Algebraic interpretation of octonion dark matter

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Abstract

The vast majority of the dark matter in the universe is believed to be nonbaryonic, which means that it contains no atoms and does not interact with ordinary matter via electromagnetic forces. The nonbaryonic dark matter includes neutrinos, and possibly hypothetical entities such as axions, or supersymmetric particles. Unlike baryonic dark matter, nonbaryonic dark matter does not contribute to the formation of the elements in the early universe (big bang nucleosynthesis) and so its presence is revealed only via its gravitational attraction. The nonbaryonic dark matter is evident through its gravitational effect only. There are two type of nonbaryonic dark matter respectively defined as hot dark matter and cold dark matter. For the existence of nonbaryonic dark matter in this universe we can used the higher dimensional division algebra. There exists four normed division algebras: the real numbers, complex numbers, quaternions, and octonions. Since, the octonions are the last division algebra, so we can easily described the octonion space as the combination of two quaternionic spaces namely gravitational G-space and electromagnetic EM-space. Thus, describing the octonion eight dimensional space as the combination of two quaternionic spaces (namely associated with the electromagnetic interaction (EM-space) and linear gravitational interaction (G-space)), we have reexamined the unified picture of EM-G space in terms of octonionic formulation in consistent manner. Consequently, we have obtained the fundamental components of angular momentum and torque for unified theory of gravi- electromagnetism. After that we relate these components in terms of octonionic dark matter and dark energy. In this formulation, it should be noted that the unified octonionic rotation energies in terms of angular momentum and torque will be responsible for the existence of dark matter and dark energy in this universe.

Key Words: octonions, angular momentum, torque, dark matter, dark energy.

1.Introduction

We have analyzed the role of division algebras (octonions) in gravity and nonbaryonic dark matter. In astronomy and cosmology, dark matter is matter that is inferred to exist from gravitational effects on visible matter and background radiation, but is undetectable by emitted or scattered electromagnetic radiation. The nonbaryonic dark matter [1, 2, 3] includes neutrinos, and possibly hypothetical entities such as axions, or supersymmetric particles. Unlike baryonic dark matter, nonbaryonic dark matter does not contribute to the formation of the elements in the early universe (big bang nucleosynthesis) and so its presence is revealed only via its gravitational attraction.). Thus, the nonbaryonic dark matter is evident through its gravitational effect only.There are two type of nonbaryonic dark matter respectively defined as hot dark matter and cold dark matter.

In mathematical points of views, the two fundamental mathematical structures (division algebras) a physicist uses in his everyday life are the real (R) and the complex (C) numbers. Complex numbers are described as pairs of real numbers with a specific multiplication laws. One can however go even further and build two other sets of numbers, known in mathematics as quaternions (H) [4, 5] and

octonions (O) [6, 7]. The quaternions, formed as pairs of complex numbers are non-commutative whereas the octonions, formed as pairs of quaternion numbers are both non-commutative and nonassociative. The four sets of numbers are mathematically known as division algebras. The octonions are the last division algebra, no further generalization being consistent with the laws of mathematics.

So, there exists four normed division algebras [8]: the real numbers, complex numbers, quaternions,and octonions, i.e.

- Scalars are represented by 1 number.
- Complex numbers are represented by 2 numbers (1 real and 1 imaginary).
- Quaternions are represented by 4 numbers (1 real and 3 imaginary).
- Octonions are represented by 8 numbers (1 real and 7 imaginary).

In this paper,we have described the octonion space as the combination of two quaternionic spaces namely gravitational G-space and electromagnetic EM-space. Consequently, we have formulated the theory of dark matter in terms of octonion variables. It is emphasized that the dark matter neither emits nor absorbs light or electromagnetic radiation at any significant level. We have also represented the generalized form of octonion angular momentum and octonion torque for gravitational space (G-space), Electromagnetic space(EM-space) and also for gravi-electromagnetic space.

2. Octonion Algebra

An octonion $X \in O$ is expressed [9, 10, 11] as a set of eight real numbers

$$X \equiv (X_0, X_1, ...X_7) \equiv X_0 e_0 + \sum_{A=1}^{7} X_A e_A, \quad (2.1)$$

Where $e_A (A = 1,2,3,.....7)$ are imaginary octonion units and e_0 is the multiplicative unit element. The octet $(e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7)$ is known as the octonion basis and its elements satisfy the following multiplication rules [12, 13, 14],

$$e_0 = 1, e_0 e_A = e_A e_0 = e_A$$
$$e_A e_B = -\delta_{AB} e_0 + f_{ABC} e_C . (A, B, C = 1,2,.....,7) \quad (2.2)$$

The structure constants f_{ABC} are completely antisymmetric and take the value 1 for the following combinations

$$f_{ABC} = +1$$
$$\forall (ABC) \mapsto (123), (471), (257), (165), (624), (543), (736) \quad (2.3)$$

The octonion algebra O is described over the algebra of rational numbers having the vector space of dimension 8. The relations among octonion basis elements are defined as

$$[e_A, e_B] = 2 f_{ABC} e_C \; ; \{e_A, e_B\} = -\delta_{AB} e_0 \; ; e_A (e_B e_C) \neq (e_A e_B) e_C \quad (2.4)$$

where brackets [ ] and { } are used respectively for commutation and the anti commutation relations while δ_{AB} is the usual Kronecker delta-Dirac symbol. Octonion conjugate is thus defined as,

$$\overline{X} = X_0 e_0 - \sum_{A=1}^{7} X_A e_A \quad (2.5)$$

The octonion can be decomposed in terms of its scalar (Sc(X)) and vector (Vec(X)) parts as

$$Sc(X) = \frac{1}{2}(X + \overline{X}) \equiv X_0 ; Vec(X) = \frac{1}{2}(X - \overline{X}) = \sum_{A=1}^{7} X_A e_A . \quad (2.6)$$

The norm of the octonion $N(X)$ is defined as

$$N(X) = \overline{X} X = X \overline{X} = \sum_{\alpha=0}^{7} X_\alpha^2 e_0 \quad (2.7)$$

which is zero if X = 0, and is always positive otherwise. It also satisfies the following property of normed algebra

$$N(XY) = N(X) N(Y) = N(Y) N(X) \quad (2.8)$$

3. Generalized Electromagnetic (EM) and Linear Gravitational (G) Space

The octonion space (eight dimensional) may be consider as the combination of two quaternionic spaces namely associated with the gravitational interaction (G-space) and electromagnetic interaction(EM-space) [15, 16]. Thus the octonionic (gravitational-electromagnetic) space defined as

$$O \mapsto (O_g, O_{em}); \quad (3.1)$$

Where $(O_{g-space})$ is linear gravitational(LG) space consists e_0, e_1, e_2, e_3 octonion basis and $(O_{em-pace})$ is octonionic electromagnetic (EM) space consists e_4, e_5, e_6, e_7. Any physical quantity X ∈ O may be expressed as

$$X = (X_g, X_{em}) \mapsto \sum_{j=o}^{3} (X_g^j e_j) + e_7 \sum_{j=0}^{3} (X_{em}^j e_j) . \quad (3.2)$$

Accordingly ,the octonionic space radius vector $\Re$ also may be written as of the two quaternionic space (G-space & EM-space) in the terms of eight dimensional space as

$$\Re_{oct} = (\Re_g, \Re_{em})$$
$$= [\Re_{em}] + \varepsilon e_7 [\Re_g] = (\Re^0_{em}, \Re^j_{em}) + \varepsilon e_7 (\Re^0_g, \Re^j_g) \quad (3.3)$$

Where $\Re_{em} = (\Re^0_{em}, \Re^j_{em}) = \Re^0_{em} + \sum_{j=1}^{3} \Re^j_{em}$ and

$$\Re_g = (\Re^0_g, \Re^j_g) = \Re^0_g + \sum_{j=0}^{3} \Re^j_g$$

In the same way the octonionic linear momentum vector P also may be written as of the two quaternionic space (G-space & EM-space) in the terms of eight dimensional space as

$$P_{oct} = (P_g, P_{em})$$
$$= [P_{em}] + \varepsilon e_7 [P_g] = (P^0_{em}, P^j_{em}) + \varepsilon e_7 (P^0_g, P^j_g) \quad (3.4)$$

Where P_{em} is the momentum in electromagnetic space , P_g is momentum in the gravitational space and represented as

$$P_{em} = (P^0_{em}, P^j_{em}) = P^0_{em} + \sum_{j=1}^{3} P^j_{em} \quad (3.5)$$

and

$$P_g = (P^0_g, P^j_g) = P^0_g + \sum_{j=0}^{3} P^j_g \quad (3.6)$$

4 Octonion Momentum in gravi-electromagnetic space

The definition of octonion angular momentum is considered to be embedded in the electromagnetic and gravitational theories .In the octonion space *O*.the octonion angular momentm *L*[17] can be defined from the octonion linear momentum $\mathbb{P}$ and radius vector $\Re$,i.e.

$L_{oct} = \Re_{oct} o P_{oct}$ may be expressed as

$$L_{oct} = L_o e_o + \sum_{i=1}^{6} L_i e_i + L_7 e_7 \quad (4.1)$$

which may be further rewritten as in the form

$$L_{oct} \equiv (L_{em}, L_g, L_{em/g-em}) \quad (4.2)$$
$$(L_{em} + \varepsilon^2 L_g) e_1 + (L_{em} + \varepsilon^2 L_g) e_2 + (L_{em} + \varepsilon^2 L_g) e_3 +$$
$$\varepsilon (L_{em-g} / L_{g-em}) e_4 + \varepsilon (L_{em-g} / L_{g-em}) e_5 +$$
$$\varepsilon (L_{em-g} / L_{g-em}) e_6 + \varepsilon (L_{em-g} / L_{g-em}) e_7$$

Where L_{em} is angular momentum with respect to electromagnetic space, L_g is angular momentum with respect to gravitational space, $L_{em/g-em}$ is angular momentum with respect to gravi-electromagnetic space.

$$\Re_{oct} = \{\Re^0_{em} e_0 + \Re^1_{em} e_1 + \Re^2_{em} e_2 + \Re^3_{em} e_3\} + \varepsilon e_7 \{\Re^0_g e_0 + \Re^1_g e_1 + \Re^2_g e_2 + \Re^3_g e_3\}$$
$$= \Re^0_{em} e_0 + \Re^1_{em} e_1 + \Re^2_{em} e_2 + \Re^3_{em} e_3 + \varepsilon \{\Re^0_g e_7 + \Re^1_g e_4 + \Re^2_g e_5 + \Re^3_g e_6\} \quad (4.3)$$

$$P_{oct} = \{P^0_{em} e_0 + P^1_{em} e_1 + P^2_{em} e_2 + P^3_{em} e_3\} + \varepsilon e_7 \{P^0_g e_0 + P^1_g e_1 + P^2_g e_2 + P^3_g e_3\}$$
$$= P^0_{em} e_0 + P^1_{em} e_1 + P^2_{em} e_2 + P^3_{em} e_3 + \varepsilon \{P^0_g e_7 + P^1_g e_4 + P^2_g e_5 + P^3_g e_6\} \quad (4.4)$$

$$\begin{aligned} L_{oct} = {} & \{\Re^0_{em} P^0_{em} - \Re^1_{em} P^1_{em} - \Re^2_{em} P^0_{em} - \Re^3_{em} P^3_{em}\} + \\ & e_1 \{\Re^0_{em} P^1_{em} + \Re^1_{em} P^0_{em} + \Re^2_{em} P^3_{em} - \Re^3_{em} P^2_{em}\} \\ & + e_2 \{\Re^0_{em} P^2_{em} + \Re^2_{em} P^0_{em} + \Re^3_{em} P^1_{em} - \Re^1_{em} P^3_{em}\} \\ & + e_3 \{\Re^0_{em} P^3_{em} + \Re^3_{em} P^0_{em} + \Re^1_{em} P^2_{em} - \Re^2_{em} P^1_{em}\} \\ & + \varepsilon \begin{pmatrix} e_4 \{\Re^0_{em} P^1_g - \Re^1_{em} P^0_g + \Re^3_{em} P^2_g - \Re^2_{em} P^3_g\} + \\ e_5 \{\Re^0_{em} P^2_g - \Re^2_{em} P^0_g + \Re^1_{em} P^3_g - \Re^3_{em} P^1_g\} + \\ e_6 \{\Re^0_{em} P^3_g - \Re^3_{em} P^0_g + \Re^1_{em} P^2_g - \Re^2_{em} P^1_g\} \end{pmatrix} \\ & + e_7 \begin{Bmatrix} \Re^0_{em} P^0_g + \Re^1_{em} P^1_g + \Re^2_{em} P^2_g + \Re^3_{em} P^3_g + \\ \Re^0_g P^0_{em} - \Re^1_g P^1_{em} - \Re^2_g P^2_{em} - \Re^3_g P^3_{em} \end{Bmatrix} \\ & + \varepsilon^2 \begin{bmatrix} e_1 \{\Re^1_g P^0_g - \Re^0_g P^1_g + \Re^3_g P^2_g - \Re^2_g P^3_g\} + \\ e_2 \{\Re^2_g P^0_g - \Re^0_g P^2_g + \Re^1_g P^3_g - \Re^3_g P^1_g\} \\ + e_3 \{\Re^3_g P^0_g - \Re^0_g P^3_g + \Re^2_g P^1_g - \Re^1_g P^2_g\} + \\ \{-\Re^0_g P^0_g - \Re^1_g P^1_g - \Re^2_g P^2_g - \Re^3_g P^3_g\} \end{bmatrix} \end{aligned} \quad (4.5)$$

Where

$$\begin{aligned} L_{em} = {} & \{\Re^0_{em} P^0_{em} - \Re^1_{em} P^1_{em} - \Re^2_{em} P^0_{em} - \Re^3_{em} P^3_{em}\} + \\ & e_1 \{\Re^0_{em} P^1_{em} + \Re^1_{em} P^0_{em} + \Re^2_{em} P^3_{em} - \Re^3_{em} P^2_{em}\} \\ & + e_2 \{\Re^0_{em} P^2_{em} + \Re^2_{em} P^0_{em} + \Re^3_{em} P^1_{em} - \Re^1_{em} P^3_{em}\} + \\ & e_3 \{\Re^0_{em} P^3_{em} + \Re^3_{em} P^0_{em} + \Re^1_{em} P^2_{em} - \Re^2_{em} P^1_{em}\} \end{aligned} \quad (4.6)$$

$$L_g = e_1\left\{\Re^1_g \mathrm{P}^0_g - \Re^0_g \mathrm{P}^1_g + \Re^3_g \mathrm{P}^2_g - \Re^2_g \mathrm{P}^3_g\right\}+ e_2\left\{\Re^2_g \mathrm{P}^0_g - \Re^0_g \mathrm{P}^2_g + \Re^1_g \mathrm{P}^3_g - \Re^3_g \mathrm{P}^1_g\right\} + e_3\left\{\Re^3_g \mathrm{P}^0_g - \Re^0_g \mathrm{P}^3_g + \Re^2_g \mathrm{P}^1_g - \Re^1_g \mathrm{P}^2_g\right\}+ \left\{-\Re^0_g \mathrm{P}^0_g - \Re^1_g \mathrm{P}^1_g - \Re^2_g \mathrm{P}^2_g - \Re^3_g \mathrm{P}^3_g\right\} \quad (4.7)$$

$$L_{em/g-em} = \begin{pmatrix} e_4\left\{\Re^0_{em} \mathrm{P}^1_g - \Re^1_{em} \mathrm{P}^0_g + \Re^3_{em} \mathrm{P}^2_g - \Re^2_{em} \mathrm{P}^3_g\right\}+ \\ e_5\left\{\Re^0_{em} \mathrm{P}^2_g - \Re^2_{em} \mathrm{P}^0_g + \Re^1_{em} \mathrm{P}^3_g - \Re^3_{em} \mathrm{P}^1_g\right\}+ \\ e_6\left\{\Re^0_{em} \mathrm{P}^3_g - \Re^3_{em} \mathrm{P}^0_g + \Re^1_{em} \mathrm{P}^2_g - \Re^2_{em} \mathrm{P}^1_g\right\} \end{pmatrix} + e_7\begin{Bmatrix} \Re^0_{em} \mathrm{P}^0_g + \Re^1_{em} \mathrm{P}^1_g + \Re^2_{em} \mathrm{P}^2_g + \Re^3_{em} \mathrm{P}^3_g + \\ \Re^0_g \mathrm{P}^0_{em} - \Re^1_g \mathrm{P}^1_{em} - \Re^2_g \mathrm{P}^2_{em} - \Re^3_g \mathrm{P}^3_{em} \end{Bmatrix} \quad (4.8)$$

5. Octonion Torque formulation in Gravi-Electromagnetc Space

In this section we have defined octonion torque[17] in the electromagnetic and gravitational space. The definition of octonion torque is supposed to contain all the information about torque,work and energy in the gravi-electromagnetic space .This discussion have some resemblance with the existing gravitational and electromagnetic theories.

Then the octonion torque is given by

$$\tau_{oct} = \Re_{oct} \circ F_{oct} \quad (5.1)$$

with

$$\begin{aligned} \Re_{\mathrm{oct}} &= \left\{\Re^0_{em} e_0 + \Re^1_{em} e_1 + \Re^2_{em} e_2 + \Re^3_{em} e_3\right\}+ \varepsilon e_7\left\{\Re^0_g e_0 + \Re^1_g e_1 + \Re^2_g e_2 + \Re^3_g e_3\right\} \\ &= \Re^0_{em} e_0 + \Re^1_{em} e_1 + \Re^2_{em} e_2 + \Re^3_{em} e_3 + \\ &\quad \varepsilon\left\{\Re^0_g e_7 + \Re^1_g e_4 + \Re^2_g e_5 + \Re^3_g e_6\right\} \quad (5.2) \end{aligned}$$

$$\begin{aligned} \mathrm{F}_{\mathrm{oct}} &= \left\{F^0_{em} e_0 + F^1_{em} e_1 + F^2_{em} e_2 + F^3_{em} e_3\right\}+ \varepsilon e_7\left\{F^0_g e_0 + F^1_g e_1 + F^2_g e_2 + F^3_g e_3\right\} \\ &= F^0_{em} e_0 + F^1_{em} e_1 + F^2_{em} e_2 + F^3_{em} e_3 \\ &+ \varepsilon\left\{F^0_g e_7 + F^1_g e_4 + F^2_g e_5 + F^3_g e_6\right\} \quad (5.3) \end{aligned}$$

Then according to the above we have generalized the torque components which are corresponding to electromagnetic , gravitational and gravito-electromagnetic space can be represented as

$$\begin{aligned} \tau_{em} = {} & \Re^0_{em} F^0_{em} - \Re^1_{em} F^0_{em} - \Re^2_{em} F^2_{em} - \Re^3_{em} F^3_{em} + \\ & e_1\left\{\Re^0_{em} F^1_{em} + \Re^1_{em} F^0_{em} + \Re^2_{em} F^3_{em} - \Re^3_{em} F^2_{em}\right\} \\ & + e_2\left\{\Re^0_{em} F^2_{em} - \Re^1_{em} F^3_{em} + \Re^2_{em} F^0_{em} + \Re^3_{em} F^1_{em}\right\} + \\ & e_3\left\{\Re^0_{em} F^3_{em} + \Re^1_{em} F^2_{em} - \Re^2_{em} F^1_{em} + \Re^3_{em} F^0_{em}\right\} \quad (5.4) \end{aligned}$$

$$\begin{aligned} \tau_g = {} & e_1\left\{\Re^1_g F^0_g - \Re^0_g F^1_g + \Re^3_g F^2_g - \Re^2_g F^3_g\right\} \\ & + e_2\left\{\Re^2_g F^0_g - \Re^0_g F^2_g + \Re^1_g F^3_g - \Re^3_g F^1_g\right\} \\ & + e_3\left\{\Re^3_g F^0_g - \Re^0_g F^3_g + \Re^2_g F^1_g - \Re^1_g F^2_g\right\} \quad (5.5) \end{aligned}$$

$$\begin{aligned} \tau_{em-g/g-em} = {} & e_4\left\{\Re^0_{em} F^1_g - \Re^1_{em} F^0_g + \Re^3_{em} F^2_g - \Re^2_{em} F^3_g\right\} \\ & + e_5\left\{\Re^0_{em} F^2_g - \Re^2_{em} F^0_g + \Re^1_{em} F^3_g - \Re^3_{em} F^1_g\right\} \\ & + e_6\left\{\Re^0_{em} F^3_g - \Re^3_{em} F^0_g + \Re^2_{em} F^1_g - \Re^1_{em} F^2_g\right\} \\ & + e_7\left\{\Re^0_{em} F^0_g + \Re^1_{em} F^1_g + \Re^2_{em} F^2_g + \Re^3_{em} F^3_g\right\} \\ & + \left\{\Re^0_g F^1_{em} + \Re^1_g F^0_{em} - \Re^2_g F^3_{em} + \Re^3_g F^3_{em}\right\} \\ & + \left\{\Re^0_g F^2_{em} + \Re^2_g F^0_{em} + \Re^1_g F^3_{em} - \Re^3_g F^1_{em}\right\} \\ & + \left\{\Re^0_g F^3_{em} + \Re^3_g F^0_{em} + \Re^2_g F^1_{em} - \Re^1_g F^2_{em}\right\} \\ & + \left\{\Re^0_g F^0_{em} - \Re^1_g F^1_{em} - \Re^2_g F^2_{em} - \Re^3_g F^3_{em}\right\} \quad (5.6) \end{aligned}$$

6. Dark Matter

The Dark Matter [18, 19, 20] is a type of matter hypothesized to account for a large part of the total mass in the universe.. Early theories of dark matter concentrated on hidden heavy normal objects, such as black holes, neutron stars, faint old white dwarfs, brown dwarfs, as the possible candidates for dark matter, collectively known as massive compact halo objects or MACHOs.. The majority of dark matter in the universe cannot be baryons, and thus does not form atoms. This nonbaryonic dark matter is evident through its gravitational effect. Consequently, the most commonly held view was that dark matter is primarily non-baryonic, made of one or more elementary particles other than the usual electrons, protons, and neutrons. The most commonly proposed particles [18, 19, 20] then became WIMPs (Weakly Interacting Massive Particles, including neutralinos), or axions, or sterile neutrinos, though many other possible candidates have been proposed.. Nonbaryonic dark matter is classified in terms of the mass of the particles that is assumed to make it up, or the typical velocity dispersion of those particles (since more massive particles move more slowly). There

are mainly two prominent hypotheses on nonbaryonic dark matter[21, 22, 23], called cold dark matter (CDM) and hot dark matter (HDM). Determining the nature of this dark matter is one of the most important problems in modern cosmology and particle physics.

Conclusion and Discusson

The dark matter has been considered as a type of matter hypothesized to account for a large part of the total mass in the universe. Dark matter cannot be seen directly with telescopes which is neither emits nor absorbs light or other electromagnetic radiation at any significant level. Instead, its existence and properties are inferred from its gravitational effects on visible matter, radiation and the large scale structure of the universe. Here matter and energy (which special relativity tells us are equivalent) are distinguished by their different dependence on the cosmic volume: matter density decreases with the inverse of the volume, while energy density remains (approximately) constant. Only about 4.6% of the mass-energy of the universe is ordinary matter, about 23% is thought to be composed of dark matter. The remaining 72% is thought to consist of dark energy, an even stranger component, distributed almost uniformly in space and with energy density non-evolving or slowly evolving with time. Thus, describing the octonion eight dimensional space as the combination of two quaternionic spaces (namely associated with the electromagnetic interaction (EMspace) and linear gravitational interaction (G-space)), we have reexamined the unified picture of EM-G space in terms of octonionic formulation in consistent manner. Consequently, we have obtained the fundamental components of angular momentum and torque for unified theory of gravi-electromagnetism.After that we relate these components in terms of octonionic dark matter and dark energy. In this formulation, it should be noted that the unified octonionic rotation energies in terms of angular momentum and torque will be responsible for the existence of dark matter and dark energy in this universe.

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