Perceptive or P-Calculus: Ordinale & Residuale Noesis

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Abstract[1]

An ordual offspring bridging a variety of otherwise remote areas.

A Succinct Exposition

Consider a structural expansion of the form:

$$\forall A, B\ \exists\ (P, P_2, P_3):\ P(A+B) \equiv P_2(A) + P_3(B)$$

This may for one be seen as an alternative to Taylor linearization, or indeed a generalized representation of the Cauchy functional equation as part of Hilbert's Fifth Problem to be addressed in subsequent research. For now, suffice it to point out that this may capture an inherently entwined nature of arbitrary objects or values (be it the arguments or the operators to be seen as dual bases or unity decompositions). *Inter alia*, this could be a straightforward illustration of what I have long referred to as the "rho-ellipse" pertaining to how the invariant sum of partial radii could be hinting at a global characteristic of a relationship (as defined on the Ψ basis) irrespective of the interim or partial relations. In a sense, this might also be akin to path-invariant action in line with the variations or functional-analytic principle. However, the present analysis is aimed at keeping things as simple and endogenous as the setup warrants while discerning some findings that might potentially prove to be of interest.

In particular, the 'ellipsoid' metaphor (somewhat along the 'amorphous polytope' lines as proposed before) could best be appreciated in terms of an alpha-distribution:

$$1.1\ P(A) = P(\alpha A + [1-\alpha]A) = P_2(\alpha A) + P_3([1-\alpha]A)$$

$$1.2\ P(A) = P([1-\alpha]A + \alpha A) = P_2([1-\alpha]A + P_3(\alpha A))$$

$$1.3\ P(A) \equiv \alpha P(A) + [1-\alpha]P(A)$$

Technically, this is not to suggest that any of the righthand-side terms can be held as equivalent pairwise. However, the line of reasoning can ironically be extended even further than that, suggesting that the irrelevance of any *particular* alpha values lends some extra relevance to its complete and unrestricted domain or very presence—indeed much akin to the case with rho in the past expositions.

[1] To Elizaveta "Dr. Liza" Glinka and her Musical and Heraldic Guards, whose names shall never be smashed into oblivion.

On the other hand, insofar as homogeneity may apply to structures that pose no well-behaved patterns of the functional type, alpha-invariance could tentatively be viewed in terms of homogeneity of degree zero (H0). Unity homogenization (H1) might also lend itself to sign transitivity of the sort,

$$2.1\ \ P(A-B)=P\big(A+(-B)\big)=P_2(A)+P_3(-B)=P_2(-B)+P_3(A)$$

$$2.2\ \ P(A+B)=P\big(A-(-B)\big)=P\big(B-(-A)\big)\equiv P(A-B')=P(B-A')$$

$$2.3\ \ P(0)=P(A-A)=P_2(A)+P_3(-A)=P_2(-A)+P_3(A)$$

$$2.4\ \ P(0)=P(\alpha A+[1-\alpha]A)=P_2(\alpha A)+P_3([1-\alpha]A)=P_2([1-\alpha]A)+P_3(\alpha A)$$

Although pairwise or one-to-one comparison is not an option, complete equivalents can still be discerned. For instance, (2.1) may suggest that:

$$2.5\ \ P_2(A)-P_3(A)=P_2(-B)-P_3(-B)=P_2(X)-P_3(X)=const\ \forall X$$

For that matter, (2.4) implies that,

$$2.6\ \ P(0)-P_3(\alpha A)-P_3([1-\alpha]A)=const\ \forall\alpha$$

One could be led to presume that the implied invariants in (2.5) and (2.6) alike hint at the exact same value, possibly *P(0)*, in which case sign transitivity literally suggests H1, or homogeneity of degree one:

$$2.7\ \ P(0)=P_2(A)-P_3(A)=P_2(A)+P_3(-A)$$

This may, however, be qualified by *A* acting as a kind of period or complete rotation, with (2.6) positing:

$$2.8\ \ P_3(\alpha A)+P_3([1-\alpha]A)=0$$

This could more rigorously be solved as a functional equation:

$$P_3(x+\Delta)=(-1)*P_3(x),\qquad x\equiv[1-\alpha]A,\qquad \Delta\equiv[2\alpha-1]A$$

$$2.9\ P_3(A)=P_3(0)*(-1)^{\frac{1}{2\alpha-1}}$$

The solution might hint at *A* invariance (irrespective of the arcane extension of complex numbers as implied by the generally-irrational power or decomposition of unity). A more neutral possibility could be as follows:

$$2.10\ P_3([1-\alpha]A)=P_3(A)*(-1)^{\frac{\alpha}{1-2\alpha}}$$

$$2.11\ P_3(\alpha A)=P_3(A)*(-1)^{\frac{1-\alpha}{1-2\alpha}}$$

From comparison of the two conjugates, it becomes evident that (2.8) is met.

Again, though, the above stems from but one possibility, with quasi-homogeneity otherwise taking on far more straightforward forms without loss of non-trivial explanatory power. To begin with, from comparing (1.1) and (1.3) it readily follows that:

$$3.1\quad P_2(\alpha A) = \alpha P(A)$$

$$3.2\quad P_3([1-\alpha]A) = (1-\alpha)P(A)$$

$$3.3\quad \frac{P_3([1-\alpha]A)}{P_2(\alpha A)} = \frac{1-\alpha}{\alpha} \sim (\rho - 1)\ \forall A$$

Now, if one were to assume away the arguments or objects or treat the very operators or relationships as such, it can be induced that:

$$P^{[2]} \equiv P(P) = P(P_2 + P_3) = P_2(P_2) + P_3(P_3) \equiv P_2^{[2]} + P_3^{[2]}$$

Or, more generally still,

$$4.1\quad P^{[n]} = P_2^{[n]} + P_3^{[n]}$$

By drawing an analogy with (3.1)-(3.2), it may follow that:

$$4.2\quad P_2(P) = \alpha P(\frac{P}{\alpha})$$

$$4.3\quad P_3(P) = [1-\alpha]P(\frac{P}{1-\alpha})$$

Insofar as it holds that, $P(0) \equiv P(P-P) = P_2(P) - P_3(P)$, it appears that

$$4.4\quad \alpha P\left(\frac{P}{\alpha}\right) - [1-\alpha]P\left(\frac{P}{1-\alpha}\right) = P(0)$$

If we are to continue drawing on the homogeneity metaphor, suppose:

$$\forall\alpha\exists\gamma: P(\alpha A) \equiv \alpha^{\gamma}P(A)$$

This convention can be extended so far as to reconsider (4.4) as,

$$5.1\quad [\alpha^{1-\gamma} - (1-\alpha)^{1-\gamma}]P(P) = P(0)$$

Moreover, simultaneously it can be proffered that,

$$5.2\quad [\alpha^{1-\gamma} + (1-\alpha)^{1-\gamma}]P(P) = P(2P) = P(P+P)$$

Following straightforward algebraic manipulations, it obtains that

$$5.3\quad \frac{P(0)}{P(P)} + \frac{P(2P)}{P(P)} = 2\alpha^{1-\gamma}$$

$$5.4\ \ \frac{P(2P)}{P(P)} - \frac{P(0)}{P(P)} = 2[1-\alpha]^{1-\gamma}$$

$$5.5\ \ \frac{P(2P)}{P(P)} = \alpha^{1-\gamma} + [1-\alpha]^{1-\gamma} = 2^{\gamma}$$

The first two accounts can be juxtaposed to arrive at,

$$6.1\ \ \frac{P(2P) - P(0)}{P(2P) + P(0)} = (\frac{1-\alpha}{\alpha})^{1-\gamma} \sim (\rho - 1)^{1-\gamma}$$

In fact, this alone shows that *P(0)* need not trivially amount to zero—except either in the cardinalcy case of rho equal 2 or 0 or H1. Even in the latter case, though, the role of *P(0)* has (and will have been) shown to be anything but superfluous.

Now, if one were to embark on (5.5), it may again refer to a kind of ellipse generalizing the aforementioned functional equation, with gamma homogeneity hinging on alpha distribution. That said, the instances of particular interest obtain under some corner cases. For instance, H1 (i.e. gamma anywhere near unity) suggests that (5.5) holds identically for any alpha—indeed genuine alpha invariance as posited at the outset. Ironically, though, largely the same holds for H0 (or gamma at zero). This does hint at either one standing out as legitimate special cases.

However, the generalized meta-relationship could be of importance in its own right. A set of equivalent interim results can be arrived at:

$$6.2\ \ 2^{\gamma} = \frac{P_2(\alpha A)}{P(\alpha A)} + \frac{P_3([1-\alpha]A)}{P([1-\alpha]A)}$$

$$6.3\ \ 2^{\gamma} = \left(1 - \frac{P_3(0)}{P(\alpha A)}\right) + \left(1 - \frac{P_2(0)}{P([1-\alpha]A)}\right)$$

$$6.4\ \ 2^{\gamma} = \left(\frac{\alpha}{P(\alpha A)} + \frac{1-\alpha}{P([1-\alpha]A)}\right) P(A)$$

In passing, note that one way to recover (6.3) could be by invoking,

$$6.5\ \ P(P_2) = P_2^{[2]} + P_3(0) \neq P_2(P)$$

$$6.6\ \ P(P_3) = P_3^{[2]} + P_2(0) \neq P_3(P)$$

Based on (6.2) through (6.4), the H1 case could border on,

$$\frac{P_2(0)}{P([1-\alpha]A)} = -\frac{P_3(0)}{P(\alpha A)}$$

In fact, this appears to be an extension of (2.8), without necessarily positing $P_2(0) = P_3(0)$ or restricting either one to a zero value. By contrast, H0 would point to $P(\alpha A) = P([1-\alpha]A)$ while reducing both to *P(A)*.

Now, combine the two trivial results as discerned previously, toward a slightly reworked notation:

$$P(A) \equiv \alpha P(A) + [1-\alpha]P(A)$$

$$P(A) = \alpha^{-\gamma}P(\alpha A) = [1-\alpha]^{-\gamma}P([1-\alpha]A)$$

The implied re-weighting suggests a functional equation alternate to the one maintained from the outset and the Cauchy as its special case:

$$7.1\ \ P(X+Y) = \alpha^{1-\gamma}P(X) + [1-\alpha]^{1-\gamma}P(Y), \qquad \alpha \equiv \frac{X}{A} = 1 - \frac{Y}{A}$$

Rather than trying to solve it by standard methods (which may either not exist or otherwise yield pathological scenarios) or reducing to recurrent representations as before, this will be revisited and reduced to some of the ordinalcy cognates. For starters,

$$7.2\ \ F(X+Y) = k_1F(X) + k_2F(Y)$$

Among other things, one may want to seek solutions along the lines of,

$$7.3\ \rho + \frac{\rho}{\rho-1} = \rho * \frac{\rho}{\rho-1}$$

For instance, $F(X){\sim}k_2a^X,\ F(Y){\sim}k_1a^Y$, in which case $F(X+Y){\sim}k_1k_2a^{X+Y}$. [Please be sure to distinguish between the lowercase *a* versus alpha] In effect, this amounts to

$$7.4\ \ P(\alpha A) = \alpha^{1-\gamma}a^{\alpha A}$$

$$7.5\ \ P([1-\alpha]A) = [1-\alpha]^{1-\gamma}a^{[1-\alpha]A}$$

$$7.6\ \ P(A) = [\alpha(1-\alpha)]^{1-\gamma}a^{A}$$

In addition, one has to keep in mind, in line with (7.3), that

$$7.7A\ \ (k_1a^X)^{-1} + (k_2a^Y)^{-1} \equiv 1$$

In terms of the proposed notations, the above elliptic-curve family (or a space of characteristic functionals) could be rendered as,

$$7.7B\ \ P^{-1}(\alpha A) + ([1-\alpha]A)^{-1} \equiv 1$$

$$7.7C\ \ \alpha^{\gamma-1}a^{-\alpha A} + [1-\alpha]^{\gamma-1}a^{[\alpha-1]A} \equiv 1$$

Among other things, it could be of importance comparing and contrasting (7.7B) against (2.8), with equivalence only obtaining in case of $P(A) = P(0) = 0$. In a sense, this amounts to a double or multi-level fixed point, bearing in mind the structural relationships obtained.

Other than that, miscellaneous peripheral results are listed below.

$$8.1\ \ P_2P = P_2^2 + P_2P_3 = PP_2$$

$$8.2\ \ P_3P = P_3^2 + P_2P_3 = PP_3$$

$$8.3\ \ P_2P + P_3P = (P_2 + P_3)P = P^2$$

Whereas (8.3) appears to be justifying the way operators are treated on par with argument objects (again in line with the ordual premises of there being but minimal hierarchy between functions versus arguments), (8.1) and (8.2) as opposed to (6.5) and (6.6) showcase how *products* are different from levels of *compositions*, not least with an eye toward [non]commutativity.

Another glimpse at sign transitivity or homogeneity can be provided from the trivial difference between addition versus subtraction:

$$8.4\ \ (P_2 + P_3)^2 - (P_2 - P_3)^2 = 4P_2P_3 \equiv P^2 - (P - 2P_3)^2 = 4P_3(P - P_3) \sim 4\alpha[1-\alpha]P^2$$

This appears to have remote resemblance to Euler's Beta density, with the variations of $P_3(0)$ making a difference even under *P(0)* held fixed as long as the two are not tantamount. Otherwise, not only would the residuale $P_2(0)$ term (or element of basis) be trivially zero, the entire setup collapses to cardinal singularity.

While at it, note that, as long as the squared difference in (8.4) resembles the discriminant of a quadratic equation (or a determinant of a quadratic form), the latter could be implied as either,

$$8.5A\ \ P_2X^2 + PX + P_3 = 0$$

$$8.5B\ \ P_3X^2 + PX + P_2 = 0$$

The implied X could be inferred at either one,

$$8.6A\ \ X = \frac{-P \pm (P_2 - P_3)}{2P_2} = -1\ OR\ \frac{P_3}{P_2}$$

$$8.6B\ \ X = \frac{-P \pm (P_2 - P_3)}{2P_3} = -\frac{P_2}{P_3}\ OR\ 1$$

Now, consider an alternative to (8.3), or a generalization of the distributed form:

$$P_2P \sim \beta P^2, \qquad P_3P \sim [1-\beta]P^2$$

It is straightforward to see that the betas amount to alphas for all practical purposes, if the above were to be reduced to:

$$P_2 \sim \beta P \sim \alpha P, \qquad P_3 \sim [1-\beta]P \sim [1-\alpha]P$$

To detrivialize this somewhat, the above could be induced to an *n>2* setting:

$$P_2 P^{n-1} = \alpha P^n,\ P_3 P^{n-1} = [1-\alpha] P^n$$

This could in turn be viewed as a recursive functional equation, reduced to parity:

$$P^n = P * (\frac{P_2}{\alpha})^{n-1} = P * (\frac{P_3}{1-\alpha})^{n-1}$$

In other words, it follows that

$$8.7A\ \ P_2^n = \alpha^n P^n = \alpha^n (P_2 + P_3)^n, \qquad P_3^n = [1-\alpha]^n P^n = [1-\alpha]^n (P_2 + P_3)^n$$

In a well-defined sense, then, a re-weighting redefines the power structure as,

$$8.7B\ \ P^n = \alpha^{1-n} P_2^n + [1-\alpha]^{1-n} P_3^n = (P_2 + P_3)^n$$

This might resemble (5.5), (7.1), and (7.7), while clearly allowing for a power generalization of the underlying definition or identity as posited from the outset. However, apart from Cauchy extension, the above may hint at that for complex numbers while straddling linearity *and* non-linearity *a la* CES or Lame functions (for which H1 readily obtains).

One alternate extension would pertain to composition levels as opposed to powers. An induction could naturally proceed as follows:

$$P\big(P(A)\big) = P\big(P_2(\alpha A) + P_3([1-\alpha]A)\big) = P_2\big(P_2(A)\big) + P_3(P_3([1-\alpha]A))$$

$$8.8A\ \ P^{[n]}(A) = P_2^{[n]}(\alpha A) + P_3^{[n]}([1-\alpha]A)$$

Now, bearing in mind the weighted-average accounting identity we have made a heavy use of thus far, it is straightforward to see H1 (homogeneity of degree one) in composition levels:

$$8.8B\ \ P_2^{[n]}(\alpha A) = \alpha P^{[n]}(A), \qquad P_3^{[n]}([1-\alpha]A) = [1-\alpha] P^{[n]}(A)$$

In the meantime, from (8.8B) it follows that (3.3) holds intact at any level.

If one were to afford a trick by embarking on the corner cases of (8.8A) and re-applying these to (8.8B), it could follow that:

$$8.8C\ \ P_2^{[n]}(\alpha A) = \alpha \left[P_2^{[n]}(A) + P_3^{[n]}(0)\right],\ P_3^{[n]}([1-\alpha]A) = [1-\alpha]\left[P_3^{[n]}(A) + P_2^{[n]}(0)\right]$$

In a sense, the implied affinity partially denies homogeneity—even though the proposed is just one possibility, and hardly the more reliable one out there. Again, for practical purposes, these very structures could posit $P_2^{[n]}(0)$ and $P_3^{[n]}(0)$ as zeros, by holding the alpha values near its respective corners. More rigorously, as before, these could be solved for as functional equations of a recurrent type. Not least, (8.8A) could mechanically be rewritten as,

$$8.8D\ \ P^{[n]}(A) \equiv \alpha^{\gamma_2(n)} P_2^{[n]}(A) + [1-\alpha]^{\gamma_3(n)} P_3^{[n]}(A)$$

Keeping in mind the trivial identity accounting, it follows that:

$$8.8E\ \alpha P^{[n]}(A) \equiv \alpha^{\gamma_2(n)} P_2^{[n]}(A), \qquad [1-\alpha]P^{[n]}(A) \equiv [1-\alpha]^{\gamma_3(n)} P_3^{[n]}(A)$$

In other words, a dual Diophantine is captured in,

$$P^{[n]}(A) = \alpha^{\gamma_2(n)-1} P_2^{[n]}(A) = [1-\alpha]^{\gamma_3(n)-1} P_3^{[n]}(A)$$

Somewhat as before with corner traces deployed, it trivially holds that

$$\alpha^{1-\gamma_2(n)} P^{[n]}(A) + P_3^{[n]}(0) = [1-\alpha]^{1-\gamma_3(n)} P^{[n]}(A) + P_2^{[n]}(0) = P^{[n]}(A)$$

In effect, it is straightforward to retrieve the implied homogeneity terms:

$$8.8F\ \ \alpha^{\gamma_2(n)} = \alpha(1 - \frac{P_3^{[n]}(0)}{P^{[n]}(A)})^{-1}$$

$$8.8G\ \ [1-\alpha]^{\gamma_3(n)} = [1-\alpha](1 - \frac{P_2^{[n]}(0)}{P^{[n]}(A)})^{-1}$$

Incidentally, the above may do a fair job reconciling linearity and nonlinearity, or *H1* versus gamma-homogeneity ($H\gamma$). Moreover, should these be substituted for (8.8E), it would turn out that,

$$8.8H\ P^{[n]}(A) \equiv (1 - \frac{P_3^{[n]}(0)}{P^{[n]}(A)})^{-1} P_2^{[n]}(A), \qquad P^{[n]}(A) \equiv (1 - \frac{P_2^{[n]}(0)}{P^{[n]}(A)})^{-1} P_3^{[n]}(A)$$

In other words, the above relationships do hold identically. The proposed *(A, H)* path might prove more of an *azimuthality* trajectory than *gradiency*, and yet it does yield some controls taking on the simplicity-in-completeness (or *plaena*) form.

It is noteworthy that H1, or homogeneity of degree one, should not be downplayed, as its parsimony offers some meaningful insights early on. Consider again a corner distribution which turns out to be a mixed or generalized one with respect to the core operator *P*.

$$P(A) = P_2(0) + P_3(A) = P_2(\alpha A) + P_3([1-\alpha]A) = \alpha[P_2(A) - P_3(A)] + P_3(A)$$

$$9.1\ \ P(A) = \alpha P(0) + P_3(A) = P_2(0) + P_3(A)$$

$$9.2\ \ P(A) = [1-\alpha]P(0) + P_2(A) = P_2(A) + P_3(0)$$

$$9.3\ \ P_2(0) = \alpha P(0), \qquad P_3(0) = \pm[1-\alpha]P(0)$$

$$9.4\ \ \pm\alpha P_3(0) = [1-\alpha]P_2(0)$$

Not only do these add up to *P(0)* in formal compliance with the original premises, they also posit a relationship among the three sub-operators (or within the basis) akin to that between natural versus real numbers, with the indexed sub-operators acting as a kind of conjugate infinitesimals or complementary differentials over and above unity orthants. Not least, it is partially on this premise that a calculus of the quasi-differential and integrative type will be introduced, with the latter deploying an operator straddling the likes of summation and set power. Moreover, as per (9.4),

$$P_3 \sim \frac{1-\alpha}{\alpha} P_2 \sim (\rho - 1) P_2$$

The above may hint at the two sub-operators being dually inter-related akin to the ordual basis:

$$(A, a)^{\rho-1} \sim (a, A)$$

The sign reversal in (9.4) may pertain to either an infinitesimal-like status or, more generally, the left versus right foci (actions, partial relationships, or *m>2* power of focal distribution). Along similar lines, it can moreover be shown that,

$$9.5 \;\; \alpha \sim \frac{1}{1 \pm P_3(0)/P_2(0)} \sim \frac{P_2(0)}{P(0)}$$

$$9.6 \;\; P^{[n]}(A) = \alpha P^{[n]}(0) + P_3^{[n]}(A)$$

When it comes to the counterparts of differentiation and integration, it is straightforward to see that:

$$9.7 \;\; \dot{P}(t) \equiv \frac{\Delta P}{\Delta} = \frac{P(t+\Delta) - P(t)}{\Delta} = \frac{P_3(\Delta) \mp P_3(0)}{\Delta}$$

$$9.8A \;\; P(\Delta) - \Delta P = P(0)$$

What (9.8) showcases is how the null-operator can act as an effective *commutator*. By maintaining a convention for integration, it becomes apparent that:

$$9.8B \;\; \Delta^{-1}\Delta P \equiv {}^{\ulcorner}_{\lrcorner}\Delta P \equiv P = {}^{\ulcorner}_{\lrcorner}[P(\Delta) - P_2(\Delta) + P_3(\Delta)]$$

As an extension of (9.7) and (9.8), it can be proposed that:

$$9.9A \;\; P^{[l]}(\Delta^k) = \Delta^{k+l}$$

$$9.9B \;\; \frac{\Delta^n P}{\Delta^n} = \frac{P^{[n+1]}(\Delta)}{P^{[n]}(\Delta)}$$

$$9.9C \;\; \Delta^{\pm i} P^{[n]} = P^{[n \pm i]}$$

The above makes use of the convention: $9.9D \;\; P^{[0]}(a) = \{a\}$, so that in particular $9.9E \;\; P^{[0]} \sim \{1\}$, and $9.9F \;\; P^{[0]}(\Delta) = \{\Delta\}$. Furthermore,

$$9.9G \;\; \frac{\Delta^\gamma P^{[0]}}{\Delta^\delta P^{[0]}} \sim \frac{\partial^\gamma}{\partial^\delta}$$

$$9.9H \quad \frac{\Delta^{\gamma} P^{[0]}(a)}{\Delta^{\delta} P^{[0]}(b)} \sim \frac{\partial^{\gamma} a}{\partial^{\delta} b}$$

$$9.9I \quad \frac{\Delta^{\gamma} P^{[0]}(a)}{\Delta^{\delta} P^{[0]}(a)} = \{\Delta^{\gamma-\delta}\}$$

Parallelisms & Extensions

The proposed preliminary survey amounts to but a partial and very minor excerpt of the remote yet closely intertwined results that have been obtained along ordual lines, in particular by building on the family of *perceptive calculi*. In more detailed and specialized papers, it will be shown how this P-calculus distinguishes between the [*residuale*] likes of zero, infinitesimals, differentials, constants, and variables. This may for one be seen in line with the '*levels of variableness*' approach as naturally spawned by an ordinalcy agenda.

On the other hand, not only does the integration cognate act as a dual of the differential, it moreover amounts to the extension of set power, in which light the latter can be rethought as follows: $\rho^{\ulcorner\lrcorner} \sim (2^{\ulcorner\lrcorner})^{\rho-1}$. Under cardinalcy, or rho tending to 2, this reduces to $2^{\aleph_0}$.

After having provided an exposition of another algebra, the RP calculus (for '*rational perception*' as an alternate to rational expectations) to accommodate the notion of *strategic rationality* I introduced back in 1999, a remarkable posterior similarity will be demonstrated between the two apparatuses, albeit centered around rather disjoint prior premises.

Apart from generalizing the Cauchy functional equation and the Taylor expansion alike, it will be proposed that a straightforward recurrent approach similar to the P-calculus can formalize the common foundations for numbers (generalized values) and their routine operators, notably integration. One other parsimonious yet ubiquitous application of the rho-approach to ordinalcy will be deployed as a natural bridging device over a variety of operations, likely to revisit the ABC conjecture. Not least, in a series of alternate calculi, aside from CES or Lame operators yet to be illustrated, the entire setup suggests how Fermat's LP can be approached alongside its extensions.