Conjecture on semiprimes n=pq related to the number of primes up to n

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Abstract. In this paper I conjecture that there exist an infinity of semiprimes n = p*q, where p = 30*k + m1 and q = 30*h + m2, m1 and m2 distinct, having one from the values 1, 7, 11, 13, 17, 19, 23, 29, such that the number of primes congruent to m1 (mod 30) up to n is equal to the number of primes congruent to m2 (mod 30) up to n. Example: for n = 91 = 7*13, there exist 3 primes of the form 30*k + 7 up to 91 (7, 37 and 67) and 3 primes of the form 30*k + 13 up to 91 (13, 43 and 73).

Conjecture:

There exist an infinity of semiprimes n = p*q, where p = 30*k + m1 and q = 30*h + m2, m1 and m2 distinct, having one from the values 1, 7, 11, 13, 17, 19, 23, 29, such that the number of primes congruent to m1 (mod 30) up to n is equal to the number of primes congruent to m2 (mod 30) up to n.

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The first six semiprimes n = pq having this property:

: 77 (= 7*11), because there exist 3 primes congruent to 7 (mod 30) up to 77 (7, 37, 67) and 3 primes congruent to 11 (mod 30) up to 91 (11, 41, 71);

: 91 (= 7*13), because there exist 3 primes congruent to 7 (mod 30) up to 91 (7, 37, 67) and 3 primes congruent to 13 (mod 30) up to 91 (13, 43, 73);

: 187 (= 11*17), because there exist 5 primes congruent to 11 (mod 30) up to 187 (11, 41, 71, 101, 131) and 5 primes congruent to 17 (mod 30) up to 187 (17, 47, 107, 137, 167);

: 221 (= 13*17), because there exist 6 primes congruent to 13 (mod 30) up to 221 (13, 43, 73, 103, 163, 193) and 6 primes congruent to 17 (mod 30) up to 221 (17, 47, 107, 137, 167, 197);

: 299 (= 13*23), because there exist 8 primes congruent to 13 (mod 30) up to 299 (13, 43, 73, 103, 163, 193, 223, 283) and 8 primes congruent to 23 (mod 30) up to 299 (23, 53, 83, 113, 173, 233, 263, 293);

: 391 (= 17*23), because there exist 10 primes congruent to 17 (mod 30) up to 391 (17, 47, 107, 137, 167, 197, 227, 257, 317, 347) and 10 primes congruent to 23 (mod 30) up to 391 (23, 53, 83, 113, 173, 233, 263, 293, 353, 383).

Comment:

The number of the primes of the form 30k + 1, 30k + 7, 30k + 11, 30k + 13, 30k + 17, 30k + 19, 30k + 23 respectively 30k + 29 seem to be very close in the case of Fermat pseudoprimes to base 2 with two prime factors (the 2-Poulet numbers, see the sequence A214305 in OEIS).

Examples : for n = 35333, we have:
: 462 primes of the form 30k + 1 up to n;
: 470 primes of the form 30k + 7 up to n;
: 476 primes of the form 30k + 11 up to n;
: 475 primes of the form 30k + 13 up to n;
: 474 primes of the form 30k + 17 up to n;
: 459 primes of the form 30k + 19 up to n;
: 470 primes of the form 30k + 23 up to n;
: 475 primes of the form 30k + 29 up to n.

: for n = 164737, we have:
: 1874 primes of the form 30k + 1 up to n;
: 1885 primes of the form 30k + 7 up to n;
: 1896 primes of the form 30k + 11 up to n;
: 1884 primes of the form 30k + 13 up to n;
: 1885 primes of the form 30k + 17 up to n;
: 1873 primes of the form 30k + 19 up to n;
: 1889 primes of the form 30k + 23 up to n;
: 1888 primes of the form 30k + 29 up to n.

Note:

This property seem to be shared by other classes of numbers also (but this paper is only about semiprimes), like for instance Carmichael numbers: in the case of Hardy-Ramanujan number 1729 four such sets from eight have the same number of primes, respectively 34 (30k + 7, 30k + 13, 30k + 17, 30k + 23). Open questions: are there numbers for which all eight sets have the same number of primes? Are there an infinity of such numbers?