On hypersurfaces $F^3 \subset E^4$ with nonzero Laplacian of the second fundamental form

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Abstract

The some properties of hypersurfaces F^3 in Euclidean space E^4 of nonzero Laplacian $\Delta b \neq 0$ of the second fundamental form b are studied in this report.

Laplacian Δb of the second fundamental form b.

Let E^{n+1} be $(n+1)$-dimensional $(n \geq 2)$ Euclidean space with Cartesian coordinates $(x_1, x_2, \ldots, x_{n+1})$. In E^{n+1}, consider hypersurface F^n given in neighborhood of each its point $x \in F^n$, by the following equations

$$x_\alpha = f_\alpha(u^1, \ldots, u^n), \quad (u^1, \ldots, u^n) \in D, \quad \alpha = \overline{1, n+1},$$

where D is some domain of parametric space $(u^1, \ldots, u^n)$, $f_\alpha \in C^3(D)$.

Let $I = g_{ij} du^i du^j$ and $II = b_{ij} du^i du^j$ are the first and the second fundamental forms of hypersurface $F^n \subset E^{n+1}$ respectively.

The symmetric covariant three-valent tensor $\Theta_{(n)}$ is determined on hypersurfaces F^n $(n \geq 2)$ with nonzero Gaussian curvature $K \neq 0$ in Euclidean space E^{n+1} [1]. The properties of hypersurfaces $F^n \subset E^{n+1}$ with $\Theta_{(n)} \equiv 0$ for $n > 2$ are studied in [2].

On $F^n \subset E^{n+1}$ we determine Laplacian Δb of the second fundamental form b by the formula:

$$(\Delta b)_{ij} = g^{km} \nabla_k \nabla_m b_{ij}, \qquad i, j, k, m = \overline{1, n},$$

where ∇_i is the operation of covariant differentiation relative to tensor g_{ij}.

There exists hypersurface F^n $(n \geq 3)$ of zero Gaussian curvature $K = 0$ and of zero mean curvature $H = 0$ such that $\Delta b \neq 0$ in E^{n+1}.

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Hypersurfaces $F^3 \subset E^4$ of nonzero Laplacian $\Delta b \neq 0$ of the second fundamental form b.

Let E^4 be Euclidean space with Cartesian coordinates (x_1, x_2, x_3, x_4). Consider in E^4, a cylindrical surface F^3 with a two-dimensional orthogonal base F^2. The base F^2 is given in E^4 by the following equation system:

$$x_1 tg x_3 - x_4 = 0, \quad x_2 = 0. \tag{1}$$

The following statements hold.

Lemma 1. *Cylindrical surface F^3 in E^4 with a two-dimensional orthogonal base F^2 given in E^4 by the equation system (1), has zero Gaussian curvature $K = 0$ and zero mean curvature $H = 0$.*

Lemma 2. *For cylindrical surface F^3 in E^4 with a two-dimensional orthogonal base F^2 given in E^4 by the equation system (1), the following formula holds:*

$$(\Delta b)_{11} = \frac{-4x_1 sec^3 x_3 tg x_3}{(1 + x_1^2 sec^2 x_3)^{5/2}}.$$

For cylindrical hypersurface F^3 in E^4 determined by the conditions of the lemma 2 the following relations hold:

$$(\Delta b)_{11} \neq 0, (\Delta b)_{13} \neq 0, (\Delta b)_{31} \neq 0, (\Delta b)_{33} \neq 0.$$

$$(\Delta b)_{12} \equiv 0, (\Delta b)_{21} \equiv 0, (\Delta b)_{22} \equiv 0, (\Delta b)_{23} \equiv 0, (\Delta b)_{32} \equiv 0.$$

The following theorem holds.

Theorem. *Cylindrical surface F^3 in E^4 with a two-dimensional orthogonal base F^2 given in E^4 by the equation system (1), has zero Gaussian curvature $K = 0$ and zero mean curvature $H = 0$, and Laplacian $\Delta b \neq 0$.*

The proof of the theorem follows from lemmas 1 and 2.

Remark.

From the theorem proved in article [3] follows the statement.

Statement. *On every minimal surface F^2 in E^3 the following equation holds:*

$$\Delta b = 2Kb,$$

where K is Gaussian curvature F^2 in E^3.

References.

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3. *Bodrenko A. I.* Surfaces with recurrent second fundamental form in E^3. OP&PM Surveys in Applied and Industrial Mathematics. 2004. V. 11. 2. P. 300–301.