On the smallest Surface Scale and Dark Energy

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In this study we want to propose a heuristic model to compute and to interpret the dark energy content of our universe. To this purpose we include the mass-energy of the static gravitational field and compute its effect at very small distances. From its analysis, we obtain for the smallest surface scale for the empty space $\tau = 2h\gamma/5c^3$. After that we show, how this result can be used to compute a natural energy cutoff k_c for all quantum fields and study its utility in computing the dark energy density and its implications on the content of fermionic and bosonic elementary fields. Indeed for the vacuum equation of state $w = p_{vac}/\rho_{vac}$ we obtain $w = -128\pi^2/(15\Delta N)$, where $\Delta N = N_f - N_b$ represents the difference between the number of species of fermions and bosons. Finally comparing our result with the measured cosmological parameters, we discus general constraints on the field content beyond the Standard Model of the elementary particles.

1 Introduction

A common aspect of many different approaches to quantum gravity such as string theory (see *e.g.* [1]), causal sets [2, 3], spin foams [4], causal dynamical triangulation (CDT) [5, 6] and loop quantum gravity [7, 8] is the presence of a smallest length scale. The experimental search of such a scale has gained in the last years a lot of importance and concrete projects have already been started [9,10]. A first phenomenological review of these approaches to quantum gravity can be found for example in [11–14].

The presence of a smallest scale has usually the advantage to solve the problems associated with infinities, if one tries to quantize a non renormalizable theory like gravity. In particular quartic divergencies emerge when one interprets dark energy as being originated by quantum fluctuations and this is independent on the curvature. The problem is that, even if the final result is finite, it turns out to be anyway many orders of magnitudes above the observed value [15].

Interesting new approaches have been developed in the last years from the point of view of supersymmetry, the renormalization procedure [16,17], the renormalization group flow [18], the holographic principle [19] and stability considerations concerning the Minkowski space-time [20]. In all such approaches the results are all improved and some of them are also able to predict the dark energy with the correct order of magnitude, even if not yet precisely.

The purpose of our work is to introduce a new method to precisely predict the dark energy density and to elucidate its nature. As we will show, our approach has implications also on the possible field content of dark matter, showing a strict connection between the two aspects.

An important point of our investigation is an idea introduced by Heim [21], which consists in considering also the effects of the field mass $\mu = E/c^2$ associated to the energy content E of the gravitational field generated by a central massive body. Accordingly one should include its contribution in a consistent way also in the phenomenological stress energy tensor $T_{\mu\nu}$ of the Einstein field equations. A first important consequence of this idea, which together with other considerations lead to an alternative approach to quantum gravity, is again the existence of a smallest scale. In [22–24] the smallest scale surface is called "metron" and Heim has found for it the result $\tau_H = 3h\gamma/8c^3$. We want here to remind the reader, that the Heim discretization is not intended to be a kind of "atomism" of elementary space-time elements as it might seem to be: indeed the elementary surface τ has to be intended as an internal physical characteristic of extended "structures" and can not be thought separated from the others. In this context it can be shown, that a particle can be viewed as a dynamical structure of specific condensed states of "metrons", which correspond to discretized eigenstates of the curvature tensor. In this approach one can also try to compute particle masses. For example for the mass of the lightest charged particle (which is interpreted as electron mass m_e) one finds the following formula (see Eq.(32) of [22]):

$$m_e = \sqrt[3]{\frac{32\,\pi^{9/4}\hbar^3}{c^3\,\sqrt{\tau}\,\eta^4 s_0^2}} \tag{1}$$

where $\eta = 1/\sqrt[4]{1 + 4/\pi^4}$, $s_0 = 1m$ is the unit length, c is the speed of light and $\hbar = h/2\pi$ with h the Planck constant. The derivation of Eq.(1) is not the aim of this work and can be found in [22, 23]. A detailed discussion and a possible phenomenological interpretation of Eq.(1) will be presented in a future work [25].

In this work we want first of all to compute the smallest surface element τ including only field mass effects. To do this we will first review the determination of the modification of the Newton potential due to field mass effects at small distances. Our result for the smallest surface scale τ can, after that, be applied to compute explicitely the high energy cutoff k_c of the quantum fluctuacions, which is generally expected to be of the order of the Planck energy. The precise determination of the cutoff k_c is what one needs, according to our approach, for a deeper understanding of dark energy and dark matter.

Indeed for the study of the current universe we exploit the usual Friedmann equations for a spacially flat universe reported in Eqs(21) and, similarly as in [20], compute the dark energy contribution to the stress tensor $T_{\mu\nu}$ entirely from quantum fluctuations on this curved background. This point will be explained in detail in Section 4 together with the Appendix. To avoid the quartic divergence of the large cutoff k_c, we also assume, that the Minkowski space is stable, *i.e.* it is imposed as a general principle to have a vanishing vacuum energy, as it should be. This implies that the contribution of the flat space-time has always to be subtracted from the vacuum energy derived from the quantum fluctuations also in a general curved space-time. We will show that according to our interpretation of the result, we can compute the dark energy density in very good agreement with the actual measurements. In addition we will also obtain a prediction for the equation of state parameter of the dark energy $w = p_{vac}/\rho_{vac}$ from first principles using the computed cutoff k_c. The comparison with the current measuremens will show, that even if it is not yet possible to discriminate between "quintessence" ($-1 \le w < -1/3$) and "phantom-energy" ($w < -1$) , it is still generally possible to constrain the field content, fixing the difference between the number of species of fermions and bosons $\Delta N = N_f - N_b$. According to the Standard Model of elementary particles we have that $\Delta_{SM} = 60$ and hence our result provides also a way to determine the minimal amount of additional degrees of freedom, which can contribute to the dark matter.

The paper is organized as follows. In Section 2 we explain in detail the computation for the modified Newtonian potential due to the inclusion of field mass effects. The derivation of the smallest surface element τ is then shown in Section 3. The application of these results for the computation and explanation of the nature of the dark energy from vacuum fluctuations and the possible influence of the result on the field content of dark matter beyond the Standard Model follows in Section 4. Finally we write our conclusions in Section 5.

2 The inclusion of the field mass in the static potential

In this section we want to include the effects of the inclusion of the field mass on the Newtonian potential $\phi_n = \gamma m_{(0)}/r$ of a central mass $m_{(0)}$, where γ is the usual gravitational constant.

We denote the field mass by μ and we then assume it produces a modification of the Newtonian potential ϕ_n, leading to a function of the form $\phi = \gamma m(r)/r$, where $m(r) = m_{(0)} + \mu(r)$. According to this definition we obtain for the Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ that

$$\nabla^2\phi = \gamma m(r)\nabla^2\left(\frac{1}{r}\right) + \frac{\gamma}{r}\frac{d^2\mu(r)}{dr^2} \tag{2}$$

$$= -4\pi\gamma m(r)\delta(r) + \frac{4\pi\gamma}{3}\left(9V\frac{d^2\mu(r)}{dV^2} + 6\frac{d\mu(r)}{dV}\right),$$

where in the second line we have used the distributional relation $\nabla^2(1/r) = -4\pi\delta$ with δ the Dirac function and where we have performed the change of variables $V = 4/3\pi r^3$. Now it seems reasonable to normalize the contribution of the field mass $\vec{\nabla}\cdot\vec{G}$ with the factor $9 + 6 = 15$. At this point we notice that if we include in Eq.(2) field mass effects, then we can put the first term to zero because $r \neq 0$ every time that $\mu(r)$ and its spacial derivative are different from zero. Inversely we have that if we could neglect the field mass ($\mu = 0$), then the case $r = 0$ becomes in a distributional sense possible and only the first term in Eq.(2) should be retained. The former case represents the situation we are interested in with the inclusion of fiedl mass effewcts, while the last one is the usual Newtonian limit.

In this way we arrive at a generalized field equation proposed for the gravitational field $\vec{G}$ in a static system:

$$\vec{\nabla}\vec{G} = \frac{\sigma}{\alpha}, \tag{3}$$

with

$$\begin{aligned}\sigma &= \frac{1}{15}\left(9V\frac{d^2\mu(r)}{dV^2} + 6\frac{d\mu(r)}{dV}\right) \\ \alpha &= (20\pi\gamma)^{-1},\end{aligned} \tag{4}$$

if the influence of the field mass is considered and with

$$\sigma_n = -m_{(0)}\delta(r); \quad \alpha_n = (4\pi\gamma)^{-1}, \tag{5}$$

recovering the usual Poisson equation of the Newtonian case, if μ and its derivatives can be considered small enough to be neglected.

The presence of a gravitational field $\vec{G}$ associated to central spherical source with radius r_0 and mass m_0 is from our point of view not a possibility but a necessity and this should be reflected in the fact that it is somehow produced by an "energetic convenience" *i.e.* a reduction of the system energy. Hence we can write for the energy ($E = m(r)c^2$) of the mass-field system up to a radial coordinate r from the center:

$$m(r)c^2 = m_0c^2 - \frac{\alpha}{2}\int_{r_0}^{r}\vec{G}^2\,dV. \tag{6}$$

The last term in this equation represents the field energy and is obtained from Eqs.(3,4) in complete analogy to the energy of the static electrical field. Now remembering that $\vec{G} = \vec{\nabla}\phi$ Eq.(6) becomes

$$m(r)c^2 = m_0c^2 - \frac{\alpha}{2}\int_{r_0}^{r}(\vec{\nabla}\phi)^2\,dV. \tag{7}$$

Now performing the first derivative, taking into account that $\phi = \gamma m(r)/r$ and that for a spherical symmetric function

$(\vec{\nabla}\phi)2 = (d\phi/dr)^2$, we obtain easily the following differential equation for the static potential ϕ:

$$\left(r\,\frac{d\phi}{dr}\right)^2 + \frac{c^2}{2\pi\gamma\alpha}\left[\left(r\,\frac{d\phi}{dr}\right) + \phi\right] = 0. \tag{8}$$

This nonlinear differential equation can be easily solved viewing it as a quadratic equation with respect to $rd\phi/dr$, whose solutions are:

$$r\,\frac{d\phi}{dr} = -\frac{c^2}{4\pi\gamma\alpha}\left(1 \pm \sqrt{1 - \frac{8\pi\gamma\alpha}{c^2}\phi}\right). \tag{9}$$

With help of the following substitution,

$$q = 1 \pm \sqrt{1 - \frac{8\pi G\alpha}{c^2}\phi}, \tag{10}$$

one can straightforward rewrite Eq.(9) as:

$$r\,\frac{d(2q - q^2)}{dr} = -2q, \tag{11}$$

which according to $dx/x = d\ln(x)$ can be simplified to

$$d\ln(rqe^{-q}) = 0, \tag{12}$$

or equivalently to

$$rqe^{-q} = A, \tag{13}$$

where the integration constant A has been introduced.

We want now to fix the sign in Eq.(10) and the constant A in Eq.(13). Assuming that for $r \to \infty$, $\phi \to 0$, we obtain immediately that the negative sign in Eq.(10) is the only possibility. This follows from the fact that with this choice $q \to 0$ when $r \to \infty$ and only in this case remains our assumption consistent with the fact that in Eq.(13) A is a numerical constant. As far as the determination of the constant A is concerned, we can fix it requiring that the classical Newton potential $\phi_n = \gamma m_{(0)}/r$ will be reproduced if $\phi/c^2 \ll 1$. Accordingly expanding Eq.(13) to the first order in ϕ/c^2, we obtain for the constant A:

$$A = rqe^{-q} = r\frac{4\pi\gamma\alpha}{c^2}\phi_n = \frac{4\pi\gamma^2\alpha}{c^2}m_{(0)}. \tag{14}$$

Summarizing, we obtain for the relativistic gravitational static potential $\phi = \gamma m(r)/r$ including field mass effects the following implicit equation:

$$\left(1 - \sqrt{1 - \frac{8\pi\gamma\alpha}{c^2}\phi}\right)e^{\sqrt{1-\frac{8\pi\gamma\alpha}{c^2}\phi}-1} = \frac{4\pi\gamma^2\alpha}{c^2}\cdot\frac{m_{(0)}}{r}. \tag{15}$$

From this last equation we can easily determine the smallest allowed r-value r_- from its reality condition. Indeed this is fulfilled by Eq.(15), when $\phi \leq c^2/(8\pi\gamma\alpha)$, which means that

$$r \geq r_- = \frac{8\pi\gamma^2\alpha m(r_-)}{c^2}. \tag{16}$$

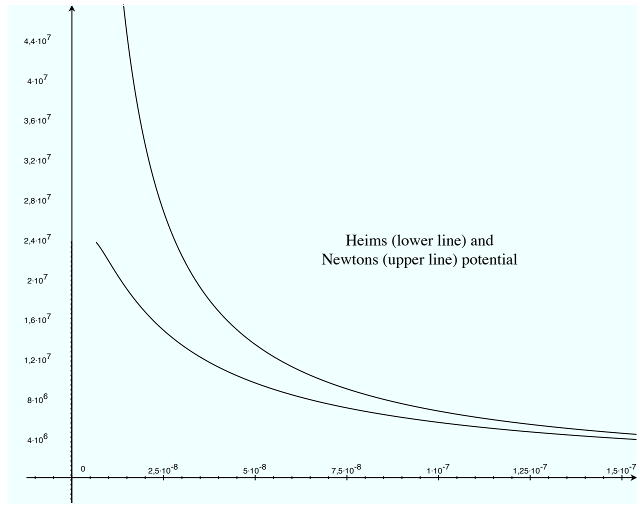

Fig. 1: Comparison of the Newtons potential ϕ_n (upper line) with the Heim potential ϕ defined implicitly by Eq.(15) for $m_{(0)} = 1kg$ including relativistic and field mass effects. The unit for the radial distance r is $10^{-10}m$.

This last result needs few words: First of all we notice that very often for a macroscopic high density collapsing system one can neglect the field mass such that $m(r) \approx m_{(0)}$ and $\alpha = \alpha_n = (4\pi\gamma)^{-1}$, according to Eq.(5). In this case we obtain for the smallest radius of a relativistic collapsing system $r_- = 2\gamma m_{(0)}/c^2$, which is equal to the well known Schwarzschield radius of the general relativity. Considering also field mass effects typical of high density microscopic systems like particles, we have for $r = r_-$ that $q = 1$ and that $\alpha = (20\pi\gamma)^{-1}$, according to Eq.(4). In this case we get from Eqs.(13,14), that $r_- = eA = e4\pi\gamma^2\alpha m_{(0)}/c^2 = e\gamma m_{(0)}/5c^2$, showing by comparison with Eq.(16) also that $m(r_-) = em_{(0)}/2$.

In Figure 1 we compare for a mass $m_{(0)} = 1kg$ the Newtons potential with the Heim potential ϕ including both relativistic and field mass effects. We notice that significant deviation accrue down to distances around $10^{-17}m$ well below current experimental limits (see *e.g.* [11, 26, 27] and references therein).

3 The smallest surface scale τ

Following the same approach adopted in [22], we want now to derive the smallest surface element τ. Eq.(16) fixes a lower limit r_- for the radial coordinate due to relativistic and gravitational field mass effects and thus does not represent a smallest length for the empty space-time, because it vanishes with the mass m. However a microscopic mass system should also be characterized by its quantum behavior, which becomes important at the scale of the corresponding Compton wavelength $\lambda_c = h/(mc)$. Conversely we have that in this case for a vanishing mass λ_c diverges. Hence if we are looking for a good definition of the smallest surface τ for the empty space with a vanishing mass m, we see that the product $r_-\tau$ turns out to be well defined. In this way we arrive finally at the following definition for τ:

$$\tau = \lim_{m\to 0} r_- \cdot \lambda_c = \frac{2h\gamma}{5c^3} = \frac{4\pi}{5}\, l_{Pl}^2, \tag{17}$$

where we have used Eq.(16) and the second of Eq.(4) and where $l_{Pl} = \sqrt{\hbar\gamma/c^3}$ is the well known Planck length. Taking the measured values of the constants c and h into Eq.(17), we find

$$\tau \approx 6,56536 \cdot 10^{-70} m^2. \tag{18}$$

Substituting this numerical value for τ into the Heim mass formula Eq.(1), we find $m_e = 0.50822 MeV/c^2$, a result which is $0,55\%$ below the measured value. We notice here that our result for the smallest surface scale of Eq.(17) differs by a factor $16/15$ from the one obtained by Heim in [22] as already mentioned in the Introduction. The correspondig prediction for the electron mass would in this case be $m_e = 0.51371 MeV/c^2$, which is this time $0,53\%$ above the measured value. The origin of this difference is due to additional assumptions concerning possible speculative contributions to the rotor of the gravitational field $\vec{G}$.

In any case there is to our knowledge no better theoretical prediction of the electron mass from first principles than the one given in Eq.(1) and for this reason we assume reasonably

$$l_{min} = \sqrt{\tau} \tag{19}$$

to be also a good definition for the minimal length, for estimating the natural cutoff for the quantum fluctuations. Indeed to this purpose we propose to substitute in the usual uncertainty principle $\Delta x \cdot \Delta p \geq \hbar/2$ the position uncertainty Δx with l_{min} of Eq.(19)and the momentum uncertainty Δp with the UV momentum cutoff k_c/c. In this way and assuming a minimal uncertaicy for the higher energy fluctuations, we obtain

$$k_c = \frac{\hbar c}{2\sqrt{\tau}} = \sqrt{\frac{5}{\pi}}\frac{E_{Pl}}{4}, \tag{20}$$

where $E_{Pl} = \sqrt{\hbar c^5/\gamma}$ is the Planck energy.

4 A possible description of the dark sector

In this Section we want to investigate the cosmological consequences of our result obtained in Eq.(20). A few years ago a new approach in considering the zero-point energy fluctuations of the quantum fields has been proposed in [20]. According to their method it was possible to obtain a consistent formula for the computation of the cosmological dark energy density ρ_{vac} entirely from vacuum energy quantum fluctuations. The basic additional principles of the authors in [20] are that the empty Minkowski space should be gravitational stable ($\rho_{vac} = 0$), that our universe is spatially flat and that the vacuum stress energy tensor should have the form $< T_{\mu\nu} >= -\rho_{vac}\, g_{\mu\nu}$ with $\dot{\rho}_{vac} = 0$. These are the usual properties assumed in the Standard Model of Cosmology, the ΛCDM-model, for the cosmological constant. Following [20], we remind the reader that according to the Friedmann equations,

$$\begin{aligned} \left(\frac{\dot a}{a}\right)^2 &= \frac{8\pi\gamma}{3\,c^2}\rho \\ \left(\frac{\dot a}{a}\right)^2 &+ 2\frac{\ddot a}{a} = -\frac{8\pi\gamma}{c^2}p, \end{aligned} \tag{21}$$

one can easily check that

$$\dot{\rho}_{vac} = -3\left(\frac{\dot a}{a}\right)(\rho_{vac} + p_{vac}), \tag{22}$$

This implies that putting $p_{vac} = w\rho_{vac}$ with $w = -1$, for the vacuum energy equation of state one satisfies simultaneously the constraints $\dot{\rho}_{vac} = 0$ and $< T_{\mu\nu} >= -\rho_{vac}\, g_{\mu\nu}$ and hence also $\nabla^{\mu} < T_{\mu\nu} >= 0$ with ∇_{μ} the usual covariant derivative. The result computed in [20] is:

$$\rho_{vac} = \frac{g\,c^2}{8\pi\gamma}\left(\left(\frac{\dot a}{a}\right)^2 + 2\frac{\ddot a}{a}\right) \tag{23}$$

with

$$g = \frac{3\gamma}{8\pi\hbar c^5}\,\Delta N k_c^2, \tag{24}$$

where $\Delta N = N_f - N_b$ is the difference between the number of species of fermions and bosons and k_c is an UV-cutoff. The reader can find in the Appendix a detailed derivation of Eq.(23). Now substituting $p_{vac} = w\rho_{vac}$ with ρ_{vac} given by Eq.(23) into the second of Eqs.(21), remembering that for non relativistic matter $p_m = 0$ and neglecting the relativistic radiation density, which is a factor $\sim 10^{-5}$ smaller than the total energy density, one can very easily check that

$$w = -\frac{1}{g}. \tag{25}$$

Clearly this result is consistent with the assumptions of [20] outlined at the beginning of this section only if $g = 1$.

In our study we relax the constraint $g = 1$ of [20] allowing more general and exotic possibilities, which deviates from the usual cosmological constant vacuum energy scenario with $w = -1$. Obviously, according to the actual experimental observations, a realistic description of the dark energy imposes that the deviations of g from the unity are expected to be small. Indeed taking recent fits from the observations of Type Ia supernovae dynamics [28, 29] of the HZSN and the the SCP collaborations we can estimate that

$$\left(\frac{\ddot a}{a}\right)_{t=t_0} \approx 0.58\, H_0^2, \tag{26}$$

where t_0 is the actual time and H_0 the actual Hubble constant. One can check this result for example computing the time derivatives of the fitting function for the scale factor $a(t)$ in Eq.(26.82a) in the book of Thomas Müller [30]. Although this is only a qualitative argument, because the specific fitted function of [30] is model dependent, it provides anyway a plausible estimation of the correct value. Substituting this result in Eq.(23), and using the usual definition for the critical density $\rho_{c\,0} = 3c^2 H_0^2/(8\pi\gamma)$, we obtain

$$\Omega_{vac\,0} = \frac{\rho_{vac\,0}}{\rho_{c\,0}} \approx g \cdot \frac{1 + 2 \cdot 0.58}{3} \approx g \cdot 0.7, \qquad (27)$$

which with $g \approx 1$ is in quite good agreement with recent analysis from CMB measurements [31, 32], considering the experimental uncertainties in Eq.(26).

We have already shown that Eq.(23) satisfies the second of Eqs.(21) with the identification $g = -1/w$. We want now to discuss the solution of the first of Eqs.(21) for the scale parameter $a(t)$ in the more general case $g \neq 1$. To this purpose we put the expression of the vacuum energy given in Eq.(23) into the first of of Eqs.(21) and thus we get the following differential equation for the scale factor $a(t)$:

$$\left(1 - \frac{g}{3}\right)\left(\frac{\dot{a}}{a}\right)^2 - \frac{2g}{3}\frac{\ddot{a}}{a} = H_0^2 \frac{\Omega_{m\,0}}{a^3}, \qquad (28)$$

where as usual $\Omega_{m\,0} = \rho_{m\,0}/\rho_{c\,0}$, $\rho_m = \rho_{m\,0}/a^3$ and where $\Omega_{rad\,0}$ has been again neglected. Performing now the change of variables,

$$w(a) = a\dot{a}^2, \qquad (29)$$

one has that Eq.(28) becomes a first order linear differential equation in w, whose solution is given by

$$w(a) = \Omega_{m\,0} H_0^2 + (1 - \Omega_{m\,0}) a^{3/g} H_0^2, \qquad (30)$$

where the usual initial conditions $a_0 = a(t_0) = 1$ and $w_0 = w(a_0) = H_0^2$ have been imposed. According to this result and treating Eq.(29) as a separable variables differential equation, we can rewrite and integrate it as follows:

$$\int_{a_0}^{a} \frac{da}{\sqrt{\Omega_{m0} a^{-1} + (1 - \Omega_{m0}) a^{-(1-3/g)}}} = H_0(t - t_0), \qquad (31)$$

again with the initial condition $a_0 = a(t_0) = 1$. This integral represents the general solution to the Friedmann equations with the presence of matter with $w_m = 0$ and dark energy with $w_\Lambda = -1/g$ as expected by consistency. To our knowledge there is not a simple general analytic expression that solves the integral in Eq.(31) for $g \neq 1$. However a very simple solution can be obtained at early times ($a \ll a_0$), when the universe was matter dominated, and at later times ($a \gg a_0$), when the universe will be dark energy dominated:

$$a_g(t) \propto t^{2/3}; \qquad a \ll a_0; \qquad (32)$$

$$a_g(t) \propto \left[1 + \frac{3(g-1)}{2g}\sqrt{1 - \Omega_{m0}} H_0(t - t_0)\right]^{\frac{2g}{3(g-1)}}; \quad a \gg a_0. \qquad (33)$$

Consistently in the limit $g \to 1$ we obtain the later times behavior of the ΛCDM model according to which $a(t) \propto \exp\left[\sqrt{1 - \Omega_{m0}} H_0(t - t_0)\right]$. For the case $0 < g < 1$ we have that the scale factor $a_g(t)$ rapidly expands and diverges in the finite time $t = 2g/[3(1 - g)\sqrt{1 - \Omega_{m0}} H_0] + t_0$, producing a "Big Rip" as it is well known in the case that dark energy is phantom energy [33].

After that we come back to the physical interpretation of Eq.(24). First of all we substitute the computed result for the cutoff k_c of Eq.(20) into Eq.(24) and with Eq.(25) we obtain that

$$w = -\frac{128\pi^2}{15\Delta N}. \qquad (34)$$

This is the main result of our paper. We can fix ΔN trying to satisfy the constraint $w = -1$ as accurately as possible. According to this point of view, we would find that,

$$w = -1.0026 \qquad \text{for} \quad \Delta N = 84, \qquad (35)$$

expecting for this case at least 24 additional fermionic degrees of freedom. In [20] the authors have shown that for the Standard Model of elementary particles $\Delta N_{SM} = 60$. This possibility would suggest a *phantom energy* interpretation of the dark energy and Eqs.(22) would give $\dot{\rho}_{vac} > 0$.

Finally we notice, that both the result for the equation of state $p_{vac} = w\rho_{vac}$ predicted in Eq.(35) and the dark energy density obtained by Eq.(27) are in agreement with the experimental measurements. However with the actual uncertainties it is not yet possible to discriminate all the possibilities above $\Delta N \sim 82$ and below $\Delta N \sim 92$. Furthermore according to the result found in [22] that we discussed below Eq.(17), we would have the range between $\Delta N \sim 78$ and $\Delta N \sim 86$. Hence, considering also this possibility, we expect at least 18 fermionic degress of freedom beyond the Standard Model. They include possibly three right-handed neutrinos responsible for the neutrino masses and three spin-1/2 fermion contributing to the dark matter. For this last discussion we have compared with the central values of w reported in chapter 27 of [35].

5 Conclusions

Summaryizing, we have firstly reviewed the computation of the modified Newtonian potential coming from the inclusion of the so called field mass effects. This fullfills also a gap in the literature, because the original works were written in German [22, 23] and we have also taken the opportunity to correct some typos present in the original version. Without any additional assumption and limiting ourselves to the small distance effects, we could find for the smallest surface element $\tau = 2h\gamma/5c^3$ of the empty space-time. Imposing this result to the scale length of the quantum fluctuations we computed a natural UV-cutoff k_c for the modes of the zero point energy finding $k_c = E_{Pl}/4\sqrt{5/\pi}$, where $E_{Pl} = \sqrt{\hbar c^5/\gamma}$ is the Planck energy. Substituting this result into the formula of Bernard and LeClair for the cosmological constant given in [20], we obtained the result $w = -128/(15\pi^2\Delta N)$

for the dark energy equation of state $p_{vac} = w\rho_{vac}$, where $\Delta N = N_f - N_b$ is the difference between the number of species of fermions and bosons. we found also that and $w = -1,0026$ with $\Delta N = 84$. More generally comparing with the recent experimental determinations of w [35], we found that the number of additional fields beyond the Standard Model should at least include 18 fermionic degrees of freedom (implying $\Delta N = 78$), just enough to have three massive neutrinos and three additional 1/2-spin, explaining the dark matter. We found also that ΔN, according to the actual experimental constraints, should be bounded from above by 92.

Appendix

In this Appendix we compute the result for the vacuum energy from quantum fluctuations given in Eq.(23). We start with the action of a single bosonic field on a curved background:

$$S^b = \int dt d^3x \sqrt{-g}\frac{1}{2}\left(-\partial^\mu\phi\,\partial_\mu\phi - m^2\phi^2\right), \qquad (36)$$

where g is the determinant of metric $g_{\mu\nu}$. We take as backgroung the FLRW-metric in the case of a spacially flat universe

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \qquad (37)$$

thus implying that $g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2)$ and that that $g = -a^6$. Before proceeding with the canonical quantization of the field, we first perform the change of variable $\phi = \chi/a^{3/2}$, in oder to remove the time dependence appearing in the measure of the integral action coming from g. Indeed in this way one obtains after some algebra that the action in Eq.(36) becomes

$$S^b = \int dt d^3x \frac{1}{2}\left((\partial_t\chi)^2 - \frac{1}{a^2}(\vec{\nabla}\chi)^2 - (m^2 - \mathcal{A})\chi^2\right), \qquad (38)$$

where

$$\mathcal{A} = \frac{3}{4}\left(\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a}\right). \qquad (39)$$

The corresponding equation of motion for the field χ is then

$$\partial_t^2\chi - \frac{1}{a^2}\nabla^2\chi + (m^2 - \mathcal{A})\chi = 0. \qquad (40)$$

We rewrite now the field χ as a Fourier integral with a relativistic invariant measure

$$\chi = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_{k/a}}}\left(a_{\vec{k}} u_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{x}} + a^\dagger_{\vec{k}} u^*_{\vec{k}}(t) e^{-i\vec{k}\cdot\vec{x}}\right), \qquad (41)$$

where

$$\omega^2_{k/a} = \frac{k^2}{a^2} + m^2 - \mathcal{A}, \qquad (42)$$

where $a^\dagger_{\vec{k}}, a_{\vec{k}}$ are the usual creation and annihilation operators of a particle state with momentum $\vec{k}$ and where $u_{\vec{k}}$ is a time dependent function. Substituting the Fourier integral into Eq.(40) one obtains for $u_{\vec{k}}$ the following equation:

$$(\partial_t^2 + \omega^2_{k/a})u_{\vec{k}}(t) = 0. \qquad (43)$$

In the so called “adiabatic limit” one assumes that the time dependence of $\omega_{k/a}$ can be neglected in our actual universe and one can easily find the solution to Eq.(43), which is

$$u_{\vec{k}}(t) = u_{\vec{k}}\, e^{-i\omega_{k/a}t}. \qquad (44)$$

After that performing a Legendre transformation of the Lagrangian in the action for χ Eq.(38) and substituting Eq.(41) together with Eq.(44) into it, one finds for the Hamiltonian

$$H^b = \int d^3x \frac{1}{2}\left((\partial_t\chi)^2 + \frac{1}{a^2}(\vec{\nabla}\chi)^2 + (m^2 - \mathcal{A})\chi^2\right) \qquad (45)$$

$$= \frac{1}{2}\int d^3k\, \omega_{k/a}(a^\dagger_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^\dagger_{\vec{k}}). \qquad (46)$$

Repeating a similar computation for a fermionic field one obtains

$$H^f = \frac{1}{2}\int d^3k\, \omega_{k/a}(b^\dagger_{\vec{k}} b_{\vec{k}} - b_{\vec{k}} b^\dagger_{\vec{k}}). \qquad (47)$$

We introduce now the usual commutation (anticommutation) relations for bosons (fermions):

$$[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = \left\{b_{\vec{k}}, b^\dagger_{\vec{k}'}\right\} = \delta_3(\vec{k} - \vec{k}') = \int \frac{d^3x}{(2\pi)^3} e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}, \qquad (48)$$

where we have also added the integral representation of the Dirac function. Remembering that $a_{\vec{k}}|vac>= b_{\vec{k}}|vac>= 0$ for the vacuum state $|vac>$ and using the relations in Eq.(48) one gets

$$\rho^{b(f)}_{vac,0} \equiv \frac{1}{a^3V_0} < vac|H^{b(f)}|vac >= \pm\frac{\delta_3(\vec{0})}{2V_0}\int d^3k\, \omega_k, \qquad (49)$$

where the change of variables $\vec{k} \to \vec{k}/a$ has been performed and where + has to be chosen for bosons and − has to be chosen for fermions. According to the last equality in Eq.(48) one has that $\delta_3(\vec{0}) = V_0/(2\pi)^3$ and Eq.(49) becomes

$$\rho^{b(f)}_{vac,0} = \pm\frac{1}{16\pi^3}\int d^3k\, \omega_k. \qquad (50)$$

Now as mentioned in the Introduction, one has to subtract from it the contribution from the flat space-time ($\mathcal{A} = 0$) and the vacuum energy contributions to dark energy becomes:

$$\rho^{b(f)}_{vac} = \pm\frac{1}{16\pi^3}\int d^3k\left(\sqrt{k^2 + m^2 - \mathcal{A}} - \sqrt{k^2 + m^2}\right), \qquad (51)$$

where Eq.(42) has been used. Finally remembering that $d^3k = k^2 dk \sin(\theta) d\theta d\phi$, introducing the large cutoff k_c to regulate the integral and considering N_f fermionic and N_b bosonic fields, one obtains at the leading order

$$\rho_{vac} = N_f \rho_{vac}^f + N_b \rho_{vac}^b = \frac{\Delta N k_c^2 \mathcal{A}}{16\pi^2} + \dots, \qquad (52)$$

where $\Delta N = N_f - N_b$ and where the additional terms are all suppresed by powers of $1/k_c^2$. As a last step one can easily check that Eq.(52) coincides with Eq.(23).

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