Infinite Sets: The Appearance of an Infinite Set Depends on the Perspective of the Observer

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Abstract

This paper discusses how an infinite set would appear to different observers and how this applies to both physics and mathematics. Consider a set, N, defined as containing an infinite number of discrete, finite-sized elements such as balls. Any one of these balls can be defined as an internal observer, O. The balls extend outward in infinite numbers relative to any location and orientation of any internal observer O. That is, wherever O is in the set and in whichever direction O is "looking", the elements of the set extend without bounds the same potentially infinite distance in all directions relative to O. To observer O, set N appears as a potentially infinite space composed of discrete, finite-sized elements. Now, consider a hypothetical second observer, P who is outside the same set N and whose size relative to internal observer O is actually infinite. That is, P is of the same size "scale" as the entire set N, which is actually infinite relative to O. To observer P, each ball O is infinitesimally small, so that P can not distinguish the boundary of each ball O. Therefore, to P, set N appears as a finite-sized object containing a smooth, infinitely divisible internal space. The implications of these differing views of the same set depending on the reference frame of the observer are discussed for both mathematics and physics.

The Appearance of Set N to Internal Observer O

In this first section, I discuss how an infinite set, N, would appear to an internal observer, O. Consider a set, N, defined as containing an infinite number of discrete, finite-sized elements such as balls. Any one of these balls can be defined as an internal observer, O. The balls extend outward in infinite numbers relative to any location and orientation of any internal observer O. That is, wherever O is in the set and in whichever direction O is "looking", the elements of the set extend without bounds the same potentially infinite distance in all directions relative to O. Given this, then:

1. O sees the contents of the set as being discrete and finite-sized (finite in size relative to O) balls.

2. O sees its size relative to the entire set as approaching, but never quite reaching, zero. It never reaches zero because no matter how far O looks, it can never see an actually infinite endpoint of the set. From O's viewpoint, the set is always potentially infinite and, thus, its size gets smaller and smaller relative to the whole set but never quite reaches zero. If O were ever able to see the whole set in its entirety, then O's size would finally reach zero relative to that whole. Luckily, for O, that can't happen!

3. No matter how far O travels within its reference frame (e.g., set, N), it will never reach the edge, boundary or "exit door" of the set due the unending nature of infinity.

4. By definition of this set, relative to any element/observer O, the other elements progress radially away from it the same potentially infinite distance in all directions, and, thus, O would view the set as a potentially infinite sphere. It will be an odd sphere because the observer, or center point, O, can be at every point inside the sphere. Of course, elements within the sphere can never view the edge, so that, for them, the shape of the overall set only approaches that of an infinite sphere.

Thus, relative to finite-sized internal observer O, set N would appear as a potentially infinite, spherical space composed of finite-sized, discrete elements.

The Appearance of Set N to External Observer P

Next, the view of set N relative to a hypothetical, external observer is discussed. Again, consider set N, which was defined above as having an infinite number of elements relative to any location and orientation of an element/observer, O, within the set. However, now assume that there is a second observer, P, outside this set and that P's size relative to O is actually infinite. That is, P is of the same size "scale" as the entire set N, which is actually infinite relative to O. Therefore, P views the entire set N itself as of finite size, which means that P can see set N in its entirety. Given this, then:

1. If P's size relative to O is actually infinite, then O's size relative to P is infinitely small. Additionally, the boundary, or edge, of each element O that defines it and separates it from the other elements, is also infinitely small relative to P. The O elements still exist, by definition of set N, but they are of infinitely small size relative to the actually infinite observer P, and, thus, individual O elements and their boundaries become indiscernible to P. These elements, therefore, don't disappear, because they do exist, but instead merge into a continuous space from P's perspective. That is, P would observe the inside of set N as a continuous space, as opposed to O's view of it as a space filled with discrete elements.

2. P can see the whole or entire amount of N and, thus, can see the edge or boundary of N, which means that set N, in its entirety, is seen as an existent, finite-sized, discrete object by P.

3. P cannot "step inside" N and hope to be able to see its elements as discrete. This is because P is of a different size scale than the elements inside the set. P's scale is the same as that of the entire set, that is, actually infinite relative to O. So, even if P tried to step inside N, it would still be infinitely big relative to the elements and would, therefore, still just see a continuous space. Thus, P is "trapped" in its reference frame, or "dimension", just as O is "trapped" in its reference frame inside the set.

Thus, relative to infinite-sized, external observer P, set N would appear as a finite-sized, discrete, existent object with an interior continuous, smooth, infinitely divisible space. An important point is that these arguments don't prove the necessity of an external observer, they just suggest how this observer, if it existed, would view the set.

Implications for Our Perception of Reality, Physics and Mathematics

How does the appearance of infinite set N to different observers relate to our perception of reality? For one, the dichotomy between finite-sized, internal observer O's view of set N as discrete and potentially infinite in size, and external observer P's view of set N as continuous is analogous to the dichotomy between the quantum physics-based view of reality as discrete and quantized and the general relativity-based view of reality as smooth and continuous. This analogy suggests that it's possible that both quantum physics and relativity can be thought of as different views from different observer perspectives of the same set. In this case, the observer is the mind of the scientist, and the set being observed is our universe/reality.

It is important for both quantum physics and relativity, as well as for any theory, to use an internally consistent perspective throughout the theory. For instance, if a theory, such as quantum physics, describes space-time as discrete, indicating that the scientist/observer's perspective is similar to that of internal observer O, then it should ideally use the same perspective in its calculations, such as in its calculations of probabilities. Assuming a continuous, real number-like distribution of probabilities while also assuming a discrete space-time would mean that the theory is switching back and forth in its perspective of reality. Perspective-switching within a theory may cause internal inconsistencies, and the scientist should be aware of these and be cautious in their use.

Another implication is that when we view space, we think of it as infinitely divisible and continuous. If this were actually the case, it would suggest that space would be composed of discrete chunks and that we are infinite in size relative to the elements inside each of these chunks. The chunks that make up space would be like the integers and the area inside the chunks would be like the continuum of the real numbers. However, it could be that space is not continuous and instead is made of discrete chunks without smaller elements inside. This would mean that we are finite-sized observers relative to these chunks, and our view of space as continuous is incorrect.

The appearance of set N also has implications for mathematics, which is another way of perceiving and describing reality. The example of set N implies, at the very least, that the cardinality of an infinite set depends on the perspective, or reference frame, of the observer relative to the set. For example, within infinite set N, observer O would assign the set's cardinality as equal to that of the set of integers, omega. However, outside set N, observer P would assign it a cardinality equal to that of the real numbers. Furthermore, the example of set N says that the perception of the integers as being a potentially infinite set of finite, discrete chunks (e.g., 0-to-1, 1-to-2, etc.) and the real numbers within an integral range as being a continuum will vary depending on the perspective of the observer. That is, if the observer of these numbers could decrease his size scale to that of the real numbers, they might appear as finite-sized and discrete elements instead of their usual external observer-based description as infinitesimally small. Additionally, a hypothetical external observer of infinite size would view the set of integers as a continuous, infinitely divisible space similar to how we observe the real numbers.

Conclusions

It is shown that the same infinite set will have a different appearance depending on the perspective of the observer. An internal observer will view an infinite set of elements, such as set N, as a potentially infinite, spherical space composed of finite-sized, discreet elements. An external observer whose size is actually infinite relative to an element in the set will view the set as a finite-sized object containing a continuous, smooth, infinitely divisible space. The implications of these differing views of the same set for both mathematics and physics are discussed. Overall, these results suggest that the appearance of reality as discrete or continuous will depend on the perspective of the observer (e.g., the scientist's mind) relativity to reality and that the perspective of the observer should be taken into account when using infinities in physics.