Discrete Relativistic Electron Dynamics

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ABSTRACT

This is the fifth report on a new research programme investigating the fundamental electromagnetic (EM) interaction. This paper completes the analysis of the interaction between **two** electrons described in the two previous papers. Each of these *electron papers* imposed a new, discrete restriction on the EM interaction. This paper now adds **two new** restrictions to the asynchronous interaction between two electrons. A pair-wise **ray**-like form of the EM **impulse** is proposed whose magnitude diminishes linearly with temporal separation to a finite, fixed value. Additionally, the discrete EM interaction is universalized by proposing a simple, new form for **quantizing** both the dynamical and kinematical **activity** between interacting pairs of electrons. This constrains both the linear momentum and kinetic energy of the two electrons to be exchanged across each interaction in **fixed** quanta. This dynamical exchange of quantized action ($h/2$) replaces Coulomb's **unproven** spherical, continuous and instantaneous 'law' of electrostatics with a new, unified, discrete and dynamical basis for electromagnetism.

This fulfills Maxwell's quest for extending Newtonian (particle) mechanics to the phenomena of electricity and magnetism but now for **all** relative distances and speeds: unifying both EM and classical and relativistic mechanics. This new EM model replaces Planck's arbitrary (mathematical) quantum of action 'rule' (first injected mathematically into EM radiation theory and then later into atomic physics) and thereby providing a universal **physical** explanation for all atomic phenomena. This paper provides the first complete analytical solution of the isolated two-electron interaction – both attraction and repulsion.

The present theory shows that negative and **positive electrons** are two complementary manifestations of the same type of entity, phase-shifted from one another in the timing of their interactions. It is proposed that both forms of the electron execute a four-step transverse, cyclic trajectory across space (in opposite directions). This new physical and mathematical EM model provides a simple, discrete mechanical explanation for the foundational (but mysterious) two-valued quantum phenomenon of electron '**spin**'. Since this motion is sub-microscopic, this may be viewed as the 'hidden quantum degree of freedom' that characterizes this most mysterious behavior of the electron. The intrinsic ('clicking') motion of the electron is proposed here as the *spatial* component (where) of the fundamental interaction between pairs of electrons. The other new component is now intimately associated with when and how an electron interacts with other electrons. These ideas result in electrons stepping cyclically through **four** interaction states (the positron goes through the same four states but in the opposite sequence). This provides novel insights into **Dirac**'s Equation of the Relativistic Electron while avoiding all taint of negative energy states.

This new two-time theory unifies an asynchronous version of Newton's physics of point particle motion (at **all** velocities) with all of the experimental phenomena of classical and quantum EM while **avoiding** use of single-time field theories and their necessary relativistic transformations of space and time. The new theory now forms the foundation for a new quantum theory of atoms, nuclei and elementary particles that is easier to visualize (*geometric*) while its finite *algebraic* basis (rather than continuous calculus) make it much easier to calculate **and extend** than prior continuum theories; the new results of these further extensive investigations will be presented in a series of subsequent papers.

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1. INTRODUCTION & OVERVIEW

1.1 INTRODUCTION

1.1.1 RESEARCH PROGRAMME

Background

This is the fifth report on a new research programme investigating the electromagnetic (EM) interaction. This programme has explicitly rejected the 'Continuum Hypothesis' that has been the implicit assumption of most of the theoretical work in physics over the last 300 years. The results of this new direction are being reported in a series of papers that demonstrate the problems with these assumptions of continuity and present an alternative model of physics that is grounded in a totally discrete viewpoint, which has been confirmed by experimentalists as describing the actual microstructure of nature.

The first paper in this series [1] introduced a new algebraic representation that appears more suitable for describing the **interaction** between pairs of electrons. This symmetric view is contrasted with the traditional separation into a 'target' particle and the forces that determine its dynamics. This representation (known as *Natural Vectors*) is based on Hamilton's greatest mathematical innovation: quaternions. The mathematical results derived in this first paper were then used in the next paper [2] to show how suitable this representation is for rapidly recovering all of the major results of classical EM. In contrast to Minkowski 4-vectors, the new algebra automatically generates all the field equations of Maxwell's theory when formulated in terms of continuous, electrical charge density. This paper also severely criticized this central assumption of spatial continuity of electrical charge as a model without physical or mathematical justification. Consequently, the full EM asynchronous interaction between two point electrons with finite electrical charge was investigated in the third paper [3]. This paper proved that all continuous interaction (**force**) theories between point particles that exhibit inertial resistance to changes in their motion are **not** consistent with asynchronous (or Gaussian) forms of action-at-a-distance or equivalently: two-particle interactions limited to points 'on their mutual light-cones'. This latter condition is always misinterpreted as a fact that there exists an independent entity called 'light' that travels through space at the same speed in all reference frames. This model of light was investigated in the fourth paper [4] from the symmetric perspective of interactions between two electrons. This analysis rejected the conclusions of Einstein's Special Theory of Relativity and provided a new mechanical explanation for Planck's 1907 Proposal for the formulae of relativistic point-particle mechanics while still preserving each electron's invariant inertial mass at all speeds. This 'classical relativistic' approach was based on the hypothesis that the universal inter-electron interaction only occurs discontinuously over time, whereas all earlier models were based on the implicit assumption of continuous interactions between the particles.

This paper now completes the analysis of the interaction between **two** electrons described in these earlier papers by further adding the hypothesis that the magnitude of the interaction itself is discrete and finite or, in other words, quantized. Each of these *electron papers* imposes a new, discrete restriction on the EM interaction. This theory now extends Newton's original, instantaneous model of physics to one centered on the asynchronous exchange of fixed impulses between the two particles.

An Ancient Metaphysical Tradition

This research programme explicitly follows in the tradition established by Leucippus and Democrates, who postulated that the world consisted exclusively of discrete particles of matter that could not be further divided; their term was 'atoms' – this is now replaced here with electrons. This discrete tradition was adopted by Isaac Newton and must be contrasted with the continuum tradition begun with Plato, re-established by DesCartes and adopted without restriction by Einstein; followers of this rival metaphysical tradition embrace the primacy of geometry as their mathematical representation of the world. The followers of the 'particle' tradition have obviously focused on the equally ancient, point representation of the world.

One of the few modern mathematical physicists to write on these issues is Roger Penrose. In his magisterial survey of the application of mathematics as 'the true representation of reality', he quotes Erwin Schrödinger as daring to suggest that in its foundations, Nature may well be discrete, with **no** experimental evidence to support the use of continuous mathematics as its fundamental representation [5]. This programme accepts the challenge (also quoted by Penrose on the same page) made by Einstein (in his last published writing) that an algebraic (discrete) theory might be the way forward for the future of physics.

Kantian Metaphysics

The present theory follows Kant philosophically, in his view of nature by agreeing that the natural world exists and changes independent of all human observations: this '*ding an sich*' is seen as a vast collection of electrons that constitute the material world all interacting amongst themselves. Humans can construct mental models of parts of this world as ideal abstractions but all our attempts to observe any part of this micro world will inevitably interfere with this normal level of activity and the results here will be described explicitly as a third level of reality. Since these interventions occur randomly in the natural cycles of microscopic systems, the results must appear statistically at the macroscopic level: this will force a probabilistic view on the results of measurements, without implying that the microscopic world itself always behaves statistically.

Ockham's Razor

The new theory of electrons presented here now unifies two competing strands in philosophy: materialism and rationality. Electrons are viewed in this research programme as the universal 'stuff' of the material world and are defined exclusively through their mutual interactions ('spirit'). In contrast to the historical approach that introduced a single 'free' material particle that had its interaction with the rest of the universe gradually 'turned-on', the electrons here are always considered in terms of their pair-wise interaction that is 'on' at all times. This integrated viewpoint reflects a powerful principle of philosophy that has stood the test of time: William of Ockham's *Principle of Parsimony*. Here there is only one ontological entity (the electron) and only one form of interaction (the EM interaction). This programme has rejected the Continuum Hypothesis that has been the unchallenged assumption behind 350 years of theoretical research in physics since Newton introduced this mathematical technique to assist him in evaluating his original discrete approach to material dynamics that was first described in his *Principia* [6]. The new theory views the role of time (not space) as central to the dynamics of the micro world. The possibility of interaction is proposed to be cyclic, not continuous, with this cycle-time (or chronon) now considered as one of the fundamental constants of nature.

1.1.2 MOTIVATION

The research underlying this paper was driven by the desire to gain a greater understanding of the principal entity on which this programme is constructed: the **electron**. There was an intense burst of activity surrounding this subject soon after the electron was 'discovered' in 1897; several models of the electron were proposed [7] that imagined very small spheres to be covered or filled with 'electrical fluid'. These soon fell out of favor and the consensus developed that the electron should simply be considered as a point particle with intrinsic properties of mass and electrical charge. Developments in quantum mechanics in the first quarter of the 20th Century focused on the atomic dynamics of a single electron, usually as part of the hydrogen atom. It was not until 1928 when Dirac published his famous pair of papers [8] that interest was revived in the nature of the electron itself. The heart of this new theory was a linear equation describing the electron's relativistic motion. This was immediately recognized as a major advance and laid the foundations for the modern theory of quantum electro-dynamics (QED). Dirac's theory will be the focus of the present paper, as it is still the best theory of the electron today but it retains some major flaws that were present from its inception. From the perspective of the present research programme, this theory suffers from three major weaknesses. Firstly, it was constructed as a description of a 'free' electron – a concept that is rejected here in favor of the interaction between pairs of electrons, so now the focus must be on two locations and two distinct times. Secondly, as a model of a single electron it uses the relativistic particle dynamics derived from Planck's 1907 Proposal for redefining momentum. This casual approach was analyzed extensively in the previous paper [9] and found to be massively deficient. The new importance of phase relationships throughout the EM interaction will be incorporated here. Finally, Dirac's approach was really a quantum field model and was not a true model of particle dynamics; worse, all of its interactions with other electrons were modeled through Maxwell's EM theory – another foundational theory that has been severely criticized in this series of papers [2]. These latter two problems can be traced back to Dirac's implicit use of the electron model of Lorentz [3], which was a deeply flawed 'marriage' of Maxwellian field theory and Newtonian particle dynamics. Ironically, it was the deficiencies in this model that drove Einstein to develop his relativity theory in 1905. As always throughout this series, two of the underlying objectives will continue to be the restoration of both philosophy and the history of science to the forefront of fundamental research to regenerate their key roles as engines of theoretical physics.

As in each of the papers, reporting on this new research programme a broad historical perspective is deliberately introduced. This is to re-establish the context from which the new ideas are developed; these are often contrasted with the assumptions that usually drove the historical development of physics at the time. In other words, in order to understand the significance of the new ideas it is important to recall the alternatives that were considered in the past. In particular, many of the most severe criticisms of what became the canonical evolution of physics were never answered – just ignored. This is especially the case with Dirac's Electron Theory: major critics like Heisenberg, Bohr and Pauli never had their objections to this theory answered – the discovery of the positron simply swept them away in 'the rush to the new'.

Today's researchers in physics are almost completely unaware of either the assumptions in the present theories or the major weaknesses underlying them since the history of science has been relegated to a small group of specialists. It is important to realize that only one of the possible pathways through the evolution of science has usually been developed. The 'pressure to publish' has increasingly compelled researchers to only focus on the latest activities – adding another 'brick in the wall' may not be the best use of one's time when the castle that is being constructed is actually floating off into the clouds.

1.2 OBJECTIVES

The primary objective of this paper is to present a new theory of the positive electron that is an alternative to Dirac's Theory of the Relativistic Electron. Orthodox physics has decided that Dirac's theory was the final explanation of this phenomenon. The key to this new electron model is to develop a detailed model of how **two** electrons interact at **all** distances and speeds. In the course of achieving this EM objective, it will prove necessary to determine the space-time solutions (or trajectories) of revised equations of motion that are defined in terms of *Discrete Natural Vectors* (DNVs). These are discrete quaternion based mathematical objects that result in much denser algebraic equations (fewer symbols) than the traditional use of scalars or vectors. Additionally, an extended set of difference operators are used instead of the tradional use of calculus to avoid the integrations that need to be summed over all of space and time. All of these new techniques and ideas are used to throw new light on the physical meaning of Dirac's relativistic electron equation. In particular, this new theory was designed to create a more understandable perspective on those particles known as positrons and, ultimately, on the idea of anti-matter. This paper is also dedicated to clarifying the concept of electron spin. This idea lies at the heart of quantum theory but its conceptual foundations are still deeply confused and contradictory: real progress in physics requires clear concepts.

This detailed revision of the history of the electron has been undertaken to create a solid foundation for investigating atomic, nuclear and elementary particle phenomena. It is time to revive the progress in physics that has now stalled for fifty years.

1.2.1 PHYSICAL MODEL OF THE ELECTRON

Following Newton, physics has moved away from the interaction between two particles to focus on only a single (target) particle (or location in space) at **one** single point in time. This approach was adequate for classical mechanics where the (implicit) instantaneous interaction could be replaced by an equivalent spatially sensitive energy potential. This technique was first found inadequate when the **asynchronous** electromagnetic interaction between charged particles was investigated. This failure to recognize that **two** distinct times were critical to an understanding of the EM interaction has resulted in two major crises for theoretical physics after the electron became the center of investigation. The first crisis was initiated in 1904 when Hendrik Lorentz merged Maxwell's continuum model of the EM field with the newly discovered point particle [10] that became known as the electron. Early 20th Century research resulted in the bizarre theories of space and time associated with Einstein's Special Theory of Relativity. The second crisis appeared at atomic scales when the interaction between the atomic electrons and the nucleus could not be viewed as a miniature model of the solar system. This focus on the behavior of a single electron continued with Dirac's Electron Theory, where the concept of 'spin' was viewed as just another property of every electron: a classical concept completely at odds with the view of the electron as a structureless, point particle.

1.2.2 AN ALTERNATIVE TO THE DIRAC EQUATION

In 1928, Paul Adrien Maurice Dirac (1902-1984) published his eponymous equation, which described the kinematics of free electrons and led to the prediction of the existence of anti-matter. This equation was later viewed as predicting the existence of the positron that was discovered in 1932; as such, it is presented as one of the greatest triumphs in mathematical physics; Pais wrote [11]: "The relativistic wave equation of the electron ranks among the highest achievements of twentieth-century science." Dirac is now considered one of the greatest theoretical physicists of the 20th Century and was awarded the Nobel Prize in Physics for 1933 with Erwin Schrödinger. This new theory will avoid all use of field concepts and thus avoid the problems of negative-energy states and mathematical infinities that result from using continuum mathematics.

1.2.3 EXPLANATION OF THE POSITRON

Ever since the Dirac theory of the electron, the vacuum has become an active participant in all electromagnetic situations, a veritable 'sea of infinite energy' acknowledging that the number of electrons in any finite volume of space is **not** fixed. In contrast, the present theory views both electrons and positrons as real, complementary forms of matter, whose individual existence is **eternal**, so their total number is always fixed and the vacuum returns to its Newtonian role, as simply **no-thing**.

1.3 OVERVIEW

In this overview the contents of this paper are summarized by including a brief description of each section and the major reasons the particular material has been included. The paper ends with a 'Summary and Conclusions' that focuses on the implications of the material covered and the new results obtained, along with brief previews of future papers in this series.

1.3.1 A NEW APPROACH TO ELECTRON DYNAMICS

When Newton invented modern physics it was quite reasonable to assume that there existed a realm of nature (the 'micro-world') much smaller than our own world (the macro-sphere) that could be seen with the human eye or even with the use of microscopes. The Principle of Simplicity (or Ockham's Razor) recommended that this micro-world was just like our own macro-sphere, so that the idea of 'particles' as very, small (far too small to ever be observed) objects of matter was quite plausible. Classical mechanics was constructed on the view that these particles behaved just like large-scale aggregates of matter that were susceptible to scientific measurements: the miniature billiard ball was the prototypical, classical particle. The modern revolution in atomic physics beginning around 1900 blew this assumption to pieces. The resulting quantum theory was a mathematical prescription for the newly discovered facts of the micro-world. None-the-less, the new quantum mechanics preserved the mathematical schema introduced to describe the macrosphere with the one exception of substituting non-commuting operators for the role of certain of the classical, dynamical variables. Although the new mathematical approach 'worked' (in very limited areas) it generated paradoxes and confusions that are still with physics today.

The present programme not only challenges this new mathematical schema but proposes alternative conceptions of the very foundations of mechanics – velocity, mass and rotation – on the grounds that each of these concepts today is still built upon the implicit assumption, referred to here as the Continuum Hypothesis. The result of this new approach is the development of a new model of the electron (referred to as the Digital Electron model) that extends Newtonian physics both conceptually and mathematically, while preserving the original metaphysical assumptions of natural philosophy (realism).

1.3.2 DIRAC'S RELATIVISTIC ELECTRON

Chapter II sets the stage for this paper: it establishes the context which defines the extent and foundations from which the new material is related; in particular, this chapter focuses on the new research done in the late 1920s and early 1930s on the relativistic motion of the electron and on the mysterious, quantum property of particles that became known as 'spin'.

Pauli & Dirac invent 'Electron Spin'

Pauli introduced the mathematical idea of spin to explain certain 'doubling' (or duplexity) of spectral lines; he required a new representational scheme that only allowed **two** equal and opposite values rather than the singular (scalar) numbers that had always appeared in both classical and quantum mechanics (QM) until that time. This new 'duplexity' was soon followed by a similar move made by Dirac, who "doubled up" once again to produce a mathematical scheme representing the high-speed motion of the electron with **four** inter-related components: this is referred to as the Dirac Equation. In analogy with classical E/M, where a rotating, electrically charged body of finite dimensions can be treated like a magnetic dipole, the 'spinning' electron was considered to possess a finite magnetic moment, whose effects could be both calculated and measured.

Dirac's Anti-Particle

The heart of Dirac's problems with his new equation of the electron was that two of the four solutions represented states of the electron with negative energy – a physical impossibility. Dirac first suggested that these two extra states represented a positively charged particle and the only candidate at that time was the heavier nuclear proton. In 1930, Dirac revived the Electronic Theory of Matter, proposing that the proton was just a different manifestation of the negative electron, since "it has always been the dream of philosophers to have all matter built up from one fundamental kind of particle." [12] This short-lived revival was the last attempt to build up matter from electrons alone. As W. Crookes said (disparagingly) in 1903 to the Congress of Applied Chemistry, alluding to all these types of basic theory beginning with Prout's Hypothesis: "this (unity of matter) has been essentially a British dream." One of the aims of this programme is to make this dream a reality.

The Positive Electron

The discovery of the positive electron is now viewed as a triumph of the predictive capabilities of the mathematical approach to theoretical physics. This revisionist view of history was sufficient to kill discussions on the deep problems with Dirac's equation. The challenge of explaining positive electricity is first introduced here as it is explored in depth later in this paper.

'Forgotten' Problems

This chapter briefly describes these radical innovations and discusses the "paradoxes and mysteries" that these new quantum concepts generated, illustrating the problems of understanding sub-atomic ideas that are introduced purely by mathematical hypotheses. These problems are rarely discussed today but were seen as major difficulties by Dirac's contemporaries. They have not been solved but as has happened too often in the evolution of theoretical physics they have just been 'forgotten'. In his correspondence with Dirac over any relativistic form of QM, Bohr [13] wrote: "I quite agree with you that we have no reason to think that we have already reached the final limit of quantum mechanics. I am only inclined to take the present difficulties as indications that we have not yet obtained the proper expression for the correspondence with classical electro-dynamics. It is just in this connection that [Oscar] Klein and I suspect that the existence of the elementary electronic charge may perhaps prove as fundamental as the existence of the quantum of action." This programme realizes and builds upon this deep intuition of a great physicist.

Other Electron Models

This chapter concludes with a brief discussion of the present view of the electron, including Feynman's space-time model and the electro-weak theory that forms a key part of the so-called "standard model" of elementary particles. The new theory differs radically from both of these orthodox models – both physically and mathematically. The final section here discusses several alternative but unorthodox models of the electron that have received little attention in mainstream physics but have contributed useful insights in the development of the new theory of the electron presented here.

1.3.3 RELATIVISTIC REPRESENTATIONS

Chapter III segregates all the mathematics that has been used in earlier approaches to describe 'spin', including Dirac's electron theory. It reviews the mathematics that has been introduced in the earlier papers in this research programme and combines elements of these into variants that will be used both here and in subsequent papers. The deliberate rejection of continuum mathematics in this programme means that it is important to establish these new **discrete** mathematical tools on a very firm foundation, particularly as so many theoretical physicists today are people with strong mathematical skills.

Traditional Spin Mathematics

The focus in this section is on Pauli's (2x2) spin matrices that were later extended to a (4 x 4) representation by Dirac. It is useful to thoroughly understand this simplest example of matrix algebra as this forms the bridge to understanding quaternions that are the basis of the new algebra referred to as *Natural Vectors* (NVs) that are used throughout this research programme. A brief discussion of other, so-called 'higher algebras', is included but only to contrast them with the algebra of NVs. Every mathematical discussion of the Dirac Equation inevitably introduces the idea of spinors – mathematical objects intimately involved in rotations in 3D (normal) space. Since magnetism is a phenomenon linked to the 3D motion of electrical charges, it is not a surprise to discover that spinors can be used to describe classical electromagnetism and hence special relativity. As a result, an extensive discussion is included here of the mathematics of spinors, which were invented by Cartan in 1913 and first introduced into physics by Pauli in his revolutionary mathematics of spin in 1927.

Discrete Natural Vectors

Continous Natural Vectors (or CNVs) were introduced in the very first paper in this research programme [1] as a specific subset of Hamilton's glorious invention: quaternions. The second paper in this series [2] applied the mathematics of single-time CNVs to a model of classical electromagnetism (CEM) that has represented electricity by a continuous, incompressible fluid (the so-called EM Helmholtz flow model). The power of NVs was demonstrated by showing there that all of the standard results of CEM were very quickly recovered by assuming that the three central quantities in CEM (current, potential and electro-kinetic momentum) were simply spatial components of comparable four-dimensional CNVs. The utility of NVs was shown in the third paper in this series [3] when the remote interaction of <u>two</u> classical electrons (each always existing at a point in space and interacting exclusively and continuously with each other) led to the extension of single-point CNVs to two-time CNVs (or 'Double-Point CNVs') as the natural representation of the *asynchronous EM interaction*. The power of this representation was that important differences between CNVs automatically result in pair-wise, 'difference' CNVs. The appropriateness of Natural Vectors, especially the 'difference' NVs to coherently link together eight (8) real numbers implies that the pair-wise interaction between two electrons at $(t_1; \underline{x}_1)$ and $(t_2; \underline{x}_2)$ is best represented by this new extended definition as *quaternions are the **maximal**, useful algebra* of all four possible multiplicative algebras. Since the third paper had proved that <u>*two inertial particles cannot interact continuously and asynchronously*</u> the idea of <u>discrete impulses</u> was explored in the fourth paper [4] which showed that the <u>time intervals</u> between these instantaneous impulses must vary with the velocity of the participating particle; this provided a physical explanation for the bizarre results of relativity (Lorentz transform) theory.

These results were generated by replacing all usage of the infinitesimal calculus with a new extended version of the calculus of finite differences; in particular, the use of point and extended finite difference operators. The present research programme rejects all continuum ideas, especially the metaphysical hypothesis that a 3D vector field represents any aspect of reality, including the EM interaction between charged particles. The key alternative hypothesis, now investigated here, is that a real electron is only considered to have the *possibility* of interacting at one of its (cyclic) interaction times (or 'ticks' t_η), which must be a multiple of the fundamental time difference or chronon (a new universal constant). This programme proposes that all quantum effects derive from this quantization of time characterizing interactions between electrons: such discrete behavior demands the use of discrete mathematics that was only imposed in QM by forcing the electron's 'wave' into a fixed 'box'. The rest of this chapter investigates several new discrete mathematical operations that are deliberately introduced to replace comparable "continuum mathematics" that have characterized physics ever since Newton. The end point of this mathematics is the introduction of two-time functions that describe the **interaction** between two remote electrons: this naturally leads to a description in terms of the differences of their locations and times when the two electrons exchange action between them. All of this extensive new mathematics is needed to establish the firm foundations for representing the discrete motion of single electrons and the motions that result from their discrete, asynchronous interactions: the model of EM interactions developed here is proposed to represent the foundational behavior of all of matter at its deepest levels of reality.

1.3.4 DISCRETE INTERACTIONS

This chapter forms the pivot where the new direction of this research programme diverges fundamentally from conventional physics. Ever since Newton the interactions between material objects have been reduced, to simplify the mathematics, to an abstraction of one object (the 'target') and a set of forces that act upon the target. This programme rejects this artificial and asymmetric view of the world and returns the focus to **the interaction between pairs of objects**: in this case, electrons. This chapter is dedicated to analyzing the key concept of interaction and builds on the insight that it consists of the exchange of **action** between the participating electrons. As continuous interactions have also been shown to lead to contradictions when asynchronous interactions (such as the electromagnetic interaction) are considered then the concept of 'force' is also rejected. Although the present approach returns to Newton's original concept of **impulse**, this is only introduced as a mathematical technique for analyzing the symmetrical effects of an interaction on each electron as it participates in each interaction.

This theory is firmly established in the Newtonian tradition, where the foundational (metaphysical) entity is the concept of the point particle, which is here identified with the **electron** (hence the label of an 'electromagnetic' theory). In contrast to most 20th Century attempts to develop purely mathematical or phenomenological theories of physics, the present particle theory firmly offers a realistic theory of reality. This is a materialist theory grounded in the reality of electrons and their interactions. However, in contrast to all earlier theories in physics that followed Newton's calculational approach, centered on the calculus (that was then *post facto* interpreted as a continuum model of reality), the fundamental idea investigated here is that the interaction between electrons does **not** occur continuously but only at periodic instants of time. This requires the introduction of a new universal constant, the *chronon,* corresponding to the invariant time interval between the consecutive instants of possible interaction. These ideas lead to the focus on the trajectories of interacting electrons, where the electrons move between consecutive interaction nodes in straight lines only changing their motion discontinuously under the influence of an instantaneous impulse exchanged with another electron. This form of asynchronous, action-at-a-distance means that the concept of velocity must be carefully re-examined since interaction events in a Newtonian passive background of space and time now takes priority. This old/new approach is contrasted with the special theory of relativity, where the mathematical concept of velocity was given fundamental significance and the ideas of space and time were consequently merged into an ephemeral role as 'bit-players' in a story of magical co-ordinate systems. This eliminated the drama of the action between electrons and focused all the attention on the deserted stage itself, where Maxwell's EM fields and their descendents can reappear every where and every when, ultimately generating all the well-known problems of quantum field theory.

The first section here lays down the philosophical foundations on which this theory is constructed. Contemporary physics has rejected **philosophy** – a position that is viewed here as totally wrong: philosophy clarifies the ideas that we use to create all our views of the world, whether this be implicit (today's Platonism in physics) or explicitly; since it is false assumptions that are the major source of errors in human affairs, this programme prefers to make its philosophical assumptions explicit. Key concepts of causality, temporal asymmetry and advanced effects are reviewed here before being used in more detail.

Although the concept of **action** has been used in physics almost as long as Newton's revolutionary concept of momentum, it has always played a relatively minor role – even after Planck's quantization of this quantity, action has still remained in the background. The present theory now gives this idea a starring role in the drama of the micro-world. Section 4.2 zeros in on this central idea, building on the experimental observations that all the electron's distinguishing characteristics are finite.

The discreteness of action implies that the timing of an interaction is no longer continuous. This leads to the intuitive idea of a "quantum of time" (or **chronon**) defining the temporal scale of all interactions and always designated here by the symbol τ; a numerical value of about 10^{-23} seconds is first estimated here for nuclear interactions but a longer "atomic chronon" (τ_A) of about 10^{-17} seconds is found for the larger spatial dimensions of atomic phenomena reflecting the deeper cyclic motion of the nuclear electrons forming the so-called nucleons. This section restores the coherent focus that interaction dynamics are determined over an extended set of interactions (i.e. over a finite time): each single interaction event is constrained by its wider context. Planck's quantization of action is analyzed herein and shown to be a mathematical 'fix' without any supporting physical mechanisms – a stratagem that has been used too often in the last 100 years of theoretical physics. This section also introduces the idea of the separation of changes in discrete action into distinct kinetic and dynamic components. Earlier, the use of *Natural Vector* algebra showed that the combination of the electron's action and angular momentum formed a CNV (here called '**activity**'), whose value is invariant while an electron is not involved in an interaction (i.e. while 'free'). Single particle activity is an uninteresting concept by itself but leads to more interesting ideas when inter-particle interactions are introduced when joint (interacting pair) electron activity becomes central. An interaction becomes manifest whenever the exhange in activity induced by the interaction is considered, particularly when this exchange is based on the difference in **velocity** that results from the interaction compared with the 'free' activity that would have occurred had the interaction not manifested itself: this idea is made mathematically specific through the new idea of an '*impact*' operator.

Section 4.3 brings these ideas together in a new two-electron discrete NV called '**interactivity**' when the concept of velocity is revisited again but now in even more detail and in a 4D context; Newton's intuitions on discrete particle dynamics under generic impulses are shown to be still valid for the extended (digital) idea of electrons subject to EM interactions investigated here. This section introduces the *quantum of activity* hypothesis by assuming the amount of unit activity exchanged during each interaction is constant and whose value is suggested by comparison with Bohr's atomic theory to be half the value of Planck's action constant *h*. The next part of this section introduces the new, universal form of the EM impulse between two interacting electrons: this is a point-to-point **impulse** whose magnitude varies inversely with their separation at short-range (less than about one mm) and is fixed at a small, constant value at distances greater than this. The long-range impulse form is responsible for classical/statistical EM momentum while the short-range form accounts for all atomic and nuclear interactions (i.e. quantum effects). This section presents an analysis of two simplified models of the neutrino and the hydrogen atom and ends with a brief discussion on the relationship of these new proposals to the standard version of wave mechanics.

1.3.5 DIGITAL ELECTRONS

Chapter 5 begins the extended investigation of the new concepts associated here with the idea of the Digital Electron Model (DEM) based on the radical notion that each electron executes an intrinsic 4-step, cyclic motion around its axis of motion. The basic hypotheses of this new theory are first presented along with their direct corollaries. The principal starting point is an analysis of the ontological concept of **existence**, with its two possibilities: exist or not-exist, these basic binary choices become the bridge to integer arithmetic in mapping reality to a human representation. The two real, physical possibilities in the interaction view – "sending or receiving an impulse" lead to a binary representation, which is shown to correspond to Pauli's concept of 'spin'. The hypothesis that the emission process can go forwards in time or backwards leads to an **extra doubling**, resulting in a new integer (4x4) matrix representation, similar but different from Dirac. This is referred to as the 'canonical phase representation' and reflects the 4 basic states that every electron cycles through over time; the old 'spin' operator now appears simply as the anti-symmetric NV complement of the existence operator suggesting an entirely different physical explanation for 'spin'. The canonical and complementary progressions through these four states proves sufficient to give a realistic model of both electrically negative and positive charged electrons without invoking metaphysical fictions like Dirac's "sea" of negative energy states.

The third section investigates how the exchange of discrete amounts of activity (the discrete interaction) affects the motion of each of the digital electrons in a series of interactions. The use of finite difference operators shows that this approach is 100% consistent with discrete classical mechanics as long as each electron's total velocity (including transverse) is included. The use of DNV algebra readily shows here that only when the non-interacting digital electron moves across a multiple of four chronons does it behave (on average) like a 'free' Newtonian particle. It is also shown that the Thomas precession factor is exactly two when the discrete rotation consists of exactly four equal steps. The interaction analysis is extended in the final section to show that the interaction between two digital electrons must be purely radial. This chapter ends by showing the connection between these new ideas and Planck's original assumption of quantized action in continuous radiation exchange.

1.3.6 POSITIVE ELECTRONS

The present theory is in the tradition of Larmor's Electronic Theory of Matter (ETM) but introduces two major differences. First, the present theory has no need for any concept of the aether, which was the primary metaphysical focus for the ETM. Secondly, the new theory drops all references to the Continuum Hypothesis [3]. However, this new theory, like the ETM, views all matter as exclusively constructed from negative and positive electrons: the ultimate form of material reductionism.

Chapter six faces the issue of positive electrons directly by building on some of Dirac's mathematical insights but rejecting totally Dirac's idea of negative energy states and his reliance on Planck's arbitrary definition of relativistic momentum. The new theory combines ideas of space and time to explain why positive electrons are as widespread (but hidden) as negative electrons and just as fundamental. This theory will propose models for ALL the elementary particles, including the proton, constructed only from positive and negatively charged point electrons, moving around each other in periodic but stable trajectories: these detailed models will be presented in later papers in this series. The present paper focuses on the positron.

The present theory views positive and negative electrons as two complementary manifestations of the same type of entity, just phase-shifted from one another in the timing of their interactions due to their complementary trajectories through space. This idea is sufficient to explain the observation that particles with similar charges repel one another while particles with opposite (or complementary) charges attract each other. This phase-shift is also a clue to how the complementary electrons differ in their motions through space: in the transverse plane, the electron 'clicks' to the left while the positron 'clicks' to the right. Like Dirac's dual-spinor electron theory, the present theory posits that every electron can exist in one of four possible states. Unlike Dirac's theory, where two of his four 'permanent' states are found to have 'negative' energy and are always assigned to the positron, the present theory introduces four unique electron states that are occupied by both electrons and positrons over time but now each type of electron *cycles* through these four states in a **fixed** but **different sequence**. These four states determine *when* the individual particle **may interact** *forwards or backwards in time* with another particle.

Chapter VI also includes an analysis of why Lorentz invariance is irrelevant in any theory, such as the present theory, that is constructed around interactions that span the two interaction events between pairs of interacting electron when both events are on each other particle's light-cone .

This chapter includes the first comprehensive analysis of the dynamics of a pair of attractive particles; this complements the analysis offered in the previous paper when only the dynamics of a pair of repulsive particles was considered. This analysis shows how non-continuous interactions can allow two point particles to occupy the same location in space at the same time; this allows real solutions to one-dimensional scattering of two attracting electrons. These trajectories never involve infinite forces and since the interaction is asynchronous it never occurs when the two particles are at their closest (same time) spatial separation.

Attractive impulses are key to building more complex, stable structures from only positive and negative electrons. This chapter includes two such simplified models describing the neutrino and the Bohr hydrogen atom – more comprehensive models are reserved for later papers. The neutrino model predicts that the neutrino will have an effective diameter of $(\pi/\alpha)\Lambda$, which is about 430 times larger than the classical electron radius Λ. The simple hydrogen model will demonstrate that the formulae developed here for the impulse-model of EM interactions are capable of providing a Newtonian-like model of the hydrogen atom, without using quantum mechanics or the paradoxical assumptions of the Bohr model of the atom.

This chapter ends with an estimate of the maximum radius of the hydrogen atom – both the Bohr model and QM predict that this is infinite (in other words, there are no restrictions on the size of the principal quantum number). In contrast, this analysis will show that the maximum distance that an electron can move around the proton and still stay bound into a neutral atom is only a little bit less than one centimeter.

The deep respect for the history of science shown in this programme will be demonstrated again by making every attempt to avoid using the lazy phrase 'well-known'; wherever possible, detailed references will be provided for readers to confirm the sources used here and allow readers to follow up with their own further reading: there is much to be gained by reading the source material directly (particularly those of the 'masters') rather than textbook summaries by later acolytes. This is why the final chapter returns to an in-depth analysis of the most influential book on QM written in the Twentieth Century.

1.3.7 DECONSTRUCTING DIRAC

Chapter VII will refocus on Dirac's most famous equation. It will attempt to deconstruct Dirac's final views on his greatest contribution described in chapter XI of the fourth and final edition of his masterpiece *Principles of Quantum Mechanics* [14]. As in previous papers in this series, the work of one of the giants of physics is used to exemplify an earlier approach to one of the central areas of modern physics. Unfortunately, the foundations of modern theoretical physics have been accepted too readily: physicists rarely study the **history** of their own subject and simply accept the canonical interpretations that usually constitutes their formal education, which is too often just a mathematical exposition (a technique that is easier to teach than physics and much easier to examine). The papers in this series demonstrate that there are alternative theories that can explain the unassailable facts discovered by experimental physicists; no one can challenge the facts but theorists forget too often today that their theories are **not** facts. The approach used throughout this programme builds on a deep respect for the history of physics and recovers several older, neglected (and often forgotten) alternatives. Natural philosophy begins with **ontology** – the identification of the fundamental objects of reality, the qualities and interactions of these objects are then investigated; finally, if possible, a mathematical scheme is then applied to generate numerical values that can be compared with standard experiments. This has **not** been the approach for 100 years. The popular modern approach has **not** been successful.

Dirac had an intuition that the electron was one of the most important constituents of material reality; this programme takes this view to its logical extreme: the only particle of material reality is the electron. A critical analysis of his brief penultimate chapter of his famous text on the relativistic electron is the appropriate subject for the penultimate chapter of this paper. Here, Dirac's approach is severely criticized both from previously unstated philosophical and mathematical perspectives. The present theory rejects the asymmetry of focusing on a single electron and restores all the original source electrons to an equal status as the target electron. Dirac, in constructing his probability density, had to revert to restoring the implied charge-density model that was the foundation for classical EM theory and was severely criticized in an earlier paper [15].

Reviews and criticisms of several features of Dirac's electron theory are covered in this chapter including the prediction of the electron's own intrinsic light-speed oscillations around its center of mass (that are given a different interpretation here). This chapter also expands on Dirac's perennial concern that Hamiltonian mechanics was too classical to be a suitable basis for a relativistic theory. This classical versus relativistic dichotomy always bothered Dirac when he discussed the role of 'observables' in any relativistic theory, particularly as this concept lies at the heart of his own formulation of non-relativistic QM. The new theory rejects this central role of the mathematics of 'observables'.

The mathematical heart of Dirac's electron theory was the use of four states (or solutions) to describe an electron; these were very embarrassing for Dirac because two of them were needed for his model of the positron but they corresponded to states of negative energy – a totally meaningless idea in physics when describing the motion of a free particle. These ideas are not needed here to explain the positron and all four states are given an entirely different interpretation without introducing any need for negative energy. The present theory returns to Newton's philosophical concept of the vacuum (contrasting with that of DesCartes *and* Dirac) that a volume of space without particles literally contains no thing: no fields, no energy; nothing. In the present theory there is no infinite 'self-energy' as the electron is not modeled here as a real object of any finite size covered with repulsive electrical 'paint' (CEM) nor does it interact with an infinite number of negative-energy electron states or its own EM field (QFT). It only interacts with another electron that is never at the same location in space at the same time. The final section addresses Dirac's mistaken idea that his four states doubly corresponded to Pauli's spatial (3D) 'spin vector'.

Physics has not yet faced up to the implications of the experimental observations of the electron – that this is a fundamental, **point** particle of matter: it has **no** spatial extent and **cannot** be viewed as a finite object rotating *in toto* around some interior axis. In the present theory, 'spin' is interpreted as the unit of interaction between **two** electrons. The origins of Dirac's errors are finally exposed as due to a direct result of starting new ventures with mathematical leaps of the imagination instead of acknowledging the primacy of philosophy – new, **visualizable** concepts must come first and **then** mathematics.

1.3.8 SUMMARY & CONCLUSIONS

The final chapter summarizes the major points brought out in the earlier points of this extensive paper. It also draws attention to the major results obtained in the heart of this paper (chapters five and six). This paper is dedicated to P. A. M. Dirac – it was reading his text as a teenager fifty years ago that attracted the author to the study of theoretical physics. Although the present theory offers an alternative approach to his famous Electron Equation, it acknowledges that the present theory could never have been constructed without the pioneering work of this *Strangest Man*. [16]

2. DIRAC'S RELATIVISTIC ELECTRON

This section will set the stage for the rest of the paper; in particular, it will review the historical context of the two major ideas that are the focus here, namely: the concepts of particle 'spin' and the relativistic motion of electrons. As described in the introduction, outstanding historical criticisms of the orthodox theories will be discussed and several new problems will be introduced. Later sections will re-examine and re-interpret these two key concepts in terms of neo-classical imagery.

2.1 ELECTRON SPIN

The concept of electron spin was invented to explain a mysterious doubling (duplexity) in the spectral lines of excited atoms and molecules. This concept rapidly moved to the center of modern quantum theory as it was seen as a special quantum property of all fundamental particles. The intrinsic mathematics of spatial rotation was incorporated into the wave equations that were used to characterize the dynamics of single particles. The orthodox view today, repeated by most physicists, is that there have been no satisfactory explanations that can be given a visual (or classical) analog of this phenomenon – from its inception onwards it has remained a purely mathematical description. This widespread opinion is not necessarily a fact of nature and an alternative view, more attractive to the imagination, will be described below.

2.1.1 PAULI INVENTS SPIN

A new (or fourth) quantum number was first introduced in the context of the emission spectra of alkali metals. In 1925, Wolfgang Pauli (1900-1958) introduced what he called a "two-valued quantum degree of freedom" associated with every electron in the atom [17]; this new idea allowed Pauli to resolve the 'duplex' inconsistencies between observed molecular spectra and the evolving new theory of quantum mechanics. In particular, this allowed Pauli to formulate his Exclusion Principle, where every electron in an atom had to have a unique set of quantum numbers. Pauli formalized the theory of spin in 1927 [18], using the modern theory of quantum mechanics created by Heisenberg and Schrödinger. He pioneered the use of Pauli matrices as a representation of the spin operators and introduced and solved his two-component QM wave function equation. Pauli acknowledged in a footnote that it was actually ('the bad Nazi of physics') Pascual Jordan (1902-1980), who had introduced these critical matrices to him and who had told him of their connection with quaternions.

2.1.2 DISCOVERING SPIN

Ralph Kronig (1904-1995) suggested in early 1925 that Pauli's new quantum degree of freedom was produced by the self-rotation of the electron, treated as a small rigid body revolving around its own axis. Pauli scornfully dismissed this idea, noting that points on the electron's equatorial surface (based on a plausible estimate of the electron's radius) would have to be moving greater than light-speed in order for it to rotate fast enough to produce the necessary angular momentum. This would then violate one of the major predictions of special relativity. Largely due to Pauli's criticism, Kronig decided not to publish his idea. In the fall of 1925 the same idea was proposed by two Dutch physicists, George Uhlenbeck (1900-1968) and Samuel Goudsmit (1904-1978), they followed the advice of their PhD supervisor (Paul Ehrenfest) and published their results, eventually winning the Max Planck Medal in 1964 for their proposal. This proposal soon met a favorable response, especially after Llewellyn Thomas (1903-1992) managed, in 1926, to resolve a disturbing discrepancy [1x] between known experimental results and Uhlenbeck and Goudsmit's calculations (and Kronig's unpublished ones) involving the spin-orbit coupling. Thomas proposed that the resolution of this discrepancy (a missing factor of two, now known as the Thomas precession) was produced by the relativistic time dilation of the electron's precessing orbit relative to the fixed nucleus. Goudsmit has provided an amusing personal recollection of this episode in an anniversary lecture [19].

2.1.3 SPIN & RELATIVITY

Pauli's theory of spin was non-relativistic. However, in 1928, Dirac published a wave equation [8], which described the relativistic electron. In the Dirac Equation, a four-component spinor (known as a 'Dirac spinor') was used for the electron wave function. Unfortunately, this was overwhelmingly a mathematical approach, so that as one historian of science wrote recently [20]: "the Dirac theory added nothing new to our knowledge of how or why this property is an intrinsic or internal feature of the electron." The major reason for the early acceptance of Dirac's theory was its exact prediction (including the Thomas factor) of the electron's magnetic moment needed to account for the observations of atomic spectra. This equation has now been extended to describe all fundamental spin-half particles (fermions), such as protons, neutrons and even quarks.

2.1.4 SPIN & MAGNETIC MOMENTS

Electron Magnetic Dipoles

The philosopher Margaret Morrison, in her contribution to the history of the electron [20], traces the first model of electron spin back to 1903 when Max Abraham provided a detailed study of the dynamics of the new electron [21], assuming it to be a spherical, rigid object with an homogenous surface or volume charge. This model figured prominently in Goudsmit and Uhlenbeck's later spin hypothesis and remains, even today, the most popular physical image of electron spin. Particles with spin can possess a magnetic dipole moment, just like a rotating electrically charged body in classical electrodynamics. These magnetic moments have been experimentally 'observed' in several ways such as by the small deflection of charged particles by inhomogeneous magnetic fields in the Stern-Gerlach experiment or by measuring the magnetic fields generated by the particles themselves. All real particles (such as electrons and protons and even neutrons) can have a permanent magnetic moment along the direction of their spin and this magnetic moment gives rise to EM interactions that depend on the spin.

Electron g-Factor

The magnetic moment of a particle is usually compared with its mechanical angular momentum through a dimensionless quantity called the g-factor. For exclusively orbital rotations this ratio is defined to be unity. The electron, despite being an elementary particle with no structure, possesses a non-zero magnetic moment. One of the major triumphs of the theory of modern QED is its accurate prediction of the electron g-factor, which has been experimentally determined to have the value 2.0023193043768(86), with the first 12 figures certain. The exact value of 2 was predicted by the Dirac Equation and the decimal remainder is thought to arise from the electron's interaction with the surrounding EM field, including its own field.

2.1.5 SPIN & STATISTICS

In 1940, Pauli proved the "Spin-Statistics" theorem [22], which states that all real particles are fermions with half-integer spin while bosons must have integer spin**.** Symmetric quantum states are occupied by bosons. Fermions can only occupy unique anti-symmetric quantum states, forbidding fermions from sharing quantum states - a restriction now known as the Pauli Exclusion principle. In terms of many-particle quantum statistical mechanics (QSM), fermions must obey Fermi-Dirac statistics if they are to be described by a causal theory of quantum fields [23] while bosons obey Bose-Einstein statistics. Pauli's paper was an analysis of continuous wave functions represented by the most general class of tensors. He focused on particles with non-zero mass, in the rest frame of the particle, where the wave function depends only on time. In fact, the analysis was limited to the second quantization of 'free' fields – that is, those satisfying the standard Klein-Gordon wave equation (see next). The only 'relativistic' feature used was the fact that measurements at two points separated by a space-like interval can never disturb each other as all real signals are limited to light-speed. Pauli concluded that the mathematical anti-commutator of two wave functions for half-integral spin particles could only be non-zero (i.e. quantized) if the fields satisfy Bose-Einstein statistics but then "the energy of the system would not be positive." But the kinetic energy of several 'free' electrons must be positive so "we must apply the Exclusion Principle in connection with Dirac's hole theory." Indeed, he referred explicitly to the paper by his student, M. Fierz [24] for "the proof that a theory with a positive total energy is possible by quantization according to the Exclusion Principle for half - integral spins."

2.1.6 THE MYSTERY OF SPIN

Standard View of Spin

The central puzzle of the concept of electron 'spin' is the reconciliation of the idea of rotation with the proposition that the electron is a point object, as confirmed by every attempt to measure the electron's size over the last 100 years. All classical ideas of angular momentum require a **rigid** object of finite size, where all its parts rotate at the same angular speed around an axis. This was recognized soon after the electron was discovered and several finite models of the electron were proposed; each of these models required a finite shape (sphere or ellipsoid) that was either filled or covered with electrical fluid [25]. The modern view is simply to say that quantum objects, like electrons, cannot be visualized but they can still be described by similar mathematics to that used to describe macroscopic (classical) objects.

"According to the received view, the spin of the electron or any other elementary particle is a mysterious internal angular momentum for which there is neither a classical analog nor a concrete physical picture." [26] Today, mathematicians are quite happy to view spin simply as a consequence of merging group theory and complex numbers – a Pythagorean, rather than a physical, perspective: a viewpoint that is nearly universal amongst modern theoretical (or mathematical) physicists.

Rotating Waves

One of the very few exceptions to the view that "spin is a mysterious angular momentum for which no physical picture is available" was the very readable paper written by Hans Ohanian in 1986 [27]. He reminded his readers that a few authors, such as W. Gordon in 1928 and F. Bellifante in 1939, had presented a physical model of spin as an intrinsic, rotating wave form (classical or quantum); unfortunately, even this fairly recent reminder has now been largely forgotten. Ohanian quite sardonically describes "Pauli pontificating that spin is an essentially quantum mechanical property" after he rejected the rotating sphere model. Even after Dirac's electron theory, this left physicists with the central concept of spin minus any physical basis, "like the grin of the Cheshire cat." The key starting point of this more physical model was the observation that the symmetrical tensor representations of the energy-momentum density can produce separable orbital and spin angular momentum – this is not possible with the canonical anti-symmetrical representations. Ohanian showed that the Poynting EM energy density vector can be separated into an obviously orbital part and a distinct 'spin' part created from the vector cross product of the electric field and vector potential densities (i.e. $\underline{E} \wedge \underline{A}$). He then transformed the momentum density formed from the Dirac electron wave function to expose the separable spin-density energy flow (i.e. $\psi^{\dagger}\, \underline{\sigma}\, \psi\, h/4\pi$). The electron's magnetic moment is then seen as a circulating flow of electric charge of the wave field, even when the electron is at rest. When moving, the electron's rotating wave describes a helical motion around its average velocity.

Up or Down?

Quantum mechanics states that the component of angular momentum measured along any direction (say along the *z*-axis) can only take on discrete values. There are exactly 2s+1 possible values of S_z , where *s* is the principal spin quantum number introduced in the previous section. For example, there can be only two possible values for any spin 1/2 particle: $S_z = +1/2$ and $S_z = -1/2$. These correspond to quantum states in which the spin is pointing in the $+z$ or $-z$ directions respectively, and are often referred to as "spin up" and "spin down". For a given quantum state, it is possible to describe a spin vector whose components are the expectation values of the spin components along each axis, i.e., $\pm\frac{1}{2}\, h$. This spin vector describes the "direction" in which the spin is pointing, corresponding to the classical concept of the axis of rotation. It turns out that the spin vector is not very useful in actual quantum mechanical calculations, because it cannot be measured directly: S_x, S_y and S_z cannot possess simultaneous definite values, because of a quantum uncertainty relation between them. For statistically large collections of particles that have been placed in the same pure quantum state, such as in a Stern-Gerlach apparatus, the spin vector does have a well-defined experimental meaning: it specifies the direction in ordinary space in which a subsequent detector must be oriented in order to achieve the maximum possible probability (100%) of detecting every single particle in the collection. For spin-1/2 particles, this maximum probability drops off smoothly as the angle between the spin vector and the detector increases, until at an angle of 180 degrees —that is, for detectors oriented in the opposite direction to the spin vector—the expectation of detecting particles from the collection reaches a minimum of 0%.

2.2 THE DIRAC RELATIVISTIC ELECTRON

2.2.1 THE QUANTUM-RELATIVITY RACE

By 1927-28, several of the 'Young Turks', like Heisenberg, Dirac and Jordan, were in an intense race to completely replace Bohr's orbital theory of atomic structure with a more general theory of quantum mechanics. The 'holy grail' was to include the special theory of relativity (SRT) in the new theory. The first milestone was to achieve at least as good a result as Arnold Sommerfeld (1868-1951), who had added relativity and elliptical orbits to Bohr's simple model in 1916 [28]. The goal was to match the fine structure prediction of the hydrogen atom spectrum that Sommerfeld had first derived in the previous year [29]. Most of Dirac's competitors were working on a wave equation that eventually became known as the Klein-Gordon (or KG) equation. This was a simple quantum mechanical operator-substitution of the energy E and momentum P in the energy momentum formula for a single particle that resulted from Planck's 1906-7 Proposal for relativistic momentum [9]; thus:

$$E^2 = P^2 c^2 + m^2 c^4 \quad \text{with} \quad \underline{P} \Rightarrow -i\, h_D\, \nabla \quad \text{and} \quad E \Rightarrow i\, h_D\, \partial / \partial t \quad \text{so that:} \quad (c^2\, \nabla^2 - \partial^2 / \partial t^2 - m^2 c^4 / h_D{}^2)\, \psi = 0 \quad \text{(KG)}$$

Here (and henceforth) the Dirac form of Planck's Constant (of action) h_D will be used to drop 2π factors; i.e. $h_D = h/2\pi$.

Dirac's intuition was that the non-relativistic form of Schrödinger's Equation, namely: $\mathbf{H}\, \psi = i\, h_D\, \partial\psi / \partial t$ was actually the most general quantum form of Hamiltonian mechanics, when **H** became the operator form of the Hamiltonian H, even for high-speed (relativistic) situations. The problem with the Klein-Gordon Equation is that it is second-order in $\partial/\partial t$.

This was Dirac's central insight: he wanted a linear form of the wave equation so that he could still preserve the interpretation of the wave function ψ as a probability density $\psi^*\psi$ and charge density $e\psi^*\psi$ at a single point in space with the idea of causality. A linear QM equation means that the wave function at any time determines the wave function at any later time. Dirac was also inspired by Pauli's new work on spin, which Dirac suspected was connected with relativity. He also knew that certain spectral results could only be interpreted by adding 'duplexity', where the observed number of stationary states in an atom was twice the theoretical number even when the electron was considered simply as a point-charge. He was well aware of the recent Pauli-Darwin spin theory [30] but knew that it had only added spin as a first-order correction to the non-relativistic Hamiltonian; as a result, it only matched Sommerfeld's fine structure formula to first order. In his famous text [14], Dirac rewrote his own history and claimed that his primary motivation was to produce a wave equation that was Lorentz-invariant; however, he correctly acknowledged that finite Lorentz transformations need to be built up from infinitesimal transforms.

2.2.2 DIRAC'S ELECTRON EQUATION

The world first heard of Dirac's revolutionary proposal in 1928 when his two-part paper [8] on a relativistic equation of the electron was published in the *Proceedings of the Royal Society* with the title *The Quantum Theory of the Electron*. Like his rivals, Dirac began with the KG equation for a single electron but now using the 4-vector form of momentum, so $P_0 = E/c$:

$$(-P_0^2 + \underline{P} \bullet \underline{P} + m^2 c^2)\,\psi = 0$$

Dirac had realized that "the symmetry between P_0 and each of the 3D momenta, P_k required by relativity shows that since the Hamiltonian we want is linear in P_0 it must also be linear in each of the P_1, P_2 and P_3." Dirac has related how he came to his famous equation for the electron [31]. He had already noticed that Pauli's 'spin' matrices σ_k (k = 1, 2, 3; see section 3.1.1) always satisfied the identity for any vector $\underline{A}$: $(\underline{\sigma} \bullet \underline{A})^2 = \underline{A} \bullet \underline{A}$. This implied that $\underline{\sigma} \bullet \underline{P}$ was in some sense a square root of P^2. So, he proposed:

$$(-P_0 + \underline{\alpha} \bullet \underline{P} + \alpha_4 m c)\,(P_0 + \underline{\alpha} \bullet \underline{P} + \alpha_4 m c)\,\psi = 0$$

All the cross-terms can be eliminated by imposing the condition: $\alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu = 2\,\delta_{\mu\nu}$ for μ & ν = 1, 2, 3, 4

Dirac knew that the Pauli spin matrices σ_k satisfy similar conditions but there are only three of them. The key breakthrough was his realization that he did not need to stay with the conventional (2 x 2) matrix representation of the σ_k's but could also use a (4 x 4) representation, which needs four basic matrices, α_μ (see section 3.1.2):

This gave him the famous free electron Relativistic Wave Equation: $(c\,\underline{\boldsymbol{\alpha}} \bullet \underline{P} + \boldsymbol{\alpha}_4 m c^2)\,\Psi_\mu = \pm E_\mu \Psi_\mu$

Here, the wave function had to be expanded to four independent (4 x 1) solutions Ψ_μ, each with its own eigenvalue, E_μ. So, in seeking to double the number of electron states, Dirac had ended up quadrupling the number of possibilities! He simply 'threw away' two of these solutions as he felt (but did not prove) that these corresponded to electrons with positive charge. Dirac then continued in his first paper to prove that this equation was Lorentz-invariant by applying the same (4 x 4) matrix transforms to all the terms in the equation, including the α_μ matrices, which still satisfied the 'anti-commutation' rule above.

2.2.3 DIRAC DISCOVERS SPIN

The heart of this paper was the appearance of electron spin in section four after Dirac created a relativistic Hamiltonian for a single electron in an electromagnetic field by "adopting the usual procedure" of substituting $(P_\mu + e_D A_\mu)$ for P_μ in the free case, where the A_μ are the four EM potentials of the external field in 4-vector notation and (following Dirac) $e_D = e / c$. Dirac also used another general spin vector identity [32] (see section 3.1.1):

$$(\underline{\sigma} \bullet \underline{r})\,(\underline{\sigma} \bullet \underline{P}) = \underline{r} \bullet \underline{P} + i\,\underline{\sigma} \bullet \underline{S} \qquad \text{using } \underline{S} = \underline{r} \wedge \underline{P} = \tfrac{1}{2}\,\underline{\sigma}\,h_D$$

$$(\underline{\sigma} \bullet (\underline{P} + e_D \underline{A}))^2\,\psi = (\underline{P} + e_D \underline{A})^2\,\psi + e_D h_D \psi\,(\underline{\sigma} \bullet \nabla \wedge \underline{A}) \quad \text{where the momentum operator form is used.}$$

So, the quadratic form of the Hamiltonian equation, including the EM field, using the (4 x 4) form of $\underline{\boldsymbol{\sigma}}$, becomes:

$$\{-(P_0 + e_D A_0)^2 + (\underline{P} + e_D \underline{A})^2 + e_D h_D\,(\underline{\boldsymbol{\sigma}} \bullet \underline{\mathcal{B}}) + i\,e_D h_D\,(\underline{\boldsymbol{\alpha}} \bullet \underline{\mathcal{E}})\}\psi = 0 \quad \text{with } \underline{\mathcal{B}} = \nabla \wedge \underline{A} \text{ and } \underline{\mathcal{E}} = \nabla A_0 + 1/c\;\partial \underline{A} / \partial t$$

This form differs from the one-dimensional KG equation for a charged particle in an EM field by the addition of two extra terms that, when divided by 2*m*, Dirac regarded as the extra potential energy of the electron due to its new degree of freedom. The electron behaves as though it has a magnetic moment $\underline{\mathcal{M}}$ and an electric moment $\underline{d}$, where:

$$\underline{\mathcal{M}} = e\,h\;\underline{\boldsymbol{\sigma}} / 4\pi\, m\, c \quad \text{and} \quad \underline{d} = i\,e\,h\;\underline{\boldsymbol{\alpha}} / 4\pi\, m\, c$$

The magnetic moment was the value assumed in the Pauli-Darwin 'spinning electron' theory (but now using a 4 x 4 matrix representation instead of the 2 x 2 Pauli form). Dirac 'threw away' the electric moment term, "as it was purely imaginary". The next step was to consider the electron's angular momentum in a static central field of force (like a Coulomb field); this was equivalent to setting the vector potential, $\underline{A} = 0$ and $e_D A_0 = e\,U(r)$. He then looked for periodic solutions of the wave equation by treating the P_0 as an energy parameter E/c. First, he showed that the electron's orbital angular momentum, $\underline{L}$ (or $\underline{r} \wedge \underline{P}$) is not a constant of the motion but the combination, $\underline{J} = \underline{L} + \underline{S}$ is a constant of the motion confirming the view that the electron has a spin angular momentum $\underline{S} = \frac{1}{2}\,\underline{\sigma}\,h_D$, which contributes to the total angular momentum $\underline{J}$. Dirac relied on the Heisenberg picture to determine the operator dependence on time (with timeless wave functions), as this picture gives a closer comparison between classical and quantum mechanics. In other words, he simply assumed that the non-relativistic form for the time-dependence of operator change (not Lorentz-invariant) applied unchanged in his relativistic version.

A further block of algebraic manipulations allowed Dirac to derive a correction to the Bohr energy levels that were identical to the approximate results obtained earlier by Darwin for the hydrogen atom. Interestingly, soon after this first paper was published (greeted by wide acclaim by his rivals), Hendrick Kramers (1894-1952) showed that his own second-order wave equation for a spinning electron was equivalent to Dirac's equation: the race was close. Walter Gordon almost immediately reported to Dirac that he had used Dirac's equation to derive Sommerfeld's exact fine structure formula; this proved that Dirac had finally reached the initial goal of the relativistic-quantum 'race'. The shorter second part of this duo of papers was published just one month later; Dirac reported on the impact of transitions between the allowed integer values of the new total angular momentum, $\underline{J}$ for atoms with single electrons. The Dirac Equation was to dominate quantum theory for nearly 20 years from the early 1930s onwards and is still regarded [33] as the legitimate ancestor of all later forms of QED.

2.2.4 EARLY HISTORY OF THE POSITIVE ELECTRON

One of the myths of modern physics is that Dirac invented the positive electron – the historical record shows a richer story. Maxwell's continental rival, Wilhelm Weber developed an extensive point model of electricity [34] that already proposed complementary unit charges of electrical matter to account for the well-known fact that there were two types of electricity, with equal types repelling and opposite types attracting each other. Weber's particle theory of EM was forgotten in the rush to accept a pervasive (spanning the universe) theory based on continuum aethers and the familiar (continuum) calculus. The failure of Weber's theory to explain radiation led to a complete focus on Maxwell's theory of electromagnetism.

2.2.5 DIRAC'S ANTI-PARTICLE

Dirac was acutely troubled by the extra, negative energy solutions to his new equation of the electron but he pointed out that this was a problem for all relativistic quantum theories constructed around Planck's energy-momentum formula since this is intrinsically quadratic in the energy. It is only in a classical theory, where energy changes act continuously, that the negative energy states are separable from the positive. In any quantum theory, random transitions between all states, with positive or negative energy, are always possible. The problem for Dirac was that all four solutions were necessary if his new theory was to be accepted – a position his main rivals were quick to point out. Most embarrassingly, the energy of any electron in the negative energy states would decrease as its velocity increased even though its acceleration would be in the opposite direction to the force impressed on it. In particular, the Klein-Nishina EM theory appeared in 1929 to correctly predict the scattering of photons by free electrons using Dirac's Equation [35] but it needed to use all the negative energy states as well as the real, positive ones. Indeed, detailed calculations showed that for low-frequency scattering, where the classical Thomson scattering formula had been confirmed, almost all the scattering resulted from intermediate states where the free electron only possessed negative energy [36]: an acutely embarrassing result.

In 1929, Dirac was inspired by a recent suggestion from Weyl to interpret the negative energy states as representing protons. These ideas appeared the following year in his next relativistic electron paper *A Theory of Electrons and Protons* [37]. Here, Dirac acknowledged that the most stable states were the negative energy ones with very high velocity but he then made the radical hypothesis that all of the negative energy states were filled and Pauli's Exclusion Principle would forbid any positive energy electrons from dropping back into these occupied states. He first rejected Weyl's suggestion that the negative energy states could represent protons, as particles with negative energy have no reality in physics.

He then used the analogy with closed-shell atoms (inert gases) that the absence of an electron from such a stable configuration (or 'hole') would behave like a particle that appeared as its complement to the rest of the universe. In particular, it would have positive charge and positive mass and energy. He identified this hole with the proton. He responded now publicly to a private criticism of Bohr that the infinite 'sea' of negative energy electrons would have an infinite divergence by the claim that only variations from the norm are observable experimentally. In reacting also to Heisenberg's private objection that the mass of the negative energy state should be identical to that of the electron rather than the much more massive proton, Dirac lamely expressed the hope that a future theory of the interaction between such disparate particles would account for the difference. Worse, other physicists soon showed that this model was unstable and protons would be rapidly annihilated by electrons. Eventually, by 1931, Dirac conceded defeat and in the brief introduction to a bizarre paper [38] that predicted the existence of a quantum magnetic monopole he agreed that a *hole* in the negative energy electron 'sea' would appear as an anti-electron: a positively charged particle with the same positive mass and energy as the corresponding negatively charged electron. Just for good measure, he also casually suggested that "the protons will have their own negative-energy states" and their 'holes' would appear as 'anti-protons'. These brief suggestions followed a long defense of his approach to quantum mechanics, where new mathematics was introduced first and only subsequently should a physical interpretation be searched for afterwards. This discussion was both an explanation for his reversal over his earlier proton suggestion and a further justification for the Weyl-like 'gauge' invariance of the wave function that Dirac believed must lead to the existence of the monopole. In the conclusion, Dirac admitted to a disappointment in not deriving the fine structure (electrical) constant rather than the never-discovered magnetic monopole.

2.2.6 DISCOVERY OF THE POSITRON

The investigation of cosmic radiation emerged as a new field of study after 1930: this now provided access to much higher energy levels than were available at that time to earth-bound scientists using particle accelerators. One of these pioneers was Carl D. Anderson (1905-1991), who used a Wilson cloud chamber to photograph the tracks of penetrating particles. In 1932, he first obtained some results that suggested positive, lightweight particles. It was only in his second paper published in 1933 [39] that Anderson suggested that he had discovered a positively charged electron, which he named the 'positron'.

At first, he did not identify this new particle with Dirac's predicted anti-electron but with a nuclear particle ejected after a high-energy photon collision. It was only after similar cosmic ray experiments conducted by Ochialini and Blackett [40] (who was a colleague of Dirac's at Cambridge) that the identification was made with Dirac's new particle. These discoveries were rewarded later with the Nobel Prize for Physics: Anderson in 1936 and Blackett in 1948.

2.3 PROBLEMS WITH DIRAC'S RELATIVISTIC DYNAMICS

Even though the positron was soon seen as a vindication of Dirac's theory, there was still serious opposition from major theoreticians, with the most concern still centering on the negative energy states. Schrödinger even created a rival theory in which negative energies did not appear; his new wave equation was identical to Dirac's in the free-field case but not in the presence of EM fields. Although this alternative theory [41] was able to reproduce the fine structure formula, as well as other results of Dirac's theory, it received no general acceptance primarily because this new wave equation was not Lorentz-invariant, which was already an article of faith in QM theory. Some physicists, following Bohr, objected on positivist grounds that Dirac's 'negative-energy sea' was inherently unobservable and therefore should not be admitted into realistic physics. Pauli was one of Dirac's fiercest critics, claiming that Dirac's original motivation for linearization of the wave equation to preserve the interpretation of the wave function was now null and void, as the total number of particles was no longer conserved [42]. Pauli and Weisskopf soon published their own quantum field version of the KG equation for scalar particles [43] that incorporated several new concepts such as 'pair creation', 'annihilation' and 'anti-particles' without introducing a vacuum filled with negative-energy particles. The biggest problem with their theory was that it only referred to spin zero particles whereas Dirac's theory applied to spin half fermions, such as electrons. Even by 1933, Dirac's 'hole' theory was still controversial; nonetheless, at the age of 31, Dirac shared the Nobel Prize in Physics with Erwin Schrödinger for "the discovery of new productive forms of atomic theory", following Heisenberg's singular award in the previous year.

2.4 OTHER ELECTRON MODELS

Soon after the discovery of the electron, there were several attempts to produce visualizable models of this new particle. These failed early 20th century E/M models have been described more extensively in an earlier paper in this series [3]. The standard approach today is to 'dispense' with so-called mechanical models and describe reality only through mathematics.

2.4.1 MAINSTREAM MODELS

Feynman backtracks the Electron

In his first relativistic space-time paper [44], Richard Feynman (1918-1988) deliberately neglected the mutual interaction between electrons and only modeled their motion in a given, external EM potential, focusing on the boundary conditions of Dirac's wave function. That paper developed Stückelberg's suggestion that positrons could be represented as electrons with their proper-time reversed relative to the normal flow of time. The popular, fixed potential model is implicitly a many-body situation that ignores the symmetry of the EM interaction and reduces this asynchronous interaction to a single-particle localized Hamiltonian formulation. Feynman proposed that electron-positron annihilation corresponded to the abrupt time reversal of a single electron's trajectory. This emphasis on electric charge conservation proves much simpler than tracking each particle, forward through time. He also emphasized that the global space-time viewpoint simplifies complex processes compared with the better-known, single-time Schrödinger approach, which only considers the future as developing both continuously and deterministically from out of the past. Existentially, it is impossible to imagine how divergent quantum waves (from multiple EM point sources) can completely converge together to annihilate electron/positron pairs.

Feynman's model works because it starts with a Newtonian point model of the 'free' electron and restricts its modifications to only a small, finite number of interactions by introducing a mathematical perturbation technique to limit the continuous interactions of the EM field. The further use of standard QM 'wave' mathematics then 'smears out' the electron's point existence to all of space (by imposing a probabilistic interpretation). Finally, restricting the continuous Hamiltonian time evolution of the wave functions to constant energy situations eliminates the explicit role of time in all dynamical models.

Standard Theory defines the Electron

Today, the Glashow-Salam-Weinberg electro-weak theory views the electron as "a composite of two mass-less parts (left- and right-handed), united to form the observed unitary particle through their interactions with a scalar Higgs field". In the latest version of the 'Standard Model' [7], the electron is seen as "a member of the first of three generations of similar leptonic particles that are related in a non-trivial way to three generations of hadronic quarks." Only a mathematician would be satisfied with these types of cryptic descriptions of one of the most fundamental foundations of matter in the universe.

Penrose's Zig-Zagging Electron

Roger Penrose is a modern master of geometry; as such, he cannot avoid proposing geometric images for what most others would leave purely as algebraic or analytical mathematical operations. His efforts have often been rewarded, as geometry provides a broad path into the visual imagery of the human mind, stimulating new, visual insights and connections. The standard (4 x 4) Dirac Equation is rewritten by Penrose as two differential (2 x 2) equations each operating only on its own Pauli spinor but coupled through their common rest-mass; in effect, each spinor acting as the 'source' for the other [45]. Penrose then proposes that each of these spinors be viewed as mass-less particles, where each one is converting itself into the other – he refers to each of these as the 'zig' and the 'zag' particles traveling at light-speed: he presents this as a picture of Dirac's '*Zitterbewegung'*. Each part has its own half-integral spin with the 'zig' being left-handed and the 'zag' right-handed and as their velocities keep reversing; as they zig-zag through space the spin remains unchanged. Penrose claims that the average rate of inter-conversion is equal to the electron's de Broglie frequency ν_e (mc^2 / h). Actually, this model is much more complicated, as there is an infinite set of zigs and zags superimposed in any finite unit of time. Penrose has the rare honesty to raise the question of how real this picture is of the electron in nature but confesses that it is real for him even though it seems he is proposing no more than a pictorial image of the electro-weak theory model of the electron.

2.4.2 UNORTHODOX MODELS

Only a few physicists today are concerned by the lack of any **physical** model of the electron. A recent book, entitled "*What is the Electron?*" [46], has summarized 17 unorthodox theories that have been offered by professional physicists over the last thirty years. Several of these papers have directly contributed ideas to the present theory of the electron offered here. Some of these models are summarized next, although none has achieved any large measure of support by mainstream physicists.

Wolff & Rivas & Cramer

There have been three other theories that share some, but not all, features with the present theory. Wolff created a space-wave model of the electron that offered an explanation of electron 'spin' in terms of converging and diverging spherical waves without any discussion of the discrete nature of the electron. The second approach is the Rivas model of the single spinning electron, which is able to derive the Dirac equation of the electron but (as always) describes continuous motion through time. The best-known model here is Cramer's 'Transactional Interpretation of Quantum Mechanics' that focused on a single electron but did not include any discussion of the actual interaction between electrons.

Wolff's Symmetric Wave

Milo Wolff has proposed a wave-based model of the electron [47] that emits and absorbs spherically symmetric real waves (the 'quantum space' of an unspecified nature:). Electron spin (like all the electron's properties in his theory) is viewed as an attribute of the underlying 'quantum space' rather than of a material particle. Wolff views every particle communicating its wave-state with every other 'mass particle' throughout the universe, at all times, so that physics becomes the behavior of the entire universe. In fact, Wolff explicitly accepts W. K. Clifford's proposition that there are no material particles – only undulations in space; in other words, Wolff accepts the metaphysical reality of the primacy of the aether. Wolff has recently summarized this model in Simulik's compendium on alternative models of the electron [48]. The present theory agrees that every electron is 'aware' of the interaction status of every other electron throughout the universe at all times but no 'wave fields' are introduced as a mechanism for this knowledge, which is viewed as an intrinsic property of every electron.

The Electron Models of Rivas & Mac Gregor

Since 1984, Martin Rivas has been developing a mathematical (abstract) model of the electron based on the kinematical group of space-time transformations. This 'spinning electron' model has been summarized by Rivas himself in the recent compendium by Simulik [46] and described in detail in his earlier monograph [49]. The major feature of this model is the spatial separation of the point charge from the center of the electron's mass. In order to produce the same results as Dirac after quantization, Rivas found it necessary to propose that the point nature of the electric charge rotates at the speed of light around the electron's center of mass. This model bears some resemblance to the earlier model first proposed in 1970 by Malcolm Mac Gregor, which is also summarized in Simulik [50] and described more fully in Mac Gregor's own book [51]. In this earlier model, the electron is viewed as a solid, spinning sphere with the total charge of electricity concentrated at a point on the sphere's equator; this point is also rotating at light-speed. Rivas has indicated [52] that he first became aware of Mac Gregor's model in 1998 after he had published several papers on his own model. Rivas's electron model is based on Levy-Leblond's 1967 classical approach that described spin ½ particles in a Galilean framework [53]. This approach uses generalized Lagrangians for a point particle that invokes an arbitrary, global evolution parameter τ; the time evolution of the particle is achieved by assigning $t(\tau) = \tau$: in other words, each particle carried its own 'proper' time. The dynamics follow only the trajectory of the point charge (not the center of mass) based on a 4^{th} order differential equation. Alternatively, the center of mass trajectory can be tracked by a 2^{nd} order differential equation and the motion of the point charge around the center of mass. In general, the electron's center of mass obeys Newtonian-like dynamical equations with respect to the total external force, which must be evaluated at the position of the charge at each moment of time. Rivas, unlike Mac Gregor, has proposed no structure to maintain the fixed separation of the charge relative to its center of mass, nor how dynamical changes effecting only the charge are communicated to simultaneous changes in the motion of the electron's center of mass. The new theory presented here shares this rotational view of the electron's sub-microscopic motion but rejects Mac Gregor's 'solid' (or spherical model – made of what?) while (contra Rivas) reintegrating the charge and mass of the electron back into a single spatial location at all times; the motion of the electron is proposed to be discretely cyclic in the transverse plane.

Both electron models of Rivas and Mac Gregor have been criticized severely for having their rotational motion occurring at light-speed, which (for all followers of Einstein) is viewed as impossible even though, in the case of Rivas his light-speed electron is only the mass-less electric charge. Surprisingly, this relativistic criticism was never applied to Dirac's model where Dirac himself calculated that the electron's average velocity (*Zitterbewegung*) was always moving at light-speed.

3. RELATIVISTIC REPRESENTATIONS

This section segregates all the mathematics that has been used in earlier approaches to describe 'spin', including Dirac's electron theory. It also reviews the mathematics that has been introduced in our earlier papers and combines elements of these into variants that will be used here and in subsequent papers. The deliberate rejection of continuum mathematics in this programme means that it is important to establish these new discrete mathematical tools on a very firm foundation.

3.1 THE TRADITIONAL MATHEMATICS OF SPIN

The mathematics of spin always involves the Pauli (2 x 2) matrices that were extended to a (4 x 4) representation by Dirac. This section will first review these forms and then show their connection to *Natural Vectors* that are the basic mathematics of the present research programme. The double Pauli matrix products are isomorphic to Hamilton's quaternion bases { **i**, **j**, **k** } is the critical link across all these representations. It was shown in the first paper in this series [1] how these are related to the real (4 x 4) bases $\mathbf{I}_\mu$ of the Natural Vectors. It was then shown in the second paper [2] in this series that these Natural Vectors form the simplest algebraic representation for describing Maxwell's standard theory of electromagnetism. It was inevitable that someone, like Dirac, would eventually link these (4 x 4) representations to a theory of electron dynamics, as the electron is the real source of all EM phenomena.

Pauli obsesses on Spin & Two

Pauli acknowledged in his 1946 Nobel Prize Lecture [54] that: "his focus on the two-valued nature of spin was driven by the problem of the closing of every electron shell." He was also well aware of Sommerfeld's Keplerian obsession with the number '8' as the number of corners in a cube. Pauli was early predisposed to seeking finite mathematical representations.

Subscript Conventions

The following subscript conventions will be followed, throughout this and all subsequent papers in this series.

1. Time: $\mu = 0$
2. All Space: $\underline{k} = \{1, 2, 3\}$
3. Time & Space: $\mu = \{0, 1, 2, 3\} = \{0; \underline{k}\}$
4. Longitudinal Space: $k = 3$
5. Rotational-Transverse Space: $\lambda = \pm 1$
6. Time, Long. & Transverse: $\nu = \{0; 3, +, -\}$

3.1.1 PAULI MATRICES

Every self-adjoint 2 x 2 matrix is defined by only two real numbers and one complex number; they can always be written as a sum of Pauli matrices. Thus, if electron 'spin' is represented by a <u>pair</u> of complex numbers then the Pauli spin matrices must result in a valid mathematical representation [55]. The canonical Pauli matrices have always been represented by σ_k.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_\lambda = \sigma_x + i\lambda\sigma_y \quad \therefore \sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \sigma_-^\dagger \quad \lambda = \pm \text{ or } \pm 1$$

The commutation and anti-commutation rules are:

$$[\,\sigma_j\,,\sigma_k\,] \equiv \sigma_j\sigma_k - \sigma_k\sigma_j = 2i\sum_n \varepsilon_{jkn}\sigma_n \quad \& \quad \{\,\sigma_j\,,\sigma_k\,\} \equiv \sigma_j\sigma_k + \sigma_k\sigma_j = 2\delta_{jk}\mathbf{I}$$

where ε_{jkn} is the Levi-Civita symbol (anti-symmetric permutation factor), δ_{jk} is the Kronecker delta symbol and **I** is the unit (2 x 2) matrix; these rules may be combined into a single algebraic (group multiplication) rule:

$$\sigma_j\sigma_k = \delta_{jk}\mathbf{I} + i\sum_n \varepsilon_{jkn}\sigma_n \qquad \text{The Pauli vector: } \underline{\sigma} \equiv \sigma_x\underline{\hat{e}}_1 + \sigma_y\underline{\hat{e}}_2 + \sigma_z\underline{\hat{e}}_3$$

For any 3D vector, $\underline{A}$ there is an important identity: $\exp\{i\,\underline{A}\cdot\underline{\sigma}\} = \cos A + i(\underline{\hat{e}}\cdot\underline{\sigma})\sin A$ where $\underline{A} = A\,\underline{\hat{e}}$
The components of the Pauli vector in an arbitrary direction $\underline{\hat{e}}$ specified by the direction cosines $\{l, m, n\}$ is :

$$\underline{\hat{e}}\cdot\underline{\sigma} = l\sigma_x + m\sigma_y + n\sigma_z = \begin{pmatrix} n & K_- \\ K_+ & -n \end{pmatrix} \quad \text{where } K_\lambda = l + i\lambda m$$

Dirac also used another spin vector identity for two 3D vectors $\underline{W}$ and $\underline{V}$, derivable from their anti-commutation definitions:

$$(\underline{\sigma} \bullet \underline{W})\,(\underline{\sigma} \bullet \underline{V}) \;=\; \underline{W} \bullet \underline{V} \;+\; i\,\underline{\sigma} \bullet \underline{W} \wedge \underline{V} \qquad \& \qquad \sigma_x\,\sigma_y\,\sigma_z = \; i\,\mathbf{I}$$

The basic set $\{\, i\,\underline{\sigma}\,\}$ are the generators of the Lie algebra SU2, which is doubly isomorphic to another Lie algebra SO3 – the group of rotations in 3D space; so that this Pauli set is the smallest matrix representation of infinitesimal 3D rotations. All (2 x 2) unitary matrices **U** can be expressed in the form:

$$\mathbf{U} = \exp\{\, i\,\alpha\,\sigma_x \,\}\,\exp\{\, i\,\beta\,\sigma_y \,\}\,\exp\{\, i\,\gamma\,\sigma_z \,\}\,\exp\{\, i\,\delta \,\} \qquad \text{for all real } \alpha,\, \beta,\, \gamma,\, \delta.$$

Here, the set $\{1,\, \sigma_x\sigma_y\,,\, \sigma_z\sigma_x\,,\, \sigma_y\sigma_z\,\}$ is isomorphic to the real quaternions $\{1,\, \mathbf{i}\,,\, \mathbf{j}\,,\, \mathbf{k}\}$, which is the largest, useful division algebra. Although the two spin 'up' and 'down' states are often visualized as the North and South poles on a 3D sphere they are actually orthogonal vectors in a 2D Hilbert (complex) space: a mathematical concept known as a '*spinor*' (see 3.1.4).

3.1.2 DIRAC MATRICES

In developing his linear Hamiltonian form for describing the dynamics of a single electron, Dirac was soon forced to move beyond Pauli's (2 x 2) matrix representation to a full (4 x 4) representation that satisfied similar multiplication rules. The choice of representation proposed by Dirac was a direct analogue of Pauli's famous duplex representation:

$$\boldsymbol{\sigma}_k = \begin{vmatrix} \sigma_k & \emptyset \\ \emptyset & \sigma_k \end{vmatrix} \qquad \boldsymbol{\alpha}_k = \begin{vmatrix} \emptyset & \sigma_k \\ \sigma_k & \emptyset \end{vmatrix} \qquad \text{and} \qquad \boldsymbol{\alpha}_0 = \begin{vmatrix} \mathbf{I} & \emptyset \\ \emptyset & -\mathbf{I} \end{vmatrix} \qquad \text{where} \quad \mathbf{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad \text{and} \quad \emptyset = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$$

The commutation and anti-commutation rules for the (4 x 4) matrices, $\boldsymbol{\sigma}_k$ are identical to those for the (2 x 2) representation; the product rules for Dirac's $\boldsymbol{\alpha}_k$ matrices are:

$$[\, \boldsymbol{\alpha}_j\,,\, \boldsymbol{\alpha}_k \,] \;=\; \boldsymbol{\alpha}_j\,\boldsymbol{\alpha}_k - \boldsymbol{\alpha}_k\,\boldsymbol{\alpha}_j \;=\; 2\,i\,\textstyle\sum_n \varepsilon_{jkn}\,\boldsymbol{\sigma}_n \qquad \& \qquad \{\, \boldsymbol{\alpha}_\mu\,,\, \boldsymbol{\alpha}_\nu \,\} \;=\; \boldsymbol{\alpha}_\mu\,\boldsymbol{\alpha}_\nu + \boldsymbol{\alpha}_\nu\,\boldsymbol{\alpha}_\mu \;=\; 2\,\delta_{\mu\nu}\,\mathbf{I} \quad \text{for} \quad \mu\ \&\ \nu \;=\; 0,\, 1,\, 2,\, 3$$

Dirac found it useful to introduce a second set of (4 x 4) matrices, which he designated as $\boldsymbol{\rho}_\mu$ and defined as: $\boldsymbol{\alpha}_k \;=\; \boldsymbol{\rho}_1\,\boldsymbol{\sigma}_k$

$$\boldsymbol{\rho}_1 = \begin{vmatrix} \emptyset & \mathbf{I} \\ \mathbf{I} & \emptyset \end{vmatrix} \qquad \boldsymbol{\rho}_2 \;=\; i\begin{vmatrix} \emptyset & -\mathbf{I} \\ \mathbf{I} & \emptyset \end{vmatrix} \qquad \text{and} \qquad \boldsymbol{\rho}_3 = \begin{vmatrix} \mathbf{I} & \emptyset \\ \emptyset & -\mathbf{I} \end{vmatrix} \;=\; \boldsymbol{\alpha}_0$$

These satisfy: $\boldsymbol{\alpha}_j\,\boldsymbol{\alpha}_k \;=\; \delta_{jk}\,\mathbf{I} \;+\; i\,\textstyle\sum_n \varepsilon_{jkn}\,\boldsymbol{\sigma}_n \qquad \& \qquad \boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_j \;=\; i\,\sigma_j\,\boldsymbol{\rho}_2 \qquad \& \qquad \boldsymbol{\alpha}_j\,\boldsymbol{\alpha}_0 \;=\; -\,i\,\sigma_j\,\boldsymbol{\rho}_2 \qquad \text{for} \quad j,\, k \;=\; 1,\, 2,\, 3$

Care must be taken with the definitions of the (4 x 4) matrices used in relativistic presentations, not all authors follow Dirac; some have their own idiosyncratic conventions, such as Feynman [56], who adds a 'negative twist'.

$$\boldsymbol{\gamma}_k \;=\; \boldsymbol{\rho}_3\,\boldsymbol{\alpha}_k \;=\; \begin{vmatrix} \emptyset & \sigma_k \\ -\sigma_k & \emptyset \end{vmatrix} \qquad \text{and} \qquad \boldsymbol{\gamma}_0 \;=\; \begin{vmatrix} \mathbf{I} & \emptyset \\ \emptyset & -\mathbf{I} \end{vmatrix} \;=\; \boldsymbol{\alpha}_0 \qquad \text{and} \qquad (\boldsymbol{\gamma}_k)^2 \;=\; -\,\mathbf{I} \;\&\; (\boldsymbol{\gamma}_0)^2 \;=\; \mathbf{I} \qquad \text{or} \qquad \boldsymbol{\gamma}_0\,\boldsymbol{\gamma}_k \;=\; \boldsymbol{\alpha}_k$$

Dirac believed that Special Relativity results in the two-state nature of the electron (i.e. 'spin'). In contrast, Tomanaga believed (but never proved) that it was the electron's two-state nature that generates the results of Special Relativity [57]. He described how Pauli tried to extend his 3D spin matrices to six dimensions but found great difficulty introducing the condition that half of them must have zero spin in the rest system. This was one of the clues pursued in developing the present two-electron view of the EM interaction. Tomonaga showed that six triplet and doublet combinations of Dirac's alpha matrices form the components of a 6D vector, these are:

$$\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_1\,\boldsymbol{\alpha}_2 \quad,\quad \boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_2\,\boldsymbol{\alpha}_3 \quad,\quad \boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_3\,\boldsymbol{\alpha}_1 \quad,\quad \boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_1 \quad,\quad \boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_2 \quad,\quad \boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_3$$

The first three are isomorphic with: $\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_1\,\boldsymbol{\alpha}_2 \;=\; i\begin{vmatrix} \sigma_3 & \emptyset \\ \emptyset & -\sigma_3 \end{vmatrix}$, $\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_2\,\boldsymbol{\alpha}_3 \;=\; i\begin{vmatrix} \sigma_1 & \emptyset \\ \emptyset & -\sigma_1 \end{vmatrix}$, $\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_3\,\boldsymbol{\alpha}_1 \;=\; i\begin{vmatrix} \sigma_2 & \emptyset \\ \emptyset & -\sigma_2 \end{vmatrix}$

The other three doublets are isomorphic: $\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_1 \;=\; \begin{vmatrix} \emptyset & \sigma_1 \\ -\sigma_1 & \emptyset \end{vmatrix}$, $\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_2 \;=\; \begin{vmatrix} \emptyset & \sigma_2 \\ -\sigma_2 & \emptyset \end{vmatrix}$, $\boldsymbol{\alpha}_0\,\boldsymbol{\alpha}_3 \;=\; \begin{vmatrix} \emptyset & \sigma_3 \\ -\sigma_3 & \emptyset \end{vmatrix}$

3.1.3 HIGHER ALGEBRAS

Clifford Algebras

Before leaving this section on fundamental algebra, it would be useful to review why this programme has rejected two other higher algebras that have attracted some attention in recent attempts by mathematical physicists to represent reality.

In addition to continuous rotations, it is normally possible to define a discrete operation, namely reflection. In three spatial dimensions, there are three independent reflection operations (represented by the operators $\boldsymbol{\rho}_k$), each of which reverses its own co-ordinate axis while leaving all the others axes unchanged [58]; obviously, reflecting any normal object twice in the same direction leaves it unchanged. Each reflection is equivalent to a rotation around an orthogonal axis by π, so a double reflection is equivalent to a full rotation of 2π. The discussion above (and section 3.1.4) showed that a full 2π rotation of a spinorial object generated its own reversal or negative, so that a double reflection of a spinorial object will also generate its own negative, like quaternion multiplication. Thus, for a spinor ψ, a double reflection along the k-axis gives:

$$\boldsymbol{\rho}_k \boldsymbol{\rho}_k \psi = -\psi \qquad \text{or, as an operator multiplication:} \qquad \boldsymbol{\rho}_k^{\,2} = -1$$

Two successive reflections, not in the same direction (say, j and k), are represented by ordered pairs of distinct reflection operators, so that these (like quaternions) generate anti-commutation products:

$$\boldsymbol{\rho}_j \boldsymbol{\rho}_k + \boldsymbol{\rho}_k \boldsymbol{\rho}_j = 0 \quad (\text{for } j \neq k) \qquad\qquad \boldsymbol{\rho}_j \boldsymbol{\rho}_k + \boldsymbol{\rho}_k \boldsymbol{\rho}_j = -2\,\delta_{jk} \quad (\text{for all j and k})$$

The three second-order products are isomorphic with Hamilton's quaternion vector bases $\{\,\mathbf{i}, \mathbf{j}, \mathbf{k}\,\}$:

$$\mathbf{i} = \boldsymbol{\rho}_2 \boldsymbol{\rho}_3 \quad , \quad \mathbf{j} = \boldsymbol{\rho}_3 \boldsymbol{\rho}_1 \quad , \quad \mathbf{k} = \boldsymbol{\rho}_1 \boldsymbol{\rho}_2$$

The $\boldsymbol{\rho}_k$ operators represent the Clifford algebra in three dimensions first introduced by the brilliant British mathematician, William Kingdon Clifford (1845-1879) one year before his very early death. Spinors may thus be thought of as the mathematical 'objects' on which the elements of a Clifford algebra act, as operators representing such operations as reflection and rotations. These elements form an N-dimensional 'spin-space', which in 4D splits into two independent 2D 'spaces'; so that a 4D spinor (like Dirac's electron spinor) is always the sum of two complementary (2 x 2) basic spinors. These two 'sub-spaces' have opposite '*chirality*' (or 'handedness') corresponding to clockwise and counter-clockwise rotations around any axis – this is the clue to the real explanation of electrons and their anti-particles ('positrons'), as will be shown in section 5. This programme rejects the direct use of Clifford algebra in favor of Natural Vectors, as it agrees with the Newtonian view that reality makes a fundamental distinction between the characteristics of time and space. This historic separation is not acknowledged in nearly all of today's 4D space-time theories that adopt a Minkowski view of nature, where time and space are blended together – a view suitable only for mathematicians in an ideal or empty world: not one that the rest of humanity inhabits. All of us will eventually take our permanent place in real space at a critical point in time.

Grassmann Algebras

Clifford algebras need to be able to 'define an axis' in space so that rotations can be constructed out of reflections. This requirement can dropped in the truly general algebra, first introduced in 1844 by Hermann Grassmann (1809-1877). In this N-dimensional algebra, the elements $\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_N$ satisfy the general anti-commutation rule:

$$\boldsymbol{\eta}_j \boldsymbol{\eta}_k + \boldsymbol{\eta}_k \boldsymbol{\eta}_j = 0$$

Here, each $\boldsymbol{\eta}_k$ is associated with an arbitrary direction in N-dimensional space but unlike Clifford algebras, these axes need not be orthogonal. Any vector in this 'space' A directed from the 'origin' can be represented by an N-tuple of real parameters $\{a_1, \ldots, a_N\}$ applied to these algebraic bases:

$$\underline{A} = \Sigma\, a_k \boldsymbol{\eta}_k$$

In order to distinguish the Grassmann algebraic products from the more conventional Clifford products, it is now quite common to use the 'wedge' symbol '$\wedge$', so that (like simple 3D vector notation), the anti-commutation rule is written:

$$\boldsymbol{\eta}_j \wedge \boldsymbol{\eta}_k = -\boldsymbol{\eta}_k \wedge \boldsymbol{\eta}_j$$

These Grassmann products are used to represent planar surfaces but our research programme is based on the hypothesis that electrons are real point objects in 3D space so that there will be no further reference to Grassmann algebras in subsequent papers. The Clifford algebra approach to Natural Vectors is acceptable, for in the present model of reality the EM interaction between two electrons always defines one privileged spatial direction in 3D space and a temporal direction in 1D time.

Hestenes' Geometric Bias

The Natural Vector (4 x 4) real matrix formulation [1] showed that Hamilton's intuition for the suitability of quaternions for representing the four independent dimensions of nature was correct. The isolation of the 3D components to form the basis of geometrical (timeless) vectors by Gibbs and Heaviside removed the natural symmetry of this representation. This move away from the dynamic has been further reinforced in recent years by David Hestenes [59], who has also created the closest approach in the scientific literature to the present theory of the electron. His attempt to minimize the distinction between different algebras was motivated by the seduction of geometric analogies but this now only leads to the traditional (2 x 2) spinor isomorphisms including double-spinors for his 'space-time' algebra. By adopting Minkowski's fourth dimension for the role of time, Hestenes was able to create a real space-time algebra that is suitable for describing the relativistic physics of fields incorporating Lorentz transformations at every point. There are many overlaps mathematically (but not physically) between Hestenes geometric approach and the Natural Vector point algebra used in this programme. Hestenes has applied his non-commutative geometric algebra to the Dirac electron [60]. His radical model also described the electron as a light-like particle with helical "*zitterbewegung*", showing that spin and phase are inseparably related and that the Dirac equation must be viewed "as some kind of average form". In keeping with standard approaches, Hestenes continues to invoke field theory and focuses exclusively on a single electron subject to external (EM) influences. Rather than deriving electron spin and relativistic momentum from first principles, Hestenes just simply posits these key concepts as mathematical definitions – a technique that explicitly assumes spin has a magnitude of one half and implicitly smuggles in Planck's Proposal for relativistic mass. These assumptions lead directly to the idea of a 'zitter radius' equal to the Compton wavelength: a magnitude that reappears in the present theory.

The Centrality of Time

David Hestenes repeated the classic Pythagorean error when he re-named his 'Space-Time Algebra' to 'Geometric Algebra' [61] and [62]. This dropped the unique contribution of time from the dynamical description of physics. Although Newton was always aware of the special role of time in 'growing' his continuous curves across space when he analyzed planetary trajectories, the scientists who further developed his mathematics could not resist their Greek heritage and converted time into an equivalent 'spatial' dimension. Generations of physics students have been educated by viewing 2D diagrams of motion where time is portrayed as just 'the other line' on the blackboard to complement one of the spatial dimensions.

3.1.4 MATHEMATICAL SPINORS

Mathematicians have classified various number 'objects' according to how these objects 'rotate' in the mathematical spaces where they are defined. The simplest objects are *scalars* that remain unchanged (or 'invariant') when their reference frame is rotated by any amount; the scalars correspond to real numbers. The next, more complicated class of objects are known as *vectors*, representing simple displacements in 3D space; or more abstractly, as ordered pairs of points {A, B} or AB. These 3D vectors can be readily rotated all around the unit sphere. The study of the vibrations of homogenous elastic spheres (or their 2D surfaces) identified certain stable modes or equivalently, points and lines on the surface with no radial movement. Mathematically, this was idealized using the Laplacian operator, ∇^2 expressed in spherical polar co-ordinates (θ , ϕ) on the unit sphere, where: $x = \sin\theta \cos\phi$, $y = \sin\theta \sin\phi$, $z = \cos\theta$. This problem was equivalent to finding the *eigenvalues*, λ of the equation: $\nabla^2\Phi = \lambda \Phi$. It was found that finite, discrete solutions occurred whenever: $\lambda = -n\,(n+1)$ with the new parameter, *n* taking all positive and zero integer values. The corresponding solutions (or *eigenfunctions*) are the spherical harmonics. It is a convention to also seek simultaneously, solutions that are invariant for rotations around the polar axis (normally taken to be the z-axis) or defined in terms of the longitudinal angle, ϕ. Here, all of the eigenvalues of the polar differential operator, $\partial/\partial\phi$ are imaginary integers, *m*; thus: $\partial\Phi / \partial\phi = i\, m\, \Phi$, where $n \geq m \geq -n$. The introduction of imaginary solutions on the unit sphere led to the introduction of new mathematical objects, known as '***spinors***'. Penrose [63] has given these a geometrical interpretation by associating a tangent direction to these special points on the unit sphere.

The spinors were first investigated algebraically by Elie J. Cartan (1869-1951), who found in 1913 [64] that certain pairs of complex numbers $\xi = \{\zeta , \eta\}$ were solutions of the equation: $\xi^*\xi = 0$ or $\zeta^*\zeta + \eta^*\eta = 0$. These have (2 x 1) dual representations:

$$\xi_1 = \begin{bmatrix} e^{i\phi/2} \cos\theta/2 \end{bmatrix} \qquad \& \qquad \xi_2 = \begin{bmatrix} e^{-i\phi/2} \sin\theta/2 \end{bmatrix}$$

$$\lfloor\, e^{\,i\,\phi/2} \sin\theta/2 \,\rfloor \qquad\qquad \lfloor\, e^{\,-i\,\phi/2} \cos\theta/2 \,\rfloor$$

This form of spinorial representation is based on the concept of *isotropic* vectors, which are complex eigenvectors of a 3D rotation. The pair of complex conjugate eigenvectors $(1, \pm i\,, 0)$ are the prototypical isotropic base vectors with zero length; all isotropic vectors $(\underline{A} \pm i\,\underline{B})$ are self-orthogonal with equal real and imaginary components; i.e. $\underline{A}\cdot\underline{A} = \underline{B}\cdot\underline{B}$ & $\underline{A}\cdot\underline{B} = 0$. Spinors may be regarded as a 'kind of square root' of isotropic vectors with real vectors isomorphic to Hermitian matrices with the spinor Hermitian product being preserved by all rotations; i.e. $\xi_1^{\dagger}\,\xi_2 = \langle \xi_1 \mid \xi_2 \rangle = \zeta_1^{*}\,\eta_1 + \zeta_2^{*}\,\eta_2$

Circular Vectors

Spinors are associated with circular vectors, where any 3D vector $\underline{X}$ can be decomposed into a pair-wise set X_3 and X_T.

$$\underline{X} \;\equiv\; \underline{X}_T + \underline{X}_3 \quad \text{where} \quad \underline{X}_3 = X_3\,\hat{\underline{e}}_3 \quad \text{and} \quad \underline{X}_T = X_1\,\hat{\underline{e}}_1 + X_2\,\hat{\underline{e}}_2 \quad \text{with} \quad \hat{\underline{e}}_j \cdot \hat{\underline{e}}_k = \delta_{jk}$$

Defining two complex unit vectors: $\hat{\underline{e}}_\lambda \;\equiv\; \hat{\underline{e}}_1 + i\,\lambda\,\hat{\underline{e}}_2$ where $\lambda = \pm$ or ± 1 then $X_\lambda \equiv \tfrac{1}{2}\,(X_1 + i\,\lambda\,X_2)$ N.B ½ factor.

This gives the transverse 3D vector form: $\underline{X}_T = X_-\,\hat{\underline{e}}_+ + X_+\,\hat{\underline{e}}_-$ These new bases are not orthonormal like the original ones.

$$\hat{\underline{e}}_3 \cdot \hat{\underline{e}}_3 = 1 \quad \text{and} \quad \hat{\underline{e}}_\lambda \cdot \hat{\underline{e}}_3 = 0 \quad \text{while} \quad \hat{\underline{e}}_\lambda \cdot \hat{\underline{e}}_\lambda = 0 \quad \text{and} \quad \hat{\underline{e}}_+ \cdot \hat{\underline{e}}_- = 2$$

Using the Einstein summation convention (for any 3D vector $\underline{A}$): $\underline{A} \;\equiv\; \underline{A}_T + \underline{A}_3$ where $\underline{A}_T = \hat{\underline{e}}_\lambda\,A_{-\lambda}$

This gives the 'dot' product of two transverse vectors: $\underline{A}_T \cdot \underline{B}_T = 2\,(A_+\,B_- + A_-\,B_+) = A_1\,B_1 + A_2\,B_2$

Isotropic Vectors

It is possible to define mathematically 3D objects ('isotropic vectors' $\underline{v}$) whose quadratic 'length' is zero; that is, there exist triplets of complex numbers $\{v_1\,,\, v_2\,,\, v_3\}$, such that:

$$\underline{v} \cdot \underline{v} = 0 \qquad \text{or} \qquad v_1^{\,2} + v_2^{\,2} + v_3^{\,2} = 0$$

The first pair of numbers is usually transformed into a 'circular' or transverse pair: $v_\lambda = \tfrac{1}{2}\,(v_1 + i\,\lambda\,v_2)$
The next step is to define the 'isotropic' parameter ξ defined as the ratio of the isotropic components, α and β: $\xi \;\equiv\; \beta / \alpha$.
The following assignments preserve the zero-length property:

$$\xi = 2\,v_+ / v_3 \quad \text{and} \quad \xi = -\,v_3 / 2\,v_- \quad \text{These have solutions:} \quad v_+ = -\,\beta^2 \;\; v_- = \alpha^2 \;\; v_3 = -\,2\,\alpha\,\beta$$

Thus, a vector $\underline{v}$ is isotropic if its components satisfy: $v_1 = (\alpha^2 - \beta^2) \quad v_2 = i\,(\alpha^2 + \beta^2) \quad v_3 = -\,2\,\alpha\,\beta$

Any real vector $\underline{W}$ can be decomposed into a complex vector $\underline{U}$ and an isotropic vector $\underline{v}$: $\underline{W} = \underline{U} + \underline{v}$

Isotropic vectors have a fundamental relationship with Pauli matrices σ_j; seen by defining: $\underline{\sigma} \;\equiv\; \hat{\underline{e}}_1\,\sigma_1 + \hat{\underline{e}}_2\,\sigma_2 + \hat{\underline{e}}_3\,\sigma_3$

These vectors satisfy the 2 x 2 matrix equation: $(\underline{v} \cdot \underline{\sigma})\,\psi = \mathbf{0}$ where $\psi = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ and $\mathbf{0} = \begin{pmatrix}0\\ 0\end{pmatrix}$

These can be used to define an equivalent form: $\underline{v} = i\,\psi^{T}\,\sigma_2\,\underline{\sigma}\,\psi$ where $\psi^{T} = (\alpha\;\; \beta)$ are spinors.
As α and β are each complex numbers, it is always possible to write: $\alpha = \alpha_1 - i\,\alpha_2$ and $\beta = \beta_1 - i\,\beta_2$ or $\psi = \psi_1 - i\,\psi_2$.

These have canonical solutions: $\psi_1^{\,T} = (1\;\; 0)$ and $\psi_2^{\,T} = (0\;\; 1)$ and always satisfy: $(\underline{v} \cdot \underline{\sigma})\,\psi_1 = i\,(\underline{v} \cdot \underline{\sigma})\,\psi_2$.

Spinors as Sequential Rotations

Penrose illustrates [65] the peculiar 'twistiness' that characterizes spinors, even going so far as to define a spinor as an object which turns into its negative when it undergoes a complete rotation through 2π, in contrast to real 3D objects that rotate back to their starting configuration. Penrose suggests that to picture a spinorial object, one can think of an ordinary object in space with 'an imaginary flexible attachment to some fixed external structure' with one end of the attachment fixed to the spinor object and the other on the fixed external structure. In order to retain the Hamiltonian multiplication rule $\mathbf{i\,j = k}$ as rotations are made around the appropriate axis, it is necessary to represent quaternions by **left-handed** rotations. This is why the basic unit of reality in this programme, the electron, is defined as rotating counter-clockwise relative to its direction of motion. Penrose solves this ambiguity by proposing the convention that the quaternion product **PQ** be interpreted as the operation **Q** to be performed first, followed by the operation **P** rather than the reverse (as in reading English). This is actually the same mathematical convention needed for pairs of differential operators or functions of functions; it is also key to the use of NVs in their application to the measurement problem in QM – this will be explored in a later paper.

Rotating a Sphere

All spinor objects have the unique characteristic that a complete rotation around the unit circle, in any direction, does not return the spinor back to its original state – only a **double** rotation of $\mathbf{4\pi}$ restores it completely. The single rotation (360°) only acts like a reflection (normally only needing a 180° rotation), in other words 2π 'multiplies' the spinor into its negative. Spinors may also be viewed as providing the microstructure of *tensors*, especially when defined as multi-linear functions of direction. Tensors have been used extensively in the mathematical analysis of idealized rigid and elastic 'solid' bodies. There is much discussion of rotation (e.g. of 'unit spheres') in texts on spinors. However, whenever such a perfect sphere is actually rotated how would anyone know since there are no distinguishing marks on such a 'perfect' object? Moreover, how would we rotate it and what timing method could we use to measure the rotation? For any object to rotate uniformly it must act as a rigid body – a concept that was severely criticized in the previous paper on Einstein's use of this non-physical concept in his creation of his Special Theory of Relativity [61]. It is sometimes suggested that instead of rotating the sphere that the whole universe is rotated around the fixed sphere; a suggestion with even less chance of ever being accomplished.

Quaternions & Spinors

Quaternions can be related to SU2 spinors through the following isomorphism [1]:

$$\mathbf{Q} = q_0 + q_1\,\mathbf{I} + q_2\,\mathbf{J} + q_3\,\mathbf{K} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} (q_0 + i\,q_1) & (q_2 + i\,q_3) \\ (-q_2 + i\,q_3) & (q_0 - i\,q_1) \end{pmatrix} = \begin{pmatrix} Z & W \\ -W^* & Z^* \end{pmatrix} \quad \text{so} \quad \mathbf{Q} \approx \mathbf{S}$$

Here i is the usual square root of minus one and (**I**, **J**, **K**) are Hamilton's famous quaternion bases: $\mathbf{I}^2 = \mathbf{I\,J\,K} = -1$

The most widely used quaternion matrix representation **Q** uses the 'doubling' of the spinor group $\mathbf{SU_2}$; i.e. $\mathbf{Q} = \mathbf{SU_2} \otimes \mathbf{SU_2}$. In terms of the complex Pauli (2x2) matrices $\underline{\boldsymbol{\sigma}}$ defined as: $\mathbf{i} = i\,\boldsymbol{\sigma}_3$, $\mathbf{j} = i\,\boldsymbol{\sigma}_2$, $\mathbf{k} = i\,\boldsymbol{\sigma}_1$, where $i^2 = -1$ and :

$$\boldsymbol{\alpha}^2 = -\mathbf{I}_0 = -\begin{pmatrix} \mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\alpha}^\dagger = -\boldsymbol{\alpha} \quad \text{this gives the standard (complex) quaternion representation:}$$

$$\mathbf{I} = \begin{pmatrix} \mathbf{j} & \mathbf{0} \\ \mathbf{0} & \mathbf{j} \end{pmatrix} \qquad \mathbf{J} = i\begin{pmatrix} \mathbf{0} & \mathbf{k} \\ -\mathbf{k} & \mathbf{0} \end{pmatrix} \qquad \mathbf{K} = i\begin{pmatrix} \mathbf{0} & \mathbf{i} \\ -\mathbf{i} & \mathbf{0} \end{pmatrix}$$

Double Spinors

The two-component complex representation ($\mathcal{C}_2$ or spinors) cannot be used to map particle kinematics, as spinors are the complex 2D representation of real 3D space. This implies that double-spinors are needed to represent the movement of point particles across 3D space over 1D time. The role of the square root of minus one is used here to retain the distinction between space and time. The state representing the absence of all particles (the so-called 'vacuum' state) cannot be the zero matrix representation $|\,0\,\rangle$ but must be a non-zero (4 x 1) column vector $|\,\text{vac}\,\rangle$ so that it can be used to generate the non-zero states corresponding to the 'send' and 'receive' states of real electrons. This is achieved (mathematically) by using the four real Clifford generators $\mathbf{C}(\lambda : \zeta)$ in their (4 x 4) matrix representation. These generate the four electron states $|\,\lambda : \zeta\,\rangle$ (where λ is either 'send' or 'receive' and ζ is either 'retarded' or 'advanced') by the adjoint multiplication, subject to the 'empty' rules:

$$\mathbf{C}^\dagger(\lambda : \zeta)\,|\,\text{vac}\,\rangle = |\,\lambda : \zeta\,\rangle \quad \& \quad \mathbf{C}(\lambda : \zeta)\,|\,\text{vac}\,\rangle = |\,0\,\rangle$$

3.1.5 PAULI SPINORS

Spinors were first applied to mathematical physics by Pauli in 1927 [18] when he introduced 'his' (2 x 2) *spin* matrices, σ_j. This was based on the isomorphism between (2 x 2) matrices $\mathbf{X}$ and every 3D vector, $\underline{X} = x\,\underline{\hat{e}}_1 + y\,\underline{\hat{e}}_2 + z\,\underline{\hat{e}}_3$, where the matrix form is:

$$\mathbf{X} = \begin{bmatrix} z & (x - i\,y) \\ (x + i\,y) & -z \end{bmatrix} \quad \text{then:} \quad \mathbf{X} = x\,\sigma_x + y\,\sigma_y + z\,\sigma_z = \underline{X} \bullet \underline{\sigma}$$

These (2 x 2) matrices satisfy: $\{\mathbf{X}_1, \mathbf{X}_2\} \equiv \mathbf{X}_1\mathbf{X}_2 + \mathbf{X}_2\mathbf{X}_1 = 2\,\underline{X}_1 \bullet \underline{X}_2\,\mathbf{I}$ & $[\mathbf{X}_1, \mathbf{X}_2] \equiv \mathbf{X}_1\mathbf{X}_2 - \mathbf{X}_2\mathbf{X}_1 = 2\,i\,\underline{X}_1 \wedge \underline{X}_2$

These matrices can also be generated from the outer-product of the (2 x 1) spinor, $\xi = \{\zeta\,,\,\eta\}$; thus: $\mathbf{X} = 2 \begin{bmatrix} \zeta \\ \eta \end{bmatrix} \begin{bmatrix} -\eta \\ \zeta \end{bmatrix}^T$

The general spin matrix form of the Rotation operator R is defined by: $\mathbf{R} = \mathbf{I} \cos\theta/2 + i\,(\underline{\hat{e}} \bullet \underline{\sigma}) \sin\theta/2$
Here, any 3D vector $\underline{A}$ is rotated into a new vector $\underline{A}'$ through an angle θ around the axis: $\underline{e} = l_x\,\underline{\hat{e}}_1 + l_y\,\underline{\hat{e}}_2 + l_z\,\underline{\hat{e}}_3$ by the simple matrix multiplication: $\mathbf{A}' = \mathbf{R}^\dagger\,\mathbf{A}\,\mathbf{R}$. Pauli's approach introduced a (2 x 1) spinor representation into quantum mechanics that had previously (like all of physics) only used scalar functions; this representation appears automatically by looking for eigenfunction solutions (Ψ) to equations using Pauli matrices: $\mathbf{B}\,\Psi = \lambda\,\Psi$ where $\mathbf{B} = \mathbf{A}\,\mathbf{R}$. The simplest eigenfunctions are those that involve only the spin matrices themselves; these are usually chosen just for the σ_z matrix; these are the famous spin 'up' $\Phi_\uparrow$ and spin 'down' $\Phi_\downarrow$ solutions, where:

$$\sigma_z\,\Phi = \lambda\,\Phi \quad \text{therefore } \lambda = \pm 1 \quad \text{and} \quad \Phi_\uparrow = (1\ 0)^T \ \&\ \Phi_\downarrow = (0\ 1)^T \quad \text{since} \quad \Phi = \Phi_\uparrow \text{ or } \Phi_\downarrow .$$

3.1.6 DIRAC SPINORS

The starting point for the relativistic equation was Dirac's desire to preserve the time-dependent Schrödinger equation.

$$\mathbf{H}\,\Psi(\underline{r}, t) = i\,h_D\,\partial\Psi(\underline{r}, t) / \partial t \qquad \text{so,} \quad \Psi(\underline{r}, t) = \Psi(\underline{r}) \exp(-i\,E\,t / h_D)$$

Here, as in non-relativistic QM, H is the Hamiltonian (or energy operator) and, as a free particle, the electron has a constant momentum $\underline{P}$ and energy E. Since the Dirac operators involve (4 x 4) matrices then $\Psi(\underline{r})$ must be a (4 x 1) column matrix, with four components, $\Psi_\mu(\underline{r})$. These four components may be re-defined into two sets$\{\,\Phi_+\,,\,\Phi_-\,\}$, where the Φ_λ are just Pauli-like (2 x 1) column matrices; in other words (and again using the transpose for simpler row typology):

$$\Psi(\underline{r}) = (\Phi_+\ \Phi_-)^T \ \text{ and } \ \Phi_+ = (\Psi_1\ \Psi_2)^T \ \ \&\ \ \Phi_- = (\Psi_3\ \Psi_4)^T$$

This gives an explicit (2 x 2) version of the relativistic equation: $\begin{bmatrix} (E - m\,c^2)\mathbf{I} & -c\,\underline{\sigma} \bullet \underline{P} \\ -c\,\underline{\sigma} \bullet \underline{P} & (E + m\,c^2)\mathbf{I} \end{bmatrix} \begin{bmatrix} \Phi_+ \\ \Phi_- \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$

There are several special solutions of these coupled equations.

a) Free electron at rest ($\underline{P} = 0$) $\begin{bmatrix} (E_0 - m\,c^2)\mathbf{I} & \emptyset \\ \emptyset & (E_0 + m\,c^2)\mathbf{I} \end{bmatrix} \begin{bmatrix} \phi_+ \\ \phi_- \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$ or $(E_0 - m\,c^2)\,\phi_+ = \emptyset$ & $(E_0 + m\,c^2)\,\phi_- = \emptyset$

Both wave functions ϕ_+ have positive energy, $E_0 = m\,c^2$, while both wave functions ϕ_- have negative energy, $E_0 = -m\,c^2$.

b) Freely moving electron ($\underline{P} \neq 0$) explicitly:

$$\begin{bmatrix} (E - m\,c^2) & 0 & -c\,P_z & -c\,P_- \\ 0 & (E - m\,c^2) & -c\,P_+ & c\,P_z \\ -c\,P_z & -c\,P_- & (E + m\,c^2) & 0 \\ -c\,P_+ & c\,P_z & 0 & (E + m\,c^2) \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using: $P_+ = P_x + i\,P_y$ & $P_- = P_x - i\,P_y$

These four simultaneous equations have a non-zero solution only if the determinant of the matrix vanishes; this gives:

$$(E^2 - c^2P^2 - m^2c^4)^2 = 0 \quad \text{So, for a given momentum: } E_+ = \sqrt{(c^2P^2 + m^2c^4)} \ \&\ E_- = -\sqrt{(c^2P^2 + m^2c^4)}$$

There are four (4 x 1) column eigenfunctions $\Psi(\mu)$, which can be solved directly by assuming partial σ_z solutions [66].

$$\Psi(1) = (1\ 0\ C_1\ D_1)^T \quad \Psi(2) = (0\ 1\ C_2\ D_2)^T \quad \Psi(3) = (A_3\ B_3\ 1\ 0)^T \quad \Psi(4) = (A_4\ B_4\ 0\ 1)^T$$

These solutions are: $\Psi(1) = (1\ 0\ Q_z\ Q_+)^T \quad \Psi(2) = (0\ 1\ Q_-\ -Q_z)^T \quad \Psi(3) = (-Q_z\ -Q_+\ 1\ 0)^T \quad \Psi(4) = (-Q_-\ Q_z\ 0\ 1)^T$

Here, the positive amount, $W = E_+ = -E_-$ is used for positive and negative energies; also: $Q_k = c\,P_k / (W + m\,c^2)$

Since the probability density $\mathscr{P}$ is the sum of the squares of its four components, it can be written as:

$$\mathscr{P} = \sum_\mu \mathscr{P}_\mu \quad \text{where} \quad \mathscr{P}_\mu = A^2 \psi_\mu^* \psi_\mu \quad \text{But} \quad Q_z\,Q_z + Q_+ Q_- = c^2P^2 / (W + m\,c^2)^2 = (W - m\,c^2) / (W + m\,c^2)$$

Each spinor $\Psi(\mu)$ must be multiplied by the same factor A for the probability density to be normalized to unit volume (and thus contributing <u>equally</u> to the total probability density); where: $A^2 = (W + m\,c^2) / 2W$

A) <u>Non-Relativistic Limit</u> ($v << c$): Here, $mc^2 >> cP$ so that $W \cong mc^2$, so that $A \cong 1$ and each $Q_k << 1$ giving:

$$\psi(1) = (1\ 0\ 0\ 0)^T \quad \psi(2) = (0\ 1\ 0\ 0)^T \quad \psi(3) = (0\ 0\ 1\ 0)^T \quad \psi(4) = (0\ 0\ 0\ 1)^T$$

B) <u>Relativistic Limit</u> ($v \cong c$): Here $mc^2 << cP$ so that $W \cong c\,P$, so that $A \cong 1/\sqrt{2}$ and each $Q_k \cong P_k / P = V_k / V$.

B1) Longitudinal Motion ($V = V_z\,\hat{e}_3$): Here, $Q_+ = Q_- = 0$ and $Q_z = 1$ giving:

$$\Psi(1)^T = (1\ 0\ 1\ 0)/\sqrt{2} \quad \Psi(2)^T = (0\ 1\ 0\ -1)/\sqrt{2} \quad \Psi(3)^T = (-1\ 0\ 1\ 0)/\sqrt{2} \quad \Psi(4)^T = (0\ 1\ 0\ 1)/\sqrt{2}$$

These are most peculiar states to interpret, as they imply an equally probable negative energy state accompanying each positive energy state, while their spin eigenvalues in the z-direction remain unchanged. They appear to be an equal merger of pairs of solutions, using symmetric and anti-symmetric combinations:

$$\Psi(1) = \{\psi(1) + \psi(3)\}/\sqrt{2} \quad \Psi(2) = \{\psi(2) - \psi(4)\}/\sqrt{2} \quad \Psi(3) = \{\psi(3) - \psi(1)\}/\sqrt{2} \quad \Psi(4) = \{\psi(4) + \psi(2)\}/\sqrt{2}$$

B2) Transverse Motion ($V_z = 0$): Here, $Q_+ = e^{+i\omega} \quad Q_- = e^{-i\omega}$ and $Q_z = 0$ giving:

$$\Psi(1)^T = (1\ 0\ 0\ Q_+)/\sqrt{2} \quad \Psi(2)^T = (0\ 1\ Q_-\ 0)/\sqrt{2} \quad \Psi(3)^T = (0\ -Q_+\ 1\ 0)/\sqrt{2} \quad \Psi(4)^T = (-Q_-\ 0\ 0\ 1)/\sqrt{2}$$

These solutions are even more difficult to interpret as none is an eigenstate of spin in the z-direction while one of the negative energy solution is merged in through a complex (rotating) factor.

$$\Psi(1) = \{\psi(1) + Q_+\psi(4)\}/\sqrt{2} \quad \Psi(2) = \{\psi(2) + Q_-\psi(3)\}/\sqrt{2} \quad \Psi(3) = \{\psi(3) - Q_+\psi(2)\}/\sqrt{2} \quad \Psi(4) = \{\psi(4) - Q_-\psi(2)\}/\sqrt{2}$$

3.2 DISCRETE NATURAL VECTORS

Obviously, 3D spatial vectors are timeless – they represent simply a directed length <u>anywhere</u> in space. In the sense that they are defined for any single time value (functions of time), they are also "free-floating" as the origin of the timescale is also arbitrary. They are only arbitrarily associated with <u>specific</u> points in space, such as the current location of a particle: they could equally be displaced to any other location at that same time. In contrast, Natural Vectors are 'pinned' in 4D (fixed in a given reference frame). It is only differences in NVs, which are independent of both space and time origins. In this regard, NVs form a better representation of the trajectory of electrons than 3D vectors, as NVs are always associated with the exact spatial location of <u>each</u> electron at <u>every</u> moment of time.

The foundational mathematics for the present research programme is an algebraic extension of quaternions that is referred to here as *Natural Vectors* or **NV**s. The first paper in this programme [1] presented, *inter alia*, a brief history of the invention of quaternions by the Irish polymath, Sir William Rowan Hamilton (1805-1865), a short review of their somewhat limited use in physics and their direct connection to NVs. The question is often raised, that if quaternions are so basic to the general structure of algebra: why have they not played a more significant role in physics? The answer suggested here is two fold.

Firstly, they were almost always approached as a quartet of real numbers multiplying the quaternion bases {1, **i**, **j**, **k**}; this gives the 'wrong signature' for their direct use in relativistic physics. The NV response is to multiply the unitary base by the standard square root of minus one (or *i*) to physically distinguish the dimension of time from space. Secondly, as was shown above, the complex bases {**i**, **j**, **k**} were almost always defined as a double representation of the Pauli spin matrices (SU2), this implied that the usual complementary operation of conjugation was extended to reverse only the signs of each of these bases. The technique used in defining NVs was to use only (4 x 4) **real** representations of all four NV bases $\mathbf{I}_{\mu}$; this meant that only standard complex conjugation had to be used which only reverses the explicit sign of the standard imaginary – this gives NVs the 'right signature' for their quadratic 'norm'. Mathematically, this means that (like a simple complex number) the NV conjugate **Q*** is independent of its real NV, **Q**; whereas the standard quaternionic conjugate of a regular quaternion is **not** independent as it can be constructed from a small set of quaternion products [67].

3.2.1 SUMMARY OF CONTINUOUS NVs

The mathematics of *Natural Vectors* (**NV**s) was introduced in the first paper [1] reporting on this research programme. This first formulation closely followed the historical evolution of physics and extended Hamilton's continuous version of his 4D quaternions (based on real numbers) to the classical physics of three dimensions of space and one of time. This produced a non-commutative algebra that extended the 3D algebra of real numbers that had been used since Newton with the analytic concept of Minkowski's 4-vectors. This simple algebra seems like a natural extension of complex numbers to the distinct four dimensions of the natural world. As in Maxwell's physics, this new mathematics was centered on Hamilton's 3D spatial vector gradient operator, producing a powerful representation of continuous change at **one** point in space and time.

The second paper in this series [2] applied the mathematics of single-time **C**ontinuous **N**atural **V**ectors (**CNV**s) to a model of classical electromagnetism (CEM) that has represented electricity by a continuous, incompressible fluid (the so-called EM Helmholtz flow model). All of the standard results of CEM were very quickly recovered by assuming that the three central quantities in CEM (current, potential and electro-kinetic momentum) were simply components of comparable CNVs. The mathematics of Continuous Natural Vectors is summarized in **Appendix I**, as this is the base model for all later results.

The focus of the third paper in this series [3] was the continuous remote interaction of two classical electrons, each always existing at a point in space and interacting exclusively and continuously with each other. This led to the extension of single-point CNVs to two-time CNVs (or 'Double-Point CNVs') as the natural representation of the asynchronous EM interaction. The power of this representation was that differences between CNVs automatically result in pair-wise, 'difference' CNVs. The appropriateness of Natural Vectors, especially the 'difference' NVs like $\mathbf{X}_{12}$, to coherently link together eight (8) real numbers implies that the pair-wise interaction between two electrons at $(t_1; \underline{x}_1)$ and $(t_2; \underline{x}_2)$ is best represented by this new extended definition as quaternions are the maximal, useful algebra of all four possible multiplicative algebras [68]. The mathematics of Double-Point Continuous Natural Vectors is summarized in **Appendix II** and the corresponding Temporal Invariants of both the single-electron and two-electron Continuous Natural Vectors are summarized in Appendix **III**.

3.2.2 SUMMARY OF DISCRETE DIFFERENCES

The focus of the fourth paper in this series [4] was the discontinuous remote interaction of two classical electrons, each always existing at a point in space and interacting asynchronously and exclusively with each other. The analysis presented in that paper was based on the key result from the third paper that *two inertial particles cannot interact continuously with each other* but only via discrete impulses. The fourth paper showed that the time intervals between these instantaneous impulses must vary with the velocity of the participating particle. These results were generated by replacing all usage of the infinitesimal calculus with a new extended version of the calculus of finite differences, in order to represent the dynamics of inter-electron interactions; no use was made of the 'Lorentz' transform. Some of the basic results of standard calculus of finite differences and these new extensions needed for representing impulse dynamics are summarized in **Appendix IV**.

3.2.3 DISCRETE NV OPERATORS

Soon after Hamilton invented the new rules for his three quaternion bases { **i** , **j** , **k** }, he combined them with the standard definitions of Cartesian partial differential operators in the three spatial directions, like $\partial/\partial x$, to define his vector partial differential operator, which he denoted by the symbol '∇' and named 'nabla' but was soon renamed 'del' by Tait [1].

$$\nabla = \mathbf{i}\,\partial/\partial x + \mathbf{j}\,\partial/\partial y + \mathbf{k}\,\partial/\partial z$$

Both Maxwell and Tait immediately recognized the value of quaternions and especially the power of this new operator to simplify the investigation of variations of continuous functions of the three variables {x, y, z} around any point $\{x_0,y_0,z_0\}$, which was the focus of Cartesian 3D analysis.

Maxwell used this new mathematics near the end of his *Treatise* [69] to try to simplify the presentation of the 20 Cartesian differential equations that summarized his theory of the electromagnetic field but these results were less than elegant and this new formulation generated little interest. After Maxwell's death, Heaviside dropped the full quaternion approach (and Maxwell's innovative potentials!) and retained only the 3D vector components by converting Hamilton's revolutionary quaternion bases to simple unit vectors in 3D space $\{\underline{i}, \underline{j}, \underline{k}\}$ with $\underline{\nabla}$ now becoming only a vector operator. Thus was born the hugely influential 3D vector calculus, which Heaviside used immediately to rewrite 16 of Maxwell's EM equations into the four vector equations that are today referred to universally as "Maxwell's Equations".

The present research programme rejects all continuum ideas, especially the metaphysical hypothesis that a 3D vector field represents any aspect of reality, including the EM interaction between charged particles. The 'power of del' was its ability to describe (investigate) variations in <u>any</u> continuous 3D function of <u>real</u> variables around <u>all</u> points in space. In the present theory, this 'global' power is no longer required; it is viewed as overkill to determine the value of functions in empty space. It is only necessary to describe the variations of an electron's properties as it moves along its trajectory. One of the central hypotheses in this research is the proposal that a real electron is only considered to have the possibility of interacting at one of its (cyclic) interaction times (or 'ticks' t_η). A further hypothesis (to be reported on later) is the idea that a measurement by humans is always viewed as a class of electron-electron interactions. The implication of these two hypotheses is that it is only necessary to check for variations 'around the tick', say in the temporal region bounded by $t_\eta \pm \tau/2$, or at locations on the electron's trajectory at two adjacent 'ticks', say t_η and $t_{\eta+1}$, which are only one time quantum (chronon) τ apart.

Discrete Electron Properties

Traditional physics, ever since Newton, has assumed that <u>all changes</u> in Nature occur 'smoothly' (this is the Continuum Hypothesis). So, if ψ is a property of an electron (arbitrarily labeled 'α') then it has always been assumed that this can be represented by an algebraic function $\psi[\alpha: t, \underline{x}]$ of its 4 space and time parameters and all such functions are assumed to vary continuously with respect to any of the four parameters. Newton's differential calculus, when applied to physical systems, is based on the assumption that a continuous succession of equal differences in time can 'go to the limit'; i.e. for all times *t* and *t′* centering their time intervals δt and $\delta t'$, then $\delta t = \delta t'$ such that, in the limit, $\delta t \rightarrow 0$: effectively $\psi^+[t] = \psi^-[t]$ for all t. In the present theory, all properties are only evaluated (mathematically or experimentally) at the universal interaction times (or 'ticks' $t_\eta = \eta\ \tau$) when electron 'α' is located in space at the position $\underline{x}[\alpha: \eta]$; the ordered (by η) set of $\{\ \underline{x}_\eta[\alpha]\ \}$ defines this electron's trajectory. The directed displacement $\Delta x_j[\alpha: \eta]$ is the spatial difference between the position of electron 'α' at two consecutive times t_η and $t_{\eta+1}$ in the spatial direction 'j' or: $\Delta x_j[\alpha: \eta] = x_j[\alpha: \eta + 1] - x_j[\alpha: \eta]$. This step now leads to the definition of the '**partial difference**' for any function (continuous or regular) $\psi[\underline{x}]$ of all three spatial variables x_j as the difference in its value between two consecutive interaction times t_η and $t_{\eta+1}$ <u>in the spatial direction</u> 'j'.

Definition: **Partial-Difference** $\Delta\psi_j[\alpha: \eta] \equiv \psi_j[\alpha: \eta + 1] - \psi_j[\alpha: \eta]$ or $\psi_j[\alpha: \eta + 1] \equiv (1 + \Delta_j)\ \psi_j[\alpha: \eta]$

This can be extended (symbolically) to the time dimension by defining: $x_0[\alpha: \eta] = c\ t_\eta = \eta\ c\ \tau$ so, $\Delta x_0[\alpha: \eta] = c\ \tau \equiv \Lambda$ for all α. Here the 'partial time difference' operator Δ_0 is only defined when the electron is stationary ($\underline{V} = 0$) at any time t_η. This enables the total-difference of any property of an electron ψ to be defined as the <u>sum</u> of any explicit time-difference and <u>all the spatially directed differences between two interaction points on the trajectory</u> of every electron.

Definition: **Total-Difference**, $\Delta\psi[\alpha: t_\eta; \underline{x}_\eta] \equiv \sum_\mu \Delta_\mu[\ \psi[\alpha: t_\eta; \underline{x}_\eta]\ ]$

This leads to the definitions of the four Digital Rate-of-Change operators Ξ_μ that replace the earlier 4D 'nabla' operator ∇.

Definition: **Partial-Rate-of-Change Operator**, $\Xi_\mu[\ \psi[\alpha: t_\eta; \underline{x}_\eta]\ ] \equiv \Delta_\mu[\ \psi[\alpha: t_\eta; \underline{x}_\eta]\ ] / \Delta x_\mu[\alpha: t_\eta; \underline{x}_\eta]$

Note, that in contrast to continuous differentials, the space and time differentials may vary with where the electron is located at different times; in other words, it is always possible that: $\Delta x_\mu[\alpha: t_\eta; \underline{x}_\eta] \neq \Delta x_\mu[\beta: t_{\eta'}; \underline{x}_{\eta'}]$; however: $\Xi_\mu[\ x_\nu\ ] = \delta_{\mu\nu}$.

This suggests that the analog of the continuous, 3D vector gradient operator $\underline{\nabla}$ is the digital gradient operator $\underline{\Xi}$, where:

Definition: ***Digital Gradient Operator***, $\underline{\Xi} \equiv \underline{\hat{e}}_x\ \Xi_x + \underline{\hat{e}}_y\ \Xi_y + \underline{\hat{e}}_z\ \Xi_z$

This finally leads to the definition of the Total-Rate-of-Change operator $\mathbf{D}$, along any electron's trajectory.

Definition: ***Digital Rate-of-Change Operator***, $\mathbf{D}[\,\psi[\alpha: t_\eta; \underline{x}_\eta]\,] \equiv \Delta\psi[\alpha: t_\eta; \underline{x}_\eta] / \Delta t[\alpha: t_\eta; \underline{x}_\eta]$

Writing $\Delta t_\eta = \Delta t[\alpha: t_\eta; \underline{x}_\eta]$ and noting that $\Delta t_\eta = \tau$ and $\Delta x_j[\alpha: \eta] \equiv v_j[\alpha: \eta]\,\tau$ while writing $x_\eta = \{\, t_\eta; \underline{x}_\eta \,\}$:

$$\mathbf{D}[\,\psi[\alpha: x_\eta]\,] = (\, c\,\boldsymbol{\Xi}_0 + \underline{v} \bullet \underline{\Xi}\,)\,\psi[\alpha: x_\eta] \quad \text{with} \quad \boldsymbol{\Xi}_0\,\psi[\alpha: x_\eta] = \{\psi[t_{\eta+1}; \underline{x}_\eta] - \psi[t_\eta; \underline{x}_\eta]\} / c\,\tau$$

This also suggests analogous definitions of both the 'Digital Divergence' (D-Div) and 'Digital Curl' (D-Curl) operators for any vector function with discrete values defined along the electron's trajectory: $\underline{F}[\alpha: x,y,z] = \hat{\underline{e}}_x F_x + \hat{\underline{e}}_y F_y + \hat{\underline{e}}_z F_z$

Definition: ***Digital Divergence Operator***, $\underline{\Xi} \bullet \underline{F} \equiv \boldsymbol{\Xi}_x F_x + \boldsymbol{\Xi}_y F_x + \boldsymbol{\Xi}_z F_z$

Definition: ***Digital Curl Operator***, $(\,\underline{\Xi} \wedge \underline{F}\,)_j \equiv \sum_{kl} \varepsilon_{jkl}\,\boldsymbol{\Xi}_k\,F_l$ where ε_{jkl} is the 3D anti-symmetric tensor.

These discrete results are directly analogous to the continuous results of differential calculus and become identical in the limit that the duration of the chronon τ goes to zero (a limit NOT taken in this programme). These analogies may be summarized in the following table.

Continuous	t	$\underline{x}[t]$	$\underline{v}[t]$	$\psi[t; \underline{x}]$	$1/c\,\partial/\partial t$	$\partial/\partial x_j$	$\underline{\nabla}$	$d\psi/dt$	$\partial/\partial t + \underline{v} \bullet \underline{\nabla}$
Discrete	t_η	$\underline{x}_\eta$	$\underline{v}_\eta$	$\psi[t_\eta; \underline{x}_\eta]$	$\boldsymbol{\Xi}_0$	$\boldsymbol{\Xi}_j$	$\underline{\Xi}$	$\mathbf{D}\psi$	$c\,\boldsymbol{\Xi}_0 + \underline{v} \bullet \underline{\Xi}$

Table 1. Correspondence between Continuous and Discrete Variables & Operators

Now, for fixed axes, just as $\underline{\nabla} v_j[t; \underline{x}] = 0$ remains the foundational step for all the differential calculus results of classical EM (or Maxwell's) theory [70] ; a similar result holds for the discrete changes in an electron's velocity.

$$\underline{\Xi}\, v_j = \underline{\Xi}\,[\Delta x_j / \Delta t_\eta] = \underline{\Xi}\,[\Delta x_j] / \tau = \underline{\Xi}\,[\boldsymbol{\Delta}_j x_j] / \tau = \boldsymbol{\Delta}_j\,[\,\underline{\Xi} x_j] / \tau = \boldsymbol{\Delta}_j\,[\,\textstyle\sum_k \Xi_k x_j\,] / \tau = \boldsymbol{\Delta}_j\,[\,\textstyle\sum_k \delta_{jk}] / \tau = \boldsymbol{\Delta}_j[1] / \tau = 0$$

A comprehensive set of useful 3D finite difference operator identities involving $\underline{\Xi}$ is provided in **Appendix V**.

These single electron discrete results can be extended to single-time Discrete Natural Vectors (DNVs) to generate analogous results to those found earlier for single-time Continuous Natural Vectors (CNVs), as was shown in earlier. [71]

3.2.4 SINGLE-POINT DNVs

In this section the Extended-Difference calculus (see 3.2.2) is merged with the single-time calculus of Continuous Natural Vectors (CNVs) developed in the first paper in this series to produce a new discrete mathematical algebra, called Discrete Natural Vectors (**DNV**s) that is suitable for representing finite changes to a single electron that occur instantaneously only at a finite number of 'interaction points' ($t_\eta = \eta\,\tau$). As in the last section, a single electron (whose identification label 'α' will be omitted here) will be characterized by its Location Natural Vector $\mathbf{X}_\eta$ at one of these interaction times when it is located at $\underline{x}_\eta$. The fundamental 'mapping hypothesis' of this programme [72] is that this set of four physical parameters can be represented by its unique locational DNV.

$$\text{Hypothesis:} \quad \{\, \underline{x}[t_\eta] \,\} \approx \mathbf{X}_\eta \equiv i\,c\,t_\eta\,\mathbf{I_0} + \underline{x}_\eta \bullet \underline{\mathbf{I}}$$

The electron's (post) velocity (i.e. after t_η) is $\underline{v}_\eta$; the corresponding DNV associated with the electron's velocity is $\mathbf{V}_\eta$.

$$\textbf{DNV Velocity}: \quad \mathbf{V}_\eta \equiv \mathbf{D}[\mathbf{X}_\eta] = i\,c\,\mathbf{I_0} + \underline{v}_\eta \bullet \underline{\mathbf{I}}$$

Continuous Natural Vectors follow Hamilton by extending his 'nabla' (or 'gradient') 3D space operator ($\underline{\nabla}$) to the 4D Natural Vector ***Gradient*** applied to any scalar function ψ that is continuous in the four space-time variables $\{t\,;\,\underline{x}\}$:

$$\textbf{Gradient}: \quad \boldsymbol{\nabla}\psi(t\,;\,\underline{x}) \equiv i\,\mathbf{I_0}\,\partial_0\psi(t\,;\,\underline{x}) + \underline{\mathbf{I}} \bullet \underline{\nabla}\psi(t\,;\,\underline{x}) \quad \& \quad \underline{\nabla} \equiv \hat{\mathbf{e}}_1\partial_1 + \hat{\mathbf{e}}_2\,\partial_2 + \hat{\mathbf{e}}_3\partial_3 \quad \partial_0 \equiv \partial/\,c\,\partial t \quad \partial_1 \equiv \partial/\,\partial x_1 \text{ etc}$$

The corresponding Discrete Natural Vector operator is the DNV operator, $\boldsymbol{\Xi}$ applicable to any discrete function $\psi[t_\eta; \underline{x}_\eta]$.

Definition: ***DNV Gradient Operator***, $\boldsymbol{\Xi}[\,\psi[x_\eta]\,] \equiv (\, i\,\mathbf{I_0}\,\Xi_0 + \underline{\mathbf{I}} \bullet \underline{\Xi}\,)\,\psi[x_\eta]$

The correspondences established above means that all the CNV results using ∇ can be extended directly to $\boldsymbol{\Xi}$. Especially, the conjugate of the DNV gradient operator can be applied to any discrete natural vector (DNV) function, $\mathbf{Q}[x_\eta]$:

$$\boldsymbol{\Xi}^*\mathbf{Q}[x_\eta] = \mathbf{I_0}\,(\Xi_0\, q_{0\eta} - \underline{\Xi} \bullet \underline{q}_\eta) + i\,\underline{\mathbf{I}} \bullet (\underline{\Xi} q_{0\eta} - \Xi_0\,\underline{q}_\eta) + \underline{\mathbf{I}} \bullet (\underline{\Xi} \wedge \underline{q}_\eta)$$

where: $\mathbf{Q}[x_\eta] = \mathbf{Q}_\eta = i\,\mathbf{I_0}\,q_0[x_\eta] + \mathbf{I}_1\,q_1[x_\eta] + \mathbf{I}_2\,q_2[x_\eta] + \mathbf{I}_3\,q_3[x_\eta] = \{\, i\,q_{0\eta}\,;\,\underline{q}_\eta\,\}$

If $\boldsymbol{\Xi}^*\mathbf{Q}_\eta = 0$ then: 1) $\underline{\Xi} \bullet \underline{q}_\eta = \Xi_0 q_{0\eta}$ 2) $\underline{\Xi} q_{0\eta} = \Xi_0 \underline{q}_\eta$ 3) $\underline{\Xi} \wedge \underline{q}_\eta = 0$

Further, the '**Pulse**' or 'discrete wave operator' $\wp$, can be defined as the analog of the d'Alembertian operator $\Box$.

$$\boldsymbol{\Xi}^*\,\boldsymbol{\Xi}\alpha_\eta = \boldsymbol{\Xi}\,\boldsymbol{\Xi}^*\,\alpha_\eta = \mathbf{I_0}\,(\Xi_0^{\,2} - \underline{\Xi}^2)\,\alpha_\eta \equiv -\mathbf{I_0}\,\wp\alpha_\eta \quad \text{since: } \nabla^*\nabla\alpha = \nabla\,\nabla^*\,\alpha = \mathbf{I_0}\,(\partial_0^{\,2} - \underline{\nabla}^2)\,\alpha \equiv -\mathbf{I_0}\,\Box\alpha$$

As with CNVs, every scalar discrete function ($\psi_\eta = \psi[x_\eta]$) has its own '**Associate**' DNV: $\mathfrak{A}_\eta \equiv \boldsymbol{\Xi}\psi_\eta$. Whenever, these functions are 'digitally harmonic' then their conjugate gradient of their associate DNV is always zero; that is to say, when:

$$\wp\psi_\eta = 0 \quad \Leftrightarrow \quad \boldsymbol{\Xi}^*\mathfrak{A}_\eta = 0$$

The "***Zero Condition***" DNV operator, $\mathbf{Z}_0$ is defined as: $\mathbf{Z}_0\psi_\eta \equiv i\,\underline{\mathbf{I}} \bullet (c\,\underline{\Xi} + \underline{v}\,\Xi_0\,)\psi_\eta - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\Xi}\psi_\eta)$

The DNV equivalent of the total-time differential for a scalar function: $\mathbf{I_0}\,\mathbf{D}\psi_\eta = -\mathbf{V}_\eta^*\boldsymbol{\Xi}^*\psi_\eta$ if $\mathbf{Z}_0\psi_\eta = 0$

The "DNV ***Flow***" equation can be derived for any DNV: $\mathbf{D}\mathbf{Q}_\eta + \mathbf{V}_\eta^*\,\boldsymbol{\Xi}^*\mathbf{Q}_\eta = 0$ if $\mathbf{Z}_0\mathbf{Q}_\eta = 0$

The 'digital zero condition' [73] is equivalent to the four NV equations involving the d-div and d-curl operators:

1) $\underline{v}_\eta \wedge \underline{\Xi} \bullet \underline{q}_\eta = 0$ or $\underline{v}_\eta \bullet (\underline{\Xi} \wedge \underline{q}_\eta) = 0$ 2) $c\,\underline{\Xi} \bullet \underline{q}_\eta + \underline{v}_\eta \bullet \Xi_0 \underline{q}_\eta = 0$

3) $\underline{\mathbf{I}} \bullet (c\,\underline{\Xi} + \underline{v}_\eta\,\Xi_0\,)q_{0\eta} - (\underline{v}_\eta \wedge \underline{\Xi}) \bullet (\underline{q}_\eta \wedge \underline{\mathbf{I}}\,) = 0$ 4) $(c\,\underline{\Xi} + \underline{v}_\eta\,\Xi_0\,) \bullet (\underline{q}_\eta \wedge \underline{\mathbf{I}}\,) - \underline{\mathbf{I}} \bullet \underline{v}_\eta \wedge \underline{\Xi} q_{0\eta} = 0$

These four CNV equations were found to be equivalent to **Maxwell's Equations** [74] when $\mathbf{Q} = \{\, -i\,\phi\,;\,\underline{A}\,\}$.

As with CNVs, the most important DNVs are the corresponding **Digital Voigt Vectors**, defined by their structure; for any discrete set function, $\alpha[x_\eta]$ these are defined as:

$$\textbf{Digital Voigt Vector}, \quad \mathfrak{V}_\eta \equiv -i\,c\,\alpha[x_\eta]\,\mathbf{I_0} + \alpha[x_\eta]\,\underline{v}_\eta \bullet \underline{\mathbf{I}} = \alpha[x_\eta]\,\mathbf{V}_\eta^* = \{\, i\,\mathfrak{V}_{0\eta}\,;\,\underline{\mathfrak{V}}_\eta\,\}$$

The scalar and vector components of a *Digital Voigt Vector* also satisfy the following equation, named in honor of the pioneer of asynchronous EM - Ludvig V. Lorenz;

$$c\,\underline{\mathfrak{V}}_\eta + \underline{v}_\eta\,\mathfrak{V}_{0\eta} = 0 \qquad (\text{The } \textbf{\textit{Lorenz Equation}}\,)$$

The most important DNV associated with a Discrete Voigt Vector ($\mathfrak{V}_\eta$) is its local gradient, $\boldsymbol{\Xi}^*\mathfrak{V}_\eta$; the gradients play the role of forces (actually, impulses) in the corresponding electromagnetic theory. This has the explicit form:

$$\boldsymbol{\Xi}^*\mathfrak{V}_\eta = -\mathbf{I_0}\,\mathbf{D}\alpha_\eta - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\Xi}\alpha_\eta + \Xi_0\underline{\mathfrak{V}}_\eta) + \underline{\mathbf{I}} \bullet \underline{\Xi} \wedge \underline{\mathfrak{V}}_\eta$$

This has three components, so the vector component of the DNV ***Gradient*** $\mathfrak{G}_\eta$ will have both real and imaginary parts.

Definition: $\Xi^*\mathfrak{V}_\eta \equiv i\,\mathfrak{G}_\eta = i\,\{i\,G_{0\eta}\,\mathbf{I_0} + \underline{\mathbf{I}}\bullet\,(\underline{G}_{R\eta} - i\,\underline{G}_{I\eta})\} = -\,G_{0\eta}\,\mathbf{I_0} + i\,\underline{\mathbf{I}}\bullet G_{R\eta} + \underline{\mathbf{I}}\bullet G_{I\eta}$

Comparing coefficients gives: $G_{0\eta} = \mathbf{D}\alpha_\eta$; $G_{R\eta} = -\,(c\,\underline{\Xi}\alpha_\eta + \Xi_0\underline{\mathfrak{V}}_\eta)$; $G_{I\eta} = \underline{\Xi}\wedge\underline{\mathfrak{V}}_\eta$

Now $\mathbf{V}_\eta{}^*\Xi^*\mathfrak{V}_\eta = \mathbf{I_0}\,(i\,c\,G_{0\eta} - i\,\underline{v}_\eta\bullet\underline{G}_{R\eta} - \underline{v}_\eta\bullet\underline{G}_{I\eta}) + \underline{\mathbf{I}}\bullet\,(c\,\underline{G}_{R\eta} + \underline{v}_\eta\wedge\underline{G}_{I\eta}) - i\,\underline{\mathbf{I}}\bullet(c\,G_{I\eta} - \underline{v}_\eta\wedge G_{R\eta})$

But if $\mathfrak{V}_\eta$ is a ***Flow Vector*** then: $\mathbf{D}[\mathfrak{V}_\eta] = -\,\mathbf{V}_\eta{}^*\Xi^*\mathfrak{V}_\eta = \mathbf{D}[-i\,c\,\alpha_\eta\,\mathbf{I_0} + \underline{\mathbf{I}}\bullet\,\underline{\mathfrak{V}}_\eta] = -\,\mathbf{I_0}\,i\,c\,\mathbf{D}[\,\alpha_\eta] + \underline{\mathbf{I}}\bullet\,\mathbf{D}[\underline{\mathfrak{V}}_\eta]$

This gives the ***Gradient Equations***:

1) $\underline{v}_\eta\bullet\underline{G}_{R\eta} = 0$
2) $\underline{v}_\eta\bullet\underline{G}_{I\eta} = 0$
3) $c\,\underline{G}_{R\eta} + \underline{v}_\eta\wedge\underline{G}_{I\eta}) = -\,\mathbf{D}[\underline{\mathfrak{V}}_\eta]$
4) $c\,G_{I\eta} - \underline{v}_\eta\wedge G_{R\eta} = 0$

Finally, in finite difference terms: $-\,i\,\mathbf{V}_\eta{}^*\,\mathfrak{G}_\eta = -\,\mathbf{V}_\eta{}^*\Xi^*\mathfrak{V}_\eta = \mathbf{D}[\underline{\mathfrak{V}}_\eta] = -\,i\,(\mathbf{D}\mathbf{X}^*)\,\mathfrak{G}_\eta \;\therefore\; i\,\boldsymbol{\Delta}\mathfrak{V}_\eta = \boldsymbol{\Delta}\mathbf{X}^*\,\mathfrak{G}_\eta$

The addition of any Associate Vector $\mathfrak{A}_\eta = \Xi\psi_\eta$ to a Voigt Vector, $\mathfrak{V}_\eta$ defines its ***Gauge Vector***, $\mathfrak{V}_\eta' \equiv \mathfrak{V}_\eta + \Xi\psi_\eta$.

$\therefore\; \mathfrak{V}_{0\eta}' = \mathfrak{V}_{0\eta} + \Xi_0\psi_\eta$ & $c\,\alpha_\eta' = c\,\alpha_\eta - \Xi_0\psi_\eta$ but since $\aleph\,\psi_\eta = 0$ $\therefore$ $\Xi^*\mathfrak{V}_\eta' = \Xi^*\mathfrak{V}_\eta$ or $\mathfrak{G}_\eta' = \mathfrak{G}_\eta$

The harmonic property of $\mathfrak{A}_\eta$ ensures that the gradient of any Voigt Vector ($\mathfrak{G}_\eta$) remains invariant under a gauge Transform.

Now, in all of these definitions, the discrete difference operator $\boldsymbol{\Delta}$ used here is defined as the sum of the point difference $\lozenge$ and the extended difference $\underline{\boldsymbol{\Delta}}$ operators that are defined in appendix A4.2. $\boldsymbol{\Delta} = \underline{\boldsymbol{\Delta}} + \lozenge$
A useful extension to multiple discrete parameters is also defined in appendix A4.3: $\boldsymbol{\Delta}F[j\,;\,k] = \partial_j\,F[j\,;\,k] + \partial_k\,F[j\,;\,k]$
A comprehensive set of useful single-point DNV identities involving the Ξ operator is provided in **Appendix VI**.

3.2.5 TWO-POINT DNVs

Double-Differences

The third paper in this series [3] analyzed the continuous interactions of two electrons. This required extending single-point Continuous Natural Vectors (CNVs) to pair-wise combinations involving the **differences** between each of the space and time parameters; these results are summarized in **Appendix II**. The central technique used in this approach was to 'spread' the infinitesimal change δt in the time difference ($T = t_1 - t_2$) equally across each 'local' time t_1 and t_2 for electrons #1 and #2 respectively, as in $(t_1 + \delta t/2)$ and $(t_2 - \delta t/2)$. This led to the definitions of the 'limit from above' and 'limit from below'; the average of these two limits (the 'average differential') was always continuous, even for discontinuous functions. The same ideas can be applied even when only finite changes (e.g. Δt) are involved; this leads to two-point *Discrete Natural Vectors* (or **DNV**s). In the present theory, electrons may only interact when <u>each</u> one is at an interaction time, say t_η and $t_{\eta'}$, where: $t_\eta = \eta\,\tau$ and $t_{\eta'} = \eta'\,\tau$. This means that each electron's smallest variation in its local time is one chronon, τ so that the smallest variation in the two electron time difference T is two chronons, i.e. $\Delta T = 2\,\tau$. The spatial 3D vector defining the <u>difference</u> between the locations $\underline{x}_\eta$ and $\underline{x}_{\eta'}$ of these two electrons at their respective interaction time t_η and $t_{\eta'}$ is $\underline{X}_{12}$.

$$\underline{X}_{12}[\,t_\eta\,,\,t_{\eta'}\,] \equiv \underline{x}_\eta[1{:}\,t_\eta\,] - \underline{x}_{\eta'}[2{:}\,t_{\eta'}\,] = \underline{X}_{12}[T_\eta] \qquad \text{where} \quad T_\eta = t_\eta - t_{\eta'}$$

This allows the 'post two-time' relative electron displacement $\Delta^+\underline{X}_{12}$ and velocity $\underline{V}_{12}{}^+$ to be defined.

$$\Delta^+\underline{X}_{12}[T_\eta] \equiv \underline{X}_{12}[T_\eta + \Delta T_\eta] - \underline{X}_{12}[T_\eta] \equiv \Delta T_\eta\,\underline{V}_{12}{}^+[T_\eta] \quad \therefore \quad \underline{V}_{12}{}^+[T_\eta] = \{\,\underline{X}_{12}[T_\eta + 2\tau] - \underline{X}_{12}[T_\eta]\,\}\,/\,2\tau$$

But each electron's local displacements define their own individual velocities, like: $\underline{v}_1[t_\eta] = \underline{v}_1{}^+[t_\eta] = \{\,\underline{x}_1[t_\eta{}^+] - \underline{x}_1[t_\eta]\,\}\,/\,\tau$

So, $\underline{x}_1[t_\eta{}^+] = \underline{x}_1[t_\eta] + \tau\,\underline{v}_1{}^+[t_\eta]$ and $\underline{x}_2[t_{\eta'}{}^-] = \underline{x}_2[t_{\eta'}] - \tau\,\underline{v}_2{}^+[t_{\eta'}]$

$\therefore$ $\underline{X}_{12}[T_\eta + 2\tau] = \underline{X}_{12}[\,t_\eta{}^+\,,\,t_{\eta'}{}^+] = \underline{x}_1[t_\eta{}^+] - \underline{x}_2[t_{\eta'}{}^-] = \underline{x}_1[t_\eta] - \underline{x}_2[t_{\eta'}] + \tau\,\{\,\underline{v}_1{}^+[t_\eta] + \underline{v}_2{}^-[t_{\eta'}]\,\}$

$$\therefore \quad \underline{V}_{12}^{+}[\,t_\eta\,,\,t_{\eta'}] \;=\; \tfrac{1}{2}\,\{\,\underline{v}_1^{+}[t_\eta] \;+\underline{v}_2^{-}[t_{\eta'}]\,\}$$

The 'prior two-time' relative electron displacement $\Delta^{-}\underline{X}_{12}$ and velocity $\underline{V}_{12}^{-}$ can be defined using: $t_\eta^{-} = t_\eta - \tau$ & $t_{\eta'}^{+} = t_{\eta'} + \tau$. This leads to the comparable result:

$$\therefore \quad \underline{V}_{12}^{-}[\,t_\eta\,,\,t_{\eta'}] \;=\; \tfrac{1}{2}\,\{\,\underline{v}_1^{-}[t_\eta] \;+\underline{v}_2^{+}[t_{\eta'}]\,\}$$

As in the continuous analysis, it is possible to define a comparable 'difference average' of any function $F[t_\eta]$ at any point t_η, whether the function changes continuously <u>or</u> discontinuously at that point.

Definition: **Difference-Average**, $\langle F[t_\eta]\rangle \;\equiv\; \tfrac{1}{2}\,\{\,F^{+}[t_\eta] \;+F^{-}[t_\eta]\,\}$

Both $\underline{v}_1[t_\eta]$ and $\underline{v}_2[t_{\eta'}]$ are spatial difference functions of $\underline{x}_1[t_\eta]$ and $\underline{x}_2[t_{\eta'}]$ and change discontinuously at t_η and $t_{\eta'}$ when they are involved in a mutual interaction; similarly, $\underline{V}_{12}[T_\eta]$ changes discontinuously when the interaction time difference is T_η.

$$\therefore \quad \langle\,\underline{V}_{12}[t_\eta - t_{\eta'}]\,\rangle \;=\; \tfrac{1}{2}\,\{\,\langle\,\underline{v}_1[t_\eta]\,\rangle + \langle\,\underline{v}_2[t_{\eta'}]\,\rangle\,\}$$

As in the continuous case, this definition is NOT the <u>difference</u> in the relative velocities at these two times ($\underline{v}_1[t_1] - \underline{v}_2[t_2]$) but the AVERAGE of these two velocities. These results for the position and velocity differentials for two particles are self-consistent and can be generalized to all anti-symmetric <u>separable</u> functions Ψ_{12} that describe the ***joint*** properties of associated pairs of electrons at two different locations in space $\underline{x}_1$ & $\underline{x}_2$ at two different times t_1 and t_2, when each particle's related (individual) property is characterized by $\psi_j[t_j;\ \underline{x}_j]$; as before, the variable ξ denotes any one of the four fundamental dimensions $\{t\,;\,x,\,y,\,z\}$.

Separable Two-Particle Property : $\Psi_{12}[\xi,\,\xi'] \;\equiv\; \psi_1[\xi] - \psi_2[\xi'] \;=\; \Psi_{12}[\xi - \xi'] \quad \&\quad \xi = \xi - \xi'$

The post and prior partial differences in any function $\psi[x_\mu]$ can be defined for finite changes in any of the differences X_μ.

$$\Delta^{+}\psi[\xi_\eta] \;\equiv\; \psi[\xi_\eta + \Delta\xi_\eta] \;-\; \psi[\xi_\eta] \;\equiv\; \Delta\psi[\xi_\eta] \quad \&\quad \Delta^{-}\psi[\xi_\eta] \;\equiv\; \psi[\xi_\eta] \;-\; \psi[\xi_\eta - \Delta\xi_\eta]$$

Here, for example, $\xi = y$ so that: $\xi_\eta^{+} = y_\eta \;+\; \Delta y_\eta \;=\; y_{\eta+1}$ and $\xi_\eta^{-} = y_\eta \;-\; \Delta y_\eta \;=\; y_{\eta-1}$ $\therefore\ \boldsymbol{\Delta}_\mu^{+}[X_\eta] \;=\; \Delta x_\mu[t_\eta] + \Delta x'_\mu[t_{\eta'}]$

$$\therefore \quad \boldsymbol{\Delta}_\mu^{+}\Psi_{12}[X_\eta] \;=\; \boldsymbol{\Delta}_\mu^{+}\psi_1[t_\eta;\,\underline{x}_\eta] \;+\; \boldsymbol{\Delta}_\mu^{-}\psi_2[t_{\eta'};\,\underline{x}_{\eta'}] \quad \&\quad \boldsymbol{\Delta}_\mu^{-}\Psi_{12}[X_\eta] \;=\; \boldsymbol{\Delta}_\mu^{-}\psi_1[t_\eta;\,\underline{x}_\eta] \;+\; \boldsymbol{\Delta}_\mu^{+}\psi_2[t_{\eta'};\,\underline{x}_{\eta'}]$$

Now, the average-difference operator can be defined just like the definition of the average-difference function above.

Definition: **Difference-Average Operator**, $\langle\,\boldsymbol{\Delta}_\mu\psi[\xi_\eta]\,\rangle \;\equiv\; \tfrac{1}{2}\,\{\,\boldsymbol{\Delta}_\mu^{+} + \boldsymbol{\Delta}_\mu^{-}\,\}\psi[\xi_\eta]$

$$\therefore \quad \langle\,\boldsymbol{\Delta}_\mu\Psi_{12}[X_\eta]\,\rangle \;=\; \boldsymbol{\Delta}_\mu\psi_1[x_\eta] + \boldsymbol{\Delta}_\mu\psi_2[x_{\eta'}]$$

N.B. The positive sign means that $\boldsymbol{\Delta}_\mu$ is **not** a linear operator on anti-symmetric functions.

Two-Electron DNVs

Just as the results of two separate CNVs could be combined into a double-difference CNV, it is possible to create a single DNV that represents the difference between the two DNVs representing the two interacting electrons. It is first necessary to define the 'one-sided' NV velocity of a single electron following the 'limits from above and below' approach.

$$\boldsymbol{\Delta}^{\lambda}[\,\psi[\xi_\eta]\,] \;=\; \Delta^{\lambda}\psi[\xi_\eta] \;=\; \Delta\psi[\xi_\eta^{\,\lambda}] \;\equiv\; \lambda\,(\,\psi[\xi_\eta + \lambda\,\Delta\xi_\eta] \;-\; \psi[\xi_\eta]\,) \quad \text{where} \quad \lambda = \pm 1 \text{ or } \pm$$

So that: $\underline{v}^{+}[t_\eta] \;=\; \underline{v}[t_\eta + \tau] \;=\; \underline{v}[t_\eta] \;=\; \Delta\underline{x}[t_\eta^{+}]\,/\,\tau \quad \&\quad \underline{v}^{-}[t_\eta] \;=\; \underline{v}[t_\eta - \tau] \;=\; \Delta\underline{x}[t_\eta^{-}]\,/\,\tau$

Since $\mathbf{X}[t_\eta] \;=\; i\,c\,t_\eta\,\mathbf{I_0} \;+\; \underline{x}_\eta \bullet \underline{\mathbf{I}}$ and $\mathbf{V}^{\lambda}[t_\eta] \;\equiv\; \Delta^{\lambda}\mathbf{X}[t_\eta]\,/\,\tau$ then $\mathbf{V}^{\lambda}[t_\eta] \;=\; i\,c\,\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet \underline{v}^{\lambda}[t_\eta]$

Using the definition of any difference-average: $\langle\,\underline{v}\,\rangle \;=\; \tfrac{1}{2}\,(\underline{v}^{+} + \underline{v}^{-})$ then:

$$< \mathbf{V}[t_\eta] > \;=\; ½\,(\,\mathbf{V}^{+}[t_\eta] + \mathbf{V}^{-}[t_\eta]\,) \quad \therefore \quad < \mathbf{V}[t_\eta] > \;=\; i\,c\,\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet < \underline{v}[t_\eta] >$$

These single electron results can now be combined to form the double-difference Velocity DNV, $\mathbf{V}_{12}[T_\eta]$ with $T_\eta = t_\eta - t_{\eta'}$.

$$\underline{V}_{12}{}^{\lambda}[\,T_\eta] \;=\; ½\,\{\,\underline{v}_1{}^{\lambda}[t_\eta] \;+ \underline{v}_2{}^{-\lambda}[t_{\eta'}]\,\} \;=\; \boldsymbol{\Delta}^{\lambda}\underline{X}_{12}[T_\eta] \,/\, \Delta T_\eta \;=\; \Delta\underline{X}_{12}[T_\eta{}^{\lambda}] \,/\, 2\tau$$

The consistent definition of the double-difference Average Velocity DNV is then: $< \mathbf{V}_{12}[T_\eta] > \;\equiv\; i\,c\,\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet < \underline{V}_{12}\,[T_\eta] >$

$$\therefore \quad < \mathbf{V}_{12}[t_\eta] > \;=\; ½\,(\,\mathbf{V}_{12}{}^{+}[t_\eta] + \mathbf{V}_{12}{}^{-}[t_\eta]\,) \quad \text{where} \quad \mathbf{V}_{12}{}^{\lambda}[T_\eta] \;=\; i\,c\,\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet\; \underline{V}_{12}{}^{\lambda}[T_\eta]$$

These velocity results can be readily extended to any 4D set of functions $\psi_\mu[\alpha\colon t_\eta;\, \underline{x}_\eta]$ for any electron 'α' contributing to a two-electron separable joint discrete function $\Psi_{12}[X_\eta]$. First, a single DNV $\boldsymbol{\psi}$ is constructed from these four components.

$$\boldsymbol{\psi}[\alpha\colon t_\eta;\, \underline{x}_\eta] \;\equiv\; i\,\psi_0[\alpha\colon t_\eta;\, \underline{x}_\eta]\;\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet\; \underline{\psi}[\alpha\colon t_\eta;\, \underline{x}_\eta] \quad \text{and} \quad \Psi_\mu[1,2\colon x_\eta,\, x_{\eta'}] \;\equiv\; \boldsymbol{\psi}_\mu[1\colon x_\eta] \;-\; \boldsymbol{\psi}_\mu[2\colon x_{\eta'}]$$

So, the generic double-difference DNV is defined as: $\quad \boldsymbol{\Psi}[1,2\colon X_\eta] \;\equiv\; i\,\Psi_0[1,2\colon X_\eta]\,\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet \underline{\Psi}[1,2\colon X_\eta]$

Post and prior partial differences can now be defined for both the one and two-electron DNVs using the operators $\boldsymbol{\Delta}_\mu{}^{\lambda}$.

$$\boldsymbol{\Delta}_\mu{}^{\lambda}\boldsymbol{\Psi}[1,2\colon X_\eta] \;=\; i\,\boldsymbol{\Delta}_\mu{}^{\lambda}\Psi_0[1,2\colon X_\eta]\,\mathbf{I_0} \;+\; \underline{\mathbf{I}} \bullet \boldsymbol{\Delta}_\mu{}^{\lambda}\underline{\Psi}[1,2\colon X_\eta] \quad \text{where} \quad \boldsymbol{\Delta}_\mu{}^{\lambda}\Psi_\nu[1,2\colon X_\eta] \;=\; \boldsymbol{\Delta}_\mu{}^{\lambda}\psi_\nu[1\colon x_\eta] \;+\; \boldsymbol{\Delta}_\mu{}^{-\lambda}\psi_\nu[2\colon x_{\eta'}]$$

Two-Electron Digital Change Operators

Section 3.2.3 introduced the single-point digital gradient operators for both 3D space and for one-electron DNVs. The total-difference of any property ψ of any one electron 'α' was defined as the <u>sum</u> of the local time-difference and <u>all</u> the spatially directed differences between <u>two</u> interaction points on the trajectory of this electron. This led to the definition of the four Digital Rate-of-Change operators $\boldsymbol{\Xi}_\mu$ for a single electron. These may be now combined together to define the two-electron partial rate of change operators Ω_μ (the 'ultimate' operators) in terms of the two differences that occur for each electron.

Definition: **Partial-Rate-of-Change Operator**, $\Omega_\mu[\,\Psi_{12}[T_\eta;\, \underline{X}_\eta]\,] \;\equiv\; \boldsymbol{\Delta}_\mu[\,\Psi_{12}[T_\eta;\, \underline{X}_\eta]\,] \,/\, \Delta X_\mu[t_\eta\,,\, t_{\eta'}]$

This suggests that the analog of the single discrete, 3D vector gradient operator $\underline{\Xi}$ is the double digital gradient operator $\underline{\Omega}$.

Definition: ***Digital Double Gradient Operator***, $\underline{\Omega} \;\equiv\; \hat{\underline{e}}_1\,\Omega_1 \;+\; \hat{\underline{e}}_2\,\Omega_2 \;+\; \hat{\underline{e}}_3\,\Omega_3$

This leads to the definition of the Double-Total-Rate-of-Change operator $\mathbf{D}_{12}$, defined along <u>both</u> electrons' trajectories.

Definition: ***Digital Double Change Operator***, $\mathbf{D}_{12}[\,\Psi_{12}[T_\eta;\, \underline{X}_\eta]\,] \;\equiv\; \Delta\Psi_{12}[T_\eta;\, \underline{X}_\eta] \,/\, \Delta T[t_\eta\,,\, t_{\eta'}]$

$$\mathbf{D}_{12}[\,\Psi_{12}[T_\eta;\, \underline{X}_\eta]\,] \;=\; \{\,c\;\Omega_0 \;+ \underline{V}_{12} \bullet \underline{\Omega}\,\}\,\Psi_{12}[T_\eta;\, \underline{X}_\eta]$$

This also suggests that the most suitable definition of the DNV Double-Gradient operator is:

Definition: ***DNV Double Gradient Operator***, $\boldsymbol{\Omega}[\,\Psi_{12}[T_\eta;\, \underline{X}_\eta]] \;\equiv\; \{\,i\,\mathbf{I_0}\,\Omega_0 + \underline{\mathbf{I}} \bullet \underline{\Omega}\,\}\Psi_{12}[T_\eta;\, \underline{X}_\eta]$

The correspondences established above means that all the DNV results using $\boldsymbol{\Xi}$ can be extended directly to $\boldsymbol{\Omega}$. Especially, the conjugate of the DNV gradient operator can be applied to any discrete natural vector (DNV) function, $\mathbf{Q}_{12}[X_\eta]$:

$$\boldsymbol{\Omega}^{*}\mathbf{Q}_{12}[X_\eta] \;=\; \mathbf{I_0}\,(\Omega_0\,Q_0 \;-\; \underline{\Omega} \bullet \underline{Q}) \;+\; i\,\underline{\mathbf{I}} \bullet (\underline{\Omega}\,Q_0 \;-\; \Omega_0\,\underline{Q}) \;+\; \underline{\mathbf{I}} \bullet (\underline{\Omega} \wedge \underline{Q})$$

where: $\quad \mathbf{Q}_{12}[X_\eta] \;=\; i\,\mathbf{I_0}\,Q_0[x_\eta] \;+ \mathbf{I}_1\,Q_1[x_\eta] \;+\; \mathbf{I}_2\,Q_2[x_\eta] \;+ \mathbf{I}_3\,Q_3[x_\eta] \;=\; \{\,i\,Q_0\,;\, \underline{Q}\,\} \;=\; \mathbf{Q}_1[x_\eta] \;-\; \mathbf{Q}_2[x_\eta]$

$$= i\,\mathbf{I_0}\,(\,q_{10}[x_\eta] - q_{20}[x_{\eta'}]\,) + \underline{\mathbf{I}} \bullet (\,\underline{q}_1[x_\eta] - \underline{q}_2[x_{\eta'}]\,)$$

If $\boldsymbol{\Omega}^*\mathbf{Q}_{12} = 0$ then: 1) $\underline{\boldsymbol{\Omega}} \bullet \underline{Q} = \Omega_0 Q_0$ 2) $\underline{\boldsymbol{\Omega}}\, Q_0 = \Omega_0 \underline{Q}$ 3) $\underline{\boldsymbol{\Omega}} \wedge \underline{Q} = 0$

Further, the '**Pulse**' (or 'double-discrete wave') operator $\boldsymbol{\aleph}$, can be defined as the analog of the d'Alembertian operator $\Box$.

$$\boldsymbol{\Omega}^*\,\boldsymbol{\Omega}\,\Psi_{12} = \boldsymbol{\Omega}\,\boldsymbol{\Omega}^*\,\Psi_{12} = \mathbf{I_0}\,(\boldsymbol{\Omega}_0^2 - \underline{\boldsymbol{\Omega}}^2)\,\Psi_{12} \equiv -\mathbf{I_0}\,\boldsymbol{\aleph}\Psi_{12} \text{ as: } \boldsymbol{\nabla}^*\boldsymbol{\nabla}\alpha = \boldsymbol{\nabla}\,\boldsymbol{\nabla}^*\,\alpha = \mathbf{I_0}\,(\partial_0^2 - \underline{\nabla}^2)\,\alpha \equiv -\mathbf{I_0}\,\Box\alpha$$

As with CNVs, every scalar, double-discrete function $\Psi_{12}[X_\eta]$ has its own '**Associate**' DNV: $\mathbf{A}_{12}[X_\eta] \equiv \boldsymbol{\Omega}\Psi_{12}[X_\eta]$. Whenever, these functions are 'digitally harmonic' then their conjugate gradient of their associate DNV is always zero; that is to say, when:

$$\boldsymbol{\aleph}\Psi_{12} = 0 \quad \Leftrightarrow \quad \boldsymbol{\Omega}^*\,\Psi_{12} = 0$$

The "***Zero Condition***" double DNV operator, $\mathbf{Z}_{12}$ is defined as: $\mathbf{Z}_{12}\Psi_{12} \equiv i\,\underline{\mathbf{I}} \bullet (c\,\underline{\Omega} + \underline{V}_{12}\,\Omega_0)\Psi_{12} - \underline{\mathbf{I}} \bullet (\underline{V}_{12} \wedge \underline{\boldsymbol{\Omega}}\Psi_{12})$

The DNV equivalent of the total-time differential for a scalar function: $\mathbf{I_0}\,\mathbf{D}_{12}\Psi_{12} = -\mathbf{V}_{12}{}^*\boldsymbol{\Omega}^*\Psi_{12}$ if $\mathbf{Z}_{12}\Psi_{12} = 0$

The "DNV ***Flow***" equation can be derived for any DNV: $\mathbf{D}_{12}\mathbf{Q}_{12} + \mathbf{V}_{12}{}^*\boldsymbol{\Omega}^*\mathbf{Q}_{12} = 0$ if $\mathbf{Z}_{12}\mathbf{Q}_{12} = 0$

The 'double-digital zero condition' is equivalent to the four NV equations involving the 2d-div and 2d-curl operators:

1) $\underline{V}_{12} \wedge \underline{\boldsymbol{\Omega}} \bullet \underline{Q} = 0$ or $\underline{V}_{12} \bullet (\underline{\boldsymbol{\Omega}} \wedge \underline{Q}) = 0$ 2) $c\,\underline{\boldsymbol{\Omega}} \bullet \underline{Q} + \underline{V}_{12} \bullet \Omega_0 \underline{Q} = 0$

3) $\underline{\mathbf{I}} \bullet (c\,\underline{\boldsymbol{\Omega}} + \underline{V}_{12}\,\Omega_0)Q_0 - (\underline{V}_{12} \wedge \underline{\boldsymbol{\Omega}}) \bullet (\underline{Q} \wedge \underline{\mathbf{I}}) = 0$ 4) $(c\,\underline{\boldsymbol{\Omega}} + \underline{V}_{12}\,\Omega_0) \bullet (\underline{Q} \wedge \underline{\mathbf{I}}) - \underline{\mathbf{I}} \bullet \underline{V}_{12} \wedge \underline{\boldsymbol{\Omega}} Q_0 = 0$

These four CNV equations are equivalent to Maxwell's Equations [74] when $\mathbf{Q}_{12} = \{-i\,\Phi_{12}\,;\,\underline{A}_{12}\}$.

As with CNVs, the most important DNVs are the corresponding **Double Digital Voigt Vectors**, defined by their structure; for any ordered discrete set, separable function, $\alpha_{12}[X_\eta]$ these are defined as:

Double-Digital Voigt Vector, $\mathbf{V}_\eta \equiv -i\,c\,\alpha_{12}[X_\eta]\,\mathbf{I_0} + \alpha_{12}[X_\eta]\,\underline{V}_{12}[X_\eta] \bullet \underline{\mathbf{I}} = \alpha_{12}[X_\eta]\,\mathbf{V}_{12}[t_\eta]^* = \{\,i\,\mathbf{V}_\eta{}^0\,;\,\underline{V}_\eta\,\}$

The scalar and vector components of a *Double-Digital Voigt Vector* also satisfy the following Lorenz equation:

$$c\,\underline{V}_\eta + \underline{V}_{12}\,\mathbf{V}_\eta{}^0 = 0$$

The most important DNV associated with a D-D Voigt Vector is its local gradient, $\boldsymbol{\Omega}^*\mathbf{V}_\eta$; these 'double-gradients' will play the role of impulses in the corresponding discrete electromagnetic theory. This has the explicit form:

$$\boldsymbol{\Omega}^*\mathbf{V}_\eta = -\mathbf{I_0}\,\mathbf{D}\alpha_\eta - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\boldsymbol{\Omega}}\alpha_\eta + \boldsymbol{\Omega}_0\,\underline{V}_\eta) + \underline{\mathbf{I}} \bullet \underline{\boldsymbol{\Omega}} \wedge \underline{V}_\eta$$

This has three components, so the vector component of the DNV ***Gradient*** $\mathfrak{G}_\eta$ will have both real and imaginary parts.

Definition: $\boldsymbol{\Omega}^*\mathbf{V}_\eta \equiv i\,\mathfrak{G}_\eta = i\,\{i\,G_{0\eta}\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet (\underline{G}_{R\eta} - i\,\underline{G}_{I\eta})\} = -G_{0\eta}\,\mathbf{I_0} + i\,\underline{\mathbf{I}} \bullet \underline{G}_{R\eta} + \underline{\mathbf{I}} \bullet \underline{G}_{I\eta}$

Comparing coefficients gives: $G_{0\eta} = \mathbf{D}\alpha_\eta$; $G_{R\eta} = -(c\,\underline{\boldsymbol{\Omega}}\alpha_\eta + \boldsymbol{\Omega}_0\,\underline{V}_\eta)$; $\underline{G}_{I\eta} = \underline{\boldsymbol{\Omega}} \wedge \underline{V}_\eta$

Now $\mathbf{V}_\eta{}^*\boldsymbol{\Omega}^*\mathbf{V}_\eta = \mathbf{I_0}\,(i\,c\,G_{0\eta} - i\,\underline{v}_\eta \bullet \underline{G}_{R\eta} - \underline{v}_\eta \bullet \underline{G}_{I\eta}) + \underline{\mathbf{I}} \bullet (c\,\underline{G}_{R\eta} + \underline{v}_\eta \wedge \underline{G}_{I\eta}) - i\,\underline{\mathbf{I}} \bullet (c\,\underline{G}_{I\eta} - \underline{v}_\eta \wedge \underline{G}_{R\eta})$

But if $\mathbf{V}_\eta$ is a ***Flow Vector*** then: $\mathbf{D}[\mathbf{V}_\eta] = -\mathbf{V}^*\boldsymbol{\Omega}^*\mathbf{V}_\eta = \mathbf{D}[-i\,c\,\alpha_\eta\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{V}_\eta] = -\mathbf{I_0}\,i\,c\,\mathbf{D}[\,\alpha_\eta] + \underline{\mathbf{I}} \bullet \mathbf{D}[\underline{V}_\eta]$

This gives the ***Gradient Equations***: 1) $\underline{v}_\eta \bullet \underline{G}_{R\eta} = 0$ 2) $\underline{v}_\eta \bullet \underline{G}_{I\eta} = 0$
3) $c\,\underline{G}_{R\eta} + \underline{v}_\eta \wedge \underline{G}_{I\eta}) = -\,\mathbf{D}[\underline{V}_\eta]$ 4) $c\,\underline{G}_{I\eta} - \underline{v}_\eta \wedge \underline{G}_{R\eta} = 0$

Finally, in finite difference terms: $-i\,\mathbf{V}_\eta^*\,\mathfrak{G}_\eta = -\,\mathbf{V}_\eta^*\boldsymbol{\Omega}^*\,\mathbf{V}_\eta = \mathbf{D}[\mathbf{V}_\eta] = -i\,(\mathbf{D}\boldsymbol{\aleph}^*)\,\mathfrak{G}_\eta \quad \therefore\; i\,\boldsymbol{\Delta}\mathbf{V}_\eta = \boldsymbol{\Delta}\boldsymbol{\aleph}^*\,\mathfrak{G}_\eta$

The addition of any Associate Vector: $\mathfrak{A}_\eta = \boldsymbol{\Omega}\psi_\eta$ to a Voigt Vector, $\mathbf{V}_\eta$ defines its ***Gauge Vector***, $\mathbf{V}_\eta' \equiv \mathbf{V}_\eta + \boldsymbol{\Omega}\psi_\eta$.

$\therefore\; \mathfrak{v}_0' = \mathfrak{v}_0 + \boldsymbol{\Omega}_0\psi_\eta \;\;\&\;\; c\,\alpha_\eta' = c\,\alpha_\eta - \boldsymbol{\Omega}_0\psi_\eta$ but since $\boldsymbol{\aleph}\psi_\eta = 0 \quad \therefore \quad \boldsymbol{\Omega}^*\,\mathbf{V}_\eta' = \boldsymbol{\Omega}^*\,\mathbf{V}_\eta$ or $\mathfrak{G}_\eta' = \mathfrak{G}_\eta$

The harmonic property of $\mathfrak{A}_\eta$ ensures that the gradient of any Voigt Vector ($\mathfrak{G}_\eta$) remains invariant under a gauge Transform.

Now, in all of these definitions, the discrete difference operator $\boldsymbol{\Delta}$ used here is defined as the sum of the point-difference $\boldsymbol{\Diamond}$ and the extended-difference $\underline{\boldsymbol{\Delta}}$ operators that are defined in appendix A4.2.

$$\boldsymbol{\Delta} = \underline{\boldsymbol{\Delta}} + \boldsymbol{\Diamond}$$

A comprehensive set of useful double-point DNV identities involving the $\boldsymbol{\Omega}$ operator is provided in **Appendix VII**.

3.2.6 MULTIPLE TIMEFRAMES

The physical insight of momentum conservation (section 4.1.8) can be merged with the mathematical concepts of Discrete Natural Vectors (DNVs) that were introduced earlier (section 3.2), first as single-electron representations (section 3.2.4) and then combined into two-electron, discrete difference DNVs (section 3.2.5). This earlier exposition focused on the more mathematical relationships of DNVs and, in fact, all references to the electron's velocity were actually only referencing its post-interaction velocity $\underline{v}^+_n$; it is now important, when examining the dynamics of electrons to explicitly include the equally important prior velocity $\underline{v}^-_n$. This type of distinction is <u>not</u> needed for simple Newtonian particles since the particle's velocity is identical with its longitudinal (inter-nodal) velocity between interactions, in other words:

$$\underline{v}_n = \underline{u}_n = \underline{v}^+_n \;\text{ and }\; \underline{v}^-_n = \underline{u}_{n-1} \;.$$

The electron's (prior &post) velocities (i.e. after t_n) are $\underline{v}^\lambda_n$ (for $\lambda = -$ or $+$) with corresponding electron velocity DNVs $\mathbf{V}^\lambda_n$.

Single Electron DNV (nodal) Velocities: $\mathbf{V}^\lambda_n \equiv i\,c\,\mathbf{I_0} + \underline{v}^\lambda_n \bullet \underline{\mathbf{I}}$

This leads to the definition of the single electron, average (nodal) velocity: $\mathbf{V}_n \equiv \tfrac{1}{2}\,(\mathbf{V}^+_n + \mathbf{V}^-_n) = i\,c\,\mathbf{I_0} + \langle \underline{v}_n \rangle \bullet \underline{\mathbf{I}}$

The digital (vector) gradient operator $\underline{\boldsymbol{\Xi}}$ was introduced earlier in section 3.2.3 to generate digital changes and which (by design) corresponded to the continuum gradient $\underline{\nabla}$ of 3D physics. This was combined with the local velocity to define the digital rate of change operator $\mathbf{D}$: this can be the analog of the continuous total time rate of change operator d/dt but now depends on the time interval being examined: this, too, will require distinctive subscripts, just like the $\boldsymbol{\Delta}$ operator. In the case of local changes at an interaction time t_n after the impact' of an impulse, then a single electron's property ψ at instants differing by one chronon τ around t_n will satisfy the following micro-difference equation:

$$\mathbf{D}_\tau[\,\psi_n\,] = (\,c\,\boldsymbol{\Xi}_{0\tau} + \underline{v}^+_n \bullet \underline{\boldsymbol{\Xi}}_\tau\,)\,\psi_n \quad \text{with} \quad \boldsymbol{\Xi}_{0\tau}\,\psi_n = \{\psi[t_n + \tau;\, \underline{x}_n] - \psi[t_n\,;\, \underline{x}_n]\,\} / c\,\tau \;\; \text{(no movement)}$$

In contrast, the electron's 'longitudinal' velocity $\underline{u}_n$ is used to define the digital rate of change between interaction nodes, so that this electron's property variable ψ will satisfy the following corresponding interaction-difference equation:

$$\mathbf{D}_I[\,\psi_n\,] = (\,c\,\boldsymbol{\Xi}_{0I} + \underline{u}_n \bullet \underline{\boldsymbol{\Xi}}_I\,)\,\psi_n \quad \text{with} \quad \boldsymbol{\Xi}_{0I}\,\psi_n = \{\psi[t_{n+1}\,;\, \underline{x}_n] - \psi[t_n\,;\, \underline{x}_n]\,\} / c\,\eta_n\,\tau \;\; \text{(no movement)}$$

Now, ψ may be the velocity of the electron itself, taken in one of the spatial directions (say, k defined by a unit vector $\hat{\underline{e}}_k$); in this case, there are two important identities that must be acknowledged (reflecting the two time-scales involved):

$$\underline{\boldsymbol{\Xi}}_\tau\,[\,\hat{\underline{e}}_k \bullet \underline{v}^+_n\,] = 0 \quad \text{and} \quad \underline{\boldsymbol{\Xi}}_I\,[\,\hat{\underline{e}}_k \bullet \underline{u}_n\,] = 0$$

This enables these two velocities to be considered the spatial components of two corresponding DNV velocities – the single electron (nodal) velocity DNV $\mathbf{V}^{+}_{n}$ (introduced above) and the inter-nodal electron velocity $\mathbf{U}_{n}$.

$$\mathbf{V}^{+}_{\eta} = (\mathbf{X}_{\eta+1} - \mathbf{X}_{\eta}) / \tau = \mathbf{D}_{\tau} [\mathbf{X}_{\eta}] \quad \text{and} \quad \mathbf{U}_{n} = (\mathbf{X}_{n+1} - \mathbf{X}_{n}) / \eta_{n} \tau = \mathbf{D}_{I} [\mathbf{X}_{n}]$$

The corresponding single electron DNV operator $\boldsymbol{\Xi}$ was also introduced in section 3.2.4 and its conjugate could be applied to any discrete natural vector function $\mathbf{Q}$, which here can be identified with either of these two forms of the electron velocity. All of this extensive new mathematics is needed to establish the firm foundations for representing the discrete motion of single electrons and the motions that result from their discrete, asynchronous interactions: the model of EM interactions developed here to represent the foundational behavior of all of matter at its deepest levels of reality.

3.2.7 ROTATING NATURAL VECTORS

It should now be obvious that standard 3D vectors are simply a subset of the four dimensional Natural Vectors but unlike Minkowski '4-vectors' there is never any possibility of converting any of the spatial dimensions into a time-like 'distance' by some kind of abstract rotation in 4D, such as the Lorentz transformation. However, it is possible to use 4-dimensional matrices (or 'rotors') to generate spatial rotations in the 3D vector components.

As in the 2D case exploited by Pauli (see §3.1.5) the general representation of the 3D Spatial Rotation, in its (4x4) matrix form $\mathbf{R}$ is defined in terms of the four basic matrices of the real quaternion group $\mathbf{I}_{\mu}$; this new, real matrix (or 'rotor') is defined by its (anti-clockwise) rotational angle θ around the axis of rotation, $\underline{\hat{e}} = l_x \underline{\hat{e}}_1 + l_y \underline{\hat{e}}_2 + l_z \underline{\hat{e}}_3$.

Definition: **3D Spatial Rotation Matrix** (Rotor) $\mathbf{R} \equiv \mathbf{I_0} \cos \theta/2 + (\underline{\hat{e}} \bullet \underline{\mathbf{I}}) \sin \theta/2$

This rotor's inverse is: $\mathbf{R}^{-1} \equiv \mathbf{I_0} \cos \theta/2 - (\underline{\hat{e}} \bullet \underline{\mathbf{I}}) \sin \theta/2$ as $\mathbf{R}\,\mathbf{R}^{-1} = \mathbf{R}^{-1}\,\mathbf{R} = \mathbf{I_0}\{\cos^2 \theta/2 + (\underline{\hat{e}} \bullet \underline{\hat{e}}) \sin^2 \theta/2\} = \mathbf{I_0}$

The 3D rotation of the vector part $\underline{Q}$ of the Natural Vector $\mathbf{Q}$ is defined by the transform: $\mathbf{Q}_R \equiv \mathbf{R}\,\mathbf{Q}\,\mathbf{R}^{-1}$

Therefore: $\mathbf{Q}_R = \mathbf{R} (i Q_0 \mathbf{I_0} + \underline{Q} \bullet \underline{\mathbf{I}}) \mathbf{R}^{-1} = i Q_0 \mathbf{R}\,\mathbf{R}^{-1} \mathbf{I_0} + \mathbf{R} (\underline{Q} \bullet \underline{\mathbf{I}}) \mathbf{R}^{-1} = i Q_{R0} \mathbf{I_0} + \underline{Q}_R \bullet \underline{\mathbf{I}}$

These NVs are equal if the following sub-transforms are defined: $Q_{R0} = Q_0$ & $(\underline{Q}_R \bullet \underline{\mathbf{I}}) = \mathbf{R} (\underline{Q} \bullet \underline{\mathbf{I}}) \mathbf{R}^{-1}$

The 'rotated' NV has the same 'norm' as the unrotated NV: $\mathbf{Q}_R^* \mathbf{Q}_R = (Q_{R0}^2 - \underline{Q}_R \bullet \underline{Q}_R) \mathbf{I_0} = Q_0^2 \mathbf{I_0} + (\underline{Q}_R \bullet \underline{\mathbf{I}}) (\underline{Q}_R \bullet \underline{\mathbf{I}})$

But $(\underline{Q}_R \bullet \underline{\mathbf{I}}) (\underline{Q}_R \bullet \underline{\mathbf{I}}) = (\mathbf{R} (\underline{Q} \bullet \underline{\mathbf{I}}) \mathbf{R}^{-1}) (\mathbf{R} (\underline{Q} \bullet \underline{\mathbf{I}}) \mathbf{R}^{-1}) = \mathbf{R}((\underline{Q} \bullet \underline{\mathbf{I}}) (\underline{Q} \bullet \underline{\mathbf{I}}))\mathbf{R}^{-1} = -\mathbf{R}(\underline{Q} \bullet \underline{Q}\,\mathbf{I_0})\mathbf{R}^{-1} = -\underline{Q} \bullet \underline{Q}\,\mathbf{I_0}$

$$\therefore \quad \mathbf{Q}_R^* \mathbf{Q}_R = (Q_0^2 - \underline{Q} \bullet \underline{Q}) \mathbf{I_0} = \mathbf{Q}^* \mathbf{Q}$$

Reflections across an axis may be achieved by selecting one axis (e.g. , $\underline{\hat{e}} = \underline{\hat{e}}_1$) and restricting the rotation to $\theta = \pi$. This creates a set of reflection (or parity) operators $\mathbf{P}_k$, which are identical to the vector bases $\mathbf{I_k}$, with $\mathbf{P}_k^{-1} = -\mathbf{I_k}$. This approach can be invoked to ensure that DNV description of two-electron digital interaction remains invariant under all rotations around the origin and reflections across the base axes in 3D space. Again, Pauli's approach can be extended from his (2 x 1) spinor representation to a (4 x 1) 'double-spinor' representation. This extended representation appears automatically by looking for eigenfunction solutions (Ψ_{μ}) to equations using only real (4 x 4) NV matrices: $\mathbf{B}\,\Psi_{\mu} = \lambda_{\mu} \Psi_{\mu}$ where $\mathbf{B} = \mathbf{Q}\,\mathbf{R}$. The simplest eigenfunctions are those that involve only the NV base matrices themselves; these are usually chosen just for the $\mathbf{I_3}$ matrix; they involve selecting one of the the famous spin 'up' $\Phi_{\uparrow}$ and spin 'down' $\Phi_{\downarrow}$ solutions, along with a binary null vector $\varnothing$, where:

$$\mathbf{I_3}\,\boldsymbol{\Phi}_{\mu} = \lambda_{\mu} \boldsymbol{\Phi}_{\mu} \quad \text{therefore } \lambda_{\mu} = \pm 1 \quad \text{where } \Phi_{\uparrow} = (1\ 0)^T \ \& \ \Phi_{\downarrow} = (0\ 1)^T \ \& \ \varnothing = (0\ 0)^T$$

$$\text{and } \boldsymbol{\Phi}_1 = (\Phi_{\uparrow}\ \varnothing)^T \ \& \ \boldsymbol{\Phi}_2 = (\varnothing\ \Phi_{\downarrow})^T \ \& \ \boldsymbol{\Phi}_3 = (\varnothing\ \Phi_{\uparrow})^T \ \& \ \boldsymbol{\Phi}_4 = (\Phi_{\downarrow}\ \varnothing)^T$$

These can be contrasted with Dirac's electron spinors {$\boldsymbol{\Phi}_+$ and $\boldsymbol{\Phi}_-$}, which are just combinations of Pauli-like solutions.

$$\Psi(\underline{r}) = (\Phi_{\uparrow}\ \Phi_{\downarrow})^T \quad \text{and} \quad \boldsymbol{\Phi}_+ = (\Psi_1\ \Psi_2)^T = \boldsymbol{\Phi}_1 + \boldsymbol{\Phi}_2 \quad \& \quad \boldsymbol{\Phi}_- = (\Psi_3\ \Psi_4)^T = \boldsymbol{\Phi}_3 + \boldsymbol{\Phi}_4$$

The fundamental (4 x 1) 'spin' eigenstates $\boldsymbol{\Phi}_{\mu}$ will re-appear in a different role later in this theory (see §5.1.3).

4. DISCRETE INTERACTIONS

4.1 INTERACTIONS: THE FOUNDATION OF PHYSICS

In traditional Greek philosophy, motion is synonymous with change - but with the triumph of Platonism, this always implied imperfection. Something truly perfect would not change, it would be eternal – neither growing or decaying or transforming. It was this metaphysical perspective that led Aristotle to the mistaken belief that immobility is the preferred state of nature and directed all Greek intellectual efforts to the study of space – the result was the magnificent mathematics of Euclidean geometry. The study of time disappeared for almost 2000 years until Galileo, at great personal risk, looked directly at nature and then challenged this ancient, scholastic assumption. Even Galileo's physical insights into the centrality of time were converted by Newton back to a geometrical perspective where trajectories through time were viewed as fixed curves in the geometry of space and time. Newton's original physical intuition of interactions between point objects was simplified into a study of the single target particle that was subject only to instantaneous forces. In classical mechanics, these were then transformed further into the spatial differentials of a new scalar concept of space alone – potential energy. Even with the study of EM, the static, spherically symmetric Coulomb potential retained its central role as the source of dynamical change. Maxwell's retention of this concept (through the Maxwell gauge: div $\underline{A} = 0$) ultimately resulted in the contortions of space and time associated with the so-called Lorentz transformation. The present theory restores time to center-stage and redirects attention away from single-particles subject to 'free-floating' forces at one point in time, back to **interactions** between **two** equally important remote, point particles, each participating in their own equally significant (but different) times.

4.1.1 PHILOSOPHICAL PREMONITIONS

In a region of space containing a finite number of eternal particles, where all the relative spatial separations never changed, it would be impossible to distinguish the absence of time (like a static image) from the absence of interaction. Indeed, it would impossible to even determine if they were there at all. Thus, our intuitions of time, change across space, existence and interaction are all intimately bound together. This cluster of ideas forms the grounds of this research programme.

The Reality of Time

The timelessness of mathematics (i.e. unchanging definitions and relationships) originated in the Western tradition when Pythagoras 'imported' Ancient Egyptian ideas that were imbued with the primacy of the unchanging 'eternal'. Kant was also wrong about time when he claimed that 'time is real' as only objects that exist (somewhere in space) **in** time are real. It is meaningless to state that something 'exists in space but not in time'. The present theory adopts the intuitive idea that only things in the 'now' exist, all past events (prior to the observer's privileged 'now') no longer exist while future events (relative to the observer's 'now') have not yet happened. Human observers always 'live in the now'. Irrespective of the observer's location in space and time, the natural world is viewed here as unitary: the total set of events characterizing the motion of all the electrons in the universe is independent of all (human, animal or even alien) observers.

Pre-Newtonian Dynamics

Aristotle's dynamics was based on the theoretical claim that: 'a thing cannot be at once mover and moved.' The present theory adopts a symmetrical view of time so that an electron interacting with another electron alternates in these two roles according to whether the impulse is emitted forwards or backwards through time. DesCartes confused natural philosophy with his view that all matter must involve spatial extension (i.e. continuous matter) rather than viewing space in terms of *differences* between the locations of matter. Conversely, his 'raw' dynamics was founded on the mechanical idea of contact action (as in collisions) with one material object directly pushing another. It required Newton to introduce the revolutionary idea of 'action-at-a-distance' with his theory of (instantaneous) gravity acting across empty space between material bodies.

Zeno's Arrow

One of Zeno's paradoxes is centered on our ideas of motion. When an arrow in motion is photographed or viewed 'in an instant', it always appears to be at rest, so motion appears impossible as all of time is just a 'series of instants'. Einstein used this 'trick' to avoid introducing acceleration in his most famous paper of 1905 [4]. Newton's calculus does not solve this problem, even when infinitesimals are divided with finite results. The present theory always views motion as spatial separations defined at finite time intervals, motion is defined as the change in position between different time intervals.

4.1.2 PHYSICS AS INTERACTIONS

The "Substantial Spirit" of DesCartes

DesCartes has had a major negative impact on the history of western philosophy resulting from several erroneous ideas. He followed the ancient Greek tradition by seeking a 'substance' explanation for the observed phenomena in nature. DesCartes viewed 'spirit' as a mysterious substance that existed independently of material substance: this led to his basic dichotomy. In the present theory, the idea of 'spirit' is subsumed under the broader, abstract concept of 'activity'. Accordingly, activity is now viewed here as an intrinsic property of matter, specifically, electrons. Their interactions (interactivity) now generate changes in their relative motion through space (more varied activity).

Newton's Momentum Conservation

Contrary to the views offered in standard presentations of Noether's theorem (e.g. [75]), it is not whether an interaction is time-invariant that leads to energy conservation or whether the interaction is spatially invariant that leads to momentum conservation. Rather, it is whether the impulses (or forces) change the momentum (Newton) or the velocity (DesCartes) of the particles involved. This can be readily illustrated from the (near) elastic collision of two small spheres of masses m and 3m when the two sphere are approaching each other head-on at speeds +3u and –u along the x-direction; after the collision they are observed to be moving at $-v_1$ and +v. Let the post-collision magnitudes have the ratio α, i.e. $v_1 = \alpha\, v$. Let the total velocity of the two masses before and after the collision be V_1 and V_2 respectively and the corresponding total momentum before and after be P_1 and P_2 (all in the positive x-direction), with corresponding kinetic energies $\mathcal{K}_1$ and $\mathcal{K}_2$; so:

$$\therefore\ V_1 = 2\,u\ ;\ V_2 = (1-\alpha)\,v\ ;\ P_1 = 0\ ;\ P_2 = m\,(\alpha-3)\,v\ ;\ \mathcal{K}_1 = 6\,m\,u^2\ ;\ \mathcal{K}_2 = \tfrac{1}{2}\,m\,(\alpha^2+3)\,v^2$$

In an elastic collision no energy is lost ($\mathcal{K}_1 = \mathcal{K}_2$) or $12\,m\,u^2 = (\alpha^2+3)\,v^2$ There are therefore two possibilities:

A) Newton: Momentum is conserved ($P_1 = P_2$) so $\alpha = 3$ then $v = u = v_1/3$

B) DesCartes: Velocity is conserved ($V_1 = V_2$) so $\alpha\,(\alpha-3) = 0$ giving two (sub) solutions:

B1) $\alpha = 3$ then $v = -u$ & $v_1 = -3\,u$ or B2) $\alpha = 0$ then $v = 2\,u$ & $v_1 = 0$

Possibility B1 only occurs if there is no interaction ('free' penetration) or the particles exchange identity; this is rejected. Now momentum is a calculated quantity, not an immediate observable, only velocity can be measured. The two remaining alternatives are markedly different in their predictions of final motion. Theory A (Newton) predicts that the two spheres exchange their initial speeds but in the opposite direction: the smaller mass rebounds with speed 3u while the larger one rebounds with speed u. Theory B2 (DesCartes) predicts that the smaller mass will be stopped dead in its tracks after the collision while the larger mass will rebound with a speed 2u. It is experiment (as Newton observed while DesCartes just imagined), which indicated that result A is observed: vindicating Newton. It is momentum that is conserved, not velocity. However, it should be noted that velocity is conserved if the two objects have identical masses, which is the case used here for electrons but not for the macro-collections available to experimentalists in Descartes' era.

4.1.3 THE ROLE OF CAUSALITY

Time and Memory

Our sense of time is intimately bound up with our intuition about changes occurring in the world. This reflects the fact that our personal sense of time is massively unsymmetrical. All animals, including humans, have a wide knowledge of our near-present; higher animals also have memories of past events that gradually dim over time and a few, cognitive creatures have a vague awareness of the future. Memory is the key and must surely be based on the vast complex web of organic synapses that evolve over time in animals, including humans. The cumulative growth of this physical structure is one example of the direction of time. Since higher animals are also intentional beings then our current actions are conducted with the intent to produce future results. This is the origin of our primitive ideas of causality in nature. Causality is a key concept in physics but is almost never discussed by professional physicists, leaving this key concept to be analyzed only by a few, highly specialized academic philosophers. This section attempts to reawaken an interest in this idea here amongst physicists.

Causality as Approximation I

Causality is the widespread human approximation of selecting the obvious output from a sub-system as the next (linear) input in the 'causal' chain constructed by our brains when attempting to anticipate the evolution of the complex natural world.

Causality as Approximation II

Causality is a human-scale mental approximation (somewhat like touching) based on the enormous number of particles that interact co-operatively over the smallest timeframes an un-enhanced human mind can perceive (less than 1/100 second) over local distances.

Causality as Approximation III

Our ancient concept of 'causality' is much too simple an idea to circumscribe the complex dynamics of reality. A linear chain of cause and effect cannot be sufficient to describe the recursive inter-relationships of nature. Causality is likely to be just the linear, first approximation in a nearby space and time proximity of the total, non-causal, dynamic network. Indeed, only the very few integratible models described in classical mechanics texts are both stable and their motions predictable. For such systems, effects are directly proportional to causes: this simple image has biased our intellectual views on causation. In a world where everything can influence everything else (as in field theories) then total knowledge of the whole of history is required. The actual stability of the real world suggests that another principle must be invoked. In the present theory, this is the hypothesis that all interactions, when they occur, are only pair-wise between any two electrons over all interactions.

Memories bias Awareness

The present theory is not in agreement with Hamilton or Kant, who both viewed 'space' as objective while our human sense of 'time' as subjective [76]. The one-dimensional nature of time and the single directional flow of time are both objective (that is agreed to by everyone) but our memories allow us to move forwards (up to the present) and backwards (for a time) in our imaginations, when reviewing past memories. However, it is the discipline of shared memories that makes our views of our shared past more than subjective, only memories of our private thoughts and experiences remain subjective. Otherwise, poor memory recall or mental illness (or drugs!) could falsify the sequence. These psychological views have a bearing on the objective nature of mathematics. For mathematics to be socially useful there must be a general consensus (at least among mathematicians) on 'the rules of the game'. Mathematics can be a useful representation of reality when a set of rules reflects the objective (shared) nature of reality; this leads to the valuable usage of algebra for the collective representation of objective memories. In contrast, the SRT admits that the relative order of distant events will vary with the position, time and relative speed of various observers. This simply leads to confusion and the need for irrational (square root) rules, such as the relativistic transformations; this degree of arbitrariness could never result in the long-term stability of nature.

Math Notation fails Causality

The standard use of algebraic equations fails to reflect the physics of reality; the standard notation needs to evolve beyond algebraic identities when notions of causality are being addressed. In particular, logical definitions should be distinguished from causality, that deserves its own binary connection symbol, such as $A \rightarrow B$ to indicate that event A 'causes' event B. Thus, Newton's metaphysics introduces physical definitions, like momentum and constant mass, symbolized by $\underline{P}$ and M and related through a particle's instantaneous velocity, $\underline{V}$ when it is located at (relative) position ($\underline{X}$) and at the (relative) time (t) through the definition: $\underline{P}[\underline{X},t] = M\ \underline{V}[\underline{X},t]$. While Newton's dynamics (the Second Law) defines how an external instantaneous impulse ($\Delta\underline{\mathbf{I}}$) **causes** a change in the particle's momentum ($\Delta\underline{P}$) through the limit definition:

$$\Delta\underline{\mathbf{I}}\,[t] \rightarrow\ \Delta\underline{P}[\underline{X},t]\ =\ \underline{P}[\underline{X} + \Delta\underline{X}, t + \Delta t]\ -\ \underline{P}[\underline{X},t]\ =\ M\ \{\ \underline{V}[\underline{X} + \Delta\underline{X},\ t + \Delta t]\ -\ \underline{V}[\underline{X},t]\ \}$$

The present theory will retain the equality notation but will always use explicit time differences to indicate past and future.

Causality not limited to Precedence

Humans have no reason to impose, at the microscopic level, the interpretation that members of one event class A that always precedes examples of another event class B as an example of directed causality; in other words, "A causes B". A more general viewpoint would be that the pair-wise set of examples 'n' and 'm' of events A and B occurring at times 't_1'and 't_2' indicate a common interaction between them, even if t_1 always occurs before t_2. Mathematically, this can be represented as the real existence of multiple, non-empty pair-wise sets: $\{\ A_n(t_1)\ ,\ B_m(t_2)\ \} \neq 0$.

Intrinsic Interactions

Philosophers have often made the mistake of confusing a complex situation with a compound one: this is referred to as the "chocolate cake" fallacy, where the difference in two related cases is always attributed to the *addition* of a distinguishing entity (like chocolate) to the simpler situation. This can easily be represented mathematically: $A = B + C$ & $A \neq B$ so that physicists have often used this approach – sometimes successfully, as in Newton's theory of gravity, where planets were viewed as inert-matter needing the extra addition of 'gravity'. The most widespread use of this 'analytic' approach today is the separation of the full Hamiltonian (H) into an non-interacting part (H_0) and an interactive component (H_I) that is treated as a small modification using perturbation theory: $H = H_0 + H_I$, as in the QED theory of interacting electrons. The present theory dismisses the concept of 'free' particles as meaningless, the interaction between electrons is seen here as an intrinsic feature of all electrons, at all times, which is not 'turned-on' as they approach each other from opposite sides of the universe (the so-called 'adiabatic hypothesis'). From this viewpoint, change is not reducible to simple arithmetic or its infinitesimal equivalence (calculus) but is the starting point for a new, causal physics: $A(t) \rightarrow B(t + \Delta t)$.

No Observer Interception

The rejection of the reality of advanced solutions to Maxwell's Equations has been based on one of two major assumptions. Walther Ritz was the first [77] to apply commonsense to reject the likelihood of coordinating many remote emitters in the future to generate a resulting exquisitely coordinated sets of waves that would travel backwards in time to exactly converge on the field point. In sharp contrast, the single emission point is all that is needed for delayed interactions ('retarded waves'). This view is based on the assumption that light is an independent existent (entity), in this case, a wave in Maxwell's aether. Similarly, an entity traveling from the future could be intercepted by a closer but still remote observer. The key assumption made in all optical theories to date is that light is an entity – wave or photon. This assumption is not made in the present theory, which equally admits advanced solutions but restricted to only pairwise interactions.

Causality is not just Retarded

The one-way human perception of the 'flow of time' reflects the retarded nature of our human brains; that is, the axon signal flows outward from the neuronal cell nucleus with the forward passage of time to activate the dendrites of nearby neurons. This forward signaling mechanism generates (in some unknown manner) memories of the past. This (macroscopic) human bias has been transferred into science where only retarded interactions are retained although the 'advanced' solutions are equally acceptable mathematically and cannot be ruled out automatically at the microscopic level.

Causality requires Two Times

Jefimenko has written two books on EM based on the concepts of retardation and causality [78], [79]. Since, like other EM authors, he always considers either the 'field' point or the EM source point at the origin, he is able to reduce his analysis to a single time variable, defined relative to the origin of this odd reference frame. In general, when the EM interaction between two electrons is being considered it is <u>not</u> possible to locate either one of the electrons at the origin <u>throughout</u> their mutual interaction as both electrons will eventually change their pre-interaction motion after the interaction is over, in other words, both electrons are eventually accelerated due to their mutual interaction. This means that, at least, two distinct times (one at each electron) are needed whenever there occurs asynchronous interactions between the particles, as is the case for the EM interaction. Since all the existing 'laws of physics' are either timeless (averages) or involve only a single time (like Newton's Laws of Motion) they can only be viewed as approximately causal, at best.

Enabling & Blocking Contributions

The idea of causation can be enhanced by introducing the idea of 'contributions' that result in a finite change in the world. Positive contributions must be present for a consequence to occur, while negative contributions must be absent. Positive or negative contributions are sufficient if a subset can impact the outcome. Contributions may be considered as major when their effects are direct while indirect contributions reinforce the major ones. An event 'A' can be said to cause an event 'B' when no other contributions are needed. A complex process 'P' consists of a series of inter-related events (over time) and contributions, including expectations, where expectations are future results that contribute to earlier activity. For example, a marriage proposal (event 'A') is a positive contribution to a possible later process 'P' (the engagement), which might be the major contribution to the desired final state or process 'Q' (the marriage). The expectation of this final situation contributed to the initial action 'A'. These contributions can be given a graphical network representation.

Contribution, not Causation

The replacement of the ancient idea of 'causation' is effected here by the new concept of 'necessary contribution'. All positive contributions must occur before a contingent event can happen, while none of the negative contributions must have occurred that could block the possible event from occurring. All other factors (that may or may not occur without impacting the planned event) are not regarded as contributions. In the complex, inter-related world of nature, there are no singular, sufficient contributions; i.e. there are no simple causes.

4.1.4 RETARDED INTERACTIONS

Retarded interactions reflect the intuition that events in the past effect the present; in the realm of human affairs this seems eminently true but this concept has only entered the realm of science in fairly recent times as discussed above and also in an earlier paper describing the historical evolution of investigations into electromagnetism [80].

Lorenz – the Pioneer of Retardation

This research programme has earlier acknowledged its debt to the first scientist to investigate retarded interactions. The much-ignored Danish physicist, Ludwig V. Lorenz was the first to introduce retarded potentials in his model of the EM interaction published in 1867 [81], only two years after Maxwell's masterpiece [82] and ten years before they are usually credited to Rayleigh in his "Theory of Sound". FitzGerald was unaware of Lorenz's work when he introduced retarded potentials into Maxwell's EM theory in 1883 but, at least, he did publicly acknowledge Lord Rayleigh's usage.

Retarded Forces

Burniston-Brown has long been one of the few advocates for action-at-a-distance explanations over the far more popular local models, exemplified by the widespread use of field theories in modern physics. In recognizing that the electromagnetic interaction does not act immediately between remote charges, he followed the earlier tradition and only accepted the normal flow-of-time direction or 'retarded' solutions. In his book, *Retarded Action-at-a-Distance* [83] he developed a heuristic theory based on Ampère's force law of 1825 concerning the mutual action between neutral (macroscopic) electric currents and magnets and other current circuits. He extended Ampère's original concept of action-at-a-distance electrodynamics to include acceleration and retardation. This was an aggregate, rather than a microscopic (particle to particle) approach as he admits: "Although on integration around a complete circuit Ampère's formula yields the correct result, it is not suitable for an inter-particle theory." Since it begins with the static Coulomb law, he could also extend his resulting final formulae to interacting, remote large mass bodies for an extended theory of gravitation. He rejected Einstein's SRT but accommodates Bucherer's results on the motion of high-speed electrons by modifying Newton's Second Law of Motion by just including Voigt's factor $\mathscr{V} = \sqrt{(1 - v^2/c^2)}$, so that the EM force on each electron is no longer the 'raw' force $\underline{F}$ but the now reduced force $\mathscr{V}\underline{F}$. Of course, mathematically, this is identical to dividing the 'rest' mass of the electron by Voigt's factor or simply multiplying it by the Lorentz factor $\mathcal{L}$ (the reciprocal of the Voigt factor) so $M = \mathcal{L}\, m$. Unfortunately, Burniston-Brown never proposed any explanation for this reduction (even failing to refer to Voigt). The present theory [4] has shown that the combination of discrete cyclic interactions and the need to reconcile the 'light-cone' condition with the finite inertia of the electron leads to a reduction in the frequency of interaction between two electrons when their relative speed increases.

Attractive Forces imply Advanced Photons

Only the repulsion between similarly charged electrons can preserve locality and causality (the propagation of an effect from the past into the future). The attraction of an electron and a positron involves the propagation of a photon to the other side of the universe in the distant future and its reappearance from the distant past at the other side of the universe so that it can then preserve mechanical momentum at the emission and absorption vertices. This interpretation is most easily seen by examining the corresponding Feynman diagram for a simple (single photon) attraction between an electron and a positron. This shows that simple Feynman diagrams are quite suitable for repulsive forces as the 'photon' propagator removes momentum from the earlier electron and transfers it to the later electron involved in the interaction. However, in attractive interactions (opposite charges), the 'exiting photon' from the earlier charged particle has to move away from the other particle and return from the other side of the universe to cause both particles to move closer to each other after the interaction is complete. Alternatively, the photon carries momentum backwards through time - this is more like Feynman's original concept of action-at-a-distance.

4.1.5 ADVANCED INTERACTIONS

Einstein's Rejection of Advanced Waves

Einstein repeated (without attribution) his rival Ritz's earlier rejection of advanced solutions of Maxwell's EM waves [84] in 1909 when he agreed that many waves originating in the future from very many emitting sources converging to a single receiving point in the past would require a massive degree of multiple correlations, which is highly improbable. However, this criticism does **not** apply for the coordinated emission and absorption between two point electrons, which is the model adopted in the present theory. In contrast to Ritz's ballistic theory, Einstein's photon model required the relativistic addition law of velocities to ensure that the photon's speed was always constant and independent of the emitter's motion.

Intentions suggest Advanced Effects

The appeal to the universal experience of human muscular activity has long familiarized students of physics to the abstract concept of '**force**'. Similarly, our human memories have formed a solid foundation for accepting the intuition of 'retarded interactions', where only the past seems to effect the present. It is now time to appeal to our universal experiences of human imagination and **intention** to accustom students to the difficult concept of 'advanced interactions', where the future plays an equally powerful role in all of human affairs: the desire for change is then the necessary pre-condition to planned action.

Forward Contracts

The automatic intuition of causality acquired by all human beings makes it difficult to appreciate advanced effects. It is only human intentionality to accomplish something in the future can give some insights into this difficult concept. One example has been present in business before the invention of long-distance communications but when overseas travel had become reliable. Merchants in 18th century London would negotiate contracts with American merchants by ship-born letter and would ship gold to New York to purchase goods at some future date; it was their expectation that these future contracts would be honored that led them to trust their gold to strangers.

Playing Tennis

The idea that the future effects the present is a little like a game of tennis where the expectation of where the opposing player will be awaiting the arrival of the ball is a major factor in where the player returns the ball.

4.1.6 TIME SYMMETRIC INTERACTIONS

Time Asymmetry as Human Construction

The asymmetry of nature across time (the statistical increase in disorder) may well be an artifact of the human mind as the mind can categorize coherent structures more readily than the myriad of 'unordered' states; so for example, it easier for us to consider an unbroken cup than one shattered into many pieces. However, each 'disordered' state is absolutely unique, just as much as the symmetric state and all transitions appear equally likely.

Fokker's Proposal

Due to the increasing number of severe logical problems in classical EM theories involving point charged particles, such as pre-acceleration, Adrien Fokker proposed in 1929 [85] that an accelerated electric charge produces both advanced, as well as retarded EM fields. This radical proposal has inspired a whole class of symmetric theories of electromagnetism.

Wheeler & Feynman's Classical EM

John Wheeler and his PhD student, Richard Feynman wrote a much-quoted paper in 1945 [77] that attempted to solve the radiation problem of self-interactions, by proposing a classical, direct-particle, action-at-a-distance theory. They proposed spherical, outgoing EM waves (retarded solutions to Maxwell's Equations) looking somewhere (on their way to infinity) for a complete set of absorbers that would retransmit advanced waves back through time to the originating electric charge. This appeared as accelerated electrons radiating to other electrons in their own future. The mathematical trick was to combine half the retarded wave from <u>one</u> accelerated charged particle with one half of the advanced wave from <u>all</u> the remote charges that had absorbed the retarded wave and then re-emitted it back in time collectively. This exact combination generated the same result as one full retarded wave. This is a very satisfactory result, that reappears in the present theory but now there is only one electron (in the future) that 'returns' all of the interaction impulse back to the original emitting electron.

The similarities and differences between the new theory and the Wheeler-Feynman EM model are summarized here.

SIMILARITIES

1. All interactions originate only from charged electrical particles.
2. There are no self-interactions, only direct *inter*-actions.
3. All effects reflect asynchronous action-at-a-distance.
4. Advanced and retarded effects are both necessary and equal.

DIFFERENCES

1. There are only complete pair-wise interactions, no partial absorptions by many absorbers.
2. Each interaction is symmetric across electron-pairs (no 'broadcasting' to find absorbers).
3. The magnitude of the momentum exchange is independent of distance, beyond atomic separations.
4. Electrons may only interact twice per cycle ('send/receive'), not unconditionally and continuously.
5. There are no EM fields (continuous forces), only discontinuous impulses at each electron.
6. All fundamental interactions are now viewed as electromagnetic, hence universal.

Csonka's Time-Symmetric Model

In 1969, Paul Csonka investigated the relationship between causal and time sequences in particle physics [86]. He began with the hypothesis that the elementary interaction has no preferred direction in time - just as it would have no preferred direction in space. Unlike Wheeler and Feynman, Csonka did not need to introduce a set of complete absorbers. He also interpreted this in field theoretical terms, so that he proposed that there would be as many waves going from a point source into the past as go into the future. This viewed the elementary interaction between all particles as time-symmetric between cause and effect – in contrast to the near universal assumption of 'retarded causality', where all effects are later in time than their causes. He also proved this symmetric result was also valid for 'massive' fields, as well as for mass-less EM (photon) interactions.

Cramer's Model

John Cramer, in 1986, added a further twist to Csonka's model by adding reflection forward in time from the 'Big Bang' if the backward wave failed to get re-emitted by any absorber on its journey to the beginning of the universe. In this relatively well-known 'transactional' model [87], the standard wave function (psi) of quantum mechanics is interpreted as mediating a transaction between two quantum particles. A real 'offer' wave is sent outwards in all directions from the source particle until it encounters one other particle that 'accepts' this offer, sending its reply 'inwards' back to the earlier particle through a 'response wave'. Although this mechanism involves only two electrons, its retarded and advanced waves are completely spherically symmetric across all space. This symmetric model (without waves) is adopted in this research programme.

Distinct Interactions imply Symmetry

The new theory explicitly rejects the concept of continuous interactions for one constructed upon the idea of a sequence of 'distinct interactions' while the time-symmetry in the theory gives equal significance to advanced as well as delayed effects.

Two Electron Symmetry

The near-universal preference for contact-action (e.g. Aristotle, DesCartes, Maxwell, Einstein) over action-at-a-distance (or far-action) style theories, such as Newtonian gravity, reflects both the illusion of personal touch (which always leaves us several atoms apart) and the mathematical seduction of the infinite (touching becomes infinitely close together). The new theory is an 'asynchronous-action' theory combining both retarded and advanced action-at-a-distance. This is distinct from Newton's instantaneous far-action or Gauss's only retarded-action or direct contact-action, whether mechanical (DesCartes) or local field theories, beloved by Maxwell and Einstein. Only an asynchronous-action type of theory offers a symmetric view of the single interaction between two particles at two remote locations at two different times. It is the introduction of a completely symmetric view of the two electrons participating in each interaction that forces a two-time view (or a single time difference parameter) into the mathematical descriptions of this new model; this feature, along with the rejection of the force model (continuous interactions) that together eliminate the near-universal reliance on single-time, continuum mathematics.

Symmetric Causality

Causal influences can occur symmetrically in time in the present theory. Standard effects travel forward in time, from one time t′ to a later time t; these are usually called 'delayed' or 'retarded' effects. Mathematically these interactions, where an impulse $\Delta\underline{\mathbf{I}}^{-}[t']$ from one electron, causes a later (at least one *chronon* later) change in momentum $\Delta\underline{P}[t]$ in another electron, can be represented using either the standard left-to-right rule of English with a causal 'arrow' or as an equation using the explicit Heaviside unit step function $\vartheta[t]$, defined as: $\vartheta[t] = 1$ whenever $t \geq 0$ and $\vartheta[t] = 0$ whenever $t < 0$

$$\Delta\underline{\mathbf{I}}^{-}[t'] \rightarrow \Delta\underline{P}[t] \qquad \text{or} \qquad \Delta\underline{\mathbf{I}}^{-}[t'] = \Delta\underline{P}[t]\ \vartheta[t - t' - \tau]$$

There is a complementary 'advanced' effect in this theory, where an impulse $\Delta\underline{\mathbf{I}}^{+}[t']$ from one electron, causes an earlier (at least one *chronon* earlier) change in momentum $\Delta\underline{P}[t]$ in another electron; this can be represented using either the reverse right-to-left rule with a reversed causal 'arrow' or as an equation using the explicit Heaviside unit step function:

$$\Delta\underline{P}[t] \leftarrow \Delta\underline{\mathbf{I}}^{+}[t'] \qquad \text{or} \qquad \Delta\underline{\mathbf{I}}^{+}[t'] = \Delta\underline{P}[t]\ \vartheta[t' - t - \tau]$$

4.1.7 REPULSIVE IMPULSES

It will be useful if it is first demonstrated that the effect of an impulse from an electron in the future (say #1) of another electron (#2) at an **earlier** time in the emitting electron's past is equivalent to a reaction of the earlier electron following the emission forwards through time to the receiving electron. The present theory views all electrons as cycling periodically (and always in phase) from a '*sendable*' state (indicated by the 'snd' or ↑ labels) to a 'receivable' state (indicated by 'rcv' or ↓) over the universal temporal constant of one quantum of time duration or chronon τ. The standard convention with two-way transmissions across time is used to label emissions that flow from the past into the future (relative to human perceptions) as '*retarded*' interactions (labeled with a negative symbol) while 'reverse' emissions that flow backwards from the future into its own past are labeled '*advance*' interactions (and signified by a plus symbol).

Retarded Repulsive Interaction

The situation investigated in the previous paper [76] only discussed the standard retarded repulsive interaction. This will first be reviewed again here but with explicit reaction displayed at the emitting electron. This is followed by the advanced repulsive interaction that is shown to be the equivalent viewpoint but seen from the other electron. The attractive interaction is analyzed in the next section. The standard retarded repulsive interaction is the easiest to understand: it corresponds to the human idea of one object (the target) being hit by an intermediate object (the missile or 'carrier') that was ejected from an originating object (the source) at an earlier time. Since the present theory posits an action-at-a-distance model then the intermediate 'carrier' has no reality – it is only offered as a conceptual convenience or crutch.

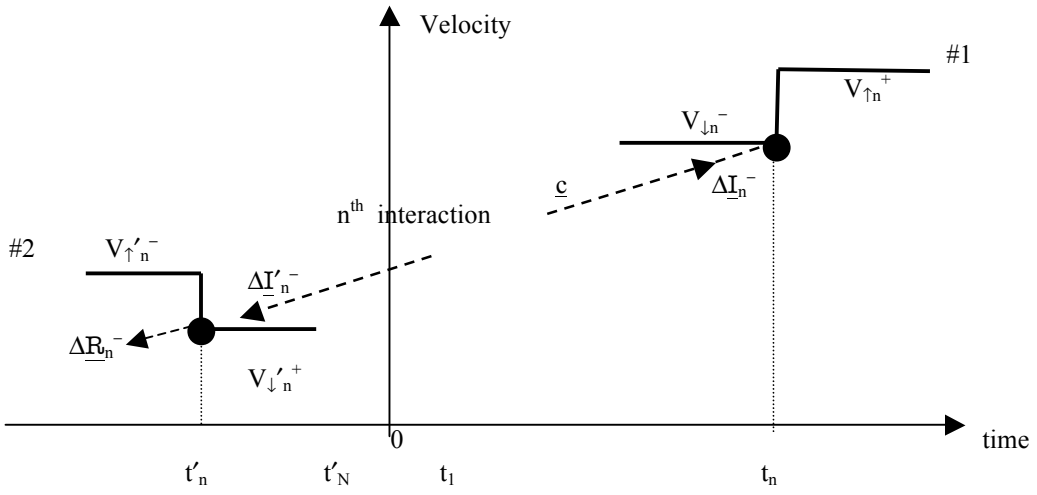


Fig. 1 Retarded-Repulsive Interaction ('ret-push photino').

Here, the earlier electron (#2) initiates the interaction at time t'_n , when it is in its 'snd' state ('↑') and its (pre-) velocity is $\underline{V}_2^-[t'_n]$ by emitting a repulsive impulse $\Delta\underline{I}_n^-$ forward in time. In the same instant, this electron suffers a reaction $\Delta\underline{R}_n^-$ against its direction of motion, slowing its motion (post-velocity] immediately to $\underline{V}_2^+[t'_n]$ and switching to its 'rcv' state ('↓'). At a later time, just before time t_n, electron #1 is moving at pre-velocity $\underline{V}_1^-[t_n]$, when it is in its 'rcv' ('↓') state. The 'arrival' of the impulse (there are no 'carriers' of the interaction in this theory) at this later time causes an immediate change in motion, increasing its post-velocity to $\underline{V}_1^+[t_n]$ and switching this electron to its 'snd' state ('↑'). The earlier analysis in [4] can now be improved somewhat by including each electron's send/receive state. Since interactions occur only through instantaneous impulses, it is sufficient to use only the local differences at each interaction time.

$$\Delta\underline{V}_1[t_n;\downarrow] = \underline{V}_1^+[t_n;\uparrow] - \underline{V}_1^-[t_n;\downarrow] \qquad \Delta\underline{V}_2[t'_n;\uparrow] = \underline{V}_2^+[t'_n;\downarrow] - \underline{V}_2^-[t'_n;\uparrow]$$

Since these are 'point' changes in velocity they can be represented by the point-difference operator, ◊ (Appendix IV), so:

$$\therefore\ \underline{V}_1^+[t_n;\uparrow] = (1+\Diamond)\,\underline{V}_1^-[t_n;\downarrow] \quad \text{and} \quad \underline{V}_2^+[t'_n;\downarrow] = (1+\Diamond)\,\underline{V}_2^-[t'_n;\uparrow]$$

Newton's Second Law (in its original form) equates the change in the momentum to the impulse received, so for electron #1

$$\Delta\underline{I}_n^- = \Delta\underline{P}_1[t_n] = (\underline{P}_1^+[t_n;\uparrow] - \underline{P}_1^-[t_n;\downarrow]) = m\,(\underline{V}_1^+[t_n;\uparrow] - \underline{V}_1^-[t_n;\downarrow]) = m\Diamond[\underline{V}_1^-[t_n;\downarrow]] = \Diamond[\underline{P}_1^-[t_n;\downarrow]]$$

This vector impulse is of magnitude ΔI_n and is parallel to the 'light-vector' $\underline{c}_n$ which tracks the difference in spatial locations from the emitter to the receiver at the two times (t_n and t'_n) defining the n^{th} consecutive interaction. The third paper proved (see [88] and section 4.1.8 here reproves) that all consecutive interactions between two electrons must remain parallel, so:

$$\text{All } \underline{c}_n = \underline{c} \quad \text{and} \quad \underline{x}_1[t_n] - \underline{x}_2[t'_n] = \underline{c}\,(t_n - t'_n) \qquad \therefore\ \underline{c}\,\Delta I_n^- = c\Diamond[\underline{P}^-[1: t_n; \underline{x}_n\downarrow]]$$

The extended version of Newton's Third Law for electron #2 equates the reaction $\Delta\underline{R}_n^-$ as equal in magnitude but opposite in direction to its 'emitted' impulse $\Delta\underline{I}_n^-$, so that: $\Delta\underline{R}_n^- = -\Delta\underline{I}_n^-$
This reaction is the actual impulse experienced by electron #2 so that Newton's II can also be used for this electron at t'_n .

$$\Delta\underline{R}_n^- = \Delta\underline{P}_2[t'_n] = (\underline{P}_2^+[t'_n;\downarrow] - \underline{P}_2^-[t'_n;\uparrow]) = m\,(\underline{V}_2^+[t'_n;\downarrow] - \underline{V}_2^-[t'_n;\uparrow]) = m\Diamond[\underline{V}_2^-[t'_n;\uparrow]] = \Diamond[\underline{P}_2^-[t'_n;\uparrow]]$$

This reaction is anti-parallel to the 'light-vector' $\underline{c}$ still with magnitude ΔI_n^-. $\therefore\ \underline{c}\,\Delta R_n^- = -c\Diamond[\underline{P}^-[2: t'_n; \underline{x}'_n\uparrow \text{ret}]]$

Advanced Repulsive Interaction

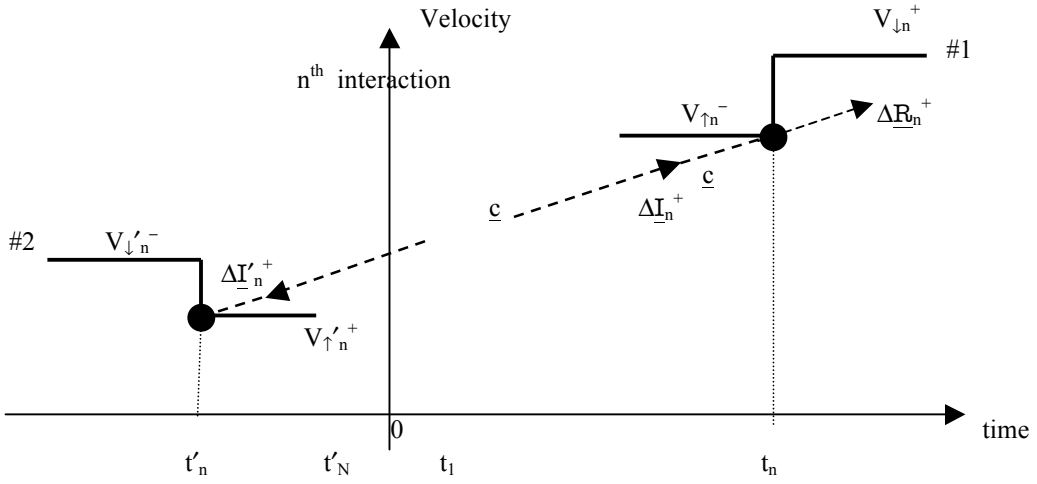


Fig. 2 Advanced-Repulsive Interaction ('adv-push photino').

Here, the later electron (#1) now initiates the interaction at time t_n , when it is in its 'snd' state ('↑') and its (pre-) velocity is $\underline{V}_1^-[t_n]$ by emitting an impulse $\Delta\underline{I}_n^+$ backwards in time. In the same instant, this electron suffers a reaction $\Delta\underline{R}_n^+$ in its own direction of motion, increasing its motion (post-velocity) immediately to $\underline{V}_1^+[t_n]$ and switching to its 'rcv' state ('↓'). At a later time, just before time t_n, electron #1 is moving at pre-velocity $\underline{V}_1^-[t_n]$, when it is in its 'rcv' ('↓') state. The 'arrival' of the impulse at electron #2 at this earlier time t'_n causes an immediate change in motion increasing its post-velocity to $\underline{V}_2^+[t'_n]$ and switching this electron to its 'snd' state ('↑'). A similar analysis as above can now be performed.

$$\Delta\underline{V}_1[t_n;\uparrow] = \underline{V}_1^+[t_n;\downarrow] - \underline{V}_1^-[t_n;\uparrow] \qquad \Delta\underline{V}_2[t'_n;\downarrow] = \underline{V}_2^+[t'_n;\uparrow] - \underline{V}_2^-[t'_n;\downarrow]$$

$$\therefore\ \underline{V}_1^+[t_n;\downarrow] = (1+\Diamond)\,\underline{V}_1^-[t_n;\uparrow] \quad \text{and} \quad \underline{V}_2^+[t'_n;\uparrow] = (1+\Diamond)\,\underline{V}_2^-[t'_n;\downarrow]$$

Newton's Second Law again equates the change in the momentum to the impulse received at t'_n, so for electron #2

$$\Delta\underline{I}'^+_n = \Delta\underline{P}_2[t'_n] = (\underline{P}_2^+[t'_n;\uparrow] - \underline{P}_2^-[t'_n;\downarrow]) = m(\underline{V}_2^+[t'_n;\uparrow] - \underline{V}_2^-[t'_n;\downarrow]) = m\Diamond[\underline{V}_2^-[t'_n;\downarrow]] = \Diamond[\underline{P}_2^-[t'_n;\downarrow]]$$

This impulse is still of magnitude $\Delta I'_n$ but anti-parallel to the 'light-vector' $\underline{c}_n$ $\therefore$ $\underline{c}\,\Delta I'^+_n = -c\Diamond[\underline{P}^-[1: t_n; \underline{x}_n\downarrow]]$

The extended version of Newton's Third Law for electron #1 equates the reaction $\Delta\underline{R}_n^+$ as equal in magnitude but opposite in direction to its 'emitted' impulse $\Delta\underline{I}'^+_n$, so that: $\Delta\underline{R}_n^+ = -\Delta\underline{I}'^+_n = \Delta\underline{I}_n^+$
This reaction is the actual impulse experienced by electron #1 so that Newton's II can also be used for this electron at t_n .

$$\Delta\underline{R}_n^+ = \Delta\underline{P}_1[t_n] = (\underline{P}_1^+[t_n;\downarrow] - \underline{P}_1^-[t_n;\uparrow]) = m(\underline{V}_1^+[t_n;\downarrow] - \underline{V}_1^-[t_n;\uparrow]) = m\Diamond[\underline{V}_1^-[t_n;\uparrow]] = \Diamond[\underline{P}_1^-[t_n;\uparrow]]$$

This advanced reaction is parallel to the 'light-vector' $\underline{c}$ still with magnitude ΔI_n. $\therefore$ $\underline{c}\,\Delta I_n^+ = c\Diamond[\underline{P}^-[1: t_n; \underline{x}_n\uparrow, \text{adv}]]$
Finally, summarizing these two situations (using $\sigma = -$ for 'retarded' and $\sigma = +$ for 'advanced') then:

$$\Delta\underline{I}_n^\sigma = -\lambda\Diamond[\underline{P}[1: t_n^-; \underline{x}_n, \uparrow, \sigma]] = \lambda\Diamond[\underline{P}[2: t'^-_n; \underline{x}'_n, \uparrow, -\sigma]]$$

But as: $\Delta\underline{I}_n^\sigma = -\sigma\,\Delta I_n\,\underline{c}\,/\,c$ then: $c\Diamond[\underline{P}[1: t_n^-; \underline{x}_n, \uparrow, \sigma]] = -c\Diamond[\underline{P}[2: t'^-_n; \underline{x}'_n, \uparrow, -\sigma]] = \underline{c}\,\Delta I_n$

This demonstrates that an 'advanced' impulse **from** the future $\Delta\underline{I}_n^+$ results in *exactly the same* kinematical changes in both electrons as a 'retarded' impulse $\Delta\underline{I}_n^-$ **into** the future. The emitting electron must always be in its 'send' state prior to the emission and the other electron must always be in its 'receive' state prior to the absorption when all times are viewed from the standard (human) direction of time. The temporal direction of the interaction is now **always** defined by the time of the receiving electron relative to the emitting electron. The present theory of EM will accordingly reflect this intrinsic time symmetry in all future analyses.

4.1.8 ATTRACTIVE IMPULSES

Retarded Attractive Interaction

The paradigmatic attractive interaction at a distance is Newton's model of gravitation, where all massive objects attract all other massive objects. Newton was fortunate that the astronomical distances he was familiar with are so small that the time for the interaction to cross these distances, even at light-speed, is very much smaller than the time for these objects to even complete one complete revolution around one another; for example, light takes about 8 minutes to travel from the Sun to the Earth while the Earth takes one year (or about 65,000 times longer). It was only the experimental investigation of 'light' and the theory that light was an EM interaction that scientists realized that all EM interactions required a finite time for the effect to span the finite distance separating the source and the target – this is referred to as 'asynchronous action-at-a-distance'. The small variation in the periodic orbit of Mercury is an example of the effect of retardation effects of gravity when viewed also as an asynchronous action-at-a-distance. The omni-present examples of EM attraction between positively and negatively electrically charged objects of all sizes, including electrons, means that asynchronous attractive interactions must also be analyzed.

The retarded attractive interaction is more difficult to comprehend at the human scale – a long stretchy elastic is one of the few examples available that require a (small) time delay; most attractive examples are more like pulling on a rigid rod. This category of asynchronous interaction is easier to understand analytically and this is first shown in the following diagram.

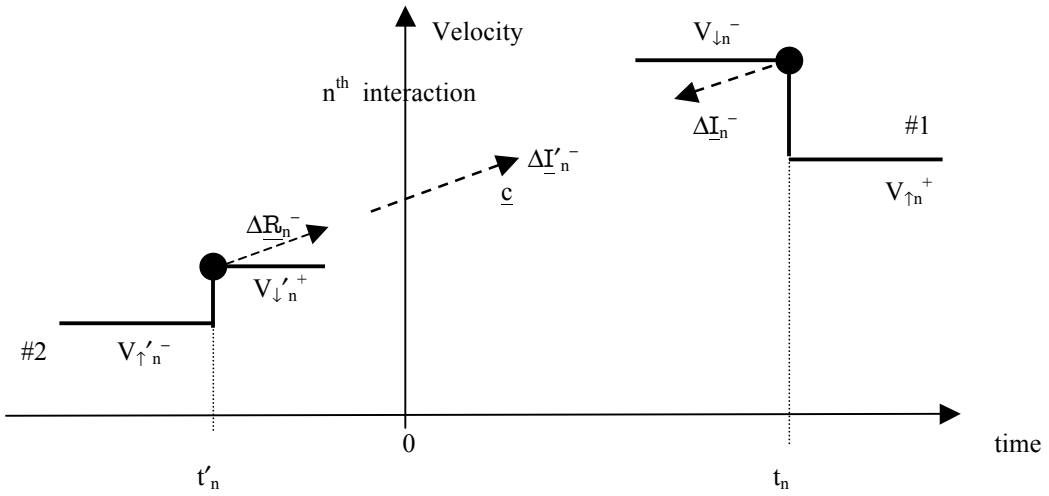


Fig. 3 Retarded-Attractive Interaction ('ret-pull photino').

Here, the earlier electron (#2) initiates the interaction at time t'_n, when it is in its 'snd' state ('↑') and its (pre-) velocity is $\underline{V}_2^{-}[t'_n]$ by emitting an attractive impulse $\Delta \underline{I}_n^{-}$ forward in time. In the same instant, this electron suffers a reaction $\Delta \underline{R}_n^{-}$ along its direction of motion, speeding up its motion (post-velocity) immediately to $\underline{V}_2^{+}[t'_n]$ and switching to its 'rcv' state ('↓'). At a later time, just before time t_n, electron #1 is moving at pre-velocity $\underline{V}_1^{-}[t_n]$, when it is in its 'rcv' ('↓') state. The 'arrival' of the impulse (remember, there are no 'carriers' of the interaction in this theory) at this later time causes an immediate change in motion, reducing its post-velocity to $\underline{V}_1^{+}[t_n]$ and switching this electron to its 'snd' state ('↑'). The earlier analysis in [4] can now be improved somewhat by including each electron's send/receive state. Since interactions occur only through instantaneous impulses, it is sufficient to use only the local differences at each interaction time.

$$\Delta \underline{V}_1[t_n ; \downarrow] = \underline{V}_1^{+}[t_n ; \uparrow] - \underline{V}_1^{-}[t_n ; \downarrow] \qquad \Delta \underline{V}_2[t'_n ; \uparrow] = \underline{V}_2^{+}[t'_n ; \downarrow] - \underline{V}_2^{-}[t'_n ; \uparrow]$$

Since these are 'point' changes in velocity they can be represented by the point-difference operator, ◊ (Appendix IV), so:

$$\therefore \quad \underline{V}_1^{+}[t_n ; \uparrow] = (1 + \Diamond)\, \underline{V}_1^{-}[t_n ; \downarrow] \quad \text{and} \quad \underline{V}_2^{+}[t'_n ; \downarrow] = (1 + \Diamond)\, \underline{V}_2^{-}[t'_n ; \uparrow]$$

Newton's Second Law (in its original form) equates the change in the momentum to the impulse received, so for electron #1

$$\Delta \underline{I}_n^{-} = \Delta \underline{P}_1[t_n] = (\underline{P}_1^{+}[t_n ; \uparrow] - \underline{P}_1^{-}[t_n ; \downarrow]) = m (\underline{V}_1^{+}[t_n ; \uparrow] - \underline{V}_1^{-}[t_n ; \downarrow]) = m \Diamond [\underline{V}_1^{-}[t_n ; \downarrow]] = \Diamond [\underline{P}_1^{-}[t_n ; \downarrow]]$$

This vector impulse is of magnitude ΔI_n and is parallel to the 'light-vector' $\underline{c}_n$ which tracks the difference in spatial locations between the two electrons at the two times (t_n and t'_n) defining the n^{th} consecutive interaction. The third paper proved (see [88] and section 4.1.8 here reproves) that all consecutive interactions between two electrons must remain parallel, so:

$$\text{All } \underline{c}_n = \underline{c} \quad \text{and} \quad \underline{x}_1[t_n] - \underline{x}_2[t_n] = \underline{c} (t_n - t'_n) \qquad \therefore \quad \underline{c}\, \Delta I_n^{-} = c \Diamond [\underline{P}^{-}[1: t_n ; \underline{x}_n \downarrow]]$$

The extended version of Newton's Third Law for electron #2 equates the reaction $\Delta \underline{R}_n^{-}$ as equal in magnitude but opposite in direction to its 'emitted' impulse $\Delta \underline{I}_n^{-}$, so that: $\Delta \underline{R}_n^{-} = - \Delta \underline{I}_n^{-}$

This reaction is the actual impulse experienced by electron #2 so that Newton's II can also be used for this electron at t'_n.

$$\Delta \underline{R}_n^{-} = \Delta \underline{P}_2[t'_n] = (\underline{P}_2^{+}[t'_n ; \downarrow] - \underline{P}_2^{-}[t'_n ; \uparrow]) = m (\underline{V}_2^{+}[t'_n ; \downarrow] - \underline{V}_2^{-}[t'_n ; \uparrow]) = m \Diamond [\underline{V}_2^{-}[t'_n ; \uparrow]] = \Diamond [\underline{P}_2^{-}[t'_n ; \uparrow]]$$

This reaction is anti-parallel to the 'light-vector' $\underline{c}$ still with magnitude ΔI_n^{-}. $\therefore \quad \underline{c}\, \Delta R_n^{-} = - c \Diamond [\underline{P}^{-}[2: t'_n ; \underline{x}'_n \uparrow \text{ret}]]$

Advanced Attractive Interaction

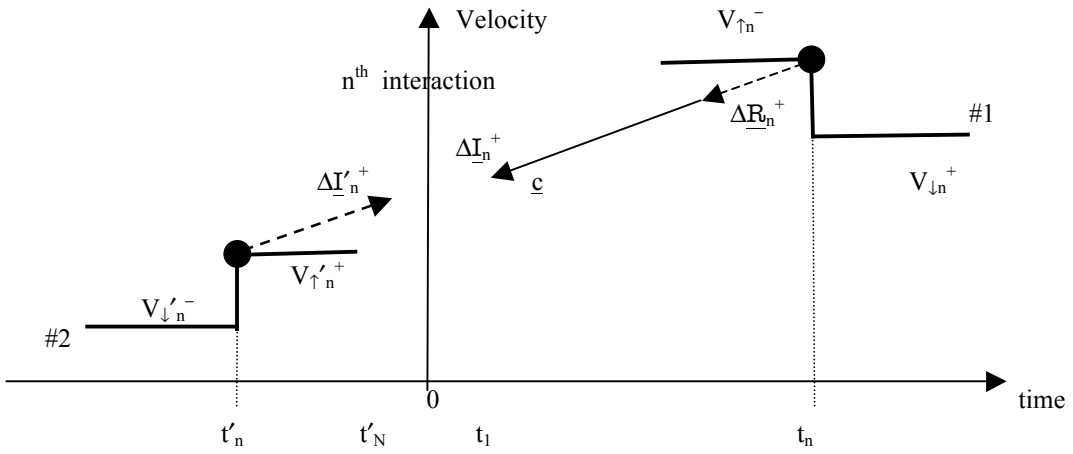


Fig. 4 Advanced-AttractiveInteraction ('adv-pull photino').

Here, the later electron (#1) now initiates the interaction at time t_n , when it is in its 'snd' state ('↑') and its (pre-) velocity is $\underline{V}_1^{-}[t_n]$ by emitting an impulse $\Delta \underline{I}'^{+}_n$ backwards in time. In the same instant, this electron suffers a reaction $\Delta \underline{R}_n^{+}$ against its own direction of motion, decreasing its motion (post-velocity) immediately to $\underline{V}_1^{+}[t_n]$ and switching to its 'rcv' state ('↓'). At an earlier time, just before time t'_n, electron #2 is moving at pre-velocity $\underline{V}_2^{-}[t_n]$, in its 'rcv' ('↓') state. The 'arrival' of the impulse at electron #2 at this earlier time t'_n causes an immediate change in motion increasing its post-velocity to $\underline{V}_2^{+}[t'_n]$ and switching this electron to its 'snd' state ('↑'). A similar analysis as above can now be performed.

$$\Delta \underline{V}_1[t_n;\uparrow] = \underline{V}_1^{+}[t_n;\downarrow] - \underline{V}_1^{-}[t_n;\uparrow] \qquad \Delta \underline{V}_2[t'_n;\downarrow] = \underline{V}_2^{+}[t'_n;\uparrow] - \underline{V}_2^{-}[t'_n;\downarrow]$$

$$\therefore\ \underline{V}_1^{+}[t_n;\downarrow] = (1+\lozenge)\,\underline{V}_1^{-}[t_n;\uparrow] \quad \text{and} \quad \underline{V}_2^{+}[t'_n;\uparrow] = (1+\lozenge)\,\underline{V}_2^{-}[t'_n;\downarrow]$$

Newton's Second Law again equates the change in the momentum to the impulse received at t'_n, so for electron #2

$$\Delta \underline{I}'^{+}_n = \Delta \underline{P}_2[t'_n] = (\underline{P}_2^{+}[t'_n;\uparrow] - \underline{P}_2^{-}[t'_n;\downarrow]) = m(\underline{V}_2^{+}[t'_n;\uparrow] - \underline{V}_2^{-}[t'_n;\downarrow]) = m\lozenge[\underline{V}_2^{-}[t'_n;\downarrow]] = \lozenge[\underline{P}_2^{-}[t'_n;\downarrow]]$$

This impulse is still of magnitude $\Delta I'_n$ but anti-parallel to the 'light-vector' $\underline{c}_n$ $\therefore$ $\underline{c}\ \Delta I'^{+}_n = -c\lozenge[\underline{P}^{-}[1: t_n; \underline{x}_n\downarrow]]$

The extended version of Newton's Third Law for electron #1 equates the reaction $\Delta \underline{R}_n^{+}$ as equal in magnitude but opposite in direction to its 'emitted' impulse $\Delta \underline{I}'^{+}_n$, so that: $\Delta \underline{R}_n^{+} = -\Delta \underline{I}'^{+}_n$
This reaction is the actual impulse experienced by electron #1 so that Newton's II can also be used for this electron at t_n .

$$\Delta \underline{R}_n^{+} = \Delta \underline{P}_1[t_n] = (\underline{P}_1^{+}[t_n;\downarrow] - \underline{P}_1^{-}[t_n;\uparrow]) = m(\underline{V}_1^{+}[t_n;\downarrow] - \underline{V}_1^{-}[t_n;\uparrow]) = m\lozenge[\underline{V}_1^{-}[t_n;\uparrow]] = \lozenge[\underline{P}_1^{-}[t_n;\uparrow]]$$

This advanced reaction is parallel to the 'light-vector' $\underline{c}$ still with magnitude ΔI_n. $\therefore$ $\underline{c}\,\Delta I_n = c\lozenge[\underline{P}^{-}[1: t_n; \underline{x}_n\uparrow, \text{adv}]]$
Finally, summarizing these two situations (using $\sigma = -$ for 'retarded' and $\sigma = +$ for 'advanced') then:

$$\Delta \underline{I}_n^{\sigma} = -\lambda\lozenge[\underline{P}[1: t_n^{-}; \underline{x}_n, \uparrow, \sigma]] = \lambda\lozenge[\underline{P}[2: t'^{-}_n; \underline{x}'_n, \uparrow, -\sigma]]$$

But as: $\Delta \underline{I}_n^{\sigma} = -\sigma\,\Delta I_n\,\underline{c}/c$ then: $c\lozenge[\underline{P}[1: t_n^{-}; \underline{x}_n, \uparrow, \sigma]] = -c\lozenge[\underline{P}[2: t'^{-}_n; \underline{x}'_n, \uparrow, -\sigma]] = \underline{c}\,\Delta I_n$

This demonstrates that an 'advanced' attractive impulse **from** the future $\Delta \underline{I}_n^{+}$ results in exactly the same kinematical changes in both electrons as a 'retarded' attractive impulse $\Delta \underline{I}_n^{-}$ **into** the future.

The emitting electron must always be in its 'send' state prior to the emission and the other electron must always be in its 'receive' state prior to the absorption when all times are viewed from the standard (human) direction of time. The temporal direction of the interaction is now **always** defined by the time of the receiving electron relative to the emitting electron. In both cases (repulsion or attraction), the reaction at the emitting electron always manifests its impact on the emitter as if it had reacted to a comparable impulse from the receiving electron, whether the emission was forwards in time (retarded) or backwards through time (advanced).

Again, whether involved in a repulsive or attractive interaction, an advanced impulse from the future when received by the receiver in the past manifests the same kinematical changes in its momentum as the reaction to the emission of a retarded impulse at the same electron when it participates in the interaction in the role of emitter. Similarly, an advanced emitting impulse produces the same change in motion on itself as if it had received a comparable retarded from an earlier emitting electron.

Electron States

This is a useful point to introduce the concept of an electron's ***state***: this is the set of parameters that completely define both how an electron is moving through space at any time and its future potential to interact at its next, possible interaction time. Obviously, this state must include the electron's position $\underline{x}$ and velocity $\underline{v}$ (or momentum $\underline{p}$) at every instant of time t but it must also indicate whether it is in a 'send' or 'receive' state and also whether the next possible interaction with any other electron will be forwards in time ('retarded') or backwards through time ('advanced'). This state can be represented by a special kind of parameter list, delimited by a vertical bar '|' and a greater-than symbol '>' (a notation first introduced by Dirac to designate a particle's quantum state) and the next interaction possibilities separated from the kinematical variables by a special separator, such as '::'; for example: $| \alpha : t ; \underline{x} , \underline{v} :: \uparrow , \text{ret} >$. In general, the two possible interaction exchange states can be distinguished by the binary variable λ, where $\lambda = -$ or '$\downarrow$' for 'retarded' and $\lambda = +$ or '$\uparrow$' for 'advanced'; the two temporal exchange directions are indicated by the binary symbol σ, where $\sigma = -$ or 'ret' for 'retarded' and $\sigma = +$ or 'adv' for 'advanced'. The unique state of any electron, labeled 'α', is then given by: $| \alpha : t ; \underline{x} , \underline{v} :: \lambda , \sigma >$. It should be noted that this notation does NOT imply that this is a mathematical representation of Hilbert vector spaces as in Dirac.

4.1.9 ALTERNATING INTERACTIONS

Alternate Advanced then Retarded Interactions

The asynchronous EM interaction is *logically* divided into a two-step (or two-event) process, which begins with one electron 'sending' an impulse $\Delta\underline{\mathbf{I}}$ (an event denoted symbolically by the letter $\mathcal{S}$) that is 'received' (an event denoted by the letter $\mathcal{R}$) at a different time by its interactive, electronic partner. This interaction can occur either forward through time (retarded $\Delta\underline{\mathbf{I}}^{-}$) or backwards through time (advanced $\Delta\underline{\mathbf{I}}^{+}$). Thus, when an electron 'sends' its impulse forward through time (which is seen as 'normal' to human observations and memory) it is defined as a 'retarded send' and denoted symbolically by $\mathcal{S}^{-}$ and the 'retarded receiving' event denoted by $\mathcal{R}^{-}$. The complementary 'advanced' interaction occurs when an electron 'sends' its impulse backwards through time (denoted by $\mathcal{S}^{+}$) that was 'received' earlier by the other electron in an advanced receive $\mathcal{R}^{+}$. These four types of event can be represented graphically on an *Interaction Diagram* as either a black node ('send') or a white node (receive') with the retarded nodes represented by circles and the advanced nodes represented by diamonds. The one-way impulse itself is represented by a directed dashed line, which leaves the 'sending' node, and is directed towards the 'receiving' node. Thus, in the previous diagram for colliding electrons during the complete inbound phase, all the 'sending' nodes are losing momentum (and energy). All these events are represented as 'retarded' nodes for **both** electrons ($\mathcal{S}^{-}$ or 'black pearls'), linked uniquely (in the normal direction of time) to all the 'retarded receiving' nodes ($\mathcal{R}^{-}$ or 'white pearls') on the outbound phase when the receiving electrons increase their momentum and energy. During the outbound phase of both electrons, all the 'sending' nodes are losing momentum but backwards in time, so they are actually gaining momentum in the normal, forward time direction. All of these advanced events can be represented graphically as 'advanced sending' nodes for both electrons ($\mathcal{S}^{+}$ or 'black diamonds'), whose impulses were earlier received from the future at 'advanced receiving' nodes ($\mathcal{R}^{+}$ or 'white diamonds') on the inbound phase. Since these events correspond to an electron receiving momentum from the future, they appear (to us) as if they are losing momentum when viewed from the forward flowing direction of time.

Whenever a node can be either a 'pearl' or a 'diamond', it will be represented by a 'box'.

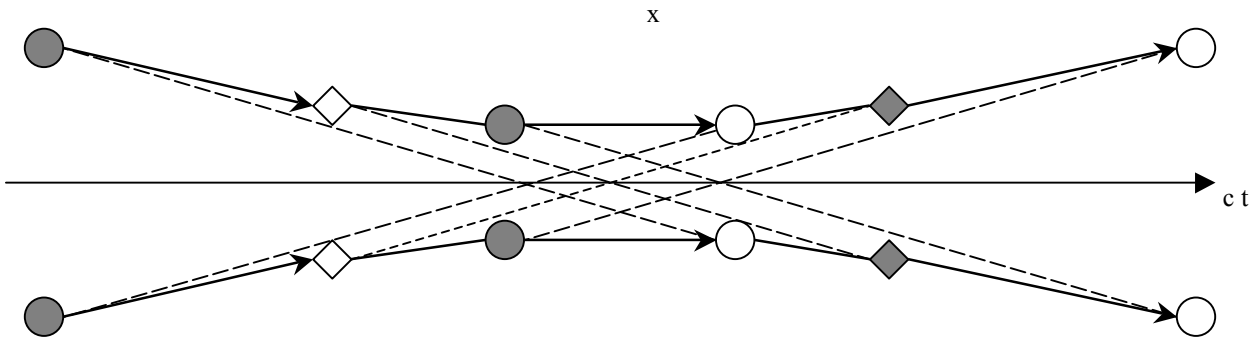


Fig. 5 Complete Two-Electron Interaction Diagram

The minimum time between consecutive interactions implies that 'diamonds' alternate with 'pearls' while 'black diamonds' only occurr in the outbound phase and 'white diamonds' occurring only in the inbound phase. Similarly, only 'black pearls' occur in the inbound phase and 'white diamonds' in the outbound phase. This rule is an example of the holistic, least-time or least-action principle along the complete trajectory of both electrons. The following mappings are useful.

Retarded Interaction: $\Delta\underline{\mathbf{I}}^{-}(A: t_1; B: t_2) \cong \mathcal{S}^{-}(A: t_1)\ \&\ \mathcal{R}^{-}(B: t_2)$ with $t_2 > t_1$

Advanced Interaction: $\Delta\underline{\mathbf{I}}^{+}(A: t_1; B: t_2) \cong \mathcal{S}^{+}(A: t_1)\ \&\ \mathcal{R}^{+}(B: t_2)$ with $t_2 < t_1$

'Sending' events: $\mathcal{S}^{-}(A: t) \cong$ BP (black pearl) $\quad \mathcal{S}^{+}(A: t) \cong$ BD (black diamond)

'Receiving' events: $\mathcal{R}^{-}(A: t) \cong$ WP (white pearl) $\quad \mathcal{R}^{+}(A: t) \cong$ WD (white diamond)

Double Interaction 'Round-Trip'

The 'light-cone' condition [89] is interpreted in the present theory as the necessary space and time co-ordination so that if the first electron 'sends' an impulse into the future ($\Delta\underline{\mathbf{I}}^{-}$) that is 'received' by the second electron then this other electron next 'sends' a return impulse back into the past ($\Delta\underline{\mathbf{I}}^{+}$) that is 'received' by the first electron. In other words, the 'round-trip' is the micro-time-average of two consecutive, but not simultaneous, interactions between a pair of electrons. These advanced effects are totally feasible in the present theory because each interaction is a point-to-point, saturated, pair-wise agreement, not a spherical, fractional 'broadcast' out to the rest of the universe, as in Maxwell's theory or even in Wheeler and Feynman's theory [77], both of which require exquisite multi-point co-ordination. This is illustrated in the next diagram:

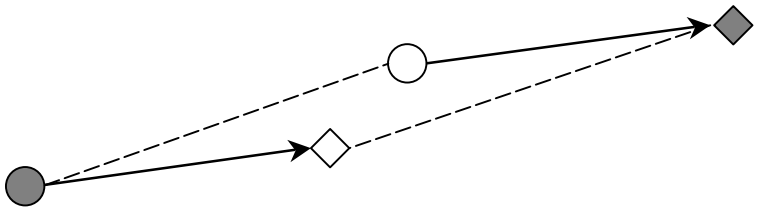


Fig. 6 'Round-Trip' Interactions

Trajectory Diagrams

In the spirit of visualization motivating the present programme, electron trajectories and their interactions can be given various visual representations. The most intuitive form is the 'Space-Time Diagram' where time is represented by the left to right horizontal axis and one of the 3D spatial dimensions (say x) is represented by the vertical axis. Electron trajectories are represented by solid lines; these are straight between interactions that are represented by dashed lines, directed from a 'send' node to a 'receiving' node using the black/white coloring convention and circles/squares ('pearls and diamonds') for denoting retarded or advanced interactions. These are illustrated below.

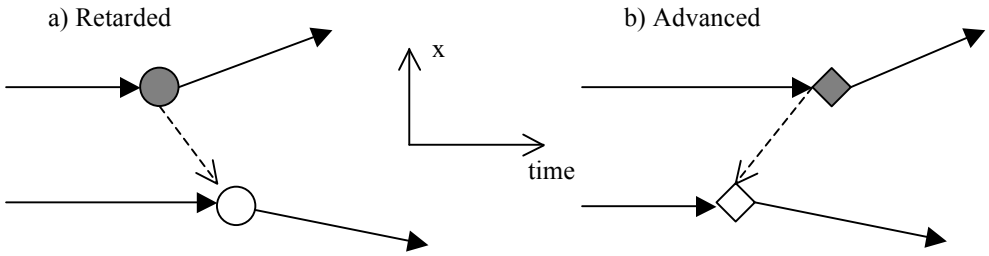


Fig. 7 Space-Time Diagrams

The other type of trajectory diagram is the 'Momentum-Time Diagram'. This is mainly used to illustrate the momentum exchanges occurring to a single electron. Here the interaction is artificially separated into two halves that show the transferred momentum during the exchange and the reaction at the emitter.

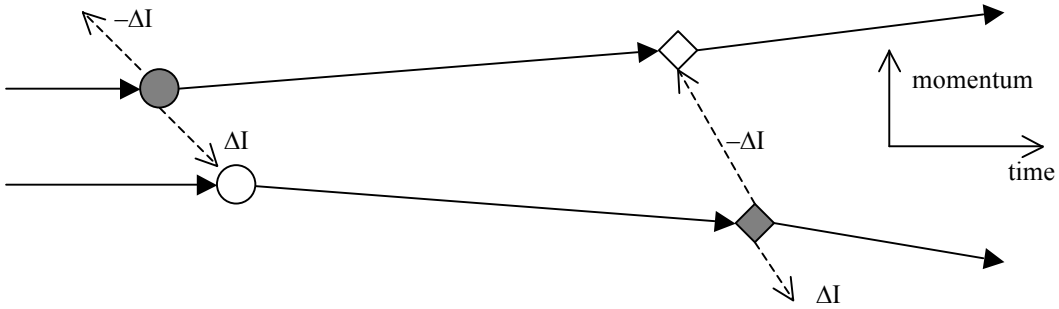


Fig. 8 Momentum-Time Diagrams

4.2 DISCRETE NATURE & ACTIVITY

4.2.1 DISCRETENESS

Electron Parameters are Finite

Contrary to conventional pedagogy, Classical Mechanics (in its calculus formulation) is not synonymous with Newton's original physics. Newton based his model of nature on the finite impulse, $\Delta\underline{I}$ as the only cause of change in a particle's momentum $\Delta\underline{P}$, so that the correct formulation of Newton's Second Law of Motion [90] is: $\Delta\underline{I} \rightarrow \Delta\underline{P}$. It was LaGrange, who merged Newton's infinitesimal calculus with Newton's physics to create what is now known as 'Classical Mechanics'. This is the implicit introduction of the Continuum Hypothesis into natural philosophy to assist in simplifying mathematical calculations whereby the time separation between impulses becomes both equal and vanishingly small ($\Delta t \rightarrow 0$). These are extra metaphysical assumptions that have always seemed reasonable at the macroscopic scale of human activity. However, experiments have discovered that the micro-world is discrete, as exemplified by the only observable, fundamental particle: the electron with its own set of discrete parameters (or qualities), charge *e*, mass *m* and interaction *h*. The dimensionless nature of Sommerfeld's Fine Structure Constant ($\alpha = e^2 / h_D c$) shows that 'light-speed' *c* is always fixed and finite. Since electricity is now known to consist of electrons then this most fundamental feature of nature is finite and non-continuous, so the 'electric fluid' model of classical EM does not reflect reality at its microscopic levels. This demonstrates that the all the 'foundational' theories of physics (LaGrange-Euler, Hamilton, Maxwell) are simply mathematical models of the continuum and do **not** provide the appropriate foundation for any physical theory of nature, which is inherently discrete or quantized.

Primacy of Processes

Classically, physics has modeled the world in terms of independent objects (or *entities*), characterized by measurable sets of universal, numerical attributes that could be enumerated by some all-seeing, no-touching 'observer'. The present theory is focused only on *relationships* and *interactions* between pairs of electrons; accordingly, interactive *processes* are considered as the primary basis of the natural world, not the motion of individual, 'target' particles or worse, empty points in space.

Newton's concept of a '**force**', or Maxwell's concept of a '**field**', were earlier attempts in the evolution of theoretical physics to 'delink' the interactions between particles. The associated linear mathematics is characteristic of 'first-order' activity. In this simple view, fields are 'single-point, single-time' models that are statistically useful in the continuum limit (or $e^2 \rightarrow 0$). A brief analysis of these earlier views was covered in section 5.4.3 of paper IV [4] and section 2 of paper II [2].

4.2.2 DISCRETE TIME

The World is Discontinuous

Physicists should not have been very surprised to find that action is quantized rather than continuous, since all real physical continuums have turned out to be discontinuous at the smallest levels of reality. This was particularly to be expected after 'solid' matter and 'continuous' liquids were realized to consist of myriads of tiny distinct exemplars (atoms and molecules) of real matter. The fact that this discreteness makes continuous mathematics only suitable as an approximation is too bad. The central physical hypothesis in this theory is that all electrons "pulsate with possibilities". This means that, unlike all traditional physics models that have followed Newton and adopted the Continuum Hypothesis (so they can continue to use the differential calculus), interactions between electrons do not occur all the time but only sometimes when the conditions are appropriate. In all cases, each electron cycles around two possibilities: each involving the exchange of one EM impulse, either receiving or sending, which changes the velocity of both electrons. This cyclic behavior is assumed to occur over a universal time duration, here referred to as one *chronon*. De Broglie proposed such a universal electron clock in 1923 [91].

Heisenberg's Universal Constant

The present theory acknowledges that Heisenberg was investigating similar discrete ideas in 1938 [92] when he was searching for a new universal constant to explain the mysterious phenomena of quantum mechanics. In contrast to these attempts, this theory does not invoke a new universal measure of space but rather one of time, characterizing how often **two** electrons may interact. This universal temporal interval, the chronon τ can be used to define a spatial measure (the luxon) Λ_0 that defines the microscopic width of the electron's transverse motion as it moves longitudinally through space: $\Lambda_0 = c\,\tau$.

Estimating the (Atomic) Chronon

The magnitude of the universal chronon τ may perhaps be estimated by assuming that Coulomb's Law is appropriate down to the smallest separation between the closest orbital electron and the hydrogen nucleus i.e. the Bohr radius $R_B = h^2/m(2\pi e)^2$. Let the magnitude of the electric attraction, at this separation, be represented by F_0 and assume that only a single equivalent impulse ΔI_B is needed to reverse the direction of the electron in this trajectory (with speed $V_B = \alpha\, c$) in only one chronon τ_A.

Newton's II: $\Delta I_B = \Delta P_0 \;\therefore\; F_0\,\tau_A = 2\,m\,V_B$ Max acceleration $\mathfrak{A}_0 = 2V_B / \tau_A = F_0 / m = e^2 / m\,R_B^{\,2} = m\,(2\pi)^4 e^6 / h^4$

The Fine Structure Constant, $\alpha = 2\pi\, e^2 / h\, c$ so that: $\tau_A = h^3 / m\,(2\pi)^3\, e^4 = h / 2\pi\,\alpha^2\, mc^2 = h / 2\pi\, m\, V_B^{\,2} = R_B / V_B$

This provides an estimate of the (atomic) chronon, $\tau_A \cong 2.4 \times 10^{-17}$ seconds.

Estimating the (Nuclear) Chronon

Around the beginning of the 20th Century, there were several attempts to produce physical models of the newly discovered electron [93]. These simple models usually posited a very small sphere filled with 'electrical fluid'. Many of these attempts derived the size of this electron, which is now referred to as the *classical electron radius* and denoted here by the symbol Λ_0. In this approach, the chronon is assumed to equal the time 'light' would take to cross the classical electron radius. The usual formula for this radius, assuming that all the electron's mass is produced by its internal electrostatic energy, is taken to be:

$$c\,\tau = \Lambda_0 = e^2 / mc^2 = \alpha\, h / 2\pi\, m\, c = \alpha\, \lambda_c / 2\pi = \alpha^2\, R_B \qquad \text{where } R_B \cong 5.3 \times 10^{-9} \text{ cm}$$

This provides an estimate of the (nuclear) chronon, $\tau = \alpha^3\, \tau_A = 9.3 \times 10^{-24}$ seconds.

In this formulation reference was made to the Compton wave-length λ_c, which is defined as: $\lambda_c = h / m\, c$ appearing in Compton scattering. In a later paper, this will be interpreted as the interaction between low-speed and high-speed electrons.

Runaway takes one Chronon

The response of the classical electron to an instantaneous, external force (arriving at time t = 0), while incorporating its own 'radiation reaction', is described in most major post-graduate EM texts, such as [94]. The analysis describes the 'runaway' solutions where the time-dependence of the charged particle's motion includes a term like exp(3 t / 2 T_0). Here the fixed parameter is defined as: $T_0 = \Lambda_0 / c = e^2 / mc^3 = \tau \cong 10^{-23}$ seconds; this is found to be equal to the nuclear chronon.

Smallest Time Intervals

The very shortest time duration that can be generated by today's technology is about 10^{-16} seconds using linear pulses. The shortest, calculated time that can be **inferred** from experiments is the mean lifetime of the 'top quark', estimated to be about $0.4 * 10^{-24}$ seconds, which is again the order of the nuclear chronon.

4.2.3 DISCRETE CHANGE

The central hypothesis of the present theory is that electrons do **not** interact continuously (the standard implicit *Continuum Hypothesis*) but that they all interact only at discrete time intervals. This fundamental aspect of nature was introduced in paper IV (see section 6.2). This universal constant is referred to as the *chronon* and designated here by the symbol τ. The universal 'ticking' of all the electrons across the universe now defines the possible rate of change in nature: the chronon is the universal invariant measure of time. One of the major results from this previous paper (section 6.4.2) was that "Planck's Proposal" for redefining the relativistic mass of a high-speed particle **cannot** be approximated by a series of **equal** impulses occurring at **every** chronon (a sequence referred to here as '*micro-time*'). The time intervals between the interactions of the high-speed electron (say Δt_n) moving with speed u_n comparable to c are much longer than those occurring between its much slower partner $\Delta t_n'$, which is moving at a significantly slower speed u_n'. The actual formula follows the space-time integrity condition (section 4.1.8) with the two electron speeds (in their center of momentum reference frame) complying with the 'complementary condition': $u_n + u_n' = c$. This time interval between successive interactions for any electron is referred to as the '*interaction-duration*'. There is also a third time frame appearing in this theory (see section 5.2.2) that corresponds to one complete cycle of 4τ for every electron as it moves through time, this is referred to here as '*cyclic time*'.

As can be seen, the role of time is much more complex in this theory than the traditional 'smooth flowing stream' that has provided the metaphor for most of the earlier discussions of this central concept. When discussing changes in the relative position of any electrons in this theory it is always important to distinguish which time frame is appropriate, particularly when averages are being considered. Much of traditional physics has focused on periodic systems, where after a certain time T (the 'period') the configuration of the constituent parts of the system return to the same values as earlier. This idea leads to computing periodic averages, so for any variable that is a function of time, say A[t] it is always possible to define its periodic average < A[t] >; in many cases, when this average is computed it is found to be independent of time.

$$\text{Definition:} \quad \textbf{Continuous Periodic Average} \quad < A[t] > \equiv 1 / T \int_{t}^{t+T} dt' \, A[t']$$

In the present theory, there are no differentials and, therefore no integrals, all sums remain finite and discrete. But it is now important to distinguish three distinct time-averages, distinguished by their characteristic subscripts: τ , 0 and I.

$$\text{Definition:} \quad \textbf{Micro-Average} \quad < A[\, t\,] >_{\tau} \equiv 1 / 2 \, \{ A[\, t - \tau /2] + A[\, t + \tau /2] \, \}$$

This average is often used here to define the average velocity of an electron at a specific interaction time, say $t_\mu = \mu \tau$.

$$\text{Definition:} \quad \textbf{Cyclic-Average} \quad < A[\, t_\mu\,] >_{0} \equiv 1 / 4 \, \{ A[t_\mu - 3\tau /2] + A[t_\mu - \tau /2] + A[t_\mu + \tau /2] + A[t_\mu + 3\tau /2] \}$$

Cyclic averages are important in analyzing 'free' electrons, when most of these cyclic averages are independent of time. A very useful, physical average is the **interaction-average** that is computed from just before one interaction time, $t_1 = t_n - \delta t$ to just before the very next interaction at $t_2 = t_{n+1} - \delta t$. Since every interaction time t_n is an integral number of chronons, say μ_n then $t_n = \mu_n \tau$, so the n^{th} interaction interval $\Delta t_n = \eta_n \tau$ where $\eta_n = \mu_{n+1} - \mu_n = \Delta \mu_n$.

$$\text{Definition:} \quad \textbf{Interaction-Average} \quad < A[\, t_n\,] >_{I} \equiv 1 / \eta_n \sum_{t1}^{t2} A\,[t_\mu]$$

Corresponding to these averages are three associated difference operators, defined as follows.

Definition: **Micro-Difference** $\Delta_\tau A[\,t\,] \equiv A[\,t+\tau/2\,] - A[\,t-\tau/2\,] \equiv \boldsymbol{\Delta}_\tau[\,A[\,t\,]\,]$

Definition: **Cyclic-Difference** $\Delta_0 A[\,t_\mu\,] \equiv A[\,t_\mu+4\tau\,] - A[\,t_\mu\,] \equiv \boldsymbol{\Delta}_0[\,A[\,t_\mu\,]\,]$

Definition: **Interaction-Difference** $\Delta_I A[\,t_n\,] \equiv A[\,t_{n+1}-\tau/2] - A[\,t_n-\tau/2] \equiv \boldsymbol{\Delta}_I[\,A[\,t_n\,]\,]$

All of these discrete averages and differences will be used later in the analysis of electron motion and their interactions.

4.2.4 DISCRETE SPACE & TIME INTEGRITY

Section 6 of the third paper in this series [3] analyzed the continuous interaction between two remotely interacting point electrons. The conventional vector analysis used, demonstrated that a continuous interaction, constrained to apply only on their mutual 'light-cones', was incompatible with the fact that the motion of these two particles was limited by inertia. This section will now revisit this situation but viewed from the perspective of two-point DNVs, showing that only instantaneous impulses 'on the light-cone' are compatible with even discrete changes in the motion of inertial particles, such as electrons. Furthermore, this earlier paper only focused on the inter-nodal (or 'longitudinal') velocity of each electron as it participated in successive interactions with its partner. It is now time to remove this restriction and, indeed, extend the analysis of the motion of the two electrons to all three dimensions of space from the specialized one-dimensional analysis performed before in the Symmetric Inertial Reference Frame (SIRF), first described in the third paper [3] in section 6.

The 'Light-Cone Condition' was first introduced in section 7.4.5 of the first paper [1]. This condition is the space and time constraint that defines **when** any two electrons **may** interact with each other – this replaces the usual metaphysical statement, which assumes that 'light' is an entity, that "electromagnetic influences always travel in *vacuo* at light-speed". Thus, for two electrons, labeled '1' and '2' interacting for the n^{th} consecutive times at t_n and t'_n, the condition becomes:

Light-Cone Condition: $(\,\underline{x}[1\!:n] - \underline{x}[2\!:n]\,) \bullet (\,\underline{x}[1\!:n] - \underline{x}[2\!:n]\,) = c^2\,(\,t_n - t'_n\,)^2$

Writing: $\underline{x}_n = \underline{x}[1\!:n]$ & $\underline{x}'_n = \underline{x}[2\!:n]$ this is equivalent to: $(\,\underline{x}_n - \underline{x}'_n\,)^2 = c^2\,(\,t_n - t'_n\,)^2$.

Defining the two 'difference' variables: $T_n \equiv t_n - t'_n$ and $\underline{S}_n \equiv \underline{x}[1\!:n] - \underline{x}[2\!:n]$ $\therefore\ \underline{S}_n \bullet \underline{S}_n = c^2\,T_n^2$ $\therefore\ S_n = c\,T_n$

This is restated in the present theory as the EM interaction between two electrons **can** occur in the present theory **only when** the 'norm' of their Space-Time Separation DNV $\mathbf{S}_n$, defined between any pair of interaction points, is zero. It will here be assumed that the n^{th} interaction occurs in the direction $\hat{\underline{e}}_n$ defining the 'light-vector' $\underline{c}_n = c\,\hat{\underline{e}}_n$. So, the condition becomes:

$$\underline{S}_n = \underline{c}_n\,T_n \quad \text{since:} \quad \underline{S}_n \bullet \underline{S}_n = (\,\underline{c}_n \bullet \underline{c}_n\,)\,T_n^2 = (\hat{\underline{e}}_n \bullet \hat{\underline{e}}_n)\,c^2\,T_n^2 = c^2\,T_n^2 \qquad \therefore\ \underline{S}_n = S_n\,\hat{\underline{e}}_n$$

Thus, the interaction impulse is defined along the 'line-of-centers' at the (different) **times** of each interaction.

$\mathbf{S}_n \equiv i\,c\,T_n\,\mathbf{I_0} + \underline{S}_n \bullet \underline{\mathbf{I}}$ The square (or 'norm') of this DNV is: $\mathbf{S}_n^*\,\mathbf{S}_n = (c^2\,T_n^2 - \underline{S}_n \bullet \underline{S}_n)\,\mathbf{I_0} = 0$

$$\mathbf{S}_n = (i\,c\,\mathbf{I_0} + \underline{c}_n \bullet \underline{\mathbf{I}})\,T_n = (i\,\mathbf{I_0} + \hat{\underline{e}}_n \bullet \underline{\mathbf{I}})\,c\,T_n \equiv \mathbf{C}_n\,T_n \qquad \therefore\ \mathbf{C}_n^*\,\mathbf{C}_n = 0$$

$$\text{Now } \boldsymbol{\Delta}\mathbf{C}_n = \boldsymbol{\Delta}[\mathbf{C}_n] = \mathbf{C}_{n+1} - \mathbf{C}_n = \underline{\mathbf{I}} \bullet \Delta\underline{c}_n = c\,\underline{\mathbf{I}} \bullet \Delta\hat{\underline{e}}_n = \boldsymbol{\Delta}[\mathbf{C}_n^*] = \boldsymbol{\Delta}\mathbf{C}_n^*$$

$$\text{While } \boldsymbol{\Delta}[\mathbf{C}_n^*\,\mathbf{C}_n] = \mathbf{C}_{n+1}^*\,\mathbf{C}_{n+1} - \mathbf{C}_n^*\,\mathbf{C}_n = \mathbf{C}_n^*\,\boldsymbol{\Delta}\mathbf{C}_n + \boldsymbol{\Delta}\mathbf{C}_n\,\mathbf{C}_n + \boldsymbol{\Delta}\mathbf{C}_n\,\boldsymbol{\Delta}\mathbf{C}_n$$

$$\therefore\ \underline{\boldsymbol{\Delta}}[\mathbf{C}_n^*\,\mathbf{C}_n] = (\underline{\mathbf{I}} \bullet \underline{c}_n)\,(\underline{\mathbf{I}} \bullet \Delta\underline{c}_n) + (\underline{\mathbf{I}} \bullet \Delta\underline{c}_n)\,(\underline{\mathbf{I}} \bullet \underline{c}_n) + (\underline{\mathbf{I}} \bullet \Delta\underline{c}_n)\,(\underline{\mathbf{I}} \bullet \Delta\underline{c}_n) = -2\,\mathbf{I_0} < \underline{c}_n > \bullet\,\Delta\underline{c}_n = 0$$

$$\therefore\ \Delta\underline{c}_n = 0 \quad \therefore\ \boldsymbol{\Delta}\mathbf{C}_n = 0 \quad \therefore\ \underline{c}_{n+1} = \underline{c}_n = \underline{c} \quad \therefore\ \mathbf{C}_n = \mathbf{C} \quad \therefore\ \mathbf{S}_n = \mathbf{C}\,T_n$$

This important result shows that **all** the **successive** interactions between two electrons always occurs in one **single** direction, defined by the difference in the spatial locations of the two electrons at the time of their **first** interaction, in other words:

$\underline{S} = \underline{x}[1:1] - \underline{x}[2:1]$ As this is independent of n, this direction remains **invariant** throughout these interactions. The next consecutive interaction after the n^{th} interaction is the $(n+1)^{th}$ at times t_{n+1} and t'_{n+1} , where $t_{n+1} > t_n$ and $t'_{n+1} > t'_n$.

This allows the definition of the next difference variables in terms of the extended difference operator $\underline{\Delta}$ (see Appendix A4).

Definition **Extended-Difference**: $\underline{\Delta}F_n \equiv F[T_{n+1} - \delta t] - F[T_n + \delta t] \equiv \underline{\boldsymbol{\Delta}}[\, F[T_n]\,]$

As was shown before [95] both the position and time variables are always analytic, that is they change continuously, so the standard difference Δ may simply be used for them; it is only electron velocities (and other related quantities) that change discontinuously at an interaction instant. Here, both Δt_n and $\Delta t'_n$ are non-zero and positive along with the one electron spatial differences $\Delta \underline{x}_n$ and $\Delta \underline{x}'_n$; it is **not** assumed *a priori* that either: $\Delta t_n = \Delta t'_n$ or $\Delta \underline{x}_n = \Delta \underline{x}'_n$.

The 'post-interaction' longitudinal velocities $\underline{u}_n$ and $\underline{u}'_n$ can be defined for each electron as '<u>joining</u>' consecutive interaction locations: $\Delta \underline{x}_n \equiv \Delta t_n \underline{u}_n$ and $\Delta \underline{x}'_n \equiv \Delta t'_n \underline{u}'_n$. It is not required that the electrons actually follow these spatial differences.

In terms of the individual electron DNVs: $\mathbf{X}_n \equiv i\,c\,t_n \mathbf{I_0} + \underline{x}_n \bullet \underline{\mathbf{I}}$ & $\boldsymbol{\Delta}\mathbf{X}_n = i\,c\,\Delta t_n \mathbf{I_0} + \Delta \underline{x}_n \bullet \underline{\mathbf{I}} = \Delta t_n\,(i\,c\,\mathbf{I_0} + \underline{u}_n \bullet \underline{\mathbf{I}})$

As $\mathbf{S}_n = \mathbf{X}_n - \mathbf{X}'_n \;\therefore\; \boldsymbol{\Delta}\mathbf{S}_n = \boldsymbol{\Delta}\mathbf{X}_n - \boldsymbol{\Delta}\mathbf{X}'_n = i\,c\,(\Delta t_n - \Delta t'_n)\,\mathbf{I_0} + (\Delta t_n \underline{u}_n - \Delta t'_n \underline{u}'_n) \bullet \underline{\mathbf{I}} = i\,c\,\Delta T_n \mathbf{I_0} + \Delta \underline{S}_n \bullet \underline{\mathbf{I}}$

From above: $\boldsymbol{\Delta}\mathbf{S}_n = \mathbf{C}\,\Delta T_n = (i\,c\,\mathbf{I_0} + \underline{c} \bullet \underline{\mathbf{I}})\,\Delta T_n \;\therefore\; \underline{c}\,\Delta T_n = \Delta \underline{S}_n$ or $\underline{c}\,(\Delta t_n - \Delta t'_n) = (\Delta t_n \underline{u}_n - \Delta t'_n \underline{u}'_n)$

This results in the universal '***Space-Time Integrity***' (STI) condition: $(\underline{c} - \underline{u}_n)\,\Delta t_n = (\underline{c} - \underline{u}'_n)\,\Delta t'_n$

This important equation (or its direct ancestor: $\mathbf{S}_n = \mathbf{C}\,T_n$) will play a key role in defining the possible motions allowed when specific interactions between electrons with both similar and dissimilar charges are considered. These results can be readily obtained using the Natural Vector formalism, which emphasizes the space-time integrity constraints in 4D, just like regular (timeless) 3D spatial vectors. This viewpoint is brought out by recognizing that there are two NV 'paths' from the earliest interaction node (say at $\underline{x}'_n$) to the latest node in the next interaction (say at $\underline{x}_{n+1}$). When these two paths are traced through both space and time using the DNVs for the two electrons, integrity demands that the same final point be reached.

Path 1: $\mathbf{X}_{n+1} = \mathbf{X}_{n+1} \pm \mathbf{X}'_n \pm \mathbf{X}_n = \mathbf{X}'_n + (\mathbf{X}_n - \mathbf{X}'_n) + (\mathbf{X}_{n+1} - \mathbf{X}_n) = \mathbf{X}'_n + \mathbf{S}_n + \boldsymbol{\Delta}\mathbf{X}_n$

Path 2: $\mathbf{X}_{n+1} = \mathbf{X}_{n+1} \pm \mathbf{X}'_n \pm \mathbf{X}'_{n+1} = \mathbf{X}'_n + (\mathbf{X}'_{n+1} - \mathbf{X}'_n) + (\mathbf{X}_{n+1} - \mathbf{X}'_{n+1}) = \mathbf{X}'_n + \boldsymbol{\Delta}\mathbf{X}'_n + \mathbf{S}_{n+1}$

$\therefore\; \mathbf{S}_n + \boldsymbol{\Delta}\mathbf{X}_n = \boldsymbol{\Delta}\mathbf{X}'_n + \mathbf{S}_{n+1}$ or $\mathbf{S}_{n+1} - \mathbf{S}_n = \boldsymbol{\Delta}\mathbf{X}_n - \boldsymbol{\Delta}\mathbf{X}'_n \;\therefore\; \boldsymbol{\Delta}\mathbf{S}_n = \boldsymbol{\Delta}_I[\mathbf{X}_n - \mathbf{X}'_n]$

This result shows that the difference between two consecutive light-vectors is equal to the 'interaction-difference' between the interaction nodes of the two interacting electrons (this notation uses the convention introduced in section 4.2.3 where the different discrete timeframes become significant, unlike the simplistic continuum case with vanishing differentials). Indeed, in particular, the **interaction difference** (Δ_I) plays a key role in the present theory, as it measures the change across one complete interaction cycle, from 'just before' one interaction time, say at $t = t_n - \delta t$, through and until 'just before' the next interaction time, say at $t = t_{n+1} - \delta t$.

N.B. $\Delta_I F_n = \Delta_I F[t_n] = F[t_{n+1} - \delta t] - F[t_n - \delta t] = F[t_{n+1} - \delta t] \pm F[t_n + \delta t] - F[t_n - \delta t]$

$= \{\, F[t_{n+1} - \delta t] - F[t_n + \delta t]\,\} + \{\, F[t_n + \delta t] - F[t_n - \delta t]\,\}$

This produces the very significant difference identity (note subscripts): $\Delta_I F_n = \underline{\Delta}F_n + \Diamond F_{n-1}$ c.f. $\boldsymbol{\Delta}F_n = \underline{\Delta}F_n + \Diamond F_n$

Returning to the STI analysis, the last DNV result can be separated into its space and time components, as:

$\boldsymbol{\Delta}\mathbf{S}_n = \mathbf{C}\,\Delta T_n = (i\,c\,\mathbf{I_0} + \underline{c} \bullet \underline{\mathbf{I}})\,\Delta T_n = \boldsymbol{\Delta}_I[\mathbf{X}_n - \mathbf{X}'_n] = \boldsymbol{\Delta}_I\,[\,(i\,c\,t_n \mathbf{I_0} + \underline{x}_n \bullet \underline{\mathbf{I}}) - (i\,c\,t'_n \mathbf{I_0} + \underline{x}'_n \bullet \underline{\mathbf{I}})\,]$

Time-Integrity: $\Delta T_n = \boldsymbol{\Delta}_I[t_n - t'_n]$ and Space-Integrity: $\underline{c}\,\Delta T_n = \boldsymbol{\Delta}_I[\underline{x}_n - \underline{x}'_n]$

Re-combining these two conditions into another form of the Space-Time Integrity condition: $\underline{c}\,\boldsymbol{\Delta}_I[t_n - t'_n] = \boldsymbol{\Delta}_I[\underline{x}_n - \underline{x}'_n]$

4.2.5 PHYSICS MANIFESTS ACTION

The present theory is based on Newton's views of invariant space and time; with particles moving through a passive space and inter-particle interactions occurring at only certain points in time. This traditional model rejects all views of both these fundamental 'background' concepts either mixing (through a change in an observer's motion) or becoming distorted due to large quantities of matter. Newton's mass particles are here seen as point electrons subject to mutual pair-wise interactions that manifest themselves as finite impulses at each participating electron. This theory is relational (or '*relative*') because the interaction is only based on their relative spatial and temporal separations while the consequence (effect) of the impulse only depends on their relative positions and velocities. This research programme's views on the fundamental nature of Nature were first introduced in the previous paper [96] and will be elaborated further in this section.

Pierre de Maupertuis (1698-1759) is credited with the first published statement of the "Least-Action Principle" [97]. In his original formulation, he defined the action $\mathcal{A}$ of a body of mass M moving with constant speed V over a distance L as the combined product of these three factors; that is: $\mathcal{A} = M\ V\ L$. He stated his principle that a body moved through space to **minimize** this quantity. Leonhard Euler generalized this equation to include variable motion over time, introducing the differential form of the body's action $d\mathcal{A}$ as it moved through an infinitesimal spatial distance $d\underline{x}$ with momentum (at that instant) of $\underline{P}[t]$, so that (in modern notation): $d\mathcal{A}[t] \equiv \underline{P}[t] \bullet d\underline{x}$. This meant that the body's total path-action during the time interval t_1 to t_2 would be $\mathcal{A}[t_1, t_2]$ defined by the sum of all these differentials and this must then be minimized.

$$\text{Definition: } \textbf{Action} \quad \mathcal{A}[t_1, t_2] = \int_{t1}^{t2} dt\ \underline{V}[t] \bullet \underline{P}[t]$$

Euler's version of this principle then became that a particle, at all times, would 'choose' its momentum and path to minimize this quantity. It was this implication of inanimate objects having choice that generated a massive resistance to this principle. While building on Hamilton's revolutionary concept of quaternions for its mathematical foundations, the present theory also builds on the Least Action principle that Hamilton himself viewed as central to mechanics, even though most scientists who constructed the magnificent edifice of classical mechanics often gave this principle short shrift due to their religious views.

It was Carl Jacobi (1804-1851), in his *Lectures on Dynamics*, who first showed that Maupertuis's Least-Action principle only holds for those cases where the energy is constant ('conservative systems'). In the present theory, where kinetic energy fluctuates during the individual interactions, this principle can only be invoked over the complete interaction between any two electrons. This illustrates the conflict between classical mechanics with its instantaneous forces and single (universal) view of time and the phenomenon of electromagnetism that always invokes time-delays for its interactions. Unfortunately, physics fell into the psychological trap of trying to preserve this single time approach, following Maxwell's EM field theory.

4.2.6 DISCRETE ACTION

Planck's Quantum of Action

Once again, with his radical hypothesis for quantizing action, Max Planck proposed a mathematical equation that led to a major surge in theoretical physics. The last 100 years has demonstrated the problematic nature of basing new, fundamental physics only on mathematics: debate still simmers about the real meaning of quantum mechanics, just as it does around the reality of relativity. A reading of Planck's seminal paper [98] indicates almost no physical insights for the introduction of the energy equation: $E_n = n\ h\ \nu$; an equation suggested to Planck, based on his explicit analogy with Boltzmann's kinetic model of random, distinct gas particles generating a **discrete**, <u>averaged</u> entropy distribution function. This mathematical approach, which used a fictitious 'sea of harmonic oscillators', will be replaced in the next paper in this series with a model of random motion of the electrons in the interior of a heated 'black-body'. Indeed, Planck made no use of the concept of action in this paper and only introduced his constant after he realized he needed to relate the fundamental frequency of each of his pure oscillators with their discrete average unit of energy. Furthermore, there was no action-minimization involved here, only a derivative of the oscillator's average entropy with respect to their average energy. With all this averaging being introduced in this mathematical model, it is somewhat difficult to see that the quantization of the action of these artificial oscillators had **any** fundamental physical significance. The vagueness and semantic disputations in the resulting 'quantum' physics illustrates the danger of building basic physical theories on such **ill-defined** (and especially mathematical) concepts. Unfortunately, the vast majority of theoretical physicists today are simply mathematicians and are not troubled at all by this 'math first' approach – particularly if it only results in philosophical problems and semantic contradictions.

Planck ignores Einstein

Planck never reconciled his conceptual breakthrough of quantized action with Albert Einstein's special theory of relativity, although both theories were constructed around Maxwell's classical theory of electromagnetism [99]. The Special Theory explicitly posited a world with no accelerations (invoking only relative inertial reference frames). In such a situation, there could be no forces (including electric and magnetic force fields) so there could be no change in momentum. This means that there could be no change in action (and it is only changes in action that were quantized); indeed, there should also be no relativistic change in mass – none of these logical contradictions prevented Planck accepting the developments in quantum theory, classical or QED. Given Planck's political power he was never called to account for these massive problems.

The Classical Limit

The experimental observations that the electron's principal characteristics (charge *e* and mass *m*) are finite should have suggested that Planck's unit of action *h* would also be finite. It was this perspective that led to the idea that the universal time between possible interactions was also discrete and resulted in the Chronon Hypothesis. This is consistent with a finite value of the Fine Structure Constant (α), which can be incorporated into the Light-Cone Hypothesis: $\Delta X = n\,\tau\,2\pi e^2/\alpha h$. This suggests that Bohr's view that the classical limit occurs when α goes to zero is more plausible than the popular view that the classical limit occurs when *h* goes to zero. In the present theory, α and *h* are always finite, so that the classical limit occurs when the interaction interval, or chronon τ, goes to zero; i.e. the continuum interaction limit of traditional physics.

Sommerfeld's Action Quantum

In his presentation in 1911 to the first Solvay Congress (dedicated to 'Radiation and the Quantum'), Arnold Sommerfeld (1868-1951) first suggested the phrase 'quantum of action' for Planck's action constant *h*. He was also the first to propose the 'Quantum Interaction' hypothesis, where in every elementary process of duration τ, the atom gains or loses an amount $h/2\pi$ of action [100], [101] [102]. Mathematically, in terms of the Lagrangian $\mathcal{L}$ (defined classically as the difference between the kinetic and potential energies), this became (where the integral extends over the duration τ of the process):

$$\Delta\mathcal{A} = \int dt\, \mathcal{L}[t] = h / 2\pi = h_D$$

Discrete Action

Although the concept of action has played a central role in the history of mechanics, its various definitions are ambiguous. This is because Maupertuis was careless in his treatment of time in his original definition. This can be resolved by bringing time back explicitly into his original definition, so that the action of a particle $\mathcal{A}[t, t_0]$ is defined between the two points in time, denoted by t and t_0 (which, often, will be set to zero), defining the particle's locations with a spatial separation $S[t, t_0]$, between which the particle moves with a constant momentum $P[t, t_0]$; thus:

$$\mathcal{A}[t, t_0] = P[t, t_0]\, S[t, t_0] = m\, V[t, t_0]\, (X[t] - X[t_0]) = m\, (X[t] - X[t_0])^2 / (t - t_0) \quad \text{or} \quad \mathcal{A}[t] = m\, V[t]\, X[t]$$

Since this theory views discrete changes in particle action induced by true impulses (instantaneous duration) as the most accurate model of the micro-world then mathematical limits become extremely important, so here the results for extended finite differences found in appendix A4.2 must be used, where T_n is defined as when the n^{th} impulse occurs at X_n.

$$\boldsymbol{\Delta}[\mathcal{A}_n] = \Delta\mathcal{A}_n = \mathcal{A}_{n+1} - \mathcal{A}_n = \mathcal{A}_{n+1}^{+} - \mathcal{A}_n^{+} = \mathcal{A}[T_{n+1} + \delta t] - \mathcal{A}[T_n + \delta t] = \underline{\Delta}\mathcal{A}_n + \Diamond\mathcal{A}_n \equiv \Delta\mathcal{A}^K_n + \Delta\mathcal{A}^D_n$$

$$\underline{\Delta}\mathcal{A}_n = m\, \underline{\boldsymbol{\Delta}}[\underline{V}_n \bullet \underline{X}_n] = m\, \underline{V}_n \bullet \underline{\boldsymbol{\Delta}}[\underline{X}_n] = m\, \underline{V}_n \bullet \Delta\underline{X}_n \quad \text{and} \quad \Diamond\mathcal{A}_n = m\, \Diamond[\underline{V}_n \bullet \underline{X}_n] = m\, \underline{X}_n \bullet \Diamond[\underline{V}_n] = m\, \underline{X}_n \bullet \Diamond\underline{V}_n$$

The corresponding discrete version of action, in the present theory, is defined between two impulses (of duration ΔT_n) as :

Definition: **Unit-Kinetic-Action** $\Delta\mathcal{A}^K_n \equiv \underline{\Delta}\mathcal{A}_n = m\, \underline{V}_n \bullet \Delta\underline{X}_n = m\, V_n^2\, \Delta T_n = 2\, \mathcal{K}_n\, \Delta T_n$

Definition: **Unit-Dynamic-Action** $\Delta\mathcal{A}^D_n \equiv \Diamond\mathcal{A}_n = m\, \underline{X}_n \bullet \Delta\underline{V}_n = \underline{X}_n \bullet \Delta\underline{I}_n$

Two Electron Action

The concept of action can be extended from a single electron to the pair of electrons involved in a series of interactions if the total times spanning the complete interaction are covered. Thus, for two electrons identified with labels '1' and '2' interacting beginning from t_1 and t_1' and finally completing at t_2 and t_2' respectively, the combined or joint action can be defined as:

$$\text{Definition: } \textbf{Joint-Action} \quad \mathcal{A}(1,2 : t_1 , t_1' ; t_2 , t_2') \equiv \sum_{k} \sum_{nk} m \, \underline{V}(k: t_k + n_k \tau) \bullet \Delta \underline{x}(k: t_k + n_k \tau)$$

Each of these electron trajectories can be segmented around each of the N interaction nodes, thus for electron #1, moving from $(t_1 - \tau/2)$ to $(t_2 + \tau/2)$. Each segment includes the change in action $\Diamond\mathcal{A}[1: t_k]$ around t_k and the constant action $\underline{\Delta}\mathcal{A}[1: t_k]$ from time $(t_k + \tau/2)$ to $(t_{k+1} - \tau/2)$ for a total segment action of $\Delta\mathcal{A}_k[1]$, where $\Delta t_k = t_{k+1} - t_k = n_k\tau$ and :

$$\Delta\mathcal{A}_k[1] = 2\,\tau < \underline{V}_k > \bullet \Delta \underline{I}_k + m\, V_k^{\,2} (\Delta t_k - \tau) = m\, V_k^{\,2}\, \Delta t_k - m\, V_{k-1}^{\,2}\, \tau$$

The first interaction changes the velocity from U_0 while the final (N^{th}) interaction changes the velocity to V_0, so the total action for electron #1 over this complete interaction from $(t_1 - \tau/2)$ to $(t_2 + \tau/2)$ is :

$$\mathcal{A}[1: t_1 - \tau/2, t_2 + \tau/2] = m\,\tau\,(V_0^{\,2} - U_0^{\,2}) + m\,\tau \sum_{k=1}^{N-1} V_k^{\,2}\,(n_k - 1) = m\,\tau\,(V_0^{\,2} - U_0^{\,2}) + \sum_{k=1}^{N-1} 2\,\mathcal{K}_k\,\tau\,(n_k - 1)$$

A similar equation results for electron #2 where all the quantities are 'dashed'. So, the complete two-electron interaction requires minimizing the sum of these two equations for the joint-action by selecting the optimum combination of interim velocities V_k and V_k' and the final velocities V_0 and V_0' while varying the two sets of interaction times $\{ t_k \}$ and $\{ t_k' \}$.

4.2.7 TWO-PARTICLE INTERACTIVITY

The concept of **interactivity** was also first introduced in the first paper [103] when it was realized that the combination of the total action and angular momentum of two electrons interacting 'on the light-cone' formed a separable Continuous Natural Vector $\mathbf{Q}_{12}$ and whose value did not change throughout the complete interaction. This CNV was defined as the product of the two-electron CNVs representing spatial separation $\mathbf{X}_{12}$ and total linear momentum $\mathbf{P}_{12}$.

$$\text{Interactivity } \mathbf{Q}_{12}[T] = i\,\mathbf{X}_{12}[T]\,\mathbf{P}_{12}[T] \quad \text{where } T = t_1 - t_2 \quad \text{and } \mathbf{P}_{12} = 2\,m\,\mathbf{V}^*_{12} = m\,(\mathbf{V}_1 + \mathbf{V}_2)^*$$

This CNV can be described in terms of the fundamental 'light-vector' CNV $\mathbf{C}$, re-introduced here in section 4.1.8, so that:

$$\mathbf{Q}_{12}[t_1 - t_2] = m\,c\,(t_1 - t_2)\,\mathbf{C}.$$

This will not be examined further now as attention from here on is shifted to discrete electron behavior. The corresponding two-electron digital interactivity natural vector $\mathbf{Q}_\eta$ is defined in terms of its double-electron components.

$$\text{Definition: } \textbf{Digital Two-electron Interactivity} \quad \mathbf{Q}_{12}[T_\eta] \equiv i\,\mathbf{X}_{12}[T_\eta]\,\mathbf{P}_{12}[T_\eta] = \mathbf{Q}_\eta = i\,\mathcal{Q}_\eta\,\mathbf{I_0} + \underline{Q}_\eta \bullet \underline{\mathbf{I}}$$

It is important to realize that there is no simple relationship between the two-particle interactivity and the single activities.

$$\mathbf{Q}_{12}[t_\eta - t'_\eta] = \mathbf{R}_1 - \mathbf{R}_2 + i\,m\,(\mathbf{X}_1[t_\eta]\mathbf{V}^*_2[t'_\eta] - \mathbf{X}_2[t'_\eta]\mathbf{V}^*_1[t_\eta])$$

Although the first two terms indicate a simple <u>difference</u> in activity between the particles, the second two terms reflect an interference-like **anti-commutation** component.

It is also important to realize that the concept of two-electron interactivity is <u>only useful in scattering experiments</u> (repulsive interactions) where the combined velocity $\mathbf{V}_{12}$ is non-zero. For conservative systems created by attractive interactions between electrons of opposite charge the combined velocity $\mathbf{V}_{12}$ is zero (see section 5.3.4), so $\mathbf{Q}_{12}$ is zero .

So, for similar electrons this result is useful as every electron has the same, intrinsic invariant inertial mass *m* when the total momentum of the two electrons at the two times t_η and t'_η is proportional to their <u>average</u> velocity at these two times:

$$\underline{V}_\eta = < \underline{V}_{12}[T_\eta] > = \tfrac{1}{2}\,(\underline{v}_\eta + \underline{v}'_\eta) \quad \text{with} \quad T_\eta = t_\eta - t'_\eta \quad \& \quad \mathbf{V}_{12}[T_\eta] \equiv i\,c\,\mathbf{I_0} + \underline{V}_\eta \bullet \underline{\mathbf{I}} = \mathbf{V}_\eta \quad \therefore \; \mathbf{P}_\eta = 2m\,\mathbf{V}_\eta^*$$

$$\therefore \; \mathbf{Q}_\eta = i\,2\,m\,(c^2\,T_\eta - \underline{S}_\eta \bullet \underline{V}_\eta)\,\mathbf{I_0} + 2\,\underline{L}_\eta \bullet \underline{\mathbf{I}} \quad \text{where} \quad \underline{L}_\eta = m\,c\,(\underline{S}_\eta - \underline{V}_\eta\,T_\eta) + i\,\underline{S}_\eta \wedge \underline{V}_\eta \quad \text{and} \quad \underline{S}_\eta = \underline{X}_{12}[T_\eta] = \underline{x}_\eta - \underline{x}'_\eta$$

Using the results from section 4.2.4 that $S_n = c\,T_n$ and $\underline{S}_n = \underline{c}\,T_n$ then the two-electron activity evaluated at each node gives:

$$\mathbf{Q}_n = i\,2\,m\,T_n\,(c^2 - \underline{c}\bullet\underline{V}_n)\,\mathbf{I_0} + 2\,m\,T_n\,\underline{\mathbf{I}}\bullet\{c\,(\underline{c} - \underline{V}_n) + i\,\underline{c}\wedge\underline{V}_n\} = 2\,m\,T_n\,(\underline{c} - \underline{V}_n)\bullet(i\,\underline{c}\,\mathbf{I_0} + c\,\underline{\mathbf{I}}) + i\,2\,m\,T_n\,(\underline{c} - \underline{V}_n)\wedge\underline{c}\bullet\underline{\mathbf{I}}$$

$$\therefore\quad \mathbf{Q}_n = -i\,2\,m\,T_n\,\mathbf{C}\,(\mathbf{C} - \mathbf{V}_{12})^* = i\,2\,m\,T_n\,\mathbf{C}\,\mathbf{V}_{12}^* \quad \text{since} \quad \mathbf{C}_n^*\,\mathbf{C}_n = 0 \quad \therefore \quad \mathbf{Q}_n = i\,\mathbf{C}\,T_n\,m\,(\mathbf{V}_1 + \mathbf{V}_2)^*$$

When these two electrons are <u>not</u> interacting with each other or any others (during any finite time period) they effectively behave like 'free' particles, so their total two-time combined velocity $\underline{V}_0$ is constant. In their Symmetric Inertial Reference Frame (SIRF) their combined velocity at any <u>single</u> time is zero but, in general, this is not true for their total two-time combined velocity. Only in this special case (when: $\underline{v}_\eta = -\underline{v}'_\eta$ and $\underline{V}_\eta = 0$), with all variables tagged with '0' then:

$$\mathbf{Q}^0[T_\eta] = i\,2\,m\,c^2\,T_\eta\,\mathbf{I_0} + 2\,m\,c\,\underline{S}_\eta\bullet\underline{\mathbf{I}} = 2\,mc(\,i\,c\,(t_\eta - t'_\eta)\,\mathbf{I_0} + (\underline{x}_\eta - \underline{x}'_\eta)\bullet\underline{\mathbf{I}}\,) = 2\,mc\,(\mathbf{X}_\eta - \mathbf{X}'_\eta) \equiv \mathbf{Q}^0[1\colon t_\eta] - \mathbf{Q}^0[2\colon t'_\eta]$$

At possible interaction nodes (i.e. $T_\eta = T_n$) when $\underline{S}_n = c\,T_n$ $\therefore$ $\mathbf{Q}^0[T_n] = 2\,mc\,T_n\,(i\,c\,\mathbf{I_0} + \underline{c}\bullet\underline{\mathbf{I}}) = 2\,mc\,\mathbf{C}\,(t_n - t'_n)$

$$\therefore\quad \mathbf{Q}^0[1\colon t_n] = 2\,mc\,t_n\,\mathbf{C} = 2\,mc\,\mathbf{X}[1\colon t_n] \quad \text{and} \quad \mathbf{Q}^0[2\colon t'_n] = 2\,mc\,t'_n\,\mathbf{C} = 2\,mc\,\mathbf{X}[2\colon t'_n]$$

The canonical situation (described in [4] §6.4) occurs when two electrons collide from "infinity"; that is to say, they each exchange the maximum number of possible impulses N_0 while reversing their motion. When viewed from within the SIRF then the combined (asynchronous) total velocity is light-speed $\underline{c}$. In this case, $\mathbf{Q}_n = 0$ so it is an invariant at all interaction nodes. When the total number of consecutive interactions is less than the maximum N_0 then: $V_\eta < c$, so that:

$$\therefore\quad \mathbf{Q}_n = i\,2\,m\,T_n\,\mathbf{C}\,\mathbf{V}_0^* \quad \therefore \quad \Delta\mathbf{Q}[T_n] = i\,2\,mc\,\mathbf{C}\,\mathbf{V}_0^*\,\Delta[T_n] = i\,2\,mc\,\mathbf{C}\,\mathbf{V}_0^*\,(\,\Delta t_n - \Delta t'_n\,)$$

The results of the Space-time Integrity analysis (see section 4.2.4) indicate that for repulsion, Δt_n is not equal to $\Delta t'_n$ since the two asynchronous electron velocities $\underline{u}_n$ and $\underline{u}'_n$ are not equal.

Interaction changes Interactivity

In this theory, interactions are represented by anti-symmetrical impulses, 'exchanged' between the two electrons. As the effects of all of these impulses are instantaneous <u>at</u> each electron, they correspond to the application of the point-difference operator $\Diamond$ (see Appendix A4.2) acting on the electron's local velocity <u>at</u> each and every one of the interaction nodes.

$\Diamond[\,\mathbf{X}_{12}[T_n]\,] = \Diamond[\,\mathbf{X}_n - \mathbf{X}'_n] = \Diamond[\,\mathbf{X}_n] - \Diamond[\,\mathbf{X}'_n] = 0$ as $\mathbf{X}_\eta$ is continuous across time; i.e. $\Diamond[\,\mathbf{X}_\eta] = 0$ and $< \mathbf{X}_{12} > = \mathbf{X}_{12}$

$$\therefore\ \Diamond\mathbf{Q}_n = i\,\Diamond[\,\mathbf{X}_{12}[T_n]\,] < \mathbf{P}_{12}[T_n] > + i < \mathbf{X}_{12}[T_n] > \Diamond[\,\mathbf{P}_{12}[T_n]\,] = i\,\mathbf{X}_{12}[T_n]\ \Diamond[\,\mathbf{P}_{12}[T_n]\,]$$

Now $\Diamond[\mathbf{V}_\eta] = \Diamond[\,i\,c\,\mathbf{I_0} + \underline{v}_\eta\bullet\underline{\mathbf{I}}\,] = \underline{\mathbf{I}}\bullet\Diamond\underline{v}_\eta$ $\therefore$ $\Diamond[m\mathbf{V}_n^*] = \Delta\underline{\mathbf{I}}[1\colon t_n] = \Delta\underline{\mathbf{I}}_n$ and $\Diamond[m\mathbf{V}'^*_n] = \Delta\underline{\mathbf{I}}_n{}'$

Here, $\Delta\underline{\mathbf{I}}_n$ is the impulse experienced by electron #1 at time t_n originating from electron #2 at t'_n while $\Delta\underline{\mathbf{I}}_n{}'$ is the impulse experienced by electron #2 at time t'_n originating from electron #1 at t_n. In the case of continuous interactions, examined in paper III [104] , momentum was conserved across the interaction; the same result for discontinuous activity will also proposed here (see later in section 4.3.2); so: $\Delta\underline{\mathbf{I}}_n{}' = -\Delta\underline{\mathbf{I}}_n$

$$\therefore\ \Diamond[\,\mathbf{P}_{12}[T_n]\,] = m\,\Diamond[\,\mathbf{V}_n^* + \mathbf{V}'^*_n\,] = \underline{\mathbf{I}}\bullet(\Delta\underline{\mathbf{I}}_n + \Delta\underline{\mathbf{I}}_n{}') \quad \therefore\ \Diamond[\,\mathbf{P}_{12}[T_n]\,] = 0 \quad \therefore\ \Diamond\mathbf{Q}_n = 0$$

This result means that as a corollary of the total (asynchronous) two-electron momentum being an invariant ($\mathbf{P}^0{}_{12}$) then the two-electron interactivity $\mathbf{Q}_{12}$ is also an invariant with respect to the effects of the interaction between the two electrons <u>at</u> each interaction node.

Thus, at all times t_η, including t_n, $(\underline{V}_1 + \underline{V}_2)$ is a constant of the motion: i.e. $\mathbf{V}_1 + \mathbf{V}_2 = 2\mathbf{V}_0$, so at every interaction node 'n' $\mathbf{C}\mathbf{V}_0^*$ is another constant, so that $\mathbf{Q}_n$ only varies with the interaction time-difference T_n.

$$\mathbf{Q}_n = i\,2\,m\,\mathbf{C}\mathbf{V}_0^*\,T_n = 2\,m\,T_n\,(\,i\,c\,\mathbf{I_0}\,(\underline{c} - \underline{V}_0) + c\,\underline{\mathbf{I}}\bullet(\underline{c} - \underline{V}_0) - i\,\underline{c}\wedge(\underline{c} - \underline{V}_0)\,)$$

Only the effect of the impulse-exchange, from 'just before' the interaction (at $t_{n+1} - \delta t$ and $t'_{n+1} - \delta t$) up to 'just before' the next pair of chronons, need be considered around the $(n+1)^{th}$ interaction. The impulse changes the electron velocities from u_n and u'_n to u_{n+1} and u'_{n+1}. The difference in interactivity across these chronon time intervals evaluates to:

$$\Delta_\tau \mathbf{Q}_n \equiv \mathbf{Q}_{12}[T_{n+1} + \tau] - \mathbf{Q}_{12}[T_n] = i\, 2\, m\, \tau\, (\underline{u}_{n+1} - \underline{u}'_{n+1}) \bullet \underline{V}_0\, \mathbf{I_0} + 2\, mc\, \tau\, (\underline{u}_{n+1} - \underline{u}'_{n+1}) \bullet \underline{\mathbf{I}} + i\, 2\, m\, \tau\, (\underline{u}_{n+1} - \underline{u}'_{n+1}) \wedge \underline{V}_0 \bullet \underline{\mathbf{I}}$$

If this interaction at t_{n+1} had <u>not</u> occurred, then the electrons would have retained their velocities and there would be $\Delta \mathbf{Q}_n^{\,0}$:

$$\therefore \quad \Delta_\tau \mathbf{Q}_n^{\,0} = i\, 2\, m\, \tau\, (\underline{u}_n - \underline{u}'_n) \bullet \underline{V}_0\, \mathbf{I_0} + 2\, mc\, \tau\, (\underline{u}_n - \underline{u}'_n) \bullet \underline{\mathbf{I}} + i\, 2\, m\, \tau\, (\underline{u}_n - \underline{u}'_n) \wedge \underline{V}_0 \bullet \underline{\mathbf{I}}$$

This suggests the introduction of a new operator – the '*impact*' operator, designated by the symbol 'δ_0', which computes the difference between two physical situations: one involving a <u>single</u> interaction and the other with <u>no</u> interaction.

Thus, if a dynamical variable $\mathbf{A}$ represents a physical situation <u>after</u> an interaction has occurred and $\mathbf{A}^0$ represents the same situation when <u>no</u> interaction has occurred, then:

$$\text{Definition: } \textbf{Impact Operator} \quad \delta_0 \mathbf{A} \equiv \mathbf{A} - \mathbf{A}^0$$

The impact operator δ_0 and the <u>single-chronon</u>, extended difference operator Δ_τ will be combined into a new operator, called the *Local Interaction Difference* operator Δ^* (and referred to colloquially as the 'Star-Diff' operator – the 'star' indicating this is the most 'physical' of all these difference operators); this will simplify the form of many of the subsequent equations.

$$\text{Definition: } \textbf{Local Interaction-Difference Operator} \quad \Delta^*[\mathbf{A}] \equiv \delta_0 \Delta_\tau[\mathbf{A}]$$

Equivalently: $$\Delta^*[A[t_n]] = \{A[t_n + \tau/2] - A[t_n - \tau/2]\} - \{A^0[t_n + \tau/2] - A^0[t_n - \tau/2]\}$$

$$\delta_0 \Delta_\tau \mathbf{Q}_n \equiv \Delta_\tau \mathbf{Q}_n - \Delta_\tau \mathbf{Q}^0{}_n = i\, \Delta_\tau^* Q_n\, \mathbf{I_0} + \Delta_\tau^* \underline{Q}_n \bullet \underline{\mathbf{I}} = \underline{\mathbf{\Delta}}_\tau (\mathbf{Q}_n - \mathbf{Q}^0{}_n) \quad \text{as} \quad \mathbf{\Delta} = \mathbf{\Diamond} + \underline{\mathbf{\Delta}} \quad \text{and} \quad \mathbf{\Diamond} \mathbf{Q}_n = 0$$

$$\therefore \quad \Delta_\tau^* Q_n = 2\, m\, \tau\, (\Delta \underline{u}_n - \Delta \underline{u}'_n) \bullet \underline{V}_0 \quad \text{and} \quad \Delta_\tau^* \underline{Q}_n = 2\, m\, \tau\, \{ c\, (\Delta \underline{u}_n - \Delta \underline{u}'_n) + i\, (\Delta \underline{u}_n - \Delta \underline{u}'_n) \wedge \underline{V}_0 \}$$

As in the discussion earlier, for a Newtonian particle, where: $\underline{u}_n = \underline{v}^-_{n+1}$ and $\underline{u}_{n+1} = \underline{v}^+_{n+1}$ and $\Delta \underline{\mathbf{I}}_{n+1} = m\, \Delta \underline{u}_n = -m\, \Delta \underline{u}'_n$

$$\text{Now} \quad 2m\, (\Delta \underline{u}_n - \Delta \underline{u}'_n) \bullet \underline{V}_0 = m\, (\underline{u}_{n+1} - \underline{u}'_{n+1}) \bullet (\underline{u}_{n+1} + \underline{u}'_{n+1}) - m\, (\underline{u}_n - \underline{u}'_n) \bullet (\underline{u}_n + \underline{u}'_n) = 2\, (\Delta \mathcal{K}_n - \Delta \mathcal{K}'_n)$$

$$\text{And} \quad 2\, (\Delta \underline{u}_n - \Delta \underline{u}'_n) \wedge \underline{V}_0 = (\underline{u}_{n+1} - \underline{u}'_{n+1}) \wedge (\underline{u}_{n+1} + \underline{u}'_{n+1}) - (\underline{u}_n - \underline{u}'_n) \wedge (\underline{u}_n + \underline{u}'_n) = 2\, (\underline{u}_{n+1} \wedge \underline{u}'_{n+1} - \underline{u}_n \wedge \underline{u}'_n)$$

$$\therefore \quad \Delta_\tau^* Q_n = 2\, \tau\, \mathbf{\Delta}(\mathcal{K}_n - \mathcal{K}'_n) \quad \text{and} \quad \Delta_\tau^* \underline{Q}_n = 2\, m\, \tau\, \{ c\, \mathbf{\Delta}(\underline{u}_n - \underline{u}'_n) + i\, \mathbf{\Delta}(\underline{u}_n \wedge \underline{u}'_n) \}$$

$$\therefore \quad \Delta_\tau^* Q_n = 4\, \tau\, \underline{V}_0 \bullet \Delta \underline{\mathbf{I}}_{n+1} \quad \text{and} \quad \Delta_\tau^* \underline{Q}_n = 4\, \tau\, (c\, \Delta \underline{\mathbf{I}}_{n+1} + i\, \Delta \underline{\mathbf{I}}_{n+1} \wedge \underline{V}_0) \quad \therefore \quad \mathbf{\Delta}_\tau^* \mathbf{Q}_n = -4\, \tau\, \mathbf{V}_{12}^* \mathbf{\Delta I}_n$$

This demonstrates that the local <u>difference</u> between the two-electron interactivity, <u>as a result of the interaction</u> and evaluated over the smallest time interval (one chronon) following the interaction, is directly proportional to this impulse. The total or combined (or average) velocity ($\underline{V}_\eta = \langle \underline{V}_{12}[t_\eta - t'_\eta] \rangle = \tfrac{1}{2} (\underline{v}_\eta + \underline{v}'_\eta)$) is a constant and reflects the initial velocities of the two electrons when they **first** interact.

4.3 INTERACTION & ACTIVITY

4.3.1 INTERACTION AS ACTION EXCHANGE

Justifying Action-at-a-Distance

The concept of 'action-at-a-distance' was briefly reviewed in the second paper [105] – this idea is no less understandable than the collision of two 'solid' bodies. The reaction of two colliding macroscopic bodies is equally due to some invisible 'force' as the interaction of two remote charged bodies but the collision has a microscopic range of effect, which is contained within our everyday experience while the impact separations remain too small and too fast to be seen by our normal, unaided senses. This human level of awareness has given us a **false** impression of both instantaneous action over time and localization of all interactions in nature. This false intuition was reinforced by Newton's original (and successful) theory of gravitation that was formulated as just a spatial sensitive force or an instantaneous interaction across immense distances of empty space.

Action is Interaction

This theory views *interaction as the difference in action*; each electron involved in an interaction changes its action around the instant of interaction t_n, when it either emits or receives the corresponding impulse $\Delta\underline{\mathbf{I}}_n$ that defines the interaction. The change in action is the difference between the unit-action that includes the results of the interaction and the unit-action that would have happened **if** the transaction did **not** occur. These two unit-actions are both calculated from time $(t_n - \tau/2)$ up to $(t_n + \tau/2)$. So, in terms of the pre-interaction velocity $\underline{V}_n^-$ and post-interaction velocity $\underline{V}_n^+$ or average velocity $< \underline{V}_n >$:

$$\Delta_\tau \mathcal{A}_n = \mathcal{A}_n^+ - \mathcal{A}_n^- = 2\,\tau\,(\mathcal{K}_n^+ - \mathcal{K}_n^-) = m\,\tau\,(\underline{V}_n^+ + \underline{V}_n^-) \bullet (\underline{V}_n^+ - \underline{V}_n^-) = \tau\,(\underline{V}_n^+ + \underline{V}_n^-) \bullet \Delta\underline{\mathbf{I}}_n = 2\,\tau < \underline{V}_n > \bullet\, \Delta\underline{\mathbf{I}}_n$$

Kinetic Energy

Consider the two electron situation around the $(n+1)^{th}$ interaction: immediately before this interaction, electron #1 has the longitudinal velocity $\underline{u}_n$ and electron #2 has the velocity $\underline{u}'_n$; after the interaction the respective velocities are $\underline{u}_{n+1}$ and $\underline{u}'_{n+1}$. The interaction conserves the total two-time momentum (or total velocity when the masses are equal), so:

$$\text{As} \quad \underline{u}_n + \underline{u}'_n = \underline{u}_{n+1} + \underline{u}'_{n+1} = 2\underline{V}_0 \quad \text{while} \quad \underline{u}_{n+1} = \underline{u}_n + \Delta\underline{u}_n \quad \text{and} \quad \underline{u}'_{n+1} = \underline{u}'_n + \Delta\underline{u}'_n \quad \therefore\ \Delta\underline{u}_n + \Delta\underline{u}'_n = 0$$

Since $\Delta\underline{\mathbf{I}}_{n+1} = m\,\Delta\underline{u}_n$ and $\Delta\underline{\mathbf{I}}'_{n+1} = m\,\Delta\underline{u}'_n$ $\therefore\ \Delta\underline{\mathbf{I}}'_n = -\,m\,\Delta\underline{u}_n$. This reduces the interaction to a single parameter $\Delta\underline{u}_n$. Defining the change in kinetic energy of electron #1 as: $\mathcal{K}_{n+1} = \mathcal{K}_n + \Delta\mathcal{K}_n$ $\therefore\ \Delta\mathcal{K}_n = \tfrac{1}{2}\,m\,(\underline{u}_{n+1} \bullet \underline{u}_{n+1} - \underline{u}_n \bullet \underline{u}_n)$

$$\therefore\ \Delta\mathcal{K}_n = \tfrac{1}{2}\,m\,\Delta\underline{u}_n \bullet \Delta\underline{u}_n + m\,\underline{u}_n \bullet \Delta\underline{u}_n \quad \text{Thus,}\ \Delta\mathcal{K}'_n = \tfrac{1}{2}\,m\,\Delta\underline{u}'_n \bullet \Delta\underline{u}'_n + m\,\underline{u}'_n \bullet \Delta\underline{u}'_n = \tfrac{1}{2}\,m\,\Delta\underline{u}_n \bullet \Delta\underline{u}_n - m\,\underline{u}'_n \bullet \Delta\underline{u}_n$$

$$\therefore\ \Delta\mathcal{K}_n + \Delta\mathcal{K}'_n = m\,\Delta\underline{u}_n \bullet \Delta\underline{u}_n + m\,(\underline{u}_n - \underline{u}'_n) \bullet \Delta\underline{u}_n \quad \text{If}\ \ \boldsymbol{\Delta}(\mathcal{K}_n + \mathcal{K}'_n) = 0 \quad \therefore\ (\underline{u}_{n+1} - \underline{u}'_n) \bullet \Delta\underline{u}_n = 0$$

Case I: $(n = 0)$ $\underline{u}_0 = 0$ $\underline{u}'_0 = c\,\hat{\underline{e}}_0$ $\therefore\ \underline{u}_1 = \Delta\underline{u}_0$ Assume $\Delta\underline{u}_0 = b\,\hat{\underline{u}}$ where $\underline{c} = c\,\hat{\underline{u}}$ $\therefore\ (\underline{u}_1 - \underline{u}'_0) \bullet \Delta\underline{u}_0 = 0$

$$\therefore\ b = c\ (\hat{\underline{u}} \bullet \hat{\underline{e}}_0) = c\cos(\pi/2 - \theta_0) = c\sin\theta_0 \cong c\,\theta_0 \quad \text{as}\ b << c$$

Here, the second (high-speed) electron is moving at light-speed c, in a direction $\hat{\underline{e}}_0$ almost orthogonal to the line-of-sight direct separation of the two electrons when they first exchange an impulse (in the direction $\hat{\underline{u}}$), offset by small angle $\theta_0 = b/c$. If the second electron is moving outside of this small angular cone (including the 'head-on' or 'Terrible-Twins' model) then there can be no interaction, which also preserves the total kinetic energy across the first interaction.

Case II: Whenever $\underline{u}_n + \underline{u}'_n = \underline{u}_{n+1} + \underline{u}'_{n+1} = 2\,\underline{V}_n$ & $\Delta\underline{u}_n + \Delta\underline{u}'_n = 0$ then $\underline{u}_n = \underline{V}_n - \tfrac{1}{2}\,\Delta\underline{u}_n$ & $\underline{u}'_n = \underline{V}_n + \tfrac{1}{2}\,\Delta\underline{u}_n$

$$\therefore\ \underline{u}_{n+1} = \underline{V}_n + \tfrac{1}{2}\,\Delta\underline{u}_n \ \ \&\ \ \underline{u}'_{n+1} = \underline{V}_n - \tfrac{1}{2}\,\Delta\underline{u}_n \quad \therefore\ (\underline{u}_{n+1} - \underline{u}'_n) = 0 \quad \therefore\ (\underline{u}_{n+1} - \underline{u}'_n) \bullet \Delta\underline{u}_n = 0 \quad \text{for all}\ \Delta\underline{u}_n$$

Thus, the 'exchange' of $\Delta\underline{u}_n$ from one electron to the other conserves the total kinetic energy across the whole interaction.

Thomson & Tait's Impulse Exchange

In their best-selling 19th century text on mechanics [106], Thomson and Tait derived the result for the change in the kinetic energy of a particle $\Delta\mathcal{K}$ when subjected to a true impulse $\Delta\mathbf{I}$ as: $\Delta\mathcal{K} = \langle \underline{V} \rangle \bullet \Delta\mathbf{I}$. Here the average velocity $\langle \underline{V} \rangle$ of the particle was computed from the initial and final velocities before $\underline{u}$ and $\underline{v}$ after the effect of the interaction: $\langle \underline{V} \rangle = \frac{1}{2}(\underline{u} + \underline{v})$. This implies that if the target particle behaves like a simple Newtonian point-particle, moving only recti-linearly between each interaction then the average velocity $\langle \underline{V} \rangle$ cannot be orthogonal to the interaction impulse $\Delta\mathbf{I}$ when kinetic energy is exchanged. Conversely, when the impulse is orthogonal to each electron's average velocity (computed at each interaction node) then only momentum is exchanged between the interacting electrons and the motion of each electron is conservative. This condition is illustrated in the following diagram.

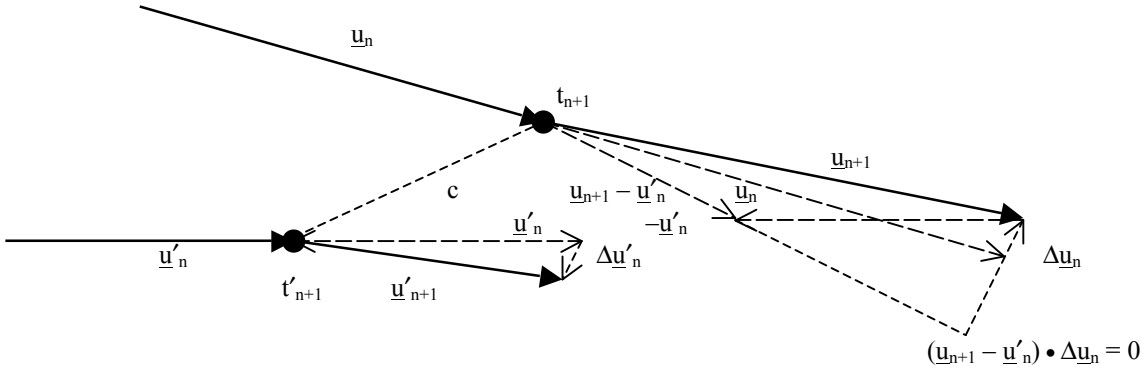


Fig. 9 Conservative Electron Interactions

4.3.2 ASYNCHRONOUS CONSERVATION

The discrete asynchronous interaction between two particles has appeared briefly in some of the earlier papers, particularly in the discussion of continuous interactions between two electrons [107] where it was shown that continuous interactions are **not** valid when the electrons are spatially separated and the interaction (like EM) requires a finite time to span the space between them. The focus will now shift permanently from here forward to the **discrete** behavior of all electrons. The corresponding single and double DNVs have already been introduced here in sections 3.2.4 and 3.2.5 for the two 'star' electrons that are located at $\underline{x}_n$ and $\underline{x}'_n$ respectively, at the two times t_n and t'_n where 'n' labels the n^{th} interaction between them:

$$\mathbf{X}_n = i\,c\,t_n\,\mathbf{I_0} + \underline{x}_n \bullet \underline{\mathbf{I}} \quad \text{and} \quad \mathbf{X}'_n = i\,c\,t'_n\,\mathbf{I_0} + \underline{x}'_n \bullet \underline{\mathbf{I}} \quad \text{while} \quad \mathbf{X}_{12}[n] = \mathbf{X}_n - \mathbf{X}'_n$$

So that: $\mathbf{X}_{12}[n] = i\,c\,T_n\,\mathbf{I_0} + \underline{X}_n \bullet \underline{\mathbf{I}}$ where $T_n = t_n - t'_n$ and $\underline{X}_n = \underline{x}_n - \underline{x}'_n$ with $t_n = \eta_n \tau$ and $t'_n = \eta'_n \tau$.

Now (from §4.1.7), $\underline{X}_n$ is the n^{th} light-separation displacement vector $\underline{S}_n$, i.e. $\underline{X}_n = \underline{S}_n$ so that: $\mathbf{X}_{12}[n] = \mathbf{S}_n = \mathbf{C}\,T_n$

It was demonstrated in the previous paper [108] that in almost all cases, there are many unit time intervals (i.e. *chronons*) between any two interaction-nodes. In this earlier paper, electrons were viewed as simple, point particles. In that simple pedagogical example motion was limited to only one spatial dimension ('the Terrible Twins'), where each electron's velocity remained constant between each pair of interaction-nodes. This simple, Newtonian model of a particle will be expanded here to include a separate transverse motion; it will be investigated in the next section. This means that it is important to discuss the ***concept of velocity*** of the electron, especially around an interaction node and now distinguish this idea from the average, longitudinal velocity between nodes. This will be done first for electron #1 around the n^{th} interaction node at time t_n. The approach that will be reprieved here for discrete NVs will follow the continuous approach that was described first in paper III [109]; specifically, the concept of finite differences, both prior- and post-interaction.

$$\text{Post-Velocity: } \underline{v}^{+}_n = (\,\underline{x}[1{:}\,t_n + \tau] - \underline{x}[1{:}\,t_n]\,)/\tau \quad \text{and Prior-Velocity:} \quad \underline{v}^{-}_n = (\,\underline{x}[1{:}\,t_n] - \underline{x}[1{:}\,t_n - \tau]\,)/\tau$$

These allow the micro-differences in the electron's location around t_n to define the average **nodal** velocity $\{\underline{v}_n\}$.

Average-Velocity: $\langle \underline{v}_n \rangle = (\underline{x}[1: t_n + \tau/2] - \underline{x}[1: t_n - \tau/2]) / \tau \;\therefore\; \langle \underline{v}_n \rangle = \frac{1}{2} (\underline{v}^+_n + \underline{v}^-_n)$ and $\langle \underline{v}'_n \rangle = \frac{1}{2} (\underline{v}'^+_n + \underline{v}'^-_n)$
Since one of the goals of the present theory is to complete "Maxwell's dream" of unifying the phenomenon of EM with Newton's theory of classical mechanics it is then important to retain as much of Newton's natural philosophy as possible. This view was elaborated in the previous paper [4], especially in section 5.4.3. That paper argued that Newton's original formulation of mechanics was actually discrete and centered on the concept of instantaneous **impulses** – the idea of force was added subsequently as a move to the continuum limit to evaluate apparently continuously smooth movement, such as is defined in perfect circular motion. The present theory retains Newton's concept of **momentum** as the 'quantity of motion', namely as the arithmetic product of a particle's intrinsic mass *m* and its velocity $\underline{v}_n$ i.e. $\underline{p}_n = m\, \underline{v}_n$. Newton's three Laws of Motion are retained (see section 6.1.1 of the previous paper), especially the idea that a particle in motion continues with its existing (longitudinal) velocity while it is not subject to any impulse and always moves in a straight line at constant speed between chronons. Newton's Second Law stated, as proposed in its original form, that an external impulse $\Delta\mathbf{I}$ **caused** a change in the particle's momentum proportional to the magnitude of the impulse and in the direction of the impulse. In the case of asynchronous interactions, the impulse received by one electron (say at t_{n+1}) originated with the other electron at a different time (say at t'_{n+1}). These impulses are designated with deltas to retain the notion of (small) change, thus:

$$\Delta\underline{\mathbf{I}}_{n+1} = \Delta\underline{\mathbf{I}}\,[2 \to 1 : t_{n+1}] \quad \text{and} \quad \Delta\underline{\mathbf{I}}'_{n+1} = \Delta\underline{\mathbf{I}}'\,[1 \to 2 : t'_{n+1}]$$

Newton's Second Law then becomes: $\Delta\underline{\mathbf{I}}_{n+1} \to \Diamond\underline{p}_n = m\,(\underline{v}^+_{n+1} - \underline{v}^-_{n+1})$ and $\Delta\underline{\mathbf{I}}'_{n+1} \to \Diamond\underline{p}'_n = m\,(\underline{v}'^+_{n+1} - \underline{v}'^-_{n+1})$

Newton's Third Law was instantaneous; now it is extended across space and time asynchronously: $\Delta\underline{\mathbf{I}}'_n[t'_n] = -\Delta\underline{\mathbf{I}}_n[t_n]$.

This results in the Conservation of Momentum **across** an interaction: $\Diamond\underline{p}_n + \Diamond\underline{p}'_n = 0$ or $\Diamond[\underline{p}_n + \underline{p}'_n] = 0$

Consequently, the total two-electron velocity after the interaction equals the total two-electron velocity before.

$$\underline{v}^+_n + \underline{v}'^+_n = \underline{v}^-_n + \underline{v}'^-_n = \underline{V}_0$$

It is very important to remember that this equation is not the one-time standard result but involves the two interaction times. This result was actually the initial hypothesis in 6.3.4 in the previous paper: either hypothesis may be considered primary. If, prior to the first interaction (n = 1) between the two electrons, their velocities were $\underline{v}_i$ and $\underline{v}'_i$ respectively and after the final interaction (n = N) between them they emerged with velocities $\underline{v}_f$ and $\underline{v}'_f$, then: $\underline{v}^-_1 = \underline{v}_i$ and $\underline{v}^+_N = \underline{v}_f$ etc.

$$\underline{v}_i + \underline{v}'_i = \underline{v}_f + \underline{v}'_f = \underline{V}_0$$

This means that the total velocity before the set of interactions is conserved across the complete set of interactions. In the repulsive case of the 1D spatial motion (the 'Terrible Twins') considered previously [110], this result remains valid for all values of n, including when n is maximal ($n = N_0$) for electron #1 where $\underline{v}^+_N = \underline{c}$ and its complement n′ is minimal (n′ = 1) where here $\underline{v}'^+_1 = 0$, so that $\underline{V}_0 = \underline{c}$ (a result only true when both electrons have similar charges). It will also be shown later (section 6.3.2) that for the more interesting attractive situation (opposite charges) the total, asynchronous combined velocity $\underline{V}_0$ (in the SIRF) is also $\underline{c}$. Thus, for both the post-interaction ($\lambda = +$) and pre-interaction ($\lambda = -$) velocities, the asynchronous conservation of electron momentum (for all charges) therefore leads to the velocity-invariance condition.

Velocity-Invariance Condition: $\underline{v}^\lambda_n + \underline{v}'^\lambda_n = \underline{V}_0$ { for all n and $\lambda = \pm$ }.

Since the present theory rejects Planck's Proposal for mass varying with speed, as a purely mathematical device to achieve agreement with Kaufmann's experimental results [9], the electron's mass follows Newton's physics and is represented by a universal, invariant scalar quantity *m*. This allows the Velocity-Invariance equation to be multiplied by *m* to define the total (two-time) momentum of the pair of interacting electrons.

Definition: Total Two-Electron Momentum $\underline{P}^\lambda_n = m\,(\underline{v}^\lambda_n + \underline{v}'^\lambda_n) = \underline{p}^\lambda_n + \underline{p}'^\lambda_n = m\,\underline{V}_0 = \underline{P}_0$

The magnitude of this quantity (P_0) is constant for any specific set of interactions ($P_0 \le mc$) – only its direction varies, based on the location of the two electrons when the series of successive interactions began. So, an alternative formulation of the Invariance Hypothesis is that each interaction leaves the total momentum unchanged.

Total.Momentum.Before = Total.Momentum.After or $\Delta[\underline{P}^\lambda_n] = 0$

The repulsive interaction between similarly charged electrons means that every interaction exchanges an absolute amount of momentum from the decelerating electron to the accelerating electron, where (for example): $v^{+}_{n} > v^{-}_{n}$ or $v_{n+1} > v_n$. This implies an increase in kinetic energy in the accelerating electron: $\Delta\mathcal{K}_{n+1} > \Delta\mathcal{K}_n$. In the case of attractive interactions, it is possible for each electron to retain its speed across each interaction ($v_{n+1} = v_n$) and only change its direction. Obviously, this maintains the kinetic energy locally (at each interaction node) and no kinetic energy is exchanged – such sets of attracting electrons form 'conservative' systems, which will be the focus of much of this research programme. Repulsive interactions can only cause 'scattering' and do not form 'closed' systems, where the configuration repeats periodically over time. The Thomson-Tait result [111] for the change in a particle's kinetic energy after receiving an impulse $\Delta\underline{\mathbf{I}}$ shows that the necessary condition for a conservative system to maintain itself is that the impulse is received symmetrically at each node; since:

$$\Delta\mathcal{K}_n = \mathcal{K}_{n+1} - \mathcal{K}_n = m \langle \underline{v}_n \rangle \bullet \Delta\underline{v}_n = \langle \underline{v}_n \rangle \bullet \Delta\underline{\mathbf{I}}_{n+1} = \tfrac{1}{2} (\underline{v}_n + \underline{v}_{n+1}) \bullet \Delta\underline{\mathbf{I}}_{n+1}$$

So, when $v_{n+1} = v_n$: $\Delta\mathcal{K}_n = \tfrac{1}{2} v_n \Delta\mathbf{I}_{n+1} \{\cos(\Delta\underline{\mathbf{I}}_{n+1}, \underline{v}_n) + \cos(\Delta\underline{\mathbf{I}}_{n+1}, \underline{v}_{n+1})\} = \tfrac{1}{2} v_n \Delta\mathbf{I}_{n+1} \{\cos\theta_n + \cos(\pi - \theta_n)\} = 0$

Asynchronous conservation becomes even more important around a single pair of interaction nodes in situations involving more than two electrons (all real systems). In these more general situations each electron may only interact with one other specific electron (say at t_{n+1}) but then each of their next interactions could be with different electrons. This situation is best addressed with the point-difference operator $\Diamond$ introduced in appendix A4.2.

For piecewise functions, *constant throughout each of their intervals*, the **positive convention** is adopted, where:

$$F^{+}_{n} \equiv F[t_n + \delta t] = F[t_{n+1} - \delta t] = F_n \quad \text{then:}$$

$$\Diamond F_n \equiv F^{+}_{n+1} - F^{-}_{n+1} = F[t_{n+1} + \delta t] - F[t_{n+1} - \delta t] = F_{n+1} - F_n = \Delta[F_n] \quad \text{or} \quad \Diamond = \Delta$$

So, for example, since only the average velocity is defined at an interaction node, the positive limit $\underline{v}^{+}_{n}$ is always chosen to characterize the kinetic energy of the electron after any interaction at t_n; that is to say: $\underline{v}^{+}_{n} = \underline{v}[t_n + \delta t] \equiv \underline{v}_n$.

$$\therefore \quad \Diamond\mathcal{K}_n = \mathcal{K}^{+}_{n+1} - \mathcal{K}^{-}_{n+1} = \Diamond[\mathcal{K}_n] = \tfrac{1}{2} m \Diamond[\underline{v}_n \,.\, \underline{v}_n] = \langle \underline{v}_n \rangle \bullet \Delta\underline{\mathbf{I}}_{n+1}$$

This is an identical form to the result obtained above for consecutive interactions between the same pair of electrons. Once again, this illustrates how each electron's kinetic energy is conserved around any interaction node when the impulse is orthogonal to its average velocity defined at that node.

This result is also valid when a 'composite' particle with mass M (greater than the electron mass m) is involved through the E/M interaction. These composite particles are stable, dynamic systems of positive and negatively charged electrons that will be investigated in much more detail in later papers, where a new **dynamic** theory of mass will be introduced.

In this case, the second particle only will be considered to be a composite particle so that the interaction deliberately involves two particles with different masses; so:

$$\therefore \quad \Diamond\mathcal{K}'_n = \Diamond[\mathcal{K}'_n] = \tfrac{1}{2} M \Diamond[\underline{v}'_n \,.\, \underline{v}'_n] = \langle \underline{v}'_n \rangle \bullet \Delta\underline{\mathbf{I}}'_{n+1}$$

This shows that Newton's intuition of the central importance of momen tum was correct, since : $\Delta\underline{\mathbf{I}}_{n+1} + \Delta\underline{\mathbf{I}}'_{n+1} = 0$

This is equivalent to the (extended) Conservation of Linear Momentum (across the two interaction times t_n and t'_n):

$$m \underline{v}^{+}_{n} + M \underline{v}'^{+}_{n} = m \underline{v}^{-}_{n} + M \underline{v}'^{-}_{n} = \underline{P}_0$$

The correlated (two-time) total kinetic energy of the two interacting particles can be defined over the next two intervals:

Definition: **Total Two-Electron Kinetic Energy** $K_{12}[\,t\,,\,t'\,] = \mathcal{K}_1[\,t\,] + \mathcal{K}_2[\,t'\,] \quad \{\, t_n \le t < t_{n+1} \,;\, t'_n \le t' < t'_{n+1} \,\}$

In these extended time intervals: $\mathcal{K}_1[\,t\,] = \mathcal{K}_n \;\&\; \mathcal{K}_2[\,t'\,] = \mathcal{K}'_n \qquad \therefore \quad K_{12}[\,t\,,\,t'\,] = K_n$

$$\therefore \quad \Diamond K_n = \Diamond[K_n] = \Diamond[\,\mathcal{K}_n + \mathcal{K}'_n\,] = \Diamond[\,\mathcal{K}_n\,] + \Diamond[\mathcal{K}'_n\,] = \langle \underline{v}_n \rangle \bullet \Delta\underline{\mathbf{I}}_{n+1} + \langle \underline{v}'_n \rangle \bullet \Delta\underline{\mathbf{I}}'_{n+1}$$

Thus, the correlated (two-time) total kinetic energy of the two interacting particles (even with different inertial masses) is still conserved ($\Diamond K_n = 0$) around each interaction event and over the next two correlated intervals whenever:

$$(\{\underline{v}_n\} - \{\underline{v}'_n\}) \bullet \Delta\underline{\mathbf{I}}_{n+1} = 0$$

In addition to the obvious solution ($\{\underline{v}_n\} = \{\underline{v}'_n\}$), there is one other very interesting general solution for two electrons, which will now be shown to be the 'exchange' solution. First, assume a general solution of the form:

$$\underline{v}_1^{+} = \underline{v}_2^{-} + \underline{W}_2 \quad \& \quad \underline{v}_2^{+} = \underline{v}_1^{-} + \underline{W}_1 \quad (\text{ or } \quad \underline{v}_1^{-} = \underline{v}_2^{+} + \underline{W}_1)$$

$$\therefore \quad \Delta\underline{\mathbf{I}}_{n+1} = \Diamond\underline{p}_1 = m\,\Diamond\underline{v}_1 = m\,(\underline{v}_1^{+} - \underline{v}_1^{-}) = -\,m\,(\underline{v}_2^{+} - \underline{v}_2^{-}) + m\,(\underline{W}_2 - \underline{W}_1) = -\,\Delta\underline{\mathbf{I}}'_{n+1} \quad \text{if} \quad \underline{W}_2 = \underline{W}_1 = \underline{W}_n$$

In other words, this solution satisfies: $\underline{P}_n^{+} = m\,(\underline{v}_1^{+} + \underline{v}_2^{+}) = m\,(\underline{v}_2^{-} + \underline{W}_n) + m\,(\underline{v}_1^{-} - \underline{W}_n) = m\,(\underline{v}_1^{-} + \underline{v}_2^{-}) = \underline{P}_n^{-}$

Since $\{\underline{v}_n\} = \tfrac{1}{2}\,(\underline{v}_1^{+} + \underline{v}_1^{-}) = \tfrac{1}{2}\,(\underline{v}_2^{+} + \underline{v}_2^{-}) + \underline{W}_n = \{\underline{v}'_n\} + \underline{W}_n \quad \therefore \quad \Diamond K_n = 0 \quad \text{if} \quad \underline{W}_n \bullet \Delta\underline{\mathbf{I}}_{n+1} = 0$

Now, $m\,\underline{W}_n = m\,(\underline{v}_1^{+} - \underline{v}_2^{-}) = m\,(\underline{v}_1^{+} \pm \underline{v}_1^{-} - \underline{v}_2^{-}) = m\,(\underline{v}_1^{+} - \underline{v}_1^{-}) + m\,(\underline{v}_1^{-} - \underline{v}_2^{-}) \quad \therefore \quad m\,\underline{W}_n = \Delta\underline{\mathbf{I}}_{n+1} + m\,(\underline{v}_1^{-} - \underline{v}_2^{-})$
Therefore, in any interaction between two electrons, the (incoming) velocities $\underline{v}_1^{-}$ and $\underline{v}_2^{-}$ are given and the impulse $\Delta\underline{\mathbf{I}}_{n+1}$ is determined purely by the space-time separations of the two halves of the single interaction between them, so the "exchange" vector $\underline{W}_n$ is fully determined. In general, since $\Delta\underline{\mathbf{I}}_{n+1}$ is non-zero, then so is $\underline{W}_n$, for:

$$\text{If} \quad \underline{W}_n = 0 \quad \text{then} \quad \underline{v}_1^{+} = \underline{v}_2^{-} \quad \& \quad \underline{v}_2^{+} = \underline{v}_1^{-} \quad \text{i.e. "exchange"} \quad \text{while} \quad \Delta\underline{\mathbf{I}}_{n+1} = m\,(\underline{v}_2^{-} - \underline{v}_1^{-}) = m\,(\underline{v}_1^{+} - \underline{v}_1^{-})$$

In the special case of planar motion when $\underline{v}_1^{-}$ and $\underline{v}_2^{-}$ are constrained to only two dimensions (say x-y) , then the central nature of the E/M interaction similarly constrains the output velocities $\underline{v}_1^{+}$ and $\underline{v}_2^{+}$ to the same (x-y) plane. By writing each velocity in terms of its horizontal (x) and vertical (y) components, using the notation: $\underline{v}_\alpha^{\lambda} = \underline{h}_\alpha^{\lambda} + \underline{g}_\alpha^{\lambda}$ and setting the impulse to be horizontal and also remembering that the combined (2-time) kinetic energy is conserved around the interaction events then the "exchange" vector $\underline{W}_n$ must be exclusively in the vertical direction. In this case:

$$\underline{W}_n = \underline{g}_1 - \underline{g}_2 \quad \text{and} \quad \underline{h}_2^{\lambda} = \underline{h}_1^{-\lambda} \quad \text{or complete exchange of horizontal velocity components.}$$

These linear concepts of momentum can be readily extended to the concept of angular momentum of a particle ($\mathcal{M}_\alpha$). Once again, the classical definitions, at a single point in time t, are the foundation for the digital (point) electron.

$$\underline{\mathcal{M}}_\alpha[\,t\,] = \underline{x}_\alpha[\,t\,] \wedge \underline{p}_\alpha[\,t\,]$$

Since the velocity and linear momentum of an electron changes discontinuously at the instance of an interaction, so can the (vector) value of the electron's angular momentum; this again requires explicit use of prior- and post-interaction values.

$$\text{Post-Ang. Mom.} \quad \underline{\mathcal{M}}^{+}[\,t\,] \equiv m\,\underline{x}[1\colon t] \wedge \underline{v}^{+}[1\colon t\,] \quad \& \quad \text{Prior-Ang. Mom.} \quad \underline{\mathcal{M}}^{-}[\,t\,] \equiv m\,\underline{x}[1\colon t] \wedge \underline{v}^{-}[1\colon t\,]$$

This defines the (average) angular momentum at an instant: **Average-Ang. Mom.** : $\{\underline{\mathcal{M}}[\,t\,]\} \equiv \tfrac{1}{2}\,(\underline{\mathcal{M}}^{+}[\,t\,] + \underline{\mathcal{M}}^{-}[\,t\,])$

Again, using the positive (or post) convention at an interaction instant: $\underline{\mathcal{M}}_n \equiv m\,\underline{x}[1\colon t_n] \wedge \underline{v}^{+}[1\colon t_n] = \underline{x}_n \wedge \underline{p}_n$

Asynchronous conservation of the combined mechanical quantities of two interacting electrons can be extended to their total angular momentum over their next two time intervals, whether this is defined between themselves or third-party electrons.

Definition: **Total Two-Electron Angular Momentum** $\underline{M}_{12}[\,t\,,t'\,] \equiv \underline{\mathcal{M}}_1[\,t\,] + \underline{\mathcal{M}}_2[\,t'\,] \quad \{\, t_n \le t < t_{n+1}\,;\, t'_n \le t' < t'_{n+1}\,\}$

But for $t = t_n + \Delta t$ {where $0 \le \Delta t < \Delta t_n$ } $\underline{\mathcal{M}}_1[\,t\,] = m\,\underline{x}_1[t] \wedge \underline{v}_1[t] = m\,(\underline{x}_n + \Delta t\,\underline{v}_n) \wedge \underline{v}_n = m\,\underline{x}_n \wedge \underline{v}_n = \underline{\mathcal{M}}_n$

So, in these extended time intervals: $\underline{\mathcal{M}}_1[\,t\,] = \underline{\mathcal{M}}_n \quad \& \quad \underline{\mathcal{M}}_2[\,t'\,] = \underline{\mathcal{M}}'_n \quad \therefore \quad \underline{M}_{12}[\,t\,,t'\,] = \underline{M}_n = \underline{\mathcal{M}}_n + \underline{\mathcal{M}}'_n$

The change in angular momentum around an interaction time t_{n+1} can be examined using the point-difference operator $\Diamond$.

$$\therefore \quad \Diamond\underline{\mathcal{M}}_n = \underline{\mathcal{M}}^{+}_{n+1} - \underline{\mathcal{M}}^{-}_{n+1} = \Diamond[\underline{\mathcal{M}}_n] = \Diamond[\,\underline{x}_n \wedge \underline{p}_n] = \{\underline{x}_n\} \wedge \Diamond\underline{p}_n + \Diamond\underline{x}_n \wedge \{\underline{p}_n\} = \underline{x}_n \wedge \Diamond\underline{p}_n = \underline{x}_n \wedge \Delta\underline{\mathbf{I}}_{n+1}$$

This is a similar form to the result obtained above for consecutive interactions between the same pair of electrons. Again, this illustrates how each electron's angular momentum is incremented around any interaction node when the impulse is orthogonal to its average velocity defined at that node. This disappears when both interacting electrons are both considered.

$$\Diamond \underline{M}_n = \Diamond[\underline{M}_n] = \Diamond[\underline{\mathcal{M}}_n + \underline{\mathcal{M}}'_n] = \Diamond[\underline{\mathcal{M}}_n] + \Diamond[\underline{\mathcal{M}}'_n] = \underline{x}_n \wedge \Delta\underline{\mathbf{I}}_{n+1} + \underline{x}'_n \wedge \Delta\underline{\mathbf{I}}'_{n+1} = (\underline{x}_n - \underline{x}'_n) \wedge \Delta\underline{\mathbf{I}}_{n+1} = \underline{X}_n \wedge \Delta\underline{\mathbf{I}}_{n+1}$$

The correlated (two-time) total angular momentum of the interacting electrons is conserved around each interaction event and over the next two intervals whenever the impulse is parallel to their spatial separation vector, as in the EM impulse.

$$\therefore \quad \Diamond \underline{M}_n = 0 \quad \text{and} \quad \Delta \underline{M}_{12}[\,t\,,t'\,] = 0 \quad \text{or} \quad \underline{\mathcal{M}}^{+}_n + \underline{\mathcal{M}}'^{+}_n = \underline{\mathcal{M}}^{-}_n + \underline{\mathcal{M}}'^{-}_n$$

All of these two electron results for asynchronous conservation will be derived in a much more elegant manner in the next section when the concepts of digital activity and interactivity will be analyzed in terms of Digital Natural Vectors (DNVs).

4.3.3 ONE-PARTICLE ACTIVITY

The concept of activity was first introduced in the first paper reporting on this programme [112] section after it was realized that the combination of the electron's action and angular momentum formed a Continuous Natural Vector $\mathbf{R}$, whose value was invariant for a free electron. This CNV was defined as the product of the separate CNVs representing the electron's spatial location $\mathbf{X}$ and linear momentum $\mathbf{P}$. Rewriting the discrete time variable as a multiple of the chronon, this gives:

$$\text{Single Particle Activity} \quad \mathbf{R}_\eta \equiv i\,\mathbf{X}[t_\eta]\,\mathbf{P}[t_\eta] = i\,m\,\mathbf{X}_\eta\,\mathbf{V}_\eta^* = i\,\mathrm{R}_\eta\,\mathbf{I_0} + \underline{R}_\eta \bullet \underline{\mathbf{I}}$$

$$\text{Where} \quad \mathrm{R}_\eta = m\,(c^2\,t_\eta - \underline{x}_\eta \bullet \underline{v}_\eta) \quad \& \quad \underline{R}_\eta = \underline{\mathbf{R}}_\eta + i\,\underline{\mathscr{R}}_\eta \quad \text{with} \quad \underline{\mathbf{R}}_\eta = m\,c\,(\underline{x}_\eta - \underline{v}_\eta\,t_\eta) \quad \& \quad \underline{\mathscr{R}}_\eta = m\,(\underline{x}_\eta \wedge \underline{v}_\eta)$$

For DNVs, the micro-difference operator Δ_τ (§4.2.3) is the closest analogue of continuous time differentiation; writing the local 'acceleration' as: $\underline{\mathcal{A}}_\eta = \Delta_\tau\,\underline{v}_\eta / \tau$ and noting that the local velocity is also defined as: $\underline{v}_\eta = \Delta_\tau\,\underline{x}_\eta / \tau$ then:

$$\Delta_\tau[\,\mathrm{R}_\eta\,] = m\,\tau\,(c^2 - \underline{v}_\eta \bullet \underline{v}_\eta - \underline{x}_\eta \bullet \underline{\mathcal{A}}_\eta) \qquad \Delta_\tau[\,\underline{\mathbf{R}}_\eta\,] = -\,m\,\tau\,c\,t_\eta\,\underline{\mathcal{A}}_\eta \qquad \Delta_\tau[\,\underline{\mathscr{R}}_\eta\,] = m\,\tau\,(\underline{x}_\eta \wedge \underline{\mathcal{A}}_\eta)$$

The requirement that $\mathbf{R}[t_\eta]$ is locally invariant is equivalent to the condition: $\Delta_\tau[\mathbf{R}^0_\eta] = 0$; this then implies:

$$\text{i)} \quad c^2 - \underline{v}_\eta \bullet \underline{v}_\eta = \underline{x}_\eta \bullet \underline{\mathcal{A}}_\eta \qquad \text{ii)} \quad t_\eta\,\underline{\mathcal{A}}_\eta = 0 \qquad \text{iii)} \quad \underline{x}_\eta \wedge \underline{\mathcal{A}}_\eta = 0$$

These three equations have the shared solution: $\underline{\mathcal{A}}_\eta = 0$ so that: $v_\eta = v_0 = c$ indicating that that the 'free' electron moves at 'light-speed'. This has the 3D spatial trajectory: $\underline{x}_\eta = \underline{x}_0 + \underline{c}\,\eta\,\tau$, where $\underline{x}_0$ is the electron's location at $t = 0$ ($\eta = 0$) and $\underline{c}$ defines its direction at 'light-speed'. The other (free) invariants are: $\underline{x}_\eta \bullet \underline{R}^0_\eta$ and $\underline{v}_\eta \bullet \underline{R}^0_\eta$ since:

$$\Delta_\tau[\,\underline{x}_\eta \bullet \underline{R}^0_\eta\,] = m\,c\,\underline{x}_0 \bullet \underline{v}_0 = \underline{v}_\eta \bullet \underline{R}^0_\eta = \text{constant (zero, when } \underline{x}_0 = 0).$$

Direct substitution of this 'free' solution (alternatively, noting that $t_0 = 0$ so that $\mathbf{X}_\eta = \underline{x}_\eta \bullet \underline{\mathbf{I}}$ and $\mathbf{P}^0_\eta = m\,\mathbf{C}^*$) demonstrates that the 'free' single electron's activity DNV is independent of time (no dependence on t_η) since:

$$\mathbf{R}^0[1{:}\,t_\eta] = -\,i\,m\,(\underline{x}_0 \bullet \underline{c})\,\mathbf{I_0} + m\,c\,\underline{x}_0 \bullet \underline{\mathbf{I}} + i\,m\,(\underline{x}_0 \wedge \underline{c}) \bullet \underline{\mathbf{I}} = i\,m\,\mathbf{X}_0\,\mathbf{C}^* = \mathbf{R}^0\,[1{:}\,t_0\,]$$

The activity of a 'free' particle is a relatively uninteresting concept but it leads onto more interesting concepts when inter-particle interactions are considered. The interaction itself is automatically introduced when the difference in activity induced by the interaction is considered. This can be seen from the fact that the interaction involves the effect on the particle's velocity induced by the impulse that constitutes one half of the complete two-electron interaction. The particle's velocity (especially its direction) plays the exclusive role in determining where in space any particle moves to after a finite time. Consider one single particle moving between two chronon points $\{t_\eta\,;\,\underline{x}_\eta\}$ and $\{t_{\eta'}\,;\,\underline{x}_{\eta'}\}$ with the absence of all interactions throughout this complete interval, so it is moving with a constant, **longitudinal** velocity $\underline{u}_n$.

$$\therefore \quad \Delta\mathbf{R}[t_{\eta'}\,,t_\eta] \equiv \mathbf{R}_{\eta'} - \mathbf{R}_\eta = i\,\Delta\mathrm{R}_\eta\,\mathbf{I_0} = i\,m\,(t_{\eta'} - t_\eta)\,(c^2 - \underline{u}_n \bullet \underline{u}_n)\,\mathbf{I_0} \qquad \text{since} \quad \underline{x}_{\eta'} = \underline{x}_\eta + (t_{\eta'} - t_\eta)\,\underline{u}_n$$

This quantity can be evaluated after any one **interaction node** ($t_\eta = t_{n+1}$) to 'just before' the next interaction node at $t_{\eta'} = t_{n+2}$.

For any Newtonian particle, the kinetic energy $\mathcal{K}_n$ is defined simply in terms of its longitudinal speed: $\mathcal{K}_n \equiv \tfrac{1}{2}\, m\, \underline{u}_n \bullet \underline{u}_n$

$$\underline{\Delta}_I \mathbf{R}_{n+1} \equiv \Delta\mathbf{R}[t_{n+2} - \delta t,\, t_{n+1} + \delta t] = i\, m\, (t_{n+2} - t_{n+1})\, (c^2 - \underline{u}_{n+1} \bullet \underline{u}_{n+1})\, \mathbf{I_0} = i\, \Delta t_{n+1}\, (m\, c^2 - 2\, \mathcal{K}_{n+1})\, \mathbf{I_0}$$

If the electron had not participated in the interaction at t_{n+1} then it would have retained the same prior-velocity up to t_{n+2} ; this 'free' activity in this interval is designated as: $\underline{\Delta}_I \mathbf{R}^0{}_{n+1} = i\, m\, \Delta t_{n+1}\, (c^2 - \underline{u}_n \bullet \underline{u}_n)\, \mathbf{I_0}$ and $\Delta t_{n+1} = t_{n+2} - t_{n+1}$

$$\therefore \quad \delta_0\, \underline{\Delta}_I \mathbf{R}_{n+1} = -\,2\, i\, \Delta t_{n+1}\, \Delta\mathcal{K}_n\, \mathbf{I_0}$$

Interaction changes Local-Activity

The impact of a single interaction on the motion of an electron is entirely focused at the interaction point, say at $\{t_{n+1}\, ;\, \underline{x}_{n+1}\}$, when the target electron instantaneously changes its longitudinal velocity from $\underline{u}_n$ to $\underline{u}_{n+1}$ (in other words, electron #1 receives from its other partner, electron #2, a non-zero impulse $\Delta\underline{\mathbf{I}}_{n+1} = m\, \Delta\underline{u}_n$). Since this theory posits that there can never be more than one interaction in a single time duration of one chronon τ it suggests that the micro-difference Δ_τ be considered. This involves calculating the difference in the electron's activity around the node t_{n+1} i.e. calculated symmetrically from half a chronon before the interaction time to half a chronon after, i.e. $(t_{n+1} - \tau/2)$ through t_{n+1} to $(t_{n+1} + \tau/2)$. However, this brings in both velocities; a more useful measure of change in activity looks only at the instantaneous change at an interaction node $\Diamond\mathbf{R}[t_n]$ while recalling that (see Appendix A4.2): $\Diamond[t_n] = 0$ & $\Diamond[\underline{x}_n] = 0$ but $m\, \Diamond\underline{u}_n = m\, \Delta\underline{u}_n = \Delta\underline{\mathbf{I}}_{n+1}$

$$\Diamond\mathbf{R}_{n+1} = \mathbf{R}[t_{n+1} + \delta t] - \mathbf{R}[t_{n+1} - \delta t] = -\,i\, m\, (\underline{x}_{n+1} \bullet \Delta\underline{u}_n)\, \mathbf{I_0} - m\, c\, t_{n+1}\, \underline{\mathbf{I}} \bullet \Delta\underline{u}_n + i\, m\, (\underline{x}_{n+1} \wedge \Delta\underline{u}_n) \bullet \underline{\mathbf{I}}$$

This leads to the key idea that the **full interval difference** Δ_I is the most appropriate measure (see §4.2.3) as this spans the full time interval: $(t_{n+1} - \delta t) \le t \le (t_{n+2} - \delta t)$, which includes the changed motion up to just before $\{t_{n+2}\, ;\, \underline{x}_{n+1}\}$ so the the impact of a single interaction is included – remember: for any full difference over time: $\Delta = \underline{\Delta} + \Diamond$ (see Appendix A4.2).

$$\Delta_I \mathbf{R}_{n+1} = \mathbf{R}[t_{n+2} - \delta t] - \mathbf{R}[t_{n+1} - \delta t] = i\, \Delta_I \mathrm{R}_{n+1}\, \mathbf{I_0} + \Delta_I \underline{\mathrm{R}}_{n+1} \bullet \underline{\mathbf{I}} + i\, \Delta_I \underline{\mathscr{R}}_{n+1} \bullet \underline{\mathbf{I}} = \underline{\Delta}_I \mathbf{R}_{n+1} + \Diamond\mathbf{R}_{n+1}$$

$$\therefore\ \Delta_I \mathrm{R}_{n+1} = m\, \{\Delta t_{n+1}\, (c^2 - \underline{u}_{n+1} \bullet \underline{u}_{n+1}) - (\underline{x}_{n+1} \bullet \Delta\underline{u}_n)\} \quad \& \quad \Delta_I \underline{\mathrm{R}}_{n+1} = -\, m\, c\, t_{n+1}\, \Delta\underline{u}_n \quad \& \quad \Delta_I \underline{\mathscr{R}}_{n+1} = m\, (\underline{x}_{n+1} \wedge \Delta\underline{u}_n)$$

This can be reformulated in terms of impulse and the electron's kinetic and dynamic actions $\Delta\mathcal{A}^K{}_n$ and $\Delta\mathcal{A}^D{}_n$ (see §4.2.6).

$$\therefore\ \Delta_I \mathrm{R}_{n+1} = \{\, m\, c^2 \Delta t_{n+1} - \Delta\mathcal{A}^K{}_{n+1} - \Delta\mathcal{A}^D{}_{n+1}\, \} \quad \& \quad \Delta_I \underline{\mathrm{R}}_{n+1} = -\, c\, t_{n+1}\, \Delta\underline{\mathbf{I}}_{n+1} \quad \& \quad \Delta_I \underline{\mathscr{R}}_{n+1} = (\underline{x}_{n+1} \wedge \Delta\underline{\mathbf{I}}_{n+1})$$

4.3.4 TWO-PARTICLE ACTIVITY

The effect on electron #1 produced by the interaction with electron #2 (that is, due to the impulse $\Delta\underline{\mathbf{I}}$ originating with #2) is matched by a comparable effect on electron #2 from its participation in the same interaction with electron #1 (that is, the impulse $\Delta\underline{\mathbf{I}}'$ originating with #1 or, equivalently, its own reaction to 'emitting the shared' impulse). If a time t_{n+1} is chosen when electron #1 is increasing its kinetic energy then it is receiving an impulse that is in the direction it is moving. In this case, its 'partner' electron (i.e. #2) must have been transferring (at t'_{n+1}) the velocity-increment $\Delta\underline{u}_n$ to the other electron #1. A similar equation defines the interval-difference in activity for the partner electron #2 (using primed quantities).

$$\Delta_I \mathbf{R}'_{n+1} = i\, m\, \{\Delta t'_{n+1}\, (c^2 - \underline{u}'_{n+1} \bullet \underline{u}'_{n+1}) - (\underline{x}'_{n+1} \bullet \Delta\underline{u}'_n)\}\, \mathbf{I_0} - m\, c\, t'_{n+1}\, \underline{\mathbf{I}} \bullet \Delta\underline{u}'_n + i\, m\, (\underline{x}'_{n+1} \wedge \Delta\underline{u}'_n) \bullet \underline{\mathbf{I}}$$

This suggests creating a new DNV representing the total two-electron activity sharing a single interaction:

Definition: **Two-Electron Total-Activity** $\mathbf{B}_{12}[T_n] \equiv \mathbf{R}_n + \mathbf{R}'_n = \mathbf{R}[1\colon t_n] + \mathbf{R}[2\colon t'_n] = \mathbf{B}_n$

This new DNV will not be investigated here but will reappear in a later section (see §5.3.5).

Scattering Interactions

Repulsive interactions between two similarly charged particles always result in open-ended situations or scattering. It is also possible to arrange the initial conditions so that even attractive interactions between two oppositely charged particles can result in scattering. These situations will be analyzed in terms of activity in later sections (see §5.3.8 and §6.3.2). Most physical cases of interest involve stable, dynamical systems, where the configurations recur over a fixed period. These will be the focus of most of the remaining investigations in this programme. Accordingly, activity is now investigated for attractive systems.

4.3.5 ATTRACTIVE INTERACTIONS

Conservative Interactions

It was shown at the end of the first paper [113] that all conserved two-particle continuous mechanical quantities correspond to the continuous exchange of both scalar and vector quantities between the two particles. The pivotal idea of the discrete interaction in this theory is that this exchange is one of momentum and also, in certain cases, kinetic energy induced by the exchange of asynchronous EM impulses $\Delta\underline{\mathbf{I}}$ and $\Delta\underline{\mathbf{I}}'$. The more interesting situation, involving conservative systems strongly suggests quantizing each electron's difference in activity between each node. This case will be examined first and the general case thereafter. This analysis extends the results from section 4.3.3 to where the impulse is node symmetric.

Consider the target particle as it is effected by the interaction with its partner when the electron located at $\{t_{n+1}\,;\,\underline{x}_{n+1}\}$ when the target electron instantaneously changes its longitudinal velocity from $\underline{u}_n$ to $\underline{u}_{n+1}$ (in other words, it receives from its partner a non-zero impulse, $\Delta\underline{\mathbf{I}}_{n+1} = m\,\Delta\underline{u}_n$). In conservative systems, the impulse interaction is symmetrical around each interaction node, which ensures that there is no increase in speed induced by the interaction – only a change in direction. This is clearly illustrated in the following diagram.

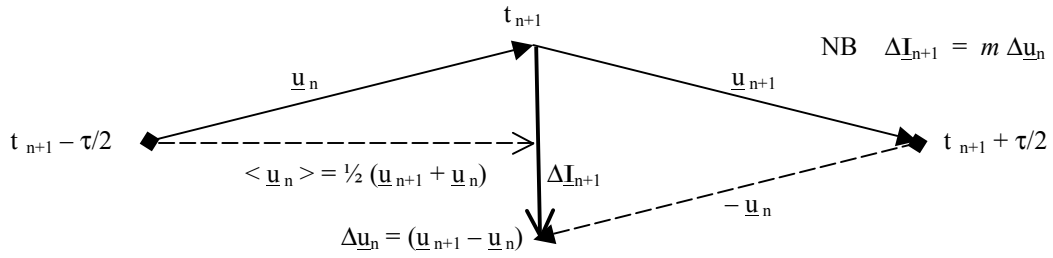


Fig. 10 Symmetric Impulse

Since the impulse at t_{n+1} is symmetrical in space it only reverses the velocity component parallel to the impulse; this means that the electron's velocity varies in direction around the node, its speed u is constant; i.e. $u_{n+1} = u_n = u$. Algebraically:

$$\therefore\ <\underline{u}_n>\bullet\,\Delta\underline{u}_n = \tfrac{1}{2}(\underline{u}_{n+1}+\underline{u}_n)\bullet(\underline{u}_{n+1}-\underline{u}_n) = \tfrac{1}{2}(\underline{u}_{n+1}\bullet\underline{u}_{n+1}-\underline{u}_n\bullet\underline{u}_n) = \tfrac{1}{2}(u^2-u^2) = 0\ \ \therefore\ \Delta\mathcal{K}_n = \Delta\underline{\mathbf{I}}_{n+1}\bullet<\underline{u}_n> = 0$$

This diagram illustrates the algebraic result that for a conservative interaction ($u_{n+1} = u_n = u$). The real difference in the electron's activity around the node t_{n+1} is the same whether calculated from just before the interaction ($t_{n+1} - \delta t$) up to the next chronon at ($t_{n+1} + \tau$) or whether calculated symmetrically from ($t_{n+1} - \tau/2$) through t_{n+1} to ($t_{n+1} + \tau/2$); in both cases:

$$\Delta_\tau\mathbf{R}_{n+1} = \Delta\mathbf{R}[t_{n+1}+\tau,\ t_{n+1}-\delta t] = \Delta\mathbf{R}[t_{n+1}+\tau/2,\ t_{n+1}-\tau/2] = i\,m\,\tau\,(c^2-u^2)\,\mathbf{I_0} + \lozenge\mathbf{R}[t_n]$$

If the interaction had not occurred at t_{n+1} then over this one chronon time interval the 'free' change in activity would be :

$$\Delta_\tau\mathbf{R}^0{}_{n+1} = i\,m\,\tau\,(c^2-u^2)\,\mathbf{I_0}$$

In the case of one of the electrons (say #1) the impact on its (one chronon) Activity as a result of the interaction at t_{n+1} is:

$$\Delta_\tau^*[\,\mathbf{R}_{n+1}\,] = \Delta_\tau\mathbf{R}_{n+1} - \Delta_\tau\mathbf{R}^0{}_{n+1} = \lozenge\mathbf{R}[t_n] = i\,\Delta_\tau^*\mathcal{R}_{n+1}\,\mathbf{I_0} + \Delta_\tau^*\underline{R}_{n+1}\bullet\underline{\mathbf{I}} + i\,\Delta_\tau^*\underline{\mathscr{R}}_{n+1}\bullet\underline{\mathbf{I}}$$

$$\therefore\ \Delta_\tau^*\mathcal{R}_{n+1} = -\,m\,\underline{x}_{n+1}\bullet\Delta\underline{u}_n \quad \&\quad \Delta_\tau^*\underline{R}_{n+1} = -\,m\,c\,t_{n+1}\,\Delta\underline{u}_n \quad \&\quad \Delta_\tau^*\underline{\mathscr{R}}_{n+1} = m\,(\underline{x}_{n+1}\wedge\Delta\underline{u}_n)$$

Although the electron's speed is unchanged at t_{n+1} , its direction is changed by this impulse $\Delta \underline{\mathbf{I}}_{n+1}$, where: $\Delta \underline{\mathbf{I}}_{n+1} = m\, \Delta \underline{u}_n$

$$\therefore\ \Delta_\tau^* \mathbb{R}_{n+1} = -\underline{x}_{n+1} \bullet \Delta \underline{\mathbf{I}}_{n+1} \quad \& \quad \Delta_\tau^* \underline{\mathbb{R}}_{n+1} = -c\, t_{n+1}\, \Delta \underline{\mathbf{I}}_{n+1} \quad \& \quad \Delta_\tau^* \underline{\mathscr{B}}_{n+1} = (\underline{x}_{n+1} \wedge \Delta \underline{\mathbf{I}}_{n+1})$$

Periodic Systems

Conservative systems often involve closed trajectories where, after a fixed number (say 2G) interactions, the particles return to their original locations ($\underline{x}_{n+2G} = \underline{x}_n$). These orbitals must exhibit at least two symmetries in space in order to close the trajectories; the choice of 2G reinforces the idea of an even number of interactions; the speed will be designated as V_G . There are two well-known situations that involve EM interactions between pairs of oppositely charged particles where the interactions all pass through the centre of symmetry of the system (which is usually taken to be the origin of the spatial coordinates). These involve mass symmetric and mass asymmetric configurations depending on whether the masses of the two particles are equal or very different – the latter case is exemplified by the neutral hydrogen atom. These models will be investigated later, simply in section 6.3.3 and more extensively in later papers in this series. At this time, only the activity of one of the particles (usually an electron) will be analyzed in this broad class of centrally symmetric interactions. When the second particle is massive ($m' >> m$) it is effectively at rest, at the origin, so $\mathbf{R}[2;\, t'] = 0$ while the target electron moves symmetrically around each interaction node. Since the speed is constant along each segment it is to be expected that the interaction rate is constant, so that: $\Delta t_{n+1} = \Delta t_n = \Delta T_G$. The time for one complete orbit is then simply: $T_G = 2G\, \Delta T_G$.

The time of the n^{th} interaction (from an arbitrary temporal origin n = 0) is: $t_n = t_0 + n\, \Delta T_G$ and $t_{2G} = t_0 + T_G$

When this impulse is radial (that is, passing through the origin) then it is anti-parallel to the electron's location $\underline{x}_{n+1}$, thus:

$$\therefore\ \Delta_I \mathbb{R}_{n+1} = m\, \{\Delta T_G\, (c^2 - V_G{}^2) + (x_{n+1}\, \Delta u_n)\} \quad \& \quad \Delta_I \underline{\mathbb{R}}_{n+1} = -m\, c\, t_{n+1}\, \Delta \underline{u}_n \quad \& \quad \Delta_I \underline{\mathscr{R}}_{n+1} = 0$$

The interaction node at t_{n+1} can be matched with its complement G interactions later at t_{n+1+G} when: $\Delta \underline{u}_{n+G} = -\Delta \underline{u}_n$

$$\therefore\ \Delta_I \underline{R}_{n+1} + \Delta_I \underline{R}_{n+1+G} = m\, c\, (t_{n+1+G} - t_{n+1})\, \Delta \underline{u}_n = G\, m\, c\, \Delta \underline{u}_n \quad \therefore\ \sum_{n=1}^{G} (\Delta_I \underline{R}_{n+1} + \Delta_I \underline{R}_{n+1+G}) = G\, m\, c \sum_{n=1}^{G} \Delta \underline{u}_n = 0$$

Thus, summing the activity over all 2G segments (intervals) around a **closed** trajectory leaves only the scalar $\Delta_I \mathbb{R}$.

Definition: **Orbital Activity** $\quad \mathbf{R}^G \equiv \mathbf{R}[t_n\, ,\, t_{n+2G}\,] = \sum_{n=1}^{2G} \boldsymbol{\Delta}_I \mathbf{R}_n = \sum_{n=1}^{2G} i\, \Delta_I \mathbb{R}_n\, \mathbf{I_0} \equiv i\, \mathbb{R}^G\, \mathbf{I_0}$

$$\therefore\ \mathbb{R}^G = m\, (c^2 - V_G{}^2) \sum_{n=1}^{2G} \Delta T_G + \sum_{n=1}^{2G} x_n\, \Delta \mathbf{I}_n = m\, (c^2 - V_G{}^2)\, T_G + \sum_{n=1}^{2G} x_n\, \Delta \mathbf{I}_n$$

It will be shown later (see §5.2.2) that the digital electron possesses an intrinsic extra degree of freedom: transverse motion, 'clicking' around its longitudinal direction of motion at 'light-speed' *c*. The electron's combined velocity is denoted by $\underline{V}$, while its longitudinal velocity is denoted by $\underline{U}$ and the transverse velocity by $\underline{W}$; thus at all times t:

$$\underline{V}[t] = \underline{U}[t] + \underline{W}[t] \qquad \text{with} \quad \underline{U}[t] \bullet \underline{W}[t] = 0 \quad \text{and} \quad \underline{W}[t] \bullet \underline{W}[t] = c^2 \qquad \therefore\ \underline{V}[t] \bullet \underline{V}[t] = c^2 + U^2$$

$$\therefore\ \mathbb{R}^G = \sum_{n=1}^{2G} \{\, x_n\, \Delta \mathbf{I}_n - m\, U_G{}^2\, \Delta T_G \,\} = -\mathcal{A}^G = -\sum_{n=1}^{2G} \{\, \Delta \mathcal{A}^D{}_n + \Delta \mathcal{A}^K{}_n \,\} = -\sum_{n=1}^{2G} \Delta_I \mathcal{A}_n$$

The final steps here use the relationship between the scalar part of the electron's activity DNV and its discrete action (§4.2.6). This summation equation is a general constraint for all closed trajectories defined by 2G attractive interactions, so it must be valid for each of its components. Every summation would only be zero if $\Delta_I \mathcal{A}_n$ is zero. This requires that each part of each term in the series is equal to a universal constant $\mathcal{a}$ with the dimension of action.

Hypothesis: **Universal Change in Action** $\quad \mathcal{a} = m\, U_G{}^2\, \Delta T_G = x_n\, \Delta \mathbf{I}_n$

For atomic systems, where the orbiting electrons are constrained within atomic dimensions, it is more than plausible that the strength of the EM impulse diminishes with distance between the interacting pair of attracting particles. This suggests that there exists a universal constant $\mathcal{a}$ that scales the EM interaction at atomic distances independent of all electron speeds. This is equivalent to a universal form of the short-range EM interaction that depends only on the distance S_n between the electrons.

$$\Delta \mathbf{I}_n = \mathcal{a} / S_n \qquad \text{where} \qquad \underline{S}_n = \underline{x}_n - \underline{x}'_n = \underline{c}\, (t_n - t'_n) = \underline{c}\, T_n$$

4.3.6 QUANTIZING THE INTERACTION

Thus one unit of activity a is exchanged from the 'partner' electron at t'_{n+1} through the EM interaction to the electron at t_{n+1} . The present theory adopts the simplest hypothesis, at this point, by assuming the amount of unit activity exchanged during each interaction is constant: this will be referred to as the "**Quantum of Activity**" hypothesis. The following examination of the impact of the repulsive interaction on the two-electron difference in activity is double the effect on a single electron's activity, so only one-half of this quantum is assumed to be exchanged in attractive impulses with a single electron.

This leads to the fundamental quantum hypothesis of this EM theory that constrains all successive interactions between any two electrons (of equal or opposite charge) such that the change in their one-electron difference-activity must take on the values defined by the universal action at their interaction locations. In the DNV formulation, this is equivalent to:

$$\text{Hypothesis: } \textbf{EM Quantum Hypothesis (I)} \quad \Delta_I \mathbf{R}_n = -\,a = -\,\Delta_I \mathbf{R}'_n \qquad \{\text{for all n}\}$$

This new DNV will be called the ***universal action*** DNV and designated by the letter 'a' (reflecting its primary role here).

$$\text{Definition: } \textbf{Universal Action} \quad a \equiv i\,a\,\mathbf{I_0} + \underline{a} \bullet \underline{\mathbf{I}} = (a / c)\,\mathbf{C} \qquad \therefore\ c\,\underline{a} = a\,\underline{c}$$

These equations have the solutions: $a = x_{n+1}\,\Delta\mathbf{I}_{n+1} = c\,t_{n+1}\,\Delta\mathbf{I}_{n+1}$ & $\underline{a} = \underline{c}\,t_{n+1}\,\Delta\mathbf{I}_{n+1} = \underline{x}_{n+1}\,\Delta\mathbf{I}_{n+1}$

These are consistent for conservative attractive interactions, as: $\underline{S}_n = \underline{x}_{n+1} - \underline{x}'_{n+1} = 2\,\underline{x}_{n+1} = 2\,\underline{c}\,t_{n+1} = \underline{c}\,(t_{n+1} - t'_{n+1}) = \underline{c}\,T_{n+1}$

Although this form of the quantization hypothesis is formulated only in terms of variables involving a single electron the second electron is always implicitly present (and subject to complementary quantization) through the relationship to the impulse $\Delta\mathbf{I}$ that originates with the 'partner' electron that generates this impulse. It is only with the disappearance of the sources of interaction into the historical concept of 'force' acting on a single particle that led to the mistaken view that quantization could be extended to the single electron. The model adopted throughout this program always requires a two-electron perspective. This becomes much more explicit when the interactions between pairs of similarly charged electrons are considered; this shows that only pairs of electrons must be quantized together.

Repulsive Interactions

Every electron is always increasing its own activity as time increases (i.e. per chronon) at a constant rate between any two adjacent interaction nodes n and (n+1), then each EM interaction must increase this rate for one electron while decreasing the rate for the other electron. The challenge is to decide which 'rule' nature chose to follow when defining the EM interaction for repulsive interactions; here both $\mathbf{B}_n$ and $\mathbf{Q}_n$ have the dimensions of action, i.e. time and energy.

The DNV equation for $\mathbf{Q}_n$ involves the combined two-electron two-time velocity $\underline{V}_{12}$ which has been shown to be constant in interactions between electrons. Implicitly, the theory so far has only considered electrons of similar charge and these are known through experiments to always repulse each other at all distances. In the next section of this paper, it will be shown that this theory can be readily extended to electrons of two complementary charges (conventionally referred to as negative and positive). Again, all experiments show that two electrons with the same type of charge always repel but electrons with opposite charge always attract. It will also be shown (in section 4.3.7) that it is possible to have very large exchanges of momentum when the electrons are very close together in space (or time) but experiments also show that when electrons are very far apart this momentum exchange is very much smaller. These experimental facts exclude Q from consideration as the basis for universal momentum exchange. This returns the focus to the combined two-electron activity B, where the impact varies explicitly with the time (and therefore spatial) separation between the two electrons, at the **two** times of their involvement with each interaction: this is the justification for focusing on each of the electron's activity DNV per segment.

This theory adopts the simplest hypothesis, at this point, by assuming that the rate of activity (i.e. per chronon) exchanged during each interaction is constant: this will be referred to as the "Quantum of Activity" hypothesis. A re-examination of the impact on B shows that in the case where the EM interaction between the two electrons is purely a linear impulse in the longitudinal direction (i.e. where the impulse $\Delta\underline{\mathbf{I}}_n$ is parallel or anti-parallel to the spatial separation $\underline{S}_n$) this impact reduces to a simple DNV. It is reasonable to assume that this new DNV is the same one as that was introduced above for attractive interactions – that is, the *universal action* DNV. Now, since: $\underline{S}_n \bullet \Delta\underline{\mathbf{I}}_n = -\,S_n\,\Delta\mathbf{I}_n$ and $c\,\Delta\underline{\mathbf{I}}_n = -\,\Delta\mathbf{I}_n\,\underline{c}$ then:

$$\Delta_I\,\mathbf{B} = 0$$

Planck's Quantum of Action

When Max Planck was trying to fit his mathematical model of Black-Body radiation to the curve determined by experiments he was forced to introduce an arbitrary relationship between the energy ε of his 'resonators' and their basic frequency ν of the oscillations. These two variables had to be related linearly through a new constant 'h' via the equation: $\varepsilon = h\,\nu$. This constant was soon recognized to be a fundamental characteristic of nature that has still to be explained. This constant has the dimensions of action and was soon referred to as Planck's Action Constant [98]; in Dirac's more modern formulation, which is found to occur more frequently in quantum mechanics, it is written as: $h_D = h/2\pi$.

It seems more than plausible that the fundamental action constant in the present theory is related to Planck's constant; this leads to the foundational quantization principle in the present theory: $a = \gamma\, h$. The form of this 'quantum' constant (a) is also confirmed by conducting a simple dimensional analysis. When looking at the impact of the interaction on such quantities, one can view the change in energy as the product of velocity and impulse (or change in momentum). Using conventional dimensional analysis, action has an unusual mix of dimensions:

$$[\text{Action}] = [\text{T}]\,[\text{K}] = [\text{T}]\,[\text{M}]\,[\text{V}]^2 = [\text{M}]\,[\text{L}]^2\,[\text{T}]^{-1}$$

Now $\Delta\underline{\mathbf{I}}_n$ is an impulse (to change motion) mutually induced in the two electrons arising solely from the fact that they each possess an electric charge q_1 and q_2. Experiments going back to Coulomb [114] 'demonstrated' that small electrified bodies repel or attract each other based on the product of the quantity of charge measured on each body, in analogy with gravity. It is therefore equally plausible to propose that the unit impulse between two electrons is proportional to the product of their unit charges, $q_1 * q_2$. Coulomb 'discovered' that the average force F between two electrified bodies varied inversely with the square of their separation. If [Q] is used to represent the dimension of an electric charge, then Coulomb found that:

$[\text{F}] = [\text{Q}]^2\,[\text{L}]^{-2}$ The definition of impulse implies that dimensionally: $[\Delta\mathbf{I}] = [\text{F}]\,[\text{T}] \;\;\therefore\;\; [\Delta\mathbf{I}] = [\text{Q}]^2\,[\text{L}]^{-2}\,[\text{T}]$

$$[a] = [\text{S}_n]\,[\Delta\mathbf{I}_n] = [\text{L}]\,[\Delta\mathbf{I}] = [\text{Q}]^2\,[\text{L}]^{-1}\,[\text{T}] = [\text{Q}]^2\,[\text{V}]^{-1} \quad\therefore\quad [\text{Q}]^2 = [\text{M}]\,[\text{L}]^3\,[\text{T}]^{-2} = [\text{Action}]\,[\text{V}]$$

There is no point in evaluating the dimensionality of an isolated charge in this theory as it always appears as a product. The dimensional argument for the universal action is the product of two charges divided by velocity. If this is to be treated (see later) as a universal '***micro-action***' constant then it suggests that these parameters might well be the electronic charge e and the speed of 'light' c. These fundamental concepts would then form a new action constant a'.

Definition: **Electron Micro-Action** $\qquad a' \equiv e^2 / \text{c} = \alpha\, h_D = \alpha\, h / 2\pi$

This possibility uses Sommerfeld's Fine Structure Constant [29]: $\qquad \alpha = 2\pi\, e^2 / h\,\text{c} = e^2 / h_D\,\text{c} \cong 1/137$

If it is assumed that $a = a'$ then this is equivalent to assuming that the proportionality constant γ has the value: $\gamma = \alpha/2\pi$. In a later section (see 6.3.3) the hydrogen atom will be examined from the viewpoint of discrete action and it will be shown that identification of the action quantization a with the electron micro-action a' leads directly to the planetary Bohr model of the hydrogen atom. It will be shown that this is due to an unstated assumption about how often the nuclear proton interacts with the orbital electron. That section will also introduce a radical alternative model that offers both a more physical explanation and a path to investigating multi-electron atoms in an analytic approach. A more satisfying philosophical hypothesis is that Planck was right: the total action generated by the two electrons across one complete interaction interval is h, this would be equally generated by each electron, so that: $\gamma = ½$ or about 430 times larger than the micro-action possibility a'. This will become the fundamental quantization hypothesis of this programme.

Hypothesis: **EM Quantization Principle** $\qquad a = h / 2 \qquad$ or $\qquad a = (\pi / \alpha)\, e^2 / c = (\pi / \alpha)\, a'$

4.3.7 THE SHORT-RANGE IMPULSE

Newton's genius was manifested through his Second Law of Motion, which was centered on his revolutionary concept of momentum [115] as the *algebraic product* of a body's mass and its (instantaneous) velocity. The present theory retains the original form of this identity, especially since the interactive impulse is retained as the <u>cause</u> of the electron's instantaneous change in velocity. The analysis above (for both attractive and repulsive interactions) resulted in the general identity:

$$a = \text{S}_n\,\Delta\mathbf{I}_n = h / 2 \qquad \text{with} \quad \underline{\text{S}}_n = \underline{\text{X}}_{12}[\text{T}_n] = \underline{\text{x}}_n - \underline{\text{x}}'_n = \underline{\text{c}}\,(\text{t}_n - \text{t}'_n) = \underline{\text{c}}\,\text{T}_n$$

This leads to the fundamental dynamical equation of the theory of "Discrete EM" (**DEM**) that relates the magnitude of the impulse between two interaction electrons to the temporal separation between them at the two times of the interaction.

Definition: **The Short-Range Electric Impulse** $\Delta \mathbf{I}_n = h / 2S_n = \pi e^2 / (\alpha c^2 T_n)$

This 'near' form resonates with the form of the nuclear chronon τ (section 4.2.2), where it was defined as: $c\tau = \Lambda = e^2/mc^2$

$$\Delta \mathbf{I}_n = (\pi / \alpha)\, mc\, (\Lambda / S_n) \quad \text{or} \quad \mathbf{\Delta I_n = (\pi / \alpha)\, m\, c\, (\tau / T_n)} \quad \text{or} \quad \Delta \mathbf{I}_n = (\pi / \alpha)\, m\, c / \eta_n \quad \text{as} \quad T_n = \eta_n \tau$$

This is a very important result, as it relates the specific impulse to the fundamental measure of electron momentum (mc). The ratio is determined by how many chronons (η_n) it takes, for the interaction to span the distance between them. This is simply an inverse (temporal) separation law, **not** a law of inverse **square** spatial separations. This reflects the fundamental physical assumption of an asynchronous 'ray-like' interaction only between pairs of charges. This can be compared with the traditional static law of EM force that is assumed to be 'broadcast' instantaneously in all directions from the emitting charge.

4.3.8 THE ELECTROMAGNETIC IMPULSE

The three dimensional nature of space means that there are multiple possibilities available when developing any impulse theory of asynchronous interactions between pairs of electrons. The most obvious way is to restrict the impulse to act only longitudinally – that is, only between the two events constituting a single interaction. The next level of complexity could introduce an additional transverse component (or torque) to the impulse that would always generate full 3D motion between the electrons. The third alternative is rather than add torque to the impulse consider adding intrinsic rotation to the motion of all electrons, so that Newton's First Law is extended to inertial angular momentum. This latter hypothesis will become the model developed here. In all cases, the starting point will be the Space-Integrity Condition described here in section 4.2.4.

It will be noted that two forms of the Impulse Law have been introduced so far. This is reminiscent of Planck's insight [116] into black-body radiation when he realized that two mathematical forms of the intensity equation applied to the short and long wavelength regimes – this led directly to his unified radiation law for all wavelengths that accurately matched the earlier results in the two limits. Section 4.3.7 here introduced the short-range form $\Delta \mathbf{I}_n$ while in a previous paper [117] the more distant ('far') one-dimensional model of electron scattering used a constant impulse model defined in terms of the **smallest** change in speed (*speed quantum b*) allowed in such an exchange: $\Delta I_0 \equiv m\, b$; it was proposed that this was a new universal constant b, which was the complement of the maximum relative velocity or 'light-speed' (universally symbolized by c). The magnitude of b was left indeterminate in that paper but was defined in terms of the maximum number of possible interactions between two far electrons N_0. An integrated proposal will now be made here in terms of the temporal separation T_n (or spatial separation $S_n = c\, T_n$) between the emission and absorption of one impulse across the interaction of the two electrons. It is proposed that when this spatial separation is less than a critical value, denoted here by D_S (where $D_S = c\, T_S$), the short-range (or 'near') form applies whereas if the inter-electron separation exceeds this value then the long-range (or 'far') form applies.

Short-Range ('near'): $\Delta \mathbf{I}_n = \boldsymbol{a} / S_n = h / (2cT_n) = (\pi / \alpha)\, m\, c\, (\tau / T_n)$ when $T_n < T_S$

Longe-Range ('far'): $\Delta \mathbf{I}_0 = m\, b = m\, c / N_0$ when $T_n \geq T_S$

These two forms can be combined into one discontinuous form, in terms of the chronon count, $T_n = \eta\tau$:

Definition: **The Electro-Magnetic Impulse** $\Delta \mathbf{I}[\eta\, \tau] = \{\, \theta[N_0 - \eta]\, (\pi / \alpha) / \eta + \theta[\eta - N_0]\, 1/N_0 \,\}\, m\, c$

Here, $\theta[n]$ is the (digital) Heaviside 'step' function whose value is unity when its argument is positive and zero otherwise. The cross-over point occurs when $\Delta \mathbf{I}_S$ equals $\Delta \mathbf{I}_0$ which occurs at (η=N_0): $c\, T_S = \boldsymbol{a} / m$ or $D_S = (\pi / \alpha)\, N_0 \Lambda$.

4.3.9 RELATIONSHIP TO WAVE MECHANICS

The hypothesis for universally quantizing the activity of an electron whenever it interacts with another electron proposed here in section 4.3.6 in terms of the activity DNV $\boldsymbol{a}$ and made specific with the hypothesis that $a = h / 2$ provides an explanation of the use of "wave mathematics" in quantum mechanics. The central idea of the activity DNV was reviewed here in §4.3.3 and was shown in §4.3.5 to be conserved in two-particle attractive interactions that generated periodic, stable configurations. The periodic trajectories are closed after 2G impulses $\Delta \mathbf{I}_j$ (each identified by a unique integer label j = 1, … , 2G). The EM quantum hypothesis was proposed for each and every of the trajectory segments of spatial extent Δx_j and temporal extent Δt_j.

EM Quantum Hypothesis: $\Delta_I \mathbf{R}_j = -\boldsymbol{a}_j = -(h / 2c)\, \mathbf{C}_j = i\, \Delta_I R_j\, \mathbf{I_0}$ {for all j}

The change in the orbital activity interval is equal to a decrease of the interval change in the orbital action: $\Delta_I \mathbb{R}_j = -\Delta\mathcal{A}_j$

This is viewed as separable into a kinetic (motion) part $\Delta\mathcal{A}^K_j$ and a dynamic (impulse) part $\Delta\mathcal{A}^D_j$: $\Delta\mathcal{A}_j = \Delta\mathcal{A}^K_j + \Delta\mathcal{A}^D_j$

Anticipating the velocity features of the digital electron (§5.2.2) and assuming conservation of orbital activity, i.e. $\Delta_I \mathbb{R}_j = 0$

$$\therefore \quad \Delta\mathcal{A}^K_j = m\, \underline{u}_j \bullet \underline{u}_j\, \Delta t_j = h/2 \quad (A) \qquad \therefore \quad \Delta\mathcal{A}^D_j = \underline{x}_j \bullet \Delta\underline{\mathbf{I}}_j = -S_j\, \Delta\mathbf{I}_j = h/2 \quad (B)$$

This last step recognizes that the impulse is central and S_j the spatial separation between the interacting electrons $\underline{S}_j = \underline{c}_j\, T_j$ while the last equation is only valid for 'near' interactions (see §4.3.8) otherwise the 'far' form is used: $\Delta\mathbf{I}_0 = m\, b$. Here $\underline{u}_j$ is the electron's longitudinal velocity along the segment $\Delta\underline{x}_j$ completed in a time of Δt_j so that: $\underline{u}_j = \Delta\underline{x}_j / \Delta t_j$. It is only the nodes (i.e. interaction events at t_j and t_{j+1}) that are significant: this can be viewed as where a periodic function has special (no 'displacement') manifestations since all periodic functions can be mapped into a Fourier series of harmonic functions or 'waves'. This allows a wavelength λ_j to be defined for use in the kinetic action equation with **half** wavelengths:

Electron Wavelength: $\lambda_j \equiv 2\, \Delta x_j \qquad \therefore \quad p_j = h / \lambda_j \quad$ or $\quad p = h / \lambda \quad$ in conservative systems ($p_j = p$).

This is the starting point of modern QM in 1923 when Louis de Broglie proposed that a free electron 'had' a wavelength [91] – he soon generalized this "fictitious associated wave" to all material particles [118]. The obvious next step is to associate a frequency ν_j with the time segment Δt_j. The group velocity of a wave U_g is then identified with u_j ; i.e. $U_g = u_j = \nu_j \lambda_j$.

Electron Frequency: $\nu_j \equiv 1 / 2\, \Delta t_j \qquad$ since $\quad u_j = \Delta x_j / \Delta t_j = \lambda_j / 2\, \Delta t_j = \lambda_j \nu_j$

This leads immediately, via the quantized kinetic action equation above (A), using $\mathcal{K}$ for the kinetic energy: $\mathcal{K}_j = \tfrac{1}{2}\, m\, \underline{u}_j \bullet \underline{u}_j$ while noting that in central, conservative systems the 'potential energy' $\mathcal{U}_j = \mathcal{K}_j$, so that the 'total' energy $\mathcal{E}_j = 2\, \mathcal{K}_j$.

Electron Energy: $\mathcal{E}_j = h\, \nu_j \qquad$ since $\quad h/2 = 2\, \mathcal{K}_j\, \Delta t_j = \mathcal{E}_j\, \Delta t_j = \mathcal{E}_j / 2\, \nu_j$

This equation was both the starting point for the quantum revolution when Planck had to quantize his oscillators in 1900 to derive the energy spectrum equation [119] and Einstein's 'photon' equation in 1904 to derive the photo-electric law [120]. The quantized kinetic action equation above (A) was also the starting hypothesis for Bohr in 1913 [121] when he succeeded in deriving the energy spectrum of the neutral Hydrogen atom using Coulomb's Law of Electrostatics (see §6.3.3). This also needed Planck's Energy/Frequency relationship to relate differences in the quantized energy levels to observed wavelengths. It can be readily seen that Bohr's approach contains an inconsistency, as 2G here is the finite number of segments, which are assumed to be infinite in number in the continuum model of standard QM.

$$\mathcal{A}^K = \sum_{n=1}^{2G} \Delta\mathcal{A}^K_n = \sum_{n=1}^{2G} h/2 = G\, h \quad \therefore \quad \sum_{n=1}^{2G} m\, \underline{u}_n \bullet \Delta\underline{x}_n = G\, h \quad \therefore \quad \oint d\underline{x} \bullet \underline{p} = G\, h \qquad \text{(In the continuum limit.)}$$

These ideas will be taken up in far more detail in a later paper proposing a new theory of particle quantum mechanics.

5. DIGITAL ELECTRONS

This core chapter explores the proposed 'digital' properties of the electron-electron interaction. This analysis will lead to the central idea that each electron executes an intrinsic 4-step, cyclic motion that results in a physical explanation for 'spin'. The digital view of the interaction between two electrons will also provide a physical explanation for the quantization of action. This 'digital rotation' is explored more extensively in the next chapter.

The Major Hypotheses

The present theory is constructed upon three central hypotheses (Hn), each of which generates several corollaries (Cn.m).

H1: The world consists only of electrons.
H2: Electrons have only one mode of interaction (electromagnetism).
H3: The electron interaction defines all the properties of the electron.

The 'Universal Electron Hypothesis' (H1) has six associated corollaries.

C1.1 Electrons have no spatial extent (only points).
C1.2 Electrons are eternal (each exists across all of time and preserves its identity).
C1.3 Reality is the totality of all electrons and their interactions across all of time and space.
C1.4 All matter consists only of electrons, including human beings and their instruments.
C1.5 Elementary particles consist of identical (small) sets of electrons, executing similar motions in time.
C1.6 Space and time are passive relationships between electrons (there are no fields).

The 'Electromagnetic Hypothesis' (H2) has several corollaries, five of which are listed here.

C2.1 The EM interaction is the only reason an electron changes its motion relative to all other electrons.
C2.2 The other three 'forces of nature' are manifestations of the EM interaction at different distances and times.
C2.3 The EM interaction is not represented by any other types (bosons) of particles, real or 'virtual'.
C2.4 The EM interaction is holistic, involving only two electrons at any pair of interaction times.
C2.5 The EM interaction has distinct short-range (longitudinal) and 'far' (transverse) components.

The 'EM Interaction Hypothesis' (H3) has the most corollaries as these are mainly defining sub-hypotheses.

H3A: Electrons only interact when they are 'on each other's light-cone" – the 'Light-Speed Hypothesis'.
H3B: Electrons alternate at a universal rate between two states: 'send' and 'receive' – the 'Chronon Hypothesis.
H3C: Electrons may only interact on the 'tick', on integral multiples of the chronon τ: - the 'Digital Hypothesis'.
H3D: Each interaction involves one 'sending' and one 'receiving' electron – the 'Saturation Hypothesis'.
H3E: Each interaction transfers one unit of micro-action asynchronously between the electrons – the 'quantum'.
H3F: Each electron also moves discretely & orthogonally to its longitudinal motion – the 'Clicking Hypothesis'.
C3.1 The interaction is time-symmetric (retarded and advanced), as any electron may 'send' across time.
C3.2 The electron's 'intrinsic' properties (a, b, c, τ) or (e, m, h) are universal characteristics of the EM interaction.
C3.3 Each electron moves longitudinally across space with a constant velocity between interactions.
C3.4 The smallest change in speed (b), at 'far' separations, is independent of relative separation and velocity.
C3.5 The interaction effects between any two electrons are independent of all other electrons in the universe.
C3.6 The electron's electric charge ($\pm e$) is a reflection of its time offset and direction of 4-step 'clicking' motion.

5.1 UNIVERSAL ELECTRON STATES

This research programme rejects the move to phenomenology that has increasingly characterized theoretical physics over the last 150 years. The ongoing invention of equations (especially differential equations that cannot be solved) to describe a simplified corner of nature has too often only resulted in the creation of contradictions and an obvious lack of progress in this most fundamental of sciences. When one cannot say what is happening in our model of the world then all progress at the higher levels of complexity suffers. The most significant step in this withdrawal from visualization was the conscious rejection of the idea of particle trajectories because "electrons could not be observed without interfering in their motion". The present theory makes a clear distinction about what happens in the world (level I) and how humans go about measuring this behavior (level III). It is our shareable imagination (level II), which acts as the great integrator between these levels. This section will return to the earliest days of natural philosophy and willingly follow Newton in imagining the motion of unobservable particles as they are proposed to move through space acting only under their mutual influences; measurement predictions will be deferred for a later paper when a replacement theory for quantum mechanics will be proposed.

5.1.1 UNIVERSAL MOTION

Macro Irreversibility

In traditional physics, all motion is represented by differential equations that are second-order in time (i.e. $\partial^2/\partial t^2$), which has been interpreted as implying universal time reversibility. The present theory recognizes that **velocity** is always determined by the finite difference in spatial location at two distinct times, which (unlike the Continuum Hypothesis) is never allowed to go to zero: the smallest temporal separation is at least one chronon τ. The velocity of electrons can only be determined by human beings (i.e. measured) by introducing an external interaction with the target electrons. This interaction always alters the original motion of the target electrons in an indeterminable manner leading to irreversibility being inserted back into the micro world from the macro (averaging) world of those giant electron collections known as humans. This is investigated in much more detail in a subsequent paper that provides a new interpretation of quantum theory in terms of measurement.

Direction of Time

In the present theory, the direction of time is absolute: it is always defined by the direction that material bodies move across space, whether these bodies be electrons or macroscopic objects; mathematically this requires: $\underline{X}(t) + \underline{V}\ \Delta t \Rightarrow \underline{X}(t + \Delta t)$. When this motion involves macroscopic collections in 'random' motion it is called 'thermodynamics' – hence the 'arrow of time'. This temporal directionality is a reflection of the impossibility of isolating any subset of electrons in the real world.

Electrons move Forward

This theory avoids any idea of 'traveling backwards in time'. Every electron has its own proper time (cumulative *chronon* count). No matter how fast any electron travels to somewhere else, it cannot return to the exact same location in space again before it was there before. In other words, the motion of <u>any</u> electron <u>defines</u> the *direction of time*. It is certainly not illogical to view an inertial object, like an electron, reaching another spatial location before a parallel <u>theoretical</u> 'light-ray' could have reached that point but it would not be on its own 'light-cone' and so could not interact with itself. Since the present theory rejects the Einsteinian view of time, then an electron reaching a point ahead of its own theoretical 'light-cone' has not gone backwards in time – it has simply moved at a speed exceeding the value *c*. This would require multiple interactions in the same direction to accumulate a sufficient change in momentum. This would require interactions with several <u>other</u> electrons, all moving at different velocities; it could not be achieved by interacting with a single set of co-moving electrons.

Electrons go through Carbon Nano-tubes

Alphenaar's experiments [122] on electron transmission through carbon nano-tubes with no change in spin is interpreted here as an indication that it is possible to have extended electron movement without any interaction with other electrons.

The World cannot be Boring

The present theory of the electron illustrates the intuition that the very structure of the world itself should not be boring. For an electron to exist it must occupy one point in space at every point in time. There can be no change without interaction and there can be no interaction if one electron can occupy the only point in space: this would be a world of no or zero spatial dimensions and only the temporal dimension or symbolically [T; 0D]. Therefore, there must be at least one extended spatial dimension or now [T; 1D]. Two electrons moving on this single line would eventually come together (attraction) or would separate forever (repulsion) – both very uninteresting results. If an electron always moved a distance Λ in a time τ before it twists through 90° and repeats this process it would generate a private square 'flatland' or again [T; 2D]. Introducing an independent 3rd spatial dimension [T; 3D] permits the electron's motion in this third dimension to define the direction of the twist at every instant of time t_n so: $\{\underline{\hat{e}}_2(t_{n+1}) = \underline{\hat{e}}_3(t_n) \wedge \underline{\hat{e}}_1(t_n)\}$ while allowing every interacting electron to travel over all of 3D space. The *Natural Vector* representation of the electron (used in this programme) forbids any two electrons occupying the same position in space at the same instant of time. This allows an existential definition of 'length' as the separation in space between any two electrons, at the same time or the 'duration' of time can be defined by the separation in time between any two electrons occupying the same location in space at different times. Thus, the present model, based on the existence of electrons and their resulting motions due to their mutual interactions, is sufficient to define a very interesting arena of one temporal and three spatial dimensions: a physical reality that is mapped exactly by the mathematics of Natural Vectors – the most complex form of division algebra.

5.1.2 ELECTRON EXISTENCE

In taking the position that philosophy precedes mathematics in theoretical physics, the present theory nails its flag without reservation to the metaphysical tradition in natural philosophy known as 'realism'. This is the view that the world exists independently of human beings and that there are objects whose unconditional existence form the basis of material reality. It has been clear from the beginning of this research programme that these real objects are considered to be those material examples now known to science as 'electrons'. This theory makes a clear distinction between the real existence of a specific electron at a given location in space at a given instant of time (symbolically designated as $x = \{t; \underline{x}\}$) and how humans detect (or 'measure') the existence of this type of particle within a narrow region of space and time. This reflects both facts that all measurements are a sub-class of all electron interactions and all humans are gigantic collections of myriads of electrons. As humans cannot keep track of the identity of individual electrons (especially when they move close together), it is inevitable that all measurements of electrons must become statistical. Standard quantum mechanics (QM) compounds the concepts of existence and measurement into **one** hybrid representation that is intrinsically statistical. The new theory presented here keeps these two concepts separate and works mostly in the realistic (existent) representation; only at the final stages, is the theory's measurement model explicitly introduced in order to compare theoretical predictions with expected measurements. This new measurement model of QM is deferred until a later paper.

Since electrons are viewed here ontologically as the foundation of reality, they are each viewed as **eternal**, existing forever throughout all of time: each electron will always be somewhere in space. Contrary to conventional theories that view matter and 'anti-matter' as disappearing into a 'flash of energy' when they come too close together, the present theory interprets this phenomenon as 'a temporary difficulty in making a measurement' of either of the components or of the composite body. Again, these models will be deferred to a later paper. The central corollary of eternal existence is that the intrinsic identity of each electron must be preserved at all times. This means that there can never be (exist) two or more electrons at the same space and time point. This means that for every point in the universe at any given time there can be either be no electrons present or only one electron present. The vacuum is then defined as any region of 3D space where there are no electrons present at a given instant of time. Since electrons are all that exist then there is no-**thing** that exists in a vacuum.

This unique binary characteristic (of existence or non-existence) at any point in space implies a correspondence between the universal human activity of counting distinct, macroscopic objects that result in our abstract conceptions of the key numbers zero and one. This is the foundation for introducing mathematics into science (and the realist's valid response to Wigner's famous essay on the effectiveness of mathematics in science [123]). In the present case, it corresponds to the isomorphism between the electron existence operator $\mathcal{E}$ and the natural number (counting) operator $\mathcal{N}$. Thus, the existence of an electron (identified by the label 'k') at the position (x) is mapped to the non-zero result of applying the number operator at the same point in space and time.

Hypothesis: **Existence implies Counting**: $\mathcal{E} \approx \mathcal{N}$

A number result of 'one' is interpreted as representing the fact that the electron exists at this point, while a result of 'zero' represents the fact that the electron does not exist at this location in space at this instant of time, *whether measured or not*.

Discrete Representations

As these are operators, it is natural to seek a representation of a state upon which these operators can operate. A linear algebra will be adopted where the eigenvalues of such equations are taken to be real numbers – these will (eventually) be mapped to the statistical results of human measurements. All of the properties characterizing the state of a specific electron will be represented in a dual-space 'bra/ket' notation. It is important to emphasize that these Dirac-like state vectors do **not** correspond to the interpretation or usage established by Dirac, as used in his approach to standard QM [124].

The ket vector $|k\rangle$ will be used to represent the existence of the k^{th} electron at the 3D space location $\underline{x}$ at the time t, when its interaction phase (send/receive) is denoted by the two-valued variable λ (where $\lambda = \pm 1$); this convention is short-hand for the fully parameterized ket:

$$|k\rangle = |k: t, \underline{x}; \lambda\rangle$$

The set of all these kets define a linear, abstract vector space [125] satisfying both the addition of vector pairs and scalar multiplication. Associated with each ket vector $|k\rangle$ is a bra vector $\langle k|$ defined in the dual vector space of the kets. These two spaces can be combined to form normalized, scalar inner-products: $\langle k|k\rangle = \langle k| * |k\rangle$ and vector outer-products: $|k\rangle\langle k|$.

All the parameter values of each ket are uniquely defined for all non-interaction times ($t \neq \eta\tau$); at an interaction time, the positive 'limit' is used, that is to say: $|k: \eta\tau, \underline{x}; \lambda\rangle = |k: \eta\tau + \delta t, \underline{x}; \lambda\rangle$ where $\delta t << \tau$. As has been pointed out earlier, since an electron's existence is forever, then its spatial location $\underline{x}$ is defined uniquely at any time t, so that its location is a continuous function of time $\underline{x}[t]$; this is not true for its velocity $\underline{v}[t]$ which is discontinuous whenever the electron commits to an interaction at any interaction time $n\tau$. Every electron ket is an eigenstate of the global existence operator $\boldsymbol{\mathcal{E}}$ that can be represented as a product of individual existence operators $\boldsymbol{\mathcal{E}}[k]$, one for each electron 'k' and (symbolically) defined for each location $\underline{x}$ and time t. It will prove sufficient to always restrict the existence operators to possible interaction times t_μ when the spatial locations are restricted to the comparable discrete value $\underline{x}_\mu = \underline{x}[t_\mu]$. Since each electron must be somewhere at all times then the eigenvalue equation becomes:

$$\boldsymbol{\mathcal{E}}\,[k': t_{\mu'}, \underline{x}_\nu; \lambda']\,|k: t_\mu, \underline{x}_\mu; \lambda\rangle = \delta_{kk'}\,\delta_{\mu\mu'}\,\delta_{\mu\nu}\,\delta_{\lambda\lambda'}\,|k: t_\mu, \underline{x}_\mu; \lambda\rangle$$

But as: $\langle k': t_{\mu'}, \underline{x}_\nu; \lambda' \,|\, k: t_\mu, \underline{x}_\mu; \lambda\rangle = \delta_{kk'}\,\delta_{\mu\mu'}\,\delta_{\mu\nu}\,\delta_{\lambda\lambda'}$ then: $\boldsymbol{\mathcal{E}}\,[k: t_\mu, \underline{x}_\mu; \lambda] = |k: t_\mu, \underline{x}_\mu; \lambda\rangle\langle k: t_\mu, \underline{x}_\mu; \lambda|$

The proposed isomorphism between the existence operator and the number operator implies that the electron ket $|k\rangle$ is also an eigenvalue of the k^{th} electron's number operator, written as: $\boldsymbol{\mathcal{N}}_\lambda[\,k: t_\mu, \underline{x}_\mu]$. At any single instant, the electron must exist in either its 'send' ($\uparrow$) state or 'receive' ($\downarrow$) state but never both, then the **addition** operand ('plus') can be used to represent this **exclusive** 'OR' condition.

$$\boldsymbol{\mathcal{N}}[\,k: t_\mu, \underline{x}_\mu] = \boldsymbol{\mathcal{N}}_\uparrow[\,k: t_\mu, \underline{x}_\mu] + \boldsymbol{\mathcal{N}}_\downarrow[\,k: t_\mu, \underline{x}_\mu].$$

The fact that the electron is in the state λ at $\{t; \underline{x}\}$ implies that electron 'k' must **exist** at $\{t; \underline{x}\}$ AND **be** in the interaction state λ: both conditions being implied by the eigenvalue of '1' of the corresponding number operator. The extended prefix notation '!' will be used to indicate the negative logical situation (or 'not'); so that '! $\underline{x}$' signifies that the electron is not at location '$\underline{x}$' (it is somewhere else) and '! λ' signifying that the electron is not in the interaction-state 'λ' (it is in state '$-\lambda$').

The remainder of this section will focus on a single electron at a given point in space and time, so the corresponding detail parameters will be temporarily omitted. The (reduced) number operator has mutually exclusive eigenvalues n_λ, so that:

$$\boldsymbol{\mathcal{N}}_\lambda\,|\,\lambda\rangle \equiv n_\lambda\,|\,\lambda\rangle \quad \text{and} \quad \boldsymbol{\mathcal{N}}_\lambda\,|\,\lambda'\rangle = \delta_{\lambda\lambda'}\,n_\lambda\,|\,\lambda\rangle$$

This has the (expected) solutions: $n_\lambda = 1$ and $n_\lambda = 0$ where both values satisfy: $n_\lambda^2 = n_\lambda$ since:

$$\boldsymbol{\mathcal{N}}_\lambda^2\,|\,\lambda'\rangle = \boldsymbol{\mathcal{N}}_\lambda\,\boldsymbol{\mathcal{N}}_\lambda\,|\,\lambda'\rangle = n_\lambda\,\delta_{\lambda\lambda'}\,\boldsymbol{\mathcal{N}}_\lambda\,|\,\lambda\rangle = n_\lambda^2\,\delta_{\lambda\lambda'}\,\boldsymbol{\mathcal{N}}_\lambda\,|\,\lambda\rangle = n_\lambda\,\delta_{\lambda\lambda'}\,|\,\lambda\rangle = \boldsymbol{\mathcal{N}}_\lambda\,|\,\lambda'\rangle \quad \therefore \quad \boldsymbol{\mathcal{N}}_\lambda\,(\boldsymbol{\mathcal{N}}_\lambda - 1) = 0$$

So, $\boldsymbol{\mathcal{N}}_\lambda$ can be represented by the outer-product (or projection operator) of its own eigenstate $|\,\lambda\rangle$.

$$\therefore \quad \boldsymbol{\mathcal{N}}_\lambda = |\,\lambda\rangle\langle\,\lambda| \qquad \& \qquad \langle\,\lambda\,|\,\lambda'\rangle = \delta_{\lambda\lambda'}$$

It is always possible to create a (2x2) matrix representation of these 'send' and 'receive' state by assigning: $\langle\uparrow| \Rightarrow (1\ 0)$ and $\langle\downarrow| \Rightarrow (0\ 1)$. This results in the introduction of the Pauli spin matrices (section 3.1.1) as the basis operators in this simplest of all vector spaces. The two number operators can each be given a binary, symmetric representation:

$$\boldsymbol{\mathcal{N}}_\lambda = \boldsymbol{\mathcal{A}}_\lambda^\dagger\,\boldsymbol{\mathcal{A}}_\lambda$$

It is found that there is only one independent representation: $\boldsymbol{\mathcal{A}}_\uparrow^\dagger = \boldsymbol{\mathcal{A}}_\downarrow = \sigma_+$ and $\boldsymbol{\mathcal{A}}_\downarrow^\dagger = \boldsymbol{\mathcal{A}}_\uparrow = \sigma_-$ and this implies that there is only one independent operator: $\boldsymbol{\mathcal{A}} = \sigma_-$ since $\sigma_+ = \sigma_-^\dagger = \boldsymbol{\mathcal{A}}^\dagger$. This illustrates Pauli's dilemma as this (2x2) representation can only be used to represent a 'spinless' electron without the required two independent states. In the present case, this would only be useful for representing a boring 'free' electron that could never interact with any other electron.

5.1.3 INTERACTION STATES

The last section ended with the conclusion that a (2x2) matrix representation could not generate the duplexity that is a basic characteristic of the electron. This was shown to be a result of the failure of this representation to provide a distinct matrix representation for the two types of electron number operators, $\mathcal{N}_{\uparrow}$ and $\mathcal{N}_{\downarrow}$. A detailed analysis shows that this was due to the identification of the two phase states $|\uparrow\rangle$ and $|\downarrow\rangle$, or $|S\rangle$ and $|R\rangle$, with the two base vectors, $|I_+\rangle$ and $|I_-\rangle$, where:

$$|I_+\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad \& \quad |I_-\rangle = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad \text{These are orthonormal, so that:} \quad \langle I_\lambda | I_{\lambda'} \rangle = \delta_{\lambda\lambda'}$$

The ortho-normality property is sufficient to define their own (2x2) projection operators: $\mathbf{I_\lambda} \equiv |I_\lambda\rangle\langle I_\lambda|$

In terms of (2x2) matrices these have the form: $\mathbf{I}_+ = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$ and $\mathbf{I}_- = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$ while $\mathbf{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \mathbf{I}_+ + \mathbf{I}_-$

Dual Spaces

Since any (2x2) matrix representation is a sub-group of any larger (2Nx2N) group, this suggests (as Dirac found) that the larger (4x4) group is a better candidate representation for the electron. Maximum flexibility arises if the matrix elements are allowed to be complex numbers defining the $\mathbb{C}4$ group. The vectors in this space are just ordered sets of 4 complex numbers arranged in column format (kets) or row format (bras). In order to distinguish these two spaces, (4x4) objects will be written in a larger, bold font from the normal fonts used for (2x2) objects, which when using complex numbers represents the $\mathcal{U}$ 2 group. The (4x4) space can be divided into two (2x2) sub-spaces, each labeled with its own index, n = 1 and 2.

$$|I_+[n]\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix}_n \quad \& \quad |I_-[n]\rangle = \begin{vmatrix} 0 \\ 1 \end{vmatrix}_n \qquad \mathbf{I}_+[n] = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}_n \quad \& \quad \mathbf{I}_-[n] = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}_n$$

The first sub-space will be represented by the first two rows of the (4x4) matrices while the second sub-space will use the last two rows of these larger matrices.

$$\therefore \quad |\mathbf{A}[1]\rangle = \begin{vmatrix} x_1 \\ x_2 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} [A] \\ [0] \end{vmatrix} \quad \& \quad |\mathbf{B}[2]\rangle = \begin{vmatrix} 0 \\ 0 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} [0] \\ [B] \end{vmatrix} \quad \text{with} \quad [A] = \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \quad \text{and} \quad [B] = \begin{vmatrix} x_3 \\ x_4 \end{vmatrix} \; \& \; [0] = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

Thus, a (4x1) column-vector $|\mathbf{X}\rangle$ can be represented as a Cartesian product: $|\mathbf{X}\rangle = |\mathbf{A}[1]\rangle \otimes |\mathbf{B}[2]\rangle = (x_1\ x_2\ x_3\ x_4)^T$

Elsewhere

The non-zero (2x2) basis vectors $|I_\lambda\rangle$ will continue to be the focus for representing the send/receive dual phase states.

$$|\uparrow[n]\rangle \cong |I_+[n]\rangle = (1\ 0)_n^T \quad \text{and} \quad |\downarrow[n]\rangle \cong |I_-[n]\rangle = (0\ 1)_n^T$$

The first sub-space can be used to represent the semantic statement: "The kth electron is **not** in its 'send' state at location x at time t." by the algebraic representation:

$$|!\uparrow[1]\rangle \cong !\,|I_+[1]\rangle = |I_-[1]\rangle = (0\ 1)_1^T$$

The second sub-space can also be used to represent the semantic statement: "The kth electron is **not** in its 'receive' state at location x at time t." by the algebraic representation:

$$|!\downarrow[2]\rangle \cong !\,|I_-[2]\rangle = |I_+[2]\rangle = (1\ 0)_2^T$$

The opposite sub-spaces could equally have been used to represent each of these 'negative' statements. So either of these two representations can be combined into new (4x1) column vectors $|\mathbf{E}_+\rangle$ and $|\mathbf{E}_-\rangle$, where 'E' signifies 'Elsewhere'.

$$|\mathbf{E}_+\rangle = |!\uparrow[1]\rangle \otimes |\emptyset[2]\rangle = (0\ 1\ 0\ 0)^T \quad \text{and} \quad |\mathbf{E}_-\rangle = |\emptyset[1]\rangle \otimes |!\downarrow[2]\rangle = (0\ 0\ 1\ 0)^T$$

The two projection operators (or their equivalent ket vectors) can be used to construct two other basis vectors.

$$| \mathbf{I}_+ \rangle = | \mathrm{I}_+[1] \rangle \otimes | \mathrm{Ø}\,[2] \rangle = (1\ 0\ 0\ 0)^{\mathrm{T}} \qquad \text{and} \qquad | \mathbf{I}_- \rangle = | \mathrm{Ø}\,[1] \rangle \otimes | \mathrm{I}_-[2] \rangle = (0\ 0\ 0\ 1)^{\mathrm{T}}$$

These four kets $\{ | \mathbf{I}_+ \rangle , | \mathbf{I}_- \rangle , | \mathbf{E}_+ \rangle , | \mathbf{E}_- \rangle \}$ form a complete basis set for this (4x4) mathematical space.

The corresponding (4x4) 'unity' operators are: $\mathbf{I}_+ = \begin{matrix} | [\mathrm{I}]\ \ [0] | \\ | [0]\ \ [0] | \end{matrix}$ and $\mathbf{I}_- = \begin{matrix} | [0]\ \ [0] | \\ | [0]\ \ [\mathrm{I}] | \end{matrix}$ with $\mathbf{I} = \mathbf{I}_+ + \mathbf{I}_- = \begin{matrix} | [\mathrm{I}]\ \ [0] | \\ | [0]\ \ [\mathrm{I}] | \end{matrix}$

These operators (i.e. 4x4 matrices) satisfy: $\mathbf{I}_\lambda \mathbf{I}_{\lambda'} = \delta_{\lambda\lambda'} \mathbf{I}_\lambda$ and $\mathbf{I}_\lambda | \mathbf{E}_{\lambda'} \rangle = \delta_{\lambda\lambda'} | \mathbf{E}_\lambda \rangle$

The following multiplication rule for the (2x2) Pauli 'raising' and 'lowering' operators $\sigma_\pm$ will be used repeatedly.

$$\sigma_\lambda | \mathrm{I}_{\lambda'} \rangle = (1 - \delta_{\lambda\lambda'}) | \mathrm{I}_\lambda \rangle \qquad \text{For example:} \qquad \sigma_+ | \mathrm{I}_+ \rangle = | 0 \rangle \qquad \& \qquad \sigma_+ | \mathrm{I}_- \rangle = | \mathrm{I}_+ \rangle$$

Possible Electron Creation/Destruction Operators

The next step requires finding an explicit (4x4) binary representation of the two 'send' and 'receive' Number operators $\mathcal{N}_\lambda$ with orthonormal eigenvectors $| \boldsymbol{\lambda} \rangle$, such that:

$$\mathcal{N}_\lambda = \mathcal{A}_\lambda^\dagger \mathcal{A}_\lambda \qquad \& \qquad \mathcal{N}_\lambda^2 = \mathcal{N}_\lambda \qquad \& \qquad \mathcal{N}_\lambda | \boldsymbol{\lambda}' \rangle = \delta_{\lambda\lambda'} | \boldsymbol{\lambda} \rangle \qquad \text{with} \quad \lambda = \pm$$

The semantic interpretation of the 'Elsewhere' ket vectors means they must both satisfy their defining property:

$$\langle \mathbf{E}_{\lambda'} | \mathcal{N}_\lambda | \mathbf{E}_{\lambda'} \rangle = 0 \quad \text{or} \quad \langle \mathbf{E}_{\lambda'} | \mathcal{A}_\lambda^\dagger \mathcal{A}_\lambda | \mathbf{E}_{\lambda'} \rangle = 0 \quad \therefore \quad \mathcal{A}_\lambda | \mathbf{E}_{\lambda'} \rangle = | \boldsymbol{\mathrm{Ø}} \rangle \qquad \text{Also} \quad \mathcal{A}_\lambda^\dagger | \mathbf{E}_{-\lambda} \rangle = | \boldsymbol{\mathrm{Ø}} \rangle$$

These equations have two independent solutions (or 4x4 matrix representations):

$$\mathcal{A}_+ = \begin{matrix} | \sigma_-\ \ \mathrm{Ø} | \\ | \mathrm{Ø}\ \ \mathrm{Ø} | \end{matrix} \qquad \& \qquad \mathcal{A}_- = \begin{matrix} | \mathrm{Ø}\ \ \mathrm{Ø} | \\ | \mathrm{Ø}\ \ \sigma_+ | \end{matrix} \qquad \therefore \quad \mathcal{A}_+^\dagger = \begin{matrix} | \sigma_+\ \ \mathrm{Ø} | \\ | \mathrm{Ø}\ \ \mathrm{Ø} | \end{matrix} \qquad \& \qquad \mathcal{A}_-^\dagger = \begin{matrix} | \mathrm{Ø}\ \ \mathrm{Ø} | \\ | \mathrm{Ø}\ \ \sigma_- | \end{matrix}$$

Explicit multiplication shows: $\mathcal{A}_\lambda \mathcal{A}_{\lambda'} = \boldsymbol{\mathrm{Ø}} \qquad \mathcal{A}_\lambda^\dagger \mathcal{A}_{-\lambda} = \boldsymbol{\mathrm{Ø}} \qquad \mathcal{A}_{-\lambda} \mathcal{A}_\lambda^\dagger = \boldsymbol{\mathrm{Ø}}$

These rules are more restrictive than the ususal fermion anti-commutation rules, which require only their sums be zero. It is also important to note that the set of all four operators are linearly independent under conjugation, unlike the Pauli matrices. This independence can be readily seen from their explicit 4x4 forms, where no non-zero number (unity) appears in more than one row or column. It is also significant that they are each purely real, integer matrices.

$$\mathcal{A}_+ = \begin{matrix} | 0\ 0\ 0\ 0 | \\ | 1\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \end{matrix} \qquad \mathcal{A}_- = \begin{matrix} | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 1 | \\ | 0\ 0\ 0\ 0 | \end{matrix} \qquad \mathcal{A}_+^\dagger = \begin{matrix} | 0\ 1\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \end{matrix} \qquad \mathcal{A}_-^\dagger = \begin{matrix} | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 1\ 0 | \end{matrix}$$

It can be easily shown by direct multiplication that these pairwise combinations of (4x4) matrices form a complete isomorphism with the Electron Number operators N; they can be combined symmetrically (for the global Number operator) or **anti-symmetrically** to form the global 'Spin' operator, M.

Definition: **Number** $\mathcal{N} \equiv \mathcal{N}_+ + \mathcal{N}_-$ & Definition: **Spin** $\mathcal{M} \equiv \mathcal{N}_+ - \mathcal{N}_-$

Explicitly:

$$\mathcal{N}_+ = \begin{matrix} | 1\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \end{matrix} \qquad \mathcal{N}_- = \begin{matrix} | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 1 | \end{matrix} \qquad \mathcal{N} = \begin{matrix} | 1\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 1 | \end{matrix} \qquad \mathcal{M} = \begin{matrix} | 1\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ 0 | \\ | 0\ 0\ 0\ -1 | \end{matrix}$$

Electron Spin Operators?

The link to traditional 'spin' ideas can be made more explicit by defining the following (4x4) 'spin' matrices.

$$\boldsymbol{\Sigma}_+ \equiv \begin{vmatrix} \sigma_3 & \emptyset \\ \emptyset & \emptyset \end{vmatrix} \quad \& \quad \boldsymbol{\Sigma}_- \equiv \begin{vmatrix} \emptyset & \emptyset \\ \emptyset & \sigma_3 \end{vmatrix} \quad \& \quad \boldsymbol{\Sigma}_3 \equiv \boldsymbol{\Sigma}_+ + \boldsymbol{\Sigma}_- = \begin{vmatrix} \sigma_3 & \emptyset \\ \emptyset & \sigma_3 \end{vmatrix} \quad \therefore \quad \boldsymbol{\Sigma}_3 = \sigma_3 \mathbf{I}$$

Explicit multiplication shows: $\boldsymbol{\Sigma}_- \boldsymbol{\Sigma}_+ = \boldsymbol{\emptyset}$ $\quad \boldsymbol{\Sigma}_+ \boldsymbol{\Sigma}_- = \boldsymbol{\emptyset}$ while $\boldsymbol{\Sigma}_+ = \sigma_3 \mathbf{I}_+$ & $\boldsymbol{\Sigma}_- = \sigma_3 \mathbf{I}_-$

As creation/destruction operators: $\mathcal{A}_\lambda \mathcal{A}_\lambda^\dagger - \mathcal{A}_\lambda^\dagger \mathcal{A}_\lambda = \boldsymbol{\Sigma}_\lambda$ and $\mathcal{A}_\lambda \mathcal{A}_\lambda^\dagger + \mathcal{A}_\lambda^\dagger \mathcal{A}_\lambda = \mathbf{I}_\lambda$

It is important to realize that these are **not** the Dirac (fermion) Number operators and their commutators are **not** zero.

In summary, the following multiplication rules often prove useful.

$$\mathcal{N}_\lambda \mathcal{N}_{\lambda'} = \delta_{\lambda\lambda'} \mathcal{N}_\lambda \quad \mathcal{A}_\lambda \mathcal{N}_{\lambda'} = \delta_{\lambda\lambda'} \mathcal{A}_\lambda \quad \mathcal{M} \mathcal{N}_\lambda = \mathcal{N}_\lambda \mathcal{M} = \lambda \mathcal{N}_\lambda \quad \mathcal{A}_\lambda \mathcal{M} = \lambda \mathcal{A}_\lambda \quad \mathcal{M} \mathcal{A}_\lambda = \boldsymbol{\emptyset}$$

Electron Spin States?

These operators and ket vectors in the (4x4) matrix representation now allow the definition of two important new vectors.

Definitions: **Positive State** $|+1\rangle \equiv \mathcal{A}_+^\dagger |\mathbf{E}_+\rangle$ & **Negative State** $|-1\rangle \equiv \mathcal{A}_-^\dagger |\mathbf{E}_-\rangle$

Thus, the four states $\{ \mathcal{A}_+^\dagger |\mathbf{E}_+\rangle , |\mathbf{E}_+\rangle , |\mathbf{E}_-\rangle , \mathcal{A}_-^\dagger |\mathbf{E}_-\rangle \}$ form a complete orthonormal set and these will be shown now to be the most useful four ket vectors for representing the characteristics of the real electron. Their explicit form shows:

$$|+1\rangle = (1\ 0\ 0\ 0)^T \ ; \ |\mathbf{E}_+\rangle = (0\ 1\ 0\ 0)^T \ ; \ |\mathbf{E}_-\rangle = (0\ 0\ 1\ 0)^T \ ; \ |-1\rangle = (0\ 0\ 0\ 1)^T$$

These four basis vectors can be used to construct the four projection operators of this mathematical space: $\mathcal{P}_\mu \equiv |\mu\rangle\langle\mu|$

$$\therefore \ |+1\rangle\langle+1| = \mathcal{A}_+^\dagger \mathcal{A}_+ = \mathcal{N}_+ \ ; \ |\mathbf{E}_+\rangle\langle\mathbf{E}_+| = \mathcal{A}_+ \mathcal{A}_+^\dagger \ ; \ |\mathbf{E}_-\rangle\langle\mathbf{E}_-| = \mathcal{A}_- \mathcal{A}_-^\dagger \ ; \ |-1\rangle\langle-1| = \mathcal{A}_-^\dagger \mathcal{A}_- = \mathcal{N}_-$$

The positive and negative kets $|\boldsymbol{\lambda}\rangle$ are eigenvectors of both N and M: $\mathcal{N}|\boldsymbol{\lambda}\rangle = |\boldsymbol{\lambda}\rangle$ & $\mathcal{M}|\boldsymbol{\lambda}\rangle = \lambda|\boldsymbol{\lambda}\rangle$

This demonstrates that these two vectors are the two eigenvectors of the 'Spin' operator $\mathcal{M}$ and are isomorphic with the 'send' and 'receive' vectors, leading to the proposal that electron 'spin' is actually a reflection of how and <u>when</u> electrons interact with each other; thus:

$$|\mathbf{S}\rangle = |\uparrow\rangle \approx |+1\rangle \quad \& \quad |\mathbf{R}\rangle = |\downarrow\rangle \approx |-1\rangle$$

Representational Problems

There are several major problems with this representation, not the least of which, is that it is fundamentally asymmetric since only two of the four states actually correspond to the 'send' and 'receive' states. The problem here is that the two 'Elsewhere' states are interpreted semantically in terms of 'other space' locations, while the phase concept is primarily about time. All attempts to construct a Time-Evolution operator $\mathbf{T}_P$ (see later) in this representation are very asymmetric. However, the four cyclic states should each appear equally as is implied by the four-step equation: $\mathbf{T}_P^4 = \mathbf{I}$. This suggests that a new representation, not involving the $\mathcal{A}_\lambda$ operators, is needed that retains some of these elements, such as the four base states. The difficulty here can be traced to the 'ansatz' solution that associated the core binary operator $\mathcal{A}_\lambda$ with the elementary Pauli spin matrix $\sigma_{-\lambda}$.

This research has identified this superior representation, known here as the '**canonical representation**'; this is reported next where the extra symmetry of time is introduced as every electron may interact forwards or backwards in time. This now introduces two possible 'send' states and two possible 'receive' states in every four possible states in a complete cycle.

5.1.4 THE CANONICAL PHASE REPRESENTATION

Phase Transition Operators

This section describes a fully symmetric representation of the four 'phase-states' that characterize the potential interaction behavior of every electron: this is refered to as the 'canonical phase representation'. The focus will be on the set of four (4x4) operators $\mathcal{C}_\nu$ representing the transitions between the four phase-states. The starting point, as before (see section 5.1.3) is with the two base operators $\mathbf{I_\lambda}$ of the (2x2) real number (matrix) space.

$$\mathbf{I_\lambda} \equiv |\, I_\lambda \rangle \langle\, I_\lambda \,| \quad \text{where } \lambda = \pm 1 \quad \text{and} \quad \langle\, I_+ \,| = (\,1 \;\; 0\,) \quad \& \quad \langle\, I_\lambda \,| = (\,0 \;\; 1\,) \quad \text{such that: } \langle\, I_\lambda \,|\, I_{\lambda'} \rangle = \delta_{\lambda\lambda'}$$

The four (4x4) base states $|\, \boldsymbol{\nu} \,\rangle$ with $\nu = 1,2,3,4$ are now defined symmetrically as Cartesian products:

$$|\, \mathbf{1} \,\rangle = |\, I_+[1] \,\rangle \otimes |\, \emptyset\,[2] \,\rangle = (\,1\;0\;0\;0\,)^T \qquad \text{and} \qquad |\, \mathbf{2} \,\rangle = |\, \emptyset\,[1] \,\rangle \otimes |\, I_-[2] \,\rangle = (\,0\;0\;0\;1\,)^T$$

$$|\, \mathbf{3} \,\rangle = |\, \emptyset\,[1] \,\rangle \otimes |\, I_+[2] \,\rangle = (\,0\;0\;1\;0\,)^T \qquad \text{and} \qquad |\, \mathbf{4} \,\rangle = |\, I_-[1] \,\rangle \otimes |\, \emptyset\,[2] \,\rangle = (\,0\;1\;0\;0\,)^T$$

Each of these states defines its own projection operator $\boldsymbol{\mathcal{E}}_\nu$ with their own eigenvalues ε_ν, remembering that: $\langle\, \boldsymbol{\nu} \,|\, \boldsymbol{\nu}' \,\rangle = \delta_{\nu\nu'}$

Definition: **Existence Operator** $\quad \boldsymbol{\mathcal{E}}_\nu \equiv |\, \boldsymbol{\nu} \,\rangle \langle\, \boldsymbol{\nu} \,| \quad \& \quad \boldsymbol{\mathcal{E}}_\nu |\, \boldsymbol{\nu} \,\rangle = \varepsilon_\nu |\, \boldsymbol{\nu} \,\rangle \quad \therefore \quad \varepsilon_\nu = 1$

Since the electron is <u>always</u> in one of these four states, then (semantically and mathematically): $\boldsymbol{\mathcal{E}}_\nu |\, \mathbf{!}\, \boldsymbol{\nu} \,\rangle = 0 |\, \mathbf{!}\, \boldsymbol{\nu} \,\rangle$
Alternatively, the 'Phase-Existence' operator $\boldsymbol{\mathcal{E}}_P$ behaves like the 'Phase-Number' or unity operator $\mathbf{I}$:

$$\boldsymbol{\mathcal{N}}_P \approx \boldsymbol{\mathcal{E}}_P \equiv \sum_1^4 \boldsymbol{\mathcal{E}}_\nu = \sum_1^4 |\, \boldsymbol{\nu} \,\rangle \langle\, \boldsymbol{\nu} \,| = \mathbf{I} \qquad \text{Since} \quad \boldsymbol{\mathcal{E}}_P |\, \boldsymbol{\nu} \,\rangle = |\, \boldsymbol{\nu} \,\rangle$$

Since the 'spin operator' $\boldsymbol{\mathcal{M}}$ is isomorphic with both (2x2) representations: $\boldsymbol{\mathcal{M}} \approx \{\, I_+[1] - I_-[1] \,\} \cong \{\, I_+[2] - I_-[2] \,\}$
This suggests the full anti-symmetric definition:

Definition: **Spin Operator** $\quad \boldsymbol{\mathcal{M}} \equiv \sum_1^4 (-1)^{\nu+1} |\, \boldsymbol{\nu} \,\rangle \langle\, \boldsymbol{\nu} \,| \quad \& \quad \boldsymbol{\mathcal{M}} |\, \boldsymbol{\nu} \,\rangle = (-1)^{\nu+1} |\, \boldsymbol{\nu} \,\rangle$

Electron Spin States

These definitions confirm the assignment of the 'send' state with the two 'odd' states ('1' and '3') and the 'receive' state with the two 'even' states ('2' and '4'). The 'send' states have an eigenvalue of plus one for the Spin operator M but an eigenvalue of minus one for the 'receive' states; hence the 'spin' assignments.

$$|\, \mathbf{S} \,\rangle = |\, \uparrow \,\rangle \approx |\, \mathbf{1} \,\rangle \approx |\, \mathbf{3} \,\rangle \quad \& \quad |\, \mathbf{R} \,\rangle = |\, \downarrow \,\rangle \approx |\, \mathbf{2} \,\rangle \approx |\, \mathbf{4} \,\rangle$$

This 'double' representation reflects the symmetric use of the two mathematical (2x2) sub-spaces. In order to distinguish between these two alternatives, the second sub-space will be given a # superscript. In other words:

$$|\, \mathbf{1} \,\rangle \approx |\, \mathbf{S} \,\rangle \qquad |\, \mathbf{2} \,\rangle \approx |\, \mathbf{R}^{\#} \,\rangle \qquad |\, \mathbf{3} \,\rangle \approx |\, \mathbf{S}^{\#} \,\rangle \qquad |\, \mathbf{4} \,\rangle \approx |\, \mathbf{R} \,\rangle$$

It will be shown later that this double-representation reflects both time-reversal and charge-conjugation. The canonical representation is based on two physical choices: the universal phase state at time zero and the *sequence* of phase-states that all electrons pass through as time progresses.

Canonical: $\quad |\, e : t_n \,\rangle \approx |\, \mathbf{1} \,\rangle \;;\; |\, e : t_n + \tau \,\rangle \approx |\, \mathbf{2} \,\rangle \;;\; |\, e : t_n + 2\tau \rangle \approx |\, \mathbf{3} \,\rangle \;;\; |\, e : t_n + 3\tau \rangle \approx |\, \mathbf{4} \,\rangle \quad \text{with} \quad t_n = 4\, n\, \tau$

The standard (negatively charged) electron is assigned to $|\, \mathbf{S} \,\rangle$ and $|\, \mathbf{R} \,\rangle$ for the possibility of retarded interactions into the future and $|\, \mathbf{S}^{\#} \,\rangle$ and $|\, \mathbf{R}^{\#} \,\rangle$ for the possibility of advanced interactions back into the past.

5.1.5 THE PHASE-STATE TRANSITIONS

The progression of time across the four phase-states is represented by the Phase Time-Evolution operator $\mathbf{T}_P$ defined by:

Definition: **Phase-Evolution Operator** $\quad \mathbf{T}_P \,|\, \nu \,\rangle \equiv |\, \nu{+}1 \,\rangle \quad (\nu = 1,2,3,4$ i.e. modulo 4)

This can be decomposed into its four orthogonal components: $\quad \mathbf{T}_P \equiv \sum_1^4 \mathbf{T}_\nu \quad \therefore \quad \mathbf{T}_\nu = |\, \nu \,\rangle\langle\, \nu{-}1 \,|$ (modulo 4)

Four 'phase-transition' matrices $\mathcal{C}_\nu$ and their adjoints $\mathcal{C}_\nu^\dagger$ can be defined as (4x4) matrices to represent all state changes.

Definition: **State Transition Operators** $\quad |\, \nu \,\rangle \equiv \mathcal{C}_\nu^\dagger \,|\, \nu{+}1 \,\rangle \quad \&\quad \langle\, \nu{+}1 \,|\, \mathcal{C}_\nu = \langle\, \nu \,|$

These have solutions: $\quad \mathcal{C}_\nu^\dagger = |\, \nu \,\rangle\langle\, \nu{+}1 \,| \quad \&\quad \mathcal{C}_\nu = |\, \nu{+}1 \,\rangle\langle\, \nu \,| \quad \therefore \quad \mathcal{C}_\nu^\dagger \mathcal{C}_\nu = |\, \nu \,\rangle\langle\, \nu \,| = \mathcal{N}_\nu \approx \mathcal{E}_\nu$

Thus, these new matrices are also a valid binary representation of the number (or existence) operators for each phase-state; they are also suitable as a decomposition of the time-evolution operators. However, there is another set of four transition matrices $\mathcal{D}_\nu$ which involve a 'double' transition between the four phase-states, these are defined next.

Definition: **Double Transition Operators** $\quad |\, \nu{+}2 \,\rangle \equiv \mathcal{D}_\nu \,|\, \nu \rangle \quad \&\quad \langle\, \nu \,|\, \mathcal{D}_\nu^\dagger = \langle\, \nu{+}2 \,|$

These have solutions: $\quad \mathcal{D}_\nu = |\, \nu{+}2 \,\rangle\langle\, \nu| \quad \&\quad \mathcal{D}_\nu^\dagger = |\, \nu\rangle\langle\, \nu{+}2 \,|$

$$\therefore \quad \mathcal{D}_1 = \mathcal{C}_2\,\mathcal{C}_1 = \mathcal{C}_3^\dagger\,\mathcal{C}_4^\dagger = \mathcal{D}_3^\dagger \quad \&\quad \mathcal{D}_4 = \mathcal{C}_1\,\mathcal{C}_4 = \mathcal{C}_2^\dagger\,\mathcal{C}_3^\dagger = \mathcal{D}_2^\dagger$$

Since: $\mathbf{T}_\nu = \mathcal{C}_\nu^\dagger\,\mathcal{D}_{\nu\text{-}1}$ then: $\mathbf{T}_\nu = \mathcal{E}_\nu\,\mathcal{C}_{\nu\text{-}1}$ i.e. electron moves from $|\, \nu{-}1 \,\rangle$ then confirms its existence at $|\, \nu \,\rangle$.

Explicitly:

$$\mathcal{C}_1^\dagger = \begin{|0\,0\,0\,1|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\end{} \quad \mathcal{C}_2^\dagger = \begin{|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,1\,0|\end{} \quad \mathcal{C}_3^\dagger = \begin{|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,1\,0\,0|\\|0\,0\,0\,0|\end{} \quad \mathcal{C}_4^\dagger = \begin{|0\,0\,0\,0|\\|1\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\end{}$$

Further:

$$\mathbf{T}_1 = \begin{|0\,1\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\end{} \quad \mathbf{T}_2 = \begin{|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\\|1\,0\,0\,0|\end{} \quad \mathbf{T}_3 = \begin{|0\,0\,0\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,1|\\|0\,0\,0\,0|\end{} \quad \mathbf{T}_4 = \begin{|0\,0\,0\,0|\\|0\,0\,1\,0|\\|0\,0\,0\,0|\\|0\,0\,0\,0|\end{}$$

This leads directly to the canonical form of the phase-evolution transition matrix.

$$\mathbf{T}_P = \begin{|\sigma_+\ \sigma_-|\\|\sigma_-\ \sigma_+|\end{} \quad \&\quad \Gamma \equiv \mathbf{T}_P\,\mathbf{T}_P = \begin{|\varnothing\ \mathbf{I}|\\|\mathbf{I}\ \varnothing|\end{} \quad \therefore \quad \Gamma\,\Gamma = \mathbf{I} \quad \therefore \quad (\mathbf{T}_P)^4 = \mathbf{I}$$

Thus, $\mathbf{T}_P$ behaves like the complex fourth roots of unity ($i^4 = 1$) with isomorphisms forming the set $\{\, 1, -1, i, -i \,\}$. The 'inversion' operator Γ (Feynman's α_0) switches the two sub-spaces, as can be confirmed by simple examples, such as:

$$\Gamma \begin{|[a]\ [b]|\\|[A]\ [B]|\end{} = \begin{|[A]\ [B]|\\|[a]\ [b]|\end{} \quad \text{and} \quad \Gamma \begin{|[a]|\\|[A]|\end{} = \begin{|[A]|\\|[a]|\end{}$$

Other useful multiplication rules involving these unitary matrices include:

$$\mathcal{D}_\nu = \Gamma\,\mathcal{N}_\nu \quad ; \quad \mathcal{C}_\nu^\dagger\,\mathcal{C}_{\nu'} = \delta_{\nu\nu'}\,\mathcal{N}_\nu \quad ; \quad \mathcal{C}_\nu\,\mathcal{C}_{\nu'}^\dagger = \delta_{\nu\nu'}\,\mathcal{N}_{\nu+1} \quad ; \quad \mathcal{C}_\nu^\dagger\,\mathcal{D}_{\nu'} = \delta_{\nu'\nu\text{-}1}\,\mathcal{C}_{\nu\text{-}1} = \delta_{\nu'\nu\text{-}1}\,\mathbf{T}_\nu$$

The symmetric phase-transition matrices ($\mathcal{C}_\nu$ and $\mathcal{D}_\nu$) are related to the earlier asymmetric 'number' matrices ($\mathcal{A}_\lambda$) through the 'inversion' (or math-space switching) operator $\mathbf{\Gamma}$. Direct comparison of their explicit (4x4) forms shows the identities:

i) $\mathcal{C}_2^\dagger = \mathcal{A}_-^\dagger$; $\mathcal{C}_2 = \mathcal{A}_-$; $\mathcal{C}_4^\dagger = \mathcal{A}_+$; $\mathcal{C}_4 = \mathcal{A}_+^\dagger$

ii) $\mathcal{C}_1^\dagger = \mathbf{\Gamma}\mathcal{A}_-$; $\mathcal{C}_1 = \mathbf{\Gamma}\mathcal{A}_+$; $\mathcal{C}_3^\dagger = \mathbf{\Gamma}\mathcal{A}_+^\dagger$; $\mathcal{C}_3 = \mathbf{\Gamma}\mathcal{A}_-^\dagger$

iii) $\mathcal{D}_1 = \mathbf{\Gamma}\,\mathcal{A}_+^\dagger\mathcal{A}_+$; $\mathcal{D}_1^\dagger = \mathbf{\Gamma}\mathcal{A}_-\mathcal{A}_-^\dagger$; $\mathcal{D}_2 = \mathbf{\Gamma}\mathcal{A}_-^\dagger\mathcal{A}_-$; $\mathcal{D}_2^\dagger = \mathbf{\Gamma}\,\mathcal{A}_+\mathcal{A}_+^\dagger$

iv) $\mathcal{C}_1^\dagger\mathcal{C}_1 = \mathcal{A}_+^\dagger\mathcal{A}_+$; $\mathcal{C}_2^\dagger\mathcal{C}_2 = \mathcal{A}_-^\dagger\mathcal{A}_-$; $\mathcal{C}_3^\dagger\mathcal{C}_3 = \mathcal{A}_-\mathcal{A}_-^\dagger$; $\mathcal{C}_4^\dagger\mathcal{C}_4 = \mathcal{A}_+\mathcal{A}_+^\dagger$

The simpler, more symmetric relationships of the 'transition' representation make this a superior choice for representing the 'send' and 'receive' states of the electron: this will the choice made here and always used here after.

Time Reversal

The canonical sequence (above) reflects the normal (human scale) flow of time i.e. $|\,\mathbf{1}\,\rangle \rightarrow |\,\mathbf{2}\,\rangle \rightarrow |\,\mathbf{3}\,\rangle \rightarrow |\,\mathbf{4}\,\rangle \rightarrow |\,\mathbf{1}\,\rangle$ etc; this corresponds to the standard definition of the individual phase-transition operators $\mathbf{T}_\nu$. This permits a complementary order that corresponds to the reverse flow of time between the electron's phase states i.e. $|\,\mathbf{4}\,\rangle \rightarrow |\,\mathbf{3}\,\rangle \rightarrow |\,\mathbf{2}\,\rangle \rightarrow |\,\mathbf{1}\,\rangle \rightarrow |\,\mathbf{4}\,\rangle$ etc. This immediately suggests that an inverse (or reverse-time) form of the transition operators.

Definition: **Time-Reverse Operator** $\mathbf{T}_P^{-1}\,|\,\boldsymbol{\nu}\mathbf{+1}\,\rangle \equiv |\,\boldsymbol{\nu}\,\rangle$ (modulo 4)

This can be decomposed into its four orthogonal components: $\mathbf{T}_P^{-1} \equiv \sum_1^4 \mathbf{T}_\nu^{-1}$ $\therefore$ $\mathbf{T}_\nu^{-1} = |\,\boldsymbol{\nu}\mathbf{-1}\,\rangle\langle\,\boldsymbol{\nu}\,|$ (modulo 4)

These obey: $\mathbf{T}_P^{-1}\,\mathbf{T}_P = \mathbf{T}_P\,\mathbf{T}_P^{-1} = \mathbf{I}$; $\mathbf{T}_\nu^{-1} = \mathbf{T}_\nu^\dagger$; $\mathbf{T}_\nu^{-1} = \mathcal{C}_{\nu\text{-}1}^\dagger$; $\mathcal{C}_\nu\,\mathcal{D}_{\nu'} = \delta_{\nu'\nu\text{-}2}\,\mathbf{T}_{\nu+2}^{-1}$

Since $\mathbf{T}_P = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3 + \mathcal{C}_4$ then $\mathbf{T}_P^{-1} = \mathcal{C}_1^\dagger + \mathcal{C}_2^\dagger + \mathcal{C}_3^\dagger + \mathcal{C}_4^\dagger$

5.1.6 COMPLEMENTARY PHASE REPRESENTATION

As was made explicit earlier (section 5.1.4) the present (4x4) matrix representation is actually a duplex form corresponding to the two (2x2) mathematical sub-spaces used to generate the complete group. The canonical representation was a direct result of defining the phase of the electron at time zero, namely $|\,\mathbf{1}\,\rangle$ and the progression through all four phases with time. There is another, complementary representation that reflects an alternative pair of key choices. Eventually, it will be shown that this complementary set form a suitable representation for the positively charged electron, or *positron*.

Since positive and negative electrons attract each other it is not possible (as it is with similarly charged electrons) to rely on 'infinite' repulsion to prevent two oppositely charged electrons from occupying the same location in space at the same time and generating an 'infinite' repulsion at any of the possible interaction times $\mu\tau$. This universal exclusion requirement can be ensured by having all electrons (irrespective of charge) synchronize their interaction phase at the same time. In other words, both positive and negative electrons are in the same 'send' state at time zero and all electrons switch phase-states every chronon. In order for both types of electrons to retain their uniqueness, it is proposed that positrons have their phase-state offset by two chronons from the negative electrons. It is also proposed that positron phase-states follow the opposite order over time from that followed by electrons.

Complementary: $|\,e^{\#} : t_n\,\rangle \approx |\,\mathbf{3}\,\rangle$; $|\,e^{\#} : t_n + \tau\,\rangle \approx |\,\mathbf{2}\,\rangle$; $|\,e^{\#} : t_n + 2\tau\rangle \approx |\,\mathbf{1}\,\rangle$; $|\,e^{\#} : t_n + 3\tau\rangle \approx |\,\mathbf{4}\,\rangle$ with $t_n = 4\,n\,\tau$

It is very important to note that in this theory time always flows in the standard manner (as experienced by humans). This is equally true for positrons as it is for electrons: e.g. the fourth phase-state is always one chronon later for positrons than their existence in the first phase-state just as this is so for the electron in its second phase-state. This leads to the definition of the complementary phase-evolution operator $\mathbf{T}^{\#}_P$.

Definition: **Complementary Phase-Evolution Operator** $\mathbf{T}^{\#}_P\,|\,\boldsymbol{\nu}\,\rangle \equiv |\,\boldsymbol{\nu}\mathbf{-1}\,\rangle$ (modulo 4)

As with electrons, this has a 4-way decomposition: $\mathbf{T}^{\#}_{P} \equiv \sum_{1}^{4} \mathbf{T}^{\#}_{\nu} \quad \therefore \quad \mathbf{T}^{\#}_{\nu} = |\,\boldsymbol{\nu}\,\rangle\langle\,\nu{+}1\,|$ (modulo 4)

Four complementary 'phase-transition' matrices $\mathcal{C}^{\#}_{\nu}$ and their adjoints $\mathcal{C}^{\#}_{\nu}{}^{\dagger}$ can also be defined as (4x4) matrices.

Definition: **Comp.State Transition Operators** $\quad |\,\boldsymbol{\nu}\,\rangle \equiv \mathcal{C}^{\#}_{\nu}{}^{\dagger}\,|\,\boldsymbol{\nu{-}1}\,\rangle \quad \& \quad \langle\,\boldsymbol{\nu{-}1}\,|\,\mathcal{C}^{\#}_{\nu} = \langle\,\boldsymbol{\nu}\,|$

These have solutions: $\quad \mathcal{C}^{\#}_{\nu}{}^{\dagger} = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu{-}1}\,| \quad \& \quad \mathcal{C}^{\#}_{\nu} = |\,\boldsymbol{\nu{-}1}\,\rangle\langle\,\boldsymbol{\nu}\,| \quad \therefore \quad \mathcal{C}^{\#}_{\nu}{}^{\dagger}\,\mathcal{C}^{\#}_{\nu} = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu}\,| = \mathcal{N}^{\#}_{\nu} \approx \mathcal{E}^{\#}_{\nu}$

Since $\mathcal{E}^{\#}_{\nu}\,|\,\boldsymbol{\nu}'\,\rangle = \delta_{\nu'\nu}\,|\,\boldsymbol{\nu}\,\rangle$ it is reasonable to interpret $\mathcal{E}^{\#}_{\nu}$ as the positron's Existence operator when it is in its ν^{th} phase-state. As before, it is useful to introduce 'double-transition' operators, here distinguished as $\mathcal{D}^{\#}_{\nu}$.

Definition: **Comp.Double Transition Operators** $\quad |\,\boldsymbol{\nu{-}2}\,\rangle \equiv \mathcal{D}^{\#}_{\nu}\,|\,\boldsymbol{\nu}\rangle \quad \& \quad \langle\,\boldsymbol{\nu}\,|\,\mathcal{D}^{\#}_{\nu}{}^{\dagger} = \langle\,\boldsymbol{\nu{-}2}\,|$

These have solutions: $\quad \mathcal{D}^{\#}_{\nu} = |\,\boldsymbol{\nu{-}2}\,\rangle\langle\,\boldsymbol{\nu}|\quad \& \quad \mathcal{D}^{\#}_{\nu}{}^{\dagger} = |\,\boldsymbol{\nu}\rangle\langle\,\boldsymbol{\nu{-}2}\,|$

Since there are only four phase-states, then: $|\,\boldsymbol{\nu{-}2}\,\rangle = |\,\boldsymbol{\nu{+}2}\,\rangle$ (modulo 4), so that: $\mathcal{D}^{\#}_{\nu} = \mathcal{D}_{\nu}$ and there is no further need to distinguish the D operators. Explicit examination leads to the general identity: $\mathbf{T}^{\#}_{\nu} = \mathcal{C}^{\#}_{\nu}{}^{\dagger}\,\mathcal{D}_{\nu+1}$ and to the following.

$$\therefore \quad \mathcal{D}_1 = \mathcal{C}^{\#}_4\,\mathcal{C}^{\#}_1 = \mathcal{C}^{\#}_3{}^{\dagger}\,\mathcal{C}^{\#}_2{}^{\dagger} = \mathcal{D}_3{}^{\dagger} \quad \& \quad \mathcal{D}_2 = \mathcal{C}^{\#}_1\,\mathcal{C}^{\#}_2 = \mathcal{C}^{\#}_4{}^{\dagger}\,\mathcal{C}^{\#}_3{}^{\dagger} = \mathcal{D}_4{}^{\dagger}$$

Since: $\mathbf{T}^{\#}_{\nu} = \mathcal{C}^{\#}_{\nu}{}^{\dagger}\,\mathcal{D}_{\nu+1}$ then: $\mathbf{T}^{\#}_{\nu} = \mathcal{E}^{\#}_{\nu}\,\mathcal{C}^{\#}_{\nu+1}$ i.e. positron moves from $|\,\boldsymbol{\nu{+}1}\,\rangle$ then confirms its existence at $|\,\boldsymbol{\nu}\,\rangle$.

Explicitly:
$$\mathcal{C}^{\#}_1{}^{\dagger} = \begin{|0\ 1\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\end{} \quad \mathcal{C}^{\#}_2{}^{\dagger} = \begin{|0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |1\ 0\ 0\ 0|\end{} \quad \mathcal{C}^{\#}_3{}^{\dagger} = \begin{|0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 1|\\ |0\ 0\ 0\ 0|\end{} \quad \mathcal{C}^{\#}_4{}^{\dagger} = \begin{|0\ 0\ 0\ 0|\\ |0\ 0\ 1\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\end{}$$

Further:
$$\mathbf{T}^{\#}_1 = \begin{|0\ 0\ 0\ 1|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\end{} \quad \mathbf{T}^{\#}_2 = \begin{|0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 1\ 0|\end{} \quad \mathbf{T}^{\#}_3 = \begin{|0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 1\ 0\ 0|\\ |0\ 0\ 0\ 0|\end{} \quad \mathbf{T}^{\#}_4 = \begin{|0\ 0\ 0\ 0|\\ |1\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\\ |0\ 0\ 0\ 0|\end{}$$

This leads directly to the complementary form of the phase-evolution transition matrix.

$$\mathbf{T}^{\#}_{P} = \begin{|\sigma_-\ \sigma_+|\\ |\sigma_+\ \sigma_-|\end{} = \mathbf{T}_P{}^{\dagger} \quad \& \quad \boldsymbol{\Gamma} \equiv \mathbf{T}^{\#}_{P}\,\mathbf{T}^{\#}_{P} = \begin{|\ \varnothing\ \mathbf{I}\ |\\ |\ \mathbf{I}\ \varnothing\ |\end{} \quad \therefore \quad (\mathbf{T}^{\#}_{P})^4 = \mathbf{I}$$

Since the canonical and complementary matrices both operate on the same set of base vectors $|\,\boldsymbol{\nu}\rangle$ and both are created from the outer products of these vectors (i.e. $|\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu}\pm\mathbf{1}\,|$) then these two representations must be inter-related.

Fundamentally:
$$|\,\boldsymbol{\nu}\,\rangle = \mathcal{C}^{\#}_{\nu}{}^{\dagger}\,|\,\boldsymbol{\nu{-}1}\,\rangle = \mathcal{C}_{\nu}{}^{\dagger}\,|\,\boldsymbol{\nu{+}1}\,\rangle \qquad \mathcal{C}^{\#}_{\nu}{}^{\dagger} = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu{-}1}\,| = \mathcal{C}_{\nu\text{-}1}$$

$$\mathcal{C}^{\#}_{\nu} = |\,\boldsymbol{\nu{-}1}\,\rangle\langle\,\boldsymbol{\nu}\,| = \mathcal{C}_{\nu\text{-}1}{}^{\dagger} \qquad \mathcal{D}_{\nu} = |\,\boldsymbol{\nu{-}2}\,\rangle\langle\,\boldsymbol{\nu}|\ = |\,\boldsymbol{\nu{+}2}\,\rangle\langle\,\boldsymbol{\nu}|\ = \mathcal{D}_{\nu+2}{}^{\dagger}$$

$$\mathcal{E}^{\#}_{\nu} = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu}\,| = \mathcal{E}_{\nu} \qquad \mathbf{T}^{\#}_{\nu} = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu{+}1}\,| = \mathbf{T}_{\nu+1}{}^{\dagger}$$

This concludes the analysis of the four phase-states of the electron, which have been shown to be a double representation of the interaction 'send' and 'receive' states of the present theory. The canonical and complementary progressions through these four states will prove sufficient to give a realistic model of both electrically negative and positive charged electrons without invoking metaphysical fictions like Dirac's "sea" of negative energy states. These will be explored forthwith.

It would be intriguing if the Natural Vector bases could be used to create the Phase Transition matrices – unfortunately this is not directly so, as can be readily seen. The real (4 x 4) bases $\mathbf{I}_\mu$ of the Natural Vectors were introduced in the first paper in this series [1], their explicit form is given by:

$$\mathbf{I_0} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \mathbf{I_1} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{vmatrix} \quad \mathbf{I_2} = \begin{vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \quad \mathbf{I_3} = \begin{vmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}$$

When these matrices are applied to the canonical (4x1) phase vectors $|\,\boldsymbol{\nu}\,\rangle$ they generate the following results.

$$\mathbf{I_1}\,|\,\mathbf{1}\,\rangle = -|\,\mathbf{4}\,\rangle \qquad \mathbf{I_1}\,|\,\mathbf{4}\,\rangle = +|\,\mathbf{1}\,\rangle \qquad \mathbf{I_1}\,|\,\mathbf{2}\,\rangle = +|\,\mathbf{3}\,\rangle \qquad \mathbf{I_1}\,|\,\mathbf{3}\,\rangle = -|\,\mathbf{2}\,\rangle$$

$$\mathbf{I_2}\,|\,\mathbf{1}\,\rangle = +|\,\mathbf{2}\,\rangle \qquad \mathbf{I_2}\,|\,\mathbf{2}\,\rangle = -|\,\mathbf{1}\,\rangle \qquad \mathbf{I_2}\,|\,\mathbf{3}\,\rangle = -|\,\mathbf{4}\,\rangle \qquad \mathbf{I_2}\,|\,\mathbf{4}\,\rangle = +|\,\mathbf{3}\,\rangle$$

$$\mathbf{I_3}\,|\,\mathbf{1}\,\rangle = +|\,\mathbf{3}\,\rangle \qquad \mathbf{I_3}\,|\,\mathbf{3}\,\rangle = -|\,\mathbf{1}\,\rangle \qquad \mathbf{I_3}\,|\,\mathbf{2}\,\rangle = +|\,\mathbf{4}\,\rangle \qquad \mathbf{I_3}\,|\,\mathbf{4}\,\rangle = -|\,\mathbf{2}\,\rangle$$

It can be seen that these natural bases produce pair-wise subsets of 'send' and 'receive' vectors with a multiplicative sign reversal introduced whenever a 'send' state is 'flipped' into its corresponding 'receive' state by the $\mathbf{I_1}$ operator while no negative is introduced when the 'receive' state is 'flipped' into its corresponding 'send' state. The $\mathbf{I_2}$ operator reverses this sign impact while also exchanging the spin state with its complementary space. The $\mathbf{I_3}$ operator does not 'flip' the spin state but does exchange the two spin spaces (from 'retarded' to 'advanced'). No single operator $\mathbf{I_k}$ can move the phase state $|\,\boldsymbol{\nu}\,\rangle$ through all four states.

The critical requirement here is the need for the 'inversion' operator $\boldsymbol{\Gamma}$ (introduced above), which switches the two 'spin' sub-spaces. This enables the phase-evolution operator to be defined in terms of sum and inverted-difference of the the two transverse bases; thus:

$$\boldsymbol{\Upsilon}_P = \tfrac{1}{2}\,(\mathbf{I_1} + \mathbf{I_2}) + \tfrac{1}{2}\,\boldsymbol{\Gamma}\,(\mathbf{I_1} - \mathbf{I_2}) = \tfrac{1}{2}\,(\mathbf{I_0} - \boldsymbol{\Gamma})\,\mathbf{I_1} + \tfrac{1}{2}\,(\mathbf{I_0} + \boldsymbol{\Gamma})\,\mathbf{I_2}$$

These insights will be used later in this paper, where the first two Hamiltonian bases ($\mathbf{I_1}$ and $\mathbf{I_2}$) imply transverse motion while $\mathbf{I_3}$ is involved in the longitudinal displacement of the electron in its local reference frame.

5.2 DISCRETE ROTATIONAL MOTION

5.2.1 ROTATIONAL MOTION

As Uhlenbeck said to Goudsmit in 1925 [19]: "This means that the electron has a spin, it rotates." This remark has usually been interpreted as implying that the electron is a small symmetric body and the whole body is rigidly rotating around its own axis. However, the present theory views the electron as a true point-like object, so it cannot rotate around itself. Rather, it is the effect of motion of the electron moving in a spiraling manner around its average, longitudinal movement through space.

Newton begins with Rotation

Newton's first two laws of motion apply only to a single body. It is his third law that links the actions of bodies together (even remotely) when he viewed this action as due only to the mutual interaction of two interacting bodies. As Newton wrote in the preface to the *Principia* [126]: "… all the difficulty of philosophy seems to consist in this – from the phenomena of motion to investigate the forces of nature and then, from these forces, to demonstrate the other phenomena." Newton's astronomical challenge was to reconcile his own priority for straight-line motion with the observed curvilinear motion of the heavenly bodies. The present theory merges these two aspects of motion with the hypothesis that electrons execute a cyclic quasi-rotational motion of sub-microscopic spatial extent that is always transverse to its average rectilinear motion through space. It will be shown that this transverse motion is far too small to be 'seen' directly but is critical to understanding the 'twisted' nature of electromagnetism, as several insightful researchers into EM have realized for almost 200 years.

Euler's Rigid Body

Euler, in his book *Theory of Solid or Rigid Bodies* (1760), showed that the motion of a rigid body could be understood in two quite different ways. The motion could be viewed either as the sum of two independent motions or seen in terms of his new concept: the 'rigid body'. The double-motion model focused on the center of mass moving as if it were a single point where all the mass of the body was concentrated and the body's orientation was along the trajectory. Alternatively, it could be viewed as if a rigid body were moving freely around its center of mass. Even this simplified theoretical model cannot in general be solved – solutions have been found only in special cases where the body satisfies special symmetry requirements. It is not just a coincidence that solid spinning tops have been the focus of 3D mechanics for so long: it is one of the very few examples in classical mechanics where, as Poincare showed, the equations of motion can actually be solved. The present theory rejects all concepts of rigid bodies as an unrealistic mathematical model; such a physical model requires mechanical constraints to propagate throughout the body instantaneously to maintain its rigidity. Mass points are the only existential objects (electrons) and their relative spatial separations are never rigidly fixed: *all is motion*. The present theory also rejects the Continuum Hypothesis, which began with the mistaken view that matter is continuous, and inevitably results in all of the natural world being modeled through differential equations that rarely have closed form (or even stable) solutions.

5.2.2 QUANTIZED ROTATION

The above analysis has demonstrated that both the phase (interaction) component and the kinematical parts of electron activity can be represented mathematically in (4x4) matrix schemes. It was also shown that these could **not** be combined together in a time-**in**dependent evolutionary manner for even a 'free' Newtonian particle that moves one dimensionally through space at a constant velocity. The most powerful reason for keeping key elements of Newton's description of matter is that all experimental evidence todate has shown that the electron is not only singular (no parts) but it has failed to reveal any finite spatial size: physically it always appears as a point in three dimensional space. Furthermore, all investigations of the fundamental electromagnetic interaction between electrons (such as Maxwell's vortex theory) indicate that 'twisting' through space ('curl') is a subtle reflection of the basic 3D nature of the world. This leads the present theory to explicily incorporate intrinsic rotational motion into the basic description of reality: this proposal is referred to as the 'Rotational' hypothesis or in its more complete, digital form, the '*Clicking Hypothesis*'.

The Electron does the 'Twist'

The present theory extends Newton's First Law of Motion by proposing that the electron is always moving in the transverse plane around its average (longitudinal) motion. This extra degree of freedom is a four-step cyclic motion (the 'Twist'), which is always at light-speed and always orthogonal to its current longitudinal direction through space. Each 'step' in this digital, or 'clicking' motion, is in a straight-line from one possible interaction time to the next; in other words, separated in time by one chronon. Note that this 'clicking' motion has nothing to do with circular polarization of so-called electric fields.

4D allows Double Equations

Since the three spatial dimensions can all be treated equally (by simply changing the arbitrary orientation of the 3D spatial reference frame) while the distinct nature of time (contra Einstein) forms a unique, linear sequence then it is always possible to reformulate the space and time equations of physics into two sets conjoined by a common (dimensionless) parameter β. The spatial and temporal parts can be distinguished dimensionally by introducing the imaginary unitary root, as in natural vectors. So, by redefining variables it is always possible to define two consolidated 4D sets of equations; in other words:

$$\text{If: } f_1(\underline{X}(\beta)) = 0 \text{ and } f_2(t(\beta)) = 0 \text{ then: } F(\underline{X}, i\,c\,t) = 0$$

Digital Spin

The present research programme has been most impressed with the rotational model of the electron proposed by Martin Rivas (see §2.4.2). However, his over-emphasis on a strictly mathematical, rather than physical approach and his reliance on the Continuum Hypothesis have meant that the full Rivas model could not form the foundation for the present theory. However, it does possess several attractive features and these have been absorbed here. The two major differences with Rivas are:

i) the electron's charge point is always co-existent with the electron's mass (not just its time-averaged value) as both electric charge and inertial mass are taken here to be intrinsic to the nature of the electron and like all the electron's properties always manifest themselves at the exact, same location in space that defines the presence of the electron;

ii) the independent, transverse motion of the electron is discrete, not continuous, and now requires four discrete moves to complete one 360° revolution around the axis defined by the electron's longitudinal motion.

This type of motion will be referred to as '***digital spin***' and is viewed here as an intrinsic property of all electrons: this local motion is now proposed as fundamental as the electron's mass or charge. In the electron's own CM frame of reference, this transverse motion always occurs at 'light-speed' c. It is this motion that relates the intrinsic distance covered in one chronon by each separate electron (the 'luxon' $\Lambda = c\,\tau$) to the spatial separation defining the interaction between two electrons. The convention adopted here will be to denote the longitudinal velocity of the electron by $\underline{u}$, the transverse velocity by $\underline{w}$ and the combined velocity by $\underline{v}$.

$$\underline{v}[t] = \underline{u}[t] + \underline{w}[t] \qquad \text{with} \qquad \underline{u}[t] \bullet \underline{w}[t] = 0 \qquad \text{and} \qquad \underline{w}[t] \bullet \underline{w}[t] = c^2$$

Each eternal, periodic change in the direction of an electron's transverse motion is referred to here as one '*click*', which can be thought of as a "tick with a twist" since each 'tick' is viewed as one discrete unit of time (the chronon) while the 'twist' (or chirality) may be seen as one unit of space (the luxon). Obviously, there are two possible directions of rotation with this type of motion. In order to achieve agreement with experiments involving nuclear interactions, such as radioactive decay, it is necessary to choose the left-hand twist for negatively charged electrons. Rivas defined this chirality as anti-parallel to the electron's motion, so that the motion of his charge-point was counter-clockwise when looking at it along the spin-direction from the center-of-mass viewpoint. Since the present Digital-Spin model is defined only in terms of relative velocities the same result can be achieved by defining the direction of the 'twist' as RIGHT-HANDED (or clockwise) with respect to the FORWARD motion of the negative electron as time progresses in its standard past-to-future direction (increasing chronon count). This has the added benefit that all rotations are defined in a standard right-handed 3D spatial reference frame. This can be seen in the following diagram.

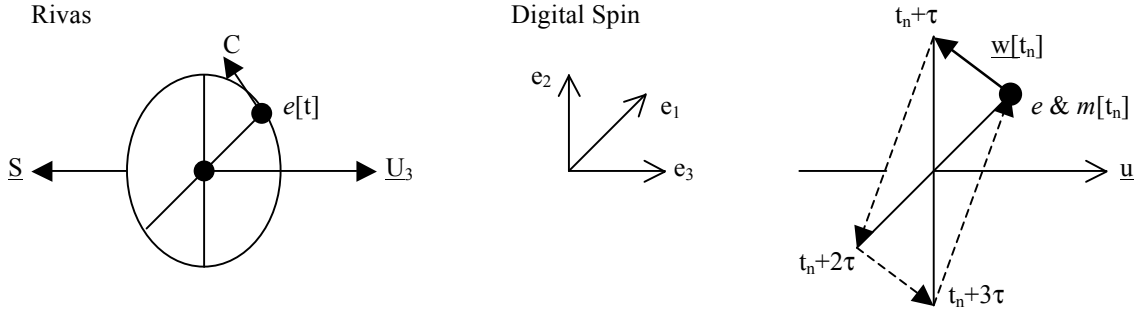


Fig. 11 Electron Chirality

Inertia is Longitudinal

The inertia of an electron is the persistence of resistance to changes in every electron's longitudinal motion. The transverse motion is intrinsic, always occurring at light-speed and does not contribute to the electron's inertial mass, as interactions do not alter this speed and its orientation is determined by its latest longitudinal direction. This new digital spin model of the electron does not contradict the macroscopic view of particle motion according to Newton's three Laws of motion especially if the diameter of rotation remains below direct experimental 'observation'. Conservation of angular momentum is given equal prominence here with conservation of linear momentum: both are viewed as foundational principles of matter.

Center-of-Mass (Electron) Reference Frame

It will often prove helpful to view the world from the perspective of the 'target' electron. When this specific electron is not interacting with any other electron, its longitudinal velocity remains constant (relative to 'distant' reference points). The origin of this reference frame can be either co-located with the electron itself or 'left-behind' somewhere on its longitudinal axis, designated $\underline{\hat{e}}_3$. The transverse axes (designated $\underline{\hat{e}}_1$ and $\underline{\hat{e}}_2$) may be fixed or they may rotate with the transverse motion of the electron. In this CM frame, the standard planar conventions will be used where the $\underline{\hat{e}}_1$ axis directed to the right and the $\underline{\hat{e}}_2$ directed vertically so that $\underline{\hat{e}}_3$ is directed outwards (to the viewer), using a right-handed co-ordinate convention. It is possible to define both a standard or '**canonical**' orientation and a '**diagonal**' orientation: the canonical convention is 'square' to the axes while the complementary convention is 'diamond-like' relative to the axes, these are described in the following tables.

Time	X_1	X_2
t_n	$-\Lambda/2$	$-\Lambda/2$
$t_n + \tau$	$+\Lambda/2$	$-\Lambda/2$
$t_n + 2\tau$	$+\Lambda/2$	$+\Lambda/2$
$t_n + 3\tau$	$-\Lambda/2$	$+\Lambda/2$

Table 2. Canonical Orientation

Time	X_1	X_2
t_n	$+\Lambda\sqrt{2}$	0
$t_n + \tau$	0	$+\Lambda\sqrt{2}$
$t_n + 2\tau$	$-\Lambda\sqrt{2}$	0
$t_n + 3\tau$	0	$-\Lambda\sqrt{2}$

Table 3. Diagonal Orientation

Bending the Rube

The 3D set of spatial locations defines the interactive possibilities of an electron: they define the twisting, rectangular path of the electron over time; the set is sometimes referred to as the '***rube***' (for **r**ectangular t**ube**) rather than a helix. As a result of interactions with other electrons, an electron's rube can be elongated or shortened in its longitudinal direction ('electric forces') or bent (or twisted) in its transverse directions ('magnetic forces') at any of its possible interaction nodes. When an electron's longitudinal speed changes then timing changes are introduced to maintain the light-cone condition; the previous paper [4] began the investigation of this phenomenon, usually analyzed by an Einstein-like distortion of space and time.

Laboratory Frame

The LAB frame-of-reference is any inertial frame that is suitable for tracking the trajectories of one or more electrons and where the frame's unit 'mileposts' do not accelerate relative to the distant 'fixed' points. In particular, the origin or the axial directions of this frame do not change over many chronons, even though the electrons may undergo multiple changes in velocity. In a LAB frame the digital electron 'clicks' around a series of spatial points forming a square orthogonal to its motion – this defines a unit 'cell' of size Λ; four adjacent cells together form a 'block' defining a full cycle of period 4τ. This rectangular tube (or '*rube*') will undergo discontinuous re-orientations at each interaction time t_n when it interacts with another electron. This is illustrated in the following diagram.

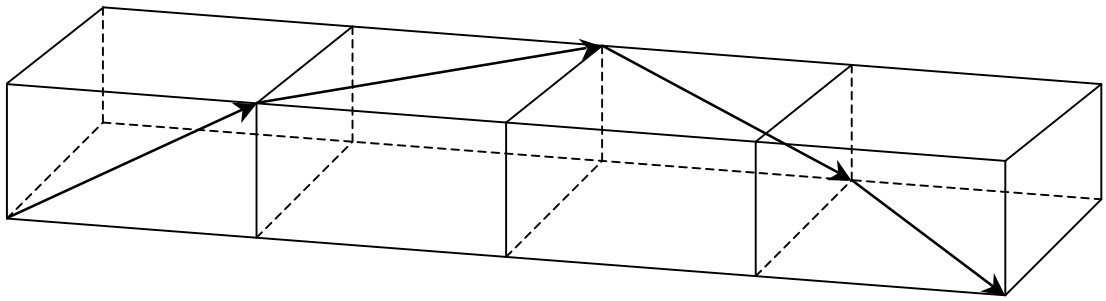


Fig. 12 The Digital Electron's "Rube"

Statistical Averages

In this theory, the electron is always and cyclically changing its direction; it is therefore useful to be able to define average values, which can be compared over longer time periods (several chronons). Local averages can then be simply defined over each electron's complete cycle of four chronons. Unfortunately, even this choice presents ambiguity: selecting either points in time corresponding to possible interaction times or choosing points mid-way between these possible interactions. The first choice will be called the 'Interaction-Average' (or *Position-Representation*) while the second choice will be called the Velocity-Average (or '*Momentum-Representation*'): the computed values in these two representations will always differ.

The *Interaction-Average* uses four consecutive times around a 'target' time of t_n: $\{t_1 = t_n - \tau,\ t_2 = t_n,\ t_3 = t_n + \tau,\ t_4 = t_n + 2\tau\}$. Any time varying function, whose value can be evaluated at any time F[t] has an Interaction-Average value defined as:

Definition: **Interaction-Average** $\langle F[\,t_n\,]\rangle_I \equiv \tfrac{1}{4}\{F[\,t_1\,] + F[\,t_2\,] + F[\,t_3\,] + F[\,t_4\,]\}$

Each 'block' of 4 cells has its own interaction time-average: $\langle t_n \rangle_I = t_n + \tau/2$. In the CM frame, the electron's position and velocity can be evaluated at each of the cell-block corners as a 'free' electron completes one full cycle.

μ	t_μ	$X_1[t_\mu]$	$X_2[t_\mu]$	$X_3[t_\mu]$	$V_1[t_\mu]$	$V_2[t_\mu]$	$V_3[t_\mu]$
1	$t_n - \tau$	$-\Lambda/2$	$-\Lambda/2$	$z_n - u\,\tau$	$+c$	0	u
2	t_n	$-\Lambda/2$	$+\Lambda/2$	z_n	0	$+c$	u
3	$t_n + \tau$	$+\Lambda/2$	$+\Lambda/2$	$z_n + u\,\tau$	$-c$	0	u
4	$t_n + 2\tau$	$+\Lambda/2$	$-\Lambda/2$	$z_n + 2u\,\tau$	0	$-c$	u

Table 4. Position-Representation (over one cycle)

Here, the finite velocity definition has been used: $\underline{V}[t_\mu] \equiv \{\underline{X}[t_{\mu+1}] - \underline{X}[t_\mu]\}/\tau$ and 'light-speed' $c = \Lambda/\tau$; giving:

$$\therefore\ \langle X_1[t_n]\rangle_I = 0\,,\ \langle X_2[t_n]\rangle_I = 0\,,\ \langle X_3[t_n]\rangle_I = z_n + u\,\tau/2\ ;\ \langle V_1[t_n]\rangle_I = 0\,,\ \langle V_2[t_n]\rangle_I = 0\,,\ \langle V_3[t_n]\rangle_I = u$$

Average 3D (vector) values are defined in terms of the unit vectors: $\langle \underline{F}[t_n]\rangle_I \equiv \hat{\underline{e}}_1\langle F_1[t_n]\rangle_I + \hat{\underline{e}}_2\langle F_2[t_n]\rangle_I + \hat{\underline{e}}_3\langle F_3[t_n]\rangle_I$

$$\therefore\ \langle \underline{X}[t_n]\rangle_I = \hat{\underline{e}}_3\,(z_n + u\,\tau/2) = \underline{z}_n + \underline{u}\,\tau/2 \qquad \text{and} \qquad \langle \underline{V}[t_n]\rangle_I = \hat{\underline{e}}_3\,u = \underline{u}$$

Thus '**cell-block**' averages appear like a simple (Newtonian) particle located **at** $\underline{X}[\langle t_n\rangle_I]$, moving **with** speed $\underline{V}[\langle t_n\rangle_I]$.

The *Velocity-Average* uses the 'mid-times' around the 'target' time t_n: $\{\mathbf{t}_1 = t_n - 3\tau/2,\ \mathbf{t}_2 = t_n - \tau/2,\ \mathbf{t}_3 = t_n + \tau/2\ ,\ \mathbf{t}_4 = t_n + 3\tau/2\}$. Any time varying function, whose value can be evaluated at any time F[t] has an Velocity-Average value defined as:

Definition: **Velocity-Average** $\langle F[\,t_n\,]\rangle_V \equiv \tfrac{1}{4}\{F[\,\mathbf{t}_1\,] + F[\,\mathbf{t}_2\,] + F[\,\mathbf{t}_3\,] + F[\,\mathbf{t}_4\,]\}$

Each 'block' of 4 cells has its own velocity time-average evaluated at each of the cell-block mid-points: $\langle t_n\rangle_V = t_n$.

μ	$\mathbf{t}_\mu$	$X_1[\mathbf{t}_\mu]$	$X_2[\mathbf{t}_\mu]$	$X_3[\mathbf{t}_\mu]$	$V_1[\mathbf{t}_\mu]$	$V_2[\mathbf{t}_\mu]$	$V_3[\mathbf{t}_\mu]$
1	$t_n - 3\tau/2$	0	$-\Lambda/2$	$z_n - u\,3\tau/2$	$-c$	0	u
2	$t_n - \tau/2$	$-\Lambda/2$	0	$z_n - u\,\tau/2$	0	$+c$	u
3	$t_n + \tau/2$	0	$+\Lambda/2$	$z_n + u\,\tau/2$	$+c$	0	u
4	$t_n + 3\tau/2$	$+\Lambda/2$	0	$z_n + u\,3\tau/2$	0	$-c$	u

Table 5. Velocity-Representation (over one cycle)

Here, the finite velocity definition has been used: $\underline{V}[\mathbf{t}_\mu] \equiv \{\underline{X}[\mathbf{t}_{\mu+1}] - \underline{X}[\mathbf{t}_\mu]\}/\tau$ and 'light-speed' $c = \Lambda/\tau$; giving:

$$\therefore\ \langle X_1[t_n]\rangle_V = 0\,,\ \langle X_2[t_n]\rangle_V = 0\,,\ \langle X_3[t_n]\rangle_V = z_n\ ;\ \langle V_1[t_n]\rangle_V = 0\,,\ \langle V_2[t_n]\rangle_V = 0\,,\ \langle V_3[t_n]\rangle_V = u$$

Average 3D (vector) 'velocity-average' values are defined: $\langle \underline{F}[t_n]\rangle_V \equiv \hat{\underline{e}}_1\langle F_1[t_n]\rangle_V + \hat{\underline{e}}_2\langle F_2[t_n]\rangle_V + \hat{\underline{e}}_3\langle F_3[t_n]\rangle_V$

$$\therefore\ \langle \underline{X}[t_n]\rangle_V = \hat{\underline{e}}_3\,z_n = \underline{z}_n \qquad \text{and} \qquad \langle \underline{V}[t_n]\rangle_V = \hat{\underline{e}}_3\,u = \underline{u}$$

Thus velocity 'cell-block' averages appear just like a simple (Newtonian) particle located **at** $\underline{X}[t_n]$, moving **with** speed $\underline{V}[t_n]$.

Comparing these two sets of averages illustrates the consequence of choosing different points ina discrete representation.

$$\therefore \quad \langle t_n \rangle_V = \langle t_n \rangle_I - \tau / 2 \quad \text{and} \quad \langle \underline{X}[t_n] \rangle_V = \langle \underline{X}[t_n] \rangle_I - u\,\tau / 2 \quad \text{and} \quad \langle \underline{V}[t_n] \rangle_V = \langle \underline{V}[t_n] \rangle_I$$

Although these two sets of average are identical in the classical limit ($\tau \to 0$) they remain distinct when the electron has a finite period for possible interactions as the averages are computed at different points in their common 4τ cycle time. The Position-Representation uses locations $\underline{X}[t_\mu]$ were the electron can be <u>measured</u> from which velocities $\underline{V}[t_\mu]$ are <u>calculated</u>. In contrast, the Momentum-Representation uses the <u>actual</u> electron velocities $\underline{V}[t_\mu]$ but these are <u>defined</u> at the mid-point positions $\underline{X}[t_\mu]$ where measurements <u>cannot</u> be made, as no interactions occur at these intermediate locations. Obviously, standard QM makes no distinction between these two types of average, as it is a continuum theory of time with $\tau \to 0$. Standard QM incorporates these hidden differences through use of an extended spatial metaphor – the "wave" representation. Unfortunately, this simple continuous mathematical representation has too often been given a physical interpretation with disastrous results in terms of understanding the meaning of quantum mechanics. Returning to the present theory, there are also some other interesting relationships between these two sets of values, such as: $V_j[t_\mu] = V_j[t_{\mu+1}]$.

The <u>instantaneous average velocity</u> at every instant of time t is always defined as: $< \underline{V}[\,t\,] > = \tfrac{1}{2} \{ \underline{V}[\,t + \delta t] + \underline{V}[\,t - \delta t] \}$.

At the possible interaction times t_μ, where the transverse velocity always changes: $< \underline{V}[\,t_\mu\,] > = \tfrac{1}{2} \{ \underline{V}[\,t_\mu^{+}] + \underline{V}[\,t^{-}] \}$.

However, everywhere else ($t \neq t_\mu$) there is no change in velocity, so: $< \underline{V}[\,t_\mu \geq t > t_{\mu+1}] > = \underline{V}[t_\mu^{+}] = \underline{V}[t_\mu + \tau / 2\,] = \underline{V}[\langle t_\mu \rangle_I]$

Effective Longitudinal Momentum & Displacement

These vector averages can be used to define vector products, such as those involving location and velocity.

$$\langle \underline{X}[t_n] \rangle_I \bullet \langle \underline{V}[t_n] \rangle_I = z_n\,u + u^2\,\tau / 2 \qquad \langle \underline{X}[t_n] \rangle_V \bullet \langle \underline{V}[t_n] \rangle_V = z_n\,u$$

$$\therefore \ \langle \underline{X}[t_n] \rangle_I \bullet \langle \underline{V}[t_n] \rangle_I - \langle \underline{X}[t_n] \rangle_V \bullet \langle \underline{V}[t_n] \rangle_V = u^2\,\tau / 2$$

Definition: **Effective Linear Momentum** $\mathcal{P}[\,t_n\,] \equiv m\,\underline{V}[t_n] \bullet \hat{\underline{e}}_3$

$$\therefore \quad \underline{\mathcal{P}}[\,t_n\,] = \mathcal{P}[\,t_n\,]\,\hat{\underline{e}}_3 = m \langle \underline{V}[t_n] \rangle_I = m \langle \underline{V}[t_n] \rangle_V = m \langle \underline{V}[t_n] \rangle \equiv m\,\underline{V}[t_n]$$

So, either the Position or Momentum-Representation's average speed behaves like a simple (Newtonian) particle's linear momentum.

$$\therefore \ \{ \langle \underline{X}[t_n] \rangle_I - \langle \underline{X}[t_n] \rangle_V \} \bullet \underline{\mathcal{P}}[\,t_n\,] = m\,u^2\,\tau / 2$$

A 'free' classical particle traveling with velocity $\underline{u}$ ($u\,\hat{\underline{e}}_3$) for a duration of 4τ has a kinetic energy $\mathcal{K}_0 = \tfrac{1}{2}\,m\,u^2$ and a cumulative total action of $4\mathcal{A}_0$, where:

$$4\,\mathcal{A}_0 = 4\,m\,u^2\,\tau = 8\,\mathcal{K}_0\,\tau \quad \therefore \mathcal{A}_0 = 2\,\mathcal{K}_0\,\tau \quad \text{or} \quad \mathcal{A}_0 = \underline{\mathcal{P}}[\,t_n\,] \bullet \Delta\underline{\mathcal{X}} \quad \text{where:} \quad \Delta\underline{\mathcal{X}} \equiv \tfrac{1}{2} (\langle \underline{X}[t_n] \rangle_I - \langle \underline{X}[t_n] \rangle_V)$$

Thus, $\Delta\underline{\mathcal{X}}[t_n]$, defined here for the digital-spin electron, behaves like the <u>average distance</u> moved by a classical Newtonian particle. Since these averages are evaluated over a time of only 4τ over a spatial displacement of only Λ then the 'clicking' (helical) motion will appear indistinguishable to human measurements from a simple Newtonian particle moving inertially in one dimension. The concept of 'digital spin' is introduced here as a simple, mechanical hypothesis that will be used to provide an easily understood explanation for various atomic and nuclear effects, that will be described in later papers.

5.2.3 DIGITAL ELECTRON KINEMATICS

It is now appropriate to return from the discussion of representing just the interaction-states of the electron to the central problem of finding a comprehensive representation of the full state of the electron, as introduced in section 5.1.2, especially including a suitable representation of its **kinematic** properties (location and velocity). It seems reasonable to separate the kinematic part of the complete electron ket vector $|k\rangle$ (introduced, in its totality, to represent the existence of the k^{th} electron at the 3D space location $\underline{x}$ at the time t) from its binary interaction phase component (send/receive phase denoted by the two-valued variable λ, where $\lambda = \pm 1$) since there are no interactions while the electron is 'free'. This implies a Cartesian product:

$$| k \rangle = | k: t, \underline{x}; \lambda \rangle = | k: t, \underline{x} \rangle \otimes | k: \lambda \rangle$$

This theory will now adopt the (4x1) canonical matrix form introduced in section 5.1 4 to represent the electron's interaction phase. It is significant that classical mechanics (CM) always represented the kinematical part (the only part acknowledged) of the particle's state simply as a continuous, real algebraic function of three **real** space variables and one **real** time variable. Standard quantum mechanics (QM), since Pauli and Dirac, has acknowledged the binary 'spin' states of the electron (see section 2.1) but QM continued to represent the kinematical state of the electron by a **complex**, continuous analytic function of the three real space variables and one time variable. This was given a 'matrix' twist by describing these states as Hilbert vectors but in practice, this simply involved infinite Fourier sums (or integrals) of complex exponential functions – a more suitable representation for infinite, plane waves covering **all** of 3D space, not localized point particles at any instance.

Standard QM: $\langle x | k: t, \underline{x}; \lambda \rangle = \psi_{\lambda}[k: t, \underline{x}]$

The present theory represents the kinematic components in space and time using discrete natural vectors (DNVs), which were introduced here in section 3.2. Thus, a single electron (labeled 'k') located at position $\underline{x}_{\eta}$ at time t_{η} is represented by:

$$| k: t, \underline{x} \rangle = \{ \underline{x}[k: t_{\eta}] \} \approx \mathbf{X}_{\eta}[k] \equiv i\, c\, t_{\eta}\, \mathbf{I_0} + \underline{x}_{\eta} \bullet \underline{\mathbf{I}} \qquad \therefore \quad | k \rangle \approx \mathbf{X}_{\eta}[k] \, | k: \nu \rangle$$

Absolute & Temporal Displacements

The introduction of time-dependency into 3D vector analysis has generated a fundamental confusion in the application of this powerful notation that has hidden its deeper ancestry – Hamilton's quaternions [1]. An attempt will now be made to clear up this confusion by introducing the concepts of absolute and temporal vectors. An *absolute* vector (in 3D space) is defined as a displacement in space that does not depend on the time at which it is determined. A *temporal* vector is defined as a directed difference in two locations in space that does depend on when this difference occurred. Physics over the last 100 years has been developed using only absolute vectors, following the tradition that time should be irrelevant and timeless geometry is everything. When a classical particle's location undergoes a sequence of absolute changes (or displacements), whether discrete or more usually infinitesimal, then the order that these changes occur does not effect the final location. The simple arithmetic features of vectors are sufficient to describe this whole process. Consider a particle constrained in a 2D plane (say z = 0) whose location at time t_0 is $\underline{R}_0$, at the corner vertex $(-\Lambda/2, -\Lambda/2, 0)$ in a Cartesian co-ordinate reference frame. Consider three possible spatial displacements at three different times, labeled $\underline{A}[t_1]$, $\underline{B}[t_2]$ and $\underline{C}[t_3]$ and defined along the $\underline{x}$ and $\underline{y}$ axes (identified by unit vectors $\hat{\underline{e}}_1$ and $\hat{\underline{e}}_2$ respectively): $\underline{A}[t_1] = \Lambda\, \hat{\underline{e}}_1$; $\underline{B}[t_2] = \Lambda\, \hat{\underline{e}}_2$; $\underline{C}[t_3] = -\Lambda\, \hat{\underline{e}}_1$. Next, consider three displacements of the particle at times t_1 then t_2 and finally t_3, reaching the locations $\underline{R}[t_1]$ then $\underline{R}[t_2]$ and $\underline{R}[t_3]$.

$$\underline{R}[t_1] = \underline{R}_0 + \underline{A}[t_1] = (\Lambda/2, -\Lambda/2, 0) \; ; \; \underline{R}[140] = \underline{R}[t_1] + \underline{B}[t_2] = (\Lambda/2, \Lambda/2, 0) \; ; \; \underline{R}[t_3] = \underline{R}[t_2] + \underline{C}[t_3] = (-\Lambda/2, \Lambda/2, 0)$$

$$\therefore \quad \underline{R}[t_3] = \underline{R}_0 + \underline{A}[t_1] + \underline{B}[t_2] + \underline{C}[t_3] \quad \text{with} \quad t_3 > t_2 > t_1$$

However, these are each absolute vectors (e.g. $\underline{A}[t_1] = \underline{A}$) as they are independent of time, with the times only acting as arbitrary labels and as absolute vectors are associative under addition ($\underline{A} + \underline{B} = \underline{B} + \underline{A}$) then the time-order is irrelevant.

$$\therefore \quad \underline{R}[t_3] = \underline{B}[t_2] + \underline{C}[t_3] + \underline{R}_0 + \underline{A}[t_1]$$

This result is unaffected by motion in the z-direction. These representations are sufficient for describing the simple motion of Newtonian (point) particles but not for describing rotational motion, when the next 'step' depends on the previous one. This will be the type of motion proposed in the next section for describing the fundamental motion of electrons and where the traditional use of absolute vectors will prove inadequate.

Although 'temporal' vectors could be developed, the appropriate mathematical description for this type of change through space over time is already available and was developed over 150 years ago by the Irish genius, William Rowan Hamilton (1805-1865). The quaternions he introduced have been slightly modified in this research programme into *natural vectors* to represent asynchronous interactions, which make them ideal for describing the EM interaction [2]. The key idea here is the need for rotations around locally moving axes so that the order of these 'twists' (especially when effected by interactions with other electrons) determines the eventual location (and hence trajectory) of each electron. This is discussed extensively in Goldstein's text on CM [127] where the sequence of rotations of a rigid body is shown to form a non-commutative group under multiplication. It is this reason, above all else, that this programme adopts a multiplicative (or **algebraic**) rather than additive (or **arithmetic**) representation for describing the movement of electrons through space. It will be shown in a later paper that this is the reason that QM had to introduce the non-commuting operator calculus ($\partial/\partial x$) or non-diagonal matrices for representing the activity of electrons i.e. atomic phenomena: hence, Dirac's q-numbers instead of classical c-numbers.

There are two complementary mathematical techniques for describing time-based changes in a multiplicative manner – both will be used here to relate electron states at different times. The first technique uses an analytic method introduced in 1828 by the (self-taught) British mathematician George Green (1793-1841) in his early investigations into EM, where he invented the concept of the EM potential. These are called *Green's Functions* and usually denoted by the symbol G. Thus, in order to relate an electron's state at one time t with another state at time t′ the Electron Green's Function is defined as:

Definition: **Electron Green's Function** $\quad | \, k: t', \underline{x}'; \nu' \, \rangle \;\equiv\; \mathcal{G}[k: t', \underline{x}', \nu' \# \, t, \underline{x}, \nu] \, | \, k: t, \underline{x}; \nu \, \rangle$

The second technique was already discussed in section 5.1.5 when the Time-Evolution operator was introduced to generate the evolution of the electron's interaction phase across time. In the present case, the Electron Time-Evolution operator $\mathbf{T}$ changes the electron's complete state vector in increments of one chronon τ, moving the electron's state from the time of one possible interaction instant $\mu\tau$ to the next possible interaction instant $(\mu+1)\tau$.

Definition: **Time-Evolution Operator** $\quad \mathbf{T} \, | \, k: \mu\tau, \underline{x}_\mu \, ; \nu \, \rangle \;\equiv\; | \, k: \mu\tau+\tau, \underline{x}_{\mu+1} \, ; \nu+1 \, \rangle \qquad$ (modulo 4)

It will also prove useful to decompose this complete electron time operator into its kinematic and phase components:

Definition: **Kinematic-Time Operator** $\quad \mathbf{T}_K \, | \, k: \mu\tau, \underline{x}_\mu \, \rangle \;\equiv\; | \, k: \mu\tau+\tau, \underline{x}_{\mu+1} \, \rangle$

where: $\quad \mathbf{T} \;\equiv\; \mathbf{T}_K \otimes \mathbf{T}_P$

This leads to the identity: $\quad \mathbf{T}_K \;=\; \mathcal{G}[\mu\tau+\tau, \underline{x}_{\mu+1} \# \, \mu\tau, \underline{x}_\mu]$ $\quad$ for all electrons 'k' and all times μ and locations $\underline{x}_\mu$.

The most general, **linear** equation satisfied by $\mathbf{T}_K$ is: $\quad \mathbf{X}_{\mu+1} \;=\; \mathbf{T}_K \, \mathbf{X}_\mu \;\equiv\; (\mathbf{I_0} + \mathbf{K}) \, \mathbf{X}_\mu \qquad \text{or} \qquad \Delta_\tau \mathbf{X}_\mu \;=\; \mathbf{K} \, \mathbf{X}_\mu$

For a Newtonian 'free' particle moving at constant speed u_0 along the z-axis starting at t = 0: $\underline{x}_\mu = \mu\tau \, u_0 \, \hat{\underline{e}}_3$. Assuming $\mathbf{K}$ can be represented by a timeless natural vector: $\mathbf{K}^0 \;=\; K_0 \, \mathbf{I_0} + \underline{K} \bullet \underline{\mathbf{I}}$ then:

$$i \, c \, \tau \, \mathbf{I_0} \;+\; u_0 \, \tau \, \mathbf{I_3} \;=\; (i \, c \, K_0 - u_0 \, K_3) \, \mu \, \tau \, \mathbf{I_0} \;+\; (u_0 \, K_0 + i \, c \, K_3) \, \mu \, \tau \, \mathbf{I_3} \qquad \therefore \quad K_1 \;=\; 0 \;\;\&\;\; K_2 \;=\; 0$$

$$\therefore \; \{i \, c - i \, c \, \mu \, K_0 + u_0 \, \mu \, K_3\} \, \mathbf{I_0} + \{u_0 - u_0 \, \mu \, K_0 - i \, c \, \mu \, K_3 \} \, \mathbf{I_3} \;=\; 0 \quad \therefore \; u_0 \;=\; c \;\;\&\;\; K_3 \;=\; 0 \;\;\&\;\; \mu \, K_0 \;=\; 1$$

$$\therefore \quad \mathbf{K}^0 \;=\; \mathbf{I_0} \, / \, \mu$$

This demonstrates that there is **no** physical solution for a 'free' Newtonian particle that permits a timeless (μ independent) evolution of both the phase-component and the kinematical part of the complete state vector. The general solution of this problem will be deferred until the 'Rotational' hypothesis has been proposed in the next section. It is also important to note that the (4x4) matrix form of $\mathbf{T}_P$ is independent of all four Natural Vector bases $\mathbf{I}_\mu$. Section 5.1.6 demonstrated that the phase time-evolution operator could only be defined in terms of these bases through the addition of the sub-space inversion operator: the phase is a true independent, mathematical 'space' from the real space of physical separations.

Free Digital-Spin Electron Kinematics

This section will now integrate two of the proposals that now form the foundation of this research, namely: the electron is best represented by a discrete natural vector (DNV) and the electron's intrinsic motion includes 'digital spin'. The case of the single, non-interacting (or 'free') electron will first be described, following the Newtonian tradition of focusing on only a single, 'target' particle. In this programme, the 'free' electron describes all those finite time durations when the electron does not interact with any other electron in the universe. These include all the situations between every possible interaction point (the discrete times t_η) and at all those possible interaction times when the electron is not on a suitable 'light-cone' to commit to an interaction. This must be contrasted with the case in standard physics where an electron's charge is active at all times and the electromagnetic interaction is universally present both in space and time (except for the fictitious, 'single-electron universe' and those infamous, 'remote' times when every electron is "infinitely far apart" from every other electron.) Thus, consider a single, 'free' electron over a time range of several ($N \geq 1$) chronons, spanning the possible (but not actual) interaction times $\{t_0 , t_1 , t_2 , \ldots , t_N\}$ or $t_\eta = t_0 + \eta\,\tau$. Let the electron's 3D spatial locations at these times be designated $\underline{X}_\eta$. In terms of a fixed set of right-handed unit spatial vectors $\{ \underline{\hat{e}}_1 , \underline{\hat{e}}_2 , \underline{\hat{e}}_3\}$ then: $\underline{X}_\eta = \underline{\hat{e}}_1 x_{1\eta} + \underline{\hat{e}}_2 x_{2\eta} + \underline{\hat{e}}_3 x_{3\eta} = \underline{\hat{e}}_T x_{T\eta} + \underline{\hat{e}}_3 x_{3\eta}$

The DNV hypothesis states that this vector is isomorphic with the spatial part of the DNV, $\mathbf{X}_\eta$: $\mathbf{X}_\eta = i\,c\,t_\eta\,\mathbf{I_0} + \underline{X}_\eta \bullet \underline{\mathbf{I}}$

In terms of local differences: $\Delta t_\eta = t_{\eta+1} - t_\eta = \tau$ $\Delta\underline{X}_\eta = \underline{X}_{\eta+1} - \underline{X}_\eta$ $\therefore\ \Delta\mathbf{X}_\eta = i\,c\,\tau\,\mathbf{I_0} + \Delta\underline{X}_\eta \bullet \underline{\mathbf{I}}$

The **local** velocity has been defined as: $\underline{V}_\eta = (\underline{X}_{\eta+1} - \underline{X}_\eta) / (t_{\eta+1} - t_\eta)$ $\therefore\ \Delta\underline{X}_\eta = \tau\,\underline{V}_\eta$ $\therefore\ \Delta\mathbf{X}_\eta = \tau\,\mathbf{V}_\eta$

The 'norm' of any NV is defined as: $\mathbf{A}^*\mathbf{A} = (A_0^2 - \underline{A} \bullet \underline{A})$ while the 'norm' of the velocity is constant: $\underline{V}_\eta \bullet \underline{V}_\eta = (V_0)^2$

This defines the invariant: $\Delta\mathbf{X}_\eta{}^*\mathbf{V}_\eta = \tau\,(c^2 - V_0^2)\,\mathbf{I_0}$

The digital-spin hypothesis proposes that the 'free' electron is moving in one direction (say $\underline{\hat{e}}_3$) with constant speed u but with a variable, transverse velocity $\underline{w}$ whose speed is *c* but its direction is changing by 90° every chronon. A CM reference frame is assumed with the origin at $t = t_0$ with the electron moving steadily at speed u along the $\underline{\hat{e}}_3$ axis. Here, the canonical orientation will be assumed (see above) with the first (in each four cycle) transverse motion parallel to the $\underline{\hat{e}}_1$ direction.

$\therefore\ \underline{x}_{1\eta} = x_{1\eta}\,\underline{\hat{e}}_1 = -\,\underline{\hat{e}}_1 \cos((2\eta+1)\pi/4)\,\Lambda/\sqrt{2}$; $\underline{x}_{2\eta} = x_{2\eta}\,\underline{\hat{e}}_2 = -\,\underline{\hat{e}}_2 \sin((2\eta+1)\pi/4)\,\Lambda/\sqrt{2}$ & $x_{3\eta} = z_0 + \eta\,u\,\tau$

$\therefore\ \underline{w}_\eta = \underline{w}_{1\eta} + \underline{w}_{2\eta}$ & $\underline{w}_{1\eta} = w_{1\eta}\,\underline{\hat{e}}_1 = c\cos(\eta\pi/2)\,\underline{\hat{e}}_1$ & $\underline{w}_{2\eta} = w_{2\eta}\,\underline{\hat{e}}_2 = c\sin(\eta\pi/2)\,\underline{\hat{e}}_2$

Since $\underline{V}_\eta = \underline{w}_\eta + \underline{u}_\eta = c\cos(\eta\pi/2)\,\underline{\hat{e}}_1 + c\sin(\eta\pi/2)\,\underline{\hat{e}}_2 + u\,\underline{\hat{e}}_3$ $\therefore\ V_0^2 = c^2 + u^2$ $\therefore\ \Delta\mathbf{X}_\eta{}^*\mathbf{P}_\eta = -\mathcal{A}_0\,\mathbf{I_0}$

Thus, the 'free' electron 'expends' a constant quantity of action $\mathcal{A}_0$ (comparable to the Newtonian particle) per chronon. The recursive equation for the transverse velocity is:

$$\underline{w}_{\eta+1} = \underline{\hat{e}}_3 \wedge \underline{w}_\eta \qquad \text{or} \qquad \underline{u}_\eta \wedge \underline{w}_\eta = u_\eta\,\underline{w}_{\eta+1}$$

As a result of the intrinsic 'clicking' motion, the electron possesses a total angular momentum $\underline{\mathcal{M}}_\eta$, which is defined in the Newtonian particle tradition as:

Definition: **Electron Total Angular Momentum** $\underline{\mathcal{M}}_\eta \equiv m\,\underline{X}_\eta \wedge \underline{V}_\eta$

$\therefore\ \underline{X}_\eta \wedge \underline{V}_\eta = (\underline{x}_{T\eta} + \underline{x}_{3\eta}) \wedge (\underline{w}_\eta + \underline{u}_\eta) = \underline{x}_{T\eta} \wedge \underline{w}_\eta + \underline{x}_{T\eta} \wedge \underline{u}_\eta + \underline{x}_{3\eta} \wedge \underline{w}_\eta$

Now $\underline{x}_{T\eta} \wedge \underline{w}_\eta = \underline{\hat{e}}_3\,c\,\Lambda/2$; $\underline{x}_{T\eta} \wedge \underline{u}_\eta = \{-\underline{\hat{e}}_1 , -\underline{\hat{e}}_2 , \underline{\hat{e}}_1 , \underline{\hat{e}}_2\}\,u\,\Lambda/2$; $\underline{x}_{3\eta} \wedge \underline{w}_\eta = \{-\underline{\hat{e}}_1 , -\underline{\hat{e}}_2 , \underline{\hat{e}}_1 , \underline{\hat{e}}_2\}\,\eta\,u\,\Lambda$

When averaged over 4 chronons: $\langle\,\underline{\mathcal{M}}_\eta\,\rangle_I = m\,\underline{x}_{T\eta} \wedge \underline{w}_\eta = \underline{\hat{e}}_3\,m\,c\,\Lambda/2 = \underline{\hat{e}}_3\,\mathcal{M}_0$ $\therefore\ \mathcal{M}_0 = \tfrac{1}{2}\,m\,c^2\,\tau$

Using the luxon definition (section 4.3.6) $\Lambda = c\,\tau = e^2/m\,c^2 = \alpha\,h_D/m\,c$ $\therefore\ \mathcal{M}_0 = e^2/2\,c = \alpha\,h_D/2$

Free Digital Electron Green's Functions

This section will explore the possibility of whether the DNV algebra of the 'free digitally spinning' electron permits Green's function-like solutions to the position and velocity representations; that is time-independent solutions of the DNV equations:

$$\mathbf{X}_{\eta+1} = \mathbf{T}_K \mathbf{X}_\eta \equiv (\mathbf{I_0} + \mathbf{K})\, \mathbf{X}_\eta \quad \text{or} \quad \Delta_\tau \mathbf{X}_\eta = \mathbf{K}\, \mathbf{X}_\eta = \tau\, \mathbf{V}_\eta \quad \text{Alternatively:} \quad \mathbf{V}_{\eta+1} = \mathbf{T}_K \mathbf{V}_\eta$$

As always: $\mathbf{X}_\eta = i\, c\, t_\eta\, \mathbf{I_0} + \underline{X}_\eta \bullet \underline{\mathbf{I}}$ and $\mathbf{V}_\eta = i\, c\, \mathbf{I_0} + \underline{V}_\eta \bullet \underline{\mathbf{I}}$ There is **no** solution like: $\mathbf{K} = K_0\, \mathbf{I_0} + \underline{K} \bullet \underline{\mathbf{I}}$

Unlike the Newtonian 'free' particle, which was constrained to move only along the z-axis at constant speed u_0, the 'digital-spin' electron introduces an additional degree of freedom by 'clicking' around the z-axis, four times per cycle ($\mu = 0,1,2,3$). The canonical orientation will also be assumed with the spatial origin defined at $t = t_0$ and $z = 0$ and with the initial motion defining the 'x' (or $\underline{\hat{e}}_1$) axis; the spatial displacement along the z-axis after η 'clicks' (one per chronon) is still: $z = \eta\, \tau\, u_0$. The (four) cyclic velocities are:

μ	0	1	2	3
$\underline{w}[\mu]$	$+ c\, \underline{\hat{e}}_1$	$+ c\, \underline{\hat{e}}_2$	$- c\, \underline{\hat{e}}_1$	$- c\, \underline{\hat{e}}_2$
$\mathbf{W}[\mu]$	$+ c\, \mathbf{I}_1$	$+ c\, \mathbf{I}_2$	$- c\, \mathbf{I}_1$	$- c\, \mathbf{I}_2$

Table 6. Transverse Velocities per Cycle

In each 4-cycle: $\underline{w}_\mu = c \cos(\mu\, \pi/2)\, \underline{\hat{e}}_1 + c \sin(\mu\, \pi/2)\, \underline{\hat{e}}_2 = \tfrac{1}{2}\, c \exp(i\, \mu\, \pi/2)\, (\underline{\hat{e}}_1 - i\, \underline{\hat{e}}_2) + \tfrac{1}{2}\, c \exp(-i\, \mu\, \pi/2)\, (\underline{\hat{e}}_1 + i\, \underline{\hat{e}}_2)$

$$\therefore \quad \mathbf{W}[\mu] = (e^{\,i\,\mu\,\pi/2}\, \mathbf{I}_- + e^{\,-i\,\mu\,\pi/2}\, \mathbf{I}_+)\, c/2 \quad \text{with} \quad \mathbf{I}_+ = \mathbf{I}_1 + i\, \mathbf{I}_2 \quad \& \quad \mathbf{I}_- = \mathbf{I}_1 - i\, \mathbf{I}_2$$

$$\therefore \quad \mathbf{W}[\mu+1] = (e^{\,i\,(\mu+1)\,\pi/2}\, \mathbf{I}_- + e^{\,-i\,(\mu+1)\,\pi/2}\, \mathbf{I}_+)\, c/2 = (i\, e^{\,i\,\mu\,\pi/2}\, \mathbf{I}_- - i\, e^{\,-i\,\mu\,\pi/2}\, \mathbf{I}_+)\, c/2 = \mathbf{I}_3\, \mathbf{W}[\mu]$$

The base $\mathbf{I}_3$ is appropriate for 'temporally moving' the transverse velocity $\mathbf{W}$ since: $\mathbf{I}_3\, \mathbf{I}_1 = +\mathbf{I}_2$ & $\mathbf{I}_3\, \mathbf{I}_2 = -\mathbf{I}_1$ but it requires a 'double' time-step to temporally move around the transverse locations; in other words:

$$\mathbf{I}_3\, \mathbf{W}[\mu] = \mathbf{W}[\mu+1] \qquad \& \qquad \mathbf{I}_3\, \mathbf{I}_3\, \mathbf{X}_T[\mu] = \mathbf{X}_T[\mu+2]$$

Worse, the base $\mathbf{I}_3$ mixes up the 'longitudinal' components ($\mathbf{V}_L = u_0\, \mathbf{I}_3 + i\, c\, \mathbf{I}_0$) and only reverses them after two steps:

$$\mathbf{I}_3\, \mathbf{V}_L[\mu=0] = i\, c\, \mathbf{I}_3 - u_0\, \mathbf{I}_0 \quad \& \quad \mathbf{I}_3\, \mathbf{I}_3\, \mathbf{V}_L[\mu] = -\mathbf{V}_L\,[\mu+2]$$

All natural vectors can be decomposed into longitudinal and transverse sub-vectors (see Appendix A1.4): $\mathbf{A} = \mathbf{A}_L + \mathbf{A}_T$

$$\mathbf{A}_L = A_3\, \mathbf{I}_3 + i\, A_0\, \mathbf{I}_0 \quad \& \quad \mathbf{A}_T = A_1\, \mathbf{I}_1 + A_2\, \mathbf{I}_2 \quad \text{or in terms of 'transverse' \& 'longitudinal' bases.}$$

Longitudinally: $\mathbf{I}_\zeta \equiv \mathbf{I}_3 + i\, \zeta\, \mathbf{I}_0$ where $\zeta = \uparrow$ or $\downarrow$ (± 1) then $A_\zeta \equiv \tfrac{1}{2}(A_3 + \zeta\, A_0)$ $\therefore$ $\mathbf{A}_L = A_\uparrow\, \mathbf{I}_\uparrow + A_\downarrow\, \mathbf{I}_\downarrow$

Transversely: $\mathbf{I}_\rho \equiv \mathbf{I}_1 + i\, \rho\, \mathbf{I}_2$ where $\rho = +$ or $-$ (± 1) then $A_\rho \equiv \tfrac{1}{2}(A_1 + i\, \rho\, A_2)$ $\therefore$ $\mathbf{A}_T = A_+\, \mathbf{I}_- + A_-\, \mathbf{I}_+$

$$\therefore \quad \mathbf{X}_L = X_\uparrow\, \mathbf{I}_\uparrow + X_\downarrow\, \mathbf{I}_\downarrow \quad \& \quad \mathbf{X}_T = X_+\, \mathbf{I}_- + X_-\, \mathbf{I}_+ \qquad \text{where} \qquad X_\zeta[\mu] = V_\zeta[\mu]\, t_\mu$$

$$\therefore \quad \mathbf{V}_L = V_\uparrow\, \mathbf{I}_\uparrow + V_\downarrow\, \mathbf{I}_\downarrow \quad \& \quad \mathbf{V}_T = V_+\, \mathbf{I}_- + V_-\, \mathbf{I}_+ \qquad \text{where} \qquad V_\zeta[\mu] = \tfrac{1}{2}(u_0 - \zeta\, c)$$

Since $\mathbf{V}_T = \mathbf{W}$ then $V_+[\mu] = e^{\,i\,\mu\,\pi/2}\, c/2$ & $V_-[\mu] = e^{\,-i\,\mu\,\pi/2}\, c/2$ but μ is cyclic-4 so $\eta = 4n + \mu$ and $e^{\,\pm i\,4n\,\pi/2} = 1$

$$\therefore \quad V_\rho[t_\eta] = \exp(\, i\, \rho\, \pi\, t_\eta\, /2\tau\,)\, c/2 \qquad \text{and} \qquad X_\rho[t_\eta] = -\exp(\, i\, \rho\, \pi\, (t_\eta + \tau/2)/2\tau\,)\, \Lambda/(2\sqrt{2})$$

The analytic continuation to 'nearby' times t ($t_\eta \le t < t_{\eta+1}$) is therefore the continuum function: $V_\rho[t] = \exp(\, i\, \rho\, \pi\, t\, /2\tau\,)\, c/2$ for the two transverse velocity functions and $X_\rho[t] = -\exp(\, i\, \rho\, \pi\, (t+\tau/2)/2\tau\,)\, \Lambda/(2\sqrt{2})$ for the transverse location functions.

The (four) transverse locations are:

μ	0	1	2	3
$\underline{X}_T[\mu]$	$-(\underline{\hat{e}}_1 + \underline{\hat{e}}_2)\Lambda/2$	$+(\underline{\hat{e}}_1 - \underline{\hat{e}}_2)\Lambda/2$	$+(\underline{\hat{e}}_1 + \underline{\hat{e}}_2)\Lambda/2$	$-(\underline{\hat{e}}_1 - \underline{\hat{e}}_2)\Lambda/2$
$\mathbf{X}_T[\mu]$	$-(\mathbf{I}_1 + \mathbf{I}_2)\Lambda/2$	$+(\mathbf{I}_1 - \mathbf{I}_2)\Lambda/2$	$+(\mathbf{I}_1 + \mathbf{I}_2)\Lambda/2$	$-(\mathbf{I}_1 - \mathbf{I}_2)\Lambda/2$

Table 7. Transverse Locations per Cycle

The (four) transverse locations X_ρ and velocities V_ρ are:

μ	0	1	2	3
$X_\rho[\mu]$	$-(1 + i\,\rho)\,\Lambda/4$	$(1 - i\,\rho)\,\Lambda/4$	$(1 + i\,\rho)\,\Lambda/4$	$-(1 - i\,\rho)\,\Lambda/4$
$V_\rho[\mu]$	c/2	$i\,\rho$ c/2	$-$ c/2	$-\,i\,\rho$ c/2
$\mathbf{V}_T[\mu]$	$+(\mathbf{I}_+ + \mathbf{I}_-)$ c/2	$-(\mathbf{I}_+ - \mathbf{I}_-)\,i$ c/2	$-(\mathbf{I}_+ + \mathbf{I}_-)$ c/2	$+(\mathbf{I}_+ - \mathbf{I}_-)\,i$ c/2

Table 8. Transverse Velocities per Cycle

Algebraically: $\mathbf{V}_T[\mu] = i^{\,\mu}\,\{\,\mathbf{I}_- + (-1)^{\,\mu}\,\mathbf{I}_+\,\}$ c/2

The general results: $\exp(\,(\mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0)\,\theta/2)\,\mathbf{I}_{\zeta'} = \delta_{\zeta\zeta'}\exp(\,i\,\zeta\,\theta\,)\,\mathbf{I}_\zeta$ & $\exp(\,(\mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0)\,\theta/2)\,\mathbf{I}_\rho = \delta_{\zeta-\rho}\exp(\,i\,\zeta\,\theta\,)\,\mathbf{I}_\rho$

This suggests another *Ansatz* : $\mathbf{T}_K = \exp(\,(\mathbf{I}_3 + i\,\mathbf{I}_0)\,\theta/2) - \exp(\,(\mathbf{I}_3 - i\,\mathbf{I}_0)\,\theta/2)$. This then operates on these bases:

$$\therefore \quad \mathbf{T}_K[\,\mathbf{I}_\zeta\,;\,\mathbf{I}_\rho\,] = [\,\{\,\delta_{\zeta\uparrow}\exp(\,i\,\theta)\,\mathbf{I}_\uparrow - \delta_{\zeta\downarrow}\exp(-i\,\theta)\,\mathbf{I}_\downarrow\,\}\;;\;\{\,\delta_{\rho-}\exp(\,i\,\theta)\,\mathbf{I}_- - \delta_{\rho+}\exp(-i\,\theta)\,\mathbf{I}_+\,\}\,]$$

$\therefore \quad \mathbf{T}_K[\,\mathbf{V}_T[\mu]\,] = i^{\,\mu}\,\{\,\exp(\,i\,\theta)\,\mathbf{I}_- - (-1)^{\,\mu}\exp(-i\,\theta)\,\mathbf{I}_+\,\}$ c/2 $= i^{\,\mu+1}\,\{\,\mathbf{I}_- + (-1)^{\,\mu+1}\,\mathbf{I}_+\,\}$ c/2 $= \mathbf{V}_T[\mu+1]$ when $\theta = \pi$.

$\therefore \quad \mathbf{T}_K[\,\mathbf{V}_L[\mu]\,] = \tfrac{1}{2}\,\mathbf{T}_K[\,(u_0 - c)\,\mathbf{I}_\downarrow + (u_0 + c)\,\mathbf{I}_\uparrow\,] = \tfrac{1}{2}\,\{-(u_0 - c)\exp(-i\,\theta)\,\mathbf{I}_\downarrow + (u_0 + c)\exp(\,i\,\theta)\,\mathbf{I}_\uparrow\,\}$

But selecting the same value for $\theta = \pi$, then: $\mathbf{T}_K[\,\mathbf{V}_L[\mu]\,] = \tfrac{1}{2}\,\{\,(u_0 - c)\,\mathbf{I}_\downarrow - (u_0 + c)\,\mathbf{I}_\uparrow\,\} \neq \mathbf{V}_L[\mu+1]$.

So, it does **not** seem possible to find a timeless Green's function that will increment the velocity part of the digital electron.

Not Single-Time Vector Models

The previous analysis has indicated that it does not seem possible to find a single multiplier, expressed only in terms of the four base elements $\mathbf{I}_\mu$ of the (4x4) natural vector mathematical space, that will temporally transform even the NV form of the **one** electron velocity DNV of either the simple Newtonian particle or the 'digital electron'. This is not surprising: the NV formulation was created to represent the asynchronous interactions between **two** electrons, not a single electron. The failure of the single electron model has continually resurfaced throughout this research, even though it is the paradigmatic model of particle physics – both classical and quantum. This deep failure of traditional physics can be traced to the many attempts to abolish the dynamic (temporal) aspects of the interactions between particles and replace them with a localized single-time model (where all the interactions have been subsumed into a spatially dependent 'potential' function). This focus on the singular particle has been compounded by the use of traditional 3D vectors. This has required the introduction of an origin in the vector space that for physical reasons must always remain arbitrary. This has two direct implications: vectors were forced to become time-dependent, even though the time-origin is also quite arbitrary; additionally only parallel vectors have any meaning. This representational technique is also fundamentally **asymmetric**: at any instant of time, the location of the particle **is** special – it is the one, unique point in 3D space occupied by a real existent (the actual electron) while the origin is not occupied by any real thing (not even the 'observer'). Real particles move along specific trajectories and only relative positions between particles have any significance. This means their **relative** motion (at least while they are interacting) is always independent of any spatial or temporal origins and, more importantly, independent of all frames of reference and their motion. In contrast, all field theories are defined at **all** points in space at **all** times. Whenever time can be eliminated, theoretical physicists have achieved the final Platonic goal of the mathematicians – all that remains is timeless geometry: "the ultimate form of truth" for these rational mystics for over 2500 years. Accordingly, field theories **cannot** be about the existence of particles (special points) but only about the possibility of events that might happen to these particles, anywhere and everywhere, at any time. As such, vector field theories (like Maxwell's EM or QED) are the worst possible combination of mathematical ideas to represent discrete reality that is exemplified by the universality of the electron. In this sense, NVs are true vectors: their spatial vector points directly at its location at that time and not any similar parallel space displacement.

Local & Global Axes

It will first prove useful to introduce the concept of fixed, global directions (or fixed axes), denoted by the set $\{\underline{\hat{e}}_1 , \underline{\hat{e}}_2 , \underline{\hat{e}}_3\}$ and local axes that move with each electron. It will still prove useful to retain the 'canonical' orientation convention for each electron so that as electron 'k' moves after the n^{th} interaction its local z-axis is still aligned with its longitudinal motion and its first transverse direction after this interaction still defines its local x-axis. This results in the triplet set of spatial axes $\{\underline{\hat{e}}_1 , \underline{\hat{e}}_2 , \underline{\hat{e}}_3\}$ for each set of possible interaction times defining the electron's trajectory between the times t_n and t_{n+1}, where:

Definition: **Local Electron Axes** $v_j \, \underline{\hat{e}}_j[k: t_n] \equiv \underline{v}_j[k: t_n]$ velocity for electron 'k' and segment 'n'.

These local, unit vectors make angles $\beta_j[k: t_n]$ with the fixed, global axes; in other words: $\underline{\hat{e}}_j[k: t_n] \bullet \underline{\hat{e}}_j = \cos \beta_j[k: t_n]$

The most useful relationship between local and global axes is defined in terms of Eulerian angles $\{\phi , \theta , \psi\}$, following the Goldstein convention [128]. This relates the local 3D unit vectors to the fixed vectors by the three (3x3) rotation matrices $\mathbf{R}_j$.

$$\underline{\hat{e}}[k: t_n] = \mathbf{R}_3[k: t_n] \, \mathbf{R}_2[k: t_n] \, \mathbf{R}_1[k: t_n] \, \underline{\hat{e}}$$

Each of the rotation matrices rotates a vector around its Euler angle: $\mathbf{R}_1[k: t_n] = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}$ with $\phi = \phi[k: t_n]$

$$\& \; \mathbf{R}_2[k: t_n] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} \; \& \; \mathbf{R}_3[k: t_n] = \begin{vmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

again, with $\theta = \theta[k: t_n]$ and $\psi = \psi[k: t_n]$

Goldstein also shows that these can also be related through (2x2) spinor matrices (see section 3.1.4): $\mathbf{e} = \mathbf{Q} \, \mathbf{E} \, \mathbf{Q}^{\dagger}$ where:

$$\mathbf{e} = \begin{bmatrix} \hat{e}_3 & \hat{e}_- \\ \hat{e}_+ & -\hat{e}_3 \end{bmatrix} \quad \& \quad \mathbf{E} = \begin{bmatrix} \hat{e}_3 & \hat{e}_- \\ \hat{e}_+ & -\hat{e}_3 \end{bmatrix} \quad \& \quad \mathbf{Q} = \begin{bmatrix} e^{\,i(\psi+\phi)/2} \cos\theta/2 & i\, e^{\,i(\psi-\phi)/2} \sin\theta/2 \\ i\, e^{\,-i(\psi-\phi)/2} \sin\theta/2 & e^{\,-i(\psi+\phi)/2} \cos\theta/2 \end{bmatrix}$$

Since each electron's spatial position can be represented by either these 3D vectors or the spatial part of natural vectors, then there is a corresponding isomorphism between the unit vectors and the NV bases. The timeless NV bases $\mathbf{I}_j$ correspond to the global unit vectors $\underline{\hat{e}}_j$ while the local unit vectors $\underline{\hat{e}}_j[k: t_n]$ map to the dynamic, local NV bases $\mathbf{I}_j[k: t_n]$. Thus, the above dynamic relationships still hold true.

$$\therefore \quad \mathbf{I}_j[k: t_n] \bullet \mathbf{I}_j = \cos \beta_j[k: t_n] \quad \& \quad \{\mathbf{I}[k: t_n]\} = \mathbf{R}_3[k: t_n] \, \mathbf{R}_2[k: t_n] \, \mathbf{R}_1[k: t_n] \{\mathbf{I}\} \quad \text{and} \quad \{\mathbf{I}\} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{I}_- \\ \mathbf{I}_+ & -\mathbf{I}_3 \end{bmatrix}$$

The spinor form of the relationship between 3D vectors is intimately related to the NV representation, so this will form the most useful form for relating the motion of pairs of electrons through 3D space, even at different times. The individual axes can be readily extracted from the composite (2 x 2) matrix **e**. This is accomplished through using either one of the Pauli base spinors $\Phi_\uparrow$ or $\Phi_\downarrow$ (see 3.1.4) and the real 'twisting' Pauli matrix σ_x to extract the transverse vectors.

$$\hat{e}_3 = \Phi_\uparrow^{\,T} \, \mathbf{e} \, \Phi_\uparrow \quad \& \quad \hat{e}_+ = \Phi_\uparrow^{\,T} \, \sigma_x \, \mathbf{e} \, \Phi_\uparrow \quad \& \quad \hat{e}_- = \Phi_\downarrow^{\,T} \, \sigma_x \, \mathbf{e} \, \Phi_\downarrow$$

$$\therefore \quad \mathbf{I}_3 = \Phi_\uparrow^{\,T} \{\mathbf{I}\} \, \Phi_\uparrow \quad \& \quad \mathbf{I}_+ = \Phi_\uparrow^{\,T} \, \sigma_x \{\mathbf{I}\} \, \Phi_\uparrow \quad \& \quad \mathbf{I}_- = \Phi_\downarrow^{\,T} \, \sigma_x \{\mathbf{I}\} \, \Phi_\downarrow$$

$$\therefore \quad \mathbf{I}_3[k: t_n] = \Phi_\uparrow^{\,T} \, \mathbf{Q} \{\mathbf{I}\} \, \mathbf{Q}^{\dagger} \Phi_\uparrow = \mathbf{I}_3 \cos\theta + i/2 \, (e^{\,-i\phi} \, \mathbf{I}_+ - e^{\,i\phi} \, \mathbf{I}_-) \sin\theta$$

$$\& \quad \mathbf{I}_+[k: t_n] = \Phi_\uparrow^{\,T} \, \sigma_x \, \mathbf{Q} \{\mathbf{I}\} \, \mathbf{Q}^{\dagger} \Phi_\uparrow = \mathbf{I}_+ \, e^{\,-i(\psi+\phi)} \cos^2\theta/2 + \mathbf{I}_- \, e^{\,-i(\psi-\phi)} \sin^2\theta/2 + i \, \mathbf{I}_3 \, e^{\,-i\psi} \sin\theta$$

$$\& \quad \mathbf{I}_-[k: t_n] = \Phi_\downarrow^{\,T} \, \mathbf{Q} \, \sigma_x \{\mathbf{I}\} \, \mathbf{Q}^{\dagger} \Phi_\downarrow = \mathbf{I}_- \, e^{\,+i(\psi+\phi)} \cos^2\theta/2 + \mathbf{I}_+ \, e^{\,+i(\psi-\phi)} \sin^2\theta/2 - i \, \mathbf{I}_3 \, e^{\,+i\psi} \sin\theta$$

These can be transformed back into the orthogonal Cartesian bases, using sum, difference and half Euler angles:

$$\mathbf{I}_1[k: t_n] = \{\cos^2\theta/2 \cos(\psi+\phi) + \sin^2\theta/2 \cos(\psi-\phi)\}\,\mathbf{I}_1 + \{\cos^2\theta/2 \sin(\psi+\phi) - \sin^2\theta/2 \sin(\psi-\phi)\}\,\mathbf{I}_2 + \sin\theta \sin\psi\,\mathbf{I}_3$$

$$\mathbf{I}_2[k: t_n] = -\{\cos^2\theta/2 \sin(\psi+\phi) + \sin^2\theta/2 \sin(\psi-\phi)\}\,\mathbf{I}_1 + \{\cos^2\theta/2 \cos(\psi+\phi) + \sin^2\theta/2 \cos(\psi-\phi)\}\,\mathbf{I}_2 + \sin\theta \cos\psi\,\mathbf{I}_3$$

Alternatively, in terms of full Euler angles:

$$\mathbf{I}_1[k: t_n] = (\cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta)\,\mathbf{I}_1 + (\sin\psi\cos\phi\cos\theta + \cos\psi\sin\phi)\,\mathbf{I}_2 + \sin\theta\sin\psi\,\mathbf{I}_3$$

$$\mathbf{I}_2[k: t_n] = (\cos\psi\cos\phi\cos\theta + \sin\psi\sin\phi)\,\mathbf{I}_1 + (\cos\psi\sin\phi\cos\theta - \sin\psi\cos\phi)\,\mathbf{I}_2 + \sin\theta\cos\psi\,\mathbf{I}_3$$

$$\mathbf{I}_3[k: t_n] = \sin\phi\sin\theta\,\mathbf{I}_1 - \cos\phi\sin\theta\,\mathbf{I}_2 + \cos\theta\,\mathbf{I}_3$$

Two Electron Green's Functions

Just as it was possible to define a Green's function for a single electron (see section 5.2.3), it is possible to define similar functions that operate on two electrons. Again, a mathematical 'vector' space is proposed to represent the state of both electrons (labeled '1' and '2'). This 'double' state is considered separable, even when the two electrons are involved in a mutual interaction, as the interaction is reduced to a finite set of momentum transfers occurring at only a discrete number of interaction times $\{t_n\}$. Each electron state is an eigenstate of the electron existence operator (see 5.1.2); accordingly:

$$|1, 2: t_1, t_2; \underline{x}_1, \underline{x}_2; \nu_1, \nu_2\rangle = |1: t_1, \underline{x}_1; \nu_1\rangle \otimes |2: t_2, \underline{x}_2; \nu_2\rangle$$

The two-electron Green's function G_{12} transforms this two-electron state, defined at times t_1 and t_2, into another two-electron state at two later times t'_1 and t'_2 when the electron have changed their location and phase.

$$|1, 2: t'_1, t'_2; \underline{x}'_1, \underline{x}'_2; \nu'_1, \nu'_2\rangle \equiv \mathcal{G}_{12}[t'_1, t'_2; \underline{x}'_1, \underline{x}'_2; \nu'_1, \nu'_2 \# : t_1, t_2; \underline{x}_1, \underline{x}_2; \nu_1, \nu_2]\,|1, 2: t_1, t_2; \underline{x}_1, \underline{x}_2; \nu_1, \nu_2\rangle$$

Although the two-electron state may be separable, the previous analysis showed this is not so for its GF: $\mathcal{G}_{12} \neq \mathcal{G}_1\,\mathcal{G}_2$ However, the effect of time on the evolution of these two electron states across a single chronon is still well defined.

$$\mathbf{T}_K\,|1, 2: \mu_1\tau, \mu_2\tau; \underline{x}_{\mu 1}, \underline{x}_{\mu 2}\rangle \equiv |1, 2: (\mu_1+1)\tau, (\mu_2+1)\tau; \underline{x}_{\mu 1+1}, \underline{x}_{\mu 2+1}\rangle$$

$$\therefore \quad \mathbf{T}_K\,|1, 2\rangle = \mathcal{G}_{12}[(\mu_1+1)\tau, (\mu_2+1)\tau; \underline{x}_{\mu 1+1}, \underline{x}_{\mu 2+1} \# : \mu_1\tau, \mu_2\tau; \underline{x}_{\mu 1}, \underline{x}_{\mu 2}]\,|1, 2\rangle$$

A special feature of the Natural Vector representation of the electron is that it 'binds' the unique time of its existence to the location of the electron in space at that moment of time. These are not independent parameters, as in traditional physics, where all 'objects' (particles or fields) share the same, independent time within an (arbitrary) inertial frame-of-reference. Although an abstract 'universal' time may be defined (characterizing all of abstract space), it is only the time-differences when the two electrons participate in an interaction, which are relevant: in this theory all (interaction) time is relative. This section will again demonstrate that it is **not** possible to define a single multiplier (the 'free' double Green's function) that temporally moves the two electrons together both at the instant of interaction and between all the subsequent times between interactions. Analysis of the 'interactive' Green's function (that moves both electrons **through** the interaction instants) will be deferred until later when it will shown to be an alternative form of representation of the fundamental EM interaction.

General 3D Solutions

The most general analysis occurs when the two interacting electrons are not co-linear in space and the local reference frames of the two electrons are oriented arbitrarily relative to each other in space. Again, consider the time segment following the n^{th} interaction between the two electrons (labeled #1 and #2) beginning at the interaction times t and t′. There is no loss in generality in allowing the local reference frame of electron #2 to be aligned with the global axes $\{\underline{\hat{e}}_j\}$, i.e. : $\underline{\hat{e}}_j[2: t'_n] = \underline{\hat{e}}_j$, while electron #1 has its own local set of axes $\{\underline{\hat{e}}_j[1: t_n]\}$ fixed relative to these global axes through this electron's own time segment and defined by the Eulerian angles $\{\phi, \theta, \psi\}$. Each electron is canonically oriented in its own reference frame, so at the instants of this interaction their transverse displacements are at $(-\Lambda/2, -\Lambda/2)$, while their longitudinal velocities are oriented along their local z-axes with speeds u_n and u'_n respectively. The 'light-vector' $\underline{c}_n$ connects these two interaction locations $\underline{x}_n$ and $\underline{x}'_n$.

$$\therefore \quad \underline{S}_n = \underline{X}_{12}[t_n; t'_n] = \underline{X}[1: t_n] - \underline{X}[2: t'_n] = \underline{X}_1[t_n] - \underline{X}_2[t'_n] = \underline{x}_n - \underline{x}'_n = \underline{c}_n\,T_n$$

After one chronon of time has elapsed, electron #2 has reached: $\underline{X}[2: t'_n+\tau] = \underline{X}_2[t'_n] + u'_n\,\tau\,\underline{\hat{e}}_3 + \Lambda\,\underline{\hat{e}}_1$

Similarly, one chronon after its interaction, electron #1 has moved by a displacement Λ in its local $\underline{\hat{e}}_1$ direction, reaching:

$$\underline{X}[1 : t_n+\tau] = \underline{X}_1[t_n] + u_n \tau\, \underline{\hat{e}}_3[1: t_n] + \Lambda\, \underline{\hat{e}}_1[1: t_n] = \underline{x}_n + \Delta\underline{X}_L[t_n] + \Delta\underline{X}_T[t_n]$$

Where $\Delta\underline{X}_L[t_n] = u_n \tau\, \underline{\hat{e}}_3[1: t_n]$ & $\Delta\underline{X}_T[t_n] = \{ \underline{\hat{e}}_+[1: t_n] + \underline{\hat{e}}_-[1: t_n] \}\Lambda/2$ with $\underline{\hat{e}}_\lambda = \underline{\hat{e}}_1 + i\,\lambda\, \underline{\hat{e}}_2$

$$\therefore\ \underline{X}_{12}[t_n+\tau;\, t'_n+\tau] = \underline{X}_1[t_n+\tau] - \underline{X}_2[t'_n+\tau] = \underline{X}_{12}[t_n;\, t'_n] + u_n \tau\, \underline{\hat{e}}_3[1: t_n] - u'_n \tau\, \underline{\hat{e}}_3 + \{ \underline{\hat{e}}_+[1: t_n] + \underline{\hat{e}}_-[1: t_n] \}\Lambda/2 - \Lambda\, \underline{\hat{e}}_1$$

$$\mathbf{X}_{12}[t_n+\tau\, ;\, t'_n+\tau] = \mathbf{X}_1[t_n+\tau] - \mathbf{X}_2[t'_n+\tau] = \mathbf{X}_1[t_n] + \Delta_\tau\mathbf{X}_1[t_n] - \mathbf{X}_2[t'_n] - \Delta_\tau\mathbf{X}_2[t'_n] = \mathbf{S}_n + \Delta_\tau\mathbf{X}_1[t_n] - \Delta_\tau\mathbf{X}_2[t'_n]$$

$$\therefore\ \mathbf{X}_{12}[t_n+\tau\, ;\, t'_n+\tau] = \mathbf{X}_{12}[t_n\, ;\, t'_n] + \Delta_\tau\mathbf{X}_{12}[t_n\, ;\, t'_n]$$

with $\mathbf{S}_n = \mathbf{X}_{12}[t_n\, ;\, t'_n] = \mathbf{C}\, T_n = i\, c\, T_n\, \mathbf{I}_0 + \underline{S}_n \bullet \underline{\mathbf{I}}$ where $\mathbf{C} = i\, c\, \mathbf{I}_0 + \underline{c}_n \bullet \underline{\mathbf{I}}$ and $\underline{S}_n = \underline{x}_n - \underline{x}'_n$ & $T_n = t_n - t'_n$

$$\therefore\ \Delta_\tau\mathbf{X}_{12}[t_n\, ;\, t'_n] = \{ u_n\, \mathbf{I}_3[1: t_n] - u'_n\, \mathbf{I}_3\}\tau + \{ \mathbf{I}_1[1: t_n] - \mathbf{I}_1 \}\, c\, \tau$$

The first factor here varies with the relative longitudinal speeds (i.e. with interaction parameter 'n') while the second factor is timeless, suggesting that it may be made equal to zero for suitable values of the Eulerian angles.

$$\therefore\ \mathbf{I}_1[1: t_n] = \mathbf{I}_1 \quad \text{or} \quad (\cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta)\,\mathbf{I}_1 + (\sin\psi\cos\phi\cos\theta + \cos\psi\sin\phi)\,\mathbf{I}_2 + \sin\theta\sin\psi\,\mathbf{I}_3 = \mathbf{I}_1$$

$$\therefore\ (\cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta) = 1 \quad \text{and} \quad (\sin\psi\cos\phi\cos\theta + \cos\psi\sin\phi) = 0 \quad \text{and} \quad \sin\theta\sin\psi = 0$$

$$\therefore\ \cos\phi = \cos\psi\cos^2\phi - (\sin\psi\cos\phi\cos\theta)\sin\phi = \cos\psi\cos^2\phi + (\cos\psi\sin\phi)\sin\phi = \cos\psi\ \ \therefore\ \phi = \psi$$

$$\therefore\ \cos\theta + 1 = 0 \quad \text{and} \quad \sin\theta\sin\psi = 0 \quad \text{with the solution:} \quad \theta = \pi \quad \text{and} \quad \psi = \phi = 0$$

$$\therefore\ \Delta_\tau\mathbf{X}_{12}[t_n\, ;\, t'_n] / \tau = u_n\, \mathbf{I}_3[1: t_n] - u'_n\, \mathbf{I}_3 = u_n\{\sin\phi\sin\theta\,\mathbf{I}_1 - \cos\phi\sin\theta\,\mathbf{I}_2 + \cos\theta\,\mathbf{I}_3\} - u'_n\,\mathbf{I}_3 = -c\,\mathbf{I}_3$$

Here the Space-Time Integrity requirement (that is: $u_n + u'_n = c$; §4.3.7) has been used to produce this attractive result. This means that the change in the spatial separation between the two electrons that interacted one chronon earlier is always only one luxon Λ apart in the longitudinal direction between the electron even though both electrons are each undergoing their own independent 'clicking' motion, provided that at the times of the interaction their local motion is consistent with the above constraints on their relative Euler orientations. This implies that when the two electrons interact their local frames of reference are effectively aligned: their $\underline{\hat{e}}_1$ and $\underline{\hat{e}}_2$ axes are parallel while their $\underline{\hat{e}}_3$ axes are anti-parallel. This means that their *local motions are highly correlated whenever they interact*.

A GF-style solution is desired: $\mathbf{X}_{12}[t_n+\tau\, ;\, t'_n+\tau] = \mathbf{T}_K\, \mathbf{X}_{12}[t_n\, ;\, t'_n] \equiv (\mathbf{I_0} + \mathbf{K})\, \mathbf{X}_{12}[t_n\, ;\, t'_n] = (\mathbf{I_0} + \mathbf{\Delta}_\tau)\, \mathbf{X}_{12}[t_n\, ;\, t'_n]$

Since $\mathbf{K} = K_0\, \mathbf{I_0} + \underline{K} \bullet \underline{\mathbf{I}}$ $\therefore$ $\mathbf{K}\mathbf{X}_{12}[t_n\, ;\, t'_n] = \mathbf{K}\mathbf{C}\, T_n = T_n\,(i\, c\, K_0 - \underline{K} \bullet \underline{C})\, \mathbf{I_0} + T_n\,(i\, c\, \underline{K} + K_0\, \underline{C}) \bullet \underline{\mathbf{I}} + T_n\, \underline{K} \wedge \underline{C} \bullet \underline{\mathbf{I}}$

Equating co-efficients of the four NV bases gives the four simultaneous (scalar and vector) equations:

$$\text{a)}\quad i\, c\, K_0 = \underline{K} \bullet \underline{C} \qquad \text{b)}\quad T_n\,(i\, c\, \underline{K} + K_0\, \underline{C} + \underline{K} \wedge \underline{C}) = -c\,\tau\, \underline{\hat{e}}_3$$

Taking the scalar product of equation b) with the vector $\underline{K}$: $T_n\,(i\, c\, \underline{K} \bullet \underline{K} + K_0\, \underline{K} \bullet \underline{C}) = i\, c\, T_n\,(\underline{K} \bullet \underline{K} + K_0\, K_0) = -c\,\tau\, K_3$

Taking the scalar product of equation b) with the vector $\underline{C}$: $T_n\,(i\, c\, \underline{C} \bullet \underline{K} + K_0\, \underline{C} \bullet \underline{C}) = -c\,\tau\, C_3 = 0$ $\therefore$ $C_3 = 0$

It is possible to make a guess (or Ansatz) at the general form of a solution to these equations.
The *Ansatz*: $K_0 = i\, K$ gives $\underline{K} \bullet \underline{K} + K_0\, K_0 = 0$ $\therefore$ $K_3 = 0$ $\therefore$ $\underline{K} \bullet \underline{C} = -c\, K$ $\therefore$ $c\, \underline{K} = -K\, \underline{C}$ $\therefore$ $\mathbf{K} = -\mathbf{C}^* K / c$

But this possibility fails the full vector equation (b): $c^2\, \tau\, \underline{\hat{e}}_3 = -T_n\,(c\, \underline{K} \wedge \underline{C}) = K\, T_n\,(\underline{C} \wedge \underline{C}) = 0$ since $\tau \neq 0$

The *Ansatz*: $K_0 = 0$ gives $\underline{K} \bullet \underline{C} = 0$ $\therefore$ $\underline{K}$ is orthogonal to $\underline{C}$ which is orthogonal to $\hat{\underline{e}}_3$. $\therefore$ $K_3 = 0$

$$\therefore \quad \underline{K} = K_1 \hat{\underline{e}}_1 + K_2 \hat{\underline{e}}_2 \quad \text{and} \quad \underline{C} = C_1 \hat{\underline{e}}_1 + C_2 \hat{\underline{e}}_2 \quad \therefore \quad c\,\tau\,\hat{\underline{e}}_3 = -T_n \{ i\,c\,(K_1 \hat{\underline{e}}_1 + K_2 \hat{\underline{e}}_2) + (K_1 C_2 - K_2 C_2)\,\hat{\underline{e}}_3 \}$$

$$\therefore \quad K_1 = K_2 = 0 \qquad \therefore \quad \mathbf{K} = 0$$

This negative result is not too surprising as each electron's position is increasing steadily over time except for those (rare) interaction instants when it undertakes a quantum jump in longitudinal velocity. In fact, the two-electron definition above indicated that the linear multiplier ($\mathbf{K}$) was mathematically identical to the micro-time difference operator ($\boldsymbol{\Delta}_\tau$) which, in general, cannot be represented by a time-independent function of the space and time variables; that is to say:

$$\mathbf{K}\ \mathbf{X}_{12}[t_n ; t'_n] = \boldsymbol{\Delta}_\tau[\ \mathbf{X}_{12}[t_n ; t'_n]\ ]$$

Similar attempts at restricted one and two-dimensional solutions also show that there are no multiplicative solutions.

5.3 INTERACTING DIGITAL ELECTRONS

5.3.1 DIGITAL ELECTRON CONSTRAINTS

Digital Electron Phase Constraints

This research programme continues to reject the new 'tradition' in modern physics of inventing equations as 'explanations' of physical phenomena. Planck's proposals for quantizing action and redefining momentum to 'curve-fit' experimental results were leading examples of this approach. Planck's Proposal for relativistic momentum was analyzed extensively in an earlier paper [9], which demonstrated that this mathematical hypothesis was **not** consistent with the physics of electromagnetism. This section will now apply Newton's original definition of momentum to the new model of the digital electron. In keeping with Newton's original definitions, the electron will retain its original invariant mass *m* and its velocity will still be defined as the difference in spatial locations across a unit of time. As in section 4.1.6, the focus here will still be on one electron (#1) when it is subject to a single inter-electron impulse $\Delta\underline{I}_n$ that occurs at one of the possible interaction times, designated as t_n. The retarded impulse will be analyzed here as it is easier to follow through time but the earlier discussion showed how the advanced case could equally apply. The time interval examined here ranges from $(t_n - \tau/2)$ to $(t_n + \tau/2)$, so that only one impulse can occur involving this electron. The impulse is assumed to act instantaneously on the 'target' electron so that all changes in velocity occur exactly (and only) at t_n, mathematically this is represented here by the point-difference operator $\Diamond$; the 'spin' states of the electron here will remain implicit. In the digital electron model (see §5.2.2), the electron moves with a constant velocity $\underline{u}_n$ between the n^{th} and $(n+1)^{th}$ interactions, while 'clicking' at 'light-speed' *c* around this axis in a four step motion, each step taking one chronon. In the post-interaction target interval, the velocities are also labeled with the 'η' subscript so the 'final' pre-interaction transverse velocity is designated by $\underline{W}_{\eta-1}$; in the post-interaction target interval, the 'first' transverse velocity is designated by $\underline{W}_\eta$ or $\underline{W}_n$ (as $t_n = \eta\tau$), so that the 'first' total velocity is $\underline{V}_n$.

For **any** chronon number ζ : $\underline{V}_\zeta = \underline{u}_\zeta + \underline{W}_\zeta$ with $\underline{u}_\zeta \bullet \underline{W}_\zeta = 0$ and $\underline{W}_\zeta \bullet \underline{W}_\zeta = c^2$ $\therefore$ $\underline{V}_n \bullet \underline{V}_n = u_n^2 + c^2$

For any dynamical variable A_n at t_n : $A^+_n = A_n$ and $A^-_n = A_{\eta-1}$ Only for the longitudinal velocity: $\underline{u}^-_n = \underline{u}_{n-1}$

It should also be noted that kinetic energy is conserved between interactions as: $\underline{V}^-_n \bullet \underline{V}^-_n = \underline{V}_{\eta-1} \bullet \underline{V}_{\eta-1} = u_{n-1}^2 + c^2$

Newton's definition of linear momentum is retained throughout this programme, thus: $\underline{P}_\eta = \underline{P}^+[t_\eta] = m\,\underline{V}^+[t_\eta] = m\,\underline{V}_\eta$

Since in the digital model, the transverse motion is viewed as eternal and intrinsic, interactions only change the longitudinal velocity – the new transverse velocity must immediately adjust itself to remain orthogonal to the new longitudinal velocity. The constraint is that the electron's existence is continuous so that the impulse is received at the true location of the electron and not at its (average) center of motion. Just as the instantaneous change in longitudinal velocity $\Diamond\underline{u}_n$ is determined by the impulse so must be the instantaneous change in the direction of the transverse velocity $\Diamond\underline{W}_n$, whose rotational 'sense' must

be conserved in the limit of negligible change in $\underline{u}_n$, which is usually the case for repulsive electron interactions at almost all distances. Determining the constraints on this 'change in phase' will be the objective of this section.

A critical question in extending Newton's schema beyond its original scope is to define the effect of the interaction on the change in motion of the participating pair of particles. Planck simply made the target particle's inertial mass a function of its velocity [9], even though this would vary with arbitrary changes in the speed of the inertial reference frame in which its motion was being determined. In fact, one of the critical assumptions that Planck made in his relativistic derivation was the equating of the change in the particle's kinetic energy $d\mathcal{K}$ with the work done $d\mathcal{W}$ by an external (constant) force $\underline{\mathcal{F}}_0$ over an infinitesimal distance $d\underline{x}$.

Planck assumed: $d\mathcal{K}[t] = d\mathcal{W}[t] = \underline{\mathcal{F}}_0 \bullet d\underline{x} = d\underline{x} \bullet d\underline{P} / dt = d\underline{P} \bullet d\underline{x} / dt = \underline{V} \bullet d\underline{P}$ and $\underline{P} = M[V]\,\underline{V}$

Newton's Second Law (in its original form) equates the change in the momentum to the impulse $\Delta\underline{\mathbf{I}}_{n+1}$ received. The only form of the instantaneous change in momentum that is consistent in the present digital model is: $\Diamond\underline{P}_n \equiv m\,\Diamond\underline{V}_n$; thus:

$\Delta\underline{\mathbf{I}}_{n+1} \rightarrow (\underline{P}^+[t_{n+1}] - \underline{P}^-[t_{n+1}]) = m (\underline{V}^+[t_{n+1}] - \underline{V}^-[t_{n+1}]) = m (\underline{V}_{n+1} - \underline{V}_n) = m \Diamond[\underline{V}[t_n]] = m \Diamond\underline{V}_n \quad \therefore \Diamond\underline{P}_n = \Delta\underline{\mathbf{I}}_{n+1}$

This impulse is of magnitude $\Delta\mathbf{I}_{n+1}$ and is parallel to the 'light-vector' $\underline{C}_{n+1}$ which tracks the difference in spatial locations between the two electrons at the two times (t_{n+1} and t'_{n+1}) defining the $(n+1)^{th}$ consecutive interaction.

The standard classical definition for the kinetic energy as the energy of motion will be retained here, thus: $\mathcal{K}_n = \tfrac{1}{2}\,\underline{V}_n \bullet \underline{P}_n$

In the digital model: $\mathcal{K}_n = \tfrac{1}{2}\,m\,\underline{V}_n \bullet \underline{V}_n = \tfrac{1}{2}\,m\,(\underline{u}_n + \underline{W}_n) \bullet (\underline{u}_n + \underline{W}_n) = \tfrac{1}{2}\,m\,(u_n^2 + c^2) = \mathcal{E}_n + \mathcal{E}_0$

This ***illustrates*** the appearance of the term mc^2 that appears everywhere in the theory of Special Relativity without use of the Lorentz transform; here it reflects the constant, intrinsic transverse motion of each electron around its own axis of motion.

The difference in kinetic energy of the electron <u>on both sides of an interaction</u>, due to the interaction with its electron partner at time t_n is $\Diamond\mathcal{K}_{n-1}$; so using the nodal average notation $\{\underline{V}_n\} \equiv \tfrac{1}{2}(\underline{V}^+_n + \underline{V}^-_n)$ then:

$2\,\Diamond\mathcal{K}_{n-1} = \Diamond[\,\underline{V}_n \bullet \underline{P}_n\,] = \underline{V}^+_n \bullet \underline{P}^+_n - \underline{V}^-_n \bullet \underline{P}^-_n = \underline{V}^-_n \bullet \Diamond\underline{P}_{n-1} + \underline{P}^+_n \bullet \Diamond\underline{V}_{n-1} = m\,(\underline{V}^+_n + \underline{V}^-_n) \bullet \Diamond\underline{V}_{n-1}$ or $\Diamond\mathcal{K}_n = \{\underline{V}_{n+1}\} \bullet \Diamond\underline{P}_n$

This is completely consistent with Thomson and Tait's classical finite difference form (see §4.3.1): $\Diamond\mathcal{K}_n = \langle \underline{V}_n \rangle \bullet \Delta\underline{\mathbf{I}}_{n+1}$

Explicit multiplication gives:

$\Diamond\mathcal{K}_{n-1} = \tfrac{1}{2}\,m\,(\underline{V}^+_n + \underline{V}^-_n) \bullet (\underline{V}^+_n - \underline{V}^-_n) = \tfrac{1}{2}\,m\,(\underline{V}^+_n \bullet \underline{V}^+_n - \underline{V}^-_n \bullet \underline{V}^-_n) = \tfrac{1}{2}\,m\,(u_n^2 - u_{n-1}^2) = \mathcal{E}_n - \mathcal{E}_{n-1} = \underline{\Delta}\mathcal{E}_{n-1} = \underline{\Delta}\mathcal{K}_{n-1}$

Note that also: $\underline{\Delta}\mathcal{K}_{n-1} = \underline{\Delta}\mathcal{E}_{n-1} = \tfrac{1}{2}\,m\,(\underline{u}_n - \underline{u}_{n-1}) \bullet (\underline{u}_n + \underline{u}_{n-1}) = \Diamond\underline{P}^{\#}_n \bullet \langle \underline{u}_{n-1} \rangle$ where $\Diamond\underline{P}^{\#}_n = m\,(\underline{u}_n - \underline{u}_{n-1}) = m\,\Diamond\underline{u}_n$

This possible definition of instantaneous momentum change, as only a change in the <u>longitudinal</u> momentum (i.e. $\Diamond\underline{P}^{\#}_n$), is **inconsistent**, as can be readily demonstrated.

Assume: $2\,\Diamond\mathcal{K}_n = \underline{V}^-_n \bullet \Diamond\underline{P}^{\#}_n + \underline{P}^+_n \bullet \Diamond\underline{V}_n = 2\,\Diamond\underline{P}^{\#}_n \bullet \langle \underline{u}_{n-1} \rangle = \Diamond\underline{P}^{\#}_n \bullet (\underline{u}_n + \underline{u}_{n-1}) \quad \therefore (\underline{W}^-_n - \underline{u}_n) \bullet \Diamond\underline{P}^{\#}_n + \underline{P}^+_n \bullet \Diamond\underline{V}_n = 0$

$\therefore \quad (\underline{W}^+_n + \underline{W}^-_n) \bullet \Diamond\underline{u}_n + \underline{V}^+_n \bullet \Diamond\underline{W}_n = 0 \qquad \therefore \quad (\underline{W}^+_n + \underline{W}^-_n) \bullet (\underline{u}_n - \underline{u}_{n-1}) + (\underline{u}_n + \underline{W}^+_n) \bullet (\underline{W}^+_n - \underline{W}^-_n) = 0$

$\therefore \quad \underline{W}^+_n \bullet \underline{V}^-_n = c^2$ alternatively $\underline{W}^+_n \bullet \Diamond\underline{V}_n = 0$

The first form only has a <u>1D solution</u> as: $\underline{V}^-_n = \underline{u}_{n-1} + \underline{W}^-_n$ & $\underline{W}^-_n = \underline{W}^+_n$ and $\underline{u}_n$ parallel to $\underline{u}_{n-1}$ so that $\underline{W}^+_n \bullet \underline{u}_{n-1} = 0$

The alternative has the solution: $\Diamond\underline{V}_n = (1+\beta_n)\,\underline{u}_n \quad \therefore \quad \Diamond\underline{W}_n = \underline{u}_{n-1} + \beta_n\,\underline{u}_n$ where β_n is a scalar that may vary with 'n'.

$\therefore \quad (\underline{W}^+_n + \underline{W}^-_n) \bullet \Diamond\underline{W}_n = (\underline{W}^+_n + \underline{W}^-_n) \bullet (\underline{W}^+_n - \underline{W}^-_n) = \underline{W}^+_n \bullet \underline{W}^+_n + \underline{W}^-_n \bullet \underline{W}^-_n = 2\,c^2 = (\underline{W}^+_n + \underline{W}^-_n) \bullet (\underline{u}_{n-1} + \beta_n\,\underline{u}_n)$

$\therefore \quad \underline{W}^+_n \bullet \underline{u}_{n-1} + \beta_n\,\underline{W}^-_n \bullet \underline{u}_n = 2\,c^2 \qquad \therefore \quad m\,\underline{W}^+_n \bullet \underline{u}_{n-1} + \beta_n\,\underline{W}^-_n \bullet \Delta\underline{\mathbf{I}}_n = 2\,m\,c^2$ Using $\Delta\underline{\mathbf{I}}_n = m\,(\underline{u}_n - \underline{u}_{n-1})$

Also, $\underline{u}_{n-1} \wedge \Diamond \underline{W}_n = \underline{u}_{n-1} \wedge \beta_n \underline{u}_n \quad \therefore\ m\, \underline{u}_{n-1} \wedge \Diamond \underline{W}_n = \beta_n \underline{u}_{n-1} \wedge \Delta \underline{\mathbf{I}}_n$ with solution $\beta_n \Delta \underline{\mathbf{I}}_n = m \Diamond \underline{W}_n$

$\therefore\ \beta_n \Diamond \underline{u}_n = \Diamond \underline{W}_n = \underline{u}_{n-1} + \beta_n \underline{u}_n = \beta_n (\underline{u}_n - \underline{u}_{n-1}) \quad \therefore\ (1 + \beta_n) \underline{u}_{n-1} = 0 \quad \therefore\ \beta_n = -1 \quad \therefore\ \Diamond \underline{V}_n = 0$

Substituting this result in the earlier equation i.e. $(\underline{W}^-_n - \underline{u}_n) \bullet \Diamond \underline{P}^{\#}_n + \underline{P}^+_n \bullet \Diamond \underline{V}_n = 0$ then requires: $(\underline{W}^-_n - \underline{u}_n) \bullet \Diamond \underline{P}^{\#}_n = 0$

This implies that either: $\underline{W}^-_n = \underline{u}_n$ or $\underline{W}^-_n = \underline{u}_n - \underline{u}_{n-1}$. This is impossible as $W^-_n = c$ while $u_n = c$ only when $n = 0$.

Returning now to the correct extension of Newton's Second Law for change in momentum (i.e. $\Diamond \underline{P}_n$) and equating the two forms of $\Diamond \mathcal{K}_n$ gives:

$\therefore\ (\underline{W}^+_n + \underline{W}^-_n) \bullet \Diamond \underline{V}_n + (\underline{u}_n + \underline{u}_{n-1}) \bullet \Diamond \underline{W}_n = 0 \quad \therefore\ \{(\underline{W}^+_n + \underline{W}^-_n) \pm (\underline{u}_n + \underline{u}_{n-1})\} \bullet \Diamond \underline{V}_n + (\underline{u}_n + \underline{u}_{n-1}) \bullet \Diamond \underline{W}_n = 0$

This leads to the necessary 3D constraint on the pre- and post velocities: $< \underline{V}_n > \bullet \Diamond \underline{V}_n = < \underline{u}_n > \bullet \Diamond \underline{u}_n$

Consider the simpler situation where the electron is moving longitudinally in the positive z-direction with speed u and also transversely in the negative x-direction prior to the interaction and the impulse from the other electron is restricted to the y-z plane. A solution will be attempted that posits that the post-interaction transverse motion is found in the x-z plane.

$\underline{u}_{n-1} = u\, \hat{\underline{e}}_3$ and $\underline{W}^-_n = -c\, \hat{\underline{e}}_1$ Assume a transverse solution: $\underline{W}^+_n = (\alpha - 1) \underline{W}^-_n + \beta \underline{u}_{n-1} = (1 - \alpha) c\, \hat{\underline{e}}_1 + \beta u\, \hat{\underline{e}}_3$

As always $\underline{W}^+_n \bullet \underline{W}^+_n = c^2 = (1 - \alpha)^2 c^2 + \beta^2 u^2 \quad \therefore\ \beta^2 u^2 = \alpha (2 - \alpha) c^2$ Since $c \geq u$ then $\alpha \leq \beta^2 / 2 << 1$

Direct substitution shows that this transverse solution satisfies the constraint: $< \underline{W}_n > \bullet \Diamond \underline{V}_n + < \underline{u}_n > \bullet \Diamond \underline{W}_n = 0$

In this case, $\underline{W}^+_n = -\underline{W}^-_n + (\alpha \underline{W}^-_n + \beta \underline{u}_{n-1}) \approx -\underline{W}^-_n$ since α and β are much less than unity. Thus, the first transverse velocity after an interaction is almost in the **opposite** direction to the last transverse velocity prior to the interaction. This means that the longitudinal velocity almost preserves its direction across the impulse while the transverse velocity "**skips a beat**", that is, instead of 'clicking' through another quarter turn, the electron effectively clicks through a full half-cycle, corresponding to the π phase-shift of EM when an atom re-emits a photon. This 'skipping a beat' at the time of interaction may be viewed as an instantaneous, torque-like contribution from the other electron that is participating in this interaction, which suffers a corresponding (but opposite) 'twist' as it emits the impulse. This is a **central result** of the DEM.

If this situation is further simplified to the 1D canonical electron-scattering model (see §4.3.4) then all longitudinal motion occurs in the z-direction and all transverse motion occurs in the x-y plane. In this model, the change in longitudinal speed Δu_n is always b (or c / N) per interaction, so that $u_n = n\, b$ and $\Delta u_n = b$ (increasing) or $u_n = c - n\, b$ and $\Delta u_n = -b$ (decreasing). In the simple (increasing) model:

$\underline{u}_{n-1} = (n - 1)\, b\, \hat{\underline{e}}_3$ and $\underline{W}^-_n = -c\, \hat{\underline{e}}_1$ while $\underline{u}_n = n\, b\, \hat{\underline{e}}_3$ so that $\Diamond \underline{u}_n = b\, \hat{\underline{e}}_3$ and $< \underline{u}_n > = (n - \tfrac{1}{2})\, b\, \hat{\underline{e}}_3$

The transverse constraint becomes: $< \underline{W}_n > \bullet (\Diamond \underline{W}_n + \Diamond \underline{u}_n) + < \underline{u}_n > \bullet \Diamond \underline{W}_n = 0$ so that after vector substitutions:

$\tfrac{1}{2} (\underline{W}^+_n - c\, \hat{\underline{e}}_1) \bullet (\underline{W}^+_n + c\, \hat{\underline{e}}_1 + b\, \hat{\underline{e}}_3) + (n - \tfrac{1}{2})\, b\, (\underline{W}^+_n + c\, \hat{\underline{e}}_1) \bullet \hat{\underline{e}}_3 = 0 \quad \therefore\ n\, \underline{W}^+_n \bullet \hat{\underline{e}}_3 = 0$

Here, the impulse: $\Delta \underline{\mathbf{I}}_n = \Diamond \underline{P}_n = m \Diamond \underline{V}_n = m \Diamond \underline{u}_n = m\, b\, \hat{\underline{e}}_3 = m (\Diamond \underline{W}_n + \Diamond \underline{u}_n) \quad \therefore\ \Diamond \underline{W}_n = 0 \quad \therefore\ \underline{W}^+_n = \underline{W}^-_n$

In order to reverse this transverse motion (which keeps the centre-of-motion continuous) it is necessary to add a torque $\Delta \underline{\Gamma}_n$ to the final form of the impulse, namely: $\Diamond \underline{P}_n = \Delta \underline{\mathbf{I}}_n + \Delta \underline{\Gamma}_n \quad \therefore\ \Diamond \underline{W}_n = \Delta \underline{\Gamma}_n = -2\, \underline{W}^-_n \quad \therefore\ \underline{W}^+_n = -\underline{W}^-_n$

In the real 3D world, this will be accomplished by an offset $\Delta \underline{r}_n$ between the two centers of motion when the two electrons 'agree' to particpate in the n^{th} interaction as long as $\Delta \underline{r}_n$ is orthogonal to the prior transverse velocity $\underline{W}^-_n$.

Digital Electron Space-Time Integrity

An earlier section (§4.2.4) analyzed the impact of the 'Light-Cone' Condition on two interacting electrons that were subject to only one of the two new properties that extended the model of a simple Newtonian point particle to a 'Digital Electron'. This prior condition was the application of the constraint that interactions only occurred at discrete time intervals ("on the chronon"). The critical combination once again is the fact that the interactions must occur on their mutual 'light-cones' and the fact that all electrons possess finite inertia (intrinsic mass) which resists changes in their longitudinal momentum. The earlier analysis showed that the 'light-vector' $\mathbf{C}_n$ connecting the n^{th} pair of interaction points $\underline{x}_n$ and $\underline{x}'_n$ on the two electrons' trajectories remained constant throughout a set of consecutive interactions; in other words: $\mathbf{C}_n = \mathbf{C}$. This was restated above as the EM interaction between two electrons **can** occur in the present theory **whenever** the 'norm' of their Space-Time Separation DNV $\mathbf{S}_n$, defined between any pair of interaction points, is zero, where the interaction impulse is defined along the 'line-of-existence nodes' at the (different) **times** of each interaction.

$$\mathbf{S}_n \equiv i\, c\, T_n\, \mathbf{I_0} + \underline{S}_n \bullet \underline{\mathbf{I}} \qquad \text{The square (or 'norm') of this DNV is:} \quad \mathbf{S}_n^* \mathbf{S}_n = (c^2 T_n^2 - \underline{S}_n \bullet \underline{S}_n)\, \mathbf{I_0} = 0$$

Using the two 'difference' variables: $T_n \equiv t_n - t'_n$ and $\underline{S}_n \equiv \underline{x}[1\colon n] - \underline{x}[2\colon n]$ $\quad \therefore\ \underline{S}_n \bullet \underline{S}_n = c^2 T_n^2 \quad \therefore\ \mathbf{S}_n = \mathbf{C}\, T_n$

Section 4.2.4 showed that the difference between two consecutive light-vectors $\Delta\mathbf{S}_n$ is equal to the 'interaction-difference' between the interaction nodes of the two interacting electrons where the next consecutive interaction after the n^{th} interaction is the $(n+1)^{th}$ at times t_{n+1} and t'_{n+1} , where $t_{n+1} > t_n$ and $t'_{n+1} > t'_n$. This approach showed that two equivalent paths could be defined through both space and time from the first two interaction points to the second two interaction points, each defined in terms of the individual electron location DNVs $\mathbf{X}_n$ and $\mathbf{X}'_n$. This confirmed the interaction-difference result:

$$\Delta\mathbf{S}_n = \Delta_I[\mathbf{X}_n - \mathbf{X}'_n] = \mathbf{C}\, \Delta T_n$$

These interaction points can now be defined in terms of the locations described in the digital model: $\mathbf{X}_n = \boldsymbol{\mathcal{X}}_n + \boldsymbol{\Lambda}_n$

Here $\underline{\mathcal{X}}_n$ is the **center-of-motion location** of electron #1 at t_n while $\underline{\Lambda}_n$ is its orthogonal offset, so: $\boldsymbol{\mathcal{X}}_\eta = i\, c\, t_\eta\, \mathbf{I_0} + \underline{\mathcal{X}}_\eta \bullet \underline{\mathbf{I}}$

The 'post-interaction' longitudinal velocities $\underline{u}_n$ and $\underline{u}'_n$ can be defined for each electron as 'joining' consecutive interaction locations: $\Delta\underline{\mathcal{X}}_n \equiv \Delta t_n\, \underline{u}_n$ and $\Delta\underline{\mathcal{X}}'_n \equiv \Delta t'_n\, \underline{u}'_n$. Note that the electrons do **not** actually follow these 'average' trajectories. In terms of the 'chronon counts' at each of the interaction times:

$$t_n = \eta_n\, \tau \quad \therefore\ \Delta t_n = (\eta_{n+1} - \eta_n)\, \tau \equiv \kappa_n\, \tau \quad \text{or} \quad \kappa_n = \Delta\eta_n = 4\, K_n + \mu_n \quad \text{where } \mu_n \text{ is the phase number (1,2,3,4).}$$

$$\text{As } \mathbf{S}_n = \mathbf{X}_n - \mathbf{X}'_n \quad \therefore\ \Delta\mathbf{S}_n = \Delta\mathbf{X}_n - \Delta\mathbf{X}'_n = i\, c\, (\Delta t_n - \Delta t'_n)\, \mathbf{I_0} + (\Delta t_n\, \underline{u}_n - \Delta t'_n\, \underline{u}'_n) \bullet \underline{\mathbf{I}} + (\Delta\underline{\Lambda}_n - \Delta\underline{\Lambda}'_n) \bullet \underline{\mathbf{I}}$$

If the two electrons' change in phase numbers remain commensurate (mod 4) then: $\Delta\underline{\Lambda}_n = \Delta\underline{\Lambda}'_n$ when $\Delta\mu_n = \Delta\mu'_n$ (mod 4).

From above: $\Delta\mathbf{S}_n = \mathbf{C}\, \Delta T_n = (i\, c\, \mathbf{I_0} + \underline{c} \bullet \underline{\mathbf{I}})\, \Delta T_n \quad \therefore\ \underline{c}\, \Delta T_n = \Delta\underline{S}_n$ or $\underline{c}\, (\Delta t_n - \Delta t'_n) = (\Delta t_n\, \underline{u}_n - \Delta t'_n\, \underline{u}'_n)$

This results once again in the universal '***Space-Time Integrity***' condition: $(\underline{c} - \underline{u}_n)\, \Delta t_n = (\underline{c} - \underline{u}'_n)\, \Delta t'_n$

Re-combining these two conditions into another form of the space-time integrity condition: $\underline{c}\, \Delta_I[t_n - t'_n] = \Delta_I[\underline{\mathcal{X}}_n - \underline{\mathcal{X}}'_n]$

So, the universal '***Space-Time Integrity***' condition remains valid for the Digital Electron model in terms of the **average** longitudinal velocities ($\underline{u}_n$ & $\underline{u}'_n$) as long as the interactions preserve the **Phase-Change Condition**: $\Delta\underline{\Lambda}_n = \Delta\underline{\Lambda}'_n$.

Impacting the Digital Electron

Section 4.3.1 introduced the idea of the Impact Operator, $\boldsymbol{\delta}_0$ to quantify the effects of any single interaction on a point particle. This was used to compute the difference in any dynamical variable describing the post-interaction effects with the situation that would have occurred had the interaction not been experienced by the particle. For pedagogical reasons, that initial presentation focused only on a simple Newtonian particle but these concepts can be readily extended to describe the impact of interactions on the more complex behaviour of the digital electron. These extensions are most easily seen by once again examining the effects on the single particle activity DNV $\mathbf{R}$ where now the electron's spatial location $\mathbf{X}$ includes the transverse offset $\boldsymbol{\Lambda}$ from the average center-of-motion location $\boldsymbol{\mathcal{X}}$ while the electron's velocity $\mathbf{V}$ adds the 4-step 'clicking' motion $\mathbf{W}$ to the longitudinal velocity $\mathbf{U}$ that is constant between any two interaction times t_n and t_{n+1}. As before, the first step is to evaluate the change in the particle's activity from any one interaction node ($t_\eta = t_{n+1}$) to immediately before the next interaction node at $t_{\eta'} = t_{n+2}$. The critical step here is to impose the 'modulo 4' condition on the number of chronons between every two consecutive interaction nodes; i.e. $\kappa_n = \eta' - \eta = 4K_n$ or $t_{\eta'} - t_\eta = 4K_n\,\tau$. Under this constraint, the difference in activity of the digital electron reduces to the difference in activity of the Newtonian particle.

$$\underline{\Delta}_I\mathbf{R}_{n+1} \equiv \Delta\mathbf{R}[t_{n+2} - \delta t,\, t_{n+1} + \delta t] = \Delta_{\eta'-\eta}[\,\mathbf{R}[t_{n+1}]\,] = \Delta_k[\,\mathbf{R}^N[t_{n+1}]\,] = \underline{\Delta}_I\mathbf{R}^N{}_{n+1}$$

$$\therefore\ \underline{\Delta}_I\mathbf{R}_{n+1} = i\,m\,(t_{n+2} - t_{n+1})\,(c^2 - \underline{u}_{n+1} \bullet \underline{u}_{n+1})\,\mathbf{I_0} = i\,m\,\Delta t_{n+1}\,(c^2 - \underline{u}_{n+1} \bullet \underline{u}_{n+1})\,\mathbf{I_0}$$

If the electron had not participated in the interaction at t_{n+1} then it would have retained the same prior-velocity up to t_{n+2} ; this 'free' activity in this interval is designated as: $\underline{\Delta}_I\mathbf{R}^0{}_{n+1} = i\,m\,\Delta t_{n+1}\,(c^2 - \underline{u}_n \bullet \underline{u}_n)\,\mathbf{I_0}$

In the case of one of the electrons (say #1), the impact across its one interval Activity as a result of the interaction at t_{n+1} is:

$$\boldsymbol{\delta}_0\,[\,\underline{\Delta}_I\mathbf{R}_{n+1}\,] = \underline{\Delta}_I\mathbf{R}_{n+1} - \underline{\Delta}_I\mathbf{R}^0{}_{n+1} = i\,\delta_0\,\underline{\Delta}_I\mathbb{R}_{n+1}\,\mathbf{I_0} \qquad \therefore\ \delta_0\,\underline{\Delta}_I\mathbb{R}_{n+1} = -\,m\,\Delta t_{n+1}\,(\underline{u}_{n+1} \bullet \underline{u}_{n+1} - \underline{u}_n \bullet \underline{u}_n)$$

Even though the digital electron's motion includes a transverse light-speed component, which contributes a fixed amount to its total kinetic energy, the change in kinetic energy across an interaction interval is still only the difference in longitudinal kinetic energy. This was shown above to be proportional to its action across this interval, so again: $\delta_0\,\underline{\Delta}_I\mathbb{R}_{n+1} = -\,\underline{\Delta}_I\mathcal{A}_{n+1}$. This shows that the impact on the digital electron's *activity* over the next time interval ($\delta_0\underline{\Delta}_I\mathrm{R}_n$), from a single (say, velocity increasing) interaction at t_n is equal to the decrease in the electron's *unit-action* ($\underline{\Delta}_I\mathcal{A}_n$) across this post-interaction interval. This critical reduction to results comparable to the simpler Newtonian particle means that all the earlier results of section 4.3 can now be imported directly to the more complex digital electron model, including the two-particle, total-activity results.

5.3.2 DIGITAL ELECTRON ACTIVITY

Hermitian (Reverse) Conjugation

The concept of *activity* was introduced in the first paper reporting on this programme [112]. This concept was extended here to discrete time in §4.3.1 where the time variable was replaced with a multiple of the chronon, this gave:

$$\text{Digital Electron Activity}\quad \mathbf{R}_\eta \equiv i\,\mathbf{X}[t_\eta]\,\mathbf{P}[t_\eta] = i\,m\,\mathbf{X}_\eta\,\mathbf{V}_\eta{}^* = i\,\mathbb{R}_\eta\,\mathbf{I_0} + \underline{R}_\eta \bullet \underline{\mathbf{I}}$$

$$\text{where}\quad \mathbb{R}_\eta = m\,(c^2\,t_\eta - \underline{X}_\eta \bullet \underline{V}_\eta)\quad \&\quad \underline{R}_\eta = \underline{\mathbf{R}}_\eta + i\,\underline{\mathscr{R}}_\eta\quad \text{with}\quad \underline{\mathbf{R}}_\eta = m\,c\,(\underline{X}_\eta - \underline{V}_\eta\,t_\eta)\quad \&\quad \underline{\mathscr{R}}_\eta = m\,(\underline{X}_\eta \wedge \underline{V}_\eta)$$

The first paper also showed that the single particle's **Activity** natural vector is a single mathematical quantity combining the four basic concepts in Newtonian physics, namely: time t_η , space $\underline{X}_\eta$, velocity $\underline{V}_\eta$ and mass m ; it can be re-stated as the sum of the three most powerful (derived) concepts in classical mechanics: action $\mathcal{A}_\eta$, Galilean momentum $\underline{\mathcal{G}}_\eta$ (or linear momentum $\underline{P}_\eta$) and angular momentum $\underline{\mathcal{M}}_\eta$.

$$\mathbf{R}_\eta \equiv i\,\mathcal{A}_\eta\,\mathbf{I_0} + c\,\underline{\mathcal{G}}_\eta \bullet \underline{\mathbf{I}} + i\,\underline{\mathcal{M}}_\eta \bullet \underline{\mathbf{I}} \qquad \text{where}\quad \mathcal{A}_\eta = \mathbb{R}_\eta\quad \&\quad \underline{\mathcal{G}}_\eta = \underline{\mathbf{R}}_\eta\quad \&\quad \underline{\mathcal{M}}_\eta = \underline{\mathscr{R}}_\eta$$

$$\text{or}\quad \mathcal{A}_\eta = (m\,c^2\,t_\eta - \underline{X}_\eta \bullet \underline{P}_\eta)\quad \&\quad \underline{\mathcal{G}}_\eta = c\,(m\,\underline{X}_\eta - \underline{P}_\eta\,t_\eta)\quad \&\quad \underline{\mathcal{M}}_\eta = \underline{X}_\eta \wedge \underline{P}_\eta$$

This is consistent with the prior definitions as can be seen by introducing the concept of the Natural Vector **Hermitian** (or reverse) **conjugate** that directly reflects the comparable matrix algebra definitions and is central to standard QM.

Def's: Natural Vector **Reversal**: $(\mathbf{A\ B\ C})^{\mathbf{R}} \equiv \mathbf{C\ B\ A}$ & Natural Vector **Hermitian**: $(\mathbf{A\ B\ C})^{\dagger} \equiv \mathbf{C^*\ B^*\ A^*}$

The Hermitian conjugate for a binary product can be easily evaluated (see A1.1) by considering two complex NVs.

$$\mathbf{A} = i\,A_0\,\mathbf{I_0} + \underline{A} \bullet \underline{\mathbf{I}} \quad \& \quad \mathbf{B} = i\,B_0\,\mathbf{I_0} + \underline{B} \bullet \underline{\mathbf{I}} \quad \& \quad \mathbf{A^*\,B} = \mathbf{I_0}\,(A_0^*\,B_0 - \underline{A}^* \bullet \underline{B}) + i\,\underline{\mathbf{I}} \bullet (\underline{A}^* B_0 - A_0^*\,\underline{B}) + (\underline{A}^* \wedge \underline{B}) \bullet \underline{\mathbf{I}}$$

$$\therefore\ (\mathbf{A^*\,B})^{\dagger} = \mathbf{B^*\,A} = \mathbf{I_0}\,(B_0^*\,A_0 - \underline{B}^* \bullet \underline{A}) + i\,\underline{\mathbf{I}} \bullet (\underline{B}^* A_0 - B_0^*\,\underline{A}) + (\underline{B}^* \wedge \underline{A}) \bullet \underline{\mathbf{I}} \quad \text{NB } (\underline{A}^* \wedge \underline{B})^{\dagger} = (\underline{B}^* \wedge \underline{A})$$

$$\mathbf{A^*\,B} \pm (\mathbf{A^*\,B})^{\dagger} = \mathbf{I_0}\,\{(A_0^*\,B_0 \pm B_0^*\,A_0) - (\underline{A}^* \bullet \underline{B} \pm \underline{B}^* \bullet \underline{A})\} + \underline{\mathbf{I}} \bullet (\underline{A}^* \wedge \underline{B} \pm \underline{B}^* \wedge \underline{A}) + i\,\underline{\mathbf{I}} \bullet \{(\underline{A}^* B_0 \pm \underline{B}^* A_0) - (A_0^*\,\underline{B} \pm B_0^*\,\underline{A})\}$$

When $\{A_0\ ,\ \underline{A}\}$ and $\{B_0\ ,\ \underline{B}\}$ are purely real four-vectors, these sum and differences reduce to scalar and complex vectors:

$$\mathbf{A^*\,B} + (\mathbf{A^*\,B})^{\dagger} = 2\,(A_0\,B_0 - \underline{A} \bullet \underline{B})\,\mathbf{I_0} \quad \text{and} \quad \mathbf{A^*\,B} - (\mathbf{A^*\,B})^{\dagger} = 2\,(\underline{A} \wedge \underline{B}) \bullet \underline{\mathbf{I}} + i\,2(\underline{A}\,B_0 - \underline{B}\,A_0) \bullet \underline{\mathbf{I}}$$

$$\therefore \quad \mathbf{R}_{\eta}^* = \{i\,m\,\mathbf{X}_{\eta}\,\mathbf{V}_{\eta}^*\}^* = -\,i\,m\,\mathbf{X}_{\eta}^*\,\mathbf{V}_{\eta} = -\,i\,\mathcal{A}_{\eta}\,\mathbf{I_0} + \underline{\mathcal{G}}_{\eta} \bullet \underline{\mathbf{I}} - i\,\underline{\mathcal{M}}_{\eta} \bullet \underline{\mathbf{I}}$$

$$\therefore \quad \mathbf{R}_{\eta}^{\dagger} = \{i\,m\,\mathbf{X}_{\eta}\,\mathbf{V}_{\eta}^*\}^{\dagger} = -\,i\,m\,\mathbf{V}_{\eta}\,\mathbf{X}_{\eta}^* = -\,i\,\mathcal{A}_{\eta}\,\mathbf{I_0} + \underline{\mathcal{G}}_{\eta} \bullet \underline{\mathbf{I}} + i\,\underline{\mathcal{M}}_{\eta} \bullet \underline{\mathbf{I}}$$

$$\therefore \quad \tfrac{1}{2}\,(\mathbf{R}_{\eta} - \mathbf{R}_{\eta}^{\dagger}) = i\,m\,(\mathbf{X}_{\eta}\,\mathbf{V}_{\eta}^* + \mathbf{V}_{\eta}\,\mathbf{X}_{\eta}^*)/2 = i\,(\mathbf{X}_{\eta}\,\mathbf{P}_{\eta} + \mathbf{P}_{\eta}^*\,\mathbf{X}_{\eta}^*)/2 = i\,\mathcal{A}_{\eta}\,\mathbf{I_0}$$

$$\therefore \quad \tfrac{1}{2}\,(\mathbf{R}_{\eta} + \mathbf{R}_{\eta}^*) = i\,(\mathbf{X}_{\eta}\,\mathbf{P}_{\eta} - \mathbf{X}_{\eta}^*\,\mathbf{P}_{\eta}^*)/2 = \underline{\mathcal{G}}_{\eta} \bullet \underline{\mathbf{I}}$$

$$\therefore \quad \tfrac{1}{2}\,(\mathbf{R}_{\eta}^{\dagger} - \mathbf{R}_{\eta}^*) = i\,(\mathbf{X}_{\eta}^*\,\mathbf{P}_{\eta}^* - \mathbf{P}_{\eta}^*\,\mathbf{X}_{\eta}^*)/2 = i\,\underline{\mathcal{M}}_{\eta} \bullet \underline{\mathbf{I}}$$

This need to extract the three basic invariants, requiring **both** the use of the standard **and** reverse conjugates, illustrates why Feynman needed to use the $\boldsymbol{\gamma}_5$ matrix (or 'adjoint' wave-function to reverse the otherwise negative signs in the 3rd and 4th rows) so that he could define a Lorentz-invariant 4-vector electron current, $\mathbf{J}_{\mu}$ (see section 3.1.2). This also explains why Hestenes needed to use his 'reversal' operation on his 4D geometric products to achieve a similar result (see section 3.1.3).

The Digital Electron DNV Model

The concept of temporal invariants was introduced in the first paper in this series and is summarized in Appendix III here. The formulation so far, has been applied to the motion of a Newtonian particle, traveling freely from a location $\underline{X}_0$ at a time t_0 at a constant velocity $\underline{u}_0$ (in the z-direction). This section will now extend this analysis to the '**digital**' electron, which **adds two** new features to the Newtonian model of a point particle. These two new features center on the intrinsic, periodic capacity of the electron to only participate in an interaction with another electron after a universal unit of time (the chronon) when it 'ticks' at time instants t_{η}. Secondly, the digital electron 'clicks' at every one of these times as it moves discretely, at 'light-speed' c, in a four-step, transverse motion around its longitudinal direction of travel. There is no loss in generality to assume that at t_0 the electron is in its canonical configuration (see section 5.2.2) and to assume that the key integer temporal variable $\eta = 4N_{\eta} + \mu$ (where $\mu = 0, 1, 2, 3$) while the phase parameter $\mu = 0$ indicates that the electron is at the start of any 4-step click-cycle (e.g. it is moving along the x-direction). The digital electron model is a direct extension of the Newtonian particle model, here labeled with an N superscript while the digital extensions are labeled by a D superscript.

$$\mathbf{X}_{\eta} = \mathbf{X}^{\mathrm{N}}{}_{\eta} + \mathbf{X}^{\mathrm{D}}{}_{\eta} \quad \text{and} \quad \mathbf{V}_{\eta} = \mathbf{V}^{\mathrm{N}}{}_{\eta} + \mathbf{V}^{\mathrm{D}}{}_{\eta} = (\mathbf{X}_{\eta+1} - \mathbf{X}_{\eta})/\tau \quad \text{Writing: } \mathbf{X}^{\mathrm{D}}{}_{\eta} = \boldsymbol{\Lambda}_{\mu} \ \& \ \mathbf{V}^{\mathrm{D}}{}_{\eta} = \mathbf{W}_{\mu}$$

$$\text{where: } \mathbf{X}^{\mathrm{N}}{}_{\eta} = \boldsymbol{\mathcal{X}}_{\eta} = i\,c\,t_{\eta}\,\mathbf{I_0} + \underline{\mathcal{X}}_{\eta} \bullet \underline{\mathbf{I}} \quad \text{with} \quad \underline{\mathcal{X}}_{\eta} = \underline{X}_0 + \eta\tau\,u_0\,\hat{\underline{e}}_3 \quad \text{and} \quad \mathbf{V}^{\mathrm{N}}{}_{\eta} = \mathbf{U} = i\,c\,\mathbf{I_0} + u_0\,\mathbf{I_3}$$

The 'digital' additions are only in the transverse plane, where it is easier to work with transverse vectors (see A1.4) and the results of section 5.2.2 can be used to describe all possible interaction times for the locations and section 5.2.3 for velocity.

$$\boldsymbol{\Lambda}_{\mu} = -\{\hat{\underline{e}}_1 \cos((2\eta+1)\pi/4) + \hat{\underline{e}}_2 \sin((2\eta+1)\pi/4)\}\Lambda/\sqrt{2} = -\{\hat{\underline{e}}_1 \cos((2\mu+1)\pi/4) + \hat{\underline{e}}_2 \sin((2\mu+1)\pi/4)\}\Lambda/\sqrt{2}$$

$\therefore\ \mathbf{\Lambda}_\mu = -(\mathbf{I}_+ e^{-i\theta} + \mathbf{I}_- e^{i\theta})\,\Lambda/(2\sqrt{2}) = \underline{\Lambda}_\mu \bullet \underline{\mathbf{I}} = \mathbf{\Lambda}_\mu^*$ and $\mathbf{W}_\mu = (\mathbf{I}_+ e^{-i\psi} + \mathbf{I}_- e^{i\psi})\,c/2 = \underline{W}_\mu \bullet \underline{\mathbf{I}} = \mathbf{W}_\mu^*$

Where $\psi = \mu\pi/2$ and $\theta = (2\mu+1)\pi/4 = \psi + \pi/4$ have been used only for visual simplification (NB not Euler angles).

$$\therefore\ \mathbf{V}_\eta = \mathbf{U} + \mathbf{W}_\mu \quad \therefore\ \mathbf{V}_\eta + \mathbf{V}_\eta^* = 2\,u_0\,\mathbf{I_3} + (\mathbf{I}_+ e^{-i\psi} + \mathbf{I}_- e^{i\psi})\,c \quad \text{and} \quad \mathbf{V}_\eta - \mathbf{V}_\eta^* = 2\,i\,c\,\mathbf{I_0}$$

The normal forms of these DNVs are: $\mathbf{\Lambda}_\mu^*\,\mathbf{\Lambda}_\mu = -\mathbf{I_0}\,\Lambda^2/2$ & $\mathbf{U}_\mu^*\,\mathbf{U}_\mu = (c^2 - u_0^2)\,\mathbf{I_0}$ & $\mathbf{W}_\mu^*\,\mathbf{W}_\mu = -c^2\,\mathbf{I_0}$

It is always possible (at least mathematically) to transform these variables into the frame of reference traveling with the center of motion of the electron; this will be called the *Electron's Reference Frame* (or **ERF**): $\underline{x}_\mu \equiv \underline{X}_\eta - \underline{\mathcal{X}}_\eta = \underline{X}^D{}_\eta = \underline{\Lambda}_\mu$

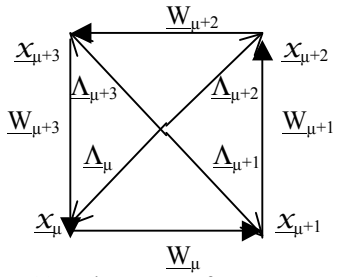


Fig. 13 Electron Reference Frame

In this frame, it can be readily seen that: $\underline{W}_\mu = \Delta\underline{x}_\mu / \tau = (\underline{x}_{\mu+1} - \underline{x}_\mu)/\tau$ or $\Delta\underline{\Lambda}_\mu = \tau\,\underline{W}_\mu$ N.B. $\underline{W}_{\mu+2} = -\underline{W}_\mu$

and $\Delta\underline{W}_\mu = \underline{W}_{\mu+1} - \underline{W}_\mu = \underline{W}_{\mu+1} + \underline{W}_{\mu+2} = \Delta\underline{x}_{\mu+1}/\tau + \Delta\underline{x}_{\mu+2}/\tau = (\underline{x}_{\mu+3} - \underline{x}_{\mu+2} + \underline{x}_{\mu+2} - \underline{x}_{\mu+1})/\tau = (\underline{x}_{\mu+3} - \underline{x}_{\mu+1})/\tau$

$\therefore\ \Delta\underline{W}_\mu = (\underline{\Lambda}_{\mu+3} - \underline{\Lambda}_{\mu+1})/\tau = -2\,\underline{\Lambda}_{\mu+1}/\tau$ This is the Thomas precession result, introducing the 'mysterious' factor 2 [1x].

This key result can **only** be deduced from **discrete** motion if the electron undertakes a **FOUR** step cycle around the axis.

Now $\mathbf{I_3}\,\mathbf{W}_\mu = \mathbf{I_3}\,(\mathbf{I}_+ e^{-i\psi} + \mathbf{I}_- e^{i\psi})\,c/2 = (-\mathbf{I}_+ i\,e^{-i\psi} + \mathbf{I}_- i\,e^{i\psi})\,c/2 = (\mathbf{I}_+ e^{-i\pi/2} e^{-i\psi} + \mathbf{I}_- e^{+i\pi/2} e^{i\psi})\,c/2$

$= (\mathbf{I}_+ e^{-i(\mu+1)\pi/2} + \mathbf{I}_- e^{i(\mu+1)\pi/2})\,c/2$ $\therefore\ \mathbf{I_3}\,\mathbf{W}_\mu = \mathbf{W}_{\mu+1}$ also $\mathbf{W}_\mu\,\mathbf{I_3} = -\mathbf{W}_{\mu+1}$

$\therefore\ \mathbf{U}_\eta^*\,\mathbf{W}_\mu = (\,i\,c\,\mathbf{I_0} + u_0\,\mathbf{I_3}\,)^*\,\mathbf{W}_\mu = -i\,c\,\mathbf{W}_\mu + u_0\,\mathbf{W}_{\mu+1}$ also $\mathbf{W}_\mu^*\,\mathbf{U}_\eta = i\,c\,\mathbf{W}_\mu - u_0\,\mathbf{W}_{\mu+1}$

$\therefore\ \mathbf{V}_\eta^*\,\mathbf{V}_\eta = (\mathbf{U}_\eta + \mathbf{W}_\mu)^*\,(\mathbf{U}_\eta + \mathbf{W}_\mu)$ $\therefore\ \mathbf{V}_\eta^*\,\mathbf{V}_\eta = -u_0^2\,\mathbf{I_0}$ (independent of t_η)

A similar analysis results in the identities: $\mathbf{I_3}\,\mathbf{\Lambda}_\mu = \mathbf{\Lambda}_{\mu+1}$ also $\mathbf{\Lambda}_\mu\,\mathbf{I_3} = -\mathbf{\Lambda}_{\mu+1}$

$\therefore\ \mathcal{X}_\eta^*\,\mathbf{\Lambda}_\mu = (\,i\,c\,t_\eta\,\mathbf{I_0} + \underline{X}_0 \bullet \underline{\mathbf{I}} + u_0\,t_\eta\,\mathbf{I_3}\,)^*\,\mathbf{\Lambda}_\mu = -i\,c\,t_\eta\,\mathbf{\Lambda}_\mu + u_0\,t_\eta\,\mathbf{\Lambda}_{\mu+1} - \mathbf{I_0}\,(\underline{X}_0 \bullet \underline{\mathbf{\Lambda}}_\mu) + i\,\underline{\mathbf{I}} \bullet (\underline{X}_0 \wedge \underline{\mathbf{\Lambda}}_\mu)$

also $\mathbf{\Lambda}_\mu^*\,\mathcal{X}_\eta = i\,c\,t_\eta\,\mathbf{\Lambda}_\mu - u_0\,t_\eta\,\mathbf{\Lambda}_{\mu+1} - \mathbf{I_0}\,(\underline{X}_0 \bullet \underline{\mathbf{\Lambda}}_\mu) - i\,\underline{\mathbf{I}} \bullet (\underline{X}_0 \wedge \underline{\mathbf{\Lambda}}_\mu)$

$\therefore\ \mathbf{X}_\eta^*\,\mathbf{X}_\eta = (c^2 t_\eta^2 - X_\eta^2)\,\mathbf{I_0}$ $\therefore\ \mathbf{X}_\eta^*\,\mathbf{X}_\eta = \{c^2 t_\eta^2 - (\underline{X}_0 + \underline{u}_0\,t_\eta + \underline{\mathbf{\Lambda}}_\mu)^2\}\,\mathbf{I_0}$

These two digital normal forms can be contrasted with those for a simple Newtonian particle:

$$\mathbf{V}^N{}_\eta^*\,\mathbf{V}^N{}_\eta = \mathbf{U}^*\mathbf{U} = (c^2 - u_0^2)\,\mathbf{I_0} \qquad \text{and} \qquad \mathbf{X}^N{}_\eta^*\,\mathbf{X}^N{}_\eta = \mathcal{X}_\eta^*\mathcal{X}_\eta = (\,c^2 t_\eta^2 - \mathcal{X}_\eta^2)\,\mathbf{I_0}$$

The potential invariants will be evaluated in terms of the *Activity* NV for the digital electron.

$$\mathbf{R}_\eta \equiv i\,\mathbf{X}[t_\eta]\,\mathbf{P}[t_\eta] = i\,m\,\mathbf{X}_\eta\,\mathbf{V}_\eta^* = i\,m\,(\boldsymbol{\mathcal{X}}_\eta + \boldsymbol{\Lambda}_\mu)\,(\mathbf{U}_\eta + \mathbf{W}_\mu)^* = i\,m\,(\boldsymbol{\mathcal{X}}_\eta\mathbf{U}_\eta^* + \boldsymbol{\mathcal{X}}_\eta\mathbf{W}_\mu^* + \boldsymbol{\Lambda}_\mu\mathbf{U}_\eta^* + \boldsymbol{\Lambda}_\mu\mathbf{W}_\mu^*)$$

This illustrates how the total activity is the sum of three parts: the first part derives completely from the Newtonian features ($\boldsymbol{\mathcal{X}}_\eta$ and $\mathbf{U}_\eta$), the final part results purely from the new Digital features ($\boldsymbol{\Lambda}_\mu$ and $\mathbf{W}_\mu$), while the other two term are 'cross-terms' involving one part from the Newtonian view and one part from the digital view. This suggests a tripartite division:

$$\mathbf{R}_\eta = \mathbf{R}^N{}_\eta + \mathbf{R}^X{}_\eta + \mathbf{R}^S{}_\eta \quad \text{so:} \quad \mathbf{R}^N{}_\eta = i\,m\,\boldsymbol{\mathcal{X}}_\eta\mathbf{U}_\eta^* \quad \& \quad \mathbf{R}^X{}_\eta = i\,m\,(\boldsymbol{\Lambda}_\mu\mathbf{U}_\eta^* + \boldsymbol{\mathcal{X}}_\eta\mathbf{W}_\mu) \quad \& \quad \mathbf{R}^S{}_\eta = i\,m\,\boldsymbol{\Lambda}_\mu\mathbf{W}_\mu$$

Direct multiplication shows: $\mathbf{R}^N{}_\eta = i\,m\,\boldsymbol{\mathcal{X}}_\eta\mathbf{U}_\eta^* = i\,m\,\{(c^2 - u_0^2)\,\eta\tau - \underline{X}_0 \bullet \underline{u}_0\}\,\mathbf{I_0} - m\,c\,\underline{\mathbf{I}} \bullet \underline{X}_0 + i\,m\,(\underline{X}_0 \wedge \underline{u}_0) \bullet \underline{\mathbf{I}}$

The transverse terms can be readily evaluated using the transverse base NV multiplication rules described in A1.4.

$$\therefore\ \boldsymbol{\Lambda}_\mu\,\mathbf{U}_\eta^* = \boldsymbol{\Lambda_\mu}\,(-i\,c\,\mathbf{I}_0 + u_0\,\mathbf{I}_3) = -(i\,c\,\boldsymbol{\Lambda}_\mu + u_0\,\boldsymbol{\Lambda}_{\mu+1}) = -i\,c\,(\underline{\boldsymbol{\Lambda}}_\mu \bullet \underline{\mathbf{I}}) - u_0\,(\underline{\boldsymbol{\Lambda}}_{\mu+1} \bullet \underline{\mathbf{I}}) = -(i\,c\,\underline{\boldsymbol{\Lambda}}_\mu + u_0\,\underline{\boldsymbol{\Lambda}}_{\mu+1}) \bullet \underline{\mathbf{I}}$$

$$\therefore\ \boldsymbol{\mathcal{X}}_\eta\,\mathbf{W}_\mu = (i\,c\,t_\eta\,\mathbf{I_0} + u_0\,t_\eta\,\mathbf{I}_3)\mathbf{W}_\mu + (\underline{\mathbf{I}} \bullet \underline{X}_0)\,(\underline{W}_\mu \bullet \underline{\mathbf{I}}) = i\,c\,t_\eta\,(\underline{W}_\mu \bullet \underline{\mathbf{I}}) + u_0\,t_\eta(\underline{W}_{\mu+1} \bullet \underline{\mathbf{I}}) - \mathbf{I}_0\,(\underline{X}_0 \bullet \underline{W}_\mu) + \underline{\mathbf{I}} \bullet (\underline{X}_0 \wedge \underline{W}_\mu)$$

The digital part can be immediately evaluated: $\therefore\ \boldsymbol{\Lambda_\mu}\,\mathbf{W}_\mu = (\underline{\boldsymbol{\Lambda}}_\mu \bullet \underline{\mathbf{I}})\,(\underline{W}_\mu \bullet \underline{\mathbf{I}}) = -(\underline{\Lambda}_\mu \bullet \underline{W}_\mu)\,\mathbf{I}_0 + \underline{\mathbf{I}} \bullet (\underline{\Lambda}_\mu \wedge \underline{W}_\mu)$

In terms of the three 'free' particle invariants $\{\mathcal{A}_\eta, \underline{\mathcal{G}}_\eta, \underline{\mathcal{M}}_\eta\}$: $\mathbf{R}_\eta \equiv i\,\mathcal{A}_\eta\,\mathbf{I_0} + \underline{\mathcal{G}}_\eta \bullet \underline{\mathbf{I}} + i\,\underline{\mathcal{M}}_\eta \bullet \underline{\mathbf{I}}$

$$\mathcal{A}^N{}_\eta = -\,m\{\underline{X}_0 \bullet \underline{u}_0 - (c^2 - u_0^2)\,\eta\tau\} \quad \& \quad \mathcal{A}^X{}_\mu = -\,m\,\underline{X}_0 \bullet \underline{W}_\mu \quad \& \quad \mathcal{A}^S{}_\mu = -\,m\,\underline{\Lambda}_\mu \bullet \underline{W}_\mu$$

$$\underline{\mathcal{G}}^N{}_\eta = -\,m\,\underline{X}_0 \quad \& \quad \underline{\mathcal{G}}^X{}_\mu = m\,c\,(\underline{\Lambda}_\mu - t_\eta\,\underline{W}_\mu) \quad \& \quad \underline{\mathcal{G}}^S{}_\mu = 0$$

$$\underline{\mathcal{M}}^N{}_\eta = m\,\underline{X}_0 \wedge \underline{u}_0 \quad \& \quad \underline{\mathcal{M}}^X{}_\mu = m\,\{(\underline{X}_0 + t_\eta\,\underline{u}_0) \wedge \underline{W}_\mu + \underline{\Lambda}_\mu \wedge \underline{u}_0\} \quad \& \quad \underline{\mathcal{M}}^S{}_\mu = m\,\underline{\Lambda}_\mu \wedge \underline{W}_\mu$$

Since $\underline{u}_\eta \wedge \underline{w}_\eta = u_\eta\,\underline{w}_{\eta+1}$ then $\underline{u}_0 \wedge \underline{W}_\mu = u_0\,\underline{W}_{\mu+1}$ and $\underline{u}_0 \wedge \underline{\Lambda}_\mu = u_0\,\underline{\Lambda}_{\mu+1}$ while $\underline{u}_0 \bullet \underline{\Lambda}_\mu = 0$ and $\underline{u}_0 \bullet \underline{W}_\mu = 0$.

Now $\underline{\boldsymbol{\Lambda}}_\mu \wedge \underline{W}_\mu = -(\,\hat{\underline{e}}_1 \cos\theta + \hat{\underline{e}}_2 \sin\theta) \wedge (\hat{\underline{e}}_1 \cos\psi + \hat{\underline{e}}_2 \sin\psi)\,c\,\Lambda/\sqrt{2} = \hat{\underline{e}}_3\,(\sin\theta\cos\psi - \cos\theta\sin\psi)\,c\,\Lambda/\sqrt{2}$
$= \hat{\underline{e}}_3 \sin(\theta - \psi)\,c\,\Lambda/\sqrt{2} = \hat{\underline{e}}_3 \sin\pi/4\ c\,\Lambda/\sqrt{2} = \hat{\underline{e}}_3\,c\,\Lambda/2 = \tfrac{1}{2}\,c^2\,\tau\ \hat{\underline{e}}_3$

and $\underline{\boldsymbol{\Lambda}}_\mu \bullet \underline{W}_\mu = -(\,\hat{\underline{e}}_1 \cos\theta + \hat{\underline{e}}_2 \sin\theta) \bullet (\hat{\underline{e}}_1 \cos\psi + \hat{\underline{e}}_2 \sin\psi)\,c\,\Lambda/\sqrt{2} = -(\cos\theta\cos\psi + \sin\theta\sin\psi)\,c\,\Lambda/\sqrt{2}$
$= -\cos(\theta - \psi)\,c\,\Lambda/\sqrt{2} = -\cos\pi/4\ c\,\Lambda/\sqrt{2} = -\,c\,\Lambda/2 = -\tfrac{1}{2}\,c^2\,\tau$

Thus, the digital components evaluate to similar constants: $\mathcal{A}^S{}_\mu = \tfrac{1}{2}\,m\,c^2\,\tau = \tfrac{1}{2}\,\alpha\,h_D$ & $\underline{\mathcal{M}}^S{}_\mu = \tfrac{1}{2}\,m\,c^2\,\tau\ \hat{\underline{e}}_3$
This means that the 'pure' digital Activity DNV $\mathbf{R}^S{}_\eta$ is a <u>constant</u> and has the following three equivalent forms:

$$\mathbf{R}^S{}_\eta = i\,m\,\boldsymbol{\Lambda}_\mu\mathbf{W}_\mu = i\,m\{-(\underline{\boldsymbol{\Lambda}}_\mu \bullet \underline{W}_\mu)\,\mathbf{I}_0 + \underline{\mathbf{I}} \bullet (\underline{\boldsymbol{\Lambda}}_\mu \wedge \underline{W}_\mu)\} = i\,(\mathbf{I}_0 + \mathbf{I}_3)\,m\,c^2\,\tau/2$$

Importantly, the 'cross-term' activity $\mathbf{R}^X{}_\eta$ is linear in $\underline{\Lambda}_\mu$ and $\underline{W}_\mu$ so its three dynamic quantities can be averaged over one full micro-cycle (see §5.2.2) when both transverse vectors average to zero, so:

$$\langle\,\underline{\Lambda}_\mu\,\rangle_I = 0 \quad \& \quad \langle\,\underline{W}_\mu\,\rangle_I = 0 \quad \therefore \quad \langle\,\underline{X}_0 \bullet \underline{W}_\mu\,\rangle_I = 0 \quad \& \quad \langle\,\underline{X}_0 \wedge \underline{W}_\mu\,\rangle_I = 0$$

$$\therefore \quad \langle\,\mathcal{A}^X{}_\eta\,\rangle_I = 0 \quad \& \quad \langle\,\underline{\mathcal{G}}^X{}_\eta\,\rangle_I = -\,m\,\langle\,t_\eta\,\underline{W}_\mu\rangle_I \quad \& \quad \langle\,\underline{\mathcal{M}}^X{}_\eta\,\rangle_I = m\,u_0\,\langle\,t_\eta\,\underline{W}_{\mu+1}\rangle_I$$

This leaves the quantities $(t_\eta\,\underline{W}_\mu)$ and $(t_\eta\,\underline{W}_{\mu+1})$ to be averaged over four consecutive chronons but since $\eta = (\eta_0 + \mu)$ then these averages reduces to evaluating only the product $(\mu\,\underline{W}_\mu)$ over any four consectutive points in the ERD.

$$\langle\,\mu\underline{W}_\mu\,\rangle_I = \tfrac{1}{4}\,(\,F[\,\mu\underline{W}_\mu\,]_1 + F[\,\mu\underline{W}_\mu\,]_2 + F[\,\mu\underline{W}_\mu\,]_3 + F[\,\mu\underline{W}_\mu\,]_4\,) = \tfrac{1}{4}\,(\,\underline{W}_1 + 2\,\underline{W}_2 + 3\,\underline{W}_3 + 4\,\underline{W}_4\,)$$

Since $\underline{W}_\mu$ is cyclic, then $\underline{W}_1 + \underline{W}_2 + \underline{W}_3 + \underline{W}_4 = 0 \quad \therefore \quad \langle \mu \underline{W}_\mu \rangle_I = \tfrac{1}{4}(\underline{W}_2 + 2\,\underline{W}_3 + 3\,\underline{W}_4) = \tfrac{1}{4}(\underline{W}_3 + 2\,\underline{W}_4 - \underline{W}_1)$

While $\underline{W}_{\mu+2} = -\underline{W}_\mu \quad \therefore \quad \langle \mu \underline{W}_\mu \rangle_I = \tfrac{1}{2}(\underline{W}_4 - \underline{W}_1) = -\tfrac{1}{2}(\underline{W}_1 + \underline{W}_2) = -(\Delta\underline{\Lambda}_1 + \Delta\underline{\Lambda}_2)/2\tau = (\underline{\Lambda}_1 - \underline{\Lambda}_3)/2\tau = \underline{\Lambda}_1/\tau$

This result was obtained by starting the set of four points with $\mu = 1$, this implies that, in general: $\tau \langle \mu \underline{W}_\mu \rangle_I = \underline{\Lambda}_\mu$

The 'cross-term' activity averages to a non-zero value ($\underline{\mathbf{\Lambda}}_\mu \mathbf{U}_\eta^*$): $\langle \mathbf{R}^X{}_\eta \rangle_I = -\mathrm{m}(\mathrm{c}\,\underline{\Lambda}_\mu - i\,\mathrm{u}_0\,\underline{\Lambda}_{\mu+1}) \bullet \underline{\mathbf{I}} = i\,\mathrm{mc}\tau\,\mathbf{U}^*$

This is consistent with the initial form of the 'cross-term' activity, so: $\langle \mathbf{R}^X{}_\eta \rangle_I^* \langle \mathbf{R}^X{}_\eta \rangle_I = (\mathrm{mc}\tau)^2 (\mathrm{c}^2 - \mathrm{u}_0^2)\,\mathbf{I}_0$

Although the quantities for the 'free' Newtonian particle are invariant at all times, this analysis has demonstrated that the three kinematical quantities describing the complete motion of the digital electron, when time averaged over a four chronon micro-cycle, are also invariant (independent of η). Note, the cross-term quantities includes small, transverse terms whose magnitude decreases to zero as the longitudinal speed approaches 'light-speed'; i.e. $|\langle \mathbf{R}^X{}_\eta \rangle_I| = \tau\,\mathrm{m}\,\mathrm{c}^2 \sqrt{(1 - \mathrm{u}_0^2/\mathrm{c}^2)}$

NB Unlike the continuous case, $\underline{\Lambda}_\mu \bullet \underline{W}_\mu \neq 0$ but the average rotational velocity is orthogonal; i.e. $\underline{\Lambda}_\mu \bullet \langle \underline{W}_{\mu-1} \rangle = 0$

The extra digital kinematical quantities usually associated with the motion of a Newtonian particle can also evaluated.

Linear Momentum DNV: $\mathbf{P}_\eta = m\,\mathbf{V}_\eta^* = \mathbf{P}^N{}_\eta + \mathbf{P}^D{}_\eta$ with $\mathbf{V}_\eta = \mathbf{U}_\eta + \mathbf{W}_\mu$ and $\mathbf{U}_\eta = i\,c\,\mathbf{I_0} + \mathrm{u}_0\,\mathbf{I_3}$

1) Digital Natural Momentum: $\mathbf{P}^D{}_\eta = m\,\mathbf{W}_\mu = \tfrac{1}{2} m\,c\,(\mathbf{I}_+ e^{-i\psi} + \mathbf{I}_- e^{i\psi})$

2) Digital Linear Momentum: $\underline{P}^D{}_\eta = m\,\underline{W}_\mu = m\,c\,(\underline{\hat{e}}_1 \cos \mu\pi/2 + \underline{\hat{e}}_2 \sin \mu\pi/2)$

3) Digital Particle Energy: $\mathcal{E}^D{}_\eta = 0$

4) Digital Kinetic Energy: $\mathcal{K}^D{}_\eta = \tfrac{1}{2} m\,c^2$

5) Digital Action: $\mathcal{A}^D{}_\mu = \tfrac{1}{2} m\,\mathrm{c}^2\,\tau$

5) Digital Angular Momentum: $\underline{\mathcal{M}}^D{}_\eta = \tfrac{1}{2} m\,c^2\,\tau\;\underline{\hat{e}}_3$

Here, the intrinsic digital action $\mathcal{A}$ and angular momentum $\underline{\mathcal{M}}$ have been included for completeness.

It is interesting to compare the digital action $\mathcal{A}^D{}_\mu$ occurring as the electron 'clicks' 4 times around its longitudinal axis at light-speed c with the quantized action at the atomic scale $\Delta\mathcal{A}^K{}_j$, characterized by the Bohr velocity $V_B = \alpha\,c$ (see §6.3.3).

Since $\Delta\mathcal{A}^K{}_j = m\,\underline{u}_j \bullet \underline{u}_j\,\Delta t_j = h/2 \quad \therefore \quad \Delta\mathcal{A}^K{}_j / \mathcal{A}^D{}_\mu = h/(m\,\mathrm{c}^2\,\tau) = h\,c/e^2 = 2\pi/\alpha \cong 860$

This ratio indicates that the electron 'clicks' many times between each atomic scale interaction.

N.B. The factor (π/α) re-appears many times in this programme and links this work to conventional QM.

Digital Electron Invariants

Earlier (section 4.3.1), it was shown that a Newtonian particle's free motion (unaccelerated) was equivalent to 3 conditions:

i) $\Delta_\tau[\,\mathrm{R}_\eta\,] = m\,\tau\,(c^2 - \underline{u}_0 \bullet \underline{u}_0)$ ii) $\Delta_\tau[\,\underline{\mathbf{R}}_\eta\,] = 0$ iii) $\Delta_\tau[\,\underline{\mathscr{R}}_\eta\,] = 0$

These conditions are equivalent to: i) $\mathcal{A}^N{}_\eta = m\,t_\eta\,(\mathrm{c}^2 - \mathrm{u}_0^2) - m\,\underline{X}_0 \bullet \underline{u}_0$ ii) $\underline{\mathcal{G}}^N{}_\eta = m\,c\,\underline{X}_0$ iii) $\underline{\mathcal{M}}^N{}_\eta = m\,\underline{X}_0 \wedge \underline{u}_0$

Explicitly, these have the solution: a) $\underline{X}^N{}_\eta = \underline{\mathcal{X}}_\eta = \underline{X}_0 + \underline{u}_0\,\eta\,\tau$ b) $\underline{P}^N{}_\eta = m\,\underline{u}_0$

Condition ii) is a re-statement of linear momentum conservation while condition iii) is a re-statement of the conservation of angular momentum. Condition i) indicates that a Newtonian particle 'generates action' at the rate of $m(c^2 - u_0^2)$ units of energy per chronon when its speed is u_0. These three Newtonian conditions can be combined into one DNV equation:

$$\Delta_\tau[\mathbf{R}^N{}_\eta] = i\, m\, \tau\, (c^2 - u_0^2)\, \mathbf{I_0} \qquad \text{or} \qquad \mathbf{R}^N{}_{\eta+1} = \mathbf{R}^N{}_\eta + i\, m\, \tau\, \mathbf{U}^*\, \mathbf{U}$$

The corresponding differences in the activity DNV will now be investigated for the digital electron. Rather than just analyzing the adjacent differences (i.e. for t_η and $t_{\eta+1}$), it is almost as easy to analyze the general differences in the digital electron model across several (k) chronons using the notion of 'super-differences' ($\Delta_k[\zeta_n]$) described in A4.1.

$$\text{In general, } \Delta_k[\zeta_n] \equiv \zeta_{n+k} - \zeta_n = \Delta_k \zeta_n \quad \therefore\ \Delta_k[t_\eta] = (\eta + k)\,\tau - \eta\tau = k\,\tau \quad \&\quad \Delta_k[\underline{\Lambda}_\mu] = \underline{\Lambda}_{\mu+k} - \underline{\Lambda}_\mu$$

The analog of the 'super-differences' ($\Delta_k[\zeta_n]$) are the 'super-averages' ($<\zeta_n>_k$) , defined in a complementary manner:

$$\text{Definition:} \quad \textbf{Super-Average}: \quad <\zeta_n>_k \equiv \tfrac{1}{2}\,(\zeta_{n+k} + \zeta_n) = \zeta_n + \tfrac{1}{2}\,\Delta_k[\,\zeta_n\,]$$

The generalization of the simple difference of a product (see A4.1) is: $\Delta_k[\alpha_\eta\, \beta_\eta] = <\alpha_\eta>_k\, \Delta_k\beta_\eta + \Delta_k\alpha_\eta\, <\beta_\eta>_k$

The evaluation of the super-difference of the digital electron's activity $\mathbf{R}_\eta$ requires the evaluation of the DNV product:

$$\Delta_k[\,\mathbf{X}_\eta\, \mathbf{V}_\eta^*\,] = <\mathbf{X}_\eta>_k\, \Delta_k\mathbf{V}_\eta^* + \Delta_k\mathbf{X}_\eta\, <\mathbf{V}_\eta^*>_k = \mathbf{X}_\eta\, \Delta_k\mathbf{V}_\eta^* + \Delta_k\mathbf{X}_\eta\, \mathbf{V}_\eta^* + \Delta_k\mathbf{X}_\eta\, \Delta_k\mathbf{V}_\eta^*$$

Using the ERF: $\mathbf{V}_\eta = \mathbf{U} + \mathbf{W}_\mu$ with $\mathbf{U} = i\,c\,\mathbf{I_0} + u_0\,\mathbf{I_3}$ $\therefore\ \Delta_k[\mathbf{U}] = 0$ $\therefore\ \Delta_k[\mathbf{V}_\eta] = \Delta_k[\mathbf{W}_\mu] = \Delta_k\underline{W}_\mu \bullet \underline{\mathbf{I}}$

$$\text{and} \quad \mathbf{X}_\eta = \boldsymbol{\mathcal{X}}_\eta + \boldsymbol{\Lambda}_\mu \quad \text{with} \quad \boldsymbol{\mathcal{X}}_\eta = i\,c\,t_\eta\,\mathbf{I_0} + \underline{\mathcal{X}}_\eta \bullet \underline{\mathbf{I}} \quad \&\quad \underline{\mathcal{X}}_\eta = \underline{X}_0 + \underline{u}_0\,t_\eta \ \therefore\ \Delta_k[\boldsymbol{\mathcal{X}}_\eta] = k\,\tau\,\mathbf{U}$$

$$\therefore\ \Delta_k[\,\mathbf{X}_\eta\, \mathbf{V}_\eta^*\,] = k\,\tau\,\mathbf{U}\,\mathbf{U}^* + (\boldsymbol{\mathcal{X}}_{\eta+k}\,\Delta_k\mathbf{W}_\mu + \Delta_k[\boldsymbol{\Lambda}_\mu]\,\mathbf{U}^*) + (\boldsymbol{\Lambda}_\mu\,\Delta_k\mathbf{W}_\mu + \Delta_k[\boldsymbol{\Lambda}_\mu]\,\mathbf{W}_{\mu+k})$$

The first term again reflects only the super-difference of the activity for the Newtonian particle: $\Delta_k[\mathbf{R}^N{}_\eta] = k\,\Delta_\tau[\mathbf{R}^N{}_\eta]$; the second term is 'mixed', involving both a 'classical' part ($\boldsymbol{\mathcal{X}}_\eta$ or $\mathbf{U}$) and a 'digital' part ($\boldsymbol{\Lambda}_\mu$ or $\mathbf{W}_\mu$) while the final term results only from purely digital considerations.

Now $\underline{\boldsymbol{\Lambda}}_\mu$ and $\underline{W}_\mu$ are both cyclic, period 4τ; that is to say: $\underline{\boldsymbol{\Lambda}}_{\mu+4} = \underline{\boldsymbol{\Lambda}}_\mu$ and $\underline{W}_{\mu+4} = \underline{W}_\mu$ then $\boldsymbol{\Delta}_4[\underline{\boldsymbol{\Lambda}}_\mu] = 0$ and $\boldsymbol{\Delta}_4[\underline{\boldsymbol{\Lambda}}_\mu] = 0$

This implies that over <u>any</u> multiple of four chronons ($4N_\eta$) the differences in the digital electron's purely digital part ($\mathbf{R}^D{}_\eta$) are <u>always zero</u> whenever there are no interactions with other electrons and the digital electron then appears to behave just like a simple, 'free' Newtonian, point particle. <u>Most importantly, for the digital part</u>: $\Delta_k[\mathbf{R}^D{}_\eta] = 0$ when k is modulo 4. This imposes a powerful constraint on **when** consecutive interactions can occur for a digital electron.

If $j = 4\,N_j + k$ then $\boldsymbol{\Delta}_j[\,\{\,\underline{\boldsymbol{\Lambda}}_\mu\,,\,\underline{W}_\mu\,\}\,] = \boldsymbol{\Delta}_k[\,\{\,\underline{\boldsymbol{\Lambda}}_\mu\,,\,\underline{W}_\mu\,\}\,]$ while $\boldsymbol{\Delta}_3[\zeta_n] = \boldsymbol{\Delta}[2\boldsymbol{\Delta}_2[\zeta_n] + \zeta_n]$ So, only need k =1 & 2.

The earlier results for the simple (single) digital differences were: $\Delta\underline{\Lambda}_\mu = \tau\,\underline{W}_\mu$ and $\Delta\underline{W}_\mu = -2\,\underline{\Lambda}_{\mu+1}/\tau$

$\therefore\ \Delta\underline{X}_\eta = \tau\,(\underline{u}_0 + \underline{\mathbf{W}}_\mu) = \tau\,\underline{V}_\eta$ $\therefore\ \underline{X}_{\eta+1} = \underline{X}_\eta + \tau\,\underline{V}_\eta$ & $\Delta\underline{V}_\eta = \Delta\underline{W}_\mu = -2\,\underline{\Lambda}_{\mu+1}/\tau$ NB $\Delta_3\underline{X}_\eta = \tau\,(\underline{u}_0 - \underline{\mathbf{W}}_\mu) \neq \tau\,\underline{V}_\eta$

$\therefore\ \Delta_k\underline{X}_\eta = \boldsymbol{\Delta}_k[\underline{X}^N{}_\eta + \underline{\boldsymbol{\Lambda}}_\mu] = \boldsymbol{\Delta}_k[\underline{X}_0 + t_\eta\,\underline{u}_0 + \underline{\boldsymbol{\Lambda}}_\mu] = k\tau\,\underline{u}_0 + \Delta_k\underline{\boldsymbol{\Lambda}}_\mu$ & $\Delta_k\underline{V}_\eta = \boldsymbol{\Delta}_k[\underline{V}^N{}_\eta + \underline{\mathbf{W}}_\mu] = \boldsymbol{\Delta}_k[\underline{u}_0 + \underline{\mathbf{W}}_\mu] = \Delta_k\underline{\mathbf{W}}_\mu$

$$\text{The other useful results are:} \quad \Delta_2\underline{X}_\eta = 2(\tau\,\underline{u}_0 - \underline{\boldsymbol{\Lambda}}_\mu) \quad \&\quad \Delta_2\underline{V}_\eta = \Delta_2\underline{W}_\mu = -2\,\underline{W}_\mu \ (\text{since } \underline{W}_{\mu+2} = -\underline{W}_\mu)$$

Other interesting identities include: $<t_\eta>_k = t_\eta + k\,\tau/2$; $<\underline{W}_\mu>_2 = 0$; $<t_\eta>_2 = t_{\eta+1}$; $<\underline{V}_\eta>_2 = \underline{V}_\eta - \underline{\mathbf{W}}_\mu = \underline{u}_0$

While the 'double' digital differences are readily found: $\Delta_2[\,\underline{\Lambda}_\mu\,] = \underline{\Lambda}_{\mu+2} - \underline{\Lambda}_\mu = -2\,\underline{\Lambda}_\mu$ & $\Delta_2[\,\underline{W}_\mu\,] = -2\,\underline{W}_\mu$

$$\Delta_k[A_\eta \otimes B_\eta] = \langle A_\eta \rangle_k \otimes \Delta_k B_\eta + \Delta_k A_\eta \otimes \langle B_\eta \rangle_k = A_\eta \otimes \Delta_k B_\eta + \Delta_k A_\eta \otimes B_\eta + \Delta_k A_\eta \otimes \Delta_k B_\eta$$

But $\Delta_k[\underline{\Lambda}_\mu \bullet \underline{W}_\mu] = \Delta_k[-\tfrac{1}{2} c^2 \tau] = 0$; similarly $\Delta_k[\underline{\Lambda}_\mu \wedge \underline{W}_\mu] = \Delta_k[\tfrac{1}{2} c^2 \tau\ \underline{\hat{e}}_3] = 0$

Another useful super-difference identity involves t_η and any discrete function:

$$\Delta_k[t_\eta \zeta_n] = \langle t_\eta \rangle_k \Delta_k\zeta_n + \Delta_k t_\eta \langle \zeta_n \rangle_k = (t_\eta + k\tau/2)\,\Delta_k\zeta_n + k\tau \langle \zeta_n \rangle_k = k\tau\,\zeta_n + t_{\eta+k}\,\Delta_k\zeta_n$$

These transformations and identities can now be applied to the component parts of the digital electron model's DNV:

Thus, $\Delta_k[\mathbf{R}^X{}_\eta] = i\,\mathbf{I_0}\,\Delta_k[\mathcal{A}^X{}_\eta] + \underline{\mathbf{I}} \bullet \Delta_k[\underline{\mathcal{G}}^X{}_\eta] + i\,\underline{\mathbf{I}} \bullet \Delta_k[\underline{\mathcal{M}}^X{}_\eta]$ with $\Delta_k[\mathcal{A}^X{}_\eta] = -m\,\underline{X}_0 \bullet \Delta_k[\underline{W}_\mu]$ and

$\Delta_k[\underline{\mathcal{G}}^X{}_\eta] = m\,c\,\Delta_k[\underline{\Lambda}_\mu] - m\,c\,\Delta_k[t_\eta\,\underline{W}_\mu]$ & $\Delta_k[\underline{\mathcal{M}}^X{}_\eta] = m\,(\underline{X}_0 \wedge \Delta_k[\underline{W}_\mu] - m\,u_0\,(\Delta_k[\underline{\Lambda}_{\mu+1}] - \Delta_k[t_\eta \underline{W}_{\mu+1}])$

a) Single Diffs: $\Delta[\mathcal{A}^X{}_\mu] = 2\,m\,\underline{X}_0 \bullet \underline{\Lambda}_{\mu+1}/\tau$; $\Delta[\underline{\mathcal{G}}^X{}_\mu] = -2\,m\,c\,t_{\eta+1}\,\underline{\Lambda}_{\mu+1}/\tau$; $\Delta[\underline{\mathcal{M}}_\mu] = -2\,m\,(\underline{X}_0 + \underline{u}_0\,t_{\eta+1}) \wedge \underline{\Lambda}_{\mu+1}/\tau$

$\therefore\ \Delta[\mathbf{R}^X{}_\eta] = -2\,i\,m\,\boldsymbol{\mathcal{X}}_{\eta+1}\,\boldsymbol{\Lambda}_{\mu+1}/\tau = i\,m\,\boldsymbol{\mathcal{X}}_{\eta+1}\,\Delta\mathbf{W}_\mu$ with $\boldsymbol{\mathcal{X}}_{\eta+1} = i\,c\,t_{\eta+1}\,\mathbf{I_0} + \underline{\mathcal{X}}_{\eta+1} \bullet \underline{\mathbf{I}}$ & $\underline{\mathcal{X}}_{\eta+1} = \underline{X}_0 + \underline{u}_0\,t_{\eta+1}$

As above, it can be similarly shown that: $\langle \mu\,\underline{\Lambda}_\mu \rangle_I = \underline{W}_{\mu+1}\,\tau/2$ $\therefore\ \langle \Delta[\underline{\mathcal{G}}^X{}_\mu] \rangle_I = m\,c\,\tau\,\underline{W}_\mu$ & $\langle \Delta[\underline{\mathcal{M}}^X{}_\mu] \rangle_I = m\tau\,\underline{u}_0 \wedge \underline{W}_\mu$

The consecutive average of this first-difference are non-zero, i.e. $\langle \Delta[\mathbf{R}^X{}_\eta] \rangle_I = m\tau\,(c\,\underline{W}_\mu - i\,u_0\,\underline{W}_{\mu+1}) = -i\,m\tau\,\mathbf{U}_\eta\mathbf{W}_\mu$

Using, from before, $\mathbf{U}\mathbf{W}_\mu = \mathbf{U}\mathbf{W}_\mu{}^* = (\mathbf{U}^*\mathbf{W}_\mu)^* = (-i\,c\,\mathbf{W}_\mu + u_0\,\mathbf{W}_{\mu+1})^* = i\,c\,\mathbf{W}_\mu + u_0\,\mathbf{W}_{\mu+1}$

b) Double Differences: $\Delta_2[\mathbf{R}^X{}_\eta] = i\,m\,(\boldsymbol{\mathcal{X}}_{\eta+2}\,\Delta_2\mathbf{W}_\mu + \Delta_2[\boldsymbol{\Lambda}_\mu]\,\mathbf{U}^*) = -2\,i\,m\,(\boldsymbol{\mathcal{X}}_{\eta+2}\mathbf{W}_\mu + \boldsymbol{\Lambda}_\mu\,\mathbf{U}^*)$

$$= -2\,i\,m\,\{(\boldsymbol{\mathcal{X}}_\eta + 2\tau\,\mathbf{U})\mathbf{W}_\mu + \boldsymbol{\Lambda}_\mu\,\mathbf{U}^*\} = -2\,\mathbf{R}^X{}_\eta - 4\,i\,m\,\tau\,\mathbf{U}\mathbf{W}_\mu$$

$\therefore\ \Delta_2[\mathbf{R}^X{}_\eta] = -2\,\mathbf{R}^X{}_\eta + 4\,m\,\tau\,(c\,\mathbf{W}_\mu - i\,u_0\,\mathbf{W}_{\mu+1})$ $\therefore\ \langle \Delta_2[\mathbf{R}^X{}_\eta] \rangle_I = -2\,i\,mc\tau\,\mathbf{U}^*$

Thus, both the local single and double differences of the digital electron model's Newtonian and Digital activity are all zero. The 4-chronon averages of both of the single and double differences of the digital electron's 'cross-term' activity DNV are both non-zero but they are time invariant. This reinforces the idea that only when the non-interacting digital electron moves across a multiple of **four chronons** does it behave (on average) like a 'free' Newtonian particle.

Summary of the Digital Rotation Model

The analysis of the Activity DNV of the digital electron model $\mathbf{R}_\eta$ indicated that this key concept separates into three parts: the first part representing the original (or 'free') Newtonian particle moving between interaction nodes only in one direction through space $\mathbf{R}^N_\eta$, the second part $\mathbf{R}^S_\eta$ represented the 2D 'clicking' motion that characterizes the digital electron model and a so-called 'cross-term' part $\mathbf{R}^X_\eta$ that combines one component ($\mathbf{\Lambda}_\mu$ or $\mathbf{W}_\mu$) from each. The first two parts, separately or together, are invariant at the micro-cycle time scale (the single chronon). The cross-term part rotates rapidly around the longitudinal motion and requires averaging over any four consecutive chronon cycle to produce any non-zero but invariant quantities. In summary, the mathematical version of the single particle digital model becomes:

$$\mathbf{R}_\eta \equiv i\,\mathbf{X}_\eta\,\mathbf{P}_\eta = i\,m\,\mathbf{X}_\eta\,\mathbf{V}_\eta^* \equiv i\,\mathcal{A}_\eta\,\mathbf{I_0} + \underline{\mathcal{G}}_\eta \bullet \underline{\mathbf{I}} + i\,\underline{\mathcal{M}}_\eta \bullet \underline{\mathbf{I}} \equiv \mathbf{R}^N_\eta + \mathbf{R}^D_\eta$$

$$\text{with} \quad \mathcal{A}_\eta = (m\,c^2\,t_\eta - \underline{X}_\eta \bullet \underline{P}_\eta) \quad \& \quad \underline{\mathcal{G}}_\eta = c\,(m\,\underline{X}_\eta - \underline{P}_\eta\,t_\eta) \quad \& \quad \underline{\mathcal{M}}_\eta = \underline{X}_\eta \wedge \underline{P}_\eta$$

$$\mathbf{R}_\eta = i\,m\,(\boldsymbol{\mathcal{X}}_\eta + \mathbf{\Lambda}_\mu)\,(\mathbf{U}_\eta + \mathbf{W}_\mu)^* = i\,m\,\{\boldsymbol{\mathcal{X}}_\eta\mathbf{U}_\eta^* + \mathbf{\Lambda}_\mu\mathbf{W}_\mu + (\boldsymbol{\mathcal{X}}_\eta\mathbf{W}_\mu + \mathbf{\Lambda}_\mu\mathbf{U}_\eta^*)\} \equiv \mathbf{R}^N_\eta + \mathbf{R}^S_\eta + \mathbf{R}^X_\eta$$

$$\mathbf{R}^N_\eta = i\,m\,\boldsymbol{\mathcal{X}}_\eta\mathbf{U}_\eta^* = -\,i\,m\{\underline{X}_0 \bullet \underline{u}_0 - (c^2 - u_0^2)\,\eta\tau\}\mathbf{I_0} - mc\,\underline{\mathbf{I}} \bullet \underline{X}_0 + i\,m\,(\underline{X}_0 \wedge \underline{u}_0) \bullet \underline{\mathbf{I}}$$

$$\mathbf{R}^S_\eta = i\,m\,\mathbf{\Lambda}_\mu\mathbf{W}_\mu = -\,i\,m\,(\underline{\mathbf{\Lambda}}_\mu \bullet \underline{W}_\mu)\,\mathbf{I}_0 + i\,m\,\underline{\mathbf{I}} \bullet (\underline{\mathbf{\Lambda}}_\mu \wedge \underline{W}_\mu) = i\,(\mathbf{I}_0 + \mathbf{I}_3)\,m\,c^2\,\tau/2$$

$$\mathbf{R}^X_\eta = -\,i\,m\,(\underline{X}_0 \bullet \underline{W}_\mu)\,\mathbf{I_0} + mc\,(\underline{\Lambda}_\mu - t_\eta\,\underline{W}_\mu) \bullet \underline{\mathbf{I}} + i\,m\{(\underline{X}_0 + t_\eta\,\underline{u}_0) \wedge \underline{W}_\mu + \underline{\Lambda}_\mu \wedge \underline{u}_0\} \bullet \underline{\mathbf{I}}$$

This means that the 'pure' digital Activity DNV $\mathbf{R}^S_\eta$ is a constant with: $\mathcal{A}^S_\mu = \tfrac{1}{2}\,m\,c^2\,\tau$ & $\underline{\mathcal{M}}^S_\mu = \tfrac{1}{2}\,m\,c^2\,\tau\,\hat{\underline{e}}_3$

The 'cross-term' activity averages to a non-zero value ($\mathbf{\Lambda}_\mu\mathbf{U}_\eta^*$): $\langle\,\mathbf{R}^X_\eta\,\rangle_I = -\,m(c\,\underline{\Lambda}_\mu - i\,u_0\,\underline{\Lambda}_{\mu+1}) \bullet \underline{\mathbf{I}} = i\,mc\tau\,\mathbf{U}^*$

Temporal invariance is defined in terms of 'super-differences' across several (k) chronons: $\mathbf{R}_{\eta+k} = \mathbf{R}_\eta + \mathbf{\Delta}_k[\mathbf{R}_\eta]$

Most importantly: $\mathbf{\Delta}_k[\mathbf{R}^D_\eta] = 0$ and $\mathbf{\Delta}_k[\mathbf{R}^N_\eta] = \mathbf{\Delta}_k[i\,\mathcal{A}^N_\eta\,\mathbf{I_0}] = k\,\mathbf{\Delta}_\tau[\mathbf{R}^N_\eta] = i\,k\,m\,\tau\,\mathbf{U}^*\,\mathbf{U} = i\,k\,m\,\tau\,(c^2 - u_0^2)\,\mathbf{I_0}$

But $\mathbf{\Delta}[\mathbf{R}^X_\eta] = -\,2\,i\,m\,\boldsymbol{\mathcal{X}}_{\eta+1}\,\mathbf{\Lambda}_{\mu+1}/\tau = i\,m\,\boldsymbol{\mathcal{X}}_{\eta+1}\,\mathbf{\Delta W}_\mu$ with $\boldsymbol{\mathcal{X}}_{\eta+1} = i\,c\,t_{\eta+1}\,\mathbf{I_0} + \underline{\mathcal{X}}_{\eta+1} \bullet \underline{\mathbf{I}}$ & $\underline{\mathcal{X}}_{\eta+1} = \underline{X}_0 + \underline{u}_0\,t_{\eta+1}$

The 4-consecutive average of this first-difference is non-zero, i.e. $\langle\,\mathbf{\Delta}[\mathbf{R}^X_\eta]\,\rangle_I = m\tau\,(c\,\underline{W}_\mu - i\,u_0\,\underline{W}_{\mu+1}) = -\,i\,m\tau\,\mathbf{U}_\eta\mathbf{W}_\mu$

The second-difference: $\mathbf{\Delta}_2[\mathbf{R}^X_\eta] = -\,2\,\mathbf{R}^X_\eta + 4\,m\,\tau\,(c\,\mathbf{W}_\mu - i\,u_0\,\mathbf{W}_{\mu+1})$ with $\langle\,\mathbf{\Delta}_2[\mathbf{R}^X_\eta]\,\rangle_I = -\,2\,i\,mc\tau\,\mathbf{U}^*$

The two constant parts of the 'pure spin' digital Activity DNV are equal in magnitude, consisting only of universal electron constants, and have the dimensions of action or angular momentum. The temptation, **not** resisted by other authors (§2.4), is to identify this magnitude with half of Planck's constant (in Dirac's form). That is, they identify the angular momentum part with the electron spin to 'derive' Dirac's electron 'spin' result; that is: $\underline{S} = \underline{\mathcal{M}}^S = \tfrac{1}{2}\,h_D\,\hat{\underline{e}}_3$ or $\tau = h/2\pi\,mc^2$. Since this programme views Dirac's Equation as a statistical average and worse, one that was derived directly from Planck's **arbitrary** proposal for redefining the electron's mass, this path will not automatically be followed. The critical decision here is how to move from continuous time differentials describing the rate of change of the **fields** describing single electrons, to discrete time intervals directly involving the interaction of two electrons. This issue has been further confused today by the analogous use of field theory, based on Maxwell's statistical electromagnetic field theory [3], to describe all quantum fields of point particles with the additional unknown significance of Planck's mathematical quantization of action. Therefore, these key decisions will be postponed until the analysis of the interaction of two digital electrons is complete; this will be the subject of the next section. A more complete model, involving the expected results of measurements on pairs of interacting electrons will be postponed until the seventh paper in this series when a replacement theory for standard quantum mechanics will be presented that is valid for all relative velocities up to 'light-speed'.

5.3.3 DIGITAL INTERACTION CONSERVATION

The discrete asynchronous interaction between two particles was discussed earlier (§4.3.2) when the continuous interaction hypothesis was replaced with the first part of the digital electron model – namely interactions occurring at discrete time intervals. The second part of the digital model (the 4-way 'clicking' motion) is now added to the interaction between the two remote electrons. The approach adopted here is simply to redefine the spatial location of the electron $\underline{X}_\eta$ as the vector sum of the average ('center-of-rotation') position $\underline{\mathcal{X}}_\eta$ and the 4-step, transverse offset $\underline{\Lambda}_\eta$; similarly the electron's velocity $\underline{V}_\eta$ now becomes the 3D vector sum of the current longitudinal velocity $\underline{u}_n$ and the electron's 'clicking' velocity $\underline{w}_\mu$. The corresponding one and two electron DNVs are also redefined here to include the new digital parts of the two interacting electrons that are located at $\underline{x}_n$ and $\underline{x}'_n$ respectively, at the two times t_n and t'_n where 'n' labels the n^{th} interaction between them:

$$\mathbf{X}_n = i\,c\,t_n\,\mathbf{I_0} + \underline{x}_n \bullet \underline{\mathbf{I}} \quad \text{and} \quad \mathbf{X}'_n = i\,c\,t'_n\,\mathbf{I_0} + \underline{x}'_n \bullet \underline{\mathbf{I}} \quad \text{while} \quad \mathbf{X}_{12}[n] = \mathbf{X}_n - \mathbf{X}'_n = \mathbf{S}_n$$

So that: $\mathbf{X}_{12}[n] = i\,c\,T_n\,\mathbf{I_0} + \underline{X}_n \bullet \underline{\mathbf{I}}$ where $T_n = t_n - t'_n$ and $\underline{X}_n = \underline{x}_n - \underline{x}'_n$ with $t_n = \eta_n\,\tau$ and $t'_n = \eta'_n\,\tau$.

$$\text{where} \quad \mathbf{X}_\eta = \boldsymbol{\mathcal{X}}_\eta + \boldsymbol{\Lambda}_\eta \quad \text{and} \quad \boldsymbol{\mathcal{X}}_\eta = i\,c\,t_\eta\,\mathbf{I_0} + \underline{\mathcal{X}}_\eta \bullet \underline{\mathbf{I}} \quad \& \quad \underline{\mathcal{X}}_\eta = \underline{x}_{n0} + \underline{u}_n\,(t_\eta - t_n)$$

$$\text{and} \quad \mathbf{V}_\eta = \mathbf{U}_\eta + \mathbf{W}_\mu \quad \text{with } \mathbf{U}_\eta = i\,c\,\mathbf{I_0} + \underline{u}_n \bullet \underline{\mathbf{I}} \qquad \& \quad \mathbf{W}_\mu = \underline{w}_\mu \bullet \underline{\mathbf{I}} \quad ; \quad \underline{x}_n = \underline{x}_{n0} + \underline{\Lambda}_n$$

Here, $\underline{x}_{n0}$ is the electron's 'center-of-rotation' in the n^{th} interaction segment at t_n, the moment in time of the n^{th} interaction; while the n^{th} initial digital 'offset' and 'clicking' vectors $\underline{\Lambda}_n$ and $\underline{w}_n$ are the values for this electron at this moment.

Section 5.3.1 above showed that the space-time integrity condition could be extended to the interaction between two digital electrons by retaining the requirement that interactions only occur when each electron is exactly on a shared chronon phase cycle and the spatial separations exactly match their time-differences (i.e. the electrons are on each other's 'light-cones'). Thus, as the 3D vector $\underline{S}_n$ is defined as the n^{th} 'separation-vector' connecting the n^{th} pair of interaction points $\underline{x}_n$ and $\underline{x}'_n$ then the complete set of consecutive EM interactions (all n) between these two interacting electrons all occur in parallel across space if each interaction separation DNV is temporally proportional to the electron-pair's interaction DNV $\mathbf{C}$; that is:

$$\mathbf{S}_n = \mathbf{C}\,T_n \quad \text{and} \quad \underline{\Delta}\mathbf{S}_n = \Delta_I[\mathbf{X}_n - \mathbf{X}'_n] = \mathbf{C}\,\underline{\Delta}T_n$$

Section 5.3.1 also investigated the importance in the digital electron model of distinguishing the complete, electron velocity around every possible interaction node $\underline{v}_n$ (including the actual interaction nodes that become manifest in the interaction) from the average, longitudinal velocity $\underline{u}_n$ between these actual interaction nodes. As before, it is necessary to isolate the activity (say, for electron #1 around the n^{th} interaction node at time t_n) by establishing a temporal interval spanning just one chronon around this node, so that only one interaction can occur during this time period; so:

$$\text{Post-Velocity: } \underline{v}^+_n = 2\,(\,\underline{x}[1{:}\,t_n + \tau/2] - \underline{x}[1{:}\,t_n]\,)/\tau \quad \text{and Prior-Velocity:} \quad \underline{v}^-_n = 2\,(\,\underline{x}[1{:}\,t_n] - \underline{x}[1{:}\,t_n - \tau/2]\,)/\tau$$

These definitions allow the micro-differences in the electron's location around t_n to define the average **nodal** velocity $\{\underline{v}_n\}$.

$$\text{Defn. } \textbf{Average-Velocity } \{\underline{v}_n\} \equiv \{\underline{x}[1{:}\,t_n + \tau/2] - \underline{x}[1{:}\,t_n - \tau/2]\}/\tau \quad \therefore \{\underline{v}_n\} = \tfrac{1}{2}\,(\,\underline{v}^+_n + \underline{v}^-_n\,) \;\&\; \{\underline{v}'_n\} = \tfrac{1}{2}\,(\,\underline{v}'^+_n + \underline{v}'^-_n\,)$$

The present theory retains Newton's concept of **momentum** as the 'quantity of motion', namely as the arithmetic product of a particle's intrinsic mass m and its **total** velocity $\underline{v}_n$ i.e. $\underline{p}_n = m\,\underline{v}_n$ as this original definition still relates its momentum to the actual trajectory followed by each electron and not just its time-averaged, mathematical motion. Section 5.3.1 explored why Newton's Second Law was retained in its original form for the digital electron, so that an external impulse $\Delta\underline{\mathbf{I}}$ **caused** a change in the particle's momentum proportional to the magnitude of the impulse and in the direction of the impulse. In the case of asynchronous interactions, the impulse received by one electron (say at t_n) originated with the partner electron at a different time (say at t'_n). These impulses are again designated with symbolic deltas to retain the notion of (a small) change:

$$\Delta\underline{\mathbf{I}}_n = \Delta\underline{\mathbf{I}}\,[2 \to 1 : t_n] \quad \text{and} \quad \Delta\underline{\mathbf{I}}'_n = \Delta\underline{\mathbf{I}}'\,[1 \to 2 : t'_n]$$

Newton's Dynamical (Second Law) becomes: $\Delta\underline{\mathbf{I}}_{n+1} \to \Diamond\underline{p}_n = m\,(\underline{v}^+_{n+1} - \underline{v}^-_{n+1})$ and $\Delta\underline{\mathbf{I}}'_{n+1} \to \Diamond\underline{p}'_n = m\,(\underline{v}'^+_{n+1} - \underline{v}'^-_{n+1})$

Newton's Third Law was instantaneous; and again is extended across space and time asynchronously: $\Delta \underline{\mathbf{I}}'_n[t'_n] = -\Delta \underline{\mathbf{I}}_n[t_n]$.

This results again in the Conservation of Momentum **across** an interaction: $\Diamond \underline{p}_n + \Diamond \underline{p}'_n = 0$ or $\Diamond[\underline{p}_n + \underline{p}'_n] = 0$

Consequently, the total two-electron velocity after the interaction equals the total two-electron velocity before.

$$\underline{v}^+_n + \underline{v}'^+_n = \underline{v}^-_n + \underline{v}'^-_n = \underline{V}_0 \quad \text{or} \quad \langle \underline{v}_n \rangle + \langle \underline{v}'_n \rangle = \underline{V}_0 \quad \text{where} \quad \langle \underline{v}_n \rangle = \langle \underline{u}_n \rangle + \langle \underline{w}_n \rangle$$

It is very important to remember that this equation is not the one-time standard result but involves the two interaction times. If, prior to the first interaction (n = 1) between the two electrons, their initial velocities were $\underline{v}_i$ and $\underline{v}'_i$ respectively and after the final interaction (n = N) between them they emerged with velocities $\underline{v}_f$ and $\underline{v}'_f$, then: $\underline{v}^-_1 = \underline{v}_i$ and $\underline{v}^+_N = \underline{v}_f$ etc.

$$\underline{v}_i + \underline{v}'_i = \underline{v}_f + \underline{v}'_f = \underline{V}_0$$

This means that the **total** velocity before the set of interactions is conserved across the complete set of interactions. Thus, for both the post-interaction ($\lambda = +$) and pre-interaction ($\lambda = -$) velocities, the asynchronous conservation of digital electron momentum (for all charges) therefore leads to the velocity-invariance condition.

Velocity-Invariance Condition: $\underline{v}^\lambda_n + \underline{v}'^\lambda_n = \underline{V}_0$ { for all n and $\lambda = \pm$ } where: $m\,\underline{v}^\lambda_n = m\langle \underline{v}_n \rangle + \lambda\,\Delta \underline{\mathbf{I}}_n / 2$

Planck's Proposal for mass varying with speed is still rejected [9] as the digital electron's mass still follows Newton's metaphysics and is always represented by an invariant scalar quantity *m*. This allows the Velocity-Invariance equation again to be multiplied by *m* to define the total (two-time) momentum of the pair of interacting electrons.

Definition: Total Two-Electron Momentum $\underline{P}^\lambda_n = m(\underline{v}^\lambda_n + \underline{v}'^\lambda_n) = \underline{p}^\lambda_n + \underline{p}'^\lambda_n = m\,\underline{V}_0 = \underline{P}_0$

The magnitude and direction of this quantity ($\underline{P}_0$) is constant for any specific set of interactions ($P_0 \le mc$) – based only on the relative location of the two electrons when the series of successive interactions began. So, an alternative formulation of the **Invariance Hypothesis** is that each interaction leaves the total momentum unchanged.

Total.Momentum.Before equals Total.Momentum.After or $\Delta[\underline{P}^\lambda_n] = 0$

The repulsive interaction between similarly charged electrons means that every interaction exchanges an absolute amount of momentum from the decelerating electron to the accelerating electron, where (for example): $v^+_n > v^-_n$ or $v_{n+1} > v_n$. This implies an increase in kinetic energy in the accelerating electron: $\Delta\mathcal{K}_{n+1} > \Delta\mathcal{K}_n$. In the case of attractive interactions, it is possible for each electron to retain its speed across each interaction ($v_{n+1} = v_n$) and only change its direction. Obviously, this maintains the kinetic energy locally (at each interaction node) and no kinetic energy is exchanged – such sets of attracting electrons form '***conservative***' systems, which will be the focus of much of this research programme. Repulsive interactions can only cause 'scattering' and do not form 'closed' systems, where the configuration repeats periodically over time. The earlier result (§5.3.1) for the change in a digital electron's kinetic energy after receiving an impulse $\Delta \underline{\mathbf{I}}_{n+1}$ shows that the necessary condition for a two electron system conserve energy is that the impulse is received symmetrically at each node.

$$\Diamond\mathcal{K}_n = \mathcal{K}_{n+1} - \mathcal{K}_n = m\langle \underline{v}_{n+1} \rangle \bullet \Diamond \underline{v}_n = \langle \underline{v}_{n+1} \rangle \bullet \Delta \underline{\mathbf{I}}_{n+1} = \tfrac{1}{2}(\underline{v}^+_{n+1} + \underline{v}^-_{n+1}) \bullet \Delta \underline{\mathbf{I}}_{n+1} \quad \text{while} \quad \underline{\Delta}[\mathcal{K}_n] = 0$$

So, if $u_{n+1} = u_n$: $\Delta\mathcal{K}_n = \tfrac{1}{2} u_n \Delta \mathbf{I}_{n+1} \{\cos(\underline{v}^+_{n+1}, \Delta \underline{\mathbf{I}}_{n+1}) + \cos(\underline{v}^-_{n+1}, \Delta \underline{\mathbf{I}}_{n+1})\} = \tfrac{1}{2} u_n \Delta \mathbf{I}_{n+1} \{\cos\theta_{n+1} + \cos(\pi - \theta_{n+1})\} = 0$

Alternatively, in terms of the longitudinal and transverse velocities just before and just after any interaction node (§5.3.1):

$$\Delta[\mathcal{K}_n - \mathcal{K}'_n] = \Diamond\mathcal{K}_n - \Diamond\mathcal{K}'_n = \langle \underline{v}_{n+1} \rangle \bullet \Delta \underline{\mathbf{I}}_{n+1} - \langle \underline{v}'_{n+1} \rangle \bullet \Delta \underline{\mathbf{I}}'_{n+1} = (\langle \underline{v}_{n+1} \rangle + \langle \underline{v}'_{n+1} \rangle) \bullet \Delta \underline{\mathbf{I}}_{n+1} = \underline{V}_0 \bullet \Delta \underline{\mathbf{I}}_{n+1} = m\,\underline{V}_0 \bullet \Diamond \underline{v}_n$$

Thus a system is "conservative", preserving the kinetic energy across the interaction, when every interaction impulse is orthogonal to the combined total velocity (before or after each interaction): just as it was for the Newtonian particle (§4.3.2).

5.3.4 DIGITAL ELECTRON INTERACTIVITY

Digital Electron Interactivity

The two-electron concept of **interactivity** was discussed earlier (§4.3.5) in the context of digitizing the times when the two electrons actually interacted with one another. This focused on the combination of the total action and angular momentum of two electrons interacting 'on their mutual light-cone' forming a separable Discrete Natural Vector $\mathbf{Q}_{12}$ whose value did not change throughout the complete interaction. The complete two-electron digital activity natural vector $\mathbf{Q}_\eta$ can also be defined in terms of its double-electron components as long as their Separation DNV $\mathbf{X}_{12}$ includes each of the electron's transverse offsets $\mathbf{\Lambda}_\eta$ and each electron's local velocity $\mathbf{V}_\alpha$ includes the electron's 'clicking' velocity $\mathbf{W}_\alpha$.

Definition: **Digital Two-electron Interactivity** $\quad \mathbf{Q}_{12}[T_\eta] \equiv i\,\mathbf{X}_{12}[T_\eta]\,\mathbf{P}_{12}[T_\eta] = \mathbf{Q}_\eta = i\,Q_\eta\,\mathbf{I_0} + \underline{Q}_\eta \bullet \underline{\mathbf{I}}$

where $\quad \mathbf{X}_{12}[T_\eta] = \mathbf{S}_\eta = \mathbf{X}_\eta - \mathbf{X}'_\eta \quad$ and $\quad \mathbf{X}_\eta = \boldsymbol{\mathcal{X}}_\eta + \mathbf{\Lambda}_\eta$

and $\quad \mathbf{P}_{12}[T_\eta] = 2m\,(\mathbf{V}_\eta + \mathbf{V}'_\eta)^* = \mathbf{P}_0 \quad$ and $\quad \mathbf{V}_\eta = \mathbf{U}_\eta + \mathbf{W}_\mu$

with $\quad \mathbf{U}_\eta = i\,c\,\mathbf{I_0} + \underline{u}_\eta \bullet \underline{\mathbf{I}} \quad$ and $\quad \boldsymbol{\mathcal{X}}_\eta = i\,c\,t_\eta\,\mathbf{I_0} + \underline{\mathcal{X}}_\eta \bullet \underline{\mathbf{I}} \quad \& \quad \underline{\mathcal{X}}_\eta = \underline{x}_{n0} + \underline{u}_n\,(t_\eta - t_n)$

$\therefore \quad \mathbf{Q}_\eta = i\,2\,m\,(c^2\,T_\eta - \underline{S}_\eta \bullet \underline{V}_0)\,\mathbf{I_0} + 2\,\underline{L}_\eta \bullet \underline{\mathbf{I}} \quad$ where $\quad \underline{L}_\eta = m\,c\,(\underline{S}_\eta - \underline{V}_0\,T_\eta) + i\,\underline{S}_\eta \wedge \underline{V}_0 \quad$ and $\quad \underline{S}_\eta = \underline{X}_{12}[T_\eta] = \underline{x}_\eta - \underline{x}'_\eta$

Using the results from section 4.2.4 that $S_n = c\,T_n$ and $\underline{S}_n = \underline{c}\,T_n$ then the two-electron activity evaluated at each node gives:

$$\mathbf{Q}_n = i\,2\,m\,T_n\,(c^2 - \underline{c} \bullet \underline{V}_0)\,\mathbf{I_0} + 2\,m\,T_n\,\underline{\mathbf{I}} \bullet \{c\,(\underline{c} - \underline{V}_0) + i\,\underline{c} \wedge \underline{V}_0\} = 2\,m\,T_n\,(\underline{c} - \underline{V}_0) \bullet \{i\,\underline{c}\,\mathbf{I_0} + c\,\underline{\mathbf{I}}\} + i\,2\,m\,T_n\,(\underline{c} - \underline{V}_0) \wedge \underline{c} \bullet \underline{\mathbf{I}}$$

$$\therefore \quad \mathbf{Q}_n = -\,i\,2\,m\,T_n\,\mathbf{C}\,(\mathbf{C} - \mathbf{V}_{12})^* = i\,2\,m\,T_n\,\mathbf{C}\,\mathbf{V}_{12}^* \quad \text{since} \quad \mathbf{C}_n^*\,\mathbf{C}_n = 0 \quad \therefore \quad \mathbf{Q}_n = i\,mT_n\,\mathbf{C}\,\mathbf{V}_0^*$$

When these two electrons are <u>not</u> interacting with each other or any others (during any finite time period) they effectively behave like 'free' particles, so their total two-time combined velocity $\underline{V}_0$ is constant. In their Symmetric Inertial Reference Frame (SIRF) their combined velocity at any <u>single</u> time is zero but, in general, this is not true for their total two-time combined velocity. Only in this special case (when: $\underline{v}_\eta = -\,\underline{v}'_\eta \;$ and $\; \underline{V}_\eta = 0$), with all variables tagged with '0' then:

$$\mathbf{Q}^0[T_\eta] = i\,2\,m\,c^2\,T_\eta\,\mathbf{I_0} + 2\,m\,c\,\underline{S}_\eta \bullet \underline{\mathbf{I}} = 2\,m\,c\{\,i\,c\,(t_\eta - t'_\eta)\,\mathbf{I_0} + (\underline{x}_\eta - \underline{x}'_\eta) \bullet \underline{\mathbf{I}}\,\} = 2\,m\,c\,(\mathbf{X}_\eta - \mathbf{X}'_\eta) \equiv \mathbf{Q}^0[1{:}\,t_\eta] - \mathbf{Q}^0[2{:}\,t'_\eta]$$

At possible interaction nodes (i.e. $T_\eta = T_n$) when $\underline{S}_n = c\,T_n \quad \therefore \quad \mathbf{Q}^0[T_n] = 2\,m\,c\,T_n\,(i\,c\,\mathbf{I_0} + \underline{c} \bullet \underline{\mathbf{I}}) = 2\,m\,c\,\mathbf{C}\,(t_n - t'_n)$

$$\therefore \quad \mathbf{Q}^0[1{:}\,t_n] = 2\,m\,c\,t_n\,\mathbf{C} = 2\,m\,c\,\mathbf{X}[1{:}\,t_n] \quad \text{and} \quad \mathbf{Q}^0[2{:}\,t'_n] = 2\,m\,c\,t'_n\,\mathbf{C} = 2\,m\,c\,\mathbf{X}[2{:}\,t'_n]$$

The canonical situation (described in [4] §6.4) occurs when two electrons collide from "infinity"; that is to say, they each exchange the maximum number of possible impulses N_0 while reversing their motion. When viewed from within the SIRF then the combined (asynchronous) total velocity is light-speed $\underline{c}$. In this case, $\mathbf{Q}_n = 0$ so it is an invariant at all interaction nodes. When the total number of consecutive interactions is less than the maximum then: $V_\eta < c$, so that:

$$\therefore \quad \mathbf{Q}_n = i\,2\,m\,T_n\,\mathbf{C}\,\mathbf{V}_0^* \quad \therefore \quad \mathbf{\Delta Q}[T_n] = i\,2\,m\,c\,\mathbf{C}\,\mathbf{V}_0^*\,\mathbf{\Delta}[T_n] = i\,2\,m\,c\,\mathbf{C}\,\mathbf{V}_0^*\,(\Delta t_n - \Delta t'_n)$$

Interactions change Interactivity

In this theory, interactions are represented by anti-symmetrical impulses, 'exchanged' between the two electrons. As the effects of all of these impulses are instantaneous <u>at</u> each electron, they correspond to the application of the point-difference operator $\Diamond$ (see Appendix A4.2) acting on the electron's local velocity <u>at</u> each and every one of the interaction nodes.

$\Diamond[\mathbf{X}_{12}[T_n]\,] = \Diamond[\mathbf{X}_n - \mathbf{X}'_n] = \Diamond[\mathbf{X}_n] - \Diamond[\mathbf{X}'_n] = 0$ as $\mathbf{X}_\eta$ is continuous across time; i.e. $\Diamond[\mathbf{X}_\eta] = 0$ and $< \mathbf{X}_{12} > = \mathbf{X}_{12}$

$$\therefore \quad \Diamond\mathbf{Q}_n = i\,\Diamond[\,\mathbf{X}_{12}[T_n]\,] < \mathbf{P}_{12}[T_n] > + i < \mathbf{X}_{12}[T_n] > \Diamond[\,\mathbf{P}_{12}[T_n]\,] = i\,\mathbf{X}_{12}[T_n]\;\Diamond[\,\mathbf{P}_{12}[T_n]\,]$$

$$\text{Now} \quad \Diamond[\mathbf{V}_\eta] = \Diamond[\, i\,c\,\mathbf{I_0} + \underline{V}_\eta \bullet \underline{\mathbf{I}}\,] = \underline{\mathbf{I}} \bullet \Diamond\underline{V}_\eta \quad \therefore \quad \Diamond[m\mathbf{V}_n{}^*] = \Delta\underline{\mathbf{I}}[1{:}\,t_{n+1}] = \Delta\underline{\mathbf{I}}_{n+1} \quad \text{and} \quad \Diamond[m\mathbf{V'}_n{}^*] = \Delta\underline{\mathbf{I}}'_{n+1}$$

Here, $\Delta\underline{\mathbf{I}}_{n+1}$ is the impulse experienced by electron #1 at time t_{n+1} originating from electron #2 at t'_{n+1} while $\Delta\underline{\mathbf{I}}'_n$ is the impulse experienced by electron #2 at time t'_{n+1} originating from electron #1 at t_{n+1}. The principal dynamical hypothesis of the digital electron interaction is that momentum is conserved across the interaction (see section 5.3.3); so: $\Delta\underline{\mathbf{I}}'_{n+1} = -\Delta\underline{\mathbf{I}}_{n+1}$

$$\therefore \Diamond[\,\mathbf{P}_{12}[T_n]\,] = m\,\Diamond[\,\mathbf{V}_n{}^* + \mathbf{V'}_n{}^*\,] = \underline{\mathbf{I}} \bullet (\Delta\underline{\mathbf{I}}_{n+1} + \Delta\underline{\mathbf{I}}'_{n+1}) \qquad \therefore \Diamond[\,\mathbf{P}_{12}[T_n]\,] = 0 \qquad \therefore \Diamond\mathbf{Q}_n = 0$$

This result means that as a corollary of the total (asynchronous) two-electron momentum being an invariant ($\mathbf{P}^0{}_{12}$) then the two-electron interactivity $\mathbf{Q}_{12}$ is also an invariant with respect to the effects of the interaction between the two electrons <u>at</u> each interaction node. Thus, at all times t_η, including t_n, $(\underline{v}_1 + \underline{v}_2)$ is a constant of the motion: i.e. $\mathbf{V}_1 + \mathbf{V}_2 = 2\mathbf{V}_0$, so at every interaction node 'n' $\mathbf{CV}_0{}^*$ is another constant, so that $\mathbf{Q}_n$ only varies with the interaction time-difference T_n.

$$\mathbf{Q}_n = i\,2\,m\;\mathbf{CV}_0{}^*\,T_n = 2\,m\,T_n\,\{\, i\,c\,\mathbf{I_0}\,(\underline{c} - \underline{V}_0) + c\,\underline{\mathbf{I}} \bullet (\underline{c} - \underline{V}_0) - i\,\underline{c} \wedge (\underline{c} - \underline{V}_0)\,\}$$

As was shown above, the interaction changes the individual digital electron's activity; a similar impact is made on the two-electron interactivity. Only the effect of the impulse-exchange around the $(n+1)^{th}$ interaction need be considered, from 'just before' the interaction (at $t_{n+1} - \tau/2$ and $t'_{n+1} - \tau/2$) up to 'just after' the next pair of chronons (at $t_{n+1} + \tau/2$ and $t'_{n+1} + \tau/2$). The impulse changes the electron velocities from $\underline{v}^-_{n+1}$ and $\underline{v'}^-_{n+1}$ to $\underline{v}^+_{n+1}$ and $\underline{v'}^+_{n+1}$; the difference in the Separation DNV:

$$\Delta_\tau\mathbf{S}_{n+1} \equiv \mathbf{X}_{12}[T_{n+1} + \tau/2] - \mathbf{X}_{12}[T_{n+1} - \tau/2] = \underline{\mathbf{I}} \bullet \{\, (\underline{v}^+_{n+1} + \underline{v'}^+_{n+1}) - (\underline{v}^-_{n+1} + \underline{v'}^-_{n+1})\,\}\,\tau/2$$

$$\therefore \text{m}\,\Delta_\tau\mathbf{S}_{n+1} = \underline{\mathbf{I}} \bullet \{\, m\,(\underline{v}^+_{n+1} - \underline{v}^-_{n+1}) + m\,(\underline{v'}^+_{n+1} - \underline{v'}^-_{n+1})\,\}\,\tau/2 = \underline{\mathbf{I}} \bullet (\Delta\underline{\mathbf{I}}_{n+1} + \Delta\underline{\mathbf{I}}'_{n+1})\,\tau/2 = 0$$

$$\therefore \quad \Delta_\tau\mathbf{Q}_{n+1} \equiv \mathbf{Q}_{12}[T_{n+1} + \tau/2] - \mathbf{Q}_{12}[T_{n+1} - \tau/2] = \Delta_\tau[\, i\,\mathbf{X}_{12}[T_{n+1}]\,\mathbf{P}_{12}[T_{n+1}]\,] = \Delta_\tau[\, i\,\mathbf{S}_{n+1}\,\mathbf{P}_0\,] = \Delta_\tau[\, i\,\mathbf{S}_{n+1}\,]\,\mathbf{P}_0 = 0$$

This demonstrates that the two-electron interactivity, for the digital electron model, is invariant across a finite time interval (one chronon) <u>spanning</u> any interaction node pair of these extended electrons, as was demonstrated earlier (§4.3.5).

5.3.5 DIGITAL ELECTRON QUANTIZATION

Quantized Digital Electron Activity

The concept of quantizing the exchange of **activity** was discussed earlier (§4.3.6) in the context of digitizing the times when the two electrons actually interacted with one another. The discreteness of this exchange was posited on the plausibility that all features of the electron interaction were discrete and not continuous. That analysis was developed around the hypothesis that electrons may only interact on an exact multiple of the unit of temporal duration – the chronon, τ. The present section now extends that approach to now include the other component of the 'digital electron model', namely the 4-way 'clicking' motion of each electron around its average longitudinal velocity. Since this second extension brings in the <u>direction</u> of this transverse motion, its relative timing between the two electrons (or phase) becomes an important issue. As before, it will be assumed that the 'just before' situation around an interaction node (labeled 'n') is identical to the fully time symmetrical situation, so this latter will be the basis adopted here; thus, for electron #1 in the temporal interval $(t_n - \tau/2)$ through t_n to $(t_n + \tau/2)$ i.e. from 'nearly before' t_n to 'nearly after'; i.e. electron state at $(t_n \pm \delta t)$ equal to state at $(t_n \pm \tau/2)$.

$$\Delta_\tau\mathbf{R}_n = \Delta\mathbf{R}[t_n + \tau/2,\, t_n - \tau/2] = \Diamond\mathbf{R}_{n-1} + \Delta^+\mathbf{R}_n + \Delta^-\mathbf{R}_n$$

$$\text{where:} \quad \Delta^+\mathbf{R}_n = \Delta\mathbf{R}[t_n + \tau/2,\, t_n] \quad \text{and} \quad \Delta^-\mathbf{R}_n = \Delta\mathbf{R}[t_n,\, t_n - \tau/2]$$

If the interaction had <u>not</u> occurred at t_{n+1} then over this one chronon time interval the 'free' change in activity would be :

$$\Delta_\tau\mathbf{R}^0{}_n = \Delta^+\mathbf{R}_n + \Delta^-\mathbf{R}_n \quad \text{as} \quad \Diamond\mathbf{R}^0{}_n = 0$$

The impact on the key single chronon Activity of one of the electrons (say #1) as a result of the interaction at t_{n+1} is (§4.3.6):

$$\Delta^*[\mathbf{R}_{n+1}] = \delta_0[\Delta_\tau[\mathbf{R}_{n+1}]] = \Delta_\tau \mathbf{R}_{n+1} - \Delta_\tau \mathbf{R}^0{}_{n+1} = \Diamond \mathbf{R}_n = i\,\Delta^*\mathrm{R}_{n+1}\,\mathbf{I_0} + \Delta^*\underline{\mathrm{R}}_{n+1} \bullet \underline{\mathbf{I}} + i\,\Delta^*\underline{\mathscr{R}}_{n+1} \bullet \underline{\mathbf{I}}$$

$$\therefore\ \Delta^*\mathrm{R}_{n+1} = -\,m\,\underline{x}_{n+1} \bullet \Diamond \underline{v}_n \quad \& \quad \Delta^*\underline{\mathrm{R}}_{n+1} = -\,m\,c\,t_{n+1}\,\Diamond \underline{v}_n \quad \& \quad \Delta^*\underline{\mathscr{R}}_{n+1} = m\,(\underline{x}_{n+1} \wedge \Diamond \underline{v}_n)$$

Even if the electron's speed is unchanged at t_{n+1} , its direction may be changed by the impulse ($\Delta\underline{\mathbf{I}}_{n+1} = m\,\Diamond \underline{v}_n$); thus:

$$\therefore\ \Delta^*\mathrm{R}_{n+1} = -\,\underline{x}_{n+1} \bullet \Delta\underline{\mathbf{I}}_{n+1} \quad \& \quad \Delta^*\underline{\mathrm{R}}_{n+1} = -\,c\,t_{n+1}\,\Delta\underline{\mathbf{I}}_{n+1} \quad \& \quad \Delta^*\underline{\mathscr{R}}_{n+1} = \underline{x}_{n+1} \wedge \Delta\underline{\mathbf{I}}_{n+1}$$

The corresponding difference in the 'partner' electron's local-activity (around the matching t'_{n+1} node) is:

$$\therefore\ \Delta^*\mathrm{R}'_{n+1} = -\,\underline{x}'_{n+1} \bullet \Delta\underline{\mathbf{I}}'_{n+1} \quad \& \quad \Delta^*\underline{\mathrm{R}}'_{n+1} = -\,c\,t'_{n+1}\,\Delta\underline{\mathbf{I}}'_{n+1} \quad \& \quad \Delta^*\underline{\mathscr{R}}'_{n+1} = \underline{x}'_{n+1} \wedge \Delta\underline{\mathbf{I}}'_{n+1}$$

This suggests re-focusing on the concept of **Two-Electron Total-Activity**, $\mathbf{B}_{12}[T_n]$ that was introduced in section 4.3.4 as the symmetric (two-time, non-local) sum of the two individual electron activities.

$$\mathbf{B}_n = \mathbf{R}_n + \mathbf{R}'_n = \mathbf{B}_{12}[T_n]$$

Invoking momentum conservation is equivalent to assuming any type of anti-symmetric interaction: $\Delta\underline{\mathbf{I}}'_n = -\,\Delta\underline{\mathbf{I}}_n$. This generates the universal result for the digital electron model:

$$\Delta^*\mathbf{B}_n = -\,i\,\mathbf{I_0}\,(\underline{x}_n - \underline{x}'_n) \bullet \Delta\underline{\mathbf{I}}_n - c\,(t_n - t'_n)\,\underline{\mathbf{I}} \bullet \Delta\underline{\mathbf{I}}_n + i\,\underline{\mathbf{I}} \bullet (\underline{x}_n - \underline{x}'_n) \wedge \Delta\underline{\mathbf{I}}_n$$

There are two extremal situations possible: either when each interaction impulse $\Delta\underline{\mathbf{I}}_n$ (or $\Delta\underline{v}_n$) is radial or transverse to the separation vector $\underline{S}_n$. Let $\underline{\hat{e}}_n$ represent the unit vector in the direction from the position of electron #2 to electron #1.

A) Radial Impulse: $\Delta\underline{\mathbf{I}}_{Rn} = \Delta\mathbf{I}_{Rn}\,\underline{\hat{e}}_n = \Delta\mathbf{I}_{Rn}\,\underline{S}_n / c\,T_n \quad \therefore\ \Delta^*\mathbf{B}_{Rn} = -\,\Delta\mathbf{I}_{Rn}\,T_n\,(i\,c\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{c}_n) = -\,T_n\,\Delta\mathbf{I}_{Rn}\,\mathbf{C}_n$

B) Transverse Impulse: $\Delta\underline{\mathbf{I}}_{Tn} = \Delta\mathbf{I}_{Tn}\,\underline{\ddot{e}}_n \quad \& \quad \underline{\hat{e}}_n \bullet \underline{\ddot{e}}_n = 0 \quad \therefore\ \Delta^*\mathbf{B}_{Tn} = i\,\underline{\mathbf{I}} \bullet \underline{S}_n \wedge \Delta\underline{\mathbf{I}}_{Tn} = i\,c\,T_n\,\Delta\mathbf{I}_{Tn}\,(\underline{\mathbf{I}} \bullet \underline{\hat{e}}_n \wedge \underline{\ddot{e}}_n)$

Since, for a single electron: $\mathbf{R}_\eta \equiv i\,\mathcal{A}_\eta\,\mathbf{I_0} + \underline{\mathcal{G}}_\eta \bullet \underline{\mathbf{I}} + i\,\underline{\mathcal{M}}_\eta \bullet \underline{\mathbf{I}}$ where $\mathcal{A}_\eta = \mathrm{R}_\eta \ \& \ \underline{\mathcal{G}}_\eta = \underline{\mathbf{R}}_\eta \ \& \ \underline{\mathcal{M}}_\eta = \underline{\mathscr{R}}_\eta$

This further suggests partitioning the two-electron total-activity $\mathbf{B}_n$ into its three equivalent mechanical components: total-action $\mathcal{A}_T$, total-Galilean-momentum $\underline{\mathcal{G}}_T$ and total-angular momentum $\underline{\mathcal{M}}_T$, which are constants of the motion for two non-interacting electrons (see [129] and §5.3.2).

Definition: **Digital Two-electron Action** $\mathcal{A}_{12}[t_n, t'_n] \equiv \mathcal{A}_{1n} + \mathcal{A}_{2n} = \mathcal{A}_{Tn} \quad \& \quad i\,\mathbf{I_0}\,\mathcal{A}_{Tn} = \tfrac{1}{2}\,(\mathbf{B}_n - \mathbf{B}_n{}^\dagger)$

Definition: **Digital Total G-Momentum** $\underline{\mathcal{G}}_{12}[t_n, t'_n] \equiv \underline{\mathcal{G}}_{1n} + \underline{\mathcal{G}}_{2n} = \underline{\mathcal{G}}_{Tn} \quad \& \quad \underline{\mathbf{I}} \bullet \underline{\mathcal{G}}_{Tn} = \tfrac{1}{2}\,(\mathbf{B}_n + \mathbf{B}_n{}^*)$

Definition: **Digital Angular-Momentum** $\underline{\mathcal{M}}_{12}[t_n, t'_n] \equiv \underline{\mathcal{M}}_{1n} + \underline{\mathcal{M}}_{2n} = \underline{\mathcal{M}}_{Tn} \quad \& \quad i\,\underline{\mathbf{I}} \bullet \underline{\mathcal{M}}_{Tn} = \tfrac{1}{2}\,(\mathbf{B}_n{}^\dagger - \mathbf{B}_n{}^*)$

In the SIRF, for all interactions: $\underline{x}'_n = -\,\underline{x}_n \ \& \ t'_n = -\,t_n \ \therefore\ \Delta^*\mathrm{R}'_n = -\,\Delta^*\mathrm{R}_n \quad \& \quad \Delta^*\underline{\mathrm{R}}_n = -\,\Delta^*\underline{\mathrm{R}}'_n \quad \& \quad \Delta^*\underline{\mathscr{R}}_n = 0$

When the impulse is purely radial (or in the SIRF, passing through the origin then it is parallel to the electron's location $\underline{x}_n$):

$$\therefore \quad \Delta^*\mathrm{R}_n = -\,x_n\,\Delta\mathbf{I}_{Rn} \quad \& \quad \Delta^*\underline{\mathrm{R}}_n = -\,\underline{c}\,t_n\,\Delta\mathbf{I}_{Rn} \quad \& \quad \Delta^*\underline{\mathscr{R}}_n = 0$$

$$\therefore\ \Delta^*\mathcal{A}_{Rn} = -\,c\,T_n\,\Delta\mathbf{I}_{Rn} \quad \& \quad \Delta^*\underline{\mathcal{G}}_{Rn} = -\,c\,T_n\,\Delta\underline{\mathbf{I}}_{Rn} \quad \& \quad \Delta^*\underline{\mathcal{M}}_{Rn} = 0$$

This means that both the total and individual angular momentum is conserved across every radial interaction, while if the magnitude of the radial impulse varies inversely with the temporal separation then the total change in the action (and the Galilean-momentum) will be a constant at each interaction; if the impulse is constant in magnitude then these quantities vary

directly with the duration of the temporal difference T_n, requiring a longer and longer time interval to accumulate the same amount of change in total action. When the impulse is purely transverse (i.e. orthogonal to their separation, $\underline{S}_n$):

$$\therefore \quad \Delta^* \mathcal{A}_{Tn} = 0 \qquad \& \qquad \Delta^* \underline{\mathcal{G}}_{Tn} = -c\, T_n\, \Delta \underline{\mathbf{I}}_{Tn} \qquad \& \qquad \Delta^* \underline{\mathcal{M}}_{Tn} = \underline{S}_n \wedge \Delta \underline{\mathbf{I}}_{Tn}$$

Since every impulse can be written as the (mathematical) sum of radial & transverse components: $\Delta \underline{\mathbf{I}}_n = \Delta \underline{\mathbf{I}}_{Rn} + \Delta \underline{\mathbf{I}}_{Tn}$

$$\therefore \quad \Delta^* \mathcal{A}_{Tn} = -T_n\, c\, \Delta \mathbf{I}_{Rn} \qquad \& \qquad \Delta^* \underline{\mathcal{G}}_{Tn} = -c\, T_n\, \Delta \underline{\mathbf{I}}_n \qquad \& \qquad \Delta^* \underline{\mathcal{M}}_{Tn} = T_n\, \underline{c}_n \wedge \Delta \underline{\mathbf{I}}_{Tn}$$

In terms of the impulse radial & transverse vectors: $\underline{S}_n \bullet \Delta \underline{\mathbf{I}}_n = S_n\, \Delta \mathbf{I}_{Rn}$ & $\underline{S}_n \wedge \Delta \underline{\mathbf{I}}_n = \underline{S}_n \wedge \Delta \underline{\mathbf{I}}_{Tn}$ and $c\, \Delta \mathbf{I}_{Rn} = \Delta \mathbf{I}_{Rn}\, \underline{c}$

$$\therefore \quad \Delta^* \mathbf{B}_n = -i\, \mathbf{I_0}\, \underline{S}_n \bullet \Delta \underline{\mathbf{I}}_n - c\, T_n\, \underline{\mathbf{I}} \bullet \Delta \underline{\mathbf{I}}_n + i\, \underline{\mathbf{I}} \bullet \underline{S}_n \wedge \Delta \underline{\mathbf{I}}_n = -T_n\, \Delta \mathbf{I}_{Rn}\, (i\, c\, \mathbf{I_0} + \underline{c} \bullet \underline{\mathbf{I}}) - T_n\, \underline{\mathbf{I}} \bullet (c\, \Delta \underline{\mathbf{I}}_{Tn} - i\, \underline{c} \wedge \Delta \underline{\mathbf{I}}_{Tn})$$

This is simplified with the **Orthogonal Complex Decomposition (OCD)**: $c\, \Delta \underline{\mathbf{I}}_{Tn} = c\, (\underline{G}_n + i\, \underline{H}_n) = c\, \underline{G}_n + i\, \underline{c} \wedge \underline{G}_n$ ($\underline{G}_n$ real).

Since $\underline{c}_n \bullet \Delta \underline{\mathbf{I}}_{Tn} = 0 \;\; \therefore \;\; \underline{c} \bullet \underline{G}_n = 0$ & $\underline{c}_n \wedge \Delta \underline{\mathbf{I}}_{Tn} = c\, \underline{\hat{e}} \wedge \Delta \underline{\mathbf{I}}_{Tn}$ while $\underline{\hat{e}} \wedge (\underline{c} \wedge \underline{G}_n) = (\underline{\hat{e}} \bullet \underline{G}_n)\, \underline{c} - (\underline{\hat{e}} \bullet \underline{c})\, \underline{G}_n = -c\, \underline{G}_n$

$$\therefore \quad c\, \Delta \underline{\mathbf{I}}_{Tn} - i\, \underline{c} \wedge \Delta \underline{\mathbf{I}}_{Tn} = (c\, \underline{G}_n + i\, \underline{c} \wedge \underline{G}_n) - i\, \underline{\hat{e}} \wedge (c\, \underline{G}_n + i\, \underline{c} \wedge \underline{G}_n) = c\, \underline{G}_n + \underline{\hat{e}} \wedge (\underline{c} \wedge \underline{G}_n) = 0$$

$$\therefore \quad \boldsymbol{\Delta}^* \mathbf{B}_n = -T_n\, \Delta \mathbf{I}_{Rn}\, \mathbf{C} = -\mathbf{S}_n\, (\underline{c} \bullet \Delta \underline{\mathbf{I}}_n) / c \quad \text{or} \quad \Delta^* \mathbf{B}_{Tn} = c\, T_n\, \boldsymbol{\Delta} \mathbf{I}_{Tn} \quad \text{(Pure transverse OCD impulse)}$$

This important result shows that it is only the **radial** impulse the changes the total-activity, so that all purely transverse impulses always preserve the total-activity at each interaction node and only exchange activity between the two electrons.

Since $\mathbf{V}_n = i\, c\, \mathbf{I_0} + \underline{v}_n \bullet \underline{\mathbf{I}}$ and $\Diamond \underline{c}_n = 0$ then $m \Diamond \mathbf{V}_n = m\, \underline{\mathbf{I}} \bullet \Diamond \underline{v}_n = \underline{\mathbf{I}} \bullet \Delta \underline{\mathbf{I}}_n \quad \therefore \quad \boldsymbol{\Delta} \mathbf{I}_n = m \Diamond \mathbf{V}_n = \underline{\mathbf{I}} \bullet \Delta \underline{\mathbf{I}}_n$

This DNV equation is covariant with respect to space and time; as such, the DNV impulse, $\boldsymbol{\Delta} \mathbf{I}_n$ may be (mathematically) rewritten in terms of an alternative scalar ($\Delta \mathbf{I}_{Sn}$) and complex transverse vector ($\Delta \underline{\mathbf{I}}_{Vn}$) decomposition in another direction $\underline{\mathbf{I}}^{\ddagger}$.

Decomposition: **Impulse** DNV $\boldsymbol{\Delta} \mathbf{I}^{\ddagger}_n = i\, \mathbf{I_0}\, \Delta \mathbf{I}_{Sn} + \underline{\mathbf{I}}^{\ddagger} \bullet \Delta \underline{\mathbf{I}}_{Vn} = \underline{\mathbf{I}} \bullet \Delta \underline{\mathbf{I}}_n$ where $c\, \Delta \underline{\mathbf{I}}_{Vn} = c\, (\underline{G}_n - i\, \underline{H}_n) = c\, \underline{G}_n - i\, \underline{c} \wedge \underline{G}_n$

This DNV's 'norm' is invariant: $\Delta \underline{\mathbf{I}}_n \bullet \Delta \underline{\mathbf{I}}_n = (\Delta \mathbf{I}_{Vn})^2 - (\Delta \mathbf{I}_{Sn})^2 = (\Delta \mathbf{I}_{Rn})^2 + (\Delta \mathbf{I}_{Tn})^2$ N.B. Note the sum & difference.

Now, both sub-components of the transverse vector component of each impulse are each still orthogonal to the light vector:

Since $\underline{c}_n \bullet \Delta \underline{\mathbf{I}}_{Vn} = 0 \;\; \therefore \;\; \underline{c} \bullet \underline{G}_n = 0$ & $\underline{c}_n \wedge \Delta \underline{\mathbf{I}}_{Vn} = c\, \underline{\hat{e}} \wedge \Delta \underline{\mathbf{I}}_{Vn}$ while $\underline{\hat{e}} \wedge (\underline{c} \wedge \underline{G}_n) = (\underline{\hat{e}} \bullet \underline{G}_n)\, \underline{c} - (\underline{\hat{e}} \bullet \underline{c})\, \underline{G}_n = -c\, \underline{G}_n$

$$\therefore \quad c\, \Delta \underline{\mathbf{I}}_{Vn} - i\, \Delta \underline{\mathbf{I}}_{Vn} \wedge \underline{c} = (c\, \underline{G}_n - i\, \underline{c} \wedge \underline{G}_n) + i\, \underline{\hat{e}} \wedge (c\, \underline{G}_n - i\, \underline{c} \wedge \underline{G}_n) = c\, \underline{G}_n + \underline{\hat{e}} \wedge (\underline{c} \wedge \underline{G}_n) = 0$$

$$\Delta \mathbf{I}^{\ddagger}_n\, \mathbf{C} = \mathbf{I_0}\, (c\, \Delta \mathbf{I}_{Sn} - \underline{c} \bullet \Delta \underline{\mathbf{I}}_{Vn}) + i\, \underline{\mathbf{I}}^{\ddagger} \bullet (c\, \Delta \underline{\mathbf{I}}_{Vn} - \underline{c}\, \Delta \mathbf{I}_{Sn}) + \underline{\mathbf{I}}^{\ddagger} \bullet \Delta \underline{\mathbf{I}}_{Vn} \wedge \underline{c} = -i\, \Delta \mathbf{I}_{Sn}\, (i\, c\, \mathbf{I_0} + \underline{c} \bullet \underline{\mathbf{I}}^{\ddagger}) + i\, \underline{\mathbf{I}}^{\ddagger} \bullet (c\, \Delta \underline{\mathbf{I}}_{Vn} - i\, \Delta \underline{\mathbf{I}}_{Vn} \wedge \underline{c})$$

$$\therefore \quad \Delta \mathbf{I}^{\ddagger}_n\, \mathbf{C} = -i\, \Delta \mathbf{I}_{Sn}\, \mathbf{C}^{\ddagger} \quad \therefore \quad \boldsymbol{\Delta}^* \mathbf{B}_n = -i\, T_n\, \Delta \mathbf{I}^{\ddagger}_n\, \mathbf{C} \quad \text{If \& only if:} \quad \Delta \mathbf{I}_{Sn} = \Delta \mathbf{I}_{Rn} \quad \& \quad \mathbf{C}^{\ddagger} = \mathbf{C}$$

This decomposition is equivalent to replacing the single impulse at t_n with two separate impulses that occur at $(t_n \pm \tau/2)$ with the electron moving at some constant intermediate velocity $\underline{v}^{\ddagger}_n$ between these two points. The basic spatial orthogonality condition continues to require that these two equivalent impulses still remain parallel to the original $\underline{S}_n$ vector. This alternate mathematical viewpoint will not be pursued further here but may prove useful elsewhere.

Quantizing Digital Electron Action

The previous section demonstrated that the change in the two-electron activity $\mathbf{B}_n$ due to each interaction between the two electrons over a unit time interval of one chronon around their interaction times (star-diff, §4.3.3) was always proportional to their common 'Light' DNV $\mathbf{C}$, provided the impulse was either purely radial or the transverse component always satisfied the *orthogonal complex decomposition* or OCD ; that is:

$$c\,\Delta^*\mathbf{B}_n = -c\,T_n\,\Delta\mathbf{I}_{Rn}\,\mathbf{C} = -\mathbf{S}_n\,(\underline{c}\bullet\Delta\underline{\mathbf{I}}_n) \quad \text{provided} \quad \Delta\underline{\mathbf{I}}_{Tn} = 0 \quad \text{or} \quad c\,\Delta\underline{\mathbf{I}}_{Tn} = c\,\underline{G}_n + i\,\underline{c}\wedge\underline{G}_n \;\; (\underline{G}_n \text{ real}).$$

The earlier analysis of the EM Impulse (§4.3.7) proposed that the form of the impulse $\Delta\mathbf{I}$ varied with the separation S_n of the two interacting electrons. When the electrons are far apart (exceeding D_S or about 10^{-1} cm - see §6.3.3) this '**far**' impulse is constant ($\Delta\mathbf{I}_0 = m\,b$) but at '**near**' separations ($S_n < D_S$) the impulse varies inversely with the separation ($\Delta\mathbf{I}_n = \pi mc\tau / \alpha T_n$). This indicates that for 'near' separations the quantity '$c\,T_n\,\Delta\mathbf{I}_{Rn}$' is invariant, defining a new **Micro-Action** constant $\mathcal{a}$.

$$\text{'}\mathbf{Near}\text{'}: \quad c\,T_n\,\Delta\mathbf{I}_{Rn} = \pi\,m\,c^2\,\tau/\alpha = \pi\,m\,c^2\,(e^2/m\,c^3)/\alpha = \pi\,e^2/\alpha\,c = h/2 \equiv \mathcal{a}$$

In order to extend this concept to include constant 'far' impulses, it is necessary to truncate the time separation variable at the critical value $T_S \equiv (\pi/\alpha)\,N_0\,\tau$, where $N_0 = c/b$ (§4.3.6), so that for 'far' separations ($S_n > D_S$), where $D_S \equiv c\,T_S$, then:

$$\text{'}\mathbf{Far}\text{'}: \quad \pi\,m\,c^2\,\tau/\alpha = m\,c^2\,(T_S/N_0) = cT_S\,(mc/N_0) = D_S\,(m\,b) = c\,T_S\,\Delta\mathbf{I}_0 = c\,T_n\,\Delta\mathbf{I}_{Rn} = c\,(\min[T_n, T_S])\,\Delta\mathbf{I}_{Rn} \equiv a_n$$

These two separation ranges can be consolidated (like the impulse) into one single parameter, $\mathcal{a}_n$ (note: $a_n = \mathcal{a}$).

Definition: The **Electro-Magnetic Dynamic Action** $\mathcal{a}_n \equiv \theta[S_0 - S_n]\,\mathcal{a} + \theta[S_n - S_0]\,a_n \equiv c\,\mathcal{T}_n\,\Delta\mathbf{I}_{Rn}$

& The **Effective Interaction Separation Time** $\mathcal{T}_n \equiv \theta[T_0 - T_n]\,T_n + \theta[T_n - T_0]\,T_0 \quad \therefore \quad c\,\Delta^*\mathbf{B}_n = -\mathcal{a}_n\,\mathbf{C}$

This extends the earlier (simpler) form of the universal action DNV into one that covers ALL inter-electron separations. This re-defines the **Micro-Action** DNV $\mathcal{a}_n$ (from §4.3.6) by a Lorenz-like relation: $c\,\mathcal{a}_n = \mathcal{a}_n\,\mathbf{C} \quad \therefore \quad \Delta^*\mathbf{B}_n = -\mathcal{a}_n$

$$\therefore \quad c\,(i\,\mathcal{a}_n\,\mathbf{I_0} + \underline{\mathcal{a}}_n\bullet\underline{\mathbf{I}}) = \mathcal{a}_n\,(i\,c\,\mathbf{I_0} + \underline{c}\bullet\underline{\mathbf{I}}) \qquad \therefore \quad c\,\underline{\mathcal{a}}_n = \mathcal{a}_n\,\underline{c} \qquad \therefore \quad \underline{\mathcal{a}}_n = c\,\mathcal{T}_n\,\Delta\underline{\mathbf{I}}_{Rn}$$

For **atomic** systems, $\mathcal{a}_n = \mathcal{a}$ so that the change in the two-electron activity $\Delta^*\mathbf{B}_n$ is always <u>independent of time</u> (invariant). Furthermore, the digital electron's 'pure' Activity DNV $\mathbf{R}^S_\eta$ is a <u>constant</u> with: $\mathcal{A}^S_\mu = \tfrac{1}{2}\,\mathcal{a} \quad \& \quad \underline{\mathcal{M}}^S_\mu = \tfrac{1}{2}\,\mathcal{a}\,\underline{\hat{e}}_3$

This is fully consistent with the earlier quantization hypothesis for the 'pulsating' electron. The extension shown here was unaffected by the implicit inclusion of the additional 'clicking' motion that together define the digital electron model. This leads to *the fundamental quantum hypothesis of this EM theory* that constrains all successive interactions between any two electrons (of equal **or** opposite charge) such that the change in their one-electron difference-activity must take on the values defined by the universal action at their interaction locations. This is equivalent to assuming that the amount of unit activity <u>exchanged</u> during <u>each</u> interaction is <u>constant</u> (i.e. at a constant rate): this is the "**Quantum of Activity**" hypothesis (§4.3.6).

Hypothesis: **EM Quantum Hypothesis** $\Delta^*\mathbf{R}_n = -\mathcal{a}_n = -\Delta^*\mathbf{R}'_n \qquad \{\text{for all } S_n \le S_0 \;\; \text{or} \;\; n \le N_0\}$

The extended version of Newton's dynamical impulse law (§5.3.1) is: $\Delta\underline{\mathbf{I}}_{n+1} \to \Diamond\underline{\mathbf{p}}_n$ or as an equation: $\Delta\underline{\mathbf{I}}_{n+1} = m\,\Diamond\underline{\mathbf{v}}_n$

$$\therefore \quad m\,\underline{c}\bullet\Diamond\underline{\mathbf{v}}_n = \underline{c}\bullet(\Delta\underline{\mathbf{I}}_{Rn+1} + \Delta\underline{\mathbf{I}}_{Tn+1}) = \underline{c}\bullet\Delta\underline{\mathbf{I}}_{Rn+1} = c\,\Delta\mathbf{I}_{Rn+1} \qquad \therefore \quad m\,\mathcal{T}_n\,\underline{c}\bullet\Diamond\underline{\mathbf{v}}_n = \mathcal{a}_n \;\; \{\text{All n}\}$$

As $\Delta^*\mathbf{B}_n = i\,\mathbf{I_0}\,\Delta^*\mathcal{A}_{\mathbb{T}n} + c\,\Delta^*\underline{\mathcal{G}}_{\mathbb{T}n}\bullet\underline{\mathbf{I}} + i\,\Delta^*\underline{\mathcal{M}}_{\mathbb{T}n}\bullet\underline{\mathbf{I}} = -\mathcal{a}_n \quad \therefore \quad \Delta^*\mathcal{A}_{\mathbb{T}n} = -\mathcal{a}_n \quad \& \quad c\,\Delta^*\underline{\mathcal{G}}_{\mathbb{T}n} = -\underline{\mathcal{a}}_n \quad \& \quad \Delta^*\underline{\mathcal{M}}_{\mathbb{T}n} = 0$

It was shown in the previous section that: $\Delta^*\mathcal{A}_{\mathbb{T}n} = -T_n\,c\,\Delta\mathbf{I}_{Rn} \quad \& \quad \Delta^*\underline{\mathcal{G}}_{\mathbb{T}n} = -T_n\,\Delta\underline{\mathbf{I}}_n \quad \& \quad \Delta^*\underline{\mathcal{M}}_{\mathbb{T}n} = T_n\,\underline{c}_n\wedge\Delta\underline{\mathbf{I}}_{Tn}$

But these can only be consistent (as $\Delta\underline{\mathbf{I}}_{Tn}$ must be orthogonal to $\underline{c}$) if $\Delta\underline{\mathbf{I}}_{Tn} = 0$. Thus <u>the interaction must be purely **radial**</u>.

Digital Electron Dynamics & Planck's Constant

This section will now bring together all the diverse results of this new theory of the digital electon and show how these ideas relate to Planck's earlier proposal for the quantization of EM radiation, which was the revolutionary starting point for the transition from classical mechanics to quantum mechanics – the central focus of 20th Century theoretical physics. The new results in this section will be shown to be valid for the 'digital' model of the electron but will remain true for the simpler 'pulsating' model as the 'clicking' motion will not introduce any differences at this point. The focus will be on only one **interaction time interval**, for one of the pair of interacting electrons; a time interval that spans the time from 'just before' the $(n+1)^{th}$ interaction to 'just before' the next interaction, where 'just' is defined as much less than one chronon. Here it will be useful to introduce another difference operator, which extends the temporal range of the 'Star-Diff' operator from one chronon around a single interaction time (see §4.3.3) to ONE complete **interaction interval**. The impact operator $\boldsymbol{\delta}_0$ and the extended, interaction-difference operator $\boldsymbol{\Delta}_I$ (see §4.2.3) will be combined into a new difference operator, here called the *Interval Difference* operator $\boldsymbol{\Delta}_I^*$ (and referred to colloquially as the 'I-Star-Diff' operator); this will simplify many of the subsequent equations.

Definition: **Interval-Difference Operator** $\quad \boldsymbol{\Delta}_I^*[\mathbf{A}] \equiv \boldsymbol{\delta}_0[\boldsymbol{\Delta}_I[\mathbf{A}]]$

Equivalently: $\quad \boldsymbol{\Delta}_I^*[A[t_n]] = \{A[t_{n+1} - \delta t] - A[t_n - \delta t]\} - \{A^0[t_{n+1} - \delta t] - A^0[t_n - \delta t]\} \qquad \therefore \quad \boldsymbol{\Delta}_I^* A_n = \underline{\boldsymbol{\Delta}}_I^* A_n + \boldsymbol{\Delta}_\tau^* A_n$

This definition separates the *dynamical* impact (Δ_τ^*) from the *kinematical* result ($\underline{\Delta}_I^*$) of moving between two interactions. For example, when this new operator is applied to a single electron's longitudinal velocity around the $(n+1)^{th}$ interaction:

$$\boldsymbol{\Delta}_I^* \underline{u}_{n+1} = \underline{\boldsymbol{\Delta}}_I^* \underline{u}_{n+1} + \boldsymbol{\Delta}_\tau^* \underline{u}_{n+1} = \boldsymbol{\Delta}_\tau^* \underline{u}_{n+1} = \boldsymbol{\delta}_0[\underline{u}[t_{n+2} - \delta t] - \underline{u}[t_{n+1} - \delta t]] = \underline{u}^+_{n+1} - \underline{u}^-_{n+1} = \Diamond \underline{u}_n \quad \therefore \; \boldsymbol{\Delta}_\tau^* \underline{u}_{n+1} = \Diamond \underline{u}_n \text{ (consistent)}$$

This new I-Star-Diff operator will now be applied to the activity of a single electron from 'just before' the $(n+1)^{th}$ interaction to 'just before' the next interaction. This will also use the notion of 'super-differences' from A4.1 (i.e. $\Delta_k \zeta_\eta = \zeta_{\eta+k} - \zeta_\eta$) to analyze the differences in the digital electron model across several (k) chronons that were used to derive some of the results in §5.3.2 from the sub-section on *Digital Electron Invariants*. In particular, over <u>any</u> multiple of four chronons ($4N_\eta$), the differences in the digital electron's purely digital part ($\mathbf{R}^D_\eta$) are <u>always zero</u> whenever there are no interactions with other electrons. The digital electron then appears to behave just like a simple, 'free' Newtonian, point particle, so <u>for the digital part</u>, spanning k chronons without any interaction: $\boldsymbol{\Delta}_k[\mathbf{R}^D_\eta] = 0$ when k is modulo 4. This powerful constraint will be assumed here **when** consecutive interactions can occur for a digital electron. Thus, for any time span t_η to $t_{\eta+k}$, where the electron is **not** directly involved in any interactions and its longitudinal velocity is $\underline{u}_n$, then:

$$\boldsymbol{\Delta}_k[\mathbf{R}_\eta] = \boldsymbol{\Delta}_k[\mathbf{R}^N_\eta] = k\, \boldsymbol{\Delta}_\tau[\mathbf{R}^N_\eta] = i\, k\, m\, \tau\, \mathbf{U}_n^* \mathbf{U}_n = i\, \mathbf{I}_0\, k\, m\, \tau\, (c^2 - u_n^2) = i\, \mathbf{I_0}\, \boldsymbol{\Delta}_k[\mathcal{A}^N_\eta]$$

Now, if $t_\eta = t_{n+1} + \delta t$ and $t_{\eta+k} = t_{n+2} - \delta t$ so $k = 4N_{n+1}$ then there are no interactions in this time interval following the last interaction at t_{n+1} when the longitudinal velocity changed from $\underline{u}_n$ to $\underline{u}_{n+1}$; so over this time interval:

$$\boldsymbol{\Delta}_k[\mathbf{R}_{n+1}] = i\, \mathbf{I}_0\, k\, m\, \tau\, (c^2 - u_{n+1}^2) \qquad \text{where} \quad k\tau = \Delta_I[t_{n+1}] = \underline{\Delta}[t_{n+1}] = \Delta t_{n+1} = t_{n+2} - t_{n+1}$$

If there had been **no** interaction at t_{n+1} then this would be: $\boldsymbol{\Delta}_k[\mathbf{R}^0_{n+1}] = i\, \mathbf{I}_0\, k\, m\, \tau\, (c^2 - u_n^2)$ The effects are separable:

$$\boldsymbol{\Delta}_I[A[t_{n+1}]] = A[t_{n+2} - \delta t] - A[t_{n+1} - \delta t] = A[t_{n+2} - \delta t] - A[t_{n+1} + \delta t] + A[t_{n+1} + \delta t] - A[t_{n+1} - \delta t] = \underline{\boldsymbol{\Delta}}_k[A_{n+1}] + \Diamond[A_n]$$

$$\therefore \quad \boldsymbol{\Delta}_I^*[\mathbf{R}_{n+1}] = \boldsymbol{\delta}_0\, \boldsymbol{\Delta}_I[\mathbf{R}_{n+1}] = \boldsymbol{\delta}_0 \underline{\boldsymbol{\Delta}}_k[\mathbf{R}_{n+1}] + \boldsymbol{\delta}_0 \boldsymbol{\Delta}_\tau[\mathbf{R}_{n+1}] = \underline{\boldsymbol{\Delta}}_I[\mathbf{R}_{n+1}] + \boldsymbol{\Delta}_\tau^* \mathbf{R}_{n+1}$$

Now, $\quad \underline{\boldsymbol{\Delta}}_I[\mathbf{R}_{n+1}] = \underline{\boldsymbol{\Delta}}_k[\mathbf{R}_{n+1}] - \underline{\boldsymbol{\Delta}}_k[\mathbf{R}^0_{n+1}] = -i\, \mathbf{I}_0\, k\, \tau\, m\, (u_{n+1}^2 - u_n^2) = -2\, i\, \mathbf{I}_0\, \Delta t_{n+1}\, \Delta \mathcal{K}_n$ and $\boldsymbol{\Delta}_\tau^* \mathbf{R}_{n+1} = -\boldsymbol{a}_{n+1}$

Focusing on the scalar component (§4.3.7): $\qquad \boldsymbol{\Delta}_I^* \mathcal{A}_{n+1} = -2\, \Delta t_{n+1}\, \Delta \mathcal{K}_n - a = \underline{\boldsymbol{\Delta}}_I^* \mathcal{A}^N_{n+1} - h/2$

Remember, the Newtonian particle analysis earlier (§4.3.3) demonstrated that: $\quad \underline{\boldsymbol{\Delta}}_I^*[\mathcal{A}^N_{n+1}] = \delta_0\, \underline{\Delta}_I \mathcal{R}_{n+1} = -2\, \Delta \mathcal{K}_n\, \Delta t_{n+1}$
The quantity $\boldsymbol{\Delta}_I^*[\mathcal{A}_{n+1}]$ is the <u>total</u> change in the action of a single electron from the time just before the interaction at t_{n+1} until the time just before the interaction at t_{n+2}. Its definition separates the fixed *dynamical* impact (Δ_τ^*) from the variable,

kinematical result ($\underline{\Delta}_I^*$) of moving between two interactions, which is proportional to the duration of the interaction interval. The analysis of the Digital Electron Activity (§5.3.2) showed that the digital electron model was a direct extension of the Newtonian particle model, here labeled with an N superscript and with the digital extensions labeled with a D superscript. This separability obviously extends to the interaction-interval for the change in the electron's action:

$$\underline{\Delta}_I^*[\mathcal{A}_{n+1}] = \underline{\Delta}_I^*[\mathcal{A}^N{}_{n+1}] + \Delta_\tau^*[\mathcal{A}^D{}_{n+1}] \qquad \text{with} \quad \Delta_\tau^*[\mathcal{A}^D{}_{n+1}] = -h/2$$

In the digital electron model, the 'clicking' action remains the same between interactions whether the electron interacts or not; therefore, its change in interaction action across one interaction interval remains invariant; i.e. $\Delta_\tau^*[\mathcal{A}^S{}_{n+1}] = \frac{1}{2}\, a$

These two views of this dynamic change for the digital electron can be equated: $\Delta_\tau^*[\mathcal{A}^D{}_{n+1}] = -2\,\Delta_\tau^*[\mathcal{A}^S{}_{n+1}]$.

A symmetric hypothesis will now be considered, suggested by the Bohr atom, that the next interaction occurs between two electrons when the cumulative *kinematical* change in action becomes equal to the instantaneous *dynamical* change in action.

Hypothesis: **Quantization of Micro-Action** – the next interaction occurs when: $\underline{\Delta}_I^*\mathcal{A}_n = \Delta_\tau^*\mathcal{A}_n = -h$

$$\therefore \quad \underline{\Delta}_I^*\mathcal{A}_{n+1} = \underline{\Delta}_I^*\mathcal{A}^N{}_{n+1} = -2\,\Delta t_{n+1}\,\Delta\mathcal{K}_n = -2a = -h \qquad \therefore \quad \Delta\mathcal{K}_n\,\Delta t_{n+1} = h/2 \qquad \therefore \quad \Delta_I^*\mathcal{A}_{n+1} = -\tfrac{1}{2}\,h$$

In 1900, Max Planck hypothesized that the minimum amount of (EM radiation) 'difference' exchanged between two linear harmonic oscillators (assumed to represent the EM field) was one quantum of action, represented by the symbol *h* [116]. This minimum amount will be designated here by the symbol 'Δ_P' (with the 'P' referring to Planck). As harmonic oscillators can be represented by two contra-rotating circular motions, this is equivalent to a single electron changing its action during one interaction by $\frac{1}{2}\, h_D$, where h_D is Dirac's (circular) version of Planck's constant ($h/2\pi$). This was the critical, foundational step in the development of quantum theory. This insight later evolved into Heisenberg's indeterminancy principle, which in its minimal, temporal (discrete) formulation can be written as:

$$\Delta_P\mathcal{K}_n\,\Delta_P t_{n+1} = \tfrac{1}{2}\,h_D \qquad \text{If it is assumed that:} \quad \Delta_P\mathcal{K}_n = \Delta\mathcal{K}_n \quad \text{then:} \quad \Delta_P t_n = \Delta t_n / 2\pi$$

This indicates that a discrete, Planckian EM energy transfer (or Einsteinian 'photon') is equivalent to one micro-exchange. The alternative possibility is that the differences in times are equal in the two models, in which case:

$$\Delta_P\mathcal{K}_n\,\Delta_P t_{n+1} = \tfrac{1}{2}\,h_D \qquad \text{If it assumed that:} \quad \Delta_P t_n = \Delta t_n \quad \text{then:} \quad \Delta_P\mathcal{K}_n = \Delta\mathcal{K}_n / 2\pi$$

This would indicate that a single, Planckian EM unit of energy transfer (or single 'photon') transfers the equivalent of two micro-exchanges of kinetic energy ('photinos') in the digital electron model. The first choice seems more plausible as the energy transfers must be the same, implying that the time differences should be detectable as differences in 'line-widths'.

5.3.6 REPULSIVE MOTION

The 'Terrible Twins' (Encircled)

Once more, the simple two-electron model will provide valuable insights into the nature of the EM interaction: specifically, repulsive scattering of one high-speed electron by another with similar charge. The one-dimensional interaction model that was first introduced in the previous paper and referred to colloquially as the 'Terrible-Twins' [110] will prove sufficient at this time. This 1D model describes the 'head-on' collision of two equally charged electrons that initially are moving directly towards each other at maximum speed (light-speed c), eventually reaching a minimum separation and reversing their motion to end up moving away from each other at the same speed. This model is based on the central space-time integrity analysis rehearsed here in section 4.2.3; the reduction to one dimension (say 'z') is always valid in the SIRF, where the two sets of space and time co-ordinates of the two electrons are reflected in the origin. Initially, the transverse motion will be neglected because the pairwise interactions are always at long-range, at distances vastly larger than the size of the transverse displacements (one luxon). In the revised version described here, interaction nodes will be designated in the dashed-symmetric notation, rather than the asymmetric notation used before. This is simply a linear transformation, which illustrates the symmetry much more clearly. Thus, the interaction nodes of electron #1 in the rebound phase are numbered sequentially in ascending (normal) time order, beginning with '1' and ending with 'N' (the final interaction).

The interaction nodes of electron #2, on its inbound phase, are given the same labels but the variables for electron #2 are distinguished by a dash; for example, the n^{th} interaction occurs between electron #2 at time t'_n and involves electron #1 at the later time t_n. In the previous exposition, the labels for electron #2 were distinguished by descending negative integers n' that ranged from '–N' to '–1', where: $n' = N - n + 1$. The speed of electron #1 following the n^{th} interaction is denoted by u_n (no vector notation is needed here as all are in one dimension).

The Space-Time Integrity (STI) Condition now becomes simply: $(c - u_n)\,\Delta t_n = (c - u'_n)\,\Delta t'_n$ for all $\{n = 1, 2, \ldots, N\}$.

In this model, the total velocity is constant, so that: $u_n + u'_n = c$ resulting in the equivalent form: $(c - u_n)\,\Delta t_n = u_n\,\Delta t'_n$
The previous paper described two possible solutions of this model: one was linear in 'n' and the other varied inversely with ever increasing values of 'n'. This second solution generated time intervals involving the integer Harmonic function H_n, so these two alternatives were referred to as the 'linear' and 'harmonic' solutions. The harmonic solution will play a very important role in this theory but its deeper investigation will be deferred until a subsequent paper investigating the nucleus.

a) **Linear**

The simplest impulse model [130] preserves the extended form of Newton's Third Law of Motion across each sub-interaction (i.e. it conserves momentum) simply by assuming that every impulse has the same magnitude $\Delta\mathbf{I}_0$, no matter how far apart the electrons are separated; in other words, this is a universal impulse characteristic of all electrons at all times. This impulse parameter is just another restatement of the finite nature of the minimum and maximum electron speeds: '*b*' and '*c*'; where '*c*' is the *Interaction Constant* and the minimum speed '*b*' has been defined as the speed each electron first achieves after its change in direction at t = 0 when n = 1: $c = u_N$ and $b = u_1$ (also implying $u_0 = 0$).

Alternatively: $N_0 = c / b$ and $\Delta\mathbf{I}_0 = \Delta p_n = m\,\Delta u_n$ (for all n including n = 1) so: $\Delta\mathbf{I}_0 = m\,b = m\,c / N_0$

The speed increments Δu_n are all equal, so: $\Delta u_n = b$. This has the generic solution: $u_n = n\,b = c\,(n / N_0)$

Substituting into the one-electron STI condition produces the *Universal Temporal Interval* equation: $(N_0 - n)\,\Delta t_n = n\,\Delta t'_n$

For n = 1: $(N_0 - 1)\,\Delta t_1 = \Delta t'_1$ It is always assumed that: $N_0 >> 1$ so that Δt_1 is much smaller than $\Delta t'_1$ or $\Delta t_1 << \Delta t'_1$.

As Δt_1 is the smallest time interval, then this suggests its identification with the *chronon*, so it is assumed: $\Delta t_1 = \tau$.
The previous paper began its analysis with an *Ansatz* for the general time interval, namely: $\Delta t_n = n\,\tau$, which resulted in nodal solutions that were quadratic in 'n', like: $t_n = \tau/2 + n\,(n-1)\,\tau/2$. The choice was made to assign $t_0 = \tau/2$ in order to preserve the constraint that all interactions occurred at an integer multiple of the chronon τ and to minimize the zero speed duration at zero time from t'_N to t_1. This also has the advantage that the electron switches from its 'send' to 'receive' state as it crosses the origin.
Using the linear transform above and recognizing that a change in notation does not change the physics, then:

$$t'_n = -t_{n'} = -t_{N-n+1} = -\{(N-n)(N-n+1)+1\}\tau/2 \qquad \therefore\ \Delta t'_n = (N-n)\,\tau \text{ which satisfies the UTI.}$$

Thus, the repulsive **Linear** Model (in the symmetric notation) can be summarized as:

$$u_n = n\,b\ ;\ \Delta u_n = b\ ;\ t_n = n\,(n-1)\,\tau/2 + \tau/2\ ;\ \Delta t_n = n\,\tau\ ;\ t'_n = -(N-n)(N-n+1)\tau/2 - \tau/2\ ;\ \Delta t'_n = (N-n)\,\tau$$

The spatial locations are derived using the Sum-Diff. Identity and the Equal-Difference Set results (see Appendix IV):

$$z_n = z_0 + \sum_{k=0}^{k=n-1} \Delta z_k = d/2 + \sum_{k=0}^{k=n-1} u_k\,\Delta t_k = d/2 + b\,\tau \sum_{k=0}^{k=n-1} k^2 = d/2 + n\,(n-1)\,(2n-1)\,c\,\tau/6N$$

These have some special points of interest, such as the first and last interaction nodes:
Minimum: $t_1 = -t'_N = \tau/2$ & $z_1 = d/2$ Maximum: $t_N = -t'_1 = \tau/2 + N\,(N-1)\,\tau/2 \cong N^2\,\tau/2$ & $z_N \cong N^2\,c\tau/3$

In 'optical' terms, the temporal difference T_n is the 'path-time' where: $T_n = t_n - t'_n = \tau + N(N+1)\,\tau - n(N-n+1)\,\tau$

The maximum path-time occurs between the first and last interaction: $T_1 = T_N = \tau + N(N-1)\,\tau/2 = t_N$ ("Max Path").

This path-time is stationary when n = M where: $dT_n / dn = 0$ this occurs at: $M = (N+1)/2 \cong N/2$. ("Mid-point").

The minimum path-time occurs at this mid-point: $T_M = \tau + N(N+1)\,\tau/2 - (N+1)^2\,\tau/4 \cong N^2\,\tau/4$ ("Min Path").

Therefore, the max-path is almost exactly twice as long as the min-path. All these results depend only on the parameter N.

The previous paper also calculated the single electron kinetic energy, in the SIRF, after time t_n is: $\mathcal{K}_n = \tfrac{1}{2}\, m\, b^2\, n^2$.

The average kinetic energy 'around' the n^{th} interaction was found to be: $\langle \mathcal{K}_n \rangle = m\, b^2 \{ n(n-1) + \tfrac{1}{2} \}$

The change in the electron's kinetic energy, in this model, is: $\Delta\mathcal{K}_n = (n + \tfrac{1}{2})\, m\, b^2$ & $\Delta\mathcal{K}'_n = -(N - n + \tfrac{1}{2})\, m\, b^2$
These equations hint at the QM SHM results and since $b = c / N$ then all of these results depend only on the key parameter N.
NB Energy is 'lost' in the first half phase (n = 1 … M) but completely 'regained' in the second half phase (n = M+1 … N).

b) **Harmonic**

The previous paper also introduced the 'harmonic' solution for possible time intervals with the constant impulse model for electron scattering. In this solution, the largest time interval (Δt_{N-1}) became the 'free' parameter of this model. The analysis showed (summarized here) that this would not generate total 'path-times' anywhere near as compact as the linear model.

In order to simplify the algebra the final interval will be replaced with a single symbol, so: $\xi_0 \equiv \Delta t_{N-1}$ & $\Delta t_n = \xi_0 / (N - n)$

$$t_{n+1} = t_1 + \sum_{k=0}^{k=n} \Delta t_k = \mathcal{T}_0 + \xi_0 \sum_{k=1}^{k=n} 1/(N-k) = \mathcal{T}_0 + \xi_0 \left(H_{N-1} - H_{N-n-1} \right) \quad \text{where} \quad H_n = \sum_{k=1}^{k=n} 1/k$$

Here, H_n is the 'Harmonic' function or n^{th} harmonic number, where: $H_n = \gamma + \ln(n) + 1/2n - 1/12n^2 + O(1/n^3)$ with the Euler-Mascharoni constant $\gamma \approx 0.577\ldots$ N.B. H_n is not defined for n = 0 but can be extended to this value with: $H_0 = 0$.

Thus, for n = N – 1 (and N >> 1) then: $\mathcal{T} = t_N = \mathcal{T}_0 + \xi_0 H_{N-1} = t_1 + \xi_0 H_{N-1} \approx \xi_0 H_N$ or $\mathcal{T} \approx \tau\, N!\, \ln[N]$

This indicates that if all the interaction intervals must be integer multiples of τ then: $\mathcal{T}_H \approx \mathcal{T}_L\, N!\, \ln[N]$; so that the harmonic solution takes vastly longer for the electrons to reach their maximum speed *c*; minimization would then indicate that the linear solution is the one found in Nature when electrons are free to participate in multiple, consecutive interactions. However, the small number range (e.g. N = 6) has some interesting properties based on the observation that $H_5 = 137/60$.

c) **Transverse**

It was show above (§5.3.1) that when the times between each interaction are a multiple of four chronons then the interaction vector ($\mathbf{c}_n$) remains unchanged in direction. This is simply the idea that each impulse pair remain parallel in 3D space over all of the pairs of interaction points. This is sufficient to keep all interaction pairs on their mutual 'light-cones' – in other words, the space-time integrity constraint is respected at all such successive points. In the present long-range (or linear) 1D model, this is easily accomplished by changing the chronon parameter from τ to 4τ as all the results followed directly from $\Delta t_1 = \tau$.
The 3D STI condition remains: $(\underline{c}_n - \underline{v}_n)\,\Delta t_n = (\underline{c}'_n - \underline{v}'_n)\,\Delta t'_n$ with $\underline{v}_n = \underline{u}_n + \underline{w}_n$ for all n interactions.

On electron #1's outbound phase: $\underline{c}_n = \underline{c}'_n = c\,\hat{\underline{e}}_3$; $\underline{u}_n = u_n\,\hat{\underline{e}}_3$ & $\underline{u}'_n = u'_n\,\hat{\underline{e}}_3$; $\underline{w}_n = \underline{w}'_n = \pm c\,(\hat{\underline{e}}_1$ or $\hat{\underline{e}}_2)$

This means that the interacting linear 1D solution has a solution where both electrons rotate around their rube, aligned along the common z-axis, remaining parallel since both electrons are interacting while they are traveling in the same direction.

5.3.7 MINIMIZATION PRINCIPLES

Historically, there have been several 'minimization' principles suggested that have been used to provide a global foundation for explaining physical phenomena. As described earlier in section 4.2.4, Maupertuis realized that by defining the quantity he called 'action' he could formulate a principle of 'Least-Action' that could then be used to establish Newtonian mechanics for a single particle. Pierre de Fermat (1601-1665) first formulated the 'Principle of Least Time' in 1662 to predict the path that light would take through an optical system. It was based on the idea that the path taken by a 'ray of light' traveling between any two points is the one that minimizes the total travel time between them. Since the present theory views light as a theory ultimately grounded in the EM interaction between two point electrons and not as a travelling entity there arises the need to determine which is the broader principle: both will be used here in this research programme. This is because in closed systems that ultimately conserve total energy minimal action implies minimal time to complete the interaction.

Symmetric Time Optimization

Any time symmetric action principle (discrete or continuous) optimizes across all of time – both past and future around the arbitrary time of 'now'. This combines both historical and teleological causality – so that "time returns to center stage and 'now' takes on the starring role" as it does throughout our personal lives.

5.3.8 MINIMIZING TWO-PARTICLE INTERACTIVITY

The present theory returns to Sommerfeld's suggestion of quantized action (§4.3.6) with the constraint that the interaction is limited to a pair of electrons over finite time durations, here related to a new universal constant, the *chronon* of time τ. These constraints cannot be satisfied by either a continuous interaction-model between point particles or by an interaction between one electron and variable sets of absorbers (the field model). Since the magnitude of the action is preserved across the pair-wise interaction then any minimization principle for interactions must be a *time minimization* of N consecutive interactions occurring between a set of electrons in the least time.

Minimal Time Difference

If two electrons can completely interact such that they go through all possible interactions together in reversing their maximal relative velocities (i.e. N_0 interactions) then both their mutual relative velocity and the interaction time difference $(t_n - t'_n)$ take on the smallest values at the 'mid-point' of the total set (i.e. when $M = N_0 / 2$) of the possible N_0 interaction pairs. In the 'Terrible-Twins' analysis (see §5.3.6), this mid-point also corresponded to the smallest difference b in the velocities of the two electrons at the times of their interaction. Thus, for 'far' electrons, such as the 1D electron-scattering model, the relative velocity of the two interacting electrons is smallest when both electrons have interacted about $N_0 / 2$ times, that is, around the 'mid-point'. The linear model can be **extended** to the digital electron model by redefining the electron's interaction interval. This is demonstrated next when the interaction interval Δt_n is defined in terms of a parameter B that is to be determined.

$$u_n = n\,b = n\,c / N_0 \;;\; \Delta u_n = b \;;\; \Delta\mathcal{K}_n = (n + \tfrac{1}{2})\,m\,b^2 \;;\; \Delta t_n = 4B\,n\,\tau \;;\; \therefore\; t_n = 2\{n\,(n-1)\,B + 1\}\tau \;\;\&\;\; M = N_0 / 2$$

$$\therefore\; \Delta\mathcal{A}^K_M = h / 2 = 2\,\Delta t_M\,\Delta\mathcal{K}_M \cong 8\,B\,M^2\,m\,b^2\,\tau = 2\,B\,m\,c^2\,\tau = 2\,B\,e^2 / c = 2\,B\,\alpha\,h_D \;\therefore\; B = \pi/2\alpha \;\therefore\; \Delta t_n = 2(\pi/\alpha)\,n\,\tau$$

Thus, the linear model requires for consistency that each interaction interval increase by $(2\pi/\alpha)\tau$ instead of τ as was assumed.

$$\therefore\; n = M + k \;\;\&\;\; n' = N_0 + 1 - n = M - (k-1) \;\therefore\; t'_n = -t_{n'} = -\{(M-k)\,(M-k+1) + 2\}\tau^* \;\therefore\; \Delta t'_n = (N_0 - n)\,(2\pi/\alpha)\tau \;\therefore\; \text{S T I}$$

$$\therefore\; u_n = c/2 + k\,b \;\;\&\;\; u'_n = n'\,b = c/2 - (k-1)\,b \;\;\therefore\; u_n - u'_n = (2k-1)\,b \;\;\therefore\; T_n = t_n - t'_n \cong \{N_0^2 + 4k(k-1)\}\,(\pi/\alpha)\,\tau$$

The interval T_n is ***minimized*** at $T_0 = N_0^2\,(\pi/\alpha)\,\tau$ when k = 0 or k = 1; equivalent to the two (symmetric mid-point) solutions:

$$\{n = N_0 / 2 \;\;\&\;\; n' = n + 1\} \quad \text{or} \quad \{n' = N_0 / 2 \;\;\&\;\; n = n' + 1\}$$

The interval T_n is ***maximized*** at $T_* = 4N_0^2\,(\pi/\alpha)\tau$ when $k = N_0 / 2$; this equivalent to the end-point solution: $n = N_0$ & $n' = 1$.

This demonstrates that the linear model satisfies the action and time minimization principles.

6. POSITIVE ELECTRONS

This section is the **focal** point of the present paper as it demonstrates that the key features of the 'digital electron' model can be used to provide a ***physical*** explanation of the phenomenon known as positive electricity. In particular, it develops both a physical and mathematical description of the fundamental particle known as the **positron** – the positively charged electron. Contrary to the standard view, where the positron is viewed as rare, unstable and ephemeral, the positron in this theory plays a major role, one of **equal** importance to that of its negative 'twin' or complement. This new model combines ideas of space and time to explain why positive electrons are as widespread (but hidden) as negative electrons and just as fundamental.

A Need for Structure

Repulsive interactions result in the interacting parts moving further apart over time - this prevents the appearance of stable structures appearing in nature, such as planets, microbes, people or even atoms. Since these examples actually occur (along with a myriad other complex structures), it is necessary that there exist attractive interactions between the constituents of matter. Electrons are found to always repulse each other so there must be other examples of matter that can attract electrons. Physicists first assigned this role to the proton, with an equal but opposite charge to that of the electron. Indeed, early atomic theory found that these two types of matter were sufficient to construct reasonable models of all the atoms in the Periodic Table. However, the inertial mass of the proton is almost 2000 times greater than the electron – an asymmetry that cannot be fundamental. This theory will propose models for ALL the elementary particles, including the proton, constructed only from positive and negatively charged point electrons, moving around each other in periodic but stable trajectories.

6.1 ELECTRIC CHARGE

Before introducing the idea of positively charged electrons, it is useful to review some of the unresolved problems found with the idea of positive electric charge in nature. Familiarity has hidden the deep mysteries underlying these phenomena.

6.1.1 ELECTRIC MYSTERIES

The Mysteries of Charge

There are two universal questions about electric charge still facing modern physics that are answered by this theory:

1) Why is the electric charge on all particles **equal** to the electron's charge, when the masses are so varied?

2) Why is the electric charge on all positive particles **exactly** opposite to the charge of the negative electron?

Unit Charge Anomalies

There has been an asymmetric problem with anti-particles, since the concept was first invented. Standard physics has no explanation for the preponderance of negative electrons in the observable universe or for the mass asymmetry of the most stable particles with unit positive and negative electric charges (protons and electrons). This theory answers these questions.

6.1.2 QUANTIZED CHARGE

Continuous Electricity

The phenomenon of electricity was first observed by the ancient Greeks. In 1601, the English philosopher William Gilbert (1544-1603) revived interest in this effect in his book *De Magnete*, inventing the term *electricus*, which soon gave rise to the English words "electrical" and "electricity." Unlike his contemporaries, he rejected the old Aristotelian approach to physics, viewing this effect as due to a special, continuous type of matter. Charles du Fay (1698-1739), a French chemist proposed in 1733 that electricity came in two varieties, which cancelled each other, expressing this in terms of a two-fluid theory. One of the leading experts on electricity in the 18th century was the polymath Benjamin Franklin (1706-1790), who argued in favour of a rival one-fluid theory of electricity. Franklin imagined electricity as being a type of invisible fluid present in all matter and that a flow of this fluid constituted an electric current. Unfortunately, from the mathematical view, he labeled the type of electricity that eventually was identified with electrons as 'negative' and its complement as 'positive'. In 1839, the great English experimentalist (and autodidact) Michael Faraday (1791-1867) showed that the apparent division between static electricity, current electricity and bio-electricity was incorrect as all of these phenomena were a consequence of the behavior of a single kind of electricity appearing in opposite polarities.

6.2 POSITRONS

Universal Electric Charge

The discrete nature of electric charge was first proposed by Michael Faraday in his electrolysis experiments but this was not directly demonstrated until 1913 by Robert Millikan (1868-1953) in his famous (Nobel prize-winning) oil-drop experiments. Millikan's measurements were within one percent of today's consensus value of 1.602×10^{-19} coulombs. This is equivalent to the view that a current of one ampere flowing for one second (one Coulomb of electricity) consists of about 6.25×10^{18} electrons. One of the ironies of science is that this most fundamental unit of electricity, the charge on each and every electron has no name; it is simply referred to as *the fundamental unit of charge* or simply by its universal symbol *e*.

This electric charge is a universal, identifying characteristic of all fundamental particles. Every type of particle (say, 'a') carries its own unique electric charge, $Q_a = \lambda_a N_a e$, where $\lambda_a = \pm 1$ and $N_a = 0, 1, 2,$ etc. For example, the electron's charge $Q_e = -e$, the proton's charge $Q_P = +e$ while the neutron's charge is zero. The fact that N_a is always an integer is known as the quantization of charge; it is the fundamental conserved property of all interactions involving elementary particles. The widely accepted 'Standard Theory of Elementary Particles' has proposed a new class of basic particles, called 'quarks', with fractional charges $\frac{1}{3}$ or $\frac{2}{3}$. However, these hypothetical objects have never been observed and form no part of the present theory of matter. Elementary charged particles are found in complementary pairs, often with one form found to vastly predominate in nature, such as the electron. The ephemeral complement (or anti-particle) of the electron is known as the *positron*. This theory reserves a primarily **nuclear r**ole for positrons – this will be reported on extensively in later papers.

6.2.1 A NEW MODEL OF THE POSITRON

The present theory views positive and negative electrons as two complementary manifestations of the same type of entity, just phase-shifted from one another in the timing of their interactions due to their complementary trajectories through space. This is to be contrasted with all the views of electricity from Charles Dufay's two-fluid model in the 18th Century to today's duplex entities of stable negative electrons and ephemeral positive electrons. Unfortunately, a study of the history of science demonstrates that physics has too often shown a tendency to invent a new, fundamental entity every time it is confronted with a novel feature of matter: a hangover from ancient Greek philosophy centered on finding the basic '*substances*' of the world.

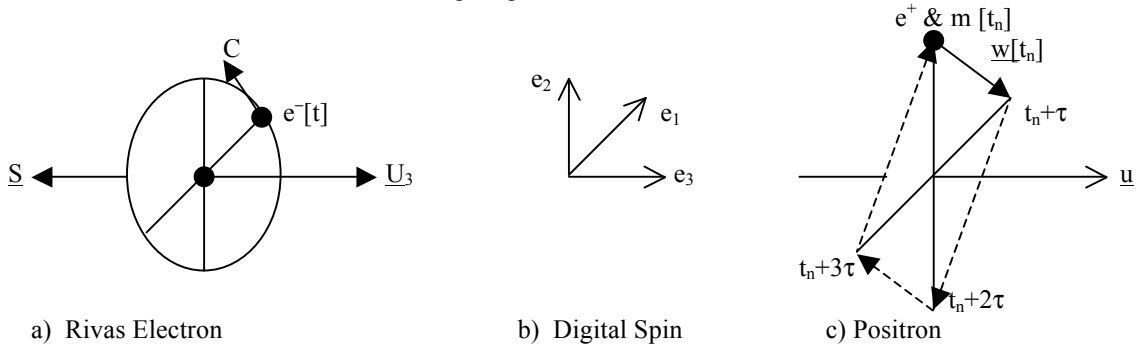
Complementary Rotations

This model is a consequence of the fact that rotation in 3D space around an axis is ambiguous: the motion can be clockwise or anti-clockwise. This simple idea is combined with the four part mathematical representation (digital electron states) earlier introduced in section 5.1 to show how similar electrons repel each other while two complementary electrons (negative and positive) will then attract each other when they participate in a common interaction. This phase-shift is also a clue to how the two complementary forms of the electron differ in their motions through space.

The earlier section on quantized rotation introduced the 'Clicking Hypothesis' (see §5.2.2). It extended Newton's First Law of Motion by proposing that the electron is always moving in the transverse plane around its average (longitudinal) motion in a four-step cyclic motion (the 'Twist'), which is always at light-speed and always orthogonal to its current longitudinal direction through space. This longitudinal motion is 'absolute' relative to all the other electrons in the universe at any instant of time. This absolute 'relative' motion has nothing to do with arbitrary, mathematical representational schemes based on so-called 'reference frames'. Each 'step' in each electron's digital or 'clicking' motion, is in a direct straight-line from one possible interaction time to the next; in other words, separated in time by one chronon. This proposal is ambiguous; there are two possible ways to rotate around a given axis – clockwise or anti-clockwise relative to its own (recent) past motion.

Experiments involving nuclear interactions, such as radioactive decay, have determined that it is is necessary to choose the left-hand twist for negatively charged electrons. The present theory follows Rivas's suggestion that the electron's chirality (or twisting direction) be defined as anti-parallel to the electron's motion, so that the motion of the electron is viewed as counter-clockwise when looked at it along the spin-direction from the center-of-mass viewpoint. Since the present Digital-Spin model is defined only in terms of absolute-relative velocities, the same result can be achieved by defining the direction of the 'twist' as LEFT-HANDED (or anti-clockwise) with respect to the local FORWARD motion of the negative electron as time progresses in its standard past-to-future direction (increasing chronon count). The 'empty' rotational direction (clockwise) is now assigned to the electron's complementary particle – the positron; this is referred to as the spatial or rotational component of the Positron Model.

The positron rotation can be seen in the following diagram.



a) Rivas Electron b) Digital Spin c) Positron

Fig. 14 Positron Chirality

The Positron Cycle

In order to fully exploit the **C**harge-**P**arity (reflection)-**T**ime reversal (CPT) symmetry view of the positron it is necessary to pick a point on the electron's trajectory that is half-way between two possible interaction times (i.e. mid-chronon). Spatial reflection can then be invoked around this point to view the positron as moving in the opposite direction through space. This implies that the interaction cycle for the positron is set one quarter cycle 'off' from that of the electron. So, assuming all negative electrons may issue a possible 'send' request whenever their clock-time is exactly $2n\tau$ then all positrons issue their 'send' request at $(2n + ½)\tau$, both for all integer values of n.

Center-of-Mass Reference Frame

It will also prove helpful to view the world from the perspective of the 'target' positron. When this specific positron is not interacting with any other particle, its longitudinal velocity remains constant (relative to 'distant' reference points). The origin of this reference frame can be either co-located with the positron itself or 'left-behind' somewhere on its longitudinal axis, designated $\underline{\hat{e}}_3$. The transverse axes (designated $\underline{\hat{e}}_1$ and $\underline{\hat{e}}_2$) may be fixed or they may rotate with the transverse motion of the positron. In this CM frame, the standard planar conventions will be used where the $\underline{\hat{e}}_1$ axis directed to the right and the $\underline{\hat{e}}_2$ directed vertically upwards so that $\underline{\hat{e}}_3$ is directed outwards (to the reader), using a right-handed co-ordinate convention. As with the electron, it is always possible to define either a square or diagonal orientation relative to the transverse axes. Since it is often necessary to display both electrons and positrons on the same diagram and it is useful to display the half-chronon 'offset' for the positron's phase, it is more useful to select the 'diagonal' orientation as the **canonical** representation for the positron and the 'square' orientation as the alternative view. These are covered in the following tables.

Time	X_1	X_2
$t_n + \tau/2$	$-\Lambda/2$	$-\Lambda/2$
$t_n + \tau + \tau/2$	$+\Lambda/2$	$-\Lambda/2$
$t_n + 2\tau + \tau/2$	$+\Lambda/2$	$+\Lambda/2$
$t_n + 3\tau + \tau/2$	$-\Lambda/2$	$+\Lambda/2$

Table 9. Positron Square Orientation

Time	X_1	X_2
$t_n + \tau/2$	$+\Lambda\sqrt{2}$	0
$t_n + \tau + \tau/2$	0	$+\Lambda\sqrt{2}$
$t_n + 2\tau + \tau/2$	$-\Lambda\sqrt{2}$	0
$t_n + 3\tau + \tau/2$	0	$-\Lambda\sqrt{2}$

Table 10. Positron Canonical Orientation

Electron & Positron Phase States

Like Dirac's dual-spinor electron theory, the present theory posits that every electron can exist in one of four possible states but these states are very different from those imagined by Dirac, where two of the states corresponded to positively charged anti-particles. Section 4.1.6 introduced the idea of phase states to characterize how and when a normal electron may interact with another electron; each state being occupied for a duration of one chronon, interactions may only occur at those instants when the electron is changing its phase. An electron can be either in the 'send' or 'receive' state but it was found that there were two possible 'send' or 'receive' states with the chosen (4x1) abstract vector representation. This difficult mathematical ambiguity (see §2.2.5) can be removed physically by associating one pair of 'snd' or 'rcv' states with an interaction forward (retarded) or backwards (advanced) across time; the standard interaction states were denoted with a negative (for retarded) superscript: S^- and R^-.

The complementary pair of advanced states are associated with the 'back in time' interaction and denoted with a positive subscript: S^+ and R^+. These ideas will now be extended here to include positively charged particles.

The four phase states of an electron are identified by the phase variable 'ν', which corresponds to specific combinations of the binary 'snd' or 'rcv' variable 'λ' and the binary interaction temporal direction variable 'σ' ; in other words: $\nu \equiv \lambda \otimes \sigma$. The canonical phase representation was introduced in section 5.1.4 for assigning specific values of ν (in the range 1, 2, 3, 4) to a unique sequence of λ and σ values (each usually represented as ±1) but sometimes explicitly as (↑, ↓) and (ret, adv) respectively. In the present theory, both electrons and positrons cycle forever, always in phase, through the same four states but in two different sequences. The convention is adopted that at time zero the electron is in the first phase. This theory will use three equivalent notations to represent the phase states of electrons and positrons: the first will use the standard notation for these particles ($e^{\pm}$) with a subscript α to distinguish the two particles ('1' and '2') involved in an interaction and a phase state variable ν. The second notation will extend the 'ket-style' vector-like notation introduced in (§4.1.6) to include the particle's charge Q (± *e*); the third representation will retain the interaction variables: 'S' for 'snd' and 'R' for 'rcv'.
Thus, $e^{Q}{}_{\alpha}(t_\alpha ; \nu) \approx | \alpha : t_\alpha :: \lambda , \sigma , Q >$ with: $| \alpha : t_\alpha :: \uparrow , \sigma , Q > \approx S^{\sigma}{}_{\alpha}(t ; Q)$ & $| \alpha : t_\alpha :: \downarrow , \sigma , Q > \approx R^{\sigma}{}_{\alpha}(t ; Q)$

All electrons here have their initial phase (ν = 1) occurring at times: $T^{-}{}_{\alpha} = 4\, N_\alpha\, \tau$, with consecutive times for {ν = 2, 3, 4} at times $\{ (T^{-}{}_{\alpha} + \tau), (T^{-}{}_{\alpha} + 2\tau), (T^{-}{}_{\alpha} + 3\tau) \}$. In order to accommodate the observed facts of electrical attraction or repulsion, it is necessary to propose that ***all the positrons are out-of-phase by one half chronon with all the electrons*** in the universe. Thus, the positrons initial phase here begins at times: $T^{+}{}_{\alpha} = (4\, N_\alpha + ½)\tau$, with consecutive times for { ν = 2, 3, 4 } at times $\{ (T^{+}{}_{\alpha} + \tau), (T^{+}{}_{\alpha} + 2\tau), (T^{+}{}_{\alpha} + 3\tau) \}$. The electron was assigned the canonical sequence (1, 2, 3, 4) while the positron is now assigned the complementary (or reversed) sequence (4, 3, 2, 1). The following table will be useful in later discussions.

Time t_n	1	2	3	4	5	6	7	8	9
e^- Phase	1	2	3	4	1	2	3	4	1
e^- State	S^-	R^+	S^+	R^-	S^-	R^+	S^+	R^-	S^-
e^+ State	S^-	R^-	S^+	R^+	S^-	R^-	S^+	R^+	S^-
e^+ Phase	1	4	3	2	1	4	3	2	1

Table 11. Electron & Positron Phase States

This table illustrates consequence of the 'reverse' sequence choice for the positron: by assigning the first phase to retarded 'snd' states of both the electron and the positron then both electron classes also share the advanced 'snd' states in the third position. This is the reason for adding a quarter-cycle (or half chronon) difference to the starting time of the positron phase in order to distinguish these two possibilities. As will be shown shortly, this allocation will have a very significant impact.
These temporal activities may be visualized on a 24 hour cycle around a standard, non-digital, 12 hour clock face. Each and every (negative) electron (e^-) may 'send' an impulse at 12 o'clock and receive an impulse at 6 o'clock. The 'snd' activity at 00:00 (or 24:00) is forward in time while the negative electron's 'snd' activity at 12:00 is backwards in time. Similarly, each negative electron may 'receive' an impulse from the past at 18:00 or it may 'receive' an impulse from the future at 06:00. The half-chronon offset for positive electrons means that every positron (e^+) may 'send' an impulse at 3 o'clock and receive an impulse at 9 o'clock. The 'snd' activity at 03:00 is forward in time while the positive electron's 'snd' activity at 15:00 is backwards in time. Similarly, each positive electron may 'receive' an impulse from the past at 09:00 or it may 'receive' an impulse from the future at 21:00. A collection of electrons and positrons will therefore always have activity occurring throughout the 24 hour 'day' at times 00:00, 03:00, 06:00, 09:00, 12:00, 15:00, 18:00, 21:00 and repeating again at 24:00.
This 'clock' imagery also illustrates the 'double-cycle' (or 4π) characteristic that is the 'weird' signature of spinors (§3.1.4).
The following figure illustrates the canonical (transverse) display of both an electron and positron at two different times.

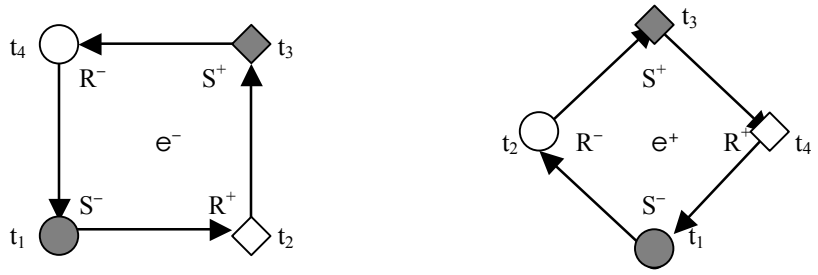


Fig. 15 Distinct e^+ / e^- Rotations

The next figure illustrates the non-canonical display of an electron and positron covering the same two locations in space.

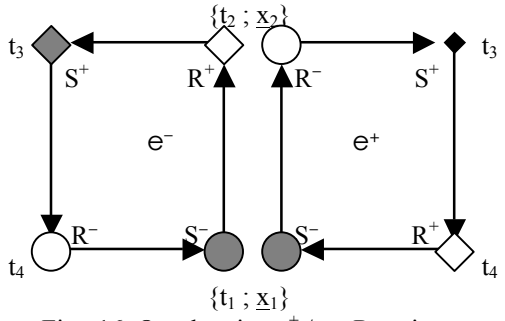


Fig. 16 Overlapping e^+ / e^- Rotations

This latter diagram also illustrates nature's solution to having two similar particles (but with negative and positive charges) moving equally through space with the same velocity (in this case, *c*) from locations $\underline{x}_1$ to $\underline{x}_2$. If both of these particles were to arrive at the same point in space ($\underline{x}_1$) at the same time then only one of these electrical particles (say, the electron) will be at its 'send' time so the other (the positron) is not yet ready to 'send', due to the critical half-chronon temporal phase offset. The half-chronon phase shift also means that if both particles are at the same point in space while entering the same part of their phase cycle (say, 'retarded-send' or S^-) then they must arrive there at different times.

Each interaction occurs between two particles: either two electrons, two positrons or one electron and one positron; they will be symbolized as $(P_1 * P_2)$; for example: $(e_1^- * e_2^+)$, since experiments indicate this is based on the product of their charges. The fundamental physical hypothesis of the present theory is that interactions always involve the 'sending' of an interaction from one particle at one time (say, t_1) to the 'receiving' at the other particle at a different time (say, t_2). Thus, there can be both retarded and advanced interactions $\Delta\underline{\mathbf{I}}^{\pm}$ (see §4.1.6-7).

Retarded Interaction: $\Delta\underline{\mathbf{I}}^-[1: t_1; 2: t_2] \cong \mathcal{S}^-[1: t_1] \,\&\, \mathcal{R}^-[2: t_2]$ with $t_2 > t_1$

Advanced Interaction: $\Delta\underline{\mathbf{I}}^+[1: t_1; 2: t_2] \cong \mathcal{S}^+[1: t_1] \,\&\, \mathcal{R}^+[2: t_2]$ with $t_2 < t_1$

where: $\Delta\mathbf{I}_n^{\sigma} = -\lambda \Diamond[\, \underline{P}[1: t_n^-; \underline{x}_n, \uparrow, \sigma]\,] = \lambda \Diamond[\, \underline{P}[2: t'^-_n; \underline{x}'_n, \uparrow, -\sigma]\,]$

The different types of interactions are summarized next (remembering that the 'adv' interaction occurs backwards in time).

A. $(e_1^- * e_2^-)$ ret: $\mathcal{S}^-[1: t_1 ; e^-] \,\&\, \mathcal{R}^-[2: t_4 ; e^-]$ adv: $\mathcal{S}^+[1: t_3 ; e^-] \,\&\, \mathcal{R}^+[2: t_2 ; e^-]$

B. $(e_1^+ * e_2^-)$ ret: $\mathcal{S}^-[1: t_1 ; e^+] \,\&\, \mathcal{R}^-[2: t_4 ; e^-]$ adv: $\mathcal{S}^+[1: t_3 ; e^+] \,\&\, \mathcal{R}^+[2: t_2 ; e^-]$

C. $(e_1^- * e_2^+)$ ret: $\mathcal{S}^-[1: t_1 ; e^-] \,\&\, \mathcal{R}^-[2: t_2 ; e^+]$ adv: $\mathcal{S}^+[1: t_7 ; e^-] \,\&\, \mathcal{R}^+[2: t_4 ; e^+]$

D. $(e_1^+ * e_2^+)$ ret: $\mathcal{S}^-[1: t_1 ; e^+] \,\&\, \mathcal{R}^-[2: t_2 ; e^+]$ adv: $\mathcal{S}^+[1: t_7 ; e^+] \,\&\, \mathcal{R}^+[2: t_4 ; e^+]$

Without the extra half-chronon offset, these four groups would separate only into two duplicate sets for the time differences needed to complete the interactions. The first set (A and B) need three chronons (modulo 4) to complete the retarded interaction, while the advanced only needs a single chronon difference (modulo 4). The opposite requirements are needed for the second set (C and D). In all cases, alternating interactions (see §4.1.9), involving sets of retarded and advanced interactions all need one chronon difference and a three chronon difference, whether viewed forward or backwards in time. This key, basic difference leads to the idea of ortho and para interactions between charges or parallel and anti-parallel 'spins'. These time differences can be used to produce a unified scheme for determining the 'sense' of the interaction (that is, either attraction or repulsion) between any pair of electrons (positive or negative charge) **based purely on their time differences**, which are always non-zero as their interaction must involve a 'send' from one electron to a 'receive' at another electron. This is achieved by assigning a '*charge phase factor*' $\xi[\alpha : t_\alpha]$ to each electron (labeled 'α') at each interaction time t_α. This factor is required to be both cyclic and unitary over the basic time duration of one chronon, so the complex exponential is a natural candidate for this universal factor as these factors are always introduced in a multiplicative manner.

Definition: **Charge Phase Factor** $\xi[\alpha : t_\alpha] \equiv \exp(-i\,2\pi\,t_\alpha / \tau)$

The 'send' activity uses this factor directly while the 'receive' activity uses its complex conjugate (to achieve a difference).

$$\mathcal{S}^\sigma[\alpha: t_\alpha ; Q] \Rightarrow \xi[\alpha : t_\alpha] \quad \text{and} \quad \mathcal{R}^\sigma[\alpha: t_\alpha ; Q] \Rightarrow \xi^*[\alpha : t_\alpha]$$

The sign of the impulse **received** $\Delta\underline{\mathbf{I}}$ at the target electron (say, #1) at time t_n that is sent by electron #2 at time t'_n is simply proportional to the product of these phase factors (where the direction is defined by their separation $\underline{\hat{e}}_n$: from #2 to #1).

$$\Delta\underline{\mathbf{I}}^\sigma[1 : t_n] = \Delta\underline{\mathbf{I}}^\sigma\,\xi[1 ; t_n] * \xi^*[2 ; t'_n] = \Delta\mathbf{I}^\sigma[1 : t_n - t'_n]\,\underline{\hat{e}}_n \quad \therefore \quad \Delta\mathbf{I}^\sigma[1 : t_n] = \Delta\mathbf{I}_n \exp(i\,2\pi\,(t_n - t'_n)/\tau)$$

Thus, for two negative electrons with the prior starting phase of electron #1 occurring at T^-_1 and the prior starting phase for electron #2 occurring at T^-_2, then for a retarded $(e_2^- * e_1^-)$ interaction, where $t_n > t'_n$:

A. $T^-_1 = 4N_1\,\tau$ & $T^-_2 = 4N_2\,\tau$ so $t_n = T^-_1 + 3\tau$ & $t'_n = T^-_2$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau + 3\tau$

Since both N_1 and N_2 are integers, while $\Delta\mathbf{I}_n$ is assumed positive, then: $\Delta\mathbf{I}^-[1 : t_n ; e_2^- * e_1^-] = \Delta\mathbf{I}_n > 0$ $\therefore$ Repulsion

When $t_n < t'_n$ then this interaction is advanced, so: $t_n = T^-_1 + \tau$ & $t'_n = T^-_2 + 2\tau$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau - \tau$

So, once again: $\Delta\mathbf{I}^+[1 : t_n ; e_2^- * e_1^-] = \Delta\mathbf{I}_n > 0$ $\therefore$ **Repulsion**. Thus, the impulse direction retains the interaction sense.

Similarly, for two positive electrons with electron #2 'sending' the impulse to electron #1 which 'receives' this impulse. In the retarded $(e_2^+ * e_1^+)$ interaction, where $t_n > t'_n$:

D. $T^+_1 = (4N_1 + ½)\tau$ & $T^+_2 = (4N_2 + ½)\tau$ so $t_n = T^+_1 + \tau$ & $t'_n = T^+_2$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau + \tau$

Again both N_1 and N_2 are integers, while $\Delta\mathbf{I}_n$ is always positive, then: $\Delta\mathbf{I}^-[1 : t_n ; e_2^+ * e_1^+] = \Delta\mathbf{I}_n > 0$ $\therefore$ Repulsion

When $t_n < t'_n$ then this interaction is advanced, so: $t_n = T^+_1 + 3\tau$ & $t'_n = T^+_2 + 6\tau$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau - 3\tau$

So, once again: $\Delta\mathbf{I}^+[1 : t_n ; e_2^+ * e_1^+] = \Delta\mathbf{I}_n > 0$ $\therefore$ **Repulsion**. Still, the impulse direction retains the interaction sense.

But now, if a negative electron 'sends' its (retarded) impulse to a later 'receiving' positive electron or $(e_2^- * e_1^+)$, then:

C. $T^-_1 = 4N_1\,\tau$ & $T^+_2 = (4N_2 + ½)\tau$ so $t_n = T^-_1 + \tau$ & $t'_n = T^+_2$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau + \tau/2$

Most importantly: $\exp(i\,2\pi\,(t_n - t'_n)/\tau) = \exp(i\pi) = -1$ $\therefore$ $\Delta\mathbf{I}^-[1 : t_n ; e_2^- * e_1^+] = -\Delta\mathbf{I}_n < 0$ $\therefore$ Attraction

In the case of the advanced version: $t_n = T^-_1 + 3\tau$ & $t'_n = T^+_2 + 6\tau$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau - 3\tau - \tau/2$

So, once again: $\Delta\mathbf{I}^+[1 : t_n ; e_2^- * e_1^+] = -\Delta\mathbf{I}_n < 0$ $\therefore$ **Attraction**. The impulse direction retains the interaction sense.

Finally, if a positive electron 'sends' its (retarded) impulse to a later 'receiving' negative electron or $(e_2^+ * e_1^-)$, then:

B. $t_n = T^-_1 + 3\tau$ & $t'_n = T^+_2$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau + 3\tau - \tau/2$ $\therefore$ $\Delta\mathbf{I}^-[1 : t_n ; e_2^+ * e_1^-] = -\Delta\mathbf{I}_n < 0$ $\therefore$ Attraction

In this case of the advanced version: $t_n = T^-_1 + \tau$ & $t'_n = T^+_2 + 2\tau$ $\therefore$ $t_n - t'_n = 4(N_1 - N_2)\,\tau - \tau - \tau/2$

So, once again: $\Delta\mathbf{I}^+[1 : t_n ; e_2^+ * e_1^-] = -\Delta\mathbf{I}_n < 0$ $\therefore$ **Attraction**. The impulse direction retains the interaction sense. Thus, this mathematical scheme represents the fact that like charges repel and unlike charges attract **WITHOUT** assigning any sign to the charge of any electron: this is critical as the impulse only depends on the square of the charge. This scheme shows that ***attraction or repulsion between electrons, whether positive or negative, is a dynamical temporal effect only.***

6.2.2 INVARIANT INTERACTIONS

Newton's physics assumed that all particles could be given positional locations against a fixed backdrop of space ("God's *sensorium*"); since both the (instantaneous) particle velocity and acceleration were defined as the limit of differences then the absolute value of these numbers was irrelevant. Further, Newton assumed that all 'forces' propagated instantaneously across all of the universe; this implied that time could be treated as a universal parameter, with its rate of evolution uniform, every where in space ("God's clock"). So-called 'frames of reference' have been introduced as a mathematical scaffold to introduce real numbers into the resulting model of Euclidean 3D geometric space. As well as needing to define standard metrics, all reference frames need to privilege their own origin, which was (implicitly) seen as the fixed location of 'the observer'. Classical mechanics was later found to preserve the form of its equations when the origin of the chosen reference frame was displaced by a fixed amount across space, symbolized as $\underline{\mathbf{\Omega}}_0$: this was also true for the initial value $\mathbf{T}_0$ on all the clocks used to establish the time of events or measurements throughout the whole of space. These transformations are now known as the Galilean transformation of co-ordinates; the mathematical equations of physical theories that preserve their form under this transformation are referred to as '*Galilean invariant*'. If unprimed variables are designated in the chosen frame and primed variables in the displaced reference frame then the **Galilean Transformation** of co-ordinates becomes:

$$\underline{x}' = \underline{x} - \underline{\mathbf{\Omega}}_0 \quad \& \quad t' = t - \mathbf{T}_0 \quad \text{or in terms of finite differences:} \quad \Delta\underline{x}' = \Delta\underline{x} \quad \& \quad \Delta t' = \Delta t$$

Newton's kinematics (velocity $\underline{v}$ and acceleration $\underline{\mathfrak{A}}$) and dynamics remained invariant if both particle masses (M) and their interactions (or forces, $\underline{F}$) remained the same whether viewed from one reference frame or another, since (if $M' = M$) :

$$\underline{v}' = \Delta\underline{x}' / \Delta t' = \Delta\underline{x} / \Delta t = \underline{v} \quad \& \quad \Delta\underline{v}' = \Delta\underline{v} \quad \& \quad \underline{\mathfrak{A}}' = \Delta\underline{v}' / \Delta t' = \Delta\underline{v} / \Delta t = \underline{\mathfrak{A}} \quad \& \quad \underline{F}' = M'\,\underline{a}' = M\,\underline{a} = F$$

These invariance ideas were extended to all frames in relative motion, as long as their relative motion was constant, say $\underline{u}_0$. Such sets of similar reference frames, for all values of $\underline{u}_0$, are known as ***inertial*** frames and inertial frames also preserve the form of the equations in classical physics. This is achieved mathematically by the substitution: $\underline{\mathbf{\Omega}}_0 \rightarrow \underline{\mathbf{\Omega}}[t] = \underline{u}_0\, t$.

$$\underline{x}' = \underline{x} - \underline{u}_0\, t \quad \& \quad t' = t - \mathbf{T}_0 \quad \text{or} \quad \Delta\underline{x}' = \Delta\underline{x} - \underline{u}_0\, \Delta t \quad \& \quad \Delta t' = \Delta t$$

$$\underline{v}' = \Delta\underline{x}' / \Delta t' = \Delta\underline{x}' / \Delta t = \underline{v} - \underline{u}_0 \quad \& \quad \Delta\underline{v}' = \Delta\underline{v} \quad \& \quad \underline{a}' = \Delta\underline{v}' / \Delta t' = \Delta\underline{v} / \Delta t = \underline{a} \quad \& \quad \underline{F}' = M'\,\underline{a}' = M\,\underline{a} = F$$

The major crisis at the beginning of the 20th Century was that while Maxwell's field equations of EM satisfied the Galilean co-ordinate transform, they did not remain invariant across inertial frames of reference, since interactions were no longer instantaneous but required a finite time to cross space at 'light-speed' c. Einstein 'solved' this problem by redefining the measurement definition of time between clocks in different inertial frames using two-way "radar averaging". This led to the re-introduction of the so-called 'Lorentz' transformation of co-ordinates for relative motion along the x-direction:

$$x' = \mathcal{L}\,(x - u_0\, t) \;\; ; \;\; y' = y \;\; ; \;\; z' = z \quad \& \quad t' = \mathcal{L}\,(t - u_0\, x / c^2) \qquad \text{where} \qquad \mathcal{L} = (1 - u_0^2 / c^2)^{-½}$$

Minkowski mathematized Einstein's theory of special relativity and introduced homogenous 4D vectors across his space-time. This style of tensor formulation has become standard throughout modern physics, which uses field theories everywhere.

The present theory is a purely 'relational' theory – that is, the only significant quantities that appear (and which eventually transform into experimental observables) are differences between the space and time variables of the two electrons that are involved in each interaction. In a later paper, it will be shown that measurements must involve many 'secondary' electrons. Unlike traditional one-particle theories of mechanics, whether classical or quantum, the present theory is always, at the very least, a two-particle theory; as it also involves asynchronous interactions, it also requires the awareness of two distinct times, characterizing each electron. The fundamental mathematical object used to describe any two interacting electrons in this theory is their Separation NV $\mathbf{X}_{12}$, defined as the difference between their own locational NVs, $\mathbf{X}_1$ and $\mathbf{X}_2$; these are then complemented by NV versions of their velocities $\mathbf{V}_{12}$, $\mathbf{V}_1$ and $\mathbf{V}_2$ (see A2.2). The dynamics of this theory are represented by instantaneous local changes in each electron's velocity $\Diamond\underline{v}[t_n]$ induced by the asynchronous impulse $\Delta\mathbf{I}_{n+1}$ generated by each interaction (see §5.3.3). The magnitude of this impulse is either constant for 'far' separations or only varies inversely with their 'near' separation (space or time), at the times of their interaction (see §4.3.8). Since time is viewed here, like Newton, as flowing uniformly everywhere across space then this theory must and does satisfy Galilean invariance in every inertial frame. By design, this theory does **not** satisfy the 'Lorentz' transformation, as it **not** a *one-time, local field theory*.

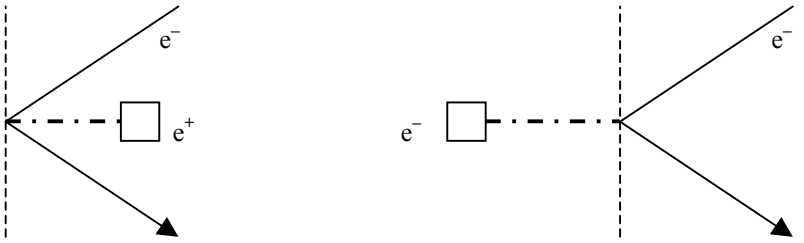
6.2.3 CPT INVARIANCE

The experience of time is intrinsically one-dimensional; nature imposes a unique order on our world that, through memory, humans can review in either a forwards (normal) or reverse direction. The mathematical representations of natural processes reflect this order through the use of a real, scalar variable that is colloquially referred to as 'time'. As was reviewed earlier [see §5.3 in A4] the interaction list $\{t_n\}$ associated with any electron can also be ordered in two time directions and it would appear more than plausible that the order in which we review the event set should not alter the totality of the real experience. This pre-amble is necessary as physicists have for too long, glibly talked about time-reversal as if this were a process subject to human experimentation. It is important to emphasize that this concept is only applicable to our theories and not to our measurements. Any experimental confirmation of 'time reversal' is always a theoretical implication. Each one of the three spatial dimensions is also represented by a single, real scalar variable and, as such its value throughout our theories, can be inverted or reversed (reflection): this corresponds to the experimental situation of re-orienting our equipment in the opposite direction to the distant, 'fixed' stars. This type of spatial binary symmetry has been given the name 'parity'. One of the deep mysteries of nature is that the phenomenon known as electric charge also appears in invertible form with elementary particles found to have equal but opposite charges. A solution to this puzzle is provided in this chapter.

It was Boltzmann who first noted in 1897 (contradicting Planck) that Maxwell's EM equations remained unchanged when both the variables representing time and all the magnetic quantities were reversed while leaving all the electric variables unchanged. Wigner was the first physicist to study parity operations in QM [131], examining the effect of unitary operators that commuted with the energy Hamiltonian. Spectral transitions could now be grouped into two parity-families ('odd' and 'even') as long as both the proton and the electron were both assigned a positive value (+1). He also noted that the parity concept had no analog in classical mechanics, unlike 3D rotational invariance that leads to the conservation of total angular momentum. Wigner went on to study time invariance in QM state transitions [132] where the use of complex numbers needs complex conjugation (introducing a significant non-linearity) and the reversal of all particle 'spins' as well as all momentum variables. Kramers first introduced the QM charge-invariance principle in 1937 [133] in his investigation of the Dirac equation demonstrating that this concept was only really applicable in a quantum field theory. Since Maxwell, localized field theories of nature have been found to be appropriate representations whenever they satisfy the simultaneous, mathematical inversion of the algebraic symbols representing charge (C), parity (P) and time (T). This was first formalized in quantum field theory by Schwinger in 1951 [134] who emphasized that these symmetries are always associated with the interactions between the entities and not intrinsically with the particles themselves, so that (for example), parity is violated when the mathematical form of the Hamiltonian cannot be made invariant under spatial reflections. The so-called CPT theorem relies on the locality of modern quantum field theories and their Lorentz invariance, which 'rotate' space into 'imaginary time'.

Mirrored Charges

The single interaction between two oppositely charged electrons (the attractive impulse) can be given an equivalent motion between two similarly charged electrons (repulsive impulse) by using the 19th Century 'mirrored images' technique. This is the underlying reality behind the 'charge-parity' or CP transformation.



Attractive Impulse Repulsive Impulse

CPT invariance reflects the fact that an electron moving in any one direction in space (as time increases) will move through space after interacting with an another electron with opposite charge in a manner that appears exactly opposite in direction across space but opposite in time from the perspective of the other charged electron. In the present theory, the parity operator P is represented by the negative unity (diagonal) matrix, the temporal operator T is represented by NV Hermitian conjugation while the charge conjugation operation C is represented simply by a sign switch ($e \rightarrow - e$) or a two chronon time displacement in the electron's *charge phase factor* $\xi[\alpha : t_\alpha]$ (see 6.2.1) $\mathbf{P}\ \mathbf{X}_{12} = -\mathbf{X}_{12}$

6.3 ATTRACTIVE INTERACTIONS

The range of motion available to systems containing both positive and negative electrons is vastly more complex than the simple repulsion that is available to collections of electrons with similar charges. The investigation of this variety will be started in this paper but will be the focus of most of the subsequent papers in this series. This section will offer an initial view of the neutrino model constructed on this theory and then briefly revisit the old Bohr model of the hydrogen atom but now viewed from the discrete perspective of the new theory; both models will be elaborated more in later papers. However, this section will first examine the phenomenon of attractive scattering, when an electron and a positron approach each other but where their motion prohibits the creation of a stable structure: all that remains is open-ended scattering, like a unique comet that only visits the solar system once and then returns forever into deep space. This attractive scattering will first be examined in the case of the one-dimensional 'toy' model of linear scattering and then in the more general, three-dimensional case. The sensitivity to initial conditions will also be explored here to determine when scattering becomes a bound system.

6.3.1 ATTRACTIVE MOTION

Momentum exchange is always easier to visualize for repulsive interactions: this was the genesis for the paradigmatic image that is used universally to introduce Feynman diagrams [56] – with the 'virtual' photon removing some momentum from the emitting electron and transferring it upon absorption to the receiving electron. This idea of virtual photons is more difficult to imagine when the two particles are attracting one another: the mathematics remains the same but now the concept alters. Now, the emitted photon must remove *negative* momentum from the emitting electron and *carry* it forwards through time to the receiving particle; this imagery is simplified if the interaction is reversed in time (this was Feynman's insight), so that the future particle emits positive momentum back through time to the earlier (but now) receiving particle. This twin directional view of asynchronous EM interactions is shared by the present theory but, as an action-at-a-distance theory, it dispenses with the idea of virtual photons, fields (both 'carriers') and all the associated problems of 'self-energy' and renormalization. In the current theory, the impulse is **not** 'broadcast' from the emitter, traveling across space until *it just happens* to hit the receiver.

So far, the present theory has dealt only with the dynamics of electrons; since their charges are equal this has only involved repulsive impulses between the two interacting electrons. The 'Terrible Twins' analysis of repulsive motion (see §5.3.6) showed that the retarded impulse, after a finite time delay, repulsed the receiving electron by increasing its momentum (and kinetic energy) while the reaction caused the emitting electron to decrease its momentum and energy. This only occurred when the two electrons were moving (at the two different interaction times) in the same direction; this same constraint was also found to be necessary for the reversed or advanced interaction. An alternative formulation of this constraint is that the accelerating particle must be on, and moving parallel to, the forward 'light-cone' of the decelerating particle (see fig.17).

When viewed from the standard direction of the flow of time, an attractive impulse must draw the two participating particles closer together; in other words, an attraction acts like a 'pull' whereas a repulsion appears as a 'push'. In this new theory all interactions are initiated by a 'snd' event from one particle (whether advanced or retarded) that is finally completed by the 'rcv' event at the receiving particle at a different time. When one particle 'sends' its attractive impulse forwards in time ($\mathcal{S}^-$) then the receiving electron must 'absorb' this at a later time ($\mathcal{R}^-$). The 'pull' at the receiver (towards the emitter) is reflected in the opposite reaction at the emitter (back towards the receiver), which is also experienced as a 'pull' at the emitter towards the receiver. This means that the receiver must be located in a direction parallel to that which the sender is moving and must also be moving in the same direction (like repulsion) as the emitter. Accordingly, the emitter here will increase its speed and kinetic energy at the expense of the receiver, while overall (two electron) momentum is conserved across the complete interaction. These complementary ideas can be illustrated in the following figures (see section 4.1.9).

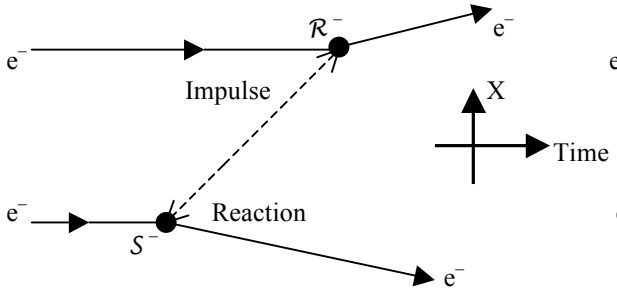


Fig. 17 Repulsion

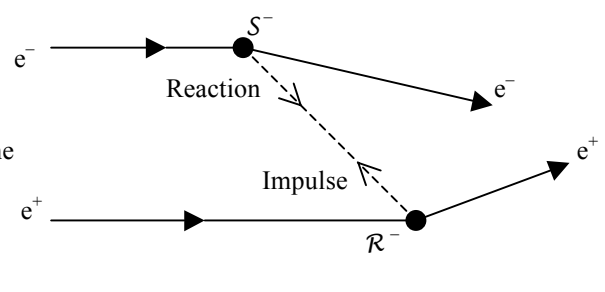


Fig. 18 Attraction

6.3.2 ATTRACTIVE SCATTERING

Attractive Penetration

Any type of one-dimensional model involving attractive particles (e.g. positive and negative electrons) has been acutely embarrassing to conventional physicists, who (since DesCartes) have viewed all material bodies as being 'solid (that is, possessing a finite physical extent) that could not be penetrated. The only solution has been to propose some invisible repulsive force that only manifests its existence when the surfaces of these two otherwise attractive bodies are ultra close. A satisfactory solution is now provided here that resolves this issue, even with point particles, by providing an equivalent attractive analysis for repulsive electrons for what was referred to before as the 'Terrible Twins' – a name that anticipated this attractive situation and for which there are no classical or quantum solutions using the point particle model.

The 'Terrible Twins' analysis demonstrated that the linear solutions are probably nature's form for describing high-speed (repulsive) 'far' electron scattering (as $S_M \approx N^2 \Lambda /3$), rather than the 'harmonic' form, which appears more suitable for describing attractive interactions between oppositely charged high-energy electrons when they are in 'close' spatial and temporal proximity. The key to the solution of this deep challenge is that the EM interaction is no longer viewed as acting continuously at all times but only at periodic, 'interaction times'. This allows the two attracting particles to **pass through each other** at instants when there is no interaction occurring between them. Moreover, since the interaction is asynchronous it never occurs when the two particles are at their closest (same time) spatial separation. As was shown in the repulsive case, the interaction always occurs at long distance since only one impulse can occur at any one interaction point along each of the electron's continuous spatial trajectory. This is why the 'far' (or fixed) interaction impulse is appropriate here and why the sub-microscopic effects of digital rotation can be ignored as the tiny transverse displacements (Λ) are vastly smaller than the longitudinal separations (proportional to $N^2 \Lambda$).

Attractive Scattering

The open scattering model of two electrons of similar charge (i.e. repulsion) leads to only one family of trajectories – those corresponding to each electron moving along a hyperbolic trajectory. In contrast, in the attractive case of oppositely charged electrons interacting consecutively with one another there are three possible families of trajectories: elliptical, parabolic and 'standard'. The elliptical family includes all situations of 'closed orbits' where the electrons are confined to a finite spatial extent and the electrons repeat the same configuration periodically. The elliptical family contains the degenerate cases of the oscillating linear (1D) trajectories and circular 'orbits' where the electrons mutually rotate around each other. These stable systems will be investigated extensively later, particularly when the electrons are never far apart and the 'harmonic' impulse form becomes dominant as it does in all nuclear models developed in this programme. The open 'standard' trajectories occur at atomic separations when the identity of the 'rebounding' electron appears to be exchanged with its partner; since these are now oppositely charged this effect can be distinguished from the parabolic case where the rebounding electrons emerge from the encounter with the same charge as they entered. These two types of attractive scattering are illustrated below.

Attractive Scattering

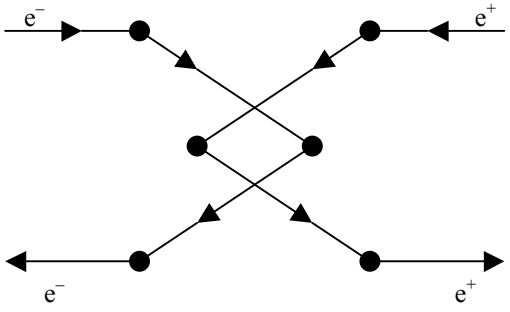


Fig. 19 Parabolic Scattering

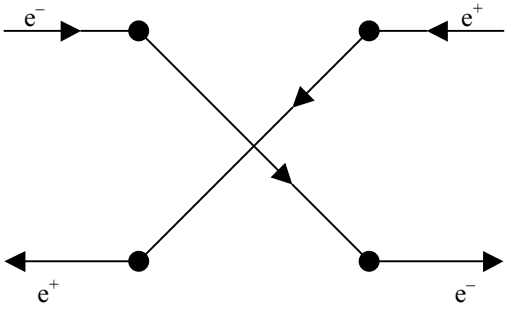


Fig. 20 Standard Scattering

The analysis of both types of attractive scattering can be most easily analyzed in one single space dimension (say z) as was investigated before as in the case of repulsion (see §5.3.6) using the symmetric reference frame (SIRF).

The Attractive 'Twins'

So far, the two electron model of EM interactions, restricted to one spatial dimension, has been limited to electrons carrying the same type of electrical charge, which has meant that only repulsive impulses are involved – this simplified the analysis. In the case of oppositely charged electrons, the interaction is always attractive. Since there are no other electrons involved the initial configuration for dissimilar electrons must have both particles in zero relative motion; from symmetry, this must also be the situation in the final configuration after all the interactions have occurred. This is the completely opposite case for repulsion when both electrons were initially moving directly towards each other at the maximum possible speed (i.e. light-speed *c*). This means that for attraction, both particles must accelerate towards each other before each reaches its maximum speed *c*, thereafter it decelerates back to zero velocity. The maximum speed must be *c* as there can be no further interactions if one of the electrons was to move faster and therefore 'off the light-cone'. The following convention will be adopted: particle #1 be assigned to the negative electron while particle #2 will be the positively charged electron, henceforth called the positron. The positive z-direction will be defined by the final velocity of the electron as it completes its motion after participating in N mutual interactions, that is, at a positive distance z_N from the origin. As it is stopping, it must be receiving 'tugs' directed back towards the origin from the positron that emits these attractive impulses at an earlier time; the positron must therefore react to its own impulse (the complementary part of this single interaction) as a tug in its forward direction, that is forwards along the z-direction and this tug increases its velocity (and kinetic energy). If this is the final impulse (ΔI_N) affecting the electron at time t_N then its subsequent velocity (v_N) is zero. Conversely, the corresponding impulse $\Delta I'_N$ affects the positron at time t'_N then its subsequent velocity (v'_N) must be *c*. The remaining analysis will differ between the parabolic and exchange model, which appears simpler so will be investigated first.

Standard Scattering

As demonstrated in a previous paper [135] the mathematical analysis of electron interactions is simplified when viewed from an inertial reference-frame that incorporates the maximum degree of symmetry in both space and time; this was called the "*Symmetric Inertial Reference Frame*" (or SIRF). Within this frame, the total (combined) velocity of the two electrons, $\underline{V}$ at any one single time t, is always zero. Since every electron has (by hypothesis) the same universal inertial mass parameter *m* then this immediately implies that the SIRF is equal to the conventional Center-of-Momentum reference frame; where an electron's momentum is defined as Newton's "quantity of motion", or algebraically: $\underline{p}_k = m\, \underline{v}_k$.

So, in the SIRF, where the origin of the SIRF is located at the center of the interaction (hence the nomenclature):

$$\underline{V}[t] = \underline{v}_1[t] + \underline{v}_2[t] = 0 \quad \text{and} \quad \underline{P}[t] = \underline{p}_1[t] + \underline{p}_2[t] = 0$$

The *inter-action* velocities $\underline{v}_k$ are defined in terms of the speed of electron #1 going **from** the interaction node at T_k to T_{k+1} :

$$\underline{v}_k \equiv (\underline{x}[1: T_{k+1} - \delta t] - \underline{x}[1: T_k + \delta t]) / (T_{k+1} - T_k)$$

It was also shown in the paper III that only "double-parallel" interactions are compatible with the electron's finite mass and the 'light-cone' condition, where only 'parallel-interactions' provide a valid momentum exchange mechanism for changing each electron's velocity during both the inbound and outbound phases of a series of pair-wise interactions. The corresponding space-time diagrams were referred to as "Canonical Interaction Diagrams" and were 'constructed' according to the following rules [136].

1) All 'links' (impulses) correspond to light-speed impulse 'propagation' (i.e. at 45º).
2) All 'links' (or 'rays') are parallel with incremental (but not equal) separation.

In the case of **repulsive** interactions between similarly charged electrons, the initial state consisted of both electrons moving towards each other at 'light-speed', coming very close together and then diverging away from one another until, again they each reached 'light-speed'. In that repulsive motion, it was found that each of the two innermost nodes of one electron linked to the outermost two nodes of the other electron forming a cross-like pattern.

In the case of **attractive** scattering, the initial state consists of both electrons being motionless at opposite points on the z-axis (at a distance D_N from the origin). As they start to interact together, they pull each other towards the origin, all the time increasing in speed until they reach 'light-speed' just before they pass through the origin at a small distance D_0. The next interactions begin again at the same distance D_0 and act to slow down both electrons until they are again motionless at D_N.

It is useful to label all the interaction nodes for each electron. The right-hand upper quadrant describes the final trajectory of the negatively charged electron e^- (referred to a particle #1); the right-hand lower quadrant describes the final trajectory of the positively charged electron e^+ (referred to a particle #2). By symmetry (and the definition of Standard Scattering) this means that particle #1 begins its journey in the lower left-hand quadrant while particle #2 begins its journey in the upper left-hand quadrant. It is also useful to use the same labeling conventions for the interaction nodes as before. Thus, each node is labeled by a unique capital letter with its own particle subscript (1 or 2); all inbound nodes are distinguished from the similar outbound nodes by a dashed superscipt. The first interaction experienced by each particle is labeled Z', the second as Y' etc until it reaches the closest point to the origin, which is given the label A'. On the outbound phase the first node is labeled A, the next B etc until the final interaction nodes that are labeled Z; for example, the first interaction involving the negative electron occurs at Z'_1 while the final interaction involving the positive electron occurs at Z_2. Now, only one node on path #2 can be on the light-cone of the negative electron starting at its first interaction node (i.e. a 45° ray from node Z'_1 reaches say node L'_2) – this interaction is notated as $\langle Z'_1 : L'_2 \rangle$ needs to communicate between node $Z'_1 \Leftrightarrow (-\underline{x}_N , -t_N)$ & $L'_2 \Leftrightarrow (\underline{x}_L , -t_L)$. The complementary first impulse $\langle Z'_2 : L'_1 \rangle$ communicates between $Z'_2 \Leftrightarrow (\underline{x}_N , -t_N)$ & $L'_1 \Leftrightarrow (-\underline{x}_L , -t_L)$. Both symmetry and the physics of this situation indicate that these L' nodes occur when both electrons have reached half of light-speed $c/2$. It is useful to label this 'mid-point' node as M, where $M = N / 2 + 1$ and N is the number of interactions, each incrementing the electron's speed by the velocity quantum b; that is: $c = N\, b$ (NB N must be an even number – pairs of nodes). This solution means that $L = M - 1 = N/2$. This analysis therefore implies that as the electrons emerge from the origin their first decelerating interaction must link to the outbound mid-point nodes; that is for the negative electron: the interaction $\langle A_1 : M_2 \rangle$ needs to communicate between node $A_1 \Leftrightarrow (D_0 , t_1)$ & $M_2 \Leftrightarrow (-D_M , t_M)$ while for the emerging positively charged electron its first interaction after emerging from the origin is $\langle A_2 : M_1 \rangle$ between node $A_2 \Leftrightarrow (-D_0 , t_1)$ & $M_1 \Leftrightarrow (D_M , t_M)$. The final interactions occur between $\langle Z_1 : L_2 \rangle$ and $\langle Z_2 : L_2 \rangle$. This is illustrated in the following diagram (contracted timescale).

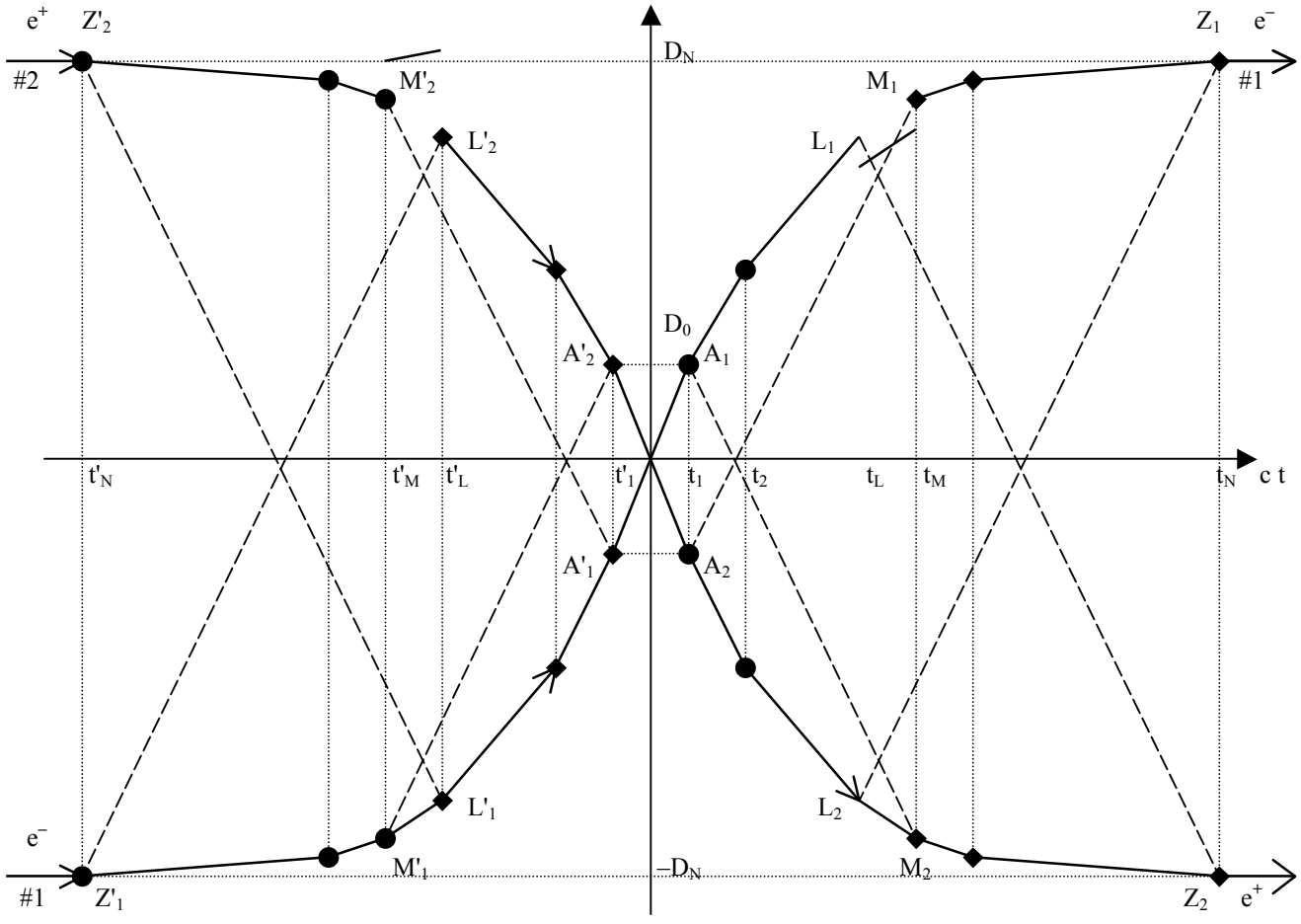


Fig. 21 Standard (Maximum Symmetric) Attractive Interaction Schematic.

In this diagram, the circular nodes represent either a retarded-snd event or an equivalent advanced-rcv event; the diamond nodes here represent either a complementary retarded-rcv event or an equivalent advanced-snd event. As discussed before, the minimal time solution (see §4.1.9) requires that the circular nodes alternate between the two possibilities (retarded and advanced) with a similar constraint on the diamond nodes. This diagram clearly shows the CT symmetry of mathematical time reversal (T) when the positron would emerge along the positive z-axis, reversing the electric charge (C) would mean that the positron would emerge from along the negative z-axis , which appears to be the original result. However, this ignores the rotation of the electrons around their direction of travel (digital twist), this requires a further rotational inversion or parity (P).

In terms of electron speeds v, the speeds in the diverging phase (right-hand side) after the n^{th} interaction in that phase v_n are steadily decreasing by *b* units of speed per interaction, while the speeds in the converging phase (left-hand side) after the n'^{th} interaction v'_n are steadily increasing by *b* units of speed per interaction. In other words, the speeds satisfy the equations:

$$v_n = (N - n)\, b \quad \text{and} \quad v'_n = (N + 1 - n)\, b = v_{n-1} \quad (\text{as } v_N = 0 \text{ and } v'_N = b)$$

A similar analysis to the repulsive 1D model (see §5.3.6) can now be carried out (here the speeds will be denoted by v not u).

Each interaction always occurs between the negative electron at the node labeled n and the positron at the node labeled k.

In the outbound phase: $k = N/2 + n$ {for $n = 1, 2, \ldots, N/2$} while $\underline{v}_n[1] = v_n \underline{e}_z$ and $\underline{v}_k[2] = -v_k \underline{e}_z$

This model is parameterized in terms of the speed V_0 as the electrons pass through the origin: $V_0 = \beta c = N b \quad (0 < \beta \leq 1)$

$$\therefore\ v_n = (N - n)\, b = \beta c - n b \quad \text{and} \quad v'_n = (N + 1 - n)\, b = \beta c - (n - 1)\, b = v_{n-1}$$

Again, in this model, the total velocity is constant (but less than half of the repulsive case): $\underline{v}_n[1] + \underline{v}_k[2] = \beta\ \underline{c} / 2$

The Space-Time Integrity (STI) Condition is actually a 3D vector constraint on the velocities and time intervals [137].

$$(\underline{c} - \underline{v}_n)\, \Delta t_n = (\underline{c} - \underline{v}_k)\, \Delta t_k \quad \text{where } \underline{c} = c\, \underline{e}_z \quad (\text{from } \underline{x}_k \text{ to } \underline{x}_n) \text{ and } c = N_0 b$$

$$\therefore\ (\underline{c} - \underline{v}_k) = (\underline{c} - (\beta\, \underline{c}/2 - \underline{v}_n)) = ((1 - \beta/2)\underline{c} + \underline{v}_n) \quad \therefore\ (c - v_n)\, \Delta t_n = ((1 - \beta/2)c + v_n)\, \Delta t_j$$

This produces the Global Time Constraint after outgoing nodes n and k: $((1 - \beta)\, N_0 + n)\, \Delta t_n = ((1 + \beta)\, N_0 - k) \Delta t_k$

A possible linear solution is suggested of the form: $\Delta t_n = (A\, n + B\, N_0)\, \Delta T_0$ for all n, where A, B and ΔT_0 are constants independent of n but may be functions of β. The Global Time Constraint transforms into:

$$((1 - \beta)\, N_0 + n)\, (A\, n + B\, N_0) = ((1 + \beta/2)\, N_0 - n)\, ((\beta\, N_0 /2 + n)\, A + B\, N_0)$$

$$\therefore\ 2A\, n^2 + N_0\, (2B + (1 - \beta/2)A)\, n = \beta\, N_0^2\, (3B + \beta\, A/2) / 2$$

There is no possible solution for $\beta = 1$ while setting $\beta = -6\, B/A$ results in $\beta = 6/5$ that is non-physical ($\beta \leq 1$); thus there are **no solutions linear in n** for the interaction interval Δt_n. This reflects the fact that the two particles can not establish an initial spatial separation that will allow them to interact asynchronously **and** consecutively on their mutual light-cones as they seem to be pulled together while they accelerate towards a zero separation and then decelerate as they separate on the other side.

There is also a difficulty with the total kinetic energy of this model (unlike repulsion) since each pair of interaction impulses has to cross the gap between them when they are on opposite sides of the z-axis and the interaction has to be restricted to the same phase of the complete set of interactions i.e. they have to ocurr all in the inbound or in the outbound phase. This means that both particles are gaining kinetic energy during the inbound phase and both particles are losing kinetic energy during the outbound phase (even asynchronously). The total kinetic energy of the two particles is not restored to zero until the complete set of interactions has occurred unlike the repulsion model where both momentum and kinetic energy are conserved across **each** asynchronous interaction. This would mean that when the electron and positron were interacting, a third electron could intervene and total energy would no longer be conserved.

Parabolic Scattering

A review of the standard scattering models indicate that in order to maintain conservation of momentum and energy across each interaction it is necessary that one particle be accelerating while the other is decelerating; i.e. they are in opposite phases of the total set of interactions. This means, that like in the repulsive scattering solution, the two particles have to be traveling in the same direction at the two times when their component impulses act on each particle. In the case of parabolic scattering, this means that ***there are no solutions where each particle begins and ends at rest***. However, there is one valid parabolic (or overshoot) solution where the two particles initially converge and finally diverge in the Symmetric Interaction Reference Frame (SIRF) at 'light-speed'. This is very similar to the repulsive solution, which was also dynamic (the initial and final states involve motion: there are no stable immobile configurations). The major difference in the attractive case is that each particle overshoots past the other and decelerates before it stops and then they each accelerate again to light-speed. When they are both stationary (t = 0) they are now far apart, separated by the maximum distance $2D_N$) unlike their closest distance apart ($2D_0$) when the interaction is repulsive. However, like the repulsive case, it is found that each of the two innermost nodes of one electron linked to the outermost two nodes of the other electron forming a cross-like pattern. This is illustrated in the following schematic diagram.

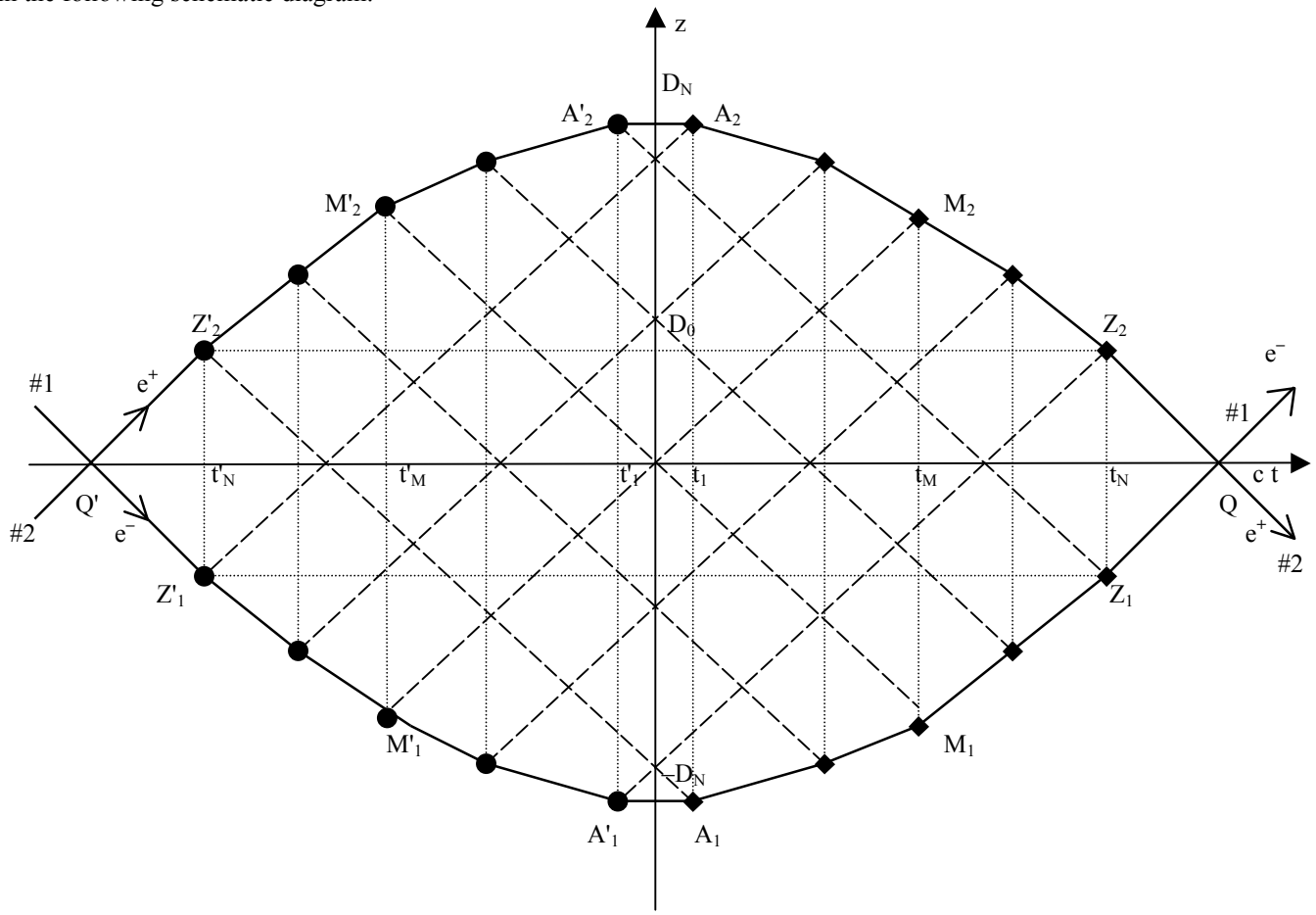


Fig. 22 Attractive Interaction: Parabolic Solution (schematic).

It is instructive to follow the trajectory of particle #1 (the negative electron). At the earliest time (before t'_N) this electron is far along the positive z-axis (above D_N) and is moving vertically downwards to the origin at light-speed. There is no possible interaction with the positron until their paths have crossed at point Q' and it has reached the first interaction point Z'_1. Here an impulse is sent upwards to the positron when it reaches the point A_2 at time t_1, where it is tugged downwards. All of these upward tugs on the electron decelerate the electron from Z'_1 to A'_1 when the electron becomes stationary (at time t'_1); it then remains stopped until time t_1. At A_1 the electron receives its first upward tug from the positron when it was at Z'_2 at time t'_N. It then receives another set of upward tugs that accelerate it to light-speed at Z_1 where it then continues upwards through Q.

In this diagram, the circular nodes represent either a retarded-snd event or an equivalent advanced-rcv event; the diamond nodes here represent either a complementary retarded-rcv event or an equivalent advanced-snd event. As discussed before, the minimal time solution (see §4.1.9) requires that the circular nodes alternate between the two possibilities (retarded and advanced) with a similar constraint on the diamond nodes.

In terms of electron speeds v, the speeds in the accelerating phase (right-hand side) after the n^{th} interaction in that phase v_n are steadily increasing by *b* units of speed per interaction, while the speeds in the decelarating phase (left-hand side) after the k^{th} interaction v'_k are steadily decreasing by *b* units of speed per interaction. Since the initial speed is *c* and the final speed is zero after N interactions then N equals N_0 (the maximal interaction count $N_0 = c / b$). Thus, the speeds satisfy the equations:

$$v_n = n\,b \quad \text{and} \quad v'_n = (N_0 - n)\,b = c - v_n \qquad \{\text{for } n = 1, 2, \dots, N_0\}$$

A similar analysis to the repulsive 1D model (see §5.3.6) can now be carried out with the focus on the accelerative phase of the electron, specifically examining the interaction that occurs between the negative electron at the node labeled n (at time t_n) and the positron at the node labeled k (at time $t'_k = -t_k$). It is readily seen that this set of interactions link the nodes:

$$\{ < A_1 : Z'_2 > < B_1 : Y'_2 > \dots < Y_1 : B'_2 > < Z_1 : A'_2 > \} \text{ or times: } \{ < t_1 : t'_N > \dots < t_n : t'_k > \dots < t_{N-1} : t'_2 > < t_N : t'_1 > \}$$

These satisfy: [n & k] where $k = N_0 + 1 - n$ $\{\text{for } n = 1, 2, \dots, N_0\}$

During the electron's accelerative phase, both velocities are parallel to the z-axis. $\underline{v}_n[1] = v_n \underline{e}_z$ and $\underline{v}_k[2] = v_k \underline{e}_z$

Again, in this model, the total velocity of the two particles **across** each interaction is constant: $\underline{v}_n[1] + \underline{v}_k[2] = \underline{c}$

It is important in using the symmetric labeling convention to notice that the labels on the increments on the negative time axis have to be adjusted to maintain equal global integrity; this stems from the particle time-inversion symmetry: $t'_k = -t_k$ but the spatial symmetry is unchanged.

Particle #1: $v_n = (z_{n+1} - z_n) / (t_{n+1} - t_n) = \Delta z_n / \Delta t_n$ Particle#2: $v'_k = (z'_{k-1} - z'_k) / (t'_{k-1} - t'_k) = \Delta z_{k-1} / \Delta t_{k-1} = v_{k-1}$

The Space-Time Integrity Condition is actually a 3D vector constraint on velocities and time intervals of particle #1 [137].

$$(\underline{c} - \underline{v}_n)\,\Delta t_n = (\underline{c} - \underline{v}_k)\,\Delta t_{k-1} \qquad \text{where } \underline{c} = c\,\underline{e}_z \quad (\text{from } \underline{x}_n \text{ to } \underline{x}_k) \text{ and } c = N_0\,b$$

$$\therefore\ (\underline{c} - \underline{v}_n)\,\Delta t_n = \underline{v}_n\,\Delta t_{k-1} \quad \therefore\ (c - v_n)\,\Delta t_n = v_n\,\Delta t_{k-1} = v_n\,\Delta t_{N-n} \quad \therefore\ (N_0 - n)\,\Delta t_n = n\,\Delta t_{N-n}$$

This produces the Global Time Constraint after accelerative nodes n and k: $k\,\Delta t_n = n\,\Delta t_k$ with solution: $\Delta t_n = n\ \Delta T_0$

Here ΔT_0 is a constant (independent of n) that is still to be determined. This is identical (in form) to the linear solution of the repulsive 1D model (see §5.3.6) so that (writing $D = D_N$ and $d = D_0$):

$$\therefore\quad z_{n+1} = z_0 + \sum^n \Delta z_k = -D + \sum^n v_k\,\Delta t_k = -D + b\,\Delta T_0 \sum^n k^2 = -D + n\,(n+1)\,(2n+1)\,c\,\Delta T_0 / 6N_0$$

$$\therefore\quad t_{n+1} = t_0 + \sum^n \Delta t_k = \Delta T_0 \sum^n k = \mathcal{T}_0 + \tau \sum^n k = \tau/2 + n\,(n+1)\,\tau/2 \quad \text{assuming } \mathcal{T}_0 = \tau/2\ .$$

The choice was made to assign $t_0 = \tau / 2$, as in the repulsive case, in order to preserve the constraint that all interactions occurred at an integer multiple of the chronon τ and to minimize the zero speed duration at zero time from t'_N to t_1. This also has the advantage that the electron switches from its 'send' to 'receive' state as it crosses the origin.

Using the linear transform above and recognizing that a change in notation does not change the physics, then:

$$t'_n = -t_{n'} = -t_{N-n+1} = -\{(N_0 - n)(N_0 - n + 1) + 1\}\tau/2 \qquad \therefore\ \Delta t'_n = (N_0 - n)\,\tau \text{ which satisfies the UTI.}$$

Now, for $n = N - 1$ then: $z_N = \mathcal{D}/2$ (and, as is likely, $\mathcal{D} >> d$), so: $\mathcal{D} \approx (N - 1)(2N - 1)\,\Lambda / 3$ or $\mathcal{D} \approx 2\,N^2\,\Lambda / 3$

Again, for $n = N - 1$ then: $t_N = \mathcal{T}$ (and, as is likely, $\mathcal{T} >> \tau$), so: $\mathcal{T} \approx N (N - 1) \tau / 2$ or $\mathcal{T} \approx N^2 \tau / 2$

This suggests an 'effective speed' (written $\mathcal{V}_{eff}$) for each electron to separate to their maximum speed 'c' from rest.

Definition: Effective-Speed $\mathcal{V}_{eff} \equiv z_N / t_N$ $\therefore \mathcal{V}_{eff} \approx 2/3\, c$

This result indicates that the electron spends most of its time at high-speeds (close to 'light-speed' c) after the 'rebound'.

The single electron kinetic energy, in the SIRF, after time t_n is $\mathcal{K}_n = ½\, m\, (v_n)^2$; remembering that $v_0 = 0$ so that $\mathcal{K}_0 = 0$.

The change in the electron's kinetic energy, in this model, is: $\Delta\mathcal{K}_n = (n + ½)\, m\, c^2 / N_0^2$ hinting at Planck's SHM result.

The 'energy quantum' (designated ε) may be defined as the <u>average</u> amount of kinetic energy transferred per interaction across the whole set of interactions, so:

$$\varepsilon \equiv < \Delta\mathcal{K}_n > = 1/ N_0 \sum^n (k + ½)\, m\, b^2 = (N_0 / 2 + 1)\, m\, b^2 \cong ½\, m\, b\, c = ½\, m\, c^2 / N_0 \quad \therefore N_0\, \varepsilon = \mathcal{K}_N$$

Since the velocity is linear with the interaction count 'n' then $p_n = m v_n = n\, m\, b$ then: $\Delta\mathcal{K}_n = b\, p_n$ for $n >> 1$.

The change in the electron's kinetic energy, in this model, is: $\Delta\mathcal{K}_n = (n + ½)\, m\, b^2$ & $\Delta\mathcal{K}'_n = -(N_0 - n + ½)\, m\, b^2$

The mid-point in the accelerating phase is designated as: $M \equiv N_0 / 2$ so redefining the (positive & negative) offsets k from M as: $n \equiv M + k$ while in the decelerating phase as: $M' \equiv N_0 / 2 + 1$ so redefining the offsets k from M as: $n' \equiv M' - k$.

$$\therefore v_k = n\, b = (M + k)\, b = (N_0/ 2 + k)\, b = c / 2 + k\, b \quad \& \quad v'_k = n' b = (M - k)\, b = (N_0/ 2 - k)\, b = c / 2 - k\, b$$

During the electron's accelerative phase, both velocities are parallel to the z-axis: $\underline{v}_k[1] = v_k\, \underline{e}_z$ and $\underline{v}'_k[2] = v'_k\, \underline{e}_z$
The accelerations are always anti-parallel: $\Delta\underline{v}_k[1] = b\, \underline{e}_z$ and $\Delta\underline{v}_k[2] = -b\, \underline{e}_z$ This conserves total velocity.

Thus, the relative velocity across each and every interaction k is: : $\underline{v}_k[1] - \underline{v}'_k[2] = 2\, k\, b\, \underline{e}_z$ This is minimized at k = 0; that is, at their midpoints M and M′ when the relative velocity is zero.

But: $\Delta\mathcal{K}_k = (M + k + ½)\, m\, b^2$ & $\Delta\mathcal{K}'_k = -(M - k + ½)\, m\, b^2$ $\therefore \Delta\mathcal{K}_k + \Delta\mathcal{K}'_k = 2\, k\, m\, b^2$

Thus, kinetic energy is only conserved across the mid-point interaction (k = 0); total kinetic energy is only conserved across the complete **set** of interactions or in symmetric pairs around the midpoints (± k and ±k′) as:

$$\Delta\mathcal{K}_k + \Delta\mathcal{K}_{-k} = m\, c^2 / N_0 \quad \& \quad \Delta\mathcal{K}'_k + \Delta\mathcal{K}'_{-k} = -m\, c^2 / N_0 \quad \therefore \Delta\mathcal{K}_k + \Delta\mathcal{K}_{-k} + \Delta\mathcal{K}'_k + \Delta\mathcal{K}'_{-k} = 0$$

NB There are no bound states when an electron and a positron approach each other at light-speed from greater than atomic distances. The next section will briefly discuss two microscopic models that result in stable bound states between positive and negative charges when they interact via the 'near' EM interaction; that is, using the harmonic form of the interaction.

6.3.3 IMPULSE QUANTUM MODELS

This section will anticipate the results of two later papers on the Hydrogen atom and the Weak Interaction and use here only simplified versions of these new models of stable configuration of the electron-positron interaction pairs at both nuclear and atomic distances. These results will confirm the choice of the nuclear chronon as the fundamental unit of temporal duration.

The Impulse Neutrino Model

In this simple model, the two particles are always separated by a spatial distance d when they are at their possible interaction nodes ($t_\mu = \mu\,\tau$). Each electron is always moving at half light-speed (c/2), sometimes in the same direction and sometimes in opposite directions along their common line-of-centers. At certain 'cross-over' locations, the two particles are occupying the same point in space (moving in opposite directions) but these points occur at times halfway between two possible interaction nodes there are no interactions between the two whenever they are at these points. The motion remains periodic if the intial positions and velocities repeat after a period of several chronons. It can be seen from the short-range impulse law that if the two electrons are interacting when their temporal separation is one chronon then the impulse will be maximal with magnitude $\Delta\mathbf{I}_1 = m\,c$. Each impulse will completely reverse the direction of each electron while maintaining its speed (a classic example of a micro conservative system that only transfers momentum and not kinetic energy). This one-dimensional model is shown in the following two graphs that plot spatial location and velocity of each electron against global-time (the History View).

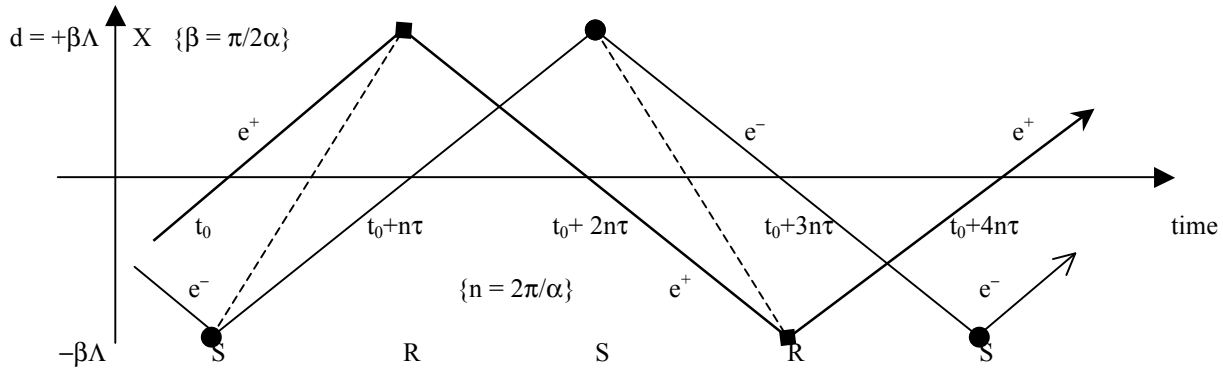


Fig. 23 Neutrino Space-Time Diagram

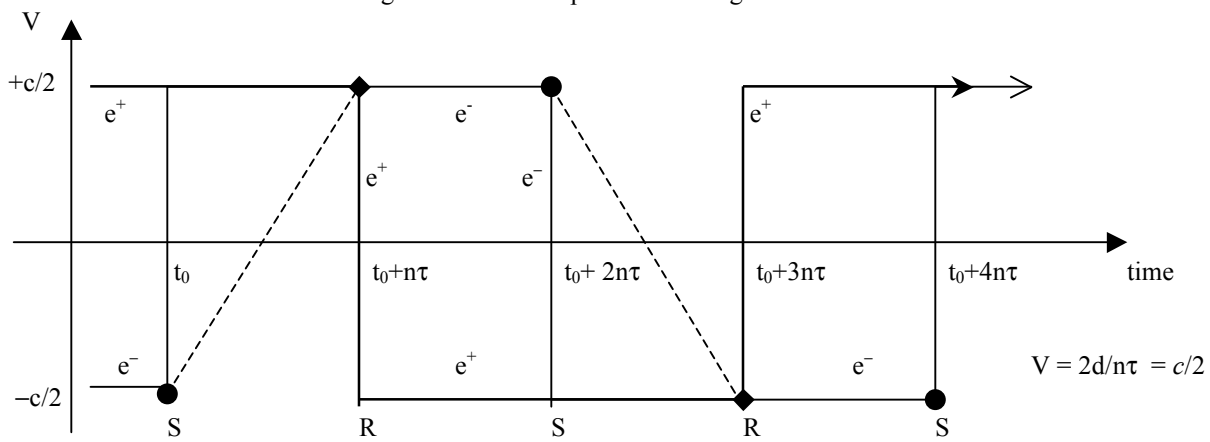


Fig. 24 Neutrino Velocity-Time Diagram

Consistent as: $\Delta\mathbf{I} = h / 2S = h / 4d = h / 4\beta\Lambda = \alpha\, h_D / \Lambda = m\,c = m\,\Delta V = \Delta P$ & $\Lambda = c\,\tau = e^2 / m\,c^2 = \alpha\, h_D / m\,c$.

The nuclear chronon τ is appropriate for this neutrino model with a $4n\tau = 4h/mc^2$ periodic cycle time. The separation distance $S = 2\beta\Lambda = (\pi/\alpha)\Lambda \cong 430\Lambda$ is a 'small' multiple of the 'classical' EM electron radius Λ, a scale suitable for intra-nuclear interactions appropriate for the 'weak' force. Only when a nuclear electron is within this range will a neutrino interact.

The Impulse Bohr Model

This section will demonstrate that the formulae developed above for the impulse-model of EM interactions are capable of providing a Newtonian model of the hydrogen atom, without using quantum mechanics or the paradoxical assumptions of the Bohr model of the atom. The Bohr model will be briefly reviewed first so that its principal features may be contrasted with the new impulse-model. A detailed critique of Bohr's model will be presented in a later paper when a comprehensive 3D model of atomic structures, based on the new theory, will be investigated.

Niels Bohr (1885-1962) was the first physicist to construct a detailed model of the hydrogen atom that predicted results that accurately matched experiments. In a series of revolutionary papers in 1913 [121], he adopted the Keplerian (or planetary) model of the atom first proposed by his colleague, Ernest Rutherford (1871-1937) in 1911 [144]. The single, orbital electron was viewed as moving in a two-dimensional plane, at a constant distance R_n around the nucleus in a circular trajectory at a constant speed V_n. In order to impose mechanical stability (and later, radiative stability), Bohr proposed that the action of the electron was constrained into a finite number of units, defined by a new parameter: the principal quantum number (n). This parameter was taken to be unity in the lowest energy (or ground) state, where the Coulomb attractive effect was the strongest, implying that R_1 was the smallest radius and therefore that V_1 was the electron's maximum speed. Bohr assumed that all the other orbits were multiples of these two values that are so important in atomic physics that they are often denoted by the special symbols $R_B = R_1$ and $V_B = V_1$ (the 'B' referring to Bohr's major contribution). The complete action of the electron in this lowest energy state $\mathcal{A}_B$ corresponded directly to Maupertuis's original definition of action defined around one complete orbit (of length l_n in a time $\mathcal{T}_n$) – this total orbital (kinetic) action was identified with Planck's new quantum of action (h).

$$\mathcal{A}_B = m\, V_B\, l_B = m\, V_B\, 2\pi\, R_B = h \qquad \text{or} \qquad m\, V_B\, R_B = h_D \qquad \text{where} \quad h_D = h / 2\pi \quad \text{with} \quad V_n = 2\pi\, R_n / \mathcal{T}_n$$

Bohr proposed that the n^{th} orbital's action $\mathcal{A}_n$ was simply 'n' times larger than $\mathcal{A}_B$: $\mathcal{A}_n = n\, \mathcal{A}_B$ or $m\, V_n\, R_n = n\, h_D$

It was later realized that this action rule was equivalent to the quantization of orbital angular momentum $\underline{L}_n$, which was also conserved. This insight became central to the subsequent quantum mechanical treatments of the hydrogen atom.

$$\underline{L}_n = \underline{R}_n \wedge \underline{P}_n \qquad \therefore \quad L_n = m\, V_n\, R_n = n\, h_D$$

In all models of the atom, the atomic electrons are attracted <u>continuously</u> to the positive nucleus via the <u>instantaneous</u>, electro-static force F_n. In the Bohr model, this force is sufficient to <u>accelerate</u> the orbital electron around its circular path.

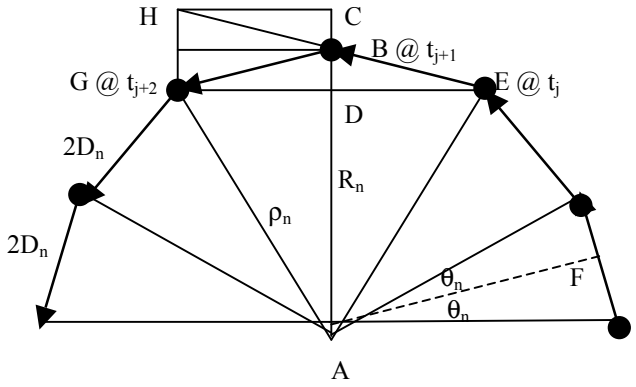
Newton's II: $F_n = m\, \underline{\mathfrak{A}}_n \;\; \therefore \;\; F_n = e^2 / R_n^2 = m\, V_n^2 / R_n \;\; \therefore \;\; e^2 = V_n\, (m\, V_n\, R_n) = n\, V_n\, h_D \;\; \therefore V_n = e^2 / (n\, h_D) = \alpha\, c / n$

The electron's total energy $\mathcal{E}_n = \mathcal{K}_n + \Phi_n$ with $\Phi_n = -\, e^2 / R_n = -\, m\, V_n^2 = -\, 2\, \mathcal{K}_n \;\; \therefore \;\; \mathcal{E}_n = -\, \mathcal{K}_n = -\, \tfrac{1}{2}\, (\alpha/n)^2\, mc^2$

It will be useful to note: $V_n = V_B / n$ & $R_n = n^2\, R_B$ & $\mathcal{T}_n = n^3\, \mathcal{T}_B$ & $\mathcal{E}_n = \mathcal{E}_B / n^2 \;\; \therefore \;\; \mathcal{T}_n^2 = (4\pi^2\, m / e^2)\, R_n^3$

Where: $V_B = \alpha\, c$ & $R_B = h_D / \alpha\, m\, c$ & $\mathcal{T}_B = 2\pi\, h_D / \alpha^2\, m\, c^2$ & $\mathcal{E}_B = -\, \tfrac{1}{2}\, \alpha^2\, m\, c^2$

In the new 'impulse' model (which will be expanded in much greater detail in subsequent papers), the circular orbits in the hydrogen atom are replaced by equal-sided polygonal trajectories. Each corner of the polygon locates a <u>single</u> interaction between the orbital electron and one high-speed positive electron in the nucleus. This EM interaction is attractive so that at each node (and only at each node), the orbital electron is 'jerked' away from its inertial (straight-line) motion to move along the next segment by a unit impulse $\Delta \mathbf{I}_n$ that is radially directed inward to the nucleus. For the present purposes, only the circular Bohr orbitals will be considered, elliptical orbits will be considered later. In this much simpler model, the nuclear positron is considered to be, on average, at the center of the nucleus, which defines the origin of the x-y plane. The discrete nature of this model means that there is no potential energy function Φ but the ground state is still the one where the orbital electron receives its maximum impulse $\Delta \mathbf{I}_1$ at the closest distance from the nucleus, here defined as ρ_1. The electron moves around the atom at a constant speed V_n crossing each segment in a time Δt_n. The impulse at each node also arrives from the nucleus after a time ΔT_n as this is seen as a delayed EM pulse 'traveling' at light-speed c: $\rho_n = c\, \Delta T_n$. The co-ordination of the time for the impulse to arrive at the next node **exactly** when the electron reaches that position in space generates the atom's stability ($\Delta t_n = \Delta T_n$). The electron motion around the interaction node (E) at time t_j is shown in the next diagram.



$EB = BH = 2D_n$
$EB = BH = \underline{v}_j \Delta t_n$
$ED = CH = \underline{u}_j \Delta t_n$
$DB = BC = \underline{w}_j \Delta t_n$
$BG = \underline{v}_{j+1} \Delta t_n$
$DG = \underline{u}_{j+1} \Delta t_n$
$BD = \underline{w}_{j+1} \Delta t_n$
$AD = AF = R_n$
$AB = AG = AE = \rho_n$
$\angle DGB = \angle BHC = \theta_n$
$\angle GAB = \angle BAE = 2\theta_n$

Fig. 25 Attractive motion around an impulse node.

In the present approach, a simple equivalence will be assumed between the Bohr model and the impulse model. A later paper will derive all of the results *ab initio*. Accordingly, in the new model discrete motion around the polygon will be taken to be comparable to the continuous, circular electron motion proposed by Bohr. Let the n^{th} orbital correspond to a polygon with $2G_n$ equal sides (or segments); symmetry suggests that this is an even numbered polygon. Each segment is defined from the origin (positioned at the nucleus) by the radial length ρ_n and the **double**, central angle θ_n , so: $\theta_n = \pi / 2G_n$.

Since the electron's speed, V_n and angular momentum L_n are assumed to be equal in the two models then the segment speed must be V_n and the length AF must be the Bohr radius, R_n. Each of the segment velocities $\underline{v}_j$ has the same speed V_n but the direction is changed at each node around the polygon by each impulse $\Delta \mathbf{I}_n$ that is directed radially inward but has the same magnitude $\Delta \mathbf{I}_n$. The velocity $\underline{v}_j$ can be decomposed into two orthogonal components $\underline{u}_j$ and $\underline{w}_j$ at the next node (t_{j+1}), where $\underline{w}_j$ is the radial component at this point.

$$\therefore \ \underline{v}_j = \underline{u}_j + \underline{w}_j \quad \text{and} \quad \underline{u}_{j+1} = \underline{u}_j \quad \& \quad \underline{w}_{j+1} = -\underline{w}_j \quad \text{where} \quad \underline{\rho}_j \wedge \underline{w}_j = 0$$

$$\therefore \ \underline{v}_{j+1} = \underline{u}_{j+1} + \underline{w}_{j+1} = \underline{u}_j - \underline{w}_j = \underline{u}_j \pm \underline{w}_j - \underline{w}_j = (\underline{u}_j + \underline{w}_j) - 2\underline{w}_j = \underline{v}_j - 2\underline{w}_j = \underline{v}_j + \Delta \underline{v}_j \quad \therefore \quad \Delta \underline{v}_j = -2\underline{w}_j$$

The impulse (orbital) electron has the same angular momentum along each segment; thus, on the segment EB its value is:

$$\underline{L}_j = \underline{R}_j \wedge \underline{P}_j \qquad \therefore \ L_j = m V_j R_j \qquad \therefore \ L_n = m V_n R_n = n h_D$$

At each node, say t_{j+1}, only the average velocity $< \underline{v}_j >$ is defined as the direction changes instantaneously, so only the average angular momentum $< \underline{L}_j >$ is defined at this instant.

$$\therefore \quad < \underline{L}_j > = < \underline{\rho}_j \wedge \underline{p}_j > = m \underline{\rho}_j \wedge < \underline{v}_j > = \tfrac{1}{2} m \underline{\rho}_j \wedge (\underline{v}_{j+1} + \underline{v}_j) = \tfrac{1}{2} m \underline{\rho}_j \wedge (\underline{u}_{j+1} + \underline{w}_{j+1}) + \tfrac{1}{2} m \underline{\rho}_j \wedge (\underline{u}_j + \underline{w}_j)$$

$$\therefore \quad < \underline{L}_j > = \tfrac{1}{2} m \underline{\rho}_j \wedge (\underline{u}_{j+1} + \underline{u}_j) + \tfrac{1}{2} m \underline{\rho}_j \wedge (\underline{w}_{j+1} + \underline{w}_j) = m \underline{\rho}_j \wedge \underline{u}_j \quad \therefore \quad < L_n > = m \rho_n u_n$$

But $u_j = V_j \cos \theta_n$ & $R_j = \rho_j \cos \theta_n$ $\therefore$ $\rho_j u_j = R_j v_j$ $\therefore$ $< L_n > = m V_n R_n$ $\therefore$ $< L_n > = L_n = n h_D$

Thus, the impulse electron's angular momentum is *conserved at all times*, even at the interaction nodes. The electron also crosses each segment of length $2D_n$ in the same time Δt_n $2 D_n / V_n = \Delta t_n$ IF $\Delta t_n = \Delta T_n = \rho_n / c$ $\therefore$ $2 D_n = \rho_n V_n / c$

For example, the segment GB: $D_n = \rho_n \sin \theta_n$ & $\sin \theta_n = BC / BH = w_n / V_n$ $\therefore$ $V_n D_n = w_n \rho_n$ $\therefore w_n = V_n^2 / 2c$

If it assumed that: $G_n >> 1$ (i.e. $\theta_n << 1$) then: $\rho_n \cong R_n$ so that: $\Delta T_n \cong R_n / c = n^2 R_B / c = n^2 h_D / \alpha m c^2$

If the physics is to remain invariant, then the time for the orbital electron to circumnavigate the nucleus should not depend on whether it traverses a continuous circular (Υ_n) or a discrete polygonal (Γ_n) trajectory receiving $2G_n$ impulses (NB $\Delta t_n = \Delta T_n$).

$\therefore\ \Gamma_n \equiv 2G_n \Delta t_n = \mathcal{T}_n\ \therefore\ 2G_n R_n / c \cong 2\pi R_n / V_n\ \therefore\ G_n \cong n\pi c / V_B\ \therefore\ G_n \cong n(\pi/\alpha) \cong 430.6\, n\ \therefore\ \theta_n \cong \alpha / 2n$

The fractional G_n implies a slowly precessing polygonal orbit in space. Nonetheless, even the lowest energy state requires a polygon of 860 sides and this number increases rapidly for higher energy states, so that Bohr's approximation of circular motion was fortunately very accurate. This accuracy can be estimated as follows by restoring ΔT to its full value:

$$2G_n \Delta T_n = \mathcal{T}_n \quad \therefore\ 2G_n \rho_n / c = 2\pi R_n / V_n \quad \therefore\ \theta_n = V_n^2 / 2c\, u_n = w_n / u_n \quad \therefore\ w_n = \alpha\, u_n / 2n$$

The radial velocity, w_n is always much smaller than the tangential velocity, u_n and $v_n = \gamma_n u_n$ with $\gamma_n = (1 + \alpha^2/4n^2)^{1/2}$

Since $\rho_n u_n = R_n v_n$ then $\rho_n = \gamma_n R_n$ so the polygonal 'radius' ρ_n is almost the same size as the Bohr radii. Although there is no potential energy in this dynamic theory, the orbital electron still has the same discrete energy spectrum as Bohr.

$$\therefore\quad \mathcal{E}_n = \mathcal{K}_n = \tfrac{1}{2} m V_n^2 = \tfrac{1}{2}(\alpha^2/n^2)\, mc^2 \qquad \text{with ground-state energy } \mathcal{E}_1 = (\alpha^2/2)\, mc^2 .$$

This is the standard ionization energy of the hydrogen atom (renormalizing the energy baseline), which is the desired result.

Bohr's quantization of kinetic action is directly equivalent to the unit kinetic action (per segment traversal) with $2D_j = V_j \Delta t_j$

$$\Delta \mathcal{A}^K_j = m V_j^2 \Delta t_j = h / 2 \qquad \text{Summing over all } 2G_n \text{ identical segments:} \qquad \therefore\ m V_n l_n = G_n h$$

All of this analysis so far has relied only on the quantization of **kinetic** action and geometry. It is now necessary to review the **dynamical** aspects associated with the impulses that in Bohr's model are replaced by the instantaneous Coulomb force.

Newton's II: $\Delta \underline{\mathcal{J}}_j = \Delta \underline{\mathbf{P}}_j$ with $\Delta \underline{\mathcal{J}}_j = -\Delta \mathcal{J}_n \hat{\underline{e}}_j$ & $\Delta \underline{\mathbf{P}}_j = m \Delta \underline{v}_j = -2m \underline{w}_j \quad \therefore\ \Delta \mathcal{J}_n = 2m w_n$

$$\therefore\ \Delta \mathcal{J}_n = 2m V_n D_n / \rho_n = m V_n^2 \Delta t_n / \rho_n = h/2\rho_n = \Delta \mathbf{I}_n = \Delta \mathcal{A}^D_n / \rho_n$$

Thus, the limit of the classical model (i.e. continuous interactions or forces) is consistent with quantized dynamic action.

The '**exact-correlation**' assumption (i.e. $\Delta t_n = \Delta T_n$) was seen above to lead to: $\therefore\ w_n = V_n^2 / 2c\ \therefore\ \Delta \mathcal{J}_n = m V_n^2 / c$

But, Bohr's atomic parameters satisfy: $e^2 = m V_n^2 R_n\ \therefore\ \Delta \mathcal{J}_n = e^2 / c R_n \cong (e^2 / \rho_n^2)(\rho_n / c) = \langle F_n \rangle \Delta T_n$

Classically, Newton defined force as the limit of impulse per unit time. In terms of the average force $\langle F_n \rangle$ over the j^{th} node applied over the same time δt_n at each node, the EM impulse is defined as: $\Delta \mathcal{J}_n = \langle F_n \rangle \delta t_n$

So, this last result implies that the exact-correlation assumption is equivalent to identifying the impulse duration δt_n with the nuclear transmission delay ΔT_n (i.e. $\delta t_n = \Delta T_n$). This also implies: $\langle F_n \rangle = m V_n^2 / \rho_n = m \langle \mathfrak{A}_n \rangle = F_n / \gamma_n$
This form of the average force uses the continuous centripetal acceleration $\langle \mathfrak{A}_n \rangle$ but evaluated at the nodal radius ρ_n not R_n.

But at a nuclear distance ρ_n the short-range impulse $\Delta \mathbf{I}_n$, predicted by this theory is:

$$\Delta \mathbf{I}_n = h / 2\rho_n = h_D V_n G_n / c R_n = h_D m V_n^2 G_n / c m V_n R_n = G_n m V_n^2 / c n = (G_n / n) \Delta \mathcal{J}_n = (\pi/\alpha) \Delta \mathcal{J}_n \cong 430\, \Delta \mathcal{J}_n$$

Bohr's $\Delta \mathcal{J}_n$ is about 430 times too small according to the new theory; the mistake is traceable to assuming: $\delta t_n = \Delta t_n = \Delta T_n$.
If instead, it is assumed that the Bohr (classical) impulse is equal to short-range impulse (i.e. $\Delta \mathcal{J}_n = \Delta \mathbf{I}_n$), then:

$$\therefore\ \langle F_n \rangle \delta t_n = h / 2\rho_n \qquad \therefore\ \delta t_n = h \rho_n / 2e^2 = \pi (h_D c / e^2)(\rho_n / c) = (\pi/\alpha) \Delta T_n \cong 430\, \Delta T_n$$

$$\therefore\ \delta t_n / \Delta t_n = (h \rho_n / 2e^2) / (h / 2m V_n^2) = m V_n^2 \rho_n / e^2 = m V_n^2 \rho_n / (m V_n^2 R_n) = \rho_n / R_n = \gamma_n \cong 1$$

Thus, for the discrete Bohr model to comply with the short-range impulse theory then the impulse duration δt_n must be spread across almost the whole of each segment time interval Δt_n but then: $\Delta t_n = (\pi / \alpha)\, \Delta T_n \cong 430\, \Delta T_n$. In this case, assuming that the polygonal orbit corresponding to the n^{th} quantized energy level consists of $2\kappa_n$ equal segments, then:

$$\Upsilon_n = 2\kappa_n \Delta t_n = 2\pi R_n / V_n \cong 2\pi \rho_n / V_n = n\, 2\pi c\, (\rho_n / c) / V_B = n\, (2\pi / \alpha)\, \Delta T_n \quad \therefore \quad 2\kappa_n\, (\pi / \alpha)\, \Delta T_n \cong n\, (2\pi / \alpha)\, \Delta T_n$$

$$\therefore \ \kappa_n \cong n$$

Thus, if the discrete Bohr model is to comply with the short-range impulse theory then the polygonal orbit corresponding to the n^{th} quantized energy level must consist of **only 2n equal segments**. This is a radically different result from the original (impulse) Bohr model that only had to comply with quantization of the kinetic action and the intuitively appealing 'exact-correlation' assumption (i.e. $\Delta t_n = \Delta T_n$) when the polygonal orbit had very many segments $2G_n \cong 860\, n$.

The unit impulsive (dynamic) action: $\Delta \mathcal{A}^D_n = \lozenge[\mathbf{R}_n] = x_n \Delta \mathbf{I}_n = \rho_n \Delta \mathbf{I}_n = \rho_n\, 2m w_n = m\, V_n\, 2D_n = m\, V_n^{\,2}\, \Delta t_n = h / 2$

The unit kinetic action (per segment traversal): $\Delta \mathcal{A}^K_n = m\, V_n\, 2D_n = h / 2 \quad \therefore \quad \Delta \mathcal{A}_n = \Delta \mathcal{A}^K_n + \Delta \mathcal{A}^D_n = h \quad \therefore \quad \mathcal{A}_n = 2n\, h$

Comparison with the total action per cycle in the Bohr model shows: $\mathcal{A}_n / n\mathcal{A}_B = 2$ or twice the action, as the Bohr model only includes the kinetic part of the action and ignores the dynamical (EM) part of the interaction (only uses electrostatics).

The total action $\mathcal{A}_n$ per cycle in the n^{th} energy state in the new theory includes the $2\kappa_n$ traversals across all the segments and the $2\kappa_n$ impulses, so that $\mathcal{A}_n = 2\kappa_n \Delta \mathcal{A}_n$, where $\Delta \mathcal{A}_n$ is the full action per segment. This can be compared with Bohr's $n\mathcal{A}_B$.

It is interesting to compare these two 'trajectory' models of the hydrogen atom. In the Bohr model, the electron is always continuously attracted by the proton in the nucleus, resulting in continuously accelerated motion in a circle. This should generate spherically symmetric EM radiation, according to Maxwell's EM theory, radiating away all the electron's energy in a fraction of a second, with the electron falling into the nucleus. The (arbitrary but necessary) quantization condition on the action acts mathematically to 'prevent' this catastrophe. In the impulse theory, there are only periodic instants of acceleration of the orbiting electron but the EM 'radiation' is directed exclusively at a nuclear positron (one of the components of the proton) in the ray-like, saturated, periodic impulse form of the EM interaction that lies at the heart of this new theory. There is therefore no external radiation unless the orbiting electron explicitly participates in an interaction with an external electron. This will be the model adopted in this programme and the results will be analyzed in much greater detail in later papers.

The problem with the Bohr model of the hydrogen atom is that every solution retains the Coulomb Law of electrostatic force that cannot be converted into an impulse model because any such attempt must spread the static (and continuous) force over a time δt_n. This impulse duration must be about as small as the segment temporal interval Δt_n. But this interval is much larger (430) than the optical duration ΔT_n which is itself $2n^2/\alpha^2$ (40000 n^2) larger than the chronon τ which is the natural minimum temporal duration. However first, the radical *linear* (or 'small segment number') model will be explored briefly.

The Impulse Linear Model

The n^{th} state of the discrete Bohr model was shown above to correspond to the impulse model using 2G segments ($G = \kappa_n$). The Linear Model rejects the EM 'broadcast' model and so adopts a radically different starting assumption, namely:

Hypothesis: The principal quantum number n equals the number of impulses needed to cross the atom G (or $G = n$).

This is called the 'Linear Model' because in the lowest energy (or ground) state only one impulse is required to reverse the orbital electron at each extremum, resulting in an oscillating one-dimensional (linear) trajectory. This orbit does go through the nucleus (as implied by modern QM where the ground state has zero angular momentum) but since the interaction is now discontinuous the electron crosses the nucleus when the nucleus is **not** ready to undergo another interaction (in contrast to QM where the EM interaction is viewed as continuous i.e. acting at all times with an infinite force at zero separation).

The universal (closed trajectory) quantization kinetic and dynamic action rules for the j^{th} trajectory segment are:

(A) $\Delta \mathcal{A}^K_j = m\, \underline{u}_j \bullet \underline{u}_j \Delta t_j = h / 2$ (B) $\Delta \mathcal{A}^D_j = \underline{x}_j \bullet \Delta \underline{\mathbf{I}}_j = -S_j \Delta \mathbf{I}_j = h / 2$

Only the linear ground-state will be examined; it will be denoted by the subscript L for the **L**inear and B for the **B**ohr model. The smallest orbital distance is then R_L corresponding to the Bohr radius R_B with corresponding orbital speeds u_L and V_B. The linear model in this presentation cannot predict the ground-state energy level so it will use a free parameter β: $u_L \equiv \beta c$. The linear ground-state trajectory consists simply of two segments (G = 1); each segment corresponds to the orbital electron moving once across an orbital diameter ($2R_L$) in a segment time Δt_L, so that the total orbital period is $T_L = 2\,\Delta t_L$.

$$\therefore \quad u_L = 2R_L / \Delta t_L \quad \therefore \quad \Delta \mathcal{A}^K{}_L = m\, \underline{u}_L \bullet \underline{u}_L\, \Delta t_L = h/2 \quad \therefore \quad 4\, m\, u_L\, R_L = h$$

This is consistent with (B).

$$\Delta \mathbf{I}_L = \Delta P_L \quad \therefore \quad h / 2\, R_L = 2\, m\, u_L \quad \therefore \quad 4\, m\, u_L\, R_L = h \quad \therefore \quad R_L = h / 4\, m\, \beta\, c$$

It is the fact that both these action rules result in the same equation for R_L that prevents this theory (at present) predicting β. But the energy levels of these two models must be identical, so the speeds must be the same; i.e. $\beta = \alpha$.

$$\therefore \quad u_L = V_B = \alpha c \quad \text{but} \quad 4\, m\, V_B\, R_B = h \quad \therefore \quad R_L = R_B \quad \therefore \quad \mathcal{K}_L = \tfrac{1}{2}\, \alpha^2 mc^2$$

Thus, the linear ground-state atomic extent is exactly the same as the one calculated from the Bohr model. In the linear model, the role of the nuclear electrons is critical – the net interactions between the nuclear electrons and positrons determine **when** the nucleus can interact with the orbital electrons. In other words, *atomic* physics is intimately tied into *nuclear* physics through the scaling factor α. An extended model of the nucleus (with a derivation of α) and a more comprehensive linear model of the atom, including multi-electron atoms, will be presented in later papers.

Estimating the Quantum Limit

These new universal parameters can be estimated from the Bohr model by assuming that the maximum radius of the impulse polygon (ρ_H) occurs (not at infinity, as in the standard Bohr model) but when the EM impulse is first reduced to the constant long-range value $\Delta \mathbf{I}_0$. Further spatial separation from the nucleus will then be insufficient to keep the orbital electron bound into the atom; this is also the value when the optical path-length (from the nucleus) to the orbital electron equals T_S. The key part of this model is the presence of a third electron (the 'ionizing' electron) that interacts with the orbital electron remotely (i.e. at a 'far' separation) to reduce the orbital electron's speed to zero when the atom is then ionized. The orbiting electron is initially in the ground-state (i.e. its speed is V_B) and is located at a distance ρ_B from the nucleus (which defines the origin). The remote electron interacts N_H times through the 'far' impulse $\Delta \mathbf{I}_0$, reducing its speed by $b = c / N_0$ each time.

$$\therefore \quad N_H\, b = V_B = \alpha c \qquad \therefore \quad N_H = \alpha\, N_0 >> 1 \quad \text{where} \quad \alpha \cong 1/137$$

By design, ionization occurs when the short-range impulse first equals the smallest value of the 'near' impulse: $\Delta \mathbf{I}_S = \Delta \mathbf{I}_0$

$$\therefore \quad \Delta \mathbf{I}_S = h / 2\rho_H = 2\, m\, b \qquad \therefore \quad \rho_H = N_0\, h / 4\, mc = D_S = c\, T_S$$

The maximal discrete Bohr orbital occurs when the principal quantum number is n_H such that: $\rho_H = (n_H)^2\, \rho_B$

$$\text{Since} \quad \rho_B = h / 2\pi\alpha\, m\, c \quad \text{then} \quad (n_H)^2 = \alpha\, (\pi/2)\, N_0 = (\pi/2)\, N_H$$

The repulsive 'Terrible-Twins' model investigated previously [110] proved, that for large n, each electron moves Z_n.

$$Z_n \cong n^3\, \Lambda / 3\, N_0 \qquad \text{where} \quad \Lambda = c\, \tau = \alpha\, h / 2\pi\, m\, c$$

$$\therefore \quad \rho_H = (N_H)^3\, \Lambda / 3\, N_0 = (N_H)^2\, \alpha\, \Lambda / 3 = (N_H)^2\, \alpha^2\, h / 6\pi\, m\, c = (N_0)^2\, \alpha^4\, h / 6\pi\, m\, c = N_0\, h / 4\, mc$$

$$\therefore \quad N_0 = 3\pi / 2\alpha^4 \quad \therefore \quad N_H = 3\pi / 2\alpha^3 \quad \therefore \quad (n_H)^2 = 3\pi^2 / 4\alpha^3 \quad \therefore \quad b = (2\alpha^4 / 3\pi)\, c \cong (c/6) \times 10^{-10}$$

Approximately: $N_0 \cong 1.65 \times 10^{11}$ $N_H \cong 1.2 \times 10^9$ $n_H \cong 4{,}360$

and: $b \cong 2$ mm/s $\rho_H \cong 2 \times 10^7\, \rho_B \cong 1$ mm $T_S \cong 3 \times 10^{-12}$ seconds.

The maximum short-range (D_S) is then about 0.1 cm so that even some macro-molecular phenomena are 'short-range'.

7. DECONSTRUCTING DIRAC

NEW INTERPRETATIONS

This section will refocus on Dirac's most famous equation. It will attempt to deconstruct Dirac's final views on his greatest contribution described in chapter XI of the fourth and final edition of his masterpiece *Principles of Quantum Mechanics* [14]. This title was consciously chosen by Dirac to allude to the most famous text in physics written by the greatest Lucasian chair, in this case, he had no need to qualify the word *principles* with the adjective *mathematical* as Dirac always viewed quantum mechanics (QM) as a mathematical description of the natural world at its deepest levels.

The term 'deconstruction' has been chosen deliberately for this chapter to confess to using some of the techniques in post-modern literary criticism that need to be applied to the arrogance of modern science. These techniques expose and challenge implicit assumptions, especially categories and concepts that are assumed to be so well-established that they are now viewed as 'true'. Historical context is a major part of this process as is the resurrection of alternatives that have been widely assumed to have been disposed of. It is appropriate that this technique was developed by philosophers: a profession dedicated to the clarification of the meaning of concepts but now rejected by most theoretical physicists. In addition, contemporary theoretical physics has degenerated into a private language, only open to those who have extensively studied arcane mathematics.

It is one of the ironies of history that Dirac's revolutionary papers published in 1928 - 1930 emphasized Lorentz invariance and only incidentally provided a mathematical discovery of 'spin' and the possibility of a positive particle. After thirty years, Dirac's final views had changed very little with the final version of his classic text on quantum mechanics still emphasizing these same features while the historical view focused almost exclusively on his 'prediction' of the positron. Dirac was also famous for his honesty and he was always unhappy with how this prediction emerged from his theory. In fact, the positron only warrants less than three pages in the final edition of Dirac's best-selling text appearing in 1958, perhaps because this particle appeared as a consequence of the anomalous, negative energy states that just 'popped out' of his innovative matrix mathematics. His final exposition on the positron is discussed at the same time as his brief review of the negative states: a feature that his fiercest rival, Pauli seized on and Dirac was always uncomfortable with.

This chapter will return to Dirac's text several times in this chapter to illustrate his final views on his most famous equation; there is value in this process because this magnificent text is rarely studied in great detail today, even by graduate students in physics who only receive summary expositions from their more modern textbooks. Like all the greats in physics, there is no substitute for reading their own words directly.

Particles, not Waves

Although Dirac had a preference for the particle view of the world he could not escape the consensus of his peers that QM was also about waves, so he developed a 'wave equation for the electron'. Again, this programme rejects this contraction between two diametrically opposed concepts and builds its fortress on the hill exclusively on the idea that reality is discrete and matter only exists as point particles – waves are viewed only as a particular mathematical representation that can be useful in certain situations but carries no ontological significance: electrons **are** particles, **not** waves.

Positrons not Anti-Matter

The new theory rejects all mystical attempts to introduce concepts such as 'anti-matter'; positrons are viewed here as just particles that are as equally real as negative electrons, they are not ephemeral but are equally eternal. The new theory also views all other fundamental particles as small dynamic sets of positive and negative electrons. This hypothesis will be explored in a later series of papers ('Nuclear Electron Mechanics').

Classical-Quantum Hybrid

Dirac wished, like almost all his contemporary (and today's) theorists, to retain Maxwell's Equations for representing EM activity by 3D vector fields. It was only later that he succeeded in quantizing these fields but since he wanted to show the magnetic effects of his new equation he chose to retain the non-operator form even in his final edition. The present theory has already demonstrated [138] that Maxwell's EM model is simply a mathematical conversion from the integral form that summarized macroscopic experiments down to the equivalent differential forms, all at the same single time. This ignores the fact that details can always be aggregated to averages but the reverse transformation is illogical: there are **no** magnetic fields.

7.1 DIRAC'S EQUATION

7.1.1 MAJOR PROBLEMS WITH DIRAC'S THEORY

Dirac's Motivation

Dirac began his short (23 pages) penultimate chapter with the confession that the theory of QM that he had been presenting up to this point is a non-relativistic theory: "an analogue of classical non-relativistic dynamics". He desired to extend this so that it would be "invariant under Lorentz transformations, conforming to the special theory of relativity". This is a common misconception: that SRT is simply a theory whose equations must conform to this mathematical transformation. It fails to recognize the classical Maxwellian roots of Einstein's theory and fails to acknowledge that this was a kinematic theory that excludes accelerations and therefore forces and dynamics. Dirac wrote that this invariance was "necessary in order that the theory may apply to high-speed particles." Ironically, as admitted by Dirac, some of the major results he developed in this chapter only apply to low-speed situations. A deeper irony is that Maxwell would have rejected any suggestion that any of the foundations of electricity had any particulate basis – the continuous field was his physical model of electromagnetism.

Once again, Dirac applied his infinitesimal approach to his transformations, now extended to the four covariant differentials dx_μ, with x_0 representing the spatialized time dimension (i.e. $x_0 = c\ t$); the four operators $\partial/\partial x^\mu$ form the 4-vector contravariant components associated with the momentum operators (i.e. $p_\mu = i\ h_D\ \partial/\partial x^\mu$). Since Dirac was implicitly making the Continuum Assumption (forces apply continuously) he was content to work with "vanishingly small" quantities in order to prove that various expressions in his theory satisfy an infinitesimal Lorentz transformation so that he could then invoke the suggestion that "a finite Lorentz transformation can be built up from infinitesimal ones".

Planck's Mistake

Planck's relativistic mass-energy formula ($E^2 = P^2 c^2 + m^2 c^4$) is usually justified [9] because its low-speed limit ($v << c$) evaluates to the Newtonian form of kinetic energy ($\mathcal{K} = ½\ m\ v^2$) plus a fixed amount, interpreted as the 'rest' energy ($m c^2$). It is rarely mentioned that SRT is a theory of systems moving at constant velocity; in other words - kinematics. This was ignored by Planck in developing this relativistic formula, which also justifies the so-called Einstein mass-energy equation: $E = M c^2$, where M is the effective or relativistic mass of the particle that uses the Lorentz factor: $M[v] = m\ (1 - v^2 / c^2)^{-½}$. Planck's Proposal for relativistic momentum ($\underline{P} = M\ \underline{v}$) was also analyzed extensively in an earlier paper [9], which clearly demonstrated that this mathematical hypothesis was **not** consistent with the physics of electromagnetism. This key equation in relativistic particle mechanics was based only on the effects of a fictitious constant force that increased the effective mass M of a point particle with speed. The embarrassing fact that this is not a feature of the electromagnetic interaction between electrons does not seem to generate any dissonance in the minds of theorists who use this equation in EM on a daily basis.

The Electron Wave Equation

Dirac wished to build on his non-relativistic Hamiltonian approach so he began with Planck's mass-energy formula for a single 'free' particle, that is, for an electron moving in the absence of an electromagnetic field. The standard quantization 'rule' in canonical QM is to just replace the momentum $\underline{P}$ with the differential operator '$-i\ h_D\ \nabla$' in the corresponding Hamiltonian $\mathbf{H}$ whose eigenvalue is the energy E. This would mean that, following the standard QM 'cookery rules', the relativistic 'free' electron Hamiltonian operator would take the form:

$$\mathbf{H} = \pm c \sqrt{(m^2 c^2 - h_D{}^2 \nabla^2)}$$

Mathematically, the expression under the square root is completely undefined; worse the root can only be expanded as a power series when $P << m\ c$, i.e. well below relativistic speeds. It can be readily seen why the founders of modern QM where relieved when Dirac produced his linear form of the Hamiltonian and Schrödinger's relativistic version was quietly dropped.

Dirac chose to multiply this 'root' equation by its positive complement to create the standard 'difference in squares' form:

$$\{P_0{}^2 - (\underline{P} \bullet \underline{P} + m^2 c^2)\}\ \psi = 0$$

Dirac accepts this expression as his fundamental starting point for his whole theory with the cavalier justification that this will be "more convenient". As described earlier (see §2.2.2) Dirac linearized the momentum quadratic form by introducing his famous (4x4) α_μ matrices. This produced the famous free electron Relativistic Wave Equation:

$$(c \underline{\boldsymbol{\alpha}} \bullet \underline{P} + \boldsymbol{\alpha}_4 m c^2) \Psi_\mu = \pm E_\mu \Psi_\mu$$

Linearization is achieved by imposing the condition: $\alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu = 2 \delta_{\mu\nu}$ for μ & $\nu = 1, 2, 3, 4$

Actually, Dirac did not view this form of his equation as the most fundamental; rather, it was the form that included the four EM potentials A_μ that he assigned this status by "following the **classical** rule of replacing P_μ by $P_\mu + e/c\, A_\mu$" to give:

$$\{ \underline{\boldsymbol{\alpha}} \bullet (\underline{P} + e/c\, \underline{A}) + \boldsymbol{\alpha}_4 m c \pm (P_0 + e/c\, A_0)\} \Psi_\mu = 0$$

Dirac's Physical Model

Once again, Dirac had created a "semi-quantized" view of the world. The 'target' electron is fully quantized by describing it by a set of continuous wave functions (the Ψ_μ) that are the object of partial differential operators (the P_μ) but all the other electrons that generate the EM field are just aggregated away into simply a set of classical fields at the electron's location. The equivalent use of potentials (to derive the fields) is similarly only a statistical approximation – there are only electrons. Dirac was mistaken in following tradition and attempting to find a **single-time** model to describe the motion of an isolated (single) electron. Electrons are inherently interacting objects, so that dynamics must always involve two or more electrons where no one electron has primacy in claiming ownership of the universal time parameter. This implies that a more accurate model of the electron must explicitly introduce both symmetric delayed and advanced interactions: or multi-time (and space) trajectories involving both past and future 'other' times relative to the 'now' of the target electron – 'free' electrons are quite illogical. The present theory rejects this asymmetry and restores the original source electrons to an equal status to the target electron. This fully symmetric viewpoint will be elaborated in several of the subsequent papers.

Probability Density

In order to retain his definition of a positive probability density $\mathscr{P}$, as the sum of the squares of its four components, Dirac had to continue to use the Hermitian conjugate $\boldsymbol{\Psi}^\dagger$; so that: $\mathscr{P}(\underline{x}; t) = \boldsymbol{\Psi}^\dagger(\underline{x}; t)\, \boldsymbol{\Psi}(\underline{x}; t) = \sum \psi_\mu^*(\underline{x}; t)\, \psi_\mu(\underline{x}; t)$. Moreover, to retain a Lorentz-invariant form of the probability density that keeps the total electric charge e constant, it is also necessary to multiply the probability density by the infinitesimal 3D spatial volume '$dx\, dy\, dz$'. However, for constant electron motion in the longitudinal direction (say, z) special relativity predicts that dz will suffer a Lorentz-contraction; this means that $\mathscr{P}$ must transform like the fourth component of a relativistic 4-vector. Feynman simplified this by realizing [139] that he could define a set of (4 x 4) matrices $\boldsymbol{\gamma}_\mu$ that transformed like a relativistic 4-vector (see §3.1.2) but this then needed a new form of the complementary wave-function called an 'adjoint' $\boldsymbol{\Psi}^\S$. This involved an additional multiplication by the scalar (or odd, 'temporal') alpha matrix $\boldsymbol{\alpha}_0$, which reverses the otherwise negative signs in the 3rd and 4th rows; thus: $\boldsymbol{\Psi}^\S = \boldsymbol{\Psi}^\dagger \boldsymbol{\alpha}_0$. This 'twisted' requirement leads immediately to the definition of a Lorentz-invariant 4-vector electron current, $\mathbf{J}_\mu$.

$$\mathbf{J}_\mu = c\, \boldsymbol{\Psi}^\S \boldsymbol{\gamma}_\mu \boldsymbol{\Psi} = c\, \boldsymbol{\Psi}^\dagger \boldsymbol{\alpha}_0 \{ \boldsymbol{\gamma}_0 ; \boldsymbol{\gamma}_k \} \boldsymbol{\Psi} = \{ c\, \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi} ;\ c\, \boldsymbol{\Psi}^\dagger \boldsymbol{\alpha}_k \boldsymbol{\Psi} \} = \{ c\, \mathscr{P};\ \mathbf{J}_k \} \quad \therefore \quad \mathbf{J}_k = c\, \boldsymbol{\Psi}^\dagger \boldsymbol{\alpha}_k \boldsymbol{\Psi}$$

This preserves the covariant form of the equation of continuity ($\partial/\partial x_\mu \bullet \mathbf{J}_\mu = 0$) but now needs the adjoint form, $\boldsymbol{\Psi}^\S$.

The Irrelevance of Relativity

Dirac's rivals were most impressed that he had created a single equation that merged the new quantum mechanics with the radical Special Relativity Theory (SRT) that was beginning to enjoy wider acceptance by 1925 [140]. Since this research programme does not accept that SRT has any relevance beyond a mathematical constraint on field theories, it plays no role in the present two-electron model of electromagnetism. This rejection is reinforced by the lack of physical content in today's presentations of SRT that are developed simply as mathematical manipulations of empty reference frame parameters. The principal approach today seems to be the investigation of transformations of the space and time co-ordinates of single points ('events') that respect light-cone invariance ('boosts'). The present theory also acknowledges the centrality of the 'Light-Cone' condition [141] but interprets this as the fundamental physical constraint on two remote electrons participating in a series of mutual interactions: no interaction can occur unless each electron is **on** the other electron's 'light-cone'.

7.1.2 TRAVELING AT LIGHT-SPEED

Free Electron Motion

Dirac demonstrated [142] that the **average** speed of the 'free' electron in his model was ± *c*. Once again, he relied on the non-relativistic Heisenberg picture of QM to define the time dependence of the electron's position operator $\underline{\mathbf{X}}$; in other words, by evaluating the velocity operator $\mathbf{V}_k$ in the kth spatial direction and noting that the eigenvalues of each $\boldsymbol{\alpha}_k$ are always ±1. Dirac used his transformation techniques of non-relativistic QM (his earlier chapter 5), where he showed:

$$i\,h_D\,d/dt\,[\,\mathbf{X}_k\,] = \mathbf{X}_k\,\mathbf{H} - \mathbf{H}\,\mathbf{X}_k = i\,h_D\;\partial/\partial\mathbf{P}_k\,[\,\mathbf{H}\,] \quad \therefore\; d/dt\,[\,\mathbf{X}_k\,] = \partial/\partial\mathbf{P}_k\,[\,\mathbf{H}\,] \quad \text{a direct classical analogue.}$$

$$\therefore\quad \mathbf{V}_k = d/dt\,[\,\mathbf{X}_k\,] = \partial/\partial\mathbf{P}_k\,[\,\mathbf{H}_0\,] = c\,\boldsymbol{\alpha}_k \qquad \text{since}\quad \mathbf{H}_0 = c\,\underline{\boldsymbol{\alpha}}\bullet\underline{\mathrm{P}} + \boldsymbol{\alpha}_4\,m\,c^2$$

Dirac concluded that: "a measurement of a component of the velocity of a free electron is certain to lead to the result ± c." He referred to this extremely rapid oscillation as an electron 'jitter' or more often by the more impressive German term '*Zitterbewegung*'; it has never been given a satisfactory explanation, so it is often just ignored, particularly as the theory of relativity forbids all real objects traveling at light-speed. Dirac tried to sweep this 'anomaly' under the rug by claiming that this is the velocity at one instant of time, while observed velocities are always average velocities through appreciable time intervals – a convenient redefinition of Newton's ancient mathematical limit. Even if the positive and negative values in any one direction average to zero, the squares of these components do not cancel indicating the average speed is $c\sqrt{3}$ – oops!

No further discussion is included, as this would lead to the embarrassing conclusion that this rapid form of local electron acceleration should (according to Maxwell's theory) generate significant radiation that would quickly deplete the energy of every electron (unless it is viewed as intrinsic). This very rapid motion is compounded by Dirac's definition of the 4-vector electron current, $\mathbf{J}_\mu$, where the spatial component $\mathbf{J}_k$ must be interpreted as the probability of the electron crossing a unit area orthogonal to the spatial direction $\underline{k}$ per unit time; only in the longitudinal direction of motion is the transverse area constant.

Interpreting Zitterbewegung

Dirac actually solved the equation of motion for the free electron in his section 69, in the non-relativistic limit of low speeds. This will be expanded here as Dirac leaves his analysis at the level of an operator / matrix equation; useful insights can be reached by pushing the analysis into average-measured (expectation) values.

Even though these $\boldsymbol{\alpha}_k$ matrices are simply constant (4 x 4) matrices in the Schrödinger picture, Dirac assumed they have explicit time-dependence by applying the Heisenberg approach to them, leading to the result that the velocity also contains a very high frequency "oscillatory part" as is shown next, using the conditions that the energy and momentum are constant:

$$i\,h_D\,d/dt\,[\,\boldsymbol{\alpha}_k\,] = \boldsymbol{\alpha}_k\,\mathbf{H}_0 - \mathbf{H}_0\,\boldsymbol{\alpha}_k = 2\,\boldsymbol{\alpha}_k\,\mathbf{H}_0 - 2\,c\,\mathbf{P}_k \qquad \therefore\quad i\,h_D\,d/dt\,[d/dt[\,\boldsymbol{\alpha}_k\,]] = 2\,d/dt[\,\boldsymbol{\alpha}_k\,]\,\mathbf{H}_0$$

$$\therefore\quad d/dt\,\boldsymbol{\alpha}_k[t] = \{d/dt\,\boldsymbol{\alpha}_k[0]\}\;\exp(-2i\,\mathbf{H}_0\,t/h_D) \quad \therefore\quad \boldsymbol{\alpha}_k[t] = (i\,h_D\,\{d/dt\,\boldsymbol{\alpha}_k[0]\}\;\exp(-2i\,\mathbf{H}_0\,t/h_D)/2 + c\,\mathbf{P}_k)\,/\,\mathbf{H}_0$$

$$\therefore\quad \mathbf{X}_k[t] = \mathbf{X}_k[0] + (\,c^2\,t\,\mathbf{P}_k - c\,h_D^{\,2}\,\{d/dt\,\boldsymbol{\alpha}_k[0]\}\,\exp(-2i\,\mathbf{H}_0\,t/h_D)\,/4\,\mathbf{H}_0)\,/\,\mathbf{H}_0$$

This was Dirac's final presentation of the free electron's equation of motion in this section of his final text edition.

$$i\,h_D\,\{d/dt\,\boldsymbol{\alpha}_k[0]\}\,\exp(-2i\,\mathbf{H}_0\,t/h_D) = 2\,\boldsymbol{\alpha}_k\,\mathbf{H}_0 - 2\,c\,\mathbf{P}_k \quad \therefore\; h_D\,\{d/dt\,\boldsymbol{\alpha}_k[0]\} = -2\,i\,\exp(2i\,\mathbf{H}_0\,t/h_D)\,(\boldsymbol{\alpha}_k\,\mathbf{H}_0 - c\,\mathbf{P}_k)$$

$$\therefore\quad \mathbf{X}_k[t] = \mathbf{X}_k[0] + (\,c^2\,t\,\mathbf{P}_k + i\,c\,h_D/2(\,\exp(2i\,\mathbf{H}_0\,t/h_D)\,\boldsymbol{\alpha}_k[t]\,\exp(-2i\,\mathbf{H}_0\,t/h_D) - c\,\mathbf{P}_k/\mathbf{H}_0)\,)\,/\,\mathbf{H}_0$$

This is both an operator and (4x4) matrix equation. When the operators are applied to any one of the four 'free' electron spinor states, Ψ_μ then the resulting matrix equation is:

$$\therefore\quad X_k[t] = X_k[0] + V_k\,t + i\,(\,\exp\{2i\,M\,c^2\,t/h_D\}\,\boldsymbol{\alpha}_k[t]\,\exp\{-2i\,M\,c^2\,t/h_D\} - V_k/c\,)\,h_D/2M\,c$$

At non-relativistic speeds: $M \approx m$ and using the classical electron radius: $\Lambda_0 \equiv e^2/m\,c^2 \equiv \alpha_D\,h/2m\,c$ & $\Lambda_0 \equiv c\,\tau$

$$\therefore \quad X_k[t] = X_k[0] + V_k\, t + i\, D_k[t] \quad \& \quad D_k[t] = \left(\exp(i\, 2\pi\, \alpha_D\, t / \tau) \boldsymbol{\alpha}_k[t] \exp(- i\, 2\pi\, \alpha_D\, t / \tau) - V_k / c \right) \Lambda_0 / 2\pi\, \alpha_D$$

The first two terms are the classical result: the electron moves along the k direction at a speed V_k from its initial location $X_k[0]$. The third term is both imaginary and the logical consequence of Dirac's initial relativistic equation; the bracketed expression is of order (1 – v/c) or approximately unity if $v << c$, while $\boldsymbol{\alpha}_k$ always has an eigenvalue of ± 1. This means (see §4.2.2) that the third term is of spatial magnitude ($\Lambda_0 / 2\pi\, \alpha_D$) or about $R_B / 274$ – that is about 2×10^{-11} cm. The exponential factors imply that the electron is oscillating at a cycle time of τ / α_D or about $430\, \tau$ – about once every 4.3×10^{-21} secs.

This should be contrasted with the digital model presented here (see §5.2) where the free electron moves along one direction at a constant speed while 'clicking' around its axis at a distance $\Lambda_0/2$ (about 137 times smaller) in a cycle time of 4τ (about 107 times shorter). Note, the factor of 2π is due to the square 'orbit' in the digital model versus Dirac's circular oscillations.

Dirac's '*Zitterbewegung*' was a continuum attempt to represent the two points in space, used to define velocity, that are here separated by a distance $c\tau$ in the limit that τ goes to zero. Dirac's 'Zitterbewegung' can now be readily interpreted in terms of the 4-step, cyclic transverse motion of every electron. Every determination of a particle's velocity actually requires two points in space to define its position and these must be possible interaction times, which are closest across a single chronon τ when the target electron is at locations x_1 and x_2 at times t_1 and t_2, where $x_2 = x_1 \pm c\,(t_1 - t_2)$ and $t_2 = t_1 + \tau$. Dirac had simply assumed the continuum limit ($\tau \rightarrow 0$) when all 4 points around the electron's basic cycle are superimposed at the same time. This is another example of how the finite duration between two consecutive interactions (explicitly recognized here through the foundational idea of the chronon) provides a physical explanation for quantum effects, whereas classical physics has always made (usually implicitly) the Continuum Assumption: in effect, assuming all velocity changes are infinitessimal.

Hamiltonian Mechanics is Non-Relativistic

The final problem of Dirac's Electron Theory as an ultimate model of nature was well known to Dirac, who frequently mentioned it (while still ignoring the problem). Dirac viewed the new QM, now using non-commuting dynamical variables, as a correspondence with classical mechanics whenever "a system is composed of a number of particles with instantaneous forces acting between them provided it is describable by a Hamiltonian function." This is obviously **not** the case with a comprehensive treatment of EM interactions with finite delays, nor is it consistent with the 4-dimensional equivalence of SRT; one particular time variable is chosen as the canonical conjugate of the Hamiltonian function. However, these basic relativistic constraints can be approximated away within the hydrogen atom when the instantaneous Coulomb interaction is regarded as the most important simplification; it is therefore not surprising that Dirac's Electron Theory could be applied to great effect to calculate the hydrogen spectrum and why it would fail, like ordinary QM, in all multi-electron situations. In an earlier paper [143], Dirac had recognized the superiority of the Lagrangian formulation of mechanics over the much more popular Hamiltonian method; this preference inspired Feynman (and, in a limited sense, this research programme).

7.1.3 NATURAL STATES

Quantum Observables

Dirac himself pointed out [14] that the general concept of a 'quantum observable' is incompatible with the central idea of superposition of states (that lies at the heart of QM) when applied to states with a relativistic space-time meaning. This is because any "observable may involve **physical things** at widely separated points at one instant of time." Fortunately, for Dirac, his theory has only been tested with the Coulomb model of the hydrogen atom, where electron velocities are so small that relativistic effects (i.e. time asynchronicities) remain only as tiny corrections to the old Bohr orbital model.

Reality vs. Measurement

Several of the most successful theoretical physicists of the Twentieth Century, including Heisenberg, Dirac and Feynman, had an intuitive preference for the *particle*-model of nature although the Zeitgeist in modern physics has always emphasized the continuous (and extended) *field* view of nature. The present asynchronous (or two-time) particle theory also drops the Lagrangrian mathematical apparatus that has been used since the mathematicians transformed Newton's model of many particle interactions into a single-time model of system components interacting via instantaneous (or local) potentials.
The present theory rejects the asymmetric interaction view implied by the merger of all the 'remote' sources of interaction with the target particle into an instantaneous, space-like potential (the preferred model in physics for 300 years).
This old perspective was criticized in greater detail in the previous paper [145]. The central idea of wave-functions (or eigenvectors of 'observables') is completely rejected in this programme. The idea of 'observables' in QM is viewed here as a

hopelessly tangled merger of the real movements of unobserved (i.e. natural) systems and the disturbing (and incalculable) effects of attempting to 'measure' such systems by gigantic human beings. This fundamental objection will be developed in later papers when a new formulation of quantum mechanics will be presented.

Asymmetric Electrons

Dirac's Relativistic Equation is deliberately asymmetric in its view of the electron. An equation is proposed to describe the 'free' motion of a single electron. All other electrons in the universe that might interact with this 'target' electron are then simply viewed from either a classical or pseudo-classical perspective, manifesting their significance only through their implied contributions to the EM fields defined by Maxwell's Equations; in this case, through the EM potentials (ϕ and $\underline{A}$). This approach has a long tradition that was reinforced by Lorentz in his Electron Theory [146], which explicitly separated out the 'target' electron from the interacting EM field carried by Maxwell's aether: this must be viewed as a weak approximation.

Send/Receive Binary Representation

One of the commonest methods to model the world is to divide it into two complementary parts, creating a so-called binary model. Arithmetic can be developed from the two concepts of 'one' and 'zero', logic can be grounded in the ideas of 'true' and 'false' (or 'not-true') while physics can be viewed as dividing the world into material things that 'exist' and everything else ('space'). The present theory also differentiates the two 'ends' of each interaction between two electrons into a 'send' action and a 'receive' action (see §4.1.8). These two states can be designated mathematically as |S> and |R>; these can also be given a simple (2 x 1) matrix representation, or in terms of their transpose (row) forms, <S| and <R|:

$$\langle S| = [\,1\;0\,] \text{ and } \langle R| = [\,0\;1\,] \quad \text{where the vector-product satisfies: } \langle a \mid b \rangle = \delta_{ab}$$

So the state of every electron at any time is either |S> or |R> ; with each electron's state alternating every chronon. At any (unknown) time this can be represented collectively as: $|S\rangle + i\,|R\rangle$. Arbitrary states can be related through a finite group of operators: { I , |S><S| , |R><R| , |S><R| , |R><S| }. This is the basic set of orthonormal (2 x 2) matrices. This binary division of nature is here extended into the temporal dimension: an electron may initiate an interaction ('send') with another electron on its own future light-cone (retarded) or it may 'send' this interaction backwards through time to an event on its historical or past light-cone (advanced). Obviously, the co-operating or 'partner' electron must play the corresponding 'rcv' role. This is the deeper, physical meaning of the **four** possibilities uncovered by Dirac with his relativistic equation.

Representing Asynchronous Send/Receive States

All (4x4) real matrices have real (4x1) eigenvectors that can be constructed from the four orthonormal base vectors.

$$|\,\mathbf{1}\,\rangle = (1\;0\;0\;0)^T \;;\; |\,\mathbf{2}\,\rangle = (0\;1\;0\;0)^T \;;\; |\,\mathbf{3}\,\rangle = (0\;0\;1\;0)^T \;;\; |\,\mathbf{4}\,\rangle = (0\;0\;0\;1)^T$$

These four states were also found by Dirac (see §3.1.6) but the correspondence is only exact in the Dirac model in the ultra classical limit of zero velocity. Two of these states resulted in negative energy solutions to the Dirac equation (see next) and arose because Dirac approached this problem from a mathematical, rather than physical basis, building on both standard QM and Planck's energy-momentum equation – each of which was driven by mathematical hypotheses rather than physics. This problem was illustrated here in section 5.1.3 when the middle two states (| 2 ⟩ and | 3 ⟩) were associated with the '*Elsewhere*' concept that was shown to be an inadequate representation of the physical situation. This was resolved by the *Canonical Phase* representation (see section 5.1.4) by combining two independent concepts: interaction and temporality, each of which involves two independent but complementary possibilities. The hypothesized EM interaction implies that the electron is always in one of two complementary states, called here 'send' or 'receive'; this cannot be represented by a single, real number. These two possibilities may be represented mathematically by either a single complex number (as in standard non-relativistic quantum mechanics) or by a (2 x 1) array of two real numbers. Additionally, the time-symmetric nature of this asynchronous interaction means that an electron can/may interact 'forward' in time (or retarded) or 'backward' in time (advanced). This doubles up the number of possible states in which any single electron can exist. These four states can be mapped into a double complex representation (quaternion) or by a (4 x 1) array of four real numbers, where the first two rows correspond to the retarded ('r' or '–' tag) states and the last two rows corresponding to the advanced ('a' or '+' tag) states.

This gives the four orthonormal states: | A : B >, where A represents 'S' or 'R' (send/ receive) and 'B' represents ' –' or '+' (retarded/advanced) interactions; these appear like:

$$\text{Retarded: } \langle S:-| = [\,1\;0\;0\;0\,] \text{ and } \langle R:-| = [\,0\;1\;0\;0\,] \;\&\; \text{Advanced: } \langle S:+| = [\,0\;0\;1\;0\,] \text{ and } \langle R:+| = [\,0\;0\;0\;1\,]$$

Where the extended vector-product satisfies: $< a : \lambda \mid b : \xi > \; = \; \delta_{ab} \; \delta_{\lambda\xi}$

These four electron states were investigated here in the *Canonical Representation* (see §5.1.4); these are the only four real possible orthonormal eigenvectors of any (4x4) mathematical 'space'. There are two distinct sequences of ordering these states that still maintain the constraint that a 'send' state must be followed by a 'receive' state. The canonical sequence is assigned to the negatively charged electron and the second or *Complementary Representation* (see §5.1.6) is assigned to the positron. In contrast, the four Dirac states are interpreted (according to standard QM) as as all 'existing' at the same time in some probabilistic sense for every electron.

These four basic states were assigned different meanings in Dirac's interpretation (see §3.1.6). The first two (in the non-relativistic limit) were interpreted as the spin up and spin down states for an electron with positive energy and the last two were interpreted as the spin up and spin down states for an electron in a negative energy state.

$$\text{Energy } E+: \quad \psi(1:\uparrow) = (1\;0\;0\;0)^T \quad \psi(2:\downarrow) = (0\;1\;0\;0)^T \quad E-: \quad \psi(3:\uparrow) = (0\;0\;1\;0)^T \quad \psi(4:\downarrow) = (0\;0\;0\;1)^T$$

The Canonical Representation was analyzed in terms of the set of four (4x4) transition matrices or transition operators $\mathcal{C}_\nu$ that transformed one phase-state vector into the next. These can be related to Dirac's four $\boldsymbol{\alpha}_\nu$ matrices (§3.1.2), as follows.

The correspondence is easier to see by using circular vectors (see §3.1.4) instead of the vectors defined along the x and y axes. This is because the transverse Pauli (2x2) 'spin' matrices (§3.1.1) take on a particularly simple form that can be decomposed.

$$\boldsymbol{\alpha}_+ = \begin{vmatrix} \emptyset & \sigma_+ \\ \sigma_+ & \emptyset \end{vmatrix} \quad \boldsymbol{\alpha}_- = \begin{vmatrix} \emptyset & \sigma_- \\ \sigma_- & \emptyset \end{vmatrix} \quad \boldsymbol{\alpha}_3 = \begin{vmatrix} \emptyset & \sigma_3 \\ \sigma_3 & \emptyset \end{vmatrix} \quad \boldsymbol{\alpha}_4 = \begin{vmatrix} \mathbf{I} & \emptyset \\ \emptyset & -\mathbf{I} \end{vmatrix} \quad \text{where} \quad \mathbf{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \;\&\; \emptyset = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$$

$$\therefore \quad \boldsymbol{\alpha}_+ = \begin{vmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \end{vmatrix} = \mathcal{C}_1^\dagger + \mathcal{C}_3^\dagger \quad \& \quad \boldsymbol{\alpha}_- = \begin{vmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{vmatrix} = \mathcal{C}_1 + \mathcal{C}_3$$

$$\therefore \quad \boldsymbol{\alpha}_3 = \begin{vmatrix} 0&0&1&0 \\ 0&0&0&0 \\ 1&0&0&0 \\ 0&0&0&0 \end{vmatrix} - \begin{vmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&1&0&0 \end{vmatrix} = (\mathcal{D}_1^\dagger + \mathcal{D}_1) - (\mathcal{D}_4^\dagger + \mathcal{D}_4) = (\mathcal{C}_1^\dagger \mathcal{C}_2^\dagger + \mathcal{C}_2 \mathcal{C}_1) - (\mathcal{C}_4^\dagger \mathcal{C}_1^\dagger + \mathcal{C}_1 \mathcal{C}_4)$$

$$\therefore \quad \boldsymbol{\alpha}_4 = \begin{vmatrix} 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{vmatrix} - \begin{vmatrix} 0&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \end{vmatrix} = (\mathcal{N}_1 + \mathcal{N}_2) - (\mathcal{N}_3 + \mathcal{N}_4) = (\mathcal{C}_1^\dagger \mathcal{C}_1 + \mathcal{C}_2^\dagger \mathcal{C}_2) - (\mathcal{C}_3^\dagger \mathcal{C}_3 + \mathcal{C}_4^\dagger \mathcal{C}_4)$$

This illustrates that Dirac's transverse $\boldsymbol{\alpha}$ matrices (1 and 2 or + and –) are moving the electron between his positive energy spin states #1 and #2 and between his negative energy spin states #3 and #4 – that is, just spin-flips, while preserving the energy signature. Dirac's longitudinal matrix $\boldsymbol{\alpha}_3$ generates two-step (or double) transitions between positive and negative energy states – that is, just energy-flips, while identifying the spin signature (with a sign change if the 'spin' is 'down'). Dirac's temporal matrix $\boldsymbol{\alpha}_4$ preserves the state but identifies if it is a positive or negative energy state. This correspondence gives new insights into the Dirac equation that was invented to solve a mathematical problem: in contrast, the present theory began with a set of physical hypotheses that were subsequently given a mathematical representation.

7.2 NOTHING THERE

7.2.1 NEGATIVE ENERGY PROBLEMS

No Escaping Negative Energy

Dirac's new equation of the electron offered a plethora of riches, now with four possible states in which an electron could be found – the problem (as Pauli was quick to point out) was that two of these solutions represented an electron with negative energy; this was a direct consequence of starting from Planck's quadratic energy-momentum equation. This negative energy state is a a physical impossibility, particularly for a 'free' electron in motion whose energy was only kinetic and since this energy of motion involved only the square of the electron's speed, it must be positive. At first, Dirac thought the positive energy gap represented by the 'rest energy' would provide a safety-barrier but again Pauli pointed out that, unlike classical particles, quantum objects could undergo large, random energy fluctuations and literally 'jump' this gap across to a negative state. Dirac was much vexed by this problem and since these anomalous states appeared to correspond to particles with a positive charge he first proposed that they represented the states of the nuclear proton – the only positively charged particle known at that time. Once again, further problems arose with this interpretation, not the least of which was that the proton's mass was almost 2000 times greater than that of the electron. The irony here was that it was relativity theory's identification of mass with energy that showed that this was impossible: the positive and negative energy states could have the same total energy but their corresponding masses had to be different: otherwise where did the proton's extra mass come from?

As was described in section two, all four solutions were necessary if Dirac's new theory was to be accepted – a position his nemesis (Pauli) was quick to point out. Worse, the faster an electron in a negative energy state moved the lower its total energy (more negative) would become and physicists were all convinced that nature preferred to seek out states of lowest energy for maximum stability. All the electrons in the universe should soon be occupying all such negative energy states. However, in spite of these difficulties, an increasing number of theorists chose to use Dirac's Equation to make electron-photon scattering calculations and the results appeared to agree well with experiments! The problem was that these new calculations had to use all four states; worse, for low-energy Thomson scattering, almost all the scattering resulted from intermediate states where the free electron only possessed negative energy [36]: an acutely embarrassing result.

7.2.2 DEEP, DARK SEA

Dirac's seething Vacuum

In 1929, Dirac was inspired by a recent suggestion from Weyl to interpret the negative energy states as representing protons. These ideas appeared the following year in his next relativistic electron paper *A Theory of Electrons and Protons* [37]. Here, Dirac acknowledged that the most stable states were the negative energy ones with very high velocity but he then made the radical hypothesis that all of the negative energy states were filled and Pauli's Exclusion Principle would forbid any positive energy electrons from dropping back into these occupied states. He first rejected Weyl's suggestion that the negative energy states could represent protons, as particles with negative energy have no reality in physics. He then used the analogy with closed-shell atoms (inert gases) that the absence of an electron from such a stable configuration (or 'hole') would behave like a particle that appeared as its complement to the rest of the universe. In particular, it would have positive charge and positive mass and energy. He identified this hole with the proton. He responded now publicly to a private criticism of Bohr that the infinite 'sea' of negative energy electrons would have an infinite divergence by the claim that only variations from the norm are observable experimentally. In reacting also to Heisenberg's private objection that the mass of the negative energy state should be identical to that of the electron rather than the much more massive proton, Dirac lamely expressed the hope that a future theory of the interaction between such disparate particles would account for the difference. Worse, other physicists soon showed that this model was unstable and protons would be rapidly annihilated by electrons. Eventually, by 1931, Dirac conceded defeat and in the brief introduction to a bizarre paper [38] that predicted the existence of a quantum magnetic monopole he agreed that a *hole* in the negative energy electron 'sea' would appear as an anti-electron: a positively charged particle with the same positive mass and energy as the corresponding negatively charged electron. Just for good measure, he also casually suggested that "the protons will have their own negative-energy states" and their 'holes' would appear as 'anti-protons'. These brief suggestions followed a long defense of his approach to quantum mechanics, where new mathematics was introduced first and only subsequently should a physical interpretation be searched for afterwards. This discussion was both an explanation for his reversal over his earlier proton suggestion and a further justification for the Weyl-like 'gauge' invariance of the wave function that Dirac believed must lead to the existence of the monopole. In the conclusion, Dirac admitted to a disappointment in not deriving the fine structure (electrical) constant rather than the never-discovered magnetic monopole.

An Invisible Sea of Electrons

Contemporary physics assumes a continuous time model for all elementary particles – with the focus always on the same single (but arbitrary) time. Dirac modeled the electron in terms of four orthogonal (4 x 1) column vectors, |υ>, normally labeled |1>, |2> , |3> and |4>. This is a double representation of the electron's spin 'up' and 'down' states. This illustrates Dirac's dilemma: once the 'spin-up' and 'spin-down' states have been assigned (to |1> and |2>) then how to interpret the other two states, |3> and |4> ? So, Dirac chose to interpret these two new states as the 'spin-up' and 'spin-down' states of the electron's 'anti-particle' at the same instant of time while realizing that both of these two states had negative energy and negative mass. Dirac circumvented this paradoxical situation with his invention of a new form of the vacuum **filled** with undetectable anti-particles that only become observable when they 'quantum-jump' from these negative states into their own complementary states with positive mass and positive charge, i.e. positrons. Later, Feynman interpreted the positron as a negative mass electron traveling backwards through time, which appears as a positive electron with positive mass and energy when viewed from our standard direction of the flow of time. Both of these well-known interpretations no longer view the vacuum as 'nothingness' but as an active medium filled with undetectable, negative energy electrons, which can be excited (at any time) into real existence.

A Hole is not an Particle

The most severe criticism of the hole theory was actually hinted at by Dirac himself when he indicated that he had to ignore all the interactions occurring between all the orbital electrons before he could propose that a missing electron in a 'closed-shell' atom was equivalent to the existence of a real electron. When interacting electrons are viewed from an ontological perspective, it is obvious that a 'hole' is not the same as the absence of a particle – the interactions of all the other negative energy electrons in the Dirac 'sea' with any one of the negative energy particles is not the same as not interacting with its absence, that is nothing. These serious objections have never been answered – just ignored or, more likely, just forgotten. Any interaction of this singular electron with remote electrons (treated as quantized EM fields) would induce further interactions with this invisible and infinite 'sea'. It is not surprising that the subsequent calculations of this theory of quantum electrodynamics (QED) always generate mathematical infinities: the so-called 'infinite self-energy' of the electron.

Dirac's variable Ontology

Fred Hoyle's admiration for Paul Dirac, who was his PhD supervisor from1939 until 1942, was understandable. However, this admiration prevented even this blunt Yorkshireman from observing a contradiction when he wrote about Dirac's views of the vacuum [147] as an "unobservable infinite sea of electrons occupying negative energy states" while a few pages earlier he approvingly quoted Dirac as saying "That which is not observable does **not** exist." Philosophy was not Dirac's strength.

No Vacuum Fluctuations

Since 'free' electrons in empty space have no quantization restrictions on their energy (which requires a finite 'box') they occupy a continuum. Once again, mathematical infinity intrudes on physics as this implies that there are an infinite number of free-electron states and therefore an infinite number of electrons in any finite space. Obviously, this is not observed with real (negatively charged) electrons because they strongly repel each other. This view forced Dirac to admit that "there be a distribution of electrons of **infinite** density everywhere in the world." [148] This presented Dirac with a quandary, so he had to define "a perfect vacuum as a region where all states of positive energy are unoccupied and all those of negative energy are occupied." Dirac fully accepted Maxwell's classical EM theory (based on a continuous medium) and stated that: "in a perfect vacuum Maxwell's equation (div $\underline{\boldsymbol{\mathcal{E}}} = 0$) must, of course, be valid." This means that the negative states have no effect on each other but they are still assumed to contribute observable effects on real electrons. The existence of this huge number of virtual 'electrons' is assumed to account only for the tiniest of QED corrections, such as the Lamb Shift; other than this, they continue to remain completely unobservable.

No Electron Self-Energy

In the present theory there is no infinite 'self-energy' as the electron is not modeled here as a real object of any finite size covered with repulsive electrical 'paint' (CEM) nor does it interact with an infinite number of negative-energy electron states or its own EM field (QFT). It only interacts with **one** other electron that is never at the same location in space at the same time.

7.3 ANTI-MATTER

Past and Future

The analysis of the four electron states in the present theory are associated with two pairs of interaction sets; electrons either request an interaction with another electron or wait for such request from another electron. Unlike Dirac, who assumed only a retarded view of the world, the present theory accepts a completely symmetric view of interactions across time. Mapping the four states in each theory (see section 7.1.3) provides an alternative interpretation of Dirac's negative energy states. The two positive mass solutions of Dirac's relativistic equation are interpreted here as the standard (negatively) charged electron either requesting a possible interaction with another electron somewhere 'on its future light-cone' or waiting for a possible interaction request from another electron somewhere 'on its past light-cone'. The 'negative mass' solutions of Dirac's theory are here interpreted as the standard (negatively) charged electron either requesting a possible interaction with another electron somewhere 'on its past light-cone' or waiting for a possible interaction with another electron somewhere 'on its future light-cone' that is capable of 'sending' a request back into its own past. In summary, standard mass ($+m$) represents the electron at its here and now participating in a *retarded* interaction while the so-called 'negative' mass ($-m$) electron can participate in an *advanced* interaction. In the present theory, both positive and negatively charged electrons always have positive mass (and positive energy) and can choose to react forward or backwards across time: there are **no** negative energy states.

7.3.1 ELECTRIC CHARGE

One Line of Force

Dirac has suggested [62] that the quantized nature of electric charge indicates that there might only be 'one line of force' between any two electrons. This indicates that Dirac considered the possibility that the EM interaction is **saturated**: at any one point in time, an electron only interacts with one other electron. This is now a **central** concept in the present theory.

Electron Charge as Rotation

Even though the present theory introduces an intrinsic rotation in the motion of every electron, this is viewed as only **part** of the mechanism of electron 'spin' (see section 7.4). This intrinsic (digital) rotation is viewed here as the basic distinction between negative and positive electrons. There are only two possible directions of rotation around the (longitudinal) axis of travel (see section 6.2.1), so there can only be **two** complementary values associated with the idea of intrinsic electric charge. This rotation is an invariant property of each one of these fundamental objects – once an electron rotates in one direction it will always rotate in that direction forever. This is why this concept is used to distinguish the electron's intrinsic charge, which also never changes whatever interactions the electron participates in – once a positron, always a positron.

7.3.2 POSITRONS

Positrons complement Electrons

Dirac's theory of positrons led to the idea that these are very rare forms of electrons that are occasionally discovered under rare circumstances and immediately disappear in a burst of EM energy ('photons') whenever they encounter a 'normal' electron. In contrast, the present theory regards positrons simply as the complement of negative electrons. The distinction is found in the 'hidden' digital motion that every electron participates in at all times. Both kinds of electrons undergo a 4-step clicking motion around their longitudinal axis of motion (see §5.2) – electrons click anti-clockwise while positrons click clockwise. The charge distinction reflects the out-of-phase offset of the interaction times of the positron relative to the time sequence of negatively charged electrons (see §6.2). It is probable that there are an equal number of electrons and positrons in the universe, which would then be electrically neutral (or symmetric) at all times but this cannot be verified experimentally so it remains only a philosophical conjecture. None-the-less, this theory predicts conservation of electric charge; it also fulfills Dirac's final hope that a new theory will be symmetrical between positive and negative electric charge.

Positrons hide in Neutrinos

In this theory, when a positron outside the nucleus encounters an electron at a close enough range then a stable configuration is created that is identified here with the particle known as the neutrino. This is an incredibly 'self-absorbed' or 'tight' mode of pair-wise interactions that persists until the neutrino comes very close to a nucleon that is able sometimes to separate them. This model is also developed in a later paper where all the fundamental particles are constructed just from electrons .

Positrons in the Nucleus

John Wheeler (1911-2008) was the first to propose [149] that the excess positive electrons are almost all inside the nuclei of atoms, which are themselves surrounded by negative electrons. In this theory, negative electrons are found in excess outside the nucleus of each atom as a consequence of the (arbitrary) choice that protons consist of pairs of positrons interacting with a single electron. That is, a consequence of the choice of electron involved in the universal interaction associated inwards to the center of each atom or outwards to the universe.

Interpreting Positron State Transitions

In section 7.1.3 an interpretation was offered for the physical meaning of Dirac's $\boldsymbol{\alpha}_\nu$ matrices in terms of transitions between the four electron states used in this theory that correspond to sending/receiving interaction requests both forwards and backwards in time. The explicit (4x4) matrices were compared with the matrices representing the four transition operators $\mathcal{C}_\nu$ that transformed one phase-state vector into the next. The canonical sequence is assigned to the negatively charged electron and the second or *Complementary Representation* (see §5.1.6) is assigned to the positron. In contrast, the four Dirac states are interpreted (according to standard QM) as all 'existing' at the same time in some probabilistic sense for every electron; the doubling of the two spin states was explained by Dirac as due to the occurrence of positive and negative energies.

The electron assignments were found to be:

$$\boldsymbol{\alpha}_+ = \mathcal{C}_1^\dagger + \mathcal{C}_3^\dagger \qquad \& \qquad \boldsymbol{\alpha}_- = \mathcal{C}_1 + \mathcal{C}_3$$

$$\boldsymbol{\alpha}_3 = (\mathcal{D}_1^\dagger + \mathcal{D}_1) - (\mathcal{D}_4^\dagger + \mathcal{D}_4) = (\mathcal{C}_1^\dagger \mathcal{C}_2^\dagger + \mathcal{C}_2 \mathcal{C}_1) - (\mathcal{C}_4^\dagger \mathcal{C}_1^\dagger + \mathcal{C}_1 \mathcal{C}_4)$$

$$\boldsymbol{\alpha}_4 = (\mathcal{N}_1 + \mathcal{N}_2) - (\mathcal{N}_3 + \mathcal{N}_4) = (\mathcal{C}_1^\dagger \mathcal{C}_1 + \mathcal{C}_2^\dagger \mathcal{C}_2) - (\mathcal{C}_3^\dagger \mathcal{C}_3 + \mathcal{C}_4^\dagger \mathcal{C}_4)$$

The previous assignments can be readily extended to the positron transitions, defined in terms of their own four transition operators $\mathcal{C}^\#_\nu$ and the earlier identities established between the two sets.

$$|\,\boldsymbol{\nu}\,\rangle = \mathcal{C}^{\#\,\dagger}_\nu\,|\,\boldsymbol{\nu-1}\,\rangle = \mathcal{C}_\nu^\dagger\,|\,\boldsymbol{\nu+1}\,\rangle \qquad \mathcal{C}^{\#\,\dagger}_\nu = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu-1}\,| = \mathcal{C}_{\nu-1}$$

$$\mathcal{C}^\#_\nu = |\,\boldsymbol{\nu-1}\,\rangle\langle\,\boldsymbol{\nu}\,| = \mathcal{C}_{\nu-1}^\dagger \qquad \mathcal{D}^\#_\nu = |\,\boldsymbol{\nu-2}\,\rangle\langle\,\boldsymbol{\nu}|\, = |\,\boldsymbol{\nu+2}\,\rangle\langle\,\boldsymbol{\nu}| = \mathcal{D}_{\nu+2}^\dagger = \mathcal{D}_\nu$$

$$\mathcal{N}^\#_\nu = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu}\,| = \mathcal{N}_\nu \qquad \mathbf{T}^\#_\nu = |\,\boldsymbol{\nu}\,\rangle\langle\,\boldsymbol{\nu+1}\,| = \mathbf{T}_{\nu+1}^\dagger$$

This means that the positron assignments are found to be:

$$\boldsymbol{\alpha}_+ = \mathcal{C}^\#_2 + \mathcal{C}^\#_4 \qquad \& \qquad \boldsymbol{\alpha}_- = \mathcal{C}^{\#\,\dagger}_2 + \mathcal{C}^{\#\,\dagger}_4$$

$$\boldsymbol{\alpha}_3 = (\mathcal{D}^{\#\,\dagger}_1 + \mathcal{D}^\#_1) - (\mathcal{D}^{\#\,\dagger}_4 + \mathcal{D}^\#_4) = (\mathcal{C}^{\#\,\dagger}_1 \mathcal{C}^{\#\,\dagger}_2 + \mathcal{C}^\#_2 \mathcal{C}^\#_1) - (\mathcal{C}^{\#\,\dagger}_4 \mathcal{C}^{\#\,\dagger}_1 + \mathcal{C}^\#_1 \mathcal{C}^\#_4)$$

$$\boldsymbol{\alpha}_4 = (\mathcal{N}^\#_1 + \mathcal{N}^\#_2) - (\mathcal{N}^\#_3 + \mathcal{N}^\#_4) = (\mathcal{C}^{\#\,\dagger}_1 \mathcal{C}^\#_1 + \mathcal{C}^{\#\,\dagger}_2 \mathcal{C}^\#_2) - (\mathcal{C}^{\#\,\dagger}_3 \mathcal{C}^\#_3 + \mathcal{C}^{\#\,\dagger}_4 \mathcal{C}^\#_4)$$

This illustrates that Dirac's transverse $\boldsymbol{\alpha}$ matrices (1 and 2 or + and –) are moving the positron between the positive energy spin states #1 and #2 and between the negative energy spin states #3 and #4 – that is, just spin-flips, while preserving the energy signature. Dirac's longitudinal matrix $\boldsymbol{\alpha}_3$ generates two-step (or double) transitions between positive and negative energy states – that is, just energy-flips, while identifying the spin signature (with a sign change if the 'spin' is 'down'). Dirac's temporal matrix $\boldsymbol{\alpha}_4$ preserves the positron's state but identifies if it is a positive or negative energy states. These are the identical physical transitions found for the electrons, demonstrating that Dirac's Equation equally describes the dynamics of both electrons and positrons. Dirac was wrong to assign two of his four states to electrons and the other two to positrons. All **four** states apply to **both** electrons and positrons – it is just the sequence of transitions that differ between them. There are **no** negative energy states and positrons are **not** 'holes' in an infinite, invisible 'sea' of negative energy states. As shown above, it is the direction of axial rotation that distinguishes electrons from positrons.

7.4 SPINNING INTO ACTION

7.4.1 DIRAC UNCOVERS SPIN

The Electron's Magnetic Moment

Since Dirac's text is mostly ignored today in the education of physicists, it is instructive to review how Dirac arrived at the idea that an electron has a magnetic moment $\underline{\mathcal{M}}$ that gives the electron a distinctive magnetic energy in a magnetic field $\underline{\mathcal{B}}$. Dirac begins in section 70 with his fundamental relativistic equation (see §7.1.1) and uses the 'rule': $\mathbf{H} = c\,\mathrm{P}_0 = i\,h_D\,\partial/\partial \mathrm{t}$.

$$\mathbf{H} = c\,\underline{\boldsymbol{\alpha}} \bullet (\underline{\mathrm{P}} + e/c\,\underline{\mathrm{A}}) + \boldsymbol{\alpha}_4\, m\,c^2 - e\,\mathrm{A}_0$$

$$\therefore\ (\mathbf{H} + e\,\mathrm{A}_0)^2 = \{c\,\underline{\boldsymbol{\alpha}} \bullet (\underline{\mathrm{P}} + e/c\,\underline{\mathrm{A}}) + \boldsymbol{\alpha}_4\, m\,c^2\}^2 = \{c\,\underline{\boldsymbol{\sigma}} \bullet (\underline{\mathrm{P}} + e/c\,\underline{\mathrm{A}})\}^2 + m^2\,c^4 = c^2\,(\underline{\mathrm{P}} + e/c\,\underline{\mathrm{A}})^2 + m^2\,c^4 + e\,h_D\,c\,(\underline{\boldsymbol{\sigma}} \bullet \underline{\mathcal{B}})$$

Here: $\boldsymbol{\alpha}_k = \boldsymbol{\rho}_1\,\boldsymbol{\sigma}_k$ where $\boldsymbol{\sigma}_k = \begin{vmatrix} \sigma_k & \mathrm{Ø} \\ \mathrm{Ø} & \sigma_k \end{vmatrix}$ & $\boldsymbol{\rho}_1 = \begin{vmatrix} \mathrm{Ø} & \mathbf{I} \\ \mathbf{I} & \mathrm{Ø} \end{vmatrix}$ and $\underline{\mathcal{B}} = \nabla \wedge \underline{\mathrm{A}}$ (see §3.1.2)

Dirac next makes the crucial approximation for a slowly moving electron (i.e. with small momentum $\mathrm{P} << m\,c$) and assumes that the Hamiltonian can be approximated by a small addition E_1 to the rest energy $m\,c^2$; i.e. $\mathbf{H} = m\,c^2 + \mathbf{H}_1$ with $\mathrm{E}_1 << m\,c^2$.

$$\therefore\ (m\,c^2 + \mathbf{H}_1 + e\,\mathrm{A}_0)^2 = m^2\,c^4 + 2\,m\,c^2\,\mathbf{H}_1 + 2\,e\,\mathrm{A}_0\,(m\,c^2 + \mathbf{H}_1) + (e\,\mathrm{A}_0)^2 + (\mathbf{H}_1)^2 \cong m\,c^2\,(m\,c^2 + 2\,\mathbf{H}_1 + 2\,e\,\mathrm{A}_0)$$

$$\therefore\ \mathbf{H}_1 + e\,\mathrm{A}_0 \cong (\underline{\mathrm{P}} + e/c\,\underline{\mathrm{A}})^2 / 2m + (\underline{\boldsymbol{\sigma}} \bullet \underline{\mathcal{B}})\,e\,h_D / 2mc = \mathbf{H}_0 + \mathbf{H}_\mathrm{M} \quad \text{with} \quad \mathbf{H}_\mathrm{M} = \underline{\mathcal{M}} \bullet \underline{\mathcal{B}} \quad \therefore\ \underline{\mathcal{M}} = -\,e\,h\ \underline{\boldsymbol{\sigma}} / 4\pi mc$$

Here, $\mathbf{H}_0$ is the classical Hamiltonian for a charged particle in an EM field while $\mathbf{H}_\mathrm{M}$ is the extra magnetic energy of the electron due to its new degree of freedom. The electron behaves as though it has a magnetic moment $\underline{\mathcal{M}}$.

The Electron's Spin Angular Momentum

After deriving this last result, Dirac admits that the spin angular momentum $\underline{\mathrm{S}}$ does **not** give rise to any potential energy, so he considers the case of a free electron or an electron in a central field of force ($\underline{\mathrm{A}} = 0$), which means using the Hamiltonian:

$$\mathbf{H} = c\,\underline{\boldsymbol{\alpha}} \bullet \underline{\mathrm{P}} + \boldsymbol{\alpha}_4\, m\,c^2 - e\,\mathrm{A}_0[\mathrm{r}]$$

Dirac uses the Heisenberg form for determining the time rate of change of an operator, in this case the L_1 part of the orbital angular momentum, $\underline{\mathbf{L}} = \underline{\mathbf{r}} \wedge \underline{\mathbf{P}}$ i.e. $\mathbf{L}_1 = \mathbf{X}_2\,\mathbf{P}_3 - \mathbf{X}_3\,\mathbf{P}_2$ and uses the $(\mathbf{X}, \mathbf{P})$ commutation and $\boldsymbol{\alpha}$ anti-commutation rules.

$$i\,h_\mathrm{D}\,\mathrm{d/dt}\,[\,\mathbf{L}_1\,] = \mathbf{L}_1\,\mathbf{H} - \mathbf{H}\,\mathbf{L}_1 = c\{\mathbf{L}_1\,(\underline{\boldsymbol{\alpha}} \bullet \underline{\mathrm{P}}) - (\underline{\boldsymbol{\alpha}} \bullet \underline{\mathrm{P}})\,\mathbf{L}_1\} = i\,h_\mathrm{D}\,c\{\boldsymbol{\alpha}_2\,\mathrm{P}_3 - \boldsymbol{\alpha}_3\,\mathrm{P}_2\} \ \therefore\ \mathrm{d/dt}\,[\,\mathbf{L}_1\,] = c\,(\underline{\boldsymbol{\alpha}} \wedge \underline{\mathrm{P}})_1$$

$$i\,h_\mathrm{D}\,\mathrm{d/dt}\,[\,\boldsymbol{\sigma}_1\,] = \boldsymbol{\sigma}_1\,\mathbf{H} - \mathbf{H}\,\boldsymbol{\sigma}_1 = c\{\boldsymbol{\rho}_1\boldsymbol{\sigma}_1\,(\underline{\boldsymbol{\sigma}} \bullet \underline{\mathrm{P}}) - \boldsymbol{\rho}_1\,(\underline{\boldsymbol{\sigma}} \bullet \underline{\mathrm{P}})\,\boldsymbol{\sigma}_1\} = -\,2c\,\boldsymbol{\rho}_1\{\boldsymbol{\sigma}_2\,\mathbf{P}_2 + \boldsymbol{\sigma}_3\,\mathbf{P}_3\}\,\boldsymbol{\sigma}_1 = -\,2\,i\,c\,(\underline{\boldsymbol{\alpha}} \wedge \underline{\mathrm{P}})_1$$

$$\therefore\ \mathrm{d/dt}\,[\,\mathbf{L}_1 + \tfrac{1}{2}\,h_\mathrm{D}\,\boldsymbol{\sigma}_1\,] = 0 \quad \text{Defining Spin } \underline{\mathbf{S}} = \tfrac{1}{2}\,h_\mathrm{D}\,\underline{\boldsymbol{\sigma}} \quad \& \quad \underline{\mathbf{J}} = \underline{\mathbf{L}} + \underline{\mathbf{S}} \quad \therefore\ \underline{\mathbf{J}}\,[\mathrm{t}] = \underline{\mathrm{J}}_0$$

This means that the sum of the orbital angular momentum $\underline{\mathbf{L}}$ and spin angular momentum $\underline{\mathbf{S}}$ is a constant of the motion $\underline{\mathrm{J}}_0$.

In recognizing that not all particles have a measured spin of $\tfrac{1}{2}\,h_\mathrm{D}$, Dirac writes that this derivation is only valid "provided the position of the particle is an observable."

The present theory re-interprets this caveat with the view that particles with non-half quantum values of the spin angular momentum are not simple point particles but composites of several electrons interacting together that cannot be described by an invariant centre of mass (i.e. not by three simple spatial variables).

7.4.2 SPIN & ROTATION

Spin still viewed as Rotation

Most physicists still seem to view particle 'spin' as the rotation of an object [150] as seen by an observer with sub-luminal speed. In the case of 'electron spin', it would require the electron to have not only a finite size but the existence a lower level of fundamental 'stuff', like electric charge density with all the attendant problems of explosive self-repulsion. This implies that the 'spinning' object has finite size and structure, an angular speed of rotation, some distinguishing surface feature and some kind of observational interaction: **NONE** of these features is exhibited by the electron – it is a true elementary particle, for which the mathematical idea of 'spin' was invented. Physics has not yet faced up to the implications of the experimental observations of the electron – that this is the fundamental, **point** particle of matter: it has **no** spatial extent and **cannot** be viewed as a finite object rotating *in toto* around some interior axis.

Thomas's Spinning Top

The Thomas precession calculation [151] suffers from the same mistake as Einstein's linear dynamics of the 'electron' in the final section of his most famous paper [152]. Both calculations introduce differential quantities d$\underline{v}$ and dt (and their effects) into a theory that is based on constant velocity. This mathematical 'trick' smuggles in acceleration by imagining an infinite series of instantaneous 'snapshots', each of which is treated as an inertial reference frame. Worse, in his calculation [153], Thomas treats the electron as if it were a classical 'spinning top': a model of the real electron almost as bizarre as Einstein's.

7.4.2 SPIN AS INTERACTION

*Spin as Interaction **and** Rotation*

The idea that the electron has only two possible 'spin states', arose from Pauli's innovative (2x2) representation that was designed the explain the duplexity of the observed spectra; in other words, magnetic interactions only distinguish two states. As Dirac first realized, there are actually four possible electron states but they are degenerate: each pair reacts in the same way to magnetic interactions. Even though the present theory introduces an intrinsic rotation in the motion of every electron, this is not proposed simply as **the** mechanism of 'spin'. This intrinsic rotation is viewed here as the basic distinction between negative and positive electrons; in other words this is the explanation for the two possible states of electric charge. However, the intrinsic ('clicking') motion of the electron is proposed here as the *spatial* component (where) of the **two physical sets of ideas** associated with the actual experiments on 'spin'. The other component is intimately associated with how (and when) and electron interacts with other electrons. Since this implies that the *timing* of each interaction is critical then all motion alters the space and time parameters involved in each interaction. These velocity effects become more prominent as the relative speed of the electrons in the interaction increases – this is the physical explanation of why 'spin' is now seen as a relativistic effect.

In the present theory, 'spin' is interpreted as the unit of interaction between two electrons. Since each electron is treated as a special point in space and as 'spin' is not viewed as a self-rotation, there is no requirement to introduce a finite radius to describe the electron. Dimensionally, both action and angular momentum are defined in terms of the product of momentum and distance. This has led to the confusion about the nature of 'spin' and the fact that the electron's 'spin' is quantized. The initial investigation of the Natural Vector motion of a classical (Newtonian) point particle clearly illustrated that 'angular momentum' and 'action' were intimately tied together through the concept of 'activity' (see section 4.3.3). This was readily extended to the Digital Electron model, where the 4-step discrete rotational motion around its own longitudinal (average) velocity (§5.3.2) introduced a fixed finite (quantum) quantity to the electron's action and its angular momentum. This latter showed that 'spin' is not just a change in the isolated electron's kinematical behavior but is a reflection of its interaction with other electrons, as was seen in the sections on interactivity (§4.3.5) and the difference between two-electron activity (§4.3.3).

Spin & Lorentz Invariance

Furthermore, since the physics of 'spin' is intimately related to the interaction between **two** electrons and **not** associated with a single electron at a single point in space and time the mathematics is **not** subject to Lorentz invariance which constrains how a $\underline{\text{single}}$ event is viewed by various observers in different mathematical frames of reference. The only constraint is that both electrons must be on each other's 'light-cone' whenever any interaction occurs between them. It has been the artificial attempt to separate interactions into sourceless 'forces' acting at a single point in space at a single instance of time that has led to this hundred year obsession with Lorentz invariance: a desperate attempt to salvage mathematical field theory.

The Electron 'Lollipop'

The present model of the electron combines a linear motion, at finite speed, through space (3D vector) with a spherically symmetric 'request' at light-speed alternating forwards and backwards through time (like outbound and inbound 'waves'). This 'lollipop' is equivalent to rotating momentum and explains why 'spin' has been confused with angular momentum.

Spin resembles Superconductivity

The model of electron spin as the exchange of 'units of interaction' with another electron is similar to the explanation of superconductivity using 'Cooper pairs'. In the BCS model of superconductivity, one electron exchanges a quantum of linear momentum with the crystal lattice that propagates this quantum without loss (at very low temperature) to another crystal atom that interacts with the other electron in the pair that has already sent a similar quantum in the opposite direction back to the first electron.

Retaining the Name 'Spin'

The present theory does not identify the concept of electron 'spin' with any form of interior angular rotation but with a 'twisting' form of directed interaction between two axially rotating electrons, where event timing differences are critical. Nonetheless, this venerable terminology will be retained here to indicate 'twisting a fiber to form a thread', an even older meaning of the term that ties back to the beginnings of every civilization.

7.4.3 SPIN & SPINORS

Spinor Invariance

The spinor invariance with respect to a double 'rotation' of 4π reflects the complete temporal symmetry of the world after each electron completes its own 'send' and 'receive' cycle after the basic interaction is completed when each electron has completed its own 'rotation' of 360 degrees. The world only returns to its original state after both electrons have completed their own 'spin-cycle'; i.e. a full 'send' and 'receive' both into the future and into the past (the four primary 'spin' states).

Interpreting Spinors

A 3D vector represents a timeless, directed magnitude, anywhere in space. A spinor represents a relationship between two specific locations in space (e.g. the unit vector from the origin to somewhere on its surrounding unit sphere) at one single instant of time. A quaternion may be represented by a pair of spinors and so would correspond to a mapping between two pairs of different points in 3D space at the same time. The addition of the unit imaginary into the definition of a Natural Vector generates a further doubling effect and indicates their suitability for representing interactions between two electrons at two different times. The spinor is also isomorphic with the lowest (half integer) spherical harmonic that corresponds to both inward and outward spherically symmetric waves, which prevents its interpretation as a real angular rotation. It is this deep isomorphism between rotation in three spatial dimensions and the complex (2x2) spinor representation (doubled-up by Dirac) that continues to divert physicists into thinking that 'spin' is simply about the rotation of a finite object in space.

Spinors as Electron Relationships

Penrose has also provided a simple visual representation of the half-angle rotations in 3D space associated with spinors by mapping vectors to oriented arcs on great circles drawn on a sphere [154]. Penrose showed how this mapping forces the rotations to be twice each arc length. The issue here is subtle: all rotations are made around the center of the sphere and the whole activity is viewed (typically) from an external 'god-like' viewpoint. In keeping with the basic relational perspective adopted in this programme it is suggested that the more appropriate viewpoint should be the opposite side of the sphere where the opposite end of the diameter traces out the complementary motion. This corresponds to the interactive nature of reality adopted here. **A single, non-interacting electron has no reality** – it is only defined by its interaction with another electron. This is why the NV mathematics is the most natural representation of displacements in real 3D space; it is not to be viewed from some 'god-like' frame-of-reference but always from the viewpoint of another, real electron. The motion of each electron contributes its spinorial perspective (SU2-like), so that the interaction is naturally double 'SU2' that is the standard representation of quaternions. Rather than use curved arcs on great circles to represent linear displacements, the NV bases are Euclidean (real, not complex) so that NVs form a more direct representation of linear displacements of the participating electrons in every interaction.

7.4.4 SPIN & FERMIONS

Only Spin Half

One of the greatest challenges to Dirac's theory was that it was first thought to apply to all charged spin half particles. It therefore predicted that the proton would have a magnetic moment of one nuclear magneton (using the proton's much larger mass instead of the electron's mass). However, Stern and Frisch measured this quantity in 1933 [155] and found that its value experimentally was almost three times larger than predicted – an anomaly that pleased Pauli. But the largest difficulty with Dirac's theory was providing a physical interpretation of its mathematical structure. Once again, Dirac's intuition provided a foreshadowing of the present theory when in an address in Leipzig he offered the following hopeful but cryptic comment [156]: "It seems that this difficulty can only be removed through a fundamental change in our previous ideas, and may even be connected with the difference between past and future."

Dirac acknowledged then that not all elementary particles (e.g. photons, some mesons) have measured spins equal to ½ h_D, so that he also decided that this theory only applied if the position of the particle is an observable. When this condition holds then "the particle must have a spin angular momentum of half a quantum. For such particles, there is still a momentum representation." This is ironic as the electron's position in the hydrogen atom is never an'observable' as Heisenberg noted.

Spin Zero, Spin One, Spin Two

Pauli and his student, Weisskopf, in their 1934 paper on the scalar relativistic wave equation [43], decided that nature has no reason to reject particles of spin zero. The present theory only posits electrons as fundamental particles, where their 'spin' is their mutual interaction. In this light (pun intended), the interaction itself could be viewed as a 'virtual' particle of spin one (the 'photon'). So, in this theory there can be no fundamental particles of spin zero (as these could never interact) but there are several, quasi-stable composites of multiple electrons but only one ever interacts with another material object. In a later paper, a theory of **gravity** will be presented that views this tiny, minute interaction as a very rare, double interaction between neutrons, appearing as another so-called 'virtual' particle (the 'graviton') with two units of spin.

Only Spin-Half Particles

Kragh quotes Dirac's erroneous use of the bogus principle of 'plenitude' in philosophy ('Anything that can occur does occur') [157] when in 1946 he attempted to justify his earlier (1936) wave equation for higher spins: "Elementary particles of higher spin have not yet been found in nature but there is no reason why they should not exist." Dirac was clearly thinking of 'spin' as angular momentum, so higher values should be possible, in principle. The present theory challenges this view by interpreting 'spin' as an *invitation* to another electron to participate in a momentum exchange while correlating their separate digital 'rotations'. Since this 'invitation' is broadcast and received spherically around each electron, it can only have two values (outward and inward). This theory therefore predicts that all real examples of matter will only exhibit a 'spin' half value but there will be many different values of magnetic moment (not explicable just by mass variations).

Universal Electron Theory

Ever since Pauli divided the microscopic world into two types of particles (fermions and bosons) and associated this basic classification scheme with the spin of each particle (see section 2.1.5) the idea of 'spin' has remained at the center of particle physics. The problem with this scheme is that it is purely mathematical: the invention of quantum field theory associated a mathematical field both with real particles (fermions), like electrons, and with the interactions between them (bosons), such as EM impulses or other 'forces'. In the present theory, all matter is constructed from stable sets of electrons and positrons interacting only via the EM interaction described in this paper: this not only simplifies the ontological view of reality but reduces the role of 'spin'. In any one system cycle, only one of these electrons (or positrons) is ever likely to interact with an electron outside the stable set. This has been interpreted as all matter consists of fermions of spin one-half. Here, it is seen as evidence that all matter consists only of electrons (of either charge). Multi-electron models of the various fundamental particles will be presented in later papers.

7.5 DIRAC'S PHILOSOPHY

7.5.1 MATH FIRST

Walking in the Dark

Dirac placed himself in the tradition established by Planck that the aim of theoretical physics is to **discover** fundamental equations of nature. This was especially unusual in that his undergraduate degree was in electrical engineering (in 1921) where mathematics is frequently seen as just another tool. Dirac went even further in his extreme commitment to the role of mathematics in physics where he believed progress in physics must be driven by intuitive mathematical imagination so that new mathematics would be introduced first and then physicists would try to interpret the mathematical features in terms of physical entities. He claimed that quantum mechanics was the first theory to be 'discovered' before anyone knew what it meant. [158] He failed to appreciate the irony that this theory had demonstrated the problems of this approach ever since his innovative, mathematical contributions in 1925-27. This programme rejects the idea of closing one's eyes (rejecting visual intuition) when physicists are trying to explore the dark jungles of nature: mathematics must be used to help perform any calculations but solid philosophical ideas (semantics) must always come first to dispel subsequent confusions. Failure to use the largest processing function that exists in mammalian brains (the visual system, including our imagination) is to fall into the arrogant, rationalist trap that conscious, linear manipulations of discrete abstract symbols is the highest form of thinking. Einstein was right to give preference to imagination over knowledge as the basis for making further progress in physics.

Mathematical Ambiguity

The argument against the primacy of the mathematics-first approach promoted by most theoretical physicists in the last 100 years, beginning with Max Planck, is illustrated by well-known examples of where two alternative mathematical schemes have been proposed to describe an area of physics and there has been no way to distinguish which is superior from either an interpretive viewpoint or pragmatically i.e. as one that generates the best way for making further progress. Three leading examples are: 1) the point mathematics of Weber versus the field mathematics of Maxwell when describing the classical EM interaction; 2) the matrix mathematics of Heisenberg versus the differential calculus mathematics of Schrödinger used in inventing the modern form of QM; 3) the continuum mathematics of Sommerfeld versus the matrix techniques of Dirac used to calculate the fine structure of the hydrogen atom. In all three cases, each scheme predicted exactly the same results and in all cases further progress came to an end. This was not the case before the Twentieth Century when scientists who studied the natural world were natural philosophers first (relying on their intuitive imagination) and mathematicians second.

The End of Infinity

Ever since Newton, both algebraic and analytic techniques have been imported from mathematics into physics, both to perform calculations and to provide mathematical representations (or *principles* as Newton and Dirac referred to them). The common foundation has been geometry – not surprisingly since Newton invented calculus to aid him in evaluating non-linear motion in his original study of planetary dynamics. The idea that material objects will always interact continuously evolved from this calculational 'trick' and became the standard background hypothesis as a feature of reality itself that it is almost never stated explicitly today – it has become a universal, implicit assumption (referred to here as the *Continuum Hypothesis*). The heart of the calculus technique in physics is to assume that interactions behave 'smoothly' when the time difference (dt) becomes infinitely small: this leads to the concept of 'force'. This programme has shown that in the case of the interaction between two point objects (like electrons) the electromagnetic interaction is not only asynchronous but must be discrete. The Continuum Hypothesis is false at the foundational level of nature as inertial mass is an intrinsic property of electrons. The search for the so-called Higgs boson is the last-ditch attempt by the continuists to preserve their massless mathematics of field and still account for the reality of inertial mass. The present theory rejects this hugely expensive quest as a fruitless task.

Algebra, not Calculus

Algebra was the first mathematical technique to permit numerical calculations in physics. It was DesCartes' genius to invent analytic geometry and modern algebra but it was Newton's genius to map these mathematical techniques together into reality with his revolutionary concepts of inertial mass and momentum. It was the mathematicians who laid the cuckoo's egg of calculus to transform and hijack classical physics as created by Newton. This takeover has had a nearly 300 year opportunity to see how far these techniques can be applied to physics – this game is now coming to a close. The present programme has shown that discrete algebra is a far superior mapping of the discrete nature of the foundational interactions of reality.

7.5.2 AN INTERACTIVE WORLD

Isolated Contradictions

The real world is a vast web of interactions, so many in fact, that the complications are overwhelming. One of Newton's innovations was focusing on only one particle at one point and furthermore to focus on only one side of all the interactions that this 'target' particle participated in; reducing all these interactions to one instance of time led to the concept of impulse. The final reduction was to compare the response of the target at one instance with another instance that was infinitely close together: hence was born the concept of force. When only two objects are interacting the mathematics of instantaneous, continuously acting forces is solvable but the addition of a third, similar interacting object generates difficulties that have remained insurmountable (the '*three body problem*') ever since Newton discovered this deadly reef in the mathematical oceans of the world. Even making the continuous interaction asychnonous, as in the delayed effects of electromagnetism, makes the equations unsolvable. It was this recognition that led Maxwell to invent the universal EM fields acting every where in space at **one** instant of time that allowed any progress to be made. This has been the implicit motivation behind the continuation of field theory in modern physics, resulting in such bizarre models as 'string theory' and 'quantum gravity'.

The only mathematical technique that has offered some promise that is based on a target particle interacting with several sources that all interact together asynchronously is known as the 'asymptotic approximation'. This starts with a single particle, so far away from all others sources of interaction that the target particle can be said to be 'free' or non-interactive. As the particle moves closer to the interaction source, the strength of the interaction increases steadily until measurable effects can be calculated. This approach can work if there are no discontinuities exposed during this process, which can lead to bifurcations of a qualitative type. Even when change occurs smoothly, small variations in the initial conditions (location or velocity) can eventually result in widely divergent behavior – an embarrassing phenomenon known as 'chaos'.

This was the reason Dirac began with his equation for the free electron. He then followed the classical EM approximation by averaging all of the remote interactions into an average set of potentials (static and dynamic) represented by his nonquantized four-scalar A_μ. This was another example of the mathematical limitations proscribing the physical model. In contrast, the present theory posits that the EM interaction is **saturated** so that it only occurs between **two** electrons at any one time. Even though it is completely asynchronous in its view of this two-electron interaction, the discrete nature removes the embarassing mathematical infinities and a purely algebraic approach proves quite sufficient. This is a satisfying return to Newton's original approach to modeling nature, replacing an averaged inverse square force with an exact inverse time impulse.

7.5.3 ONE WORLD

ETM Redux

Several times in the history of physics, attempts have been made to achieve the great simplification – a single, unified theory of everything. All material objects would be viewed as manifestations of only one basic substance; all forces would be seen as different manifestation of the same basic interaction. Larmor was one of the last scientists to pursue this dream with his Electronic Theory of Matter (ETM) that has all but been forgotten [159]. The present programme is pursuing this same dream and ironically focusing on the same basic 'stuff' (electrons) and the same universal interaction (the EM impulse) as Larmor. The final papers in this series will describe this vision in far greater detail, providing modls of all the 'fundamental' particles.

One Kind (No Waves)

One of the principal motivations of this programme is to bring the rigor of philosophy back into natural philosophy (physics). Philosophy has a long tradition of clarifying loose thinking, especially the elimination of contradictions. Unfortunately, as modern physics has been taken over by mathematicians, new theories have arisen that are riven with contradictions – physicists realize that this word is still anathema so they have covered up the problems with words like 'paradox' or better yet, chosen to completely ignore such contradictions. Probably, the most philosophically embarrassing problem brought up by modern physics is the idea that real examples of the material world, such as electrons, can be described as either particles **or** waves. These two concepts are diametrically opposed to one another: particles invoke the idea of existence only at one **point** in space at any one time, whereas the idea of a wave extends over **all** of space at all times. This conflation of ideas is called duality or the wave-particle paradox – it is associated with the Copenhagen Interpretation of QM. The core of this 'paradox' is the Platonist idea that a mathematical representation is a one-to-one mapping of the world. If the mathematics of waves proves useful in QM then particles **are** waves. This view is rejected 100% – all electrons exist only as point particles. This perspective was used here and will be elaborated in a later paper offering an all-particle alternative to standard QM.

One Time Difference

The present theory has cut the Gordian knot that has tied the physics of asynchronous interactions to the single-time field model of physics that includes Dirac's relativistic equation of the electron. The key, new idea here has been to reject the Continuum Assumption and replace it with the hypotheses that the EM interaction is both **saturated** (limited to only two electrons at any time) and **periodic** – only occurring once per quantized unit of time (called here the *chronon*). This explicitly recognizes that there is always a finite time difference between the two events that define the extent of each interaction. This also means that neither electron in the interaction has precedence, each is of equal significance. This implies that this type of theory is always a two-time, not single-time theory. It is actually the difference between these two times that is the critical temporal parameter here, just as is the two spatial points create the single, spatial difference parameter. Such **difference** parameters are independent of any frame of reference and show that the special theory of relativity has no relevance here.

A New Approach

There are two major differences between the present theory and Dirac's relativistic model of the electron. Firstly, 'spin' is not treated as it is some form of angular momentum (Dirac's 4D extension of Pauli's 3D spin matrices) but as the directed interaction between two electrons 'coupled' through their complementary 'send' and 'receive' states. Secondly, the concept of time is treated in a Newtonian sense as the universal, monotonically increasing parameter that defines change through the universe, defining when each electron may participate in an interaction with another electron: this introduces the radically new concept of the 'quantum of time' or *chronon*. In contrast, Dirac explicitly constructed his equation from the classical results of Einstein's special theory of relativity and, especially, from Planck's Proposal for the re-definition of momentum. As a result, in this theory there are no negative energy states – these are the advanced interactions with the future. Positrons are real electrons, whose time of interaction is always out-of-phase with the negatively charged electrons: therefore appearing as positively charged. There is certainly no need here to introduce an invisible filled 'sea' of negative-energy electron states.

New Math

The Dirac Equation combines mathematics from several sources. Its obvious innovation was the introduction of (4x4) finite matrices α_μ , which were the necessary generalization of Pauli's radical usage of the 3 (2x2) 'spin' matrices $\underline{\sigma}$. The other key component was the retention (from the then new quantum mechanics) of continuous 'wave' functions Ψ_μ , which required four continuous functions of the target electron's space and time parameters. The other (more implicit) mathematics retained the idea of Hamiltonian functions, based on partial differential operators, to specify the dynamics of point particles, subject to instantaneous forces and the usage of Planck's quadratic energy-momentum equation, which assumed continuous increases in momentum, energy and apparent mass. The current theory rejects almost all of these diverse features and replaces them with a single, discrete algebra that relies only on discrete changes in the activity of the pairs of electrons interacting together. Time after time in twentieth century physics, mathematicians have 'discovered' a new equation that (like the magician's spell, has extraordinary power to relate to reality) but unfortunately, like the Sorcerer's Apprentice, they fail to understand the meaning of the symbols involved. This demonstrates the necessity for the priority for physical intuition <u>before</u> mathematics. Even Dirac confessed that his intuition functioned geometrically (i.e. visually) <u>before</u> he moved onto to algebraic and analytic formulations [102]; ironically, it is only his algebraic contributions to electron theory that have remained in canonical physics.

A Tragic Hero

Paul Dirac was the quintessential theoretical physicist of the Twentieth Century – his contributions to the foundations of quantum mechanics and quantum field theory have remained at the centre of modern physics for almost a hundred years. He was also the ultimate rationalist, displaying an honesty about his own work that can only be seen as unique. It was this sense of honesty that led him to write to Bohr in 1929 [16] that he believed that quantum mechanics would ultimately be replaced by a better theory. He was also deeply disturbed by the infinities that arose in his own quantum field theory and, even in 1945, he rejected the 'triumphant' quantum electrodynamics as "ugly mathematical tricks" and that the exquisite agreement between the predictions of this theory and experiments were only a coincidence. [160] For the rest of his life, he continued to seek a better theory but in 1982 (only 18 months before his death) confessed [161]: "My life has been a failure." Specifically, he felt that theoretical physics was unable to describe even the interaction of a single electron with a photon. Sadly, this theory would have re-oriented Dirac's thoughts back to just electrons – photons are simply a mathematical fiction for only part of the complete EM interaction. Nonetheless, Paul Adrien Maurice Dirac remains one of the author's greatest heroes.

8. SUMMARY & CONCLUSIONS

In this final section, the results and conclusions from this paper will be briefly summarized in order to draw out the major implications from the material. The paper concludes with summaries of the future papers in this programme. We will let the comparison of the analytic predictions of this theory with experiment be the justification for the assumptions made here.

Classical Two-Electron Relativistic Dynamics

The previous two papers in this series investigated the dynamics of two remote electrons interacting asynchronously across time. The first replaced spatial continuity (the classical model of continuous 'charge-density') with point electrons with finite electrical charge. That paper proved that all continuous interaction (**force**) theories between point particles that exhibit inertial resistance to changes in their motion are **not** consistent with *asynchronous* (or Gaussian) forms of action-at-a-distance or equivalently: two-particle interactions limited to points 'on their mutual light-cones'. The next paper then investigated the hypothesis that the universal inter-electron interaction only occurs discontinuously over time. This was a return to Newton's original idea of fixed **impulses** between the two particles and gave a new mechanical explanation for Planck's 1907 Proposal for the formulae of relativistic point-particle mechanics while preserving each electron's invariant inertial mass at all speeds. This result limited Lorentz invariance to a mathematical constraint on universal, single-time **field** theories. The present theory extends the Newtonian approach to asynchronous interactions between pairs of particles interacting through a new form of the electromagnetic (EM) interaction: it offers a relativistic model of positive and negative electrons.

8.1 OBJECTIVES

The research underlying this paper was driven by the desire to gain a greater understanding of the principal entity on which this programme is constructed: the **electron**. In line with this programme's intention to retain as much of the Newtonian natural philosophy as possible, the electron is viewed as a real point particle: a view confirmed by the latest experiments.

The objective of this paper was to present an alternative theory of the *positive* electron. Orthodox physics has decided that Dirac's Theory of the Relativistic Electron was the final explanation of this phenomenon. This erroneous conclusion only became possible because the problems and contradictions of Dirac's theory, raised by Dirac's own contemporaries, have been forgotten by physicists in their ongoing obsession with novelty and resulting lack of real interest in the history of their own subject. This series of papers have shown (*inter alia*) that Planck's arbitrary derivation of the basic quadratic relationship between the energy and momentum of an isolated particle is the root source of all problems in relativistic mechanics. Since this was also Dirac's starting point for developing his own relativistic equation of the electron's quantum mechanical field function, it is not surprising that spurious "negative energy" solutions also emerged from his famous equation.

Both classical physics and quantum theory have been constructed on the implicit Continuum Hypothesis. This is the idea that all interactions between the foundational entities of nature occur continuously at all times. Contrary to another widely held myth of physics, this was not Newton's original physical assumption but was added later by him as a mathematical step to compute a solution to the problem of many bodies acting collectively together, as in his gravitational model. This research programme has rejected this assumption and returns to the problem of the **interaction** of two fundamental material objects; in this theory always taken to be electrons, on the hypothesis that this interaction is the explanation for all interactions in physics.

The necessary step in creating a comprehensive theory of the electron-electron interaction was developing a detailed model of how **two** electrons interact at **all** distances and speeds. The rejection of the Continuum Hypothesis meant that all continuum mathematics had also to be rejected as it seems very plausible that the key characteristics of the mathematics used to represent reality should share some of the foundational features. Accordingly, a systematic use of discrete mathematics has been used from the beginning of this programme so that the principal feature of the particle concept – it's unique location at any time – is represented by *Discrete Natural Vectors* (DNVs). These are quaternion based mathematical objects that result in much denser algebraic equations (fewer symbols) than the traditional use of scalars or vectors (and aslo maintain the foundational difference between space and time). Additionally, an extended set of difference operators are used instead of the tradional use of calculus that usually results in integrations that need to be summed over all of space and time. This paper has also clarified the concept of "electron spin". This strange idea lies at the heart of quantum theory but its conceptual foundations are still deeply confused and contradictory: real progress in physics requires clear, extensible concepts.

Simple Metaphysics

All fundamental theories must make their metaphysical assumptions explicit so that the reader may understand the conceptual foundations on which the resulting theory is constructed. Metaphysics is viewed here as those statements about reality that cannot (as yet) be determined by experiment. Ontology is the foundation of physics – this is a set of statements that posit one or more distinct types of entities, whose exemplars exist in the nature. These entities are defined in terms of their intrinsic properties and the possible relationships that may occur between them. All statements about the existence of entities are metaphysical and are logically distinct from statements about their properties, which are physical. In particular, existence claims are either true or false: an entity exists in reality or it does not. Accurate statements about properties of such entities may be accumulated over time through the objective methods of science; when these methods result in numbers they become measurements. As science progresses, these property statements evolve into facts, available to everyone. The central entity in this theory is the **electron**, viewed here as the only basic component of matter – its set of interactions define its properties and, historically, arose through the investigations of electromagnetism (EM).

The present theory now extends Newton's dynamical model of corpuscles and massive bodies to microscopic electrons. Newton's classic laws were sufficient to describe the (relatively) slow motions of quasi-rigid massive objects, where the whole object (independent of its size) moved cohesively through time; consequently, its total collective motion could be characterized by only its center-of-mass location. The heart of Newton's dynamics was his Second Law that is interpreted here as the particle's change in momentum is caused by (and, therefore, linearly related to) the instantaneous impulse that it receives. The Third Law was introduced by Newton specifically to identify the source of this impulse with another particle and to identify this impulse with reaction of the 'sending' particle thereby preserving total momentum across the interaction. These views become the core of the present theory but are extended here from the instantaneous impulses used to describe contact collisions or the effects of gravitational attraction between remote masses to the asynchronous interactions of EM. This theory proposes to view the electron as more than a simple, point particle with only inertial proerties that continuously interacts with all the other particles everywhere else throughout the whole of space. The first proposed extension here is that every electron '*pulsates*' on a universal cycle (the chronon), which defines **when** every electron may interact with any other electron. The second hypothesis is that the electron moves inertially in a 4-step motion transversely to its average velocity.

This new theory of the electron is based on a very simple, 'relational' metaphysical model. The basic electronic charge (e) establishes what IS (i.e. what is special about any point in space where an electron exists relative to all the 'empty' points in space where there are no electrons). The interaction between pairs of electrons defines the magnitude of the change in their relative motion (e^2), while the difference or similarity in sign of the charge is associated with the direction that each electron is moving transversely relative to its longitudinal direction: this now determines whether the two electron attract or repel each other during the interaction. The universal measure of interaction (or 'spin' $h/2\pi$) defines the degree of change in motion per interaction, while the electron's inertial mass (m) limits the amount of change in relative velocity of each electron induced by each interaction with its temporary 'partner'. The universal space-time parameter (c) defines which nearest electron may interact at each point in the discrete, interaction time cycle. The electron-pair's relative spatial separation and their relative temporal separation define the relative locations and times (the where and when) each individual interaction occurs 'on the mutual light-cones' of the two electrons. The negative electron's 'anti-particle' or positron is the same kind of corpuscle as the electron. The positron here does not represent a transformed electron (or a 'hole' in a negative energy 'sea'): the positron in this theory does not represent an electron with negative mass or negative energy, nor does it represent an electron traveling backwards through time (here, all matter moves forward through time). The positron now 'clicks' through space but with a 'twist' opposite to that found with the electron. The interaction effects are still the result of electric charge but now the sign of the charge (positive rather than negative) are viewed as due to a permanent half-chronon offset in the timing of its 'send' and 'receive' phase transitions compared to electrons.

8.2 DIRAC'S RELATIVISTIC ELECTRON

Since modern physicists, at best, skim over the history of the relativistic electron, the second chapter provided a more critical review of Dirac's innovative theory and Pauli's key role in this development, especially his introduction of the idea of 'spin', which was a consequence of focusing on 'doubleness' (or duplexity). This new 'duplexity' was soon followed by a similar move made by Dirac, who "doubled up" once again to produce a mathematical scheme representing the high-speed motion of the electron with **four** inter-related components. Chapter two also briefly described these radical innovations and discussed the "paradoxes and mysteries" that these new quantum concepts generated, illustrating the problems of understanding sub-atomic ideas that are first introduced purely by mathematical hypotheses.

This chapter concluded with a brief discussion of the present views of the electron, including Feynman's space-time model and the electro-weak theory that forms a key part of the so-called "standard model" of elementary particles. The new electron theory was shown to differ radically from both of these orthodox models – both physically and mathematically.

This research has shown that field theory resulted from the deliberate attempt to produce a single-time theory of interactions that occur between two remote locations (the source and the target – or field point), which always had the desired objective of retaining the single parameter ('time') techniques of Newtonian calculus. The present theory explicitly accepts the two-time (asynchronous) nature of the electromagnetic interaction between remote electrons and constructs mathematical representations that reflect this experimental fact. This approach also implies that any theory based on timeless, spatially dependent potentials is at best a statistical approximation for the dynamical world of nature. Since the focus here is on the interaction itself, then the idea of a single particle becomes only half of the complete picture. Indeed, the idea of a 'free' electron that "gradually turns on the interaction" does not correspond to reality and must be viewed only as another calculational technique; unfortunately, this was the approach adopted by Dirac in his electron theory. One major consequence of the present theory is that the vacuum returns to its Newtonian empty role: it is simply the space between particles, it truly is **no-thing** and contributes nothing here.

8.3 RELATIVISTIC REPRESENTATIONS

Chapter III summarized all the mathematics that has been used in earlier QM approaches to describe 'spin', including Dirac's electron theory. It also reviewed the mathematics that has been introduced in the earlier papers in this research programme and combined elements of these into variants that were used both here and in subsequent papers. The deliberate rejection of continuum mathematics in this programme means that it is important to establish these new discrete mathematical tools on a very firm foundation, particularly as so many theoretical physicists today are people with strong, critical mathematical skills.

The introduction of Pauli 'spin' matrices was the first intrusion of finite mathematics into physics. Ever since Newton, as an eminent mathematical physicist recently wrote [162] : "calculus has been absolutely essential for a proper understanding of physics!" This reflects the great simplicity of Newton's mathematical approach to representing reality: the focus has always been on the single-point in space at any time: differentiation has reduced the viewpoint to how changes occur only locally in terms of the structure or behavior in the tiniest neighborhoods of single points. The asynchronous interaction between point-like electric charges means that this viewpoint is far too narrow, its retention has required the equal retention of continuous properties (fields) of the empty spaces in between – the resulting mathematical infinities have become an embarrassment that can no longer be tolerated. This has been the motivation here for constructing a finite model of physics that can be then closely represented by finite forms of mathematics – matrices and the calculus of finite differences. The interplay of physics and mathematics is always most effective when the characteristics of the mathematics mirrors the characteristics of its objects.

This chapter began with a review of Pauli's (2x2) matrices as they form the starting point for Dirac's extension to the (4x4) representations. The double Pauli matrix products are isomorphic to Hamilton's quaternion bases { **i**, **j**, **k** } and are the basic link across all these representations including their connection to the real (4 x 4) bases $\mathbf{I}_{\mu}$ of the *Natural Vectors* that are the foundational mathematics of the present research programme. The difficult subject of spinors was covered here because these mathematical objects always arise in relativistic or, indeed, all 4D models of the electron and are become important later.

These topics were introduced as a logical lead-in to the new foundational mathematics for the present research programme, referred to here as *Natural Vectors* (or **NV**s), which are a simple algebraic extension of quaternions and appear as a natural extension of complex numbers to the distinct four dimensions of the natural world (hence their name). The mathematics of NVs were briefly reviewed here and linked to the first three appendices where more extensive mathematical summaries can be found. A similar approach was also used to introduce the new extended forms of finite difference calculus, with a link to appendices IV and V for more information. This mathematical preamble became the basis for a fresh examination of the concept of **velocity** when particles are subject to several discrete impulses rather than continuous forces. As with Newton's calculus, differences become the central focus of the mathematical analysis; here the pair-wise two-point differences between consecutive, discrete interactions play the major role in the form of Discrete Natural Vectors (DNVs). When discrete impulses are exchanged between particles the concept of acceleration (and continuous derivatives of position) disappear. Of course, this new emphasis on discreteness in time also abolishes the idea of 'force' that has been at the center of dynamics since Newton. This restores Newton's revolutionary concept of momentum to the focus of mechanics – an idea that was over-simplified by Planck when he tried to extend Newton's instantaneous increase in momentum to very high-speed material particles.

8.4 DISCRETE INTERACTIONS

Chapter IV formed the pivot where the new direction of this research programme diverged fundamentally from conventional physics. Ever since Newton the interactions between material objects have been reduced, to simplify the mathematics, to an abstraction of one object (the 'target') and a set of forces that act upon the target. This programme rejects this artificial and asymmetric view of the world and returns the focus to **the interaction between pairs of objects** - in this case, electrons. This chapter analyzed the key concept of interaction and builds on the insight that this consists of the exchange of **action** between the participating electrons. In contrast to all earlier theories in physics that have followed Newton's calculational approach, centered on the calculus (that was then *post facto* interpreted as a continuum model of reality), the fundamental idea here is that the interaction between electrons does **not** occur continuously but only at periodic instants of time.

The first section here laid down the philosophical foundations on which this theory is constructed. Contemporary physics has rejected philosophy – a position that is viewed here as totally wrong: philosophy clarifies the ideas that we all use to create all our views of the world, whether this be implicit (today's Platonism in physics) or explicitly; since it is false assumptions that are the major source of errors in human affairs, this programme prefers to make its philosophical assumptions explicit. Key concepts of causality, temporal asymmetry and advanced effects were reviewed here before being used in more detail. The concept of action has been used in physics almost as long as Newton's revolutionary concept of momentum, it has always played a relatively minor role – even after Planck's quantization of this quantity, action has still remained in the background. The present theory now gives this idea a starring role in the drama of the micro-world. Section 4.2 focused in on this central idea, building on the experimental observations that all the electron's distinguishing characteristics are finite. The observed discreteness of action implies that the timing of an interaction is no longer continuous. This leads to the intuitive idea of a "quantum of time" (or **chronon**) defining the temporal scale of all interactions and always designated here by the symbol τ; a numerical value of about 10^{-23} seconds is first estimated here for nuclear interactions but a longer "atomic chronon" (τ_A) of about 10^{-17} seconds is found for the larger spatial dimensions of atomic phenomena reflecting the deeper cyclic motion of the nuclear electrons forming the so-called 'nucleons'. This section restored the coheherent focus that interaction dynamics are determined over an extended set of interactions (i.e. over a finite time): each single interaction event is constrained by its wider context. Planck's quantization of action was analyzed here and shown to be a mathematical 'fix' without any explicit supporting physical mechanisms – a stratagem that has been used too often in the last 100 years of theoretical physics. This section also introduced the idea of the separation of changes in discrete action into distinct kinetic and dynamic components. Earlier, the use of Natural Vector algebra showed that the combination of the electron's own action and angular momentum formed a CNV (here called '*activity*'), whose value is invariant while an electron is not involved in an interaction (i.e. while 'free'). Single particle activity is an uninteresting concept by itself but leads to more interesting ideas when inter-particle interactions are introduced when joint (interacting pair) electron activity becomes central. An interaction becomes manifest whenever the exhange in activity induced by the interaction is considered, particularly when this exchange is based on the difference in **velocity** that results from the interaction compared with the 'free' activity that would have occurred had the interaction not manifested itself: this idea was made specific here through the introduction of the '*impact*' operator.

Section 4.3 brought these ideas together in a new two-electron discrete NV called '**interactivity**' when the rich concept of velocity is revisited again but now in even more detail and in a 4D context; Newton's intuition on discrete particle dynamics under generic impulses was shown to be still valid for the extended (digital) idea of electrons subject to the EM investigated here. The discrete form of the two-electron interactivity is defined in terms of the product of the two-electron difference DNV and the two-electron total momentum DNV, this combination is equivalent to the difference in single partle activities and an anti-commutation cross-product; as a result, interactivity is only non-zero for open electron scattering (interactions between similar charges). This section introduced the *quantum of activity* hypothesis by assuming the amount of unit activity exchanged during each interaction is constant (here denoted by a new universal constant of action a – the comparison with Bohr's atomic theory indicated that $a = h / 2$ where h is Planck's action constant). The final part of this section introduced the new, universal form of the EM **impulse** between any two interacting electrons: this is a point-to-point impulse whose magnitude varies inversely with their separation at short-range (less than about 10^{-3} cm) and is fixed at a small, constant value at distances greater than this. The long-range form is responsible for classical EM, while the short-range form accounts for all atomic and nuclear interactions (i.e. quantum effects). The magnitude of this fixed quantum of momentum, ΔI_0 is independent of their relative separation or relative motion and this is defined in terms the smallest change in speed b allowed in such an exchange. This 'speed quantum' is the complement of the maximum relative velocity or 'light-speed' (universally now symbolized by c). The magnitude of b was left indeterminate in the first paper [3] where it was introduced but was now defined here by its relationship to the smallest quantum of action a. The physics of this new theory of electromagnetism is therefore reduced to finding the relationships between these universal electron constants: a, b and c.

8.5 DIGITAL ELECTRONS

Chapter V introduced and analyzed the new concept of the **Digital** Electron Model (DEM), which **added two** new features to the Newtonian model of a point particle. These two new features center on the intrinsic, periodic capacity of the electron to only participate in an interaction with another electron after a universal unit of time (the chronon) when it 'ticks' at certain time instants. Secondly, the digital electron 'clicks' at every one of these times as it moves discretely, at 'light-speed' *c*, in a four-step, transverse motion around its (usually slower) longitudinal direction of travel.

The starting point here was the analysis of the ontological concept of **existence**, with its two possibilities: exist or not-exist; these binary choices became the bridge to integer arithmetic in mapping reality to a human representation. This focus resulted from the definition of a real particle, whose unique location in space, *at every point in time*, defines the reality of its existence. The two real, physical possibilities in the interaction view – "sending or receiving an impulse" lead to a binary representation, which was shown here to fail to reflect the known duplexity effects in atomic spectra. The hypothesis that the emission process can go forwards in time or backwards led to an **extra doubling**, resulting in an integer (4x4) matrix representation, similar but quite different from Dirac. This is referred to as the '*canonical phase* representation' and reflects the 4 basic states that every electron cycles through over time; the 'spin' operator now appeared simply as the natural anti-symmetric NV complement of the existence operator. The canonical and complementary progressions through these four states proved sufficient to give a realistic model of both electrically negative and positive charged electrons without invoking metaphysical fictions like Dirac's "sea" of negative energy states. The second section (§5.2) investigated the idea of quantized rotation, viewed here as intrinsic for electrons as linear motion. Newton's First Law of Motion was extended here by proposing that the electron is always moving in the transverse plane around its average (longitudinal) motion. This extra degree of freedom is a four-step cyclic motion (the 'Twist'), which is always at light-speed and always orthogonal to its current longitudinal direction through space. Each 'step' in this digital, or 'clicking' motion, is in a straight-line from one possible interaction time to the next; in other words, separated in time by one chronon. The inertia of an electron is the persistence of resistance to changes in every electron's longitudinal motion. The transverse motion is intrinsic, always occurring at light-speed and **never** contributes to the electron's inertial mass, as all EM interactions do not alter this speed and its orientation is determined by its latest longitudinal direction. The discrete motion of an electron now means that definitions of temporal averages are more complicated than for simple particles and these ideas were used to clearly distinguish the position representation from the velocity representation without using Fourier transforms. The kinematics of the digital electron were investigated and averages found that behaved like free Newtonian point particles. This analysis led to the conclusion that field theories **cannot** be about the existence of particles (special points) but can only be about the possibility of events that might happen to these particles, anywhere and everywhere, at any time. As such, vector field theories (like Maxwell's EM or QED) are the worst possible combination of mathematical ideas to represent discrete reality that is exemplified by the universality of the electron. But NVs are true vectors: an electron's spatial vector points directly at its location at that time (and not any similar parallel space displacement) while all interactions are fully relational.

The third section began with a detailed investigation of the correct form of the digital electron's momentum definition and showed that the inclusion of the new transverse component directly provided (without use of the Lorentz transform) for the appearance of the mc^2 term that appears everywhere in the theory of Special Relativity. This analysis resulted in a powerful 3D constraint on the electron's velocities across the interaction interval, leading to the idea that the longitudinal velocity almost preserves its direction across the impulse while the transverse velocity "skips a beat", that is, instead of 'clicking' through another quarter turn, the electron effectively clicks through a full half-cycle, corresponding to the π phase-shift of EM when an 'atom re-emits a photon' (using conventional imagery). Further analysis showed that the digital electron model also satisfied the universal *Space-Time Integrity* condition in terms of the average longitudinal velocities of the two electrons as long as both electrons synchronized their transverse displacements (i.e. preserved the Phase-Change Condition).

The concept of *activity*, introduced in the first paper, was shown there to be central to the core concepts of particle mechanics such that the single particle's **Activity** natural vector is a single mathematical quantity combining the four basic concepts in Newtonian physics, namely: time, space, velocity and mass. The third section herein also investigated how the exchange of discrete amounts of activity (or discrete interaction) affected the motion of each of the digital electrons in a series of pairwise interactions. The use of Hermitian conjugation (central to the mathematics of quantum mechanics) was shown to play a key role also for the motion of digital electrons even when they are following 'realistic' trajectories. The NV Hermitian operation was shown to be necessary, in addition to standard complex conjugation, to extract individually from the activity each one of the three most powerful, derived concepts in classical mechanics: **action**, Galilean momentum (or **linear momentum**) and **angular momentum**.

This extra ('digital') motion also adds both a constant term to the Newtonian particle's action and to its angular momentum (both proportional to mc^2) and a small, transverse 'cross-term' whose magnitude decreases to zero as the longitudinal speed u_0 approaches 'light-speed' c, i.e. it includes the Voigt factor $\mathcal{V} = \sqrt{(1 - u_0^2 / c^2)}$. This analysis also showed why Feynman needed to use his $\boldsymbol{\gamma}_5$ matrix (or 'adjoint' wave-function) so that he could define a Lorentz-invariant 4-vector electron current. This also explained why Hestenes needed to use his 'reversal' operation on his 4D geometric products to achieve this result.

The use of finite difference operators showed that the 'clicking' motion was 100% consistent with the discrete formulation of classical mechanics as long as each electron's total velocity (including its transverse motion) was included. The use of DNV algebra readily showed here that only when the non-interacting digital electron moves across a multiple of four chronons does it behave (on average) like a 'free' Newtonian particle. Without introducing any of the features from the theory of Special Relativity, the 'clicking' motion (analyzed within the Electron's Reference Frame) directly generated the Thomas 'precession' factor of two when the discrete rotation consists of exactly four equal steps. The interaction analysis was further extended here to show that the interaction between two digital electrons must be purely radial and this then preserved both the individual and total angular momentum; when the magnitude of the radial impulse varies inversely with the temporal separation then the total change in the action (and the Galilean-momentum) is a constant at each interaction; if the impulse is constant in magnitude ('far distance') then these quantities vary directly with the duration of the temporal difference T_n, requiring a longer and longer time interval to accumulate the same amount of change in total action. This chapter ended with a discussion that showed the connection between these new ideas and Planck's original assumption of quantized action in continuous radiation exchange.

8.6 POSITIVE ELECTRONS

The discovery of the electron around 1900 totally destroyed all the continuous theories of electricity that had been used up until that time – both fluid models and elastic solids (or aethers). Unfortunately, the EM theorists were so enamored of their use of calculus that every effort was made to retain the canonical equations of the electric and magnetic field densities that had evolved since Maxwell's pioneering efforts based on his model of the EM aether. The result was that Helmholtz's fluid model was adopted that could be readily transformed into the canonical field equations. The critical step here was centered on the idea of charge density that could take on both negative and positive values. This led first to the erroneous idea that the proton was just a positive form of the electron (repeated later by Dirac) although there was a massive discrepancy in mass. Electro-dynamic theories at the end of the 19th Century, such as Lorentz's theory of electrons in a stationary aether [98] and Larmor's singularities in a rotationally elastic aether [25], assumed fully symmetric positive and negative charges. Cathode ray experiments discovered that there was no positively charged counterpart to the electron; all positive particles were found to be atomic and molecular ions; in other words, the electrification is fundamentally **asymmetric**. By the second decade of the 20th Century, as a result of Ernest Rutherford's work on the atom, it became the accepted view that the 'positive electron' was just the nuclear proton – the mass discrepancy was not considered an important issue! One of the influential theoretical publications in this area was the unified field theory [163] of Hermann Weyl in 1918, which also predicted symmetrically charged electrical particles with identical mass values.

Chapter six faced the issue of positive electrons directly by building on some of Dirac's mathematical insights but rejecting totally Dirac's idea of negative energy states and his reliance on Planck's arbitrary definition of relativistic momentum. The new theory combines ideas of space and time to explain why positive electrons are as widespread (but hidden) as negative electrons and just as fundamental. Before developing a theory of the positive electron, this chapter began by addressing the mysteries of electric charge – the defining property of interacting particles, especially that this quality manifests in integral multiples of the charge found on the electron. Electric charge is only found to have two complementary qualities that have been given the conventional assignments: postive and negative. The present theory, developed in section two of this chapter, proposed that this universal binary feature could be explained by associating this key characteristic with the two rotational directions that a particle could move around its (large-scale) motional axis.

The present theory views positive and negative electrons as two complementary manifestations of the same type of entity, just phase-shifted from one another in the timing of their interactions due to their complementary trajectories through space. This idea is sufficient to explain the observation that particles with similar charges repel one another while particles with opposite (or complementary) charges attract each other. This phase-shift is also a clue to how the complementary electrons differ in their motions through space: in the transverse plane, the electron 'clicks' to the left while the positron 'clicks' to the right. Like Dirac's dual-spinor electron theory, the present theory posits that every electron can exist in one of four possible states. The present theory introduces four unique electron states that are occupied by both electrons and positrons over time but now each type of electron *cycles* through these four states in a **fixed** but **different sequence**. These four states determine *when* the individual particle **may interact** *forwards or backwards in time* with another particle.

Chapter VI also includes an analysis of why Lorentz invariance is irrelevant in any theory, such as the present theory, that is constructed around interactions that span the two interaction events between pairs of interacting electrons when both events are on each other particle's light-cone . This programme does not take the position that the mathematical formulation of a physical theory must be Lorentz invariant; this is viewed only as a requirement imposed on all field theories, as it imposes constraints on every location in space throughout all of time, mapping each such logical point (with their fields) with points in inertial frames of reference. The present theory (again like Newton's) focuses on only those points occupied at any time by material particles (electrons) with inertial mass. The resulting equations describing their dynamics must then be Galilean invariant as their relative behaviour must appear the same to any inertial observer. The present theory is purely relational between the two interacting electrons, so their relative motion must be independent of the inertial motion of any third-party electrons (or 'observers'): reference frames without 'observers' are simply mathematical or implied 'realist' schemes. Again, CPT is not a constraint on nature itself but on the mathematical representations used to describe nature: in particular, on the **single-time** theories using continuum mathematics, such as constructed around Hamiltonian or Lagrangian dynamics.

Section 6.3 introduced the analysis of **attractive** interactions that until here in this programme was limited only to repulsion. This chapter includes the first comprehensive analysis of the electro-dynamics of a pair of attractive particles (electrons); this complements a similar analysis presented in the previous paper when only the dynamics of a pair of repulsive particles was considered. This analysis showed how non-continuous interactions allow two point particles to occupy the same location in space at the same time; this allows real solutions to one-dimensional scattering of two attracting electrons. These trajectories never involve infinite forces snd since the interaction is asynchronous and not instantaneous it **never** occurs when the two particles are at their closest (same time) spatial separation. These new analyses are made possible by rejecting the traditional one-time, continuum calculations that have tried to use the statistical approximation represented by Maxwell's Equations. This scattering analysis showed that there were two possible attractive solutions but proved that only the 'rebound' (or parabolic) motion was physically possible while proving there are no solutions where each particle begins and ends at rest.

The present theory now views matter as consisting of only positive and negative electrons, differing only in their interaction phase. This ontology gives meaning to the 'Secrets of Heraclitus (500 BC) whose intuition saw beneath the apparent stability of the macroscopic view of the world to an interior world where everything is in flux: a veritable 'battlefield' of conflicting opposites. Attractive impulses are key to building more complex, stable structures from only positive and negative electrons. This chapter included two such simplified models describing the neutrino and the Bohr hydrogen atom - more comprehensive models are reserved for later papers. The neutrino model predicts that the neutrino will have an effective diameter of $(\pi/\alpha)\Lambda$, which is about 430 times larger than the classical electron radius Λ while each particle is moving at half light-speed in 1D.

The simple hydrogen model using polygonal trajectories showed that the formulae developed here for the impulse-model of EM interactions are capable of providing a Newtonian-like model of the hydrogen atom, without using the 'rules' of quantum mechanics or the paradoxical assumptions of the Bohr model of the atom. Once again, the new approach rejected Maxwell's Equations, especially any use of the venerable Coulomb Law of electrostatic attraction that plays a key role in all other models. Again, the factor (π/α) appears as a scaling factor between conventional QM and the present theory. This analysis showed that Bohr was essentially only working with quantized kinetic action and geometric constraints since the Coulomb 'force' is seen as instantaneous. At atomic separations, the EM impulse is defined by its short-range form, which ensures that the dynamic action equals the kinetic action. This perspective drastically reduces the number of sides in the polygon from $2(\pi/\alpha)n$ to $2n$. This analysis was extended to a one-dimensional (or 'linear') model when Coulomb's 3D "broadcast" model of the EM interaction was reduced to a direct point-to-point impulse so that the principal quantum number became the number of impulses needed to cross the atom. In the lowest energy (or ground) state only one impulse is now required to reverse the orbital electron at each extremum, resulting in an oscillating, one dimensional (linear) trajectory going through the nucleus.

This chapter ended with an estimate of the maximum radius of the hydrogen atom – both the Bohr model and QM predict that this is infinite (in other words, there are no restrictions on the size of the principal quantum number). In contrast, this analysis showed that the maximum distance that an electron can move around the proton and still stay bound into a neutral atom is of the order of one mm. This result also implied that the smallest increment in **speed** (the *velocity-quantum*) that an electron can exhibit is about two mm per second.

8.7 DECONSTRUCTING DIRAC

Chapter VII refocused on Dirac's most famous equation. It was a positively motivated attempt to deconstruct Dirac's final views on his greatest contribution he described in chapter XI of the fourth and final edition of his masterpiece *Principles of Quantum Mechanics*. Nonetheless, Dirac's approach is severely criticized both from previously unstated philosophical and mathematical perspectives. The present theory rejects the asymmetry of focusing on a single electron and restores all the original source electrons to an equal status as the target electron. Dirac, in constructing his probability density, had to revert to restoring the implied charge-density model that was the foundation for classical EM theory and was severely criticized in an earlier paper. Criticisms of several major features of Dirac's electron theory were covered in this chapter, including his desire to make his non-relativistic quantum theory (90% of the text) Lorentz invariant and his prediction of the electron's own intrinsic light-speed oscillations around its center of mass (*Zitterbewegung*). Dirac's 'Zitterbewegung' is here readily interpreted in terms of the intrinsic 4-step, cyclic transverse motion of every electron. This chapter also expanded on Dirac's perennial concern that Hamiltonian mechanics (used for all versions of quantum theory) was inherently too classical to be a suitable basis for a relativistic theory. This classical versus relativistic dichotomy always bothered Dirac when he discussed the role of 'observables' in any relativistic theory, particularly as this concept lies at the heart of his own formulation of non-relativistic QM. The new theory being developed here rejects this central role of the mathematics of 'observables'.

Irrelevant Relativity

It is ironic that Dirac always thought that making his equation of the electron Lorentz invariant was the most important feature of his theory as this has not been the judgment of history. Although Einstein constructed his Special Relativity Theory (SRT) on Maxwell's classical theory of EM, SRT is <u>not</u> about EM forces at one field point in time (a single event) only viewed from different inertial reference frames. In fact, the introduction of forces into SRT contradicts the physical model of empty space that is supporting optical transmissions between 'observers' in different inertial reference frames with their own measuring 'clocks' and 'rods'. Rather the basis of EM, as developed here, is the quantized interaction between two electrons, which is independent of **all** reference frames, even accelerated ones. The real physical requirement is to develop a theory of electron interactions at <u>any</u> speed. The basic error made by Dirac was to ignore the totally false assumptions that Planck had used in developing his relativistic mass-energy formula in 1907. Planck's Proposal for relativistic momentum was also analyzed extensively in an earlier paper [9], which clearly demonstrated that this mathematical hypothesis was **not** consistent with the physics of electro-magnetism.

Anti-Matter

The mathematical heart of Dirac's electron theory was the use of four states (or solutions) to describe an electron; these were very embarrassing for Dirac because two of them were needed for his model of the positron but they corresponded to states of negative energy – a totally false idea in physics when describing the motion of a free particle. These ideas are not needed in the new theory to explain the positron and all four states are given an entirely different interpretation without introducing any need for negative energy. The present theory returns to Newton's philosophical concept of the vacuum (contrasting with that of DesCartes *and* Dirac) that a volume of space without particles literally contains no thing: no fields, no energy; nothing. In the present theory there is no infinite 'self-energy' as the electron is not modeled here as a real object of any <u>finite</u> size covered with repulsive electrical 'paint' (CEM) nor does it interact with an infinite number of vacuum negative-energy electron states or its own EM field (QFT). It only interacts with another electron that is never at the same location in space at the same time. In the present theory, these four states are seen as the fundamental cyclic basis that describes the sub-microscopic motion of each electron as it 'clicks' around its longitudinal (observed) motion. As well as a kinetic role, these four states are basic to the dynamic behavior of the electron – they define where and when the electron may participate in an interaction with another electron. A mapping between the explicit (4 x 4) matrix forms of Dirac's famous alpha matrices and the phase-transition matrices introduced here show that Dirac's transverse α matrices (1 and 2 or + and –) are moving the electron between his positive energy spin states #1 and #2 and between his negative energy spin states #3 and #4 – that is, just spin-flips, while preserving the electron's energy signature. Dirac's longitudinal matrix α_3 generates transitions between positive and negative energy states – that is, energy-flips, while identifying the spin signature (with a sign change if the 'spin' is 'down'). Dirac's temporal matrix α_4 preserves the electron's state but identifies if it is a positive or negative energy state. As a physicist, these negative energy states worried Dirac very much (mathematicians just dismiss them as a 'sign problem'); eventually he came down to viewing these states as real but unobservable unless they lost an electron leaving behind an observable positive electron. When interacting electrons are viewed from an ontological perspective, it is obvious that a 'hole' is not the same as the absence of a particle – the interactions of all the other negative energy electrons in the Dirac 'sea' with any one of the negative energy particles is <u>not</u> the same as <u>not</u> interacting with its absence, that is nothing: infinities are to be expected.

Positrons

The actual discovery of positively charged electrons (i.e. positrons) ended all criticism of Dirac's theory, with theorists soon viewing this as the ultimate vindication of the mathematical approach to generating progress in foundational physics. The fact that this 'prediction' emerged out of the embarrassment of Dirac's negative energy states was soon forgotten, although few physicists now study this theory in any critical detail – like so much else in theoretical physics it is now taught just as a mathematical development. The present theory offers a much simpler model of the positron – one involving just a 'clicking' motion but in the opposite direction around its longitudinal axis of motion. The other distinctions involve how the two forms of the electron evolve over time, with critical phase (timing) differences in how and when they interact. This is now a fully discrete theory of the electron – one physically and mathematically suitable for the ultimate discrete particle.

Obviously, the new model of the positron implies that they should be as common as the electron but this is apparently not so. The resolution of this puzzle will be explored in subsequent papers where it will be shown that the vast majority of positrons are intimately involved in close-range interactions with negative electrons forming configurations that are very stable in time.

Spin

The authors of one of the most famous graduate texts in EM [164] admitted that: "It is impossible that the electron's charge has zero physical extent without introducing various mathematical divergences. … the idea of a 'rigid' dimension for an object is relativistically self-contradictory." This is the principal reason why most experts in SRT reject the idea that 'spin' is associated with the self-rotation of a small rigid body. Incidentally, these authors have no difficulty in assuming that electrons occur as a charge-density type of substance such that mathematical continuity still makes sense at very small distances where the Coulomb forces become infinite. Fortunately, the present theory is discrete and rejects this possibility. In spite of a few such caveats, the majority of physicists accepts the terminology at face value and assumes that electron 'spin' is still rotation.

Although Dirac's relativistic theory of the electron exhibited similar energy characteristics as any small, rotating magnetic dipole that was interacting with a magnetic field this was only valid at non-relativistic speeds. The fact that the concepts of magnetic dipoles and magnetic fields are macroscopic analogs with no correspondence at the level of electrons does not seem to bother theorists who continue to view Dirac's Equation as the ultimate description of matter. It was actually Dirac's own derivation showing that total angular momentum was conserved (in an electrostatic or central field only) that indicated that the electron's 'spin' corresponded to the mathematical operator ½ h_D $\boldsymbol{\sigma}$ even though not one of these angular momentum quantities is directly measurable in the hydrogen atom (the only solvable model). In contrast, the present model shows that 'spin' is **not** an invariant scalar property of every real particle but a constraint on the interaction between particles (like all dynamic properties) so that its value in a composite system, such as a nucleon, is not simply the additive sum of the 'spins' of its component particles. The fact that this value is still ½ h_D for all real, stable fermions will also be demonstrated in a later paper. The first appearance of the concept of 'spin' in a relativistic theory, such as Dirac's, is an implicit acknowledgement that at very high relative speeds, advanced as well as retarded time differences are significant when particles interact.

In the present theory, 'spin' is interpreted as the unit of interaction between two electrons. Since each electron is treated here as a special point in space and as 'spin' is not viewed as a self-rotation, there is no requirement to introduce a finite radius to describe the electron. Dimensionally, both action and angular momentum are defined in terms of the product of momentum and distance. This has led to the confusion about the nature of 'spin' and the fact that the electron's 'spin' is quantized. The initial investigation of the Natural Vector motion of a classical (Newtonian) point particle clearly illustrated that the 'angular momentum' and 'action' were intimately tied together through the concept of 'activity'. This was readily extended to the Digital Electron model, where the 4-step discrete rotational motion around its own longitudinal (average) velocity introduced a fixed, finite (quantum) quantity to the electron's action and its angular momentum. This showed that electron 'spin' is not just a change in the isolated electron's kinematical behavior but is a reflection of its 'twisting' form of directed interaction between two axially rotating electrons, where differences in event timings are critical.

Physics has not yet faced up to the implications of the experimental observations of the electron – that this is a fundamental, **point** particle of matter: it has **no** spatial extent and **cannot** be viewed as a finite object rotating *in toto* around some interior axis. In the present theory, 'spin' is interpreted as the unit of interaction between **two** electrons. The origins of Dirac's errors are finally exposed as due to a direct result of starting new ventures with mathematical leaps of the imagination instead of acknowledging the primacy of philosophy – new, **visualizable** concepts must come first and **then** mathematics.

8.8 CONCLUDING REMARKS

Scientific History

As in previous papers in this series, the work of one of the giants of physics is used to exemplify an earlier approach to one of the central areas of modern physics. Unfortunately, the foundations of modern theoretical physics have been accepted too readily: physicists rarely study the **history** of their own subject and simply accept the canonical interpretations that usually constitute their formal education, which is too often just a mathematical exposition (a technique that is easier to teach than physics and much easier to examine). The papers in this series demonstrate that there are alternative theories that can explain the unassailable facts discovered by experimental physicists; no one can challenge these facts but theorists forget too often today that their theories are **not** facts. The approach used throughout this programme builds on a deep respect for the history of physics and recovers several older, neglected (and often forgotten) alternatives. The academic world today, like so much of modern life, has retreated from quality into the easy comparisons of quantity. This has accelerated the pressure to 'publish or perish' that is trivializing the noble traditions of the academy to honor its past and build on firm foundations. As a result, graduate students, particularly in the exploding sciences, are rushed to the frontiers of one tiny aspect of their subject so that they can add another new brick to the wall – no matter how trivial. In physics, this means both the historical foundations and the philosophical implications are almost totally ignored throughout the career of academic physicists. The consequence is that assumptions are almost never challenged; this is particularly true of old assumptions, like the *Continuity Assumption*. This research programme rejects this modernist style and builds on challenging the 'truths' that have taken on the power of **groupthink**; unfortunately this results in the orthodoxies that characterize religions rather than the openness of science.

Natural Philosophy

Natural philosophy has always begun with **ontology** – the identification of the fundamental objects of reality, the qualities and interactions of these objects are then investigated; finally, if possible, a mathematical scheme is then applied to generate numerical values that can be compared with standard experiments. This has not been the approach for over 100 years. The popular modernist approach has not been successful – theoretical physics has spiraled into an expensive dead-end. It has got lost in the stars (astrophysics and cosmology) and is building multi-billion machines to investigate energy scales that have no bearing on human existence. Public support for these ***theological*** activities will soon come to an end when nothing results.

Dirac had an intuition that the electron was one of the most important constituents of material reality; this programme takes this view to its logical extreme: **the only particle of material reality is the electron**. The modernist approach of inventing a new particle to explain new sets of phenomena is firmly rejected. This paper continues the investigation of the evolution of the foundational ideas of material reality. This realist research programme is firmly entrenched in the particle view of matter, rejecting all continuum concepts. Newton was the first natural philosopher to focus on the quantitative properties of material **point particles**, aggregating the interactions between all the particles into a single, simultaneous 'force' acting on a single, 'target' corpuscle. This programme identifies the basic particle with the electron, experimentally found to exist only as a point particle. In contrast to Newton's inert matter-spheres, these electrons are now viewed as "pulsating with possibilities", interacting **asynchronously** over a universal time-cycle when pairs of electrons are separated across space and time that form an integral multiple of the foundational units of time and space (referred to here as the chronon and luxon) and when both electrons are at their appropriate phase-points. This grand simplification promises to break the ancient log-jam that began when Newton realized that continuum mathematics could not even solve the **three-body** problem.

This research programme rejects the move to phenomenology that has increasingly characterized theoretical physics over the last 150 years. The ongoing invention of equations (especially differential equations that cannot be solved) to describe a simplified corner of nature has too often only resulted in the creation of contradictions and an obvious lack of progress in this most fundamental of sciences. When one cannot say what is happening in our model of the world then all progress at the higher levels of complexity suffers. The most significant step in this withdrawal from visualization was the conscious rejection of the idea of particle trajectories because "electrons could not be observed without interfering in their motion". The present theory makes a clear distinction about what happens in the world (level I) and how humans go about measuring this behavior (level III). It is our shareable **imagination** (level II), which acts as the great integrator between these levels. This programme returns to the earliest days of natural philosophy and willingly follows Newton in imagining the motion of unobservable particles as they are proposed to move through space acting only under their mutual influences; measurement predictions will be deferred for a later paper when a replacement theory for quantum mechanics will be proposed.

Invisible Assumptions

In taking the position that philosophy precedes mathematics in theoretical physics, the present theory nails its flag without reservation to the metaphysical tradition in natural philosophy known as 'realism'. This is the view that the world exists independently of human beings and that there are objects whose unconditional existence form the basis of material reality. It has been clear from the beginning of this research programme that these real objects are considered to be those material examples now known to science as 'electrons'. This theory makes a clear distinction between the real existence of a specific electron at a given location in space at a given instant of time (symbolically designated as $x = \{t; \underline{x}\}$) and how humans detect (or 'measure') the existence of this type of particle within a narrow region of space and time. This reflects both facts that all measurements are a sub-class of all electron interactions and all humans are gigantic collections of myriads of electrons. As humans cannot keep track of the identity of individual electrons (especially when they move close together), it is inevitable that all measurements of electrons must become **statistical**. Standard quantum mechanics (QM) compounds the concepts of existence and measurement into **one hybrid** representation that is intrinsically statistical. The new theory presented here keeps these two concepts separate and works mostly in the realistic (existent) representation; only at the final stages, is the theory's measurement model explicitly introduced in order to compare theoretical predictions with expected measurements.

This will demonstrate that quantum mechanics was an attempt to construct a theory of nature only in terms of how humans interact with the world rather than the traditional, realist view that the world 'goes on its merry way' without any need for humans even to exist. This has resulted, not in a description of nature 'doing its thing', but only of those situations where human measurement has altered the unencumbered activity of the world by its explicit interference. It is the highest level of arrogance to assume that the world is constructed in terms of how we humans decide to measure it. This is just the hidden **Platonic** agenda: only what we measure (assign a number) can exist – this defines the world in terms of number. The public intuitively rejects this ultimate form of reductionism. They know that their lives are far too rich too be reduced to such a single, one-dimensional perspective and suspect that this is true at all levels of reality.

Relativistic Electrons

The present paper has described an alternative theory to the standard mathematical model of the electron (the Dirac Equation) that rejects both the field theoretical model of EM and the Hilbert-Space model of QM. It is, by design, in the Newtonian tradition that is now considered obsolete by those theoreticians that are committed to purely mathematical descriptions of the micro-world. It emphasizes the interaction between **pairs** of electrons that is both **discrete** and **saturated**. As such, it avoids all those features that have contributed to the infinities of both classical and quantum descriptions of the electron. This now provides a firm foundation for revisiting all those aspects of atomic and molecular physics that have been left behind in the rush to 'understand' the universe at its largest and smallest scales that have no significance to human technology or progress.

The new theory explicitly recognizes that electrons not only move at 'light-speed' (transversely, relative to the longitudinal motion) but may also exceed this 'speed limit' relative to one another. The theory acknowledges that in this latter situation there can be no mutual interaction between such high-speed electrons but completely rejects the notion that this is forbidden "because of the special relativity theory" since this theory has rejected this so-called foundational physics theory as a only a mathematical consequence of adopting its own mathematical representation of reality – field theory. It is unscientific to reject one theory on the basis of another theory: it is only a logical contradiction or experimental evidence that is a sufficient ground to dismiss any theory in physics.

Future Research

This paper has focused on some of the work of Paul Dirac – his relativistic theory of the electron. A later paper will return to his great masterpiece when modern quantum mechanics (an area he rationalized) will also be deconstructed and an alternative theory will be presented based on a realistic, point-particle model of the micro-world that is always disrupted by our clumsy, macro-scale attempts to measure the hidden levels of activity that form the foundation of the world.

The next paper will revisit the phenomenon of '**light**' that has become both the model for all wave activity and the primary subject of all field theories. This paper will demonstrate that both of these features, along with the idea of the '**photon**', are irrelevant when the focus shifts to the interaction between **electrons**. The final paper in the quantized electro-magnetism (QEM) series will use the discrete algebra of natural vectors and the *Saturation Hypothesis* to calculate the dynamical effects of multi-electron atoms and simple molecules. The final set of papers in the nuclear electro-magnetism (NEM) series will develop all-electron models of the nucleons and principal particles found in the 'particle zoo' known as the *Standard Model*.

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A.1 APPENDIX I: SUMMARY OF SINGLE-POINT CNVs

A1.1 DEFINITION OF SINGLE-POINT CNVs

The definition of a Natural Vector [165] is viewed as an *imaginary scalar quaternion*, symbolized by $\mathbf{Q}$.

$$\textbf{Natural Vector}: \quad \mathbf{Q} \equiv i\,\mathbf{I_0}\,q_0 + \mathbf{I_1}\,q_1 + \mathbf{I_2}\,q_2 + \mathbf{I_3}q_3 \equiv \mathbf{Q}_0 + \sum_j \mathbf{Q_j} \equiv \{i\,q_0\ ;\ \underline{q}\} = i\,q_0\,\mathbf{I_0} + \sum_j q_j\,\mathbf{I_j}$$

Here the $\mathbf{I_\mu}$ are defined in terms of real 4x4 matrices and $\mathbf{I_0}$ is isomorphic with the unit number, while the $\{\mathbf{I_1}, \mathbf{I_2}, \mathbf{I_3}\}$ are isomorphic with Hamilton's three linearly independent imaginary quantities $\{\mathbf{i, j, k}\}$. So these bases satisfy the group multiplication rules, using the indices j, k, l = 1, 2, 3 and μ = 0, j :

$$\mathbf{I_0}\,\mathbf{I_\mu} = \mathbf{I_\mu}\,\mathbf{I_0} = \mathbf{I_\mu} \qquad \mathbf{I_j}\,\mathbf{I_k} = -\delta_{jk}\,\mathbf{I_0} + \varepsilon_{jkl}\,\mathbf{I_l}$$

Here, δ_{ij} is the Kronecker delta symbol with value +1 when both indices are equal or zero otherwise and ε_{jkl} is the cyclic permutation tensor whose value is zero unless all three indices are different when its value is +1 if the indices are cyclic (even permutation of 1,2,3) or –1 if anti-cyclic (odd permutation). The rules for (conjugate) addition and multiplication of two Natural Vectors $\mathbf{A}^*$ and $\mathbf{B}$ become:

$$\text{Addition:} \quad \mathbf{A}^* + \mathbf{B} = -i\,\mathbf{I_0}\,(a_0 - b_0) + (\underline{a} + \underline{b}) \bullet \underline{\mathbf{I}}$$

$$\text{Multiplication:} \quad \mathbf{A}^*\mathbf{B} = \mathbf{I_0}\,(a_0\,b_0 - \underline{a} \bullet \underline{b}) + i\,\underline{\mathbf{I}} \bullet (b_0\,\underline{a} - a_0\,\underline{b}) + (\underline{a} \wedge \underline{b}) \bullet \underline{\mathbf{I}}$$

Since electrons are each considered both unique and eternal they can be 'labeled' by a unique positive integer identifier 'k'. It was shown earlier (§I-5.1.4) that electrons are fermions, so each electron will have a unique position in space ($\underline{x}$) at any time, t. The fundamental hypothesis of this research programme that these two parameters of every electron can be mapped into their own individual Natural Vector, $\mathbf{X}(k)$ or $\mathbf{X}_k$, rather than two separate traditional algebraic functions $\{t, \underline{x}(t)\}$.

$$\textbf{Electron Hypothesis}: \quad \{\,\underline{x}(k: t_k)\,\} \cong \mathbf{X}_k \equiv i\,c\,t_k\,\mathbf{I_0} + \underline{x}_k \bullet \underline{\mathbf{I}}$$

The square (or 'norm') of this 'positional' NV is: $\mathbf{X}(t)^*\,\mathbf{X}(t) = (c^2t^2 - x^2)\,\mathbf{I_0}$

Similarly,

$$\textbf{Velocity}: \quad \mathbf{V}(t) \equiv \lim_{\delta t \to 0} \{\,(\mathbf{X}(t + \delta t) - \mathbf{X}(t))\,/\,\delta t\,\} = d\mathbf{X}(t)/dt = i\,c\,\mathbf{I_0} + \underline{v} \bullet \underline{\mathbf{I}}$$

The "norm" of this velocity NV is: $\mathbf{V}(t)^*\,\mathbf{V}(t) = (c^2 - v^2)\,\mathbf{I_0}$

A1.2 SUMMARY OF CNV FLOW VECTORS

The theory of Natural Vectors follows Hamilton by extending his 'nabla' (or 'gradient') 3D space operator ($\underline{\nabla}$) to the 4D Natural Vector ***Gradient*** applied to any scalar function ψ that is continuous in the four space-time variables $\{t\ ;\ \underline{x}\}$:

$$\textbf{Gradient}: \quad \nabla\psi(t\ ;\ \underline{x}) \equiv i\,\mathbf{I_0}\,\partial_0\psi(t\ ;\ \underline{x}) + \underline{\mathbf{I}} \bullet \underline{\nabla}\psi(t\ ;\ \underline{x}) \quad \& \quad \underline{\nabla} \equiv \hat{\mathbf{e}}_1\partial_1 + \hat{\mathbf{e}}_2\,\partial_2 + \hat{\mathbf{e}}_3\partial_3 \quad \partial_0 \equiv \partial/\,c\,\partial t \quad \partial_1 \equiv \partial/\,\partial x_1 \text{ etc}$$

The conjugate of the NV gradient operator can be applied to any continuous natural vector (CNV) function, $\mathbf{Q}(t\ ;\ \underline{x})$:

$$\nabla^*\mathbf{Q}(t\ ;\ \underline{x}) = \mathbf{I_0}\,(\partial_0 q_0 - \underline{\nabla} \bullet \underline{q}) + i\,\underline{\mathbf{I}} \bullet (\underline{\nabla} q_0 - \partial_0 \underline{q}) + \underline{\mathbf{I}} \bullet (\underline{\nabla} \wedge \underline{q})$$

If $\nabla^*\mathbf{Q} = 0$ then: 1) $\underline{\nabla} \bullet \underline{q} = \partial_0 q_0$ 2) $\underline{\nabla} q_0 = \partial_0 \underline{q}$ 3) $\underline{\nabla} \wedge \underline{q} = 0$

The "***Zero Condition***" CNV operator, $\mathbf{Z}$ is defined as: $\mathbf{Z}\psi \equiv i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla} + \underline{v}\,\partial_0)\psi - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\psi)$

The CNV equivalent of the total-time differential for a scalar function: $\mathbf{I_0}\,d\psi/dt = -\mathbf{V}^*\nabla^*\psi$ if $\mathbf{Z}\psi = 0$

The "CNV ***Flow***" equation can be derived for any continuous CNV: $d\mathbf{Q}/dt + \mathbf{V}^*\nabla^*\mathbf{Q} = 0$ if $\mathbf{ZQ} = 0$

A1.3 SUMMARY OF CNV VOIGT VECTORS

All CNVs satisfy several useful identities, presented here; Q_0 and α are scalar functions and $\underline{Q}$ is a vector function.

1. $\nabla^* \mathbf{X} = -2\,\mathbf{I_0}$ 2. $\nabla^* \mathbf{V} = -i\,\underline{\mathbf{I}} \bullet \partial_0 \underline{v}$ 3. $\nabla^* \alpha = -i\,\mathbf{I_0}\,\partial_0 \alpha + \underline{\mathbf{I}} \bullet \underline{\nabla}\alpha$

4. $\nabla^*(\alpha\,\mathbf{Q}) = \alpha\,(\nabla^* \mathbf{Q}) + (\nabla^* \alpha)\mathbf{Q}$ 5. $\nabla^* \nabla \alpha = \nabla\,\nabla^*\,\alpha = \mathbf{I_0}\,(\partial_0^2 - \underline{\nabla}^2)\,\alpha \equiv -\mathbf{I_0}\,\Box\alpha$

6. $\mathbf{V}^* \nabla^*(\alpha) = -\mathbf{I_0}\,(c\,\partial_0 + \underline{v} \bullet \underline{\nabla})\alpha - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla} + \underline{v}\,\partial_0)\,\alpha + \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\alpha)$

7. $\mathbf{Q}\,\nabla^* \alpha = \mathbf{I_0}\,(Q_0\,\partial_0 \alpha - \underline{Q} \bullet \underline{\nabla}\alpha) + i\,\underline{\mathbf{I}} \bullet (Q_0\,\underline{\nabla}\alpha - \underline{Q}\,\partial_0 \alpha) + \underline{\mathbf{I}} \bullet (\underline{Q} \wedge \underline{\nabla}\alpha)$

8. $\nabla^*(\alpha\,\mathbf{V}) = \mathbf{I_0}\,(c\,\partial_0 \alpha - \underline{v} \bullet \underline{\nabla}\alpha) + i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla}\alpha - \underline{v}\,\partial_0 \alpha - \alpha\,\partial_0 \underline{v}) - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\alpha)$

For any continuous function, $\alpha(t; \underline{x})$ we can define generic *Voigt Vectors* as these prove central in this programme.

Voigt Vector: $\mathfrak{V} \equiv -i\,c\,\alpha(t; \underline{x})\,\mathbf{I_0} + \alpha(t; \underline{x})\,\underline{v}(t) \bullet \underline{\mathbf{I}} = \alpha(t; \underline{x})\,\mathbf{V}^* = \{\, i\,\mathfrak{V}_0; \underline{\mathfrak{V}} \,\}$

The scalar and vector components of a *Voigt Vector* satisfy the following equation, named in honor of the pioneer of asynchronous EM - Ludvig V. Lorenz; this equation will also re-appear in many forms throughout this programme.

The ***Lorenz Equation***: $c\,\underline{\mathfrak{V}} + \underline{v}\,\mathfrak{V}_0 = 0$

The most important CNV associated with a continuous Voigt Vector ($\mathfrak{V}$) is its local gradient, $\nabla^* \mathfrak{V}$; the gradients also play the role of forces in the corresponding electromagnetic theory. This has the explicit form:

$$\nabla^* \mathfrak{V} = -\mathbf{I_0}\,d\alpha/dt - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla}\alpha + \partial_0 \underline{\mathfrak{V}}) + \underline{\mathbf{I}} \bullet \underline{\nabla} \wedge \underline{\mathfrak{V}}$$

This has three components, so the vector component of the CNV ***Gradient*** $\mathfrak{G}$ will have both real and imaginary parts.

Definition: $\nabla^* \mathfrak{V} \equiv i\,\mathfrak{G} = i\,[i\,G_0\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet (\underline{G}_R - i\,\underline{G}_I)] = -G_0\,\mathbf{I_0} + i\,\underline{\mathbf{I}} \bullet G_R + \underline{\mathbf{I}} \bullet G_I$

Comparing coefficients gives: $G_0 = d\alpha/dt$; $G_R = -(c\,\underline{\nabla}\alpha + \partial_0 \underline{\mathfrak{V}})$; $G_I = \underline{\nabla} \wedge \underline{\mathfrak{V}}$

Now $\mathbf{V}^* \nabla^* \mathfrak{V} = \mathbf{I_0}\,(i\,c\,G_0 - i\,\underline{v} \bullet \underline{G}_R - \underline{v} \bullet \underline{G}_I) + \underline{\mathbf{I}} \bullet (c\,\underline{G}_R + \underline{v} \wedge \underline{G}_I) - i\,\underline{\mathbf{I}} \bullet (c\,G_I - \underline{v} \wedge G_R)$

But if $\mathfrak{V}$ is a ***Flow Vector*** then: $d/dt\,(\mathfrak{V}) = -\mathbf{V}^* \nabla^* \mathfrak{V} = d/dt\,(-i\,c\,\alpha\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{\mathfrak{V}}) = -\mathbf{I_0}\,i\,c\,d\alpha/dt + \underline{\mathbf{I}} \bullet d/dt\,(\underline{\mathfrak{V}})$

This gives the ***Gradient Equations***: 1) $\underline{v} \bullet \underline{G}_R = 0$ 2) $\underline{v} \bullet \underline{G}_I = 0$
3) $c\,\underline{G}_R + \underline{v} \wedge \underline{G}_I) = -\,d/dt\,(\underline{\mathfrak{V}})$ 4) $c\,G_I - \underline{v} \wedge G_R = 0$

Finally, in differential terms: $-i\,\mathbf{V}^*\,\mathfrak{G} = -\mathbf{V}^* \nabla^* \mathfrak{V} = d/dt\,(\mathfrak{V}) = -i\,(d/dt\,\mathbf{X}^*)\,\mathfrak{G}$ $\therefore\ i\,d\mathfrak{V} = d\mathbf{X}^*\,\mathfrak{G}$

There are a class of "harmonic" functions $\psi(t; \underline{x})$ that always satisfy the *Wave Equation* : $\Box\psi(t; \underline{x}) = 0$
Each such function has its own corresponding ***Associate*** CNV, defined as (note, not conjugate): $\mathfrak{A} \equiv \nabla\psi(t; \underline{x})$
The addition of any Associate Vector to a Voigt Vector, $\mathfrak{V}$ defines its corresponding ***Gauge Vector***, $\mathfrak{V}'$.

Definition: ***Gauge Transform*** $\mathfrak{V}' \equiv \mathfrak{V} + \nabla\psi$ where $\Box\psi = 0$

Since $\nabla\psi = i\,\mathbf{I_0}\,\partial_0\psi + \underline{\mathbf{I}} \bullet \underline{\nabla}\psi$ then $\underline{\mathfrak{V}}' = \underline{\mathfrak{V}} + \underline{\nabla}\psi$; $\mathfrak{V}_0' = \mathfrak{V}_0 + \partial_0\psi$ $\therefore\ c\,\alpha' = c\,\alpha - \partial_0\psi$

The conjugate gradient of this Associate CNV ($\mathfrak{A}$) is zero, since: $\nabla^* \nabla\psi = -\mathbf{I_0}\,\Box\psi = 0$ $\therefore\ \nabla^* \mathfrak{V}' = \nabla^* \mathfrak{V}$ or $\mathfrak{G}' = \mathfrak{G}$

The harmonic property of $\mathfrak{A}$ ensures that the Gradient of any Voigt Vector ($\mathfrak{G}$) remains invariant under a Gauge Transform.

A1.4 SUMMARY OF CNV SUB-VECTORS

All natural vectors can be decomposed into longitudinal and transverse sub-vectors or components: $\mathbf{A} = \mathbf{A}_L + \mathbf{A}_T$ where:

$\mathbf{A}_L = A_3\,\mathbf{I}_3 + i\,A_0\,\mathbf{I}_0$ & $\mathbf{A}_T = A_1\,\mathbf{I}_1 + A_2\,\mathbf{I}_2$ or in terms of 'transverse' bases (see §3.1.4)

Transversely: $\mathbf{I}_\rho \equiv \mathbf{I}_1 + i\,\rho\,\mathbf{I}_2$ where $\rho = \pm$ or ± 1 then $A_\rho \equiv \tfrac{1}{2}(A_1 + i\,\rho\,A_2)$ N.B ½ & i factors.

$\therefore\ \mathbf{A}_T = A_+\,\mathbf{I}_- + A_-\,\mathbf{I}_+$ & $\mathbf{A}_T^* = A_+^*\,\mathbf{I}_+ + A_-^*\,\mathbf{I}_-$ as all $\mathbf{I}_j$ real.

Longitudinally: $\mathbf{I}_\zeta \equiv \mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0$ where $\zeta = \uparrow$ or $\downarrow$ (± 1) then $A_\zeta \equiv \tfrac{1}{2}(A_3 + \zeta\,A_0)$ N.B. No i factor.

$\therefore\ \mathbf{A}_L = A_\uparrow\,\mathbf{I}_\uparrow + A_\downarrow\,\mathbf{I}_\downarrow$ & $\mathbf{A}_L^* = A_\uparrow^*\,\mathbf{I}_\downarrow + A_\downarrow^*\,\mathbf{I}_\uparrow$

For real NVs: $X_\mu^* = X_\mu$ $\therefore\ \mathbf{X}_L^*\,\mathbf{X}_L = (X_0\,X_0 - X_3\,X_3)\,\mathbf{I}_0$ & $\mathbf{X}_T^*\,\mathbf{X}_T = -(X_1\,X_1 + X_2\,X_2)\,\mathbf{I}_0$

$\therefore\ \mathbf{X}^*\,\mathbf{X} = \mathbf{X}_L^*\,\mathbf{X}_L + \mathbf{X}_T^*\,\mathbf{X}_T = -4\,(X_\uparrow\,X_\downarrow + X_+\,X_-)\,\mathbf{I}_0$ (a sum of two invariants).

Here the mnemonic rules for these binary variables have been used: rho (ρ) for rotation (transverse) and zeta (ζ) for z-axis (longitudinal). The original 4D 'Cartesian' NV bases follow the standard rules ($\mathbf{I_0}\,\mathbf{I_\mu} = \mathbf{I_\mu}\,\mathbf{I_0} = \mathbf{I_\mu}$ & $\mathbf{I_j}\,\mathbf{I_k} = -\delta_{jk}\,\mathbf{I_0} + \varepsilon_{jkl}\,\mathbf{I_l}$). The new NV bases no longer obey these symmetrical multiplication rules but still introduce the 'twist' as can be seen.

$$\therefore\ \mathbf{I}_\rho\,\mathbf{I}_\rho = 0\ ;\ \mathbf{I}_\zeta\,\mathbf{I}_{-\zeta} = 0\ ;\ \mathbf{I}_\rho\,\mathbf{I}_3 = i\,\rho\,\mathbf{I}_\rho\ ;\ \mathbf{I}_3\,\mathbf{I}_\rho = -i\,\rho\,\mathbf{I}_\rho\ ;\ \mathbf{I}_\zeta\,\mathbf{I}_3 = \mathbf{I}_3\,\mathbf{I}_\zeta = i\,\zeta\,\mathbf{I}_\zeta$$

$$\therefore\ \mathbf{I}_+\,\mathbf{I}_- = -2i\,\mathbf{I}_\downarrow\ ;\ \mathbf{I}_-\,\mathbf{I}_+ = 2i\,\mathbf{I}_\uparrow\ ;\ \mathbf{I}_\zeta\,\mathbf{I}_\zeta = 2i\,\zeta\,\mathbf{I}_\zeta\ ;\ \mathbf{I}_\rho\,\mathbf{I}_\zeta = i\,(\zeta + \rho)\,\mathbf{I}_\rho\ ;\ \mathbf{I}_\zeta\,\mathbf{I}_\rho = i\,(\zeta - \rho)\,\mathbf{I}_\rho$$

Notice the appearance of the factor '2' – this ubiquitous factor always appears when dealing with spinors and 3D rotations suggesting that this sub-vector decomposition is implying a dual spinor representation for 'twisted' motion in 3D space.

These multiplication rules can be re-expressed in terms of anti-symmetric commutators and symmetric anti-commutators:

Definitions: **Commutator** $[A\,,B]_- \equiv A\,B - B\,A$ and **Anti-Commutator** $[A\,,B]_+ \equiv A\,B + B\,A$

$$[\mathbf{I}_\rho\,,\mathbf{I}_{\rho'}]_- = -4i\,\rho\,\mathbf{I}_3\,\delta_{\rho-\rho'}\ ;\ [\mathbf{I}_\rho\,,\mathbf{I}_{\rho'}]_+ = -4\,\mathbf{I}_0\,\delta_{\rho-\rho'}\ \&\ [\mathbf{I}_\zeta\,,\mathbf{I}_{\zeta'}]_- = 0\ ;\ [\mathbf{I}_\zeta\,,\mathbf{I}_{\zeta'}]_+ = 4i\,\zeta\,\mathbf{I}_\zeta\,\delta_{\zeta\zeta'}$$

$$[\mathbf{I}_\rho\,,\mathbf{I}_3]_- = 2i\,\rho\,\mathbf{I}_3\ ;\ [\mathbf{I}_\rho\,,\mathbf{I}_3]_+ = 0\ \&\ [\mathbf{I}_\zeta\,,\mathbf{I}_3]_- = 0\ ;\ [\mathbf{I}_\zeta\,,\mathbf{I}_3]_+ = 2i\,\zeta\,\mathbf{I}_\zeta$$

$$[\mathbf{I}_\rho\,,\mathbf{I}_\zeta]_- = 2i\,\rho\,\mathbf{I}_\rho\ ;\ [\mathbf{I}_\rho\,,\mathbf{I}_\zeta]_+ = 2i\,\zeta\,\mathbf{I}_\rho$$

Combinations of these operators have some interesting and useful properties, especially involving repetitive multiplication.

$$\therefore\ (\mathbf{I}_3)^{2n} = (-1)^n\,\mathbf{I}_0\ ;\ (\mathbf{I}_3)^{2n+1} = (-1)^n\,\mathbf{I}_3\ ;\ (\mathbf{I}_3)^n\,\mathbf{I}_\rho = (-i\,\rho)^n\,\mathbf{I}_\rho\ ;\ \mathbf{I}_\rho\,(\mathbf{I}_3)^n = (i\,\rho)^n\,\mathbf{I}_\rho$$

$$\therefore\ (\mathbf{I}_\zeta/2)^{n+1} = (i\,\zeta)^n\,(\mathbf{I}_\zeta/2)\ ;\ (\mathbf{I}_\zeta/2)^{n+1}\,\mathbf{I}_{\zeta'} = (i\,\zeta)^n\,(\mathbf{I}_\zeta/2)\,\delta_{\zeta-\zeta'}\ ;\ (\mathbf{I}_\zeta/2)^n\,\mathbf{I}_\rho = (i\,\zeta)^n\,\mathbf{I}_\rho\,\delta_{\rho-\zeta}$$

Now $(\mathbf{I}_\zeta\,\theta/2)\exp(\mathbf{I}_\zeta\,\theta/2) = \sum_n (\mathbf{I}_\zeta\,\theta/2)^{n+1}/n! = \sum_n \theta\,(i\,\zeta\,\theta)^n\,(\mathbf{I}_\zeta/2)/n! = (\mathbf{I}_\zeta\,\theta/2)\sum_n (i\,\zeta\,\theta)^n/n! = (\mathbf{I}_\zeta\,\theta/2)\,e^{i\,\zeta\,\theta}$

$\therefore\ \exp((\mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0)\,\theta/2)\,\mathbf{I}_{\zeta'} = \delta_{\zeta\zeta'}\exp(i\,\zeta\,\theta)\,\mathbf{I}_\zeta$ Similarly: $\exp((\mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0)\,\theta/2)\,\mathbf{I}_\rho = \delta_{\zeta-\rho}\exp(i\,\zeta\,\theta)\,\mathbf{I}_\rho$

Note: Although $\mathbf{I}_3$ commutes with $\mathbf{I}_0$ the exponential cannot be factored: $\exp((\mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0)\,\theta/2) \neq \exp(i\,\zeta\,\theta/2)\exp(\mathbf{I}_3\,\theta/2)$

So, for example $\theta = 2\pi$: $\exp((\mathbf{I}_3 + i\,\zeta\,\mathbf{I}_0)\,\pi)\,\{\mathbf{I}_{\zeta'}\ ;\ \mathbf{I}_\rho\} = \{\delta_{\zeta\zeta'}\,\mathbf{I}_\zeta\ ;\ \delta_{\zeta-\rho}\,\mathbf{I}_\rho\}$

A.2 APPENDIX II: SUMMARY OF DOUBLE-POINT CNVs

A2.1 TWO-PARTICLE TWO-TIME DERIVATIVES

In terms of any continuous or discontinuous function F of a single parameter, ξ subject to a small change Δ, it is always possible to define a 'limit from above', F^+ and a 'limit from below', F^- for any specific value of the argument, ξ.

$$\textit{Above.Limit}: F^+[\xi] \equiv \underset{\Delta \to 0}{\text{Limit}} \{ F[\xi + \Delta] \} \;\; \& \;\; \textit{Below.Limit}: F^-[\xi] \equiv \underset{\Delta \to 0}{\text{Limit}} \{ F[\xi - \Delta] \}$$

In standard calculus, the Continuum Assumption is almost always made; namely, for all values of a continuous function's argument that the 'limit from above' is equal to the 'limit from below', or the function is "smooth"; in other words:

$$\textit{Continuum Assumption}: \;\; F^+[\xi] = F^-[\xi] = F[\xi]$$

The definition of the total change of a function of the time-difference, $T = t_1 - t_2$ as the differential time-difference changes, is ambiguous; the notation $\mathscr{D}/\mathscr{D}T$ is used to distinguish this type of '**double-change**' from the single (or local) total time derivative (d/dt). Standard calculus allows defining the limiting (or differential) differences, from 'above' and 'below':

$$\textit{Above.Diff}: \mathscr{D}^+F[\xi] / \mathscr{D}\xi \equiv \underset{\Delta \to 0}{\text{Limit}} \{(F[\xi + \Delta] - F[\xi]) / \Delta\} \;\; \& \;\; \textit{Below.Diff}: \mathscr{D}^-F[\xi] / \mathscr{D}\xi \equiv \underset{\Delta \to 0}{\text{Limit}} \{(F[\xi] - F[\xi - \Delta]) / \Delta\}$$

The average value of even discontinuous functions can be defined unambiguously and this average is always "smooth":

$$\textit{Average.Differential}: \; < \mathscr{D}F[\xi] / \mathscr{D}\xi > \equiv \mathscr{D}F[\xi] / \mathscr{D}\xi \equiv \tfrac{1}{2} (\mathscr{D}^+F[\xi] / \mathscr{D}\xi + \mathscr{D}^-F[\xi] / \mathscr{D}\xi)$$

$$\textit{Total } \textbf{\textit{Two-Time}}\textit{-Difference Differential}: \mathscr{D}F[T] / \mathscr{D}T = \underset{\Delta \to 0}{\text{Limit}} \{ (F[T + \Delta] - F[T - \Delta]) / \mathbf{2}\Delta \}$$

Note the factor of '2' as this is an average; this definition is used for both the average velocity of each electron, $\underline{v}_j(t)$ and the average velocity of two interacting electrons $\underline{V}_{jk}(t_j - t_k)$. Any small positive change Δ in the total time difference T between the electrons (labeled #1 & #2 i.e. $T = t_1 - t_2$) can be 'shared' unequally, like $(t_1 + \Delta)$ or $(t_2 - \Delta)$ or equally between the two times, as $(t_1 + \Delta/2)$ and $(t_2 - \Delta/2)$. The increase in the difference $(T + \Delta)$ may be due to either a 'half increase' in t_1 and a 'half decrease' in t_2 or a 'half decrease' in t_1 and a 'half increase' in t_2. This ambiguity is eliminated by using the so-called 'equal contribution' solution: the changes in time at the two electrons occur in opposite directions. This allows the difference in locations of the two electrons to define the 'above' and 'below' two-electron velocities:

$$\underline{V}^{\lambda}{}_{12}[T] \equiv \mathscr{D}^{\lambda}\underline{X}_{12}[T] / \mathscr{D}T = \tfrac{1}{2} (\underline{v}^{\lambda}{}_1[t_1] + \underline{v}^{-\lambda}{}_2[140]) \qquad \text{where } \lambda = \pm$$

The definition of the *Total Two-Time-Difference Positional Differential*, or two-electron joint-velocity, can be defined, as:

$$\underline{V}_{12}[T] \equiv \mathscr{D}\underline{X}_{12}[T] / \mathscr{D}T \equiv < \mathscr{D}\underline{X}_{12}[T] / \mathscr{D}T > = \tfrac{1}{2} (\mathscr{D}^+\underline{X}_{12}[T] / \mathscr{D}T + \mathscr{D}^-\underline{X}_{12}[T] / \mathscr{D}T)$$

Thus,

$$<\underline{V}_{12}[t_1 - t_2]> = \tfrac{1}{2} (<\underline{v}_1[t_1]> + <\underline{v}_2[140]>) \qquad \text{or} \quad \underline{V}_{12}[t_1 - t_2] = \tfrac{1}{2} (\underline{v}_1[t_1] + \underline{v}_2[140])$$

$\underline{X}_{12}(T)$ is the difference in the positions of the two electrons at two different times, the total differential of this positional difference with respect to this difference in times has the dimensions of a velocity; note this definition is NOT the difference in the relative velocities at these two times ($\underline{v}_1[t_1] - \underline{v}_2[t_2]$) but the AVERAGE of these two velocities. These results for the position and velocity differentials for two particles are self-consistent and can be generalized to all anti-symmetric separable functions F_{12} that describe the ***joint*** properties of associated pairs of electrons at two different locations in space $\underline{x}_1$ & $\underline{x}_2$ at two different times t_1 and t_2, when each particle's related (individual) property is characterized by $f_j[t_j; \underline{x}_j]$; the variable ξ denotes any one of the four fundamental dimensions {t ; x, y, z}.

$$\textit{Separable Two-Particle Property}: \; F_{12}[\xi_1, \xi_2] \equiv f_1[\xi_1] - f_2[\xi_2] = F_{12}[\xi_1 - \xi_2] = F_{12}[\xi]$$

The anti-symmetry of these separable functions determines that electrons are fermions, as $F_{12}(0) = 0$. The pre- and post-derivatives must first be defined for any scalar function of a single variable, $\psi[\xi]$.

$$\Delta^{\lambda}\psi[\xi] \equiv \lambda\,(\,\psi[\xi + \lambda\,\Delta] - \psi[\xi]\,) \quad \text{and} \quad \partial^{\lambda}\psi[\xi]/\partial\xi \equiv \underset{\Delta \to 0}{\text{Limit}}\,\{\,\Delta^{\lambda}\psi[\xi]/\Delta\,\} \qquad \text{where} \quad \lambda = \pm 1 \text{ or } \pm$$

A similar manipulation for deriving $\underline{V}^{\lambda}{}_{12}[T]$ generates the results: $\partial^{\lambda}F_{12}[\xi]/\partial\xi = \tfrac{1}{2}\,(\,\partial^{\lambda}f_1[\xi_1]/\partial\xi_1 + \partial^{-\lambda}f_2[\xi_2]/\partial\xi_2\,)$
Average values for all functions can be defined at any point or value ξ whether the function is discontinuous or not.

$$< F[\xi] > \equiv \tfrac{1}{2}\,(\,F^{+}[\xi] + F^{-}[\xi]\,) = \underset{\Delta \to 0}{\text{Limit}}\{\,\tfrac{1}{2}\,(\,F[\xi + \Delta] - F[\xi - \Delta]\,)\,\} = F[\xi]$$

Thus,

$$\partial F_{12}[\xi]/\partial\xi = \tfrac{1}{2}\,(\,\partial f_1[\xi_1]/\partial\xi_1 + \partial f_2[\xi_2]/\partial\xi_2\,) \qquad \text{N.B. <brackets> often omitted}$$

Thus, the partial difference-derivative of an anti-symmetric, separable function $F_{12}[\xi]$, with respect to its single difference argument ξ is equal to the average value of the sum of the single-value derivatives $\partial f_j[\xi_j]/\partial\xi_j$ of its functional decomposition. These extended 'difference' definitions of partial difference-derivatives allow two-electron definitions to be formulated that bear marked resemblances to single value functions of simple vector calculus; for example, introducing the two-electron (spatial) difference-gradient operator, $\underline{\nabla}_{12}$ and its corresponding two-electron equivalent divergence and curl operators.

$$\underline{\nabla}_{12}F_{12}\,[\underline{x}_1 - \underline{x}_2] = \tfrac{1}{2}\,(\,\underline{\nabla}_1 f_1[\underline{x}_1] + \underline{\nabla}_2 f_2[\underline{x}_2]\,) \quad \text{where} \quad \underline{\nabla}_1 f_1[\underline{x}_1] \equiv \hat{\mathbf{e}}_x\,\partial_x f_1[x] + \hat{\mathbf{e}}_y\,\partial_y f_1[y] + \hat{\mathbf{e}}_z\,\partial_z f_1[z]$$
$$\underline{\nabla}_{12} \bullet \underline{F}_{12}[\underline{x}_1 - \underline{x}_2] = \tfrac{1}{2}\,(\,\underline{\nabla}_1 \bullet \underline{f}_1[\underline{x}_1] + \underline{\nabla}_2 \bullet \underline{f}_2[\underline{x}_2]\,) \text{ and } \underline{\nabla}_{12} \wedge \underline{F}_{12}[\underline{x}_1 - \underline{x}_2] = \tfrac{1}{2}\,(\,\underline{\nabla}_1 \wedge \underline{f}_1[\underline{x}_1] + \underline{\nabla}_2 \wedge \underline{f}_2[\underline{x}_2]\,)$$

A2.2 TWO-PARTICLE DIFFERENCE CNVs

The two-particle *Difference* CNV, $\mathbf{X}_{12}$ is defined as the difference between the single-particle CNVs of two point particles (arbitrarily labeled #1 & #2) at two locations in space $\underline{x}_k$ at times t_k, where k = 1 & 2.

Difference Vector: $\qquad \mathbf{X}_{12} \equiv \mathbf{X}_1 - \mathbf{X}_2$

The spatial and temporal differences can also be defined: $\underline{X} \equiv \underline{x}_1 - \underline{x}_2$ and $T \equiv t_1 - t_2$ so $\mathbf{X}_{12}[T; \underline{X}] = \mathbf{X}_{12}[t_1 - t_2; \underline{x}_1 - \underline{x}_2]$

Two-particle *separable* NVs are inherently anti-symmetric with respect to the exchanging their space and time arguments:

$$\mathbf{X}_{12}[-T; -\underline{X}] = -\,\mathbf{X}_{12}[T; \underline{X}] \qquad \text{At the same time } (t_1 = t_2): \quad \mathbf{X}_{12}[0; \underline{0}] = 0$$

The NV definition of a single electron's velocity can be used to define the CNV of the **Joint-Velocity** of two electrons.

$$\mathbf{V}_k[t] = d\mathbf{X}_k[t_k]/dt_k = i\,c\,\mathbf{I_0} + \underline{v}_k \bullet \underline{\mathbf{I}} \quad \text{and} \quad \mathbf{V}^{\lambda}{}_{12}[T] = \mathscr{D}^{\lambda}\mathbf{X}_{12}[T]/\mathscr{D}T = i\,c\,\mathbf{I_0} + \underline{V}^{\lambda}{}_{12} \bullet \underline{\mathbf{I}}$$

$$\therefore\ \mathbf{V}_{12}[T] = i\,c\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{V}_{12} = i\,c\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{V}_{12}[t_1 - t_2] = i\,c\,\mathbf{I_0} + \tfrac{1}{2}\,(\,\underline{v}_1[t_1] + \underline{v}_2[140]\,) \bullet \underline{\mathbf{I}}$$

Two-Electron Difference Velocity CNV: $\mathbf{V}_{12}[t_1 - t_2] = \tfrac{1}{2}\,(\,\mathbf{V}_1[t_1] + \mathbf{V}_2[140]\,) = \mathscr{D}T\,\mathbf{X}_{12}[T]/\mathscr{D}T$

$$<\mathbf{V}_{12}[t_1 - t_2]> = \tfrac{1}{2}<\,(\,\mathbf{V}^{+}{}_{12}[t_1 - t_2] + \mathbf{V}^{-}{}_{12}\,[t_1 - t_2]\,)>$$

This last identity is consistent with the definition of single-particle velocity invariants: $\mathbf{V}^{*}{}_{12}\,\mathbf{V}_{12} = (c^2 - \underline{V}_{12} \bullet \underline{V}_{12})\,\mathbf{I_0}$

These results can be generalized to any CNV representing separable two-electron properties, like $\mathbf{Q}_k$.

$$\mathbf{Q}_{12}[t_1 - t_2] = \tfrac{1}{2}\,(\,\mathbf{Q}_1[t_1] + \mathbf{Q}_2[140]\,) \quad \text{and} \quad <\mathbf{Q}_{12}[t_1 - t_2]> = \tfrac{1}{2}<\,(\,\mathbf{Q}^{+}{}_{12}[t_1] + \mathbf{Q}^{-}{}_{12}[140]\,)>$$

These results lead to the definition of the **total** time-difference differential, D/DT using the two-electron difference velocity.

Total Time-Difference Differential : $D\psi[T; \underline{X}]/DT \equiv (\,\mathscr{D}/\mathscr{D}T + \underline{V}_{12} \bullet \underline{\nabla}_{12}\,)\,\psi[T; \underline{X}]$

The comparable one-electron vector identities are still valid with these extended two-electron difference operators.

Most importantly: a) $\mathscr{D}X_j / \mathscr{D}X_k = \delta_{jk}$ b) $\mathscr{D}X_j / \mathscr{D}T = 0$ c) $\underline{\nabla}_{12}\underline{V}_{12} = 0$ d) $\underline{\nabla}_{12} \bullet \underline{V}_{12} = 0$ e) $\underline{\nabla}_{12} \wedge \underline{V}_{12} = 0$

These definitions can be extended to the two-electron CNV difference gradient operator, defining $\mathscr{D}_0 \equiv \mathscr{D} / c\mathscr{D}T$

Two-Electron Difference Gradient CNV: $\nabla_{12}\psi[T ; \underline{X}] \equiv (i \mathbf{I}_0 \mathscr{D}_0 + \underline{\mathbf{I}} \bullet \underline{\nabla}_{12}) \psi[T ; \underline{X}]$

$$\nabla_{12}{}^*\mathbf{Q}_{12}[T; \underline{X}] = \mathbf{I}_0 (\mathscr{D}_0 Q^0{}_{12} - \underline{\nabla}_{12} \bullet \underline{Q}_{12}) + i \underline{\mathbf{I}} \bullet (\underline{\nabla}_{12} Q^0{}_{12} - \mathscr{D}_0 \underline{Q}_{12}) + \underline{\mathbf{I}} \bullet (\underline{\nabla}_{12} \wedge \underline{Q}_{12})$$

If $\nabla_{12}{}^*\mathbf{Q}_{12} = 0$ then: 1) $\underline{\nabla}_{12} \bullet \underline{Q}_{12} = \mathscr{D}_0 Q^0{}_{12}$ 2) $\underline{\nabla}_{12} Q^0{}_{12} = \mathscr{D}_0 \underline{Q}_{12}$ 3) $\underline{\nabla}_{12} \wedge \underline{Q}_{12} = 0$

All the results for one-electron CNV *Voigt Vectors* can be extended to <u>two-electron difference Voigt</u> vectors:

Two-Electron Voigt Vector: $\mathfrak{V}_{12} \equiv - i c \alpha_{12}[T; \underline{X}] \mathbf{I}_0 + \alpha_{12}[T; \underline{X}] \underline{V}_{12}[T] \bullet \underline{\mathbf{I}} = \alpha_{12}[T; \underline{X}] \mathbf{V}_{12}{}^* = \{ i \mathfrak{V}^0{}_{12}; \underline{\mathfrak{V}}_{12} \}$

The components of these two-electron *Voigt Vectors* also satisfy their own Lorenz equation: $c \underline{\mathfrak{V}}_{12} + \underline{V}_{12} \mathfrak{V}^0{}_{12} = 0$

All of the single-electron Voigt results can now be reproduced with these comparable two-electron CNVs and operators. In particular, they satisfy similar definitions of **Gauge** vectors, **Flow** vectors and the **Wave** equation. If there exists a scalar, separable function $\psi_{12}[T; \underline{X}]$ associated with the two electrons and if it is 'harmonic' (satisfies the 'double' Wave equation):

Double-Difference Wave Equation: $\Box_{12}\psi_{12}(T; \underline{X}) = 0$ where $\nabla_{12}{}^*\nabla_{12} \equiv - \mathbf{I}_0 \Box_{12}$ and $\Box_{12} = \underline{\nabla}_{12} \bullet \underline{\nabla}_{12} - \mathscr{D}_0 \mathscr{D}_0$

Similarly: $\mathbf{V}_{12}{}^*\nabla_{12}{}^*\psi_{12}[T; \underline{X}] = - \mathbf{I}_0 (\mathscr{D} / \mathscr{D}T + \underline{V}_{12} \bullet \underline{\nabla}_{12}) \psi - i \underline{\mathbf{I}} \bullet (c \underline{\nabla}_{12} + \underline{V}_{12}\mathscr{D}_0) \psi + \underline{\mathbf{I}} \bullet (\underline{V}_{12} \wedge \underline{\nabla}_{12}) \psi$

Defining the Extended (Double-Difference) ***Zero*** CNV: $\mathbf{Z}_{12} \equiv i \underline{\mathbf{I}} \bullet (c \underline{\nabla}_{12} + \underline{V}_{12}\mathscr{D}_0) - \underline{\mathbf{I}} \bullet (\underline{V}_{12} \wedge \underline{\nabla}_{12})$

This gives the Extended CNV ***Flow Equation***: $D \mathbf{Q}_{12} / DT + \mathbf{V}_{12}{}^*\nabla_{12}{}^* \mathbf{Q}_{12} = 0$ whenever $\mathbf{Z}_{12}{}^* \mathbf{Q}_{12} = 0$

Defining the Double-Difference Acceleration as the extended time-difference of the velocity:

Two-Electron Difference Acceleration CNV: $\mathcal{A}_{12}[t_1 - t_2] = \frac{1}{2} (\mathcal{A}_1[t_1] + \mathcal{A}_2[140]) = \mathscr{D}\mathbf{V}_{12}[T] / \mathscr{D}T$

The Two-Electron CNV Difference Velocity, $\mathbf{V}_{12}$ is a 'Flow Vector' ($\alpha_{12} = 1$) when its 'Zero Conditions' result in:

ii) $\underline{V}_{12} \bullet \underline{\mathcal{A}}_{12} = 0$ iv) $\underline{V}_{12} \wedge \underline{\mathcal{A}}_{12} = 0$

It was proved [166] that $\mathbf{V}_{12}[T]$ is a constant of the motion, i.e. $D\mathbf{V}_{12}[T]/DT = 0$, or equivalently, when the Joint Current is considered to be 'harmonic' so that: $\Box_{12}\underline{V}_{12} = 0$ or equivalently: $\nabla_{12}{}^*\mathbf{V}_{12} = 0$, so:

1) $\underline{\nabla}_{12} \bullet \underline{V}_{12} = 0$ 2) $\mathscr{D}_0 \underline{V}_{12} = 0$ 3) $\underline{\nabla}_{12} \wedge \underline{V}_{12} = 0$

Each harmonic function has its own corresponding ***Associate*** CNV, defined as: $\boldsymbol{\psi}'_{12} \equiv \nabla_{12}\psi_{12}[T; \underline{X}]$ NB <u>not</u> conjugate. The addition of any Associate Natural Vector to a Voigt Vector, $\mathfrak{V}$ defines its corresponding ***Gauge Vector***, $\mathfrak{V}'$.

Definition: ***Gauge Transform*** $\mathfrak{V}_{12}' \equiv \mathfrak{V}_{12} + \nabla_{12}\psi_{12}$ where $\Box_{12}\psi_{12} = 0$

The conjugate gradient of this Associate CNV is zero, since: $\nabla_{12}{}^* \boldsymbol{\psi}'_{12} = - \mathbf{I}_0 \Box_{12}\psi_{12} = 0 \quad \therefore \nabla_{12}{}^*\mathfrak{V}_{12}' = \nabla_{12}{}^*\mathfrak{V}_{12}$

A.3 APPENDIX III: SUMMARY OF CNV INVARIANTS

A3.1 TEMPORAL INVARIANTS

Natural Vectors have been found to be a powerful representation of asynchronous interactions, such as the EM interaction, between two electrons separated across space and time. The most useful NVs are *separable* across the co-ordinates of the two points; that is, an NV $\mathbf{Q}_{12}$ is separable whenever it satisfies the following decomposition (like the position difference):

$$\text{Definition. } \textbf{\textit{Separable}} \text{ NV: } \quad \mathbf{Q}_{12}[T; \underline{X}] = \mathbf{Q}_{12}[t_1 - t_2; \underline{x}_1 - \underline{x}_2] = \mathbf{Q}_1[t_1; \underline{x}_1] - \mathbf{Q}_2[t_2; \underline{x}_2]$$

This definition reflects the intrinsic, anti-symmetric nature of the interacting pair of objects; since this is true for the basic separation NV $\mathbf{X}_{12}$ between two electrons, this leads to the conclusion that even 'classical' electrons act like *fermions*.

Some CNVs have values that overall do not change when their <u>temporal</u> parameter changes, these CNVs are referred to as 'temporal invariants'; they obviously satisfy the defining condition:

$$\text{Definition. } \textbf{\textit{Temporal Invariant}} \text{ CNV: } \quad d/dt\, \mathbf{Q}_0[t; \underline{x}] = 0 \quad \text{or} \quad \mathbf{Q}_0[t'; \underline{x}] = \mathbf{Q}_0[t; \underline{x}]$$

A3.2 CONSERVED QUANTITIES

Whenever a Natural Vector $\mathbf{A}$ is both separable and temporally invariant, then it will represent two (both scalar and vector) conserved physical quantities; thus:

$$\text{Whenever } \mathbf{A}_0[t_1 - t_2; \underline{x}_1 - \underline{x}_2] = \mathbf{A}_1[t_1; \underline{x}_1] - \mathbf{A}_2[t_2; \underline{x}_2] \quad \text{and} \quad d/dT\, \mathbf{A}_0[T; \underline{X}] = 0$$

$$\text{then:} \quad d/dt_1\, \mathbf{A}_1[t_1] = -\, d/dt_2\, \mathbf{A}_2[140] = \mathbf{B}_0 = i\, B_0\, \mathbf{I_0} + \underline{B} \bullet \underline{\mathbf{I}}$$

This represents the ***transfer*** of a joint pair of properties (A_0 and <u>A</u> such as energy and momentum), across space over a finite duration of time between one interacting point object at (t_1; $\underline{x}_1$) and the other at (t_2; $\underline{x}_2$). One object 'loses' these properties at **rates** of B_0 and <u>B</u>, while the other point object 'gains' these properties at matching rates. This 'transfer' between the two objects is simply the <u>asynchronous</u> action-at-a-distance (AAAD) generated by their mutual interaction but this does **NOT** imply the physical existence of some 'third-party' object (like a 'field') that 'carries' these properties and whose existence was <u>only</u> proposed to preserve the conservation of these properties through the duration of the interaction. Now, the ***Velocity*** Natural Vector $\mathbf{V}$ is a *Flow Vector* when it satisfies the *Zero Conditions* (see Appendix I); these imply that the acceleration, <u>a</u> is zero or equivalently that the ***Acceleration*** Natural Vector $\mathbf{A}$ is zero, so the velocity is a constant, $\underline{V}_0$.

$$\mathbf{X}^* \mathbf{V} = (c^2\, t - \underline{v} \bullet \underline{x})\, \mathbf{I_0} + i\, c\, \underline{\mathbf{I}} \bullet (\underline{x} - \underline{v}\, t) + \underline{\mathbf{I}} \bullet (\underline{x} \wedge \underline{v}) \qquad \mathbf{V}^* \mathbf{X} = (c^2\, t - \underline{v}.\underline{x})\, \mathbf{I_0} - i\, c\, \underline{\mathbf{I}} \bullet (\underline{x} - \underline{v}\, t) - \underline{\mathbf{I}} \bullet (\underline{x} \wedge \underline{v})$$

$$\therefore \; d/dt\, (\mathbf{X}^* \mathbf{V} + \mathbf{V}^* \mathbf{X}) = 2\, d/dt\, (c^2\, t - \underline{v} \bullet \underline{x})\, \mathbf{I_0} = 2(c^2 - \underline{v} \bullet \underline{v})\, \mathbf{I_0} = 2(c^2 - V_0^2)\, \mathbf{I_0} \quad \therefore \; \underline{v} \bullet \underline{x} = t\, V_0^2 + \underline{V}_0 \bullet \underline{X}_0$$

For a Galilean transformation: $\underline{x} = \underline{X}_0 + \underline{v}\, t$ then $(\underline{x} - \underline{v}\, t) = \underline{X}_0$ (constant) $\therefore$ $(\underline{x} \wedge \underline{v}) = \underline{X}_0 \wedge \underline{V}_0$ (constant)

$$\therefore \; \mathbf{X}^* \mathbf{V} - \mathbf{V}^* \mathbf{X} = 2\, \mathbf{L}_0 \text{ where } \mathbf{L}_0 = \underline{\mathbf{I}} \bullet (i\, c\, \underline{X}_0 + \underline{X}_0 \wedge \underline{V}_0) \text{ (constant) and } \mathbf{V}^* \mathbf{V} = (c^2 - v^2)\, \mathbf{I_0} = 0 \text{ when } v = c.$$

In the present Newtonian-like theory [167], where every electron has the same intrinsic, invariant inertial mass *m*, the one-electron temporal (mechanical) invariants, in the absence of external forces (interactions), are the following CNVs:

1) Natural Velocity $\mathbf{V}$

2) Natural Momentum $\mathrm{P} = m\mathbf{V}^*$

3) Linear Momentum $\underline{\mathbf{I}} \bullet \underline{\mathcal{P}} = \tfrac{1}{2}(\mathrm{P}^* + \mathrm{P})$

4) Particle Energy $\mathcal{E}\, \mathbf{I_0} = \tfrac{1}{4}(\mathrm{P}^* - \mathrm{P})(\mathbf{V}^* - \mathbf{V})$

5) Angular Momentum $\underline{\mathbf{I}} \bullet \underline{\mathcal{M}} = \tfrac{1}{2}(\mathbf{X}^* \mathrm{P}^* - \mathrm{P}\, \mathbf{X})$

6) Kinetic Energy $\mathcal{K}\, \mathbf{I_0} = \tfrac{1}{4}(\mathrm{P}\, \mathbf{V} + \mathbf{V}\, \mathrm{P})$

7) Particle Action $\mathcal{A}\, \mathbf{I_0} = \tfrac{1}{2}(\mathbf{X}^* \mathrm{P}^* + \mathrm{P}\, \mathbf{X})$

8) Galilean Momentum $\underline{\mathbf{I}} \bullet \underline{\mathcal{G}} = m\, \mathbf{X} - \mathrm{P}^*\, t$

The corresponding continuous two-electron invariants, in the absence of mutual interactions, have a similar structure.

1) Joint Natural Velocity $\mathbf{V}_{12} = \frac{1}{2}(\mathbf{V}_1 + \mathbf{V}_2)$ 2) Joint Natural Momentum $\mathbf{P}_{12} = 2\,m\mathbf{V}_{12}^*$

3) Linear Momentum $\underline{\mathbf{I}} \bullet \underline{\mathcal{P}}_{12} = \frac{1}{2}(\mathbf{P}_{12}^* + \mathbf{P}_{12})$ 4) Total Energy $\mathcal{E}_{12}\,\mathbf{I_0} = \frac{1}{4}(\mathbf{P}_{12}^* - \mathbf{P}_{12})(\mathbf{V}_{12}^* - \mathbf{V}_{12})$

5) Angular Momentum $\underline{\mathbf{I}} \bullet \underline{\mathcal{M}}_{12} = \frac{1}{2}(\mathbf{X}_{12}^*\mathbf{P}_{12}^* - \mathbf{P}_{12}\mathbf{X}_{12})$

6) Kinetic Energy $\mathcal{K}_{12}\,\mathbf{I_0} = \frac{1}{4}(\mathbf{P}_{12}\mathbf{V}_{12} + \mathbf{V}_{12}\mathbf{P}_{12})$

7) Joint Action $\mathcal{A}_{12}\,\mathbf{I_0} = \frac{1}{2}(\mathbf{X}_{12}^*\mathbf{P}_{12}^* + \mathbf{P}_{12}\mathbf{X}_{12})$

8) Galilean Momentum $\underline{\mathbf{I}} \bullet \underline{\mathcal{G}}_{12} = m\,\mathbf{X}_{12} - \mathbf{P}_{12}^*T_{12}$

A.4 APPENDIX IV: SUMMARY OF EXTENDED DIFFERENCES

A4.1 STANDARD FINITE DIFFERENCES

Piecewise Regular Functions

The foundational function in the calculus of finite differences is the Heaviside 'unit' (or step) function $\theta[t]$, which is defined as zero when t is negative and one otherwise [84]. This can be used to define the more useful '*block*' function $\Theta[t; T_1, T_2]$ as the offset difference between two step functions, which has the value one inside the **interval** $(T_1 ; T_2)$ and zero otherwise:

Definitions: The **Block** Function $\Theta[\, t; T_1, T_2] \equiv \theta[t - T_1] - \theta[t - T_2]$ and $\Theta_n[\, t\,] \equiv \Theta[\, t; T_n, T_{n+1}]$

A function is considered '*regular*' if it can be defined at a finite number of points t_n in its complete range. A function will be a *piecewise*, regular function if it can be viewed as a finite sum of regular functions that are each continuous within their separate interaction-intervals.

Definition: **Piecewise-Regular** Function $F[\, t\,] \equiv \Sigma_n F_n[\, t\,] \Theta_n[\, t\,]$

The physical time variable 't' is itself an analytic function as it continuous, defined everywhere in the range and its above and below limits are finite and equal. The '**free particle**' location function (moving along any one dimension, say Z, at a constant speed u_0 and found at Z_0 at time T_0) is: $z_0[\, t\,] = Z_0 + u_0 (t - T_0)$. This is also continuous and analytic. If this particle receives an instantaneous impulse at a finite number of times T_n then its subsequent (constant) speed will be u_n in the next 'interaction interval'; this can be represented by the following piecewise-regular function:

$$X[\, t\,] \equiv \Sigma_n X_n[\, t\,] \Theta_n[\, t\,] \quad \text{where} \quad X_n[\, t\,] = X_n + u_n (t - T_n)$$

Finite Difference Definitions

A *discrete function* is defined as a *set* of (N + 1) values $\{\zeta_n\}$ *ordered* by a unique *identifier* n, spanning the positive integers from 0 to N. A discrete function is either *point-like* or *piecewise continuous*. A point-like discrete function f_n is defined only **at** its set of identification points $\{f_n\}$. A piece-wise continuous discrete function F[t] is defined as a set of (N + 1) functions $\{F_n[t]\}$, whose values vary according to a continuous parameter t. Each individual function $F_n[t]$ is a **single** value F_n defined in the range T_n to $(T_{n+1} - \delta t)$, where δt << any $(T_{n+1} - T_n)$, and zero outside this range. Each of the critical values T_n are referred to as 'interaction points', which are not usually equally distributed. Each range is characterized by its mid-point value $\mathscr{T}_n$, defined as:

$$\mathscr{T}_n \equiv \tfrac{1}{2} (T_{n+1} + T_n) \;\; ; \;\; F_n \equiv F[\mathscr{T}_n].$$

Two new classes of discrete functions can be derived from any discrete function, namely the sum and differences of adjacent values, which could be given their own unique symbol but are usually designated by a (prefix) modifier of the original function. The most commonly used derived discrete functions are the average of adjacent values designated by surrounding angular brackets and the difference designated by a small delta prefix.

Definitions: Average $\langle \zeta_n \rangle \equiv \tfrac{1}{2} (\zeta_{n+1} + \zeta_n)$ Difference $\Delta\zeta_n \equiv \zeta_{n+1} - \zeta_n$

Differences can be generated by defining the difference linear operator ('diff') designated by a larger, **bold** delta symbol.

Definition: **DIFF.OPERATOR**: $\boldsymbol{\Delta}[\zeta_n] \equiv \zeta_{n+1} - \zeta_n$

The functional (square) brackets can often be omitted when there is no ambiguity: $\boldsymbol{\Delta}\zeta_n = \boldsymbol{\Delta}[\zeta_n] = \Delta\zeta_n$. The symbol $\boldsymbol{\Delta}$ acts like a linear operator as it can be applied repeatedly e.g. $\boldsymbol{\Delta}^3[\zeta_n] = \boldsymbol{\Delta}[\boldsymbol{\Delta}[\boldsymbol{\Delta}[\zeta_n]]]$ and may be applied to sums and differences of series; thus:

$$\boldsymbol{\Delta}[\alpha_n \pm \beta_n] \equiv \boldsymbol{\Delta}\alpha_n \pm \boldsymbol{\Delta}\beta_n \qquad \boldsymbol{\Delta}[\alpha_n * \beta_n] \equiv \alpha_{n+1}\beta_{n+1} - \alpha_n \beta_n$$

It is sometimes useful to define a 'super-difference' series where non-adjacent values are used to compute the new series.

Super-Differences: $\boldsymbol{\Delta}_k[\zeta_n] \equiv \zeta_{n+k} - \zeta_n = \Delta_k\zeta_n$ Note: $\boldsymbol{\Delta}_k[\zeta_n] \neq (\boldsymbol{\Delta}\zeta_n)^k$

Repeated use of these operators leads to results that sometimes (but not always) resemble the results of differential calculus.

$$\Delta^2[\zeta_n] = \zeta_{n+2} - 2\,\zeta_{n+1} + \zeta_n = \Delta_2[\zeta_n] - 2\,\Delta[\zeta_n] \qquad \therefore \quad \text{'double-diff'} \quad \Delta_2 = \Delta^2 + 2\,\Delta$$

Also: $\Delta_2[\zeta_n] = \Delta[\zeta_{n+1}] + \Delta[\zeta_n] = 2 < \Delta\zeta_n >$ and $\Delta_2[\zeta_{n-1}] = \Delta[\zeta_n] + \Delta[\zeta_{n-1}] = < \zeta_n > - < \zeta_{n-1} >$

These latter two identities illustrate that the double-diff of the prior value equals the sum of the last two differences or the difference of the last two averages. The **product rule** also displays some useful results.

$$\Delta[\alpha_n * \beta_n] = < \alpha_n > \Delta[\beta_n] + \Delta[\alpha_n] < \beta_n > \qquad \text{So} \qquad \Delta[\alpha_n * \alpha_n] = 2 < \alpha_n > \Delta[\alpha_n]$$

There is another useful approximation that was used in analyzing Planck's model of relativistic momentum.

$$\Delta[\ln(\zeta_n)] = \ln(\zeta_{n+1}) - \ln(\zeta_n) = \ln(\zeta_{n+1} / \zeta_n) = \ln((\zeta_n + \Delta\zeta_n) / \zeta_n) = \ln((1 + \Delta\zeta_n) / \zeta_n) = \sum_{k=1}^{k=\infty} (-\Delta\zeta_k / \zeta_k)^k / k$$

So, if $\Delta\zeta_n << \zeta_n$ then: $\Delta\zeta_n / \zeta_n \cong \Delta[\ln(\zeta_n)]$ and $\ln(\zeta_n / \zeta_0) \cong \sum_{k=0}^{k=n-1} \Delta\zeta_k / \zeta_k$ ("Log-Diff Approx.")

Equal Difference Sets

An *equal-difference* set is a discrete function $\{\beta_n\}$ where every pair of adjacent values, when ordered by n, has an equal difference; in other words, all differences are equal: $\Delta[\beta_n] = \Delta\beta_0$ for all n.

$$\Delta^k[\beta_n] = 0 \text{ for } k > 1 \text{ and } < \Delta\beta_n > = \Delta\beta_0 \text{ and } \Delta[\beta_n * \beta_n] = 2 < \beta_n > \Delta\beta_0 \text{ and } \beta_n = \tfrac{1}{2}(\beta_{n+1} + \beta_{n-1})$$

This last result shows that for an equal-difference set, each value is the average of both its neighbors in the sequence. When the first term in an equal-difference set is zero (called a 'zero-based, equal-difference set') then $\beta_0 = 0$; giving:

$$\beta_n = n\,\Delta\beta_0 = n\,\beta_1 \quad \text{and} \quad \Delta_2[\beta_{n-1}] = 4\,n\,(\beta_1)^2 \quad \text{and} \quad < \beta_n > = (n + \tfrac{1}{2})\,\beta_1$$

A zero-based, equal-difference set $\{\beta_n\}$ combined with a standard, discrete set $\{\alpha_n\}$ generates the following identities:

$$\Delta_2[\alpha_{n-1}\beta_{n-1}] = ((\alpha_{n+1} + \alpha_{n-1}) + n\,\Delta_2[\alpha_{n-1}])\,\beta_1 \quad \text{and} \quad \Delta[\alpha_n\beta_n] = ((n+1)\,\alpha_{n+1} - n\,\alpha_n)\,\beta_1$$

The inverse of the difference operator Δ is the discrete summation 'operator', designated by the conventional sigma Σ.

"**Sum-Diff**" $\quad f_{n+1} = f_0 + \Sigma^n \Delta f_k \quad$ where the sum extends from k = 0 up to n.

Advanced & Retarded Differences

It is usual to progress through a set of discrete values in the ascending order of its identification index; i.e. $n \to n + 1$. The normal flow will also be referred to as 'retarded', reflecting $F_{n-k} \to F_n$ where $k > 0$. The alternate viewpoint of progressing through an ordered list in descending order (i.e. $n \to n - 1$) is obviously mathematically equivalent and any physical result should only depend on the actual list of data values and not depend on the direction of processing the list. In mapping these ideas to physical reality, this alternative is described as 'the future impacting the past', or symbolically: $F_{n+k} \to F_n$; this 'direction of influence' is conventionally referred to as 'advanced' as the effect is advanced ahead of the cause (occurs first in time). The standard (or historical) convention is followed here with 'retarded' activity denoted by a minus and 'advanced' by a plus sign. This gives a global time symmetry to be introduced with the sign convention $\lambda = \pm$ or $\lambda = \pm 1$.

Definitions: **Retarded.Diff** $\Delta^-[\zeta_n] \equiv \zeta_{n+1} - \zeta_n$ and **Advanced.Diff** $\Delta^+[\zeta_n] \equiv \zeta_{n-1} - \zeta_n$

Generically: $\Delta_k^{\lambda}[\zeta_n] \equiv \zeta_{n-\lambda k} - \zeta_n$ with $k > 0$ and $\lambda = \pm$. Obviously: $\Delta^- = \Delta = \Delta_1^-$

Finite Generator Definitions

The 'Generator' operator Γ can be defined (related to the difference operator Δ) with retarded and advanced versions.

Definitions: **Retarded.Gen** $\Gamma_k^-[\zeta_n] \equiv \zeta_{n+k}$ and **Advanced.Gen** $\Gamma_k^+[\zeta_n] \equiv \zeta_{n-k}$

Since the adjacent values are the default (k=1), the following convention will often be used: $\Gamma^\lambda = \Gamma_1^\lambda$.

$$\Gamma_k^\lambda = 1 + \Delta_k^\lambda \quad \text{and} \quad \Gamma^\lambda = 1 + \Delta^\lambda \text{ while } \Gamma_k^\lambda = (\Gamma^\lambda)^k = (1 + \Delta^\lambda)^k$$

The basic definitions also give the universal identity: $\zeta_m = \Gamma^\lambda_{\lambda(n-m)}\, \zeta_n$ for all n & m subject to $\lambda(n - m) \geq 1$.

The 'signed' generators are their mutual reciprocals: $\Gamma_k^\lambda\, \Gamma_k^{-\lambda} = 1$ or $(1 + \Delta^\lambda)^k (1 + \Delta^{-\lambda})^k = 1$ for all $k > 0$.

Discrete Vector Functions

When several sets of discrete values (each with the same cardinality) can be ordered by the same unique identifier (i.e. n) they can be aggregated into an ordered superset, known as a discrete vector function, usually denoted here by an underlined symbol. This is only meaningful when the values across all these sets are homogenous, that is, mathematically of the same number type (e.g. integers, real-numbers, complex numbers, etc). Triple supersets may then represent 3D directed physical quantities, such as velocity or momentum. Each set is referred to as a component of the vector (superset). These component sets are usually labeled with superscripted identifiers, numbered 1 through 3, so that the superset can be manipulated as a single algebraic object. If $\{A_n\}$, $\{B_n\}$ and $\{C_n\}$ be the components of the vector (superset) $\{\underline{F}_n\}$:

$\{\underline{F}_n\} = \{ \{A_n\}, \{B_n\}, \{C_n\} \} = \{ \{F^1{}_n\}, \{F^2{}_n\}, \{F^3{}_n\} \}$ these can also be ordered in row and column (matrix) format.

The three components of a 3D spatial vector function can be mapped to a three component orthogonal abstract vector space defined by its three unit vectors $\{ \underline{\hat{e}}_1 , \underline{\hat{e}}_2 , \underline{\hat{e}}_3 \}$ where their (scalar) product is defined by: $\underline{\hat{e}}_j \bullet \underline{\hat{e}}_k = \delta_{jk}$ with j, k = 1, 2, 3.

So, isomorphically: $\underline{F}_n \Leftrightarrow F^1{}_n \underline{\hat{e}}_1 + F^2{}_n \underline{\hat{e}}_2 + F^3{}_n \underline{\hat{e}}_3 = \Sigma_n F^j{}_n \underline{\hat{e}}_j$

Periodic Discrete Functions

A discrete function set $\{F_n\}$ is *periodic*, with cycle κ, if every value has the same value as another member in the set when offset by the same difference identifier κ.

Definition: **Periodic Discrete Set** $\{F_n\}$ when $F_{n+\kappa} = F_n$ for all n where κ is a positive integer less than the cardinality.

Alternatively, this may be defined in terms of the discrete generator: $\Gamma_\kappa [F_n] = F_n$.

Spatial periodic, discrete vector functions may have up to 3 distinct periods κ_j (with j = 1, 2, 3), so: $F^1{}_{n+\kappa 1} = F^1{}_n$ etc. The three cyclic values $\{\kappa_1 , \kappa_2 , \kappa_3\}$ can themselves form their own abstract 3 component 'frequency' vector $\underline{\kappa}$.

$$\underline{\kappa} \Leftrightarrow \kappa_1 \underline{\hat{e}}_1 + \kappa_2 \underline{\hat{e}}_2 + \kappa_3 \underline{\hat{e}}_3 = \Sigma\, \kappa_j \underline{\hat{e}}_j$$

A4.2 ENHANCED FINITE DIFFERENCES

Point Finite Differences

The value of a piecewise, **discontinuous** function $\{F_n[t]\}$ changes discontinuously at each of its 'transition' points. In the limit of small variations δt around any such point (say T_n) the value of the function goes from $F[T_n - \delta t]$ to $F[T_n + \delta t]$ at T_n.

Definitions **Above.Value**: $F[T_n + \delta t] \equiv F_n^+ = F[T_n] = F_n$ & **Below.Value**: $F[T_n - \delta t] \equiv F_n^-$

These definitions may be combined into the definition of a '**Point-Difference**' that is represented by a new, linear finite difference operator, colloquially referred to as the **Diff-Dot** operator, written long-hand as a delta surrounding a dot or as $\Diamond$.

Definition **Point-Difference** (diff-dot) $\Diamond F_{n-1} \equiv F_n^+ - F_n^- = F[T_n + \delta t] - F[T_n - \delta t]$ N.B. **not** $\Diamond F_n$

If, and only if, the piecewise functions are constant throughout their intervals then: $F[T_{n-1} + \delta t] = F[T_n - \delta t] = F_{n-1}$ then:

$$\Diamond F_{n-1} = F_n - F_{n-1} = \Delta^-[F_{n-1}] \quad \text{or} \quad \Diamond = \Delta$$

The purpose of introducing this operator is to <u>isolate</u> the possible discontinuities around the interaction points from the rest of the smooth (or analytic) region between such points. If, and only if, the piecewise functions are <u>continuous</u> over their whole range then:

$$F_n^+ = F_n^- \text{ so: } \Diamond F_n = 0.$$

Thus, for the 'time' function itself: $t[\,t\,] = t$ then: $\Diamond t = (t + \delta t) - (t - \delta t) = 2\,\delta t$. In the 'infinitesimal' limit: $\delta t \to 0$.

The one-dimensional 'free particle' location function $z_0[\,t\,] = Z_0 + u_0\,(t - T_0)$ is continuous throughout its range, so:

$$\Diamond z_0 = \Diamond z_0[\,t = T_n] = z_0[T_n + \delta t] - z_0[T_n - \delta t] = 2\,u_0\,\delta t \quad \therefore \Diamond z_0 = u_0 \Diamond t \quad \text{or} \quad \Diamond z_0[\,t\,] = u_0 \Diamond t$$

If this new point-difference operator is applied to the continuous, piecewise, regular function $X[t]$ that is used to describe the dynamics of a classical particle, no longer subject to any continuous forces but now only subject to a series of irregular impulses applied at the interactions times T_n then within each segment the value of the function is no longer constant but increases linearly with time, so:

$$X_n[t] = X_n + u_n\,(t - T_n) \text{ then: } \Diamond t = 2\,\delta t \text{ and } \Diamond T_n = 2\,\delta t$$

$$\Diamond X_{n-1}[t = T_n] = X[T_n + \delta t] - X[T_n - \delta t] = (X_n + u_n\,\delta t) - (X_n + u_{n-1}\,\delta t) = (u_n + u_{n-1})\,\delta t = \tfrac{1}{2}\,(u_n + u_{n-1})\,2\delta t = \langle u_{n-1} \rangle \Diamond T_n$$

This demonstrates that the classical 'point' definition: $dX[t]/dt = u[t]$ corresponds now to: $\Diamond X_n / \Diamond T_n = \langle u_n \rangle$.

Extended Finite Differences

In any continuous interval $(T_n\,;\,T_{n+1})$ for any value of the time parameter t it is possible to define the '**extended-difference**' $\underline{\Delta}F[t]$ in terms of the corresponding **diff-dash** operator, written as an underlined delta $\underline{\Delta}$ (the underline symbolizing extent).

Definition **Extended-Difference**: $\underline{\Delta}F_{n-1} \equiv F[T_n - \delta t] - F[T_{n-1} + \delta t] \equiv \underline{\Delta}[\,F[T_{n-1}]\,]$ N.B. **not** $\underline{\Delta}F_n$.

The purpose of this definition is to move disjointly through a set of discrete function values from *just beyond* the last interaction point, where there may have been a discontinuous change in the value of the function, to *just before* the next interaction point, where again there may have been another discontinuous change. The definition of 'interaction points' means that there can be no discontinuous change during this interval. This definition deliberately excludes the critical regions around each interaction point.

The result of the application of this extended-difference operator to a piece-wise discontinuous discrete function F[t] is analytic as long as the temporal parameter t is outside the '**proximity**' of each interaction point T_n, where the proximity is defined as the temporal interval: $(T_n - \delta t/2) < t < (T_n + \delta t/2)$.

Thus, for the 'time' function itself: t[t] = t then: $\underline{\Delta} T_{n-1} = (T_n - \delta t) - (T_{n-1} + \delta t) = \Delta T_{n-1} - 2\,\delta t \;\therefore\; \Delta T_n = \underline{\Delta} T_n + \Diamond T_n$

The 1D 'free particle' location function (see above) is continuous throughout its range, where: $z_0[\,t\,] = Z_0 + u_0\,(t - T_0)$

$$\Diamond z_0 = \Diamond z_0[\,t = T_n\,] = z_0[T_n + \delta t] - z_0[T_n - \delta t] = 2\,u_0\,\delta t \;\;\therefore\;\; \Diamond z_0 = u_0\,\Diamond t \quad \text{or} \quad \Diamond z_0[\,t\,] = u_0\,\Diamond t$$

When this new extended-difference operator is applied to the continuous, piecewise, regular function X[t] that describes the dynamics of a **classical particle**, then:

$$\underline{\Delta} X_{n-1}[t = T_{n-1}] = X[T_n - \delta t] - X[T_{n-1} + \delta t] = (X_n - u_n\,\delta t) - (X_{n-1} + u_{n-1}\,\delta t) = (X_n - X_{n-1}) - (u_n + u_{n-1})\,\delta t = \Delta X_{n-1} - \langle u_{n-1} \rangle \Diamond t$$

$$\therefore\; \Delta X_n = \underline{\Delta} X_n + \langle u_n \rangle \Diamond T_n = \underline{\Delta} X_n + \Diamond X_n \qquad \therefore\; \boldsymbol{\Delta}[X_n] = \underline{\boldsymbol{\Delta}}[X_n] + \Diamond[X_n]$$

These three results suggest that, in general, the difference operator can be separated into two components: $\therefore\; \boldsymbol{\Delta} = \underline{\boldsymbol{\Delta}} + \Diamond$

Combined Finite Differences

Let the ordered set $\{F_n[t]\}$ represent a piecewise, discontinuous discrete function F[t] whose value changes discretely at N 'interaction points' T_n. A **finite time duration** τ is introduced that is smaller than any of the adjacent temporal differences such that each difference is an integer multiple η_n of this smallest difference; in other words, $\Delta T_n = T_{n+1} - T_n = \eta_n\,\tau$ where η_n is a positive integer. If the *equally* spaced time values t_μ are defined to be commensurate with integer multiples of the fundamental time duration τ (that is: $t_\mu = \mu\,\tau$) then F[t] is also a **regular**, piecewise, discontinuous, discrete function.

$$F[T_{n+1} + \delta t)] = (F[T_{n+1} + \delta t] - F[T_{n+1} - \delta t]) + (F[T_{n+1} - \delta t] - F[T_n + \delta t]) + F[T_n + \delta t] = \Diamond F_n + \underline{\Delta} F_n + F[T_n + \delta t]$$

$$\boldsymbol{\Delta}[F_n] = \Delta F_n = F_{n+1} - F_n = F_{n+1}^{+} - F_n^{+} = F[T_{n+1} + \delta t] - F[T_n + \delta t] = \Diamond F_n + \underline{\Delta} F_n \qquad \therefore\; \boldsymbol{\Delta} = \underline{\boldsymbol{\Delta}} + \Diamond$$

This last result, that the application of the difference operator is equivalent to the application of the point-difference operator and the extended-difference operator (colloquially: diff = diff.dot plus diff.dash) means that the finite difference calculus, as developed here, can be applied to regular, piece-wise discontinuous discrete functions. This is the necessary mathematical step to extend Newtonian mechanics to asynchronous, discrete interactions, where the time intervals between interactions are both finite and no longer identical. It is this last feature that prevents the use of the infinitesimal limit and the reduction to continuous classical mechanics. When the extended-difference operators are applied consecutively: $\underline{\boldsymbol{\Delta}} \Diamond = \Diamond \underline{\boldsymbol{\Delta}} = -1$

In summary: $\boldsymbol{\Delta}[X_n] = \underline{\boldsymbol{\Delta}}[X_n]$ and $\Delta[T_n] = \underline{\Delta} T_n$ but $\boldsymbol{\Delta}[\underline{V}_n] = \underline{\boldsymbol{\Delta}}[\underline{V}_n] + \Diamond[\underline{V}_n]$

Enhanced Advanced & Retarded Differences

Just as it was possible to introduce so-called 'advanced' and 'retarded' distinctions for the standard definition of the finite difference operator, with the default always defining the retarded operator (i.e. $\boldsymbol{\Delta} = \boldsymbol{\Delta}^{-}$), a similar set of extensions can be defined for the component difference operators $\underline{\boldsymbol{\Delta}}$ and $\Diamond$; the lambda convention will continue to be used: $\lambda = \pm$ or $\lambda = \pm 1$.

Definitions: **Advanced.Point.Diff** $\Diamond^{+}[F_n] \equiv F[T_n - \delta t] - F[T_n + \delta t] = F_n^{-} - F_n^{+}$ N.B. Advanced is $\Diamond F_n$

Definitions: **Advanced.Extended.Diff** $\underline{\Delta}^{+}[F_n] \equiv F[T_{n-1} + \delta t] - F[T_n - \delta t] = \underline{\Delta}[\,F[T_n]\,]$

These advanced component versions are related anti-symmetrically to the previous retarded definitions by:

$$\Diamond^{+}[F_n] = -\Diamond^{-}[F_{n-1}] \qquad \text{and} \qquad \underline{\Delta}^{+}[F_n] = -\underline{\Delta}^{-}[F_{n-1}] \quad \text{and} \quad \Delta^{\lambda} = \underline{\Delta}^{\lambda} + \Diamond^{\lambda}\,.$$

A4.3 MULTIPLE FINITE DIFFERENCES

Partial Finite Differences

Just as mathematicians expanded calculus from continuous functions of one variable to functions of two variables [x,y], which led to an explosion of new mathematics, called Complex Analysis, so too we have encountered situations that require more than one discrete parameter (say n) to ones involving two independent parameters, say j and k. Complex analysis led immediately to the idea of independent partial derivatives; so too we will need partial finite differences; we retain the similarity by inventing a similar symbol: such as ∂_k defined on a discrete 2D function F[j ; k] as follows:

$$\text{Definitions:} \quad \partial_j F[j\,;\,k] \equiv F[j+1\,;\,k] - F[j\,;\,k] \quad ; \quad \partial_k F[j\,;\,k] \equiv F[j\,;\,k+1] - F[j\,;\,k]$$

Thus, the total difference of a 2D discrete function, reuires all variations to be considered (like d/dt in 1D functions):

$$\Delta F[j\,;\,k] = \partial_j F[j\,;\,k] + \partial_k F[j\,;\,k]$$

It must be noticed that as Δ can be replaced by either the point $\Diamond$ or extended difference $\underline{\Delta}$, so can the partial operator ∂_k.

Obviously, this approach can br generalized to any number of independent discrete parameters.

A.5 APPENDIX V: SOME EXTENDED DIFFERENCES IDENTITIES

This appendix summarizes a series of finite difference vector identities centered on the new extended vector difference operator (or digital gradient operator) $\underline{\Xi}$ that was introduced in section 3.2.3 (this is the analog of the continuous or standard 3D spatial vector gradient operator $\underline{\nabla}$); it is defined as:

$$\text{Definition: } \textbf{\textit{Digital Gradient Operator}}, \quad \underline{\Xi} \equiv \hat{\underline{e}}_x \Xi_x + \hat{\underline{e}}_y \Xi_y + \hat{\underline{e}}_z \Xi_z$$

Here, the individual (directional) difference operators Ξ_j are defined at every point $x_\eta\{ t_\eta; \underline{x}_\eta \}$ on an electron's trajectory as:

$$\text{Definition: } \textbf{Partial-Rate-of-Change Operator}, \quad \Xi_j[\, \psi[x_\eta] \,] \equiv \Delta_j[\, \psi[\, x_\eta] \,] \,/\, \Delta x_j[x_\eta]$$

The Ξ_j operator is defined in terms of the local spatial difference $\Delta x_j[x_\eta]$ and the linear, extended difference operator Δ, both introduced in appendix IV for any ordered discrete set $\{ \zeta_n \}$:

$$\text{Definitions:} \quad \textbf{Average } < \zeta_n > \equiv \tfrac{1}{2} (\zeta_{n+1} + \zeta_n) \qquad \textbf{Difference } \Delta\zeta_n \equiv \zeta_{n+1} - \zeta_n$$

Differences are generated by defining the linear difference operator ('diff') designated by the larger, **bold** delta symbol:

$$\text{Definition:} \quad \textbf{DIFF.OPERATOR}: \quad \Delta[\zeta_n] \equiv \zeta_{n+1} - \zeta_n$$

The functional (square) brackets can often be omitted when there is no ambiguity: $\Delta\zeta_n = \Delta[\zeta_n] = \Delta\zeta_n$.
The product rule for differences (unlike calculus) involves the local averages.

$$\Delta[\alpha_n * \beta_n] = < \alpha_n > \Delta[\beta_n] + \Delta[\alpha_n] < \beta_n > \qquad \text{So} \qquad \Delta[\alpha_n * \alpha_n] = 2 < \alpha_n > \Delta[\alpha_n]$$

The directed displacement $\Delta x_j[\alpha: \eta]$ is the spatial difference between the position of electron 'α' at two consecutive times t_η and $t_{\eta+1}$ in the spatial direction 'j' or: $\Delta x_j[\alpha: \eta] = x_j[\alpha: \eta + 1] - x_j[\alpha: \eta]$. This results in the definition of the '**partial difference**' for any function (continuous or regular) $\psi[\underline{x}]$ of all three spatial variables x_j as the difference in its value between two consecutive interaction times t_η and $t_{\eta+1}$ in the spatial direction 'j', where $t_\eta = \eta\, \tau$.

$$\text{Definition: } \textbf{Partial-Difference } \Delta\psi_j[\alpha: \eta] \equiv \psi_j[\alpha: \eta + 1] - \psi_j[\alpha: \eta] \quad \text{or} \quad \psi_j[\alpha: \eta + 1] \equiv (\, 1 + \Delta_j \,)\, \psi_j[\alpha: \eta]$$

In contrast to continuous differentials, the space differences $\Delta x_j[x_\eta]$ may vary with where the electron is located at different times; in other words, it is always possible that: $\Delta x_j[x_\eta] \neq \Delta x_k[x_{\eta'}]$. The electron's (post) local velocity (at t_η), in the spatial direction 'j', is designated by v_j, where: $\Delta x_j[\alpha: \eta] = v_j[\alpha: \eta]\, \tau$. These definitions have been introduced to analyze directly the discrete kinematics of electrons as they move along their trajectory from one possible interaction time t_η to the next $t_{\eta+1}$. Like Newton's particles, the electrons here move in straight-line motion between each possible interaction time. However, in anticipation of the impact of an impulse, the extended difference operator (see Appendix 4.2) is always being considered.

$$\text{Definition } \textbf{Point-Difference} \text{ (diff-dot) } \Diamond F_{n-1} \equiv F_n^{+} - F_n^{-} = F[T_n + \delta t] - F[T_n - \delta t] \qquad \text{N.B. } \textbf{not } \Diamond F_n$$

$$\text{Definition } \textbf{Extended-Difference}: \ \underline{\Delta} F_{n-1} \equiv F[T_n - \delta t] - F[T_{n-1} + \delta t] \equiv \underline{\Delta}[\, F[T_{n-1}] \,] \qquad \text{N.B. } \textbf{not } \underline{\Delta} F_n.$$

$$\text{Newton's I: } \Delta\, x_j[x_\eta] = \underline{\Delta}\, x_j[x_\eta] + < v_j[x_\eta] > \Diamond t_\eta = \underline{\Delta}\, x_j[x_\eta] + \Diamond\, x_j[x_\eta] \qquad \therefore\ \Delta[\underline{x}[x_\eta]] = (\, \underline{\Delta} + \Diamond \,)[\underline{x}_\eta]$$

The most general form of the difference operator can be separated into two components: $\Delta = \underline{\Delta} + \Diamond$

This set of difference definitions can be used to define analogs of the standard vector calculus; analogs using the 'Digital Gradient' (D-Grad) such as 'Digital Divergence' (D-Div) and 'Digital Curl' (D-Curl) operators for any vector function with discrete values defined along the electron's trajectory: $\underline{F}[\alpha: x_\eta] = \hat{\underline{e}}_x F_x + \hat{\underline{e}}_y F_y + \hat{\underline{e}}_z F_z$

Definition: ***Digital Divergence Operator***, $\underline{\Xi} \bullet \underline{F} \equiv \Xi_x F_x + \Xi_y F_x + \Xi_z F_z$

Definition: ***Digital Curl Operator***, $(\underline{\Xi} \wedge \underline{F})_j \equiv \sum_{kl} \varepsilon_{jkl} \Xi_k F_l$ where ε_{jkl} is the 3D anti-symmetric tensor.

Since the digital gradient is defined through the extended difference operator, it also satisfied the product rule.

$$\underline{\Xi} [\alpha_n \ \beta_n] = < \alpha_n > \underline{\Xi} [\beta_n] + \underline{\Xi} [\alpha_n] < \beta_n >$$

The starting point for this difference calculus is the critical observation that: a) $\Xi_j[x_k] = \delta_{jk}$ so that: b) $\Xi_j[v_k] = 0$

c) $\underline{\Xi} \bullet \underline{x}_\eta = 3$ d) $\underline{\Xi} \wedge \underline{x}_\eta = 0$ e) $\underline{\Xi} v_k = 0$ f) $\underline{\Xi} \bullet \underline{v}_\eta = 0$ g) $\underline{\Xi} \wedge \underline{v}_\eta = 0$ h) $(\underline{v}_\eta \bullet \underline{\Xi}) \underline{x}_\eta = \underline{v}$

These basic identities are used for more complicated identities with scalar functions like $\alpha[\underline{x}_\eta]$ and vector functions $\underline{F}[\underline{x}_\eta]$:

1. $\underline{\Xi} (\alpha_\eta v_j) = < v_j > \underline{\Xi} \alpha_\eta$
2. $\underline{\Xi} \bullet (\alpha_\eta \underline{v}_\eta) = < \underline{v}_\eta > \bullet \underline{\Xi} \alpha_\eta$
3. $\underline{\Xi} \wedge (\alpha_\eta \underline{v}_\eta) = - < \underline{v}_\eta > \wedge \underline{\Xi} \alpha_\eta$
4. $\underline{\Xi} (\underline{x}_\eta \bullet \underline{x}_\eta) = 2 < \underline{x}_\eta >$
5. $\underline{\Xi} (\underline{v}_\eta \bullet \underline{v}_\eta) = 0$
6. $\underline{\Xi} (\underline{x}_\eta \bullet \underline{F}_\eta) = < \underline{F}_\eta > + < \underline{x}_\eta > \bullet \underline{\Xi} \underline{F}_\eta$
7. $\underline{\Xi} (\underline{x}_\eta \bullet \underline{v}_\eta) = < \underline{v}_\eta >$
8. $\underline{\Xi} (\underline{v}_\eta \bullet \underline{F}_\eta) = < \underline{v}_\eta > \bullet \underline{\Xi} \underline{F}_\eta$
9. $\underline{\Xi} \bullet (\alpha_\eta \underline{F}_\eta) = < \underline{F}_\eta > \bullet \underline{\Xi} \alpha_\eta + < \alpha_\eta > \underline{\Xi} \bullet \underline{F}_\eta$
10. $\underline{\Xi} \wedge (\alpha_\eta \underline{v}_\eta) = - < \underline{v}_\eta > \wedge \underline{\Xi} \alpha_\eta$
11. $\underline{\Xi} \wedge (\alpha_\eta \underline{x}_\eta) = - < \underline{x}_\eta > \wedge \underline{\Xi} \alpha_\eta$
12. $\underline{\Xi} \bullet (\alpha_\eta \underline{x}_\eta) = < \underline{x}_\eta > \bullet \underline{\Xi} \alpha_\eta + 3 < \alpha_\eta >$
13. $\underline{\Xi} \wedge (\alpha_\eta \underline{F}_\eta) = \underline{\Xi} \alpha_\eta \wedge < \underline{F}_\eta > + < \alpha_\eta > \underline{\Xi} \wedge \underline{F}_\eta$
14. $\underline{\Xi} \wedge (\beta[v_\eta^2] \underline{v}_\eta) = 0$

All of these identities hold when the operand is itself a discrete average, like $< \underline{x}_\eta >$ and this generates comparable equations.

c′) $\underline{\Xi} \bullet < \underline{x}_\eta > = 3$ d′) $\underline{\Xi} \wedge < \underline{x}_\eta > = 0$ e′) $\underline{\Xi} < v_k > = 0$ f′) $\underline{\Xi} \bullet < \underline{v}_\eta > = 0$ g′) $\underline{\Xi} \wedge < \underline{v}_\eta > = 0$

It can be readily seen that when the digital gradient operates on a product of two terms it always generates averages, thus when it operates on an average it generates a ‘double-average’, which is defined here as (note the ‘double’ subscript):

Definition: ***Discrete Double Average***, $<< \zeta_n >> \equiv \tfrac{1}{2} <(\zeta_{n+1} + \zeta_n)> \equiv < \zeta_n >_2$

These generate comparable ‘double’ difference equations, such as:

9′. $\underline{\Xi} \bullet (< \alpha_\eta > \underline{F}_\eta) = < \underline{F}_\eta > \bullet \underline{\Xi} < \alpha_\eta > + < \alpha_\eta >_2 \underline{\Xi} \bullet \underline{F}_\eta$

13. $\underline{\Xi} \wedge (\alpha_\eta < \underline{F}_\eta >) = \underline{\Xi} \alpha_\eta \wedge << \underline{F}_\eta >> + < \alpha_\eta > \underline{\Xi} \wedge < \underline{F}_\eta >$

Definition **Digital-LaPlacian**: $\Xi^2 \psi_\eta \equiv \underline{\Xi} \bullet \underline{\Xi} \psi_\eta = \sum_k \Xi_k \Xi_k \psi_\eta = \sum_k \Delta_k^2[\psi[x_\eta]] / \Delta^2 x_j[x_\eta]$

15. $\Xi^2[x_k] = 0$
16. $\Xi^2[v_k] = 0$
17. $\Xi^2[\underline{v}_\eta \bullet \underline{x}_\eta] = 0$
18. $\Xi^2 (\alpha_\eta v_j) = << v_j >> \Xi^2[\alpha_\eta]$
19. $\Xi^2 (\alpha_\eta F_j) = << \alpha_\eta >> \Xi^2[F_j] + << F_j >> \Xi^2[\alpha_\eta] + 2 \underline{\Xi} <\alpha_\eta > \bullet \underline{\Xi} < \underline{F}_\eta >$

The ‘box-product’ retains its essential form. 20. $\underline{\Xi} \bullet (\underline{P}_\eta \wedge \underline{Q}_\eta) = < \underline{Q}_\eta > \bullet (\underline{\Xi} \wedge \underline{P}_\eta) - < \underline{P}_\eta > \bullet (\underline{\Xi} \wedge \underline{Q}_\eta)$

21. $\underline{\Xi} \bullet (\underline{x}_\eta \wedge \underline{F}_\eta) = - < \underline{x}_\eta > \bullet (\underline{\Xi} \wedge \underline{F}_\eta)$
22. $\underline{\Xi} \bullet (\underline{x}_\eta \wedge \underline{v}_\eta) = 0$
23. $\underline{\Xi} \bullet (\underline{v}_\eta \wedge \underline{F}_\eta) = - < \underline{v}_\eta > \bullet (\underline{\Xi} \wedge \underline{F}_\eta)$
24. $\underline{\Xi} \bullet (\underline{v}_\eta \wedge \alpha_\eta \underline{Q}_\eta) = < \underline{v}_\eta > \bullet \{< \alpha_\eta \underline{Q}_\eta > \wedge \underline{\Xi} \alpha_\eta - < \underline{v}_\eta > (\underline{\Xi} \wedge \underline{Q}_\eta)\}$
25. $\underline{\Xi} \wedge (\underline{\Xi} \wedge \underline{F}_\eta) = - \Xi^2 \underline{F}_\eta + \underline{\Xi} (\underline{\Xi} \bullet \underline{F}_\eta)$

A.6 APPENDIX VI: SINGLE DNV DIFFERENCE IDENTITIES

A single electron is characterized by its Natural Location Vector $\mathbf{X}_\eta$ at one of the interaction times, t_η when it is located at $\underline{x}_\eta$. The electron's (post) velocity (i.e. after t_η) is $\underline{v}_\eta$; the corresponding DNV associated with the electron's velocity is $\mathbf{V}_\eta$.

DNV Location: $\mathbf{X}_\eta \equiv i\,c\,t_\eta\,\mathbf{I_0} + \underline{x}_\eta \bullet \underline{\mathbf{I}} \cong \{\underline{x}[t_\eta]\}$ & **DNV Velocity**: $\mathbf{V}_\eta \equiv \mathbf{D}[\mathbf{X}_\eta] = i\,c\,\mathbf{I_0} + \underline{v}_\eta \bullet \underline{\mathbf{I}} \cong \{\underline{v}[t_\eta]\}$

The corresponding Discrete Natural Vector operator is the DNV operator, $\overline{\Xi}$ applicable to any discrete function $\psi[t_\eta; \underline{x}_\eta]$.

Definition: ***DNV Gradient Operator***, $\overline{\Xi}[\,\psi[x_\eta]\,] \equiv \{\, i\,\mathbf{I_0}\,\Xi_0 + \underline{\mathbf{I}} \bullet \underline{\Xi}\,\}\psi[x_\eta]$

The conjugate of the DNV gradient operator Ξ can be applied to any discrete natural vector (DNV) function, $\mathbf{Q}[x_\eta]$:

$$\overline{\Xi}^*\mathbf{Q}_\eta = \mathbf{I_0}\,(\Xi_0\,Q_{0\eta} - \underline{\Xi} \bullet \underline{Q}_\eta) + i\,\underline{\mathbf{I}} \bullet (\underline{\Xi} Q_{0\eta} - \Xi_0\,\underline{Q}_\eta) + \underline{\mathbf{I}} \bullet (\underline{\Xi} \wedge \underline{Q}_\eta)$$

where: $\mathbf{Q}_\eta = \mathbf{Q}[x_\eta] = i\,\mathbf{I_0}\,Q_0[x_\eta] + \mathbf{I}_1\,Q_1[x_\eta] + \mathbf{I}_2\,Q_2[x_\eta] + \mathbf{I}_3\,Q_3[x_\eta] = \{\, i\,Q_{0\eta}\,;\,\underline{Q}_\eta\,\}$

These DNV identities all use the basic conjugate multiplication DNV form described here in section 3.2.4; these formulae are evaluated using the 3D difference vector calculus results of Appendix V; here α_η and $\underline{Q}_\eta$ are scalar and vector functions.

1. $\overline{\Xi}^*\,\alpha_\eta = -i\,\mathbf{I_0}\,\Xi_0\alpha_\eta + \underline{\mathbf{I}} \bullet \underline{\Xi}\alpha_\eta$
2. $\overline{\Xi}^*\,v_{k\eta} = -i/c\,\mathbf{I_0}\,a_{\eta k}$ with $a_{\eta k} = \Diamond[\,v_{k\eta}\,]/\tau$
3. $\overline{\Xi}^*\,x_{k\eta} = \mathbf{I}_k$
4. $\overline{\Xi}^*\,(\,\underline{x}_\eta \bullet \underline{v}_\eta\,) = -i/c\,\mathbf{I_0} < \underline{x}_\eta > \bullet\, \underline{a}_\eta + \underline{\mathbf{I}} \bullet < \underline{v}_\eta >$
5. $\overline{\Xi}^*\mathbf{X}_\eta = -2\,\mathbf{I_0}$
6. $\overline{\Xi}^*\mathbf{V}_\eta = -i\,\underline{\mathbf{I}} \bullet \Xi_0\underline{v}_\eta = -i/c\,\underline{\mathbf{I}} \bullet \underline{a}_\eta$
7. $\overline{\Xi}^*\,(\mathbf{X}_\eta{}^*\mathbf{X}_\eta) = -2 < \mathbf{X}_\eta >$
8. $\overline{\Xi}^*\,(\mathbf{V}_\eta{}^*\mathbf{V}_\eta) = 2\,i/c < \underline{v}_\eta > \bullet\, \underline{a}_\eta\,\mathbf{I_0}$
9. $\overline{\Xi}^*\,(\alpha_\eta\mathbf{Q}_\eta) = < \alpha_\eta > (\overline{\Xi}^*\mathbf{Q}_\eta) + (\overline{\Xi}^*\alpha_\eta) < \mathbf{Q}_\eta >$
10. $\overline{\Xi}^*\,(x_{k\eta}\mathbf{V}_\eta) = \mathbf{I}_k < \mathbf{V}_\eta >$
11. $\overline{\Xi}^*\,(\alpha_\eta\mathbf{V}_\eta) = (\overline{\Xi}^*\alpha_\eta) < \mathbf{V}_\eta > + < \alpha_\eta > \overline{\Xi}^*\mathbf{V}_\eta$
12. $\overline{\Xi}^*\,(\mathbf{X}_\eta{}^*\mathbf{V}_\eta) = -2\,c\,\mathbf{I_0}$
13. $\overline{\Xi}^*\,(\alpha_\eta\mathbf{X}_\eta) = -2 < \alpha_\eta > \mathbf{I}_0 + (\overline{\Xi}^*\alpha_\eta) < \mathbf{X}_\eta >$
14. $\overline{\Xi}^*\,(\alpha_\eta\mathbf{V}_\eta) = \mathbf{I_0}\,(\,c\,\Xi_0\,\alpha_\eta - < \underline{v}_\eta > \bullet\, \underline{\Xi}\alpha_\eta\,) + i\,\underline{\mathbf{I}} \bullet (\,c\,\underline{\Xi}\alpha_\eta - < \underline{v}_\eta > \Xi_0\,\alpha_\eta\,) - \underline{\mathbf{I}} \bullet (< \underline{v}_\eta > \wedge \underline{\Xi}\alpha_\eta)$
15. $\mathbf{V}_\eta{}^*\,\overline{\Xi}^*\alpha_\eta = -\mathbf{I_0}\,(c\,\Xi_0 + \underline{v}_\eta \bullet \underline{\Xi})\alpha_\eta - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\Xi} + \underline{v}_\eta\,\Xi_0\,)\alpha_\eta + \underline{\mathbf{I}} \bullet (\underline{v}_\eta \wedge \underline{\Xi}\alpha_\eta)$
16. $\mathbf{Q}_\eta\,\overline{\Xi}^*\alpha_\eta = \mathbf{I_0}\,(Q_{0\eta}\,\Xi_0\alpha_\eta - \underline{Q}_\eta \bullet \underline{\Xi}\alpha_\eta) + i\,\underline{\mathbf{I}} \bullet (Q_{0\eta}\,\underline{\Xi}\alpha_\eta - \underline{Q}_\eta\,\Xi_0\alpha_\eta) + \underline{\mathbf{I}} \bullet (\underline{Q}_\eta \wedge \underline{\Xi}\alpha_\eta)$
17. $\overline{\Xi}^*\,\underline{\Xi}\alpha_\eta = \underline{\Xi}\,\overline{\Xi}^*\,\alpha_\eta = \mathbf{I_0}\,(\,\Xi_0\,\Xi_0 - \underline{\Xi} \bullet \underline{\Xi}\,)\alpha_\eta = \mathbf{I_0}\,(\,\Xi_0{}^2 - \underline{\Xi}^2\,)\alpha_\eta \equiv -\mathbf{I_0}\,\wp\alpha_\eta$
18. $\overline{\Xi}^*\,\underline{\Xi}(\underline{x}_\eta) = 0$
19. $\overline{\Xi}^*\,\underline{\Xi}(\underline{v}_\eta) = 0$ (if $\underline{a} = 0$)

A.7 APPENDIX VII: DOUBLE DNV DIFFERENCE IDENTITIES

A single electron is characterized by its Natural Location Vector $\mathbf{X}_\eta$ at one of the interaction times, t_η when it is located at $\underline{x}_\eta$. The electron's (post) velocity (i.e. after t_η) is $\underline{v}_\eta$; the corresponding DNV associated with the electron's velocity is $\mathbf{V}_\eta$.

DNV Location: $\mathbf{X}_\eta \equiv i\,c\,t_\eta\,\mathbf{I_0} + \underline{x}_\eta \bullet \underline{\mathbf{I}} \cong \{\underline{x}[t_\eta]\}$ & **DNV Velocity**: $\mathbf{V}_\eta \equiv \mathbf{D}[\mathbf{X}_\eta] = i\,c\,\mathbf{I_0} + \underline{v}_\eta \bullet \underline{\mathbf{I}} \cong \{\underline{v}[t_\eta]\}$

The corresponding Discrete Natural Vector operator is the DNV operator, $\boldsymbol{\Xi}$ applicable to any discrete function $\psi[t_\eta; \underline{x}_\eta]$.

Definition: ***DNV Gradient Operator***, $\boldsymbol{\Xi}[\,\psi[x_\eta]\,] \equiv \{\, i\,\mathbf{I_0}\,\Xi_0 + \underline{\mathbf{I}} \bullet \underline{\boldsymbol{\Xi}}\,\}\psi[x_\eta]$

The conjugate of the DNV gradient operator $\boldsymbol{\Xi}$ can be applied to any discrete natural vector (DNV) function, $\mathbf{Q}[x_\eta]$:

$$\boldsymbol{\Omega}^*\mathbf{Q}_\eta = \mathbf{I_0}\,(\Xi_0\,Q_{0\eta} - \underline{\Xi} \bullet \underline{Q}_\eta) + i\,\underline{\mathbf{I}} \bullet (\underline{\Xi} Q_{0\eta} - \Xi_0\,\underline{Q}_\eta) + \underline{\mathbf{I}} \bullet (\underline{\Xi} \wedge \underline{Q}_\eta)$$

where: $\mathbf{Q}_\eta = \mathbf{Q}[x_\eta] = i\,\mathbf{I_0}\,Q_0[x_\eta] + \mathbf{I}_1\,Q_1[x_\eta] + \mathbf{I}_2\,Q_2[x_\eta] + \mathbf{I}_3\,Q_3[x_\eta] = \{\, i\,Q_{0\eta}\,;\,\underline{Q}_\eta\,\}$

These DNV identities all use the basic conjugate multiplication DNV form described here in section 3.2.4; these formulae are evaluated using the 3D difference vector calculus results of Appendix V; here α_η and $\underline{Q}_\eta$ are scalar and vector functions.

1. $\boldsymbol{\Xi}^*\,\alpha_\eta = -i\,\mathbf{I_0}\,\Xi_0\alpha_\eta + \underline{\mathbf{I}} \bullet \underline{\boldsymbol{\Xi}}\alpha_\eta$
2. $\boldsymbol{\Xi}^*\,v_{k\eta} = -i/c\,\mathbf{I_0}\,a_{\eta k}$ with $a_{\eta k} = \Diamond[\,v_{k\eta}\,]/\tau$
3. $\boldsymbol{\Xi}^*\,x_{k\eta} = \mathbf{I}_k$
4. $\boldsymbol{\Xi}^*\,(\,\underline{x}_\eta \bullet \underline{v}_\eta\,) = -i/c\,\mathbf{I_0} < \underline{x}_\eta > \bullet\, \underline{a}_\eta + \underline{\mathbf{I}} \bullet < \underline{v}_\eta >$
5. $\boldsymbol{\Xi}^*\mathbf{X}_\eta = -2\,\mathbf{I_0}$
6. $\boldsymbol{\Xi}^*\mathbf{V}_\eta = -i\,\underline{\mathbf{I}} \bullet\, \Xi_0\underline{v}_\eta = -i/c\,\underline{\mathbf{I}} \bullet \underline{a}_\eta$
7. $\boldsymbol{\Xi}^*\,(\mathbf{X}_\eta{}^*\mathbf{X}_\eta) = -2 < \mathbf{X}_\eta >$
8. $\boldsymbol{\Xi}^*\,(\mathbf{V}_\eta{}^*\mathbf{V}_\eta) = 2\,i/c < \underline{v}_\eta > \bullet\, \underline{a}_\eta\,\mathbf{I_0}$
9. $\boldsymbol{\Xi}^*\,(\alpha_\eta \mathbf{Q}_\eta) = < \alpha_\eta > (\boldsymbol{\Xi}^*\mathbf{Q}_\eta) + (\boldsymbol{\Xi}^*\alpha_\eta) < \mathbf{Q}_\eta >$
10. $\boldsymbol{\Xi}^*\,(x_{k\eta}\mathbf{V}_\eta) = \mathbf{I}_k < \mathbf{V}_\eta >$
11. $\boldsymbol{\Xi}^*\,(\alpha_\eta \mathbf{V}_\eta) = (\boldsymbol{\Xi}^*\alpha_\eta) < \mathbf{V}_\eta > + < \alpha_\eta > \boldsymbol{\Xi}^*\mathbf{V}_\eta$
12. $\boldsymbol{\Xi}^*\,(\mathbf{X}_\eta{}^*\mathbf{V}_\eta) = -2\,c\,\mathbf{I_0}$
13. $\boldsymbol{\Xi}^*\,(\alpha_\eta \mathbf{X}_\eta) = -2 < \alpha_\eta > \mathbf{I}_0 + (\boldsymbol{\Xi}^*\alpha_\eta) < \mathbf{X}_\eta >$
14. $\boldsymbol{\Xi}^*\,(\alpha_\eta \mathbf{V}_\eta) = \mathbf{I_0}\,(\,c\,\Xi_0\,\alpha_\eta - < \underline{v}_\eta > \bullet\, \underline{\boldsymbol{\Xi}}\alpha_\eta\,) + i\,\underline{\mathbf{I}} \bullet (\,c\,\underline{\Xi}\alpha_\eta - < \underline{v}_\eta > \Xi_0\,\alpha_\eta\,) - \underline{\mathbf{I}} \bullet (< \underline{v}_\eta > \wedge \underline{\Xi}\alpha_\eta)$
15. $\mathbf{V}_\eta{}^*\,\boldsymbol{\Xi}^*\alpha_\eta = -\mathbf{I_0}\,(c\,\Xi_0 + \underline{v}_\eta \bullet \underline{\boldsymbol{\Xi}})\alpha_\eta - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\Xi} + \underline{v}_\eta\,\Xi_0\,)\alpha_\eta + \underline{\mathbf{I}} \bullet (\underline{v}_\eta \wedge \underline{\Xi}\alpha_\eta)$
16. $\mathbf{Q}_\eta\,\boldsymbol{\Xi}^*\alpha_\eta = \mathbf{I_0}\,(Q_{0\eta}\,\Xi_0\alpha_\eta - \underline{Q}_\eta \bullet \underline{\boldsymbol{\Xi}}\alpha_\eta) + i\,\underline{\mathbf{I}} \bullet (Q_{0\eta}\,\underline{\Xi}\alpha_\eta - \underline{Q}_\eta\,\Xi_0\alpha_\eta) + \underline{\mathbf{I}} \bullet (\underline{Q}_\eta \wedge \underline{\Xi}\alpha_\eta)$
17. $\boldsymbol{\Xi}^*\,\boldsymbol{\Xi}\alpha_\eta = \boldsymbol{\Xi}\,\boldsymbol{\Xi}^*\,\alpha_\eta = \mathbf{I_0}\,(\,\Xi_0\,\Xi_0 - \underline{\boldsymbol{\Xi}} \bullet \underline{\boldsymbol{\Xi}}\,)\alpha_\eta = \mathbf{I_0}\,(\,\Xi_0{}^2 - \underline{\Xi}^2\,)\alpha_\eta \equiv -\mathbf{I_0}\,\wp\,\alpha_\eta$
18. $\boldsymbol{\Xi}^*\,\boldsymbol{\Xi}(\underline{x}_\eta) = 0$
19. $\boldsymbol{\Xi}^*\,\boldsymbol{\Xi}(\underline{v}_\eta) = 0$ (if $\underline{a} = 0$)

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