should belong to the fault diagnosis pattern D_k.

The block diagram of the proposed fault diagnosis approach based on the dimension root similarity measure of SVNSs is shown in Fig. 1.

*4.2. **Vibration fault diagnosis of hydraulic turbine***

In this subsection, the proposed fault diagnosis method is applied to the vibration fault diagnosis of hydraulic turbine to illustrate its effectiveness.

In hydraulic turbine-generator sets, interaction effects in the factors such as the unbalance and offset center of rotor, the bearing clearance show the vibration of the turbine-generator sets. In the vibration fault diagnosis of hydraulic turbine, the relation between the cause and the fault symptoms of the hydraulic turbine has been established by means of the analyses of frequency features in the fault frequency spectrum [2, 5]. Now, we investigate the vibration fault diagnosis of hydraulic turbine by use of the proposed similarity measure of SVNSs to demonstrate the effectiveness and rationality of the fault diagnosis method.

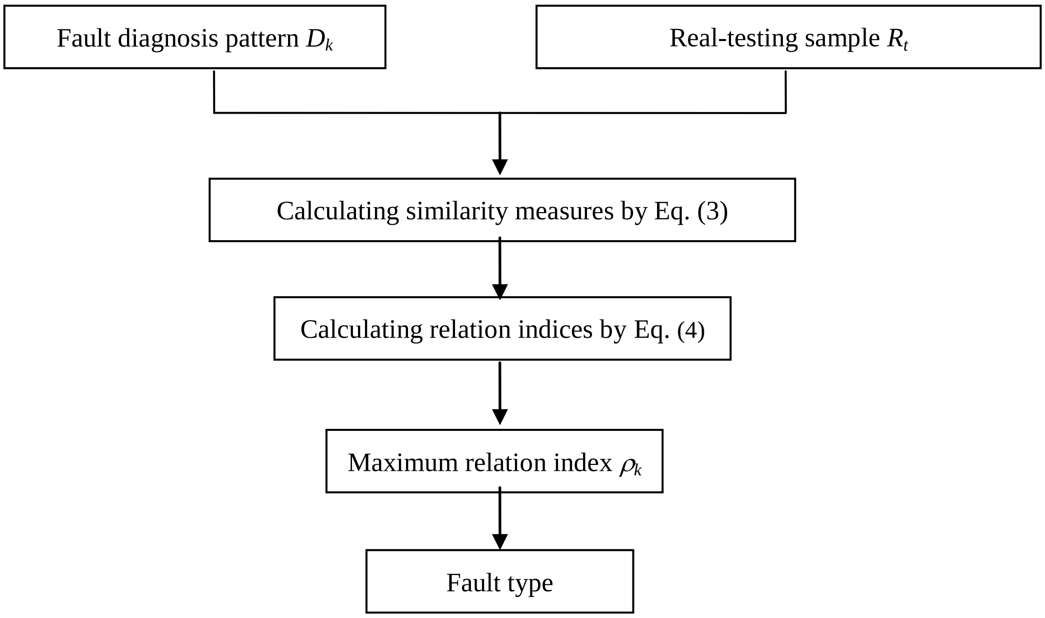

Fig. 1 The block diagram of the proposed fault diagnosis approach

Let us consider a set of four fault diagnosis patterns D = {D_1(unbalance of rotor), D_2(offset center of rotor), D_3(bigger bearing clearance), D_4(vortex band of tail water pipe)} as the fault diagnosis knowledge and a set of five frequency features in fault frequency spectrum S = {s_1(0.5f), s_2(f), s_3(2f), s_4(3f), s_5(>3f)} under operating frequency f as a frequency feature set [2, 5]. Then, the information of the fault diagnosis pattern D_k (k = 1, 2, 3, 4) with respect to the frequency feature s_i (i = 1, 2, 3, 4, 5) can be expressed by the form of SVNSs, as shown in Table 1. Assume that the weight of each feature s_j is w_j = 1/5 for j = 1, 2, 3, 4, 5.

Table 1 Vibration fault diagnosis knowledge

Fault diagnosis pattern	Frequency feature				
	$S_1(0.5f)$	$S_2(f)$	$S_3(2f)$	$S_4(3f)$	$S_5(>3f)$
D_1 (Unbalance of rotor)	[0.01, 0.05]	[0.9, 1]	[0.08, 0.11]	[0.02, 0.05]	[0.15, 0.21]
D_2 (Offset center of rotor)	[0.01, 0.09]	[0.69, 0.71]	[0.88, 0.98]	[0.65, 0.75]	[0.1, 0.21]
D_3 (Bigger bearing clearance)	[0.04, 0.07]	[0.13, 0.16]	[0.08, 0.15]	[0.02, 0.06]	[0.93, 0.99]
D_4 (Vortex band of tail water pipe)	[0.88, 0.92]	[0.2, 0.21]	[0.03, 0.12]	[0.02, 0.23]	[0.1, 0.28]

From Table 1, the fault diagnosis patterns can be expressed as the form of SVNSs:

D_1 = {<s_1, 0.01, 0.04, 0.95>, <s_2, 0.9, 0.1, 0>, <s_3, 0.08, 0.03, 0.89>, <s_4, 0.02, 0.03, 0. 95>, <s_5, 0.15, 0.06, 0.79>},

D_2 = {<s_1, 0.01, 0.08, 0.91>, <s_2, 0.69, 0.02, 0.29>, <s_3, 0.88, 0.1, 0.02>, <s_4, 0.65, 0.1, 0. 25>, <s_5, 0.1, 0.11, 0.79>},

D_3 = {<s_1, 0.04, 0.03, 0.93>, <s_2, 0.13, 0.03, 0.84>, <s_3, 0.08, 0.07, 0.85>, <s_4, 0.02, 0.04, 0. 94>, <s_5, 0.93, 0.06, 0.01>},

D_4 = {<s_1, 0.88, 0.04, 0.08>, <s_2, 0.2, 0.01, 0.79>, <s_3, 0.03, 0.09, 0.88>, <s_4, 0.02, 0.21, 0. 77>, <s_5, 0.1, 0.18, 0.72>},

For the vibration fault diagnosis of hydraulic turbine, eight real-testing samples are given as actual examples, which are expressed by the following SVNSs:

R_1 = {<s_1, 0.03, 0, 0.97>, <s_2, 0.95, 0, 0.05>, <s_3, 0.1, 0, 0.9>, <s_4, 0.04, 0, 0. 96>, <s_5, 0.2, 0, 0.8>},

R_2 = {<s_1, 0.07, 0, 0.93>, <s_2, 0.4, 0, 0.6>, <s_3, 0.12, 0, 0.88>, <s_4, 0.03, 0, 0. 97>, <s_5, 0.96, 0, 0.04>},

R_3 = {<s_1, 0.04, 0, 0.96>, <s_2, 0.16, 0, 0.84>, <s_3, 0.08, 0, 0.92>, <s_4, 0.06, 0, 0. 94>, <s_5, 0.93, 0, 0.07 >},

R_4 = {<s_1, 0.01, 0, 0.99>, <s_2, 0.85, 0, 0.15>, <s_3, 0.98, 0, 0.02>, <s_4, 0.71, 0, 0. 29>, <s_5, 0.07, 0, 0.93>},

R_5 = {<s_1, 0.9, 0, 0.1>, <s_2, 0.2, 0, 0.8>, <s_3, 0.05, 0, 0.95>, <s_4, 0.02, 0, 0. 98>, <s_5, 0.18, 0, 0.82>},

R_6 = {<s_1, 0.02, 0, 0.98>, <s_2, 1, 0, 0>, <s_3, 0.08, 0, 0.92>, <s_4, 0.03, 0, 0. 97>, <s_5, 0.18, 0, 0.82>},

R_7 = {<s_1, 0.09, 0, 0.91>, <s_2, 0.71, 0, 0.29>, <s_3, 0.88, 0, 0.12>, <s_4, 0.65, 0, 0. 35>, <s_5, 0.1, 0, 0.9>},

R_8 = {<s_1, 0.88, 0, 0.12>, <s_2, 0.21, 0, 0.79>, <s_3, 0.12, 0, 0.88>, <s_4, 0.04, 0, 0. 96>, <s_5, 0.1, 0, 0.9>}.

For the fault diagnoses of the real-testing samples of R_t for t = 1, 2, …, 8, the similarity measures and relation indices between D_k (k = 1, 2, 3, 4) and R_t (t = 1, 2) are calculated by Eqs. (3) and (4), then the calculating results are shown in Table 2.

Table 2 Results of the relation indices and fault diagnosis based on the dimension root similarity measure

R_t	Relation indices (ρ_k)				Fault diagnosis results	Actual faults
	D_1	D_2	D_3	D_4		
R_1	1.0000	–0.7579	–0.7824	–1.0000	D_1	D_1
R_2	0.0529	–1.0000	1.0000	–0.4695	D_3	D_3
R_3	–0.1641	–1.0000	1.0000	–0.3188	D_3	D_3
R_4	0.0050	1.0000	–1.0000	–0.9600	D_2	D_2
R_5	–0.1633	–1.0000	–0.1676	1.0000	D_4	D_4
R_6	1.0000	–0.7430	–0.8233	–1.0000	D_1	D_1
R_7	–0.0425	1.0000	–1.0000	–0.8805	D_2	D_2
R_8	–0.2255	–1.0000	–0.3712	1.0000	D_4	D_4

To show the diagnosis process in detail, we give the fault diagnosis process of the two testing samples R_1 and R_2 as illustration.

As for the first real-testing sample R_1, from Table 2, we can see that the fault type of the hydraulic turbine is D_1 (unbalance of rotor) according to the maximum relation index (1.0000), which is in agreement with the actual fault. Obviously, the fault types of D_2, D_3, D_4 have very low possibility due to the negative relation indices. Therefore, the unbalance of rotor causes the violent vibration of the hydraulic turbine. Hereby, the ranking order of all faults is $D_1 \to D_2 \to D_3 \to D_4$.

As for the second real-testing sample R_2, from Table 2, we can see that the fault type of the hydraulic turbine is D_3 (bigger bearing clearance) according to the maximum relation index (1.0000), which is in agreement with the actual fault. Then, the unbalance of rotor (D_1) may exit low possibility because its relation index is 0.0529, while the fault types D_2 and D_4 have very low possibility due to the negative relation indices. Therefore, the bigger bearing clearance is the main cause of producing the violent vibration of the hydraulic turbine. Hereby, the ranking order of all faults is $D_3 \to D_1 \to D_4 \to D_2$.

As for the rest of real-testing samples, similar diagnosis process can be given according to the relation indices. Obviously, we can see from Table 2 that all diagnosis results based on the dimension root similarity measure are in agreement with the actual faults of the hydraulic turbine.

The fault diagnosis results of the hydraulic turbine show that the proposed diagnosis method not only indicates the main fault type of the hydraulic turbine, but also provides useful information for multi-fault analyses and future fault trends. Therefore, the fault diagnosis method developed in this paper is effective and reasonable in the vibration fault diagnoses of hydraulic turbine.

4.3. Comparative analyses of related fault diagnosis methods

Because similarity measure is a key tool in the fault diagnosis and in existing literature there is not any diagnosis method of hydraulic turbine in a neutrosophic environment till now, to show the effectiveness of the fault diagnosis method of hydraulic turbine based on the proposed dimension root similarity measure, we compare the proposed dimension root similarity measure with the cotangent similarity measure of SVNSs proposed by Ye [21] in the fault diagnosis problems.

For the convenient comparison of the fault diagnosis problems, assume that the dimension root similarity measure of Eq. (3) is replaced by the cotangent similarity measure of SVNSs proposed in [21] as the following form:

$$\delta_k = C_w(D_k, R_t) = \sum_{j=1}^{n} w_j \cot\left[\frac{\pi}{4} + \frac{\pi}{12}\begin{pmatrix} \left|T_{D_k}(s_j) - T_{R_t}(s_j)\right| + \\ \left|I_{D_k}(s_j) - I_{R_t}(s_j)\right| + \\ \left|F_{D_k}(s_j) - F_{R_t}(s_j)\right| \end{pmatrix}\right]. \tag{5}$$

Then by using Eqs. (5) and (4), we can calculate the cotangent similarity measures and relation indices between D_k (k = 1, 2, 3, 4) and R_t (t = 1, 2, …, 8) respectively, and then the results are shown in Table 3.

Table 3 Results of the relation indices and fault diagnosis based on the cotangent similarity measure

R_t	Relation index (ρ_k)				Fault diagnosis result	Actual fault
	D_1	D_2	D_3	D_4		
R_1	1.0000	–0.9150	–0.5518	–1.0000	D_1	D_1
R_2	0.1093	–1.0000	1.0000	–0.4877	D_3	D_3
R_3	–0.0391	–1.0000	1.0000	–0.2987	D_3	D_3
R_4	0.0632	1.0000	–0.8881	–1.0000	D_2	D_2
R_5	0.0563	–1.0000	0.0083	1.0000	D_4	D_4
R_6	1.0000	–0.9468	–0.5833	–1.0000	D_1	D_1
R_7	–0.1818	1.0000	–0.9308	–1.0000	D_2	D_2
R_8	0.0094	–1.0000	–0.0465	1.0000	D_4	D_4

From Table 2 and Table 3, we can see that all diagnosis results based on the dimension root similarity measures and the cotangent similarity measure are identical in the fault diagnoses of the hydraulic turbine and in agreement with actual faults. Hence, the diagnosis results show the effectiveness of both the proposed diagnosis method for the hydraulic turbine and the proposed dimension root similarity measure of SVNSs. Hence, the diagnosis method proposed in this paper provides a new way for the fault diagnoses of hydraulic turbines.

Furthermore, the fault diagnosis method of hydraulic turbine proposed in this paper is simpler and more effective than the diagnosis methods of hydraulic turbines based on neural networks [3] and support vector machine [1, 2], where their diagnostic results are within the accurate range of 86.32% and 92.68% [1, 2], and then can overcome the disadvantages of diagnosis methods based on neural networks and support vector machine, which are only suitable for unique fault diagnosis and require learning process for updating fault knowledge; while the fault diagnosis method in this paper does not require learning process for updating fault knowledge and can provide multiple fault diagnosis information and future fault trends. Therefore, the proposed diagnosis method in this paper can be easily implemented by personal computer software and provide a new effective way for the fault diagnoses of hydraulic turbines.

5. Conclusion

This paper proposed the dimension root distance and its similarity measure between SVNSs. Then, the dimension root similarity measure was applied to the vibration fault diagnosis of hydraulic turbine under single-valued neutrosophic environment. The fault diagnosis results demonstrated the effectiveness and rationality of the proposed method. The proposed method can not only indicate the main fault type of hydraulic turbine but also predict future fault trends according to the relation indices. Furthermore, the proposed method can deal with the fault diagnosis problems with incomplete, uncertain and inconsistent information, which are not handled by existing fault diagnosis methods of hydraulic turbine. Therefore, the proposed diagnosis method in this paper extends existing fault diagnosis methods and provides a new method for the fault diagnoses of hydraulic turbine. In the future, the developed diagnosis method will be extended to other fault diagnoses, such as vibration faults of aircraft engines and wind turbines.

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Table(1)

Table 1 Vibration fault diagnosis knowledge

Fault diagnosis pattern	Frequency feature				
	$S_1(0.5f)$	$S_2(f)$	$S_3(2f)$	$S_4(3f)$	$S_5(>3f)$
D_1 (Unbalance of rotor)	[0.01, 0.05]	[0.9, 1]	[0.08, 0.11]	[0.02, 0.05]	[0.15, 0.21]
D_2 (Offset center of rotor)	[0.01, 0.09]	[0.69, 0.71]	[0.88, 0.98]	[0.65, 0.75]	[0.1, 0.21]
D_3 (Bigger bearing clearance)	[0.04, 0.07]	[0.13, 0.16]	[0.08, 0.15]	[0.02, 0.06]	[0.93, 0.99]
D_4 (Vortex band of tail water pipe)	[0.88, 0.92]	[0.2, 0.21]	[0.03, 0.12]	[0.02, 0.23]	[0.1, 0.28]

Table(2)

Table 2 Results of the relation indices and fault diagnosis based on the dimension root similarity measure

R_t	Relation indices (ρ_k)				Fault diagnosis results	Actual faults
	D_1	D_2	D_3	D_4		
R_1	1.0000	–0.7579	–0.7824	–1.0000	D_1	D_1
R_2	0.0529	–1.0000	1.0000	–0.4695	D_3	D_3
R_3	–0.1641	–1.0000	1.0000	–0.3188	D_3	D_3
R_4	0.0050	1.0000	–1.0000	–0.9600	D_2	D_2
R_5	–0.1633	–1.0000	–0.1676	1.0000	D_4	D_4
R_6	1.0000	–0.7430	–0.8233	–1.0000	D_1	D_1
R_7	–0.0425	1.0000	–1.0000	–0.8805	D_2	D_2
R_8	–0.2255	–1.0000	–0.3712	1.0000	D_4	D_4

Table(3)

Table 3 Results of the relation indices and fault diagnosis based on the cotangent similarity measure

R_t	Relation index (ρ_k)				Fault diagnosis result	Actual fault
	D_1	D_2	D_3	D_4		
R_1	1.0000	–0.9150	–0.5518	–1.0000	D_1	D_1
R_2	0.1093	–1.0000	1.0000	–0.4877	D_3	D_3
R_3	–0.0391	–1.0000	1.0000	–0.2987	D_3	D_3
R_4	0.0632	1.0000	–0.8881	–1.0000	D_2	D_2
R_5	0.0563	–1.0000	0.0083	1.0000	D_4	D_4
R_6	1.0000	–0.9468	–0.5833	–1.0000	D_1	D_1
R_7	–0.1818	1.0000	–0.9308	–1.0000	D_2	D_2
R_8	0.0094	–1.0000	–0.0465	1.0000	D_4	D_4