Decision-Making Method based on Neutrosophic Soft Expert Graphs

[1]Vakkas Uluçay, [1]Mehmet Şahin and [2]Said Broumi, [3]Assia Bakali , [2]Mohamed Talea
[4] Florentin Smarandache

[1]Department of Mathematics, Gaziantep University, Gaziantep27310-Turkey
Email: vulucay27@gmail.com, mesahin@gantep.edu.tr

[2] Laboratory of Information processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com, taleamohamed@yahoo.fr
[3]Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco.

assiabakali@yahoo.fr

[4]Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA
fsmarandache@gmail.com

AbstractIn this paper, we first define the concept of neutrosophic soft expert graph. We have established a link between graphs and neutrosophic soft expert sets. Basic operations of neutrosophic soft expert graphs such as union, intersection and complement are defined here. The concept of neutrosophic soft expert soft graph is also discussed in this paper. The new concept is called neutrosophic soft expert graph-based multi-criteria decision making method (NSEGMCDM for short). Finally, an illustrative example is given and a comparison analysis is conducted between the proposed approach and other existing methods, to verify the feasibility and effectiveness of the developed approach.

Keywords:graph, soft expert set, neutrosophic soft set, neutrosophic soft expert set,neutrosophic soft expert graph

1. Introduction

The concept of fuzzy set theory was introduced by Zadeh [44] to solve difficulties in dealing with uncertainties. Since then the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real life problems involving ambiguous and uncertain environment.Atanassov [8] introduced the concept of intuitionistic fuzzy sets as an extension of Zadeh's fuzzy set [44].The theory of neutrosophic set is introduced by Smarandache [38,39] which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. The theory is generalization of classical sets and fuzzy sets and is applied in decision making problems, control theory, medicines, topology and in many more real life problems.Molodtsov [31] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Molodtsov's soft sets give us new technique for dealing with uncertainty from the viewpoint of parameters.Maji*et al* [28,29] proposed fuzzy soft sets and neutrosophic soft sets. Broumi and Smarandache [9] proposed intuitionistic neutrosophic soft set and its application in decision making problem. Alkhazaleh and Salleh [3,4] defined the concept of soft expert set, which were later extended to vague soft expert set theory [23], generalized vague soft expert set [6] and multi Q-fuzzy soft expert set [1]. Şahin et al. [36] introduced neutrosophic soft expert sets, while Al-Quran and Hassan [7] extended it further to neutrosophic vague soft expert set.

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. The graph is a widely used tool for solving combinatorial problems in different areas, such as geometry, algebra, number theory, topology, optimization and computer science. When the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs, intuitionistic fuzzy graphs and their extensions [18-22,24-27,32,33].Smarandache [23, 24, 25] defined four main categories of neutrosophic graphs. Two of them, called I-edge neutrosophic graph and I-vertex neutrosophic graph, are based on literal indeterminacy (I); these concepts are deeply studied and gained popularity among the researchers due to applications via real world problems [41-43].More related works can be seen in [2,10-17, 30, 34, 35, 37, 40].

We have discussed different operations defined on neutrosophic soft expert graphs using examples to make the concept easier. The concept of strong neutrosophic soft expert graphs and the complement of strong neutrosophic soft graphs is also discussed. Neutrosophic soft expert graphs are pictorial representation in which each vertex and each edge is an element of neutrosophic soft sets. This paper has been arranged as the following;

In section 2, some basic concepts about graphs and neutrosophic soft sets are presented which will be employed in later sections. In section 3, concept of neutrosophic soft expert graphs is given and some of their fundamental properties have been studied. In section 4, the concept of strong neutrosophic soft expert graphs and its complement is studied. In section 5, we present an application of neutrosophic soft expert graphs in decision making and then an illustrative example is given.In section 6, a comparison analysis is conducted between the proposed approach and other existing methods, in order to verify its feasibility and effectiveness. Finally, the conclusions are drawn in section 7.

2. Preliminaries

In this section, we have given some definitions aboutgraphs and neutrosophic soft sets. These will be helpful inlater sections.

Definition 2.1.(see [38])Let U be a universe of discourse, with a generic element in U denoted by u, then a neutrosophic (NS) set A is an object having the form

$$A = \{<u: T_A(u), I_A(u), F_A(u)>, u \in U\}$$

where the functions T, I, F : $U \rightarrow\ ]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $u \in U$ to the set A with the condition.

$$^-0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3^+$$

Definition 2.2.(see [28]) Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $NS(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by $F: A \rightarrow NS(U)$.

Definition 2.3.(see [36])A pair (F, A) is called a neutrosophic soft expert set over U, where F is mapping given by

$$F: A \rightarrow P(U)$$

where$P(U)$ denotes the power neutrosophic set of U.

Definition 2.4.(see [22])A fuzzy graph is pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation onσ. i.e. $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. $\sigma$is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

Definition 2.5.(see [22]) The fuzzy subgraph $H = (\tau, \rho)$ is called a fuzzy subgraph of $G = (\sigma, \mu)$, if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u.v) \leq \mu(u, v)$ for all $u, v \in V$.

Definition 2.6.(see [23]) An intuitionistic fuzzy graph is of the form $G = (V, E)$ where

i. $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V, (i = 1,2, \dots, n)$,
ii. $E \subseteq V \times V$where$\mu_2: V \times V \rightarrow [0,1]$ and $\gamma_2: V \times V \rightarrow [0,1]$ are such that $\mu_2(v_i, v_j) \leq min[\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq max[\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E, (i, j = 1,2, \dots, n)$.

Definition 2.7.(see [15])Let $G^* = (V, E)$ be a simple graph and A be the set of parameters. Let $N(V)$ be the set of all neutrosophic sets in V. By a neutrosophic soft graph NSG, we mean a 4-tuple $G = (G^*, A, f, g)$where $f: A \rightarrow N(V)$, $g: A \rightarrow N(V \times V)$ defined as $f(e) = f_e = \{\langle x, T_{f_e}(x), I_{f_e}(x), F_{f_e}(x)\rangle : x \in V\}$ and $g(e) = g_e = \{\langle (x, y), T_{f_e}(x, y), I_{f_e}(x, y), F_{f_e}(x, y)\rangle : (x, y) \in V \times V\}$ are neutrosophic sets over V and $V \times V$ respectively, such that

$$T_{g_e}(x, y) \leq min\{T_{f_e}(x), T_{f_e}(y)\},$$

$$I_{g_e}(x, y) \leq min\{I_{f_e}(x), I_{f_e}(y)\},$$

$$F_{g_e}(x, y) \geq max\{F_{f_e}(x), F_{f_e}(y)\}.$$

For all $(x, y) \in V \times V$and $e \in A$. We can also denote a NSG by $G = (G^*, A, f, g) = \{N(e): e \in A\}$ which is a parameterized family of graphs $N(e)$ we call them Neutrosophic graphs.

3. Neutrosophic Soft Expert Graph

In this section, we introduce the definition of a neutrosophic soft expert graph and give basic properties of this concept.

Let V be a universe, Y a set of parameters,X a set of experts (agents), and $O = \{1 = agree\ , 0 = disagree\}$a set of opinions. Let $Z = Y \times X \times O$ and $A \subseteq Z$.

3.1 Definition Let $G^* = (V, E)$ be a simple graph and A be the set of parameters. Let $N(V)$ be the set of all neutrosophic sets in V. By a neutrosophic soft expert graph NSEG, we mean a 4-tuple $G = (G^*, A, f, g)$where $f: A \to N(V)$, $g: A \to N(V \times V)$ defined as $f(\alpha) = f_\alpha = \{\langle x, \mu_{f_\alpha}(x), \vartheta_{f_\alpha}(x), w_{f_\alpha}(x)\rangle : x \in V\}$ and
$g(\alpha) = g_\alpha = \{\langle (x, y), \mu_{f_\alpha}(x, y), \vartheta_{f_\alpha}(x, y), w_{f_\alpha}(x, y)\rangle : (x, y) \in V \times V\}$ are neutrosophic sets over V and $V \times V$ respectively, such that

$$\mu_{g_\alpha}(x, y) \leq min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\},$$

$$\vartheta_{g_\alpha}(x, y) \leq min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\},$$

$$w_{g_\alpha}(x, y) \geq max\{w_{f_\alpha}(x), w_{f_\alpha}(y)\}.$$

For all $(x, y) \in V \times V$and $\alpha \in A$. We can also denote a NSEG by $G = (G^*, A, f, g) = \{N(\alpha): \alpha \in A\}$ which is a parameterized family of graphs $N(\alpha)$ we call them Neutrosophic graphs.

3.2 Example Suppose that $G^* = (V, E)$ be a simple graph with $V = \{x_1, x_2, x_3\}, Y = \{e_1, e_2, e_3\}$ be a set of parameters and $X = \{p, q\}$ be a set of experts. A NSEG is given in Table 1 below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V \times V \backslash \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all $\alpha \in A$.

Table 1

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1, q, 1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_2, p, 1)$	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
$(e_2, q, 1)$	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
$(e_3, p, 1)$	(0.1,0.1,0.5)	(0.1,0.2,0.4)	(0.1,0.2,0.6)
$(e_3, q, 1)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_2, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, q, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
$(e_3, p, 0)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_3, q, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)

$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_3, p, 1)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_3, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, p, 0)$	(0.1,0.2,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, q, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)

3.3 Definition Aneutrosophic soft expert graph $G = (G^*, A^1, f^1, g^1)$ is called a neutrosophic soft expert subgraph of $G = (G^*, A, f, g)$ if

i. $A^1 \subseteq A$

ii. ${f_\alpha}^1 \subseteq f$, that is, $\mu_{{f^1}_\alpha}(x) \leq \mu_{f_\alpha}(x), \vartheta_{{f^1}_\alpha}(x) \leq \vartheta_{f_\alpha}(x), w_{{f^1}_\alpha}(x) \leq w_{f_\alpha}(x)$.

iii. ${g_\alpha}^1 \subseteq g$, that is, $\mu_{{f^1}_\alpha}(x, y) \leq \mu_{f_\alpha}(x, y), \vartheta_{{f^1}_\alpha}(x, y) \leq \vartheta_{f_\alpha}(x, y), w_{{f^1}_\alpha}(x, y) \leq w_{f_\alpha}(x, y)$.

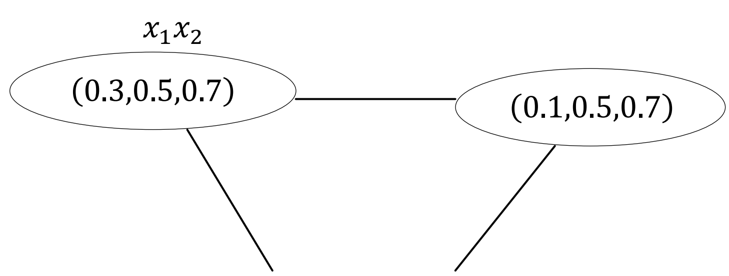
forall $\alpha \in A^1$.

3.4Example Suppose that $G^* = (V, E)$ be a simple graph with $V = \{x_1, x_2, x_3\}, Y = \{e_1\}$ be a set of parameters and $X = \{p\}$ be a set of experts. A neutrosophic soft expert subgraph of example 3.2 is given in Table 2 below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V \times V \backslash \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all $\alpha \in A$.

Table 2

f^1	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1, q, 1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
g^1	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_1, p, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)

$N(e_1, q, 1)$ Corresponding to $(e_1, q, 1)$



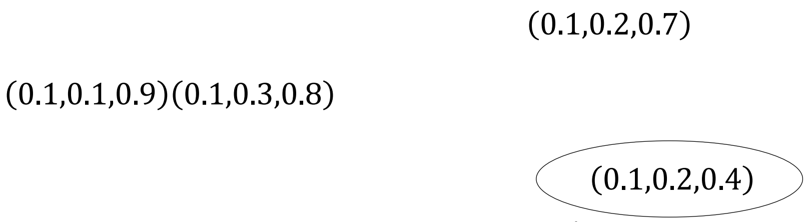


Figure 1

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

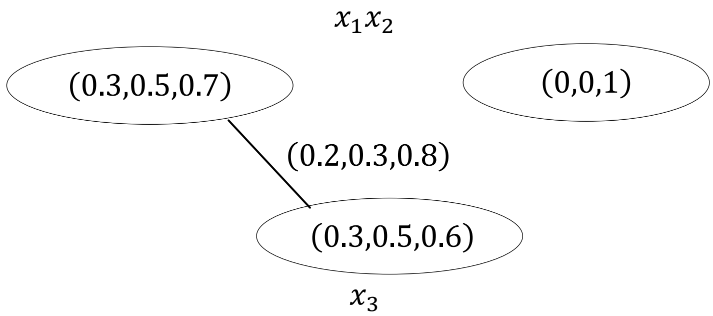


Figure 2

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

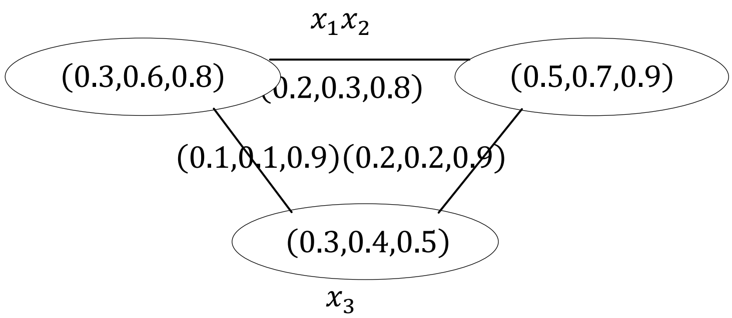


Figure 3

$N(e_1, q, 0)$ Corresponding to $(e_1, q, 0)$

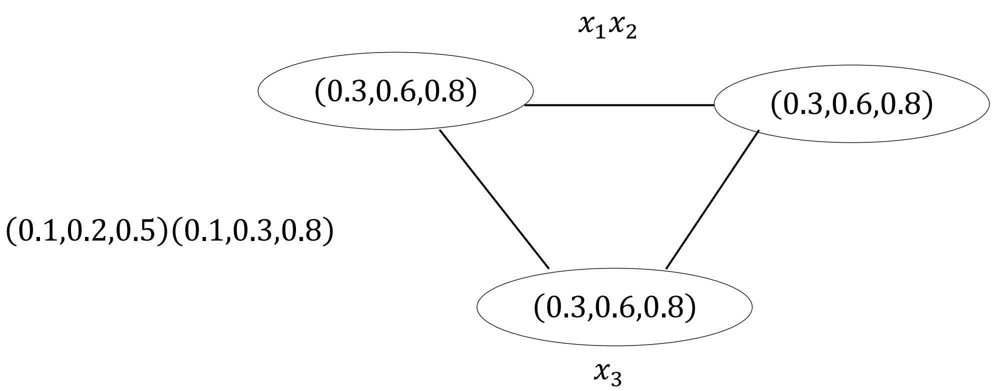


Figure 4

3.5 DefinitionA neutrosophic soft expert subgraph $G = (G^*, A^1, f^1, g^1)$ is said to be spanning neutrosophic soft expert subgraph of $G = (G^*, A, f, g)$ if ${f_\alpha}^1(x) = f(x)$, for all $x \in V, \alpha \in A^1$.

3.6 Definition An agree-neutrosophic soft expert graph $G_1 = (G^*, A, f_1, g_1)$ over $G^* = (V, E)$is a neutrosophic soft expert subgraph of $G = (G^*, A, f, g)$ defined as follow

$$G_1 = (G^*, A, f_1, g_1) = \{f_1(\alpha), g_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

3.7 Example Consider Example 3.2. Then the agree-neutrosophic soft expert graph $G_1 = (G^*, A, f_1, g_1)$over $G^* = (V, E)$.

Table 3

f_1	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1, q, 1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_2, p, 1)$	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
$(e_2, q, 1)$	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
$(e_3, p, 1)$	(0.1,0.1,0.5)	(0.1,0.2,0.4)	(0.1,0.2,0.6)
$(e_3, q, 1)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
g_1	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_3, p, 1)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_3, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)

3.8 Definition Andisagree-neutrosophic soft expert graph $G_0 = (G^*, A, f_0, g_0)$ over $G^* = (V, E)$ is a neutrosophic soft expert subgraph of $G = (G^*, A, f, g)$ defined as follow

$$G_0 = (G^*, A, f_0, g_0) = \{f_0(\alpha), g_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

3.9 Example Consider Example 3.2. Then the disagree-neutrosophic soft expert graph $G_0 = (G^*, A, f_0, g_0)$over $G^* = (V, E)$.

Table 4

f_2	x_1	x_2	x_3
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_2, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, q, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
$(e_3, p, 0)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_3, q, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
g_2	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)

$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, p, 0)$	(0.1,0.2,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, q, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)

3.10 Definition Theunion of two neutrosophic soft expert graphs $G^1 = (G^*, A^1, f^1, g^1)$ and $G^2 = (G^*, A^2, f^2, g^2)$ is denoted by $G = (G^*, A, f, g)$ with $A = A^1 \cup A^2$ where the truth-membership, indeterminacy-membership and falsity-membership of union are as follows

$$\mu_{f_\alpha}(x) = \begin{cases} \mu_{f_\alpha{}^1}(x) = \begin{cases} \text{if } e \in A^1 - A^2 \text{and } x \in V^1 - V^2 \text{or} \\ \text{if } e \in A^1 - A^2 \text{and } x \in V^1 \cap V^2 \text{or} \\ \text{if } e \in A^1 \cap A^2 \text{and } x \in V^1 - V^2. \end{cases} \\ \mu_{f_\alpha{}^2}(x) = \begin{cases} \text{if } e \in A^2 - A^1 \text{and } x \in V^2 - V^1 \text{or} \\ \text{if } e \in A^2 - A^1 \text{and } x \in V^1 \cap V^2 \text{or} \\ \text{if } e \in A^2 \cap A^1 \text{and } x \in V^2 - V^1. \end{cases} \\ max\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\}\{\text{if } e \in A^1 \cap A^2 \text{and } x \in V^1 \cap V^2\} \\ 0, \quad \text{otherwise} \end{cases}$$

$$\vartheta_{f_\alpha}(x) = \begin{cases} \vartheta_{f_\alpha{}^1}(x) = \begin{cases} \text{if } e \in A^1 - A^2 \text{and } x \in V^1 - V^2 \text{or} \\ \text{if } e \in A^1 - A^2 \text{and } x \in V^1 \cap V^2 \text{or} \\ \text{if } e \in A^1 \cap A^2 \text{and } x \in V^1 - V^2. \end{cases} \\ \vartheta_{f_\alpha{}^2}(x) = \begin{cases} \text{if } e \in A^2 - A^1 \text{and } x \in V^2 - V^1 \text{or} \\ \text{if } e \in A^2 - A^1 \text{and } x \in V^1 \cap V^2 \text{or} \\ \text{if } e \in A^2 \cap A^1 \text{and } x \in V^2 - V^1. \end{cases} \\ max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^2}(x)\right\}\{\text{if } e \in A^1 \cap A^2 \text{and } x \in V^1 \cap V^2\} \\ 0, \quad \text{otherwise} \end{cases}$$

$$w_{f_\alpha}(x) = \begin{cases} w_{f_\alpha{}^1}(x) = \begin{cases} \text{if } e \in A^1 - A^2 \text{and } x \in V^1 - V^2 \text{or} \\ \text{if } e \in A^1 - A^2 \text{and } x \in V^1 \cap V^2 \text{or} \\ \text{if } e \in A^1 \cap A^2 \text{and } x \in V^1 - V^2. \end{cases} \\ w_{f_\alpha{}^2}(x) = \begin{cases} \text{if } e \in A^2 - A^1 \text{and } x \in V^2 - V^1 \text{or} \\ \text{if } e \in A^2 - A^1 \text{and } x \in V^1 \cap V^2 \text{or} \\ \text{if } e \in A^2 \cap A^1 \text{and } x \in V^2 - V^1. \end{cases} \\ min\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^2}(x)\right\}\{\text{if } e \in A^1 \cap A^2 \text{and } x \in V^1 \cap V^2\} \\ 0, \quad \text{otherwise} \end{cases}$$

$$\mu_{g_\alpha}(x,y)=\begin{cases} \mu_{g_\alpha{}^1}(x,y)=\begin{cases}\text{if } e\in A^1-A^2\text{and}(x,y)\in(V^1\times V^1)-(V^2\times V^2)\text{or}\\ \text{if } e\in A^1-A^2\text{and}(x,y)\in(V^1\times V^1)\cap(V^2\times V^2)\text{or}\\ \text{if } e\in A^1\cap A^2\text{and}(x,y)\in(V^1\times V^1)-(V^2\times V^2).\end{cases}\\ \mu_{g_\alpha{}^2}(x,y)=\begin{cases}\text{if } e\in A^2-A^1\text{and}(x,y)\in(V^2\times V^2)-(V^1\times V^1)\text{or}\\ \text{if } e\in A^2-A^1\text{and}(x,y)\in(V^2\times V^2)\cap(V^1\times V^1)\text{or}\\ \text{if } e\in A^2\cap A^1\text{and}(x,y)\in(V^2\times V^2)-(V^1\times V^1).\end{cases}\\ max\{\mu_{g_\alpha{}^1}(x,y),\mu_{g_\alpha{}^2}(x,y)\}\{\text{if } e\in A^1\cap A^2\text{and}(x,y)\in(V^1\times V^1)\cap(V^2\times V^2)\}\\ 0,\quad \text{otherwise}\end{cases}$$

$$\vartheta_{g_\alpha}(x,y)=\begin{cases} \vartheta_{g_\alpha{}^1}(x,y)=\begin{cases}\text{if } e\in A^1-A^2\text{and}(x,y)\in(V^1\times V^1)-(V^2\times V^2)\text{or}\\ \text{if } e\in A^1-A^2\text{and}(x,y)\in(V^1\times V^1)\cap(V^2\times V^2)\text{or}\\ \text{if } e\in A^1\cap A^2\text{and}(x,y)\in(V^1\times V^1)-(V^2\times V^2).\end{cases}\\ \vartheta_{g_\alpha{}^2}(x,y)=\begin{cases}\text{if } e\in A^2-A^1\text{and}(x,y)\in(V^2\times V^2)-(V^1\times V^1)\text{or}\\ \text{if } e\in A^2-A^1\text{and}(x,y)\in(V^2\times V^2)\cap(V^1\times V^1)\text{or}\\ \text{if } e\in A^2\cap A^1\text{and}(x,y)\in(V^2\times V^2)-(V^1\times V^1).\end{cases}\\ max\{\vartheta_{g_\alpha{}^1}(x,y),\vartheta_{g_\alpha{}^2}(x,y)\}\{\text{if } e\in A^1\cap A^2\text{and}(x,y)\in(V^1\times V^1)\cap(V^2\times V^2)\}\\ 0,\quad \text{otherwise}\end{cases}$$

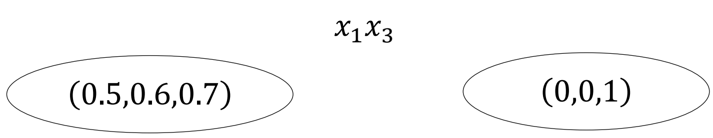
$$w_{g_\alpha}(x,y)=\begin{cases} w_{g_\alpha{}^1}(x,y)=\begin{cases}\text{if } e\in A^1-A^2\text{and}(x,y)\in(V^1\times V^1)-(V^2\times V^2)\text{or}\\ \text{if } e\in A^1-A^2\text{and}(x,y)\in(V^1\times V^1)\cap(V^2\times V^2)\text{or}\\ \text{if } e\in A^1\cap A^2\text{and}(x,y)\in(V^1\times V^1)-(V^2\times V^2).\end{cases}\\ w_{g_\alpha{}^2}(x,y)=\begin{cases}\text{if } e\in A^2-A^1\text{and}(x,y)\in(V^2\times V^2)-(V^1\times V^1)\text{or}\\ \text{if } e\in A^2-A^1\text{and}(x,y)\in(V^2\times V^2)\cap(V^1\times V^1)\text{or}\\ \text{if } e\in A^2\cap A^1\text{and}(x,y)\in(V^2\times V^2)-(V^1\times V^1).\end{cases}\\ min\{w_{g_\alpha{}^1}(x,y),w_{g_\alpha{}^2}(x,y)\}\{\text{if } e\in A^1\cap A^2\text{and}(x,y)\in(V^1\times V^1)\cap(V^2\times V^2)\}\\ 0,\quad \text{otherwise}\end{cases}$$

3.11 Example Suppose that ${G^1}^* = (V^1, E^1)$ be a simple graph with $V = \{x_1, x_3, x_5\}, Y = \{e_1\}$ be a set of parameters and $X = \{p\}$ be a set of experts. Let ${G^2}^* = (V^2, E^2)$ be a simple graph with $V = \{x_2, x_4, x_5\}, Y = \{e_1, e_2\}$ be a set of parameters and $X = \{p\}$ be a set of experts. ANSEG is given in Table 5below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V^1 \times V^1 \backslash \{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$ and for all $\alpha \in A^1$.

Table 5

f_1	x_1	x_3	x_5
$(e_1, p, 1)$	(0.5,0.6,0.7)	(0,0,1)	(0.3,0.4,0.6)
$(e_1, p, 0)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
g_1	(x_1, x_3)	(x_3, x_5)	(x_1, x_5)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.1,0.3,0.8)
$(e_1, p, 0)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$



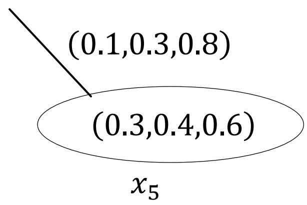


Figure 5

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

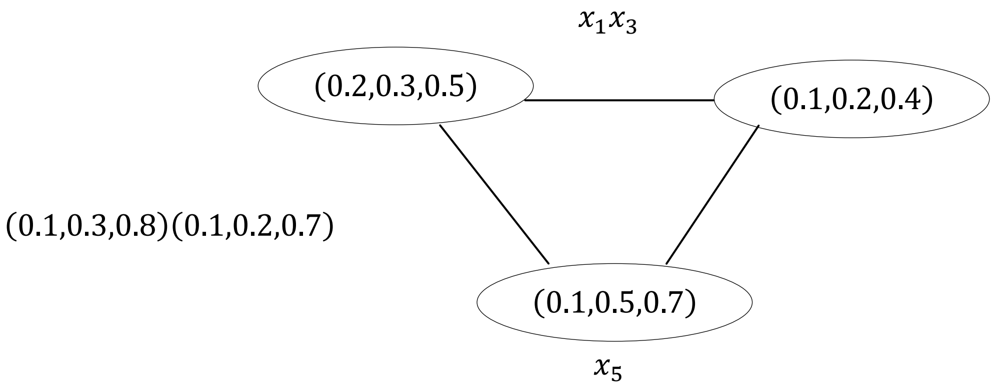


Figure 6

A NSEG $G^2 = (G^*, A^2, f^2, g^2)$ is given in Table 6 below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V^2 \times V^2 \backslash \{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$ and for all $\alpha \in A^2$.

Table 6

f_2	x_2	x_4	x_5
$(e_1, p, 1)$	(0,0,1)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_2, p, 1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_1, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, p, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
g_2	(x_2, x_4)	(x_4, x_5)	(x_2, x_5)
$(e_1, p, 1)$	(0,0,1)	(0.2,0.2,0.9)	(0,0,1)
$(e_2, p, 1)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_1, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, p, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

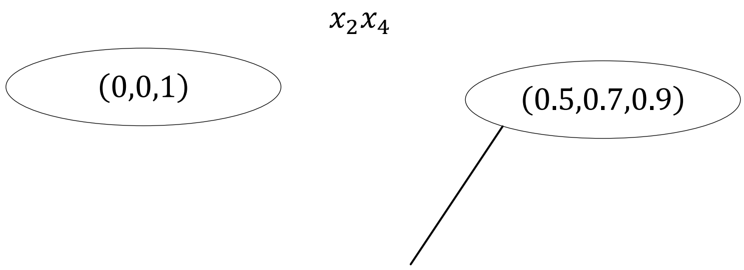


Figure 7

$N(e_2, p, 1)$ Corresponding to $(e_2, p, 1)$

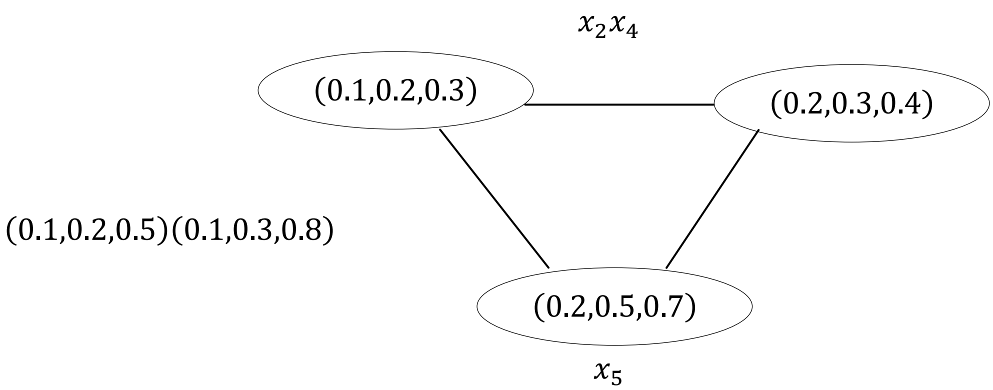


Figure 8

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

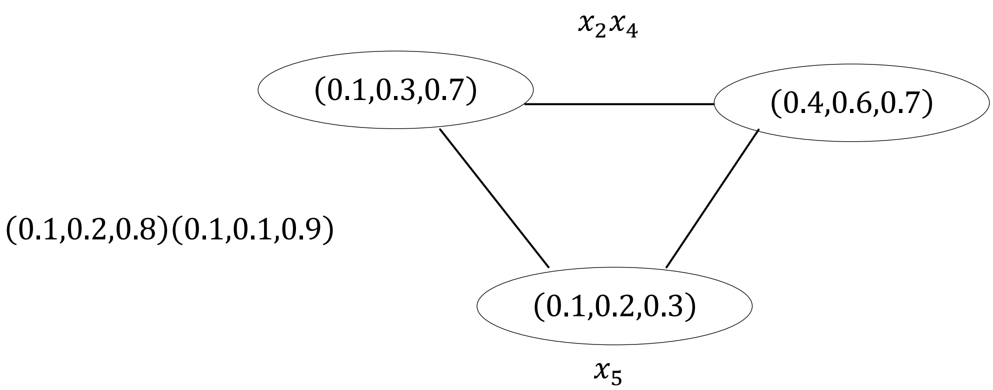


Figure 9

$N(e_2, p, 0)$ Corresponding to $(e_2, p, 0)$

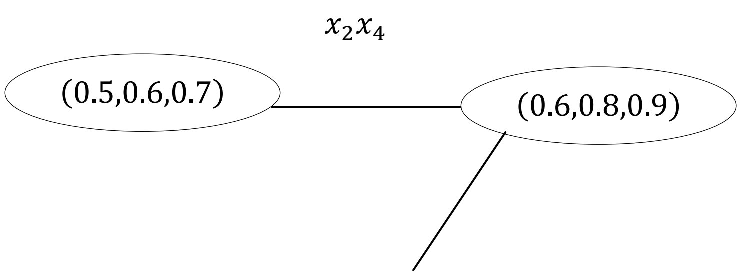


Figure 10

Table 7

f	x_1	x_2	x_3	x_4	x_5
$(e_1, p, 1)$	(0,0,1)	(0.5,0.7,0.9)	(0.3,0.4,0.5)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_2, p, 1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_1, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, p, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)	(0.6,0.8,0.9)	(0.3,0.4,0.6)

g	(x_1, x_3)	(x_3, x_5)	(x_1, x_5)	(x_2, x_4)	(x_4, x_5)	(x_2, x_5)
$(e_1, p, 1)$	(0,0,1)	(0.2,0.2,0.9)	(0,0,1)	(0,0,1)	(0.2,0.2,0.9)	(0,0,1)
$(e_2, p, 1)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_1, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, p, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

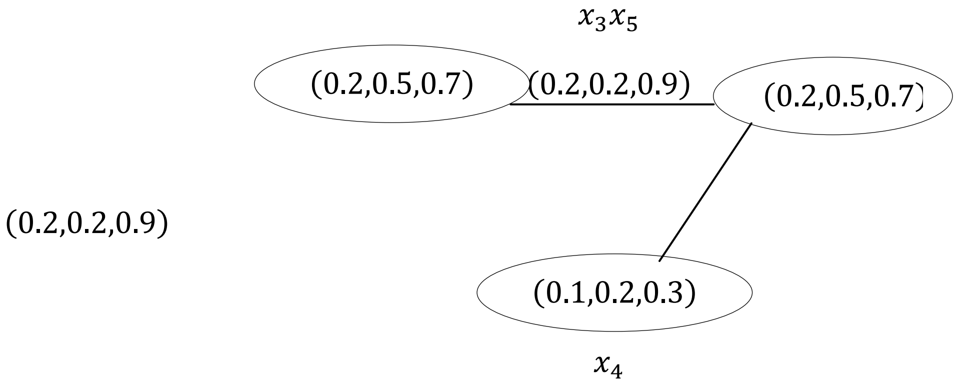
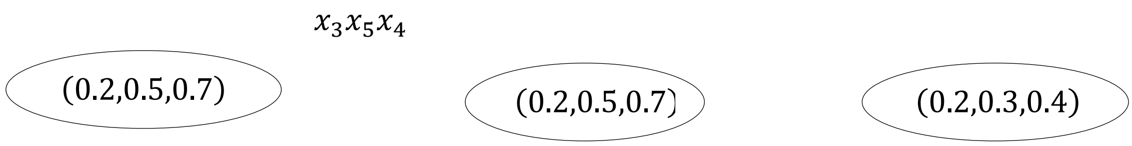


Figure 11

$N(e_2, p, 1)$ Corresponding to $(e_2, p, 1)$



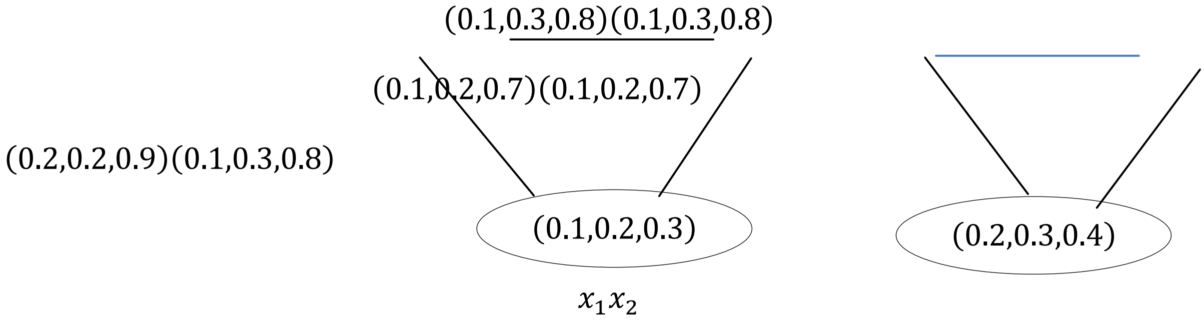


Figure 12

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

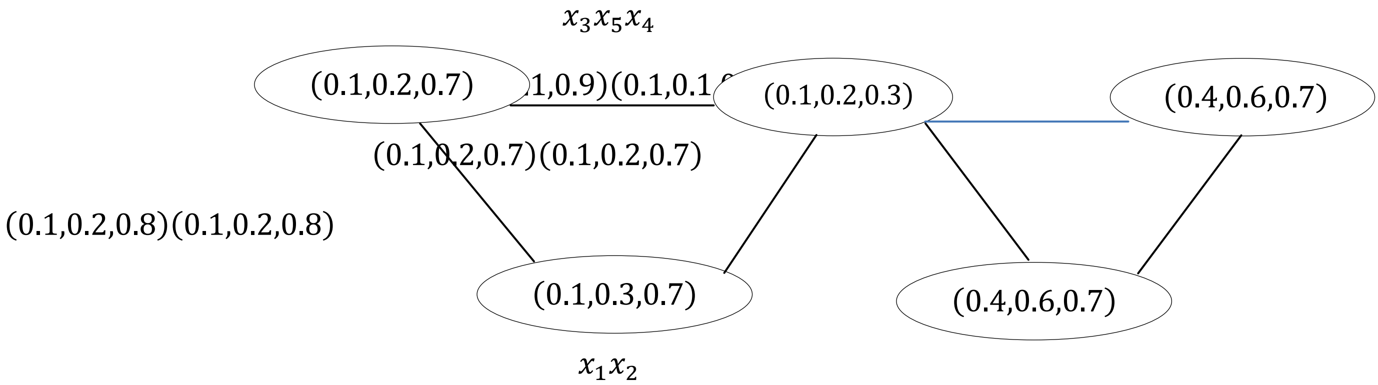


Figure 13

$N(e_2, p, 0)$ Corresponding to $(e_2, p, 0)$

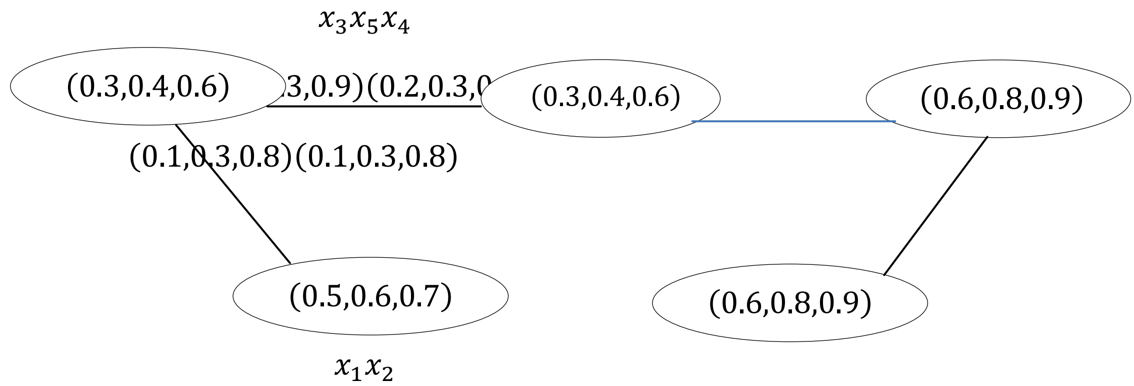


Figure 14

3.12Proposition The union $G = (G^*, A, f, g)$ of two neutrosophic soft expert graph $G^1 = (G^*, A^1, f^1, g^1)$ and $G^2 = (G^*, A^2, f^2, g^2)$ is a neutrosophic soft expert graph.

Proof i. if $e \in A^1 - A^2$and$(x, y) \in (V^1 \times V^1) - (V^2 \times V^2)$, then

$$\mu_{g_\alpha}(x,y) = \mu_{g_\alpha{}^1}(x,y) \le min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}$$

$$= min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}$$

So $\mu_{g_\alpha}(x,y) \le min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$

Also

$$\vartheta_{g_\alpha}(x,y) = \vartheta_{g_\alpha{}^1}(x,y) \le min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}$$

$$= min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}$$

So $\vartheta_{g_\alpha}(x,y) \le min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\}$

Now$w_{g_\alpha}(x,y) = w_{g_\alpha{}^1}(x,y) \ge max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}$

$$= max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}$$

So

$$w_{g_\alpha}(x,y) \ge max\{w_{f_\alpha}(x), w_{f_\alpha}(y)\}$$

Similarlyif$\{e \in A^1 - A^2$and$(x,y) \in (V^1 \times V^1) \cap (V^2 \times V^2)\}$, or

if$\{e \in A^1 \cap A^2$and$(x,y) \in (V^1 \times V^1) - (V^2 \times V^2)\}$, we can show the same as done above.

ii. if $e \in A^1 \cap A^2$and$(x,y) \in (V^1 \times V^1) \cap (V^2 \times V^2)$, then

$$\mu_{g_\alpha}(x,y) = max\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}$$

$$\le max\left\{min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}, min\left\{\mu_{f_\alpha{}^2}(x), \mu_{f_\alpha{}^2}(y)\right\}\right\}$$

$$\le min\left\{max\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\}, max\left\{\mu_{f_\alpha{}^1}(y), \mu_{f_\alpha{}^2}(y)\right\}\right\}$$

$$= min\left\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\right\}$$

Also

$$\vartheta_{g_\alpha}(x,y) = max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}$$

$$\le max\left\{min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}, min\left\{\vartheta_{f_\alpha{}^2}(x), \vartheta_{f_\alpha{}^2}(y)\right\}\right\}$$

$$\le min\left\{max\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^2}(x)\right\}, max\left\{\vartheta_{f_\alpha{}^1}(y), \vartheta_{f_\alpha{}^2}(y)\right\}\right\}$$

$$= min\left\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\right\}$$

Now

$$w_{g_\alpha}(x,y) = min\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}$$

$$\geq min\left\{max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}, max\left\{w_{f_\alpha{}^2}(x), w_{f_\alpha{}^2}(y)\right\}\right\}$$

$$\geq max\left\{min\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^2}(x)\right\}, min\left\{w_{f_\alpha{}^1}(y), w_{f_\alpha{}^2}(y)\right\}\right\}$$

$$= max\left\{w_{f_\alpha}(x), w_{f_\alpha}(y)\right\}$$

Hence the union $\mathrm{G} = G^1 \cup G^2$ is a neutrosophic soft expert graph.

3.13 Definition Theintersectionof two neutrosophic soft expert graphs $G^1 = \left(G^{1^*}, A^1, f^1, g^1\right)$ and $G^2 = \left(G^{2^*}, A^2, f^2, g^2\right)$ is denoted by $G = (G^*, A, f, g)$ with $A = A^1 \cap A^2, V = V^1 \cap V^2$and the truth-membership, indeterminacy-membership and falsity-membership ofintersection are as follows

$$\mu_{f_\alpha} = \begin{cases} \mu_{f_\alpha{}^1}(x) \text{if } e \in A^1 - A^2 \\ \mu_{f_\alpha{}^2}(x) \text{if } e \in A^2 - A^1 \\ min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\} \text{if } e \in A^1 \cap A^2 \end{cases}$$

$$\vartheta_{f_\alpha} = \begin{cases} \vartheta_{f_\alpha{}^1}(x) \text{if } e \in A^1 - A^2 \\ \vartheta_{f_\alpha{}^2}(x) \text{if } e \in A^2 - A^1 \\ min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^2}(x)\right\} \text{if } e \in A^1 \cap A^2 \end{cases}$$

$$w_{f_\alpha} = \begin{cases} w_{f_\alpha{}^1}(x) \text{if } e \in A^1 - A^2 \\ w_{f_\alpha{}^2}(x) \text{if } e \in A^2 - A^1 \\ max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^2}(x)\right\} \text{if } e \in A^1 \cap A^2 \end{cases}$$

$$\mu_{g_\alpha} = \begin{cases} \mu_{g_\alpha{}^1}(x,y) \text{if } e \in A^1 - A^2 \\ \mu_{g_\alpha{}^2}(x,y) \text{if } e \in A^2 - A^1 \\ min\{\mu_{g_\alpha{}^1}(x,y), \mu_{g_\alpha{}^2}(x,y)\} \text{if } e \in A^1 \cap A^2 \end{cases}$$

$$\vartheta_{g_\alpha} = \begin{cases} \vartheta_{g_\alpha{}^1}(x,y) \text{if } e \in A^1 - A^2 \\ \vartheta_{g_\alpha{}^2}(x,y) \text{if } e \in A^2 - A^1 \\ min\{\vartheta_{g_\alpha{}^1}(x,y), \vartheta_{g_\alpha{}^2}(x,y)\} \text{if } e \in A^1 \cap A^2 \end{cases}$$

$$w_{g_\alpha} = \begin{cases} w_{g_\alpha{}^1}(x,y) \text{if } e \in A^1 - A^2 \\ w_{g_\alpha{}^2}(x,y) \text{if } e \in A^2 - A^1 \\ max\{w_{g_\alpha{}^1}(x,y), w_{g_\alpha{}^2}(x,y)\} \text{if } e \in A^1 \cap A^2 \end{cases}$$

3.14 Example Suppose that $G^{1^*} = (V^1, E^1)$ be a simple graph with $V^1 = \{x_2, x_3, x_5\}, Y = \{e_1\}$ be a set of parameters and $X = \{p\}$ be a set of experts. A NSEG is given in Table 8 below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V^1 \times V^1 \backslash \{(x_2, x_3), (x_3, x_5), (x_2, x_5)\}$ and for all $\alpha \in A^1$.

Table 8

f_1	x_2	x_3	x_5
$(e_1, p, 1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_1, p, 0)$	(0.1,0.3,0.7)	(0.2,0.4,0.4)	(0.4,0.6,0.7)
g_1	(x_2, x_3)	(x_3, x_5)	(x_2, x_5)
$(e_1, p, 1)$	(0.1,0,2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.8)
$(e_1, p, 0)$	(0.1,0.2,0.8)	(0.2,0.3,0.9)	(0,0,1)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

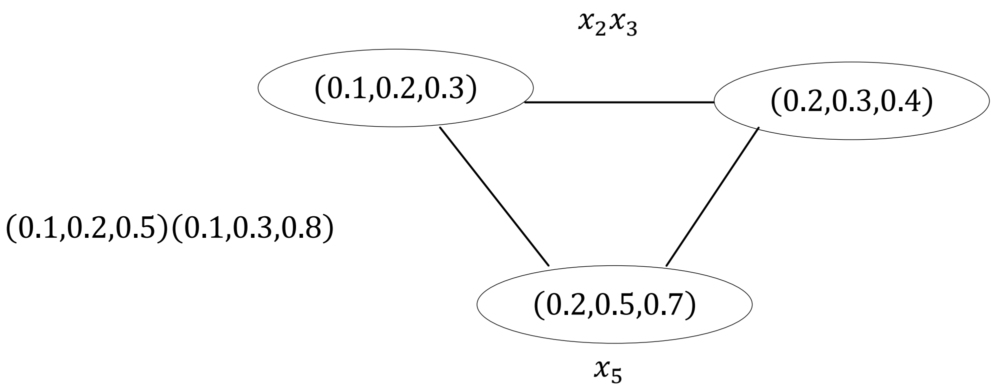


Figure 15

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

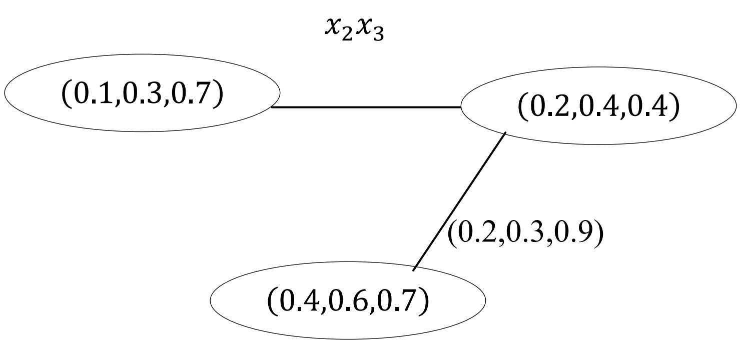


Figure 16

Let $G^{2^*} = (V^2, E^2)$ be a simple graph with $V^2 = \{x_1, x_3, x_5\}, Y = \{e_1\}$ be a set of parameters and $X = \{p\}$ be a set of experts. A NSEG is given in Table 8 below and $\mu_{g_\alpha}(x_i, x_j) =$

$0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V^2 \times V^2 \backslash \{(x_1, x_3), (x_3, x_5), (x_1, x_5)\}$ and for all $\alpha \in A^2$.

Table 9

f_2	x_1	x_3	x_5
$(e_1, p, 1)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
g_2	(x_1, x_3)	(x_3, x_5)	(x_1, x_5)
$(e_1, p, 1)$	(0.1,0,2,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

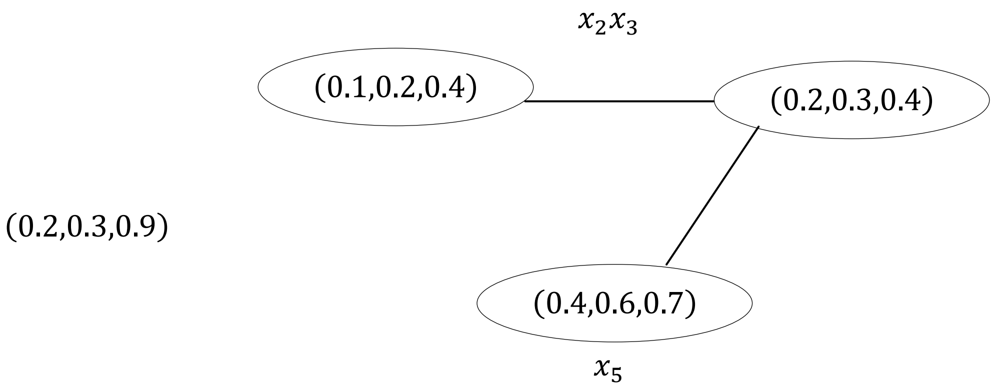


Figure 17

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

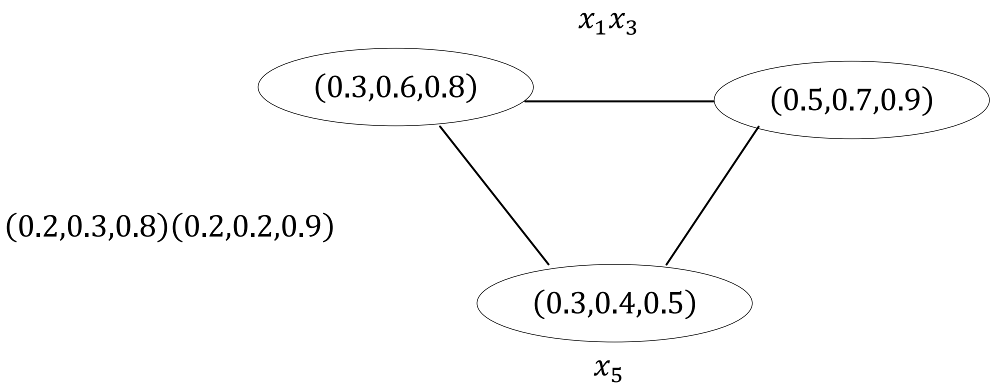


Figure 18

Let $V = V^1 \cap V^2 = \{x_3, x_5\}, A = A^1 \cap A^2 = \{(e_1, p, 1), (e_1, p, 0)\}$.The intersection of two neutrosophic soft expert graphs $G^1 = (G^{1^*}, A^1, f^1, g^1)$ and $G^2 = (G^{2^*}, A^2, f^2, g^2)$ is given in Table 10.

Table 10

f	x_2	x_5	g	(x_2, x_5)
$(e_1, p, 1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	$(e_1, p, 1)$	(0.1,0.2,0.8)
$(e_1, p, 0)$	(0.1,0.3,0.7)	(0.2,0.4,0.4)	$(e_1, p, 0)$	(0,0,1)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

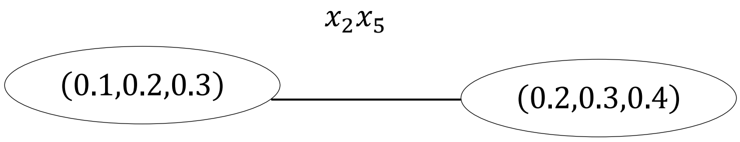


Figure 19

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

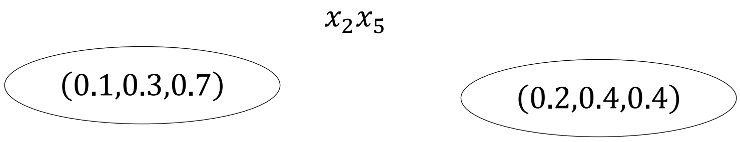


Figure 20

3.15Proposition The intersection $G = (G^*, A, f, g)$ of two neutrosophic soft expert graph $G^1 = (G^*, A^1, f^1, g^1)$ and $G^2 = (G^*, A^2, f^2, g^2)$ is a neutrosophic soft expert graph where $A = A^1 \cap A^2, V = V^1 \cap V^2$.

Proof i. if $e \in A^1 - A^2$, then

$$\mu_{g_\alpha}(x, y) = \mu_{g_\alpha{}^1}(x, y) \leq min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}$$

$$= min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$$

So $\mu_{g_\alpha}(x, y) \leq min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$

Also

$$\vartheta_{g_\alpha}(x, y) = \vartheta_{g_\alpha{}^1}(x, y) \leq min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}$$

$$= min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\}$$

So $\vartheta_{g_\alpha}(x, y) \leq min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\}$

now

$$w_{g_\alpha}(x,y) = w_{g_\alpha{}^1}(x,y) \geq max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}$$

$$= max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}$$

so

$$w_{g_\alpha}(x,y) \geq max\left\{w_{f_\alpha}(x), w_{f_\alpha}(y)\right\}$$

similarlyif$\{e \in A^1 - A^2\}$ we can show the same as done above.

ii. if $e \in A^1 \cap A^2$, then $\mu_{g_\alpha}(x,y) = max\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}$

$$\leq min\left\{min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^1}(y)\right\}, min\left\{\mu_{f_\alpha{}^2}(x), \mu_{f_\alpha{}^2}(y)\right\}\right\}$$

$$\leq min\left\{min\left\{\mu_{f_\alpha{}^1}(x), \mu_{f_\alpha{}^2}(x)\right\}, min\left\{\mu_{f_\alpha{}^1}(y), \mu_{f_\alpha{}^2}(y)\right\}\right\}$$

$$= min\left\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\right\}$$

Also

$\vartheta_{g_\alpha}(x,y) = min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}$

$$\leq min\left\{min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^1}(y)\right\}, min\left\{\vartheta_{f_\alpha{}^2}(x), \vartheta_{f_\alpha{}^2}(y)\right\}\right\}$$

$$\leq min\left\{min\left\{\vartheta_{f_\alpha{}^1}(x), \vartheta_{f_\alpha{}^2}(x)\right\}, min\left\{\vartheta_{f_\alpha{}^1}(y), \vartheta_{f_\alpha{}^2}(y)\right\}\right\}$$

$$= min\left\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\right\}$$

Now

$w_{g_\alpha}(x,y) = max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}$

$$\geq max\left\{max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^1}(y)\right\}, max\left\{w_{f_\alpha{}^2}(x), w_{f_\alpha{}^2}(y)\right\}\right\}$$

$$\geq max\left\{max\left\{w_{f_\alpha{}^1}(x), w_{f_\alpha{}^2}(x)\right\}, max\left\{w_{f_\alpha{}^1}(y), w_{f_\alpha{}^2}(y)\right\}\right\}$$

$$= max\left\{w_{f_\alpha}(x), w_{f_\alpha}(y)\right\}$$

Hence the intersectionG $= G^1 \cap G^2$ is a neutrosophic soft expert graph.

4. Strong Neutrosophic Soft Expert Graph

4.1 Definition A neutrosophic soft expert graph $G = (G^*, A, f, g)$ is called strong if $g_\alpha(x, y) = f_\alpha(x) \cap f_\alpha(y)$, for all $x, y \in V, \alpha \in A$. That is if

$$\mu_{g_\alpha}(x, y) = min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\},$$

$$\vartheta_{g_\alpha}(x, y) = min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\},$$

$$w_{g_\alpha}(x, y) = max\{w_{f_\alpha}(x), w_{f_\alpha}(y)\},$$

for all $(x, y) \in E$.

4.2 Example Suppose that $G^* = (V, E)$ be a simple graph with $V = \{x_1, x_2, x_3\}, Y = \{e_1\}$ be a set of parameters and $X = \{p\}$ be a set of experts. A NSEG is given in Table 11 below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V \times V\setminus$

$\{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$and for all $\alpha \in A$.

Table 11

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.6)	(0.2,0.4,0.6)	(0.4,0.5,0.9)
$(e_1, p, 0)$	(0.2,0.4,0.5)	(0.1,0.2,0.6)	(0.1,0.5,0.7)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0.1,0,3,0.7)	(0.2,0.4,0.9)	(0.2,0.4,0.9)
$(e_1, p, 0)$	(0.1,0.2,0.8)	(0.1,0.2,0.9)	(0.1,0.4,0.8)

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

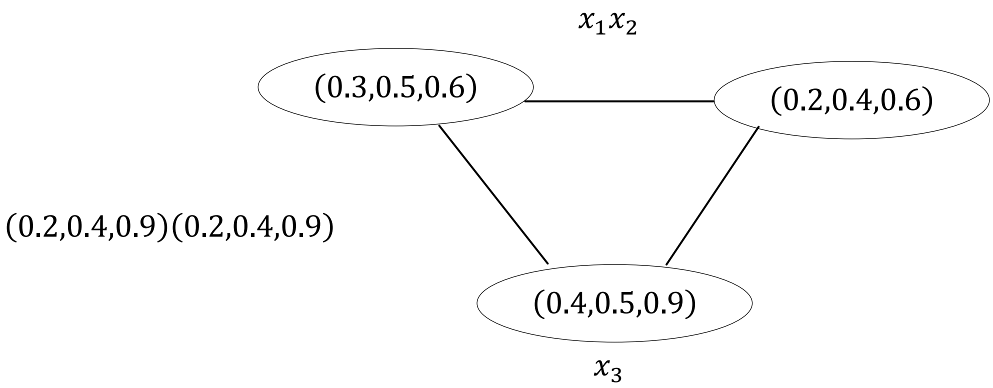
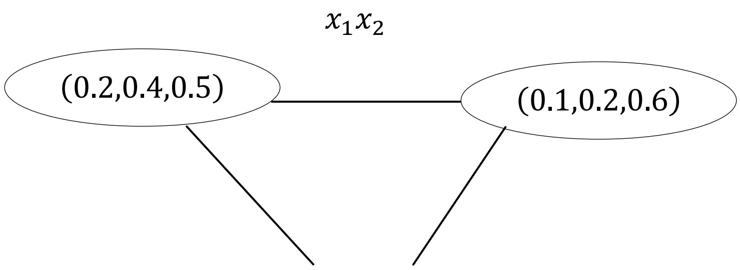
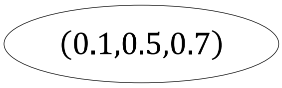


Figure 21

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$



(0.1,0.4,0.8)(0.1,0.2,0.9)



x_3

Figure 22

4.3 Definition Let$G = (G^*, A, f, g)$ be a strong neutrosophic soft expert graph that is $g_\alpha(x, y) = f_\alpha(x) \cap f_\alpha(y)$, for all $x, y \in V, \alpha \in A$. The complement $\bar{G} = \left(\bar{G}^*, \bar{A}, \bar{f}, \bar{g}\right)$ of strong neutrosophic soft expert graph $G = (G^*, A, f, g)$ is neutrosophic soft expert expert graph where

i.$\bar{A} = A$

ii. $\mu_{f_\alpha}(x) = \overline{\mu_{f_\alpha}}(x), \vartheta_{f_\alpha}(x) = \overline{\vartheta_{f_\alpha}}(x),\ w_{f_\alpha}(x) = \overline{w_{f_\alpha}}(x)$ for all $x \in V$

iii. $\overline{\mu_{f_\alpha}}(x) = \begin{cases} min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\} \text{if} \mu_{g_\alpha}(x, y) = 0 \\ 0, \quad \text{otherwise} \end{cases}$

$$\overline{\vartheta_{f_\alpha}}(x) = \begin{cases} min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\} \text{if} \vartheta_{g_\alpha}(x, y) = 0 \\ 0, \quad \text{otherwise} \end{cases}$$

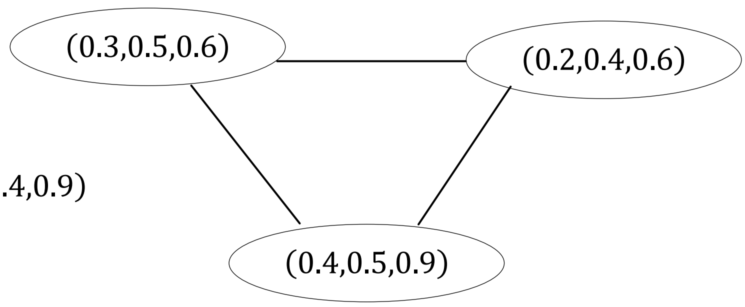
$\overline{w_{f_\alpha}}(x) = \begin{cases} min\{w_{f_\alpha}(x), w_{f_\alpha}(y)\} \text{if} w_{g_\alpha}(x, y) = 0 \\ 0, \quad \text{otherwise} \end{cases}$

4.4 Example Forthe strong neutrosophic soft graph in previous example, the complements are given below for $(e_1, p, 1)$ and $(e_1, p, 0)$

Corresponding to$(e_1, p, 1)$, the complement of

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

$x_1 x_2$



(0.2,0.4,0.9)(0.2,0.4,0.9)

x_3

Figure 23

is given by

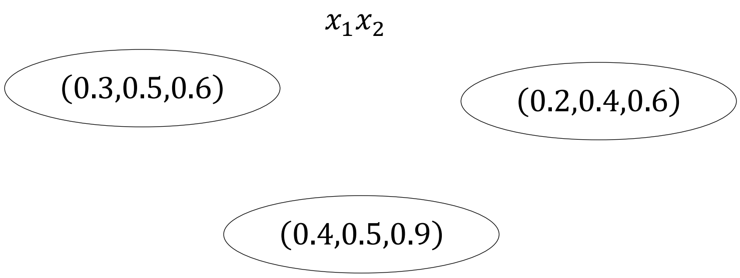


Figure 24

Corresponding to$(e_1, p, 1)$, the complement of

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

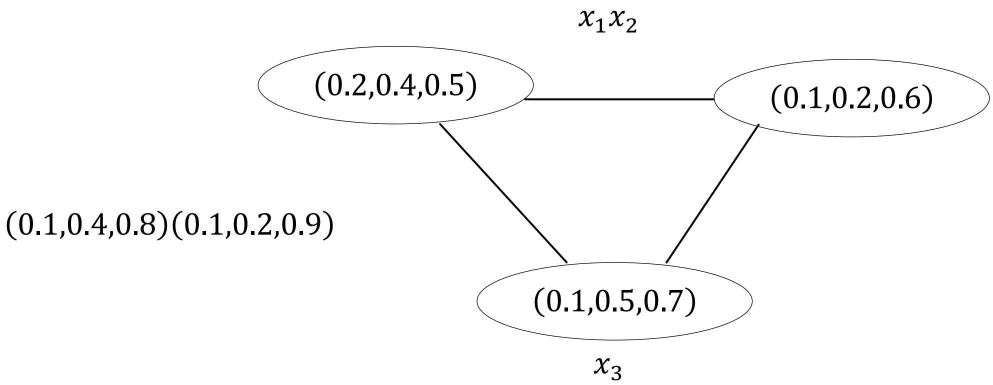


Figure 25

is given by

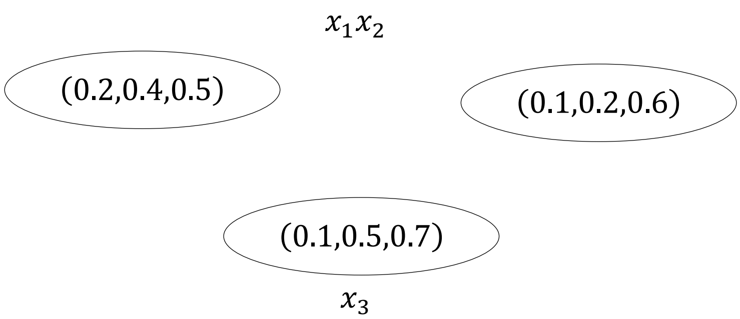


Figure 26

5. Applications of Neutrosophic Soft Expert Graph

In what follows, let us consider an illustrative example adopted from Adam et al. [1] and Shahzadi et al. [33].

5.1 Application in decision-making problem

Assume that a hospital wants to fill a position to be chosen by an expert committee. Suppose that $G^* = (V, E)$ be a simple graph with $V = \{x_1, x_2, x_3\}, Y = \{e_1, e_2\}$ be a set of parameters computer knowledge and language fluency respectively. Let $X = \{p, q\}$ be a set of two expert committee members. A NSEG is given in Table 12 below and $\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$ and $w_{g_\alpha}(x_i, x_j) = 1$, for all $(x_i, x_j) \in V \times V \backslash \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all $\alpha \in A$.After a serious deliberation the committee constructs the following neutrosophicsoft expert graph.

Table 12

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1, q, 1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_2, p, 1)$	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
$(e_2, q, 1)$	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_2, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, q, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

The following algorithm may be followed by thehospital to fill the position.

1. Input the NSEG.
2. Find the mean of each neutrosophic soft expert edges according to the relationship amongcriteria for each alternative.
3. Find an agree-NSEGand a disagree-NSEG.
4. Find $C_j = \sum_i x_{ij}$ for agree-NSEG.
5. Find $K_j = \sum_i x_{ij}$ for disagree-NSEG.
6. Find $S_j = C_j - K_j$.

7. Find r, for which $s_r = maxs_j$, where,s_r is the optimal choice object. If r has more than one value, then any one of them could be chosen by the hospital using its option.

1- Neutrosophic soft expert edges according to the relationship among criteria for each alternative.

Table 13

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

2-Tables 13 present the agree-NSEGby using the mean of each NSEG.

Table 14: Tabular presentation of theagree-NSEG

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	0,333	0,333	0,433
$(e_1, q, 1)$	0,366	0,333	0,4
$(e_2, p, 1)$	0,366	0,333	0,4
$(e_2, q, 1)$	0,366	0,333	0,333

3-Tables 15 present the disagree-NSEGrespectively by using the mean of each NSEG.

Table 15: Tabular presentation of thedisagree-NSEG

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	0,266	0,333	0,3
$(e_1, q, 0)$	0,333	0,4	0,266
$(e_2, p, 0)$	0,333	0,366	0,333
$(e_2, q, 0)$	0,4	0,466	0,333

4-$C_j = \sum_i x_{ij}$ for agree-NSEG

Table 16

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	0,333	0,333	0,433

$(e_1, q, 1)$	0,366	0,333	0,4
$(e_2, p, 1)$	0,366	0,333	0,4
$(e_2, q, 1)$	0,366	0,333	0,333
$C_j = \sum_i x_{ij}$	1,431	1,332	1,566

5- $K_j = \sum_i x_{ij}$ for disagree-NSEG

Table 17

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	0,266	0,333	0,3
$(e_1, q, 0)$	0,333	0,4	0,266
$(e_2, p, 0)$	0,333	0,366	0,333
$(e_2, q, 0)$	0,4	0,466	0,333
$K_j = \sum_i x_{ij}$	1,332	1,565	1,232

6- From Tables 16 and 17 we are able to compute the values of $S_j = C_j - K_j$as in Table 18.

Table 18:$S_j = C_j - K_j$

j	X	C_j	K_j	S_j
1	x_1	1,431	1,332	0,099
2	x_2	1,332	1,565	−0,233
3	x_3	1,566	1,232	0,334

7-Since max$S_j = 0{,}334$, hence the committee will choose candidate x_3 with a masters degree for the job.

5.2 Application in communication network

A communication network model is used in an organization to manage, regulate information flows through proper channels. These networks form a pattern of person-to-person relationship by which information flows in an organization. In an organization, information is communicated through proper channels. We use graph to represent the communication networks. We consider a company in which company members share a common purpose to achieve specific goals. We can find the most useful channel for a company employee by considering a set of attributes or channels $M = \{e_1 = \text{electronic}, e_2 = \text{print}\}$. Consider the graph G∗ with vertex set V = $\{x_1$ =managing director(M.D), x_2 =marketing manager(M.M), x_3 =operation manager, x_4 =accountant, x_5 =sale staff$\}$ as shown in Table 7. The vertices represent company employees and edges represent any kind of communication relationship between them, if there is no edge between any two employees it means that there is no communication between them (Figs. 27, 28, 29, 30).

An NSEG $G = \{N(e_1, p, 1), N(e_2, p, 1), N(e_1, p, 0), N(e_2, p, 0)\}$ of G^* corresponding to the attributes electronic and print is represented in Table 7.

NSEG$N(e_1, p, 1)$w.r.t electroniccommunication

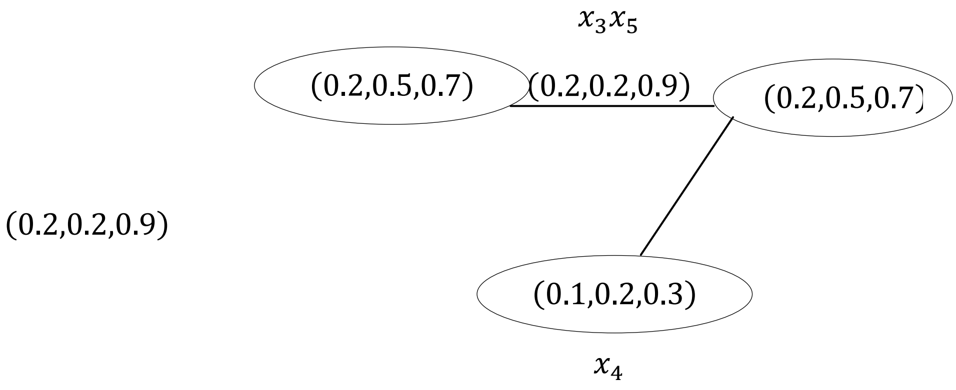


Figure 27

NSEG$N(e_2, p, 1)$w.r.t printcommunication

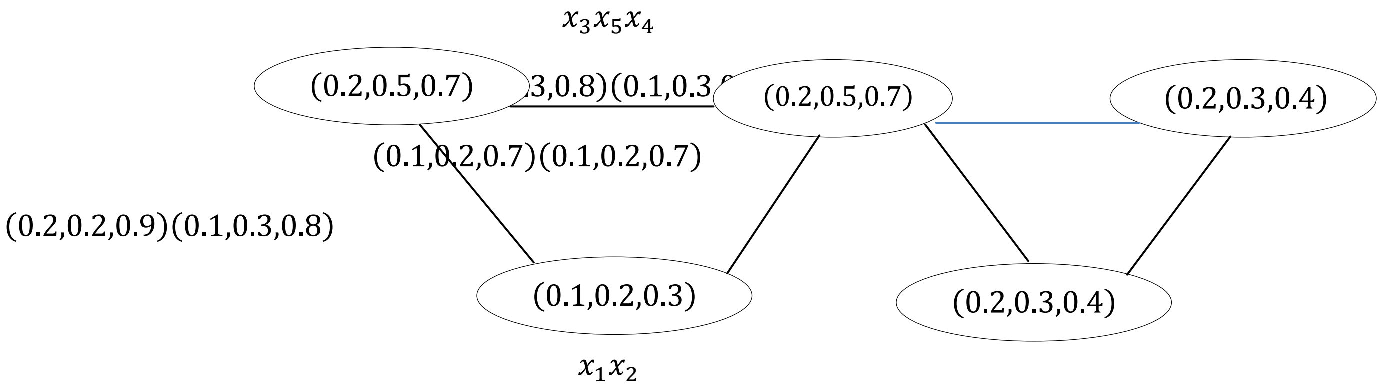


Figure 28

NSEG$N(e_1, p, 0)$w.r.t electroniccommunication

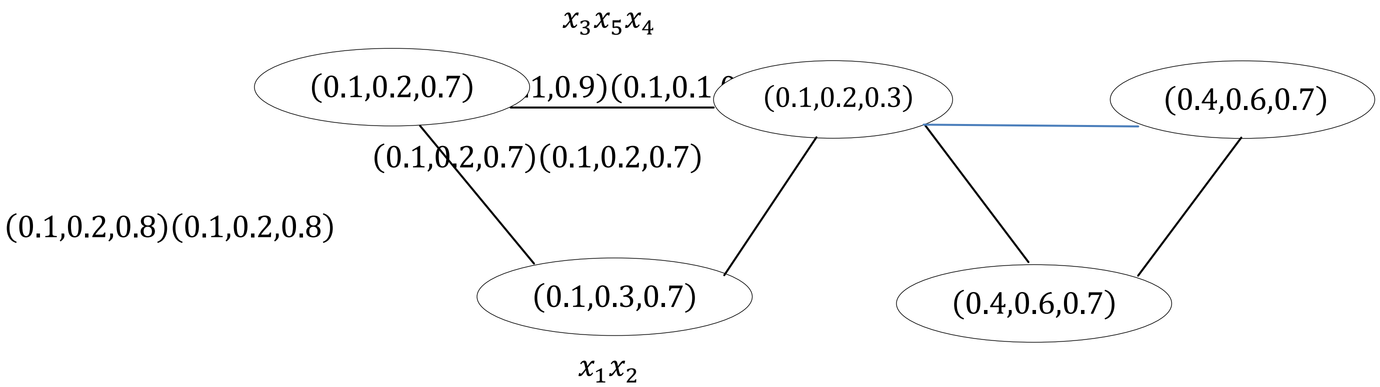


Figure 29

NSEG$N(e_2, p, 0)$w.r.t printcommunication

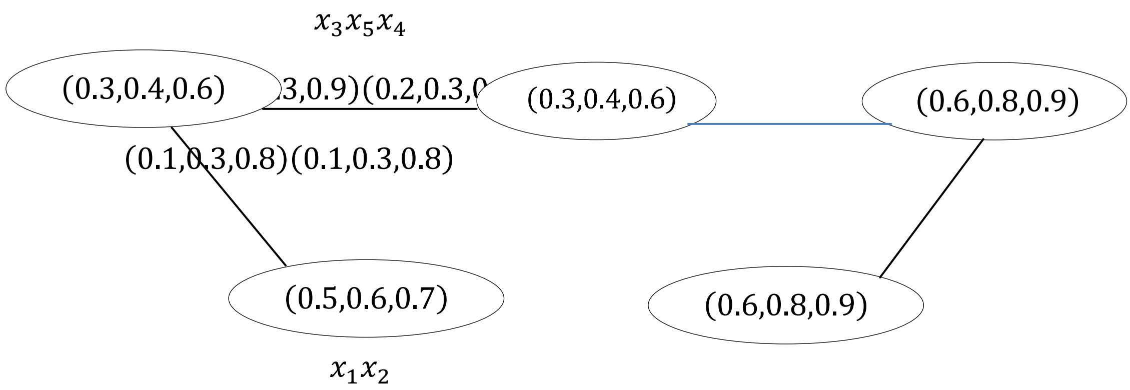


Figure 30

In the view of above NSEGs $N(e_1, p, 1), N(e_2, p, 1), N(e_1, p, 0)$and $N(e_2, p, 0)$, we can see that the precise evaluation for each employee on each attributes is unknown while the lower and the upper limits for best communication device are given.

The neutrosophic soft expert graph, as a concept generalized of neutrosophic graph, fuzzy graph and intuitionistic fuzzy graph, provides additional capability to deal with uncertainty, inconsistent, incomplete and imprecise information by including a truth-membership, an indeterminacy-membership and a falsity membership with expert. Therefore, it plays a significant role in the network systems.Neutrosophic soft expert graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Neutrosophic models are becoming useful because of their aim of reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems.

6. Comparison Analysis

In order to verify the feasibility and effectiveness of the proposed decision-making approach, a comparison analysis with interval valued neutrosophic decision method, used by Broumi et al. [16],Akram et al.'s method [2] and Shahzadi et al.'s method [33],are given, based on the same illustrative example.

Clearly, the ranking order results are consistent with the result obtained in [16]; however, the best alternative is the same as x_3, because the ranking principle is different, these four methods produced the same best alternatives.

Neutrosophic soft set is a generalization of the notion of fuzzy soft sets and intuitionistic fuzzy soft sets. Fuzzy soft graph theory is soft computing models in combination to study vagueness and uncertainty in graphs. Neutrosophic soft graphs are pictorial representation in which each vertex and each edge is an element of neutrosophic soft sets. Neutrosophic soft expert models give more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy and/or intuitionistic fuzzy models. Neutrosophic soft expert models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems.

Table 19.Comparison of fuzzysoft setanditsextensiveset theory

Methods	Fuzzy soft	intuitionistic fuzzy soft	İnterval- Valued neutrosophic	Neutrosophic Soft	Neutrosophic soft expert
Methods		Shahzadi et al.'s method [33]	Broumi et al.'s method [16]	Akram et al.'s method [2]	Proposed Method
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Co-domain	Single-value in in [0,1]	Two-value in [0,1]	Unipolarintervalin [0,1]	$[0,1]^3$	$[0,1]^3$
Parameter	Yes	Yes	No	Yes	Yes
Uncertainty	Yes	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes	Yes
Falsity	No	Yes	Yes	Yes	Yes
Indeterminacy	No	No	Yes	Yes	Yes
Expert	No	No	No	No	Yes
Edge	Yes	Yes	Yes	Yes	Yes
Vertex	No	No	No	Yes	Yes
Ranking	-	$x_3 > x_2 > x_1$	$x_3 > x_1 > x_2$	$x_3 > x_2 > x_1$	$x_3 > x_1 > x_2$

So, we think the proposed method developed in this paper is more suitable to handle this application example.

7.Conclusion

In this paper, we have introduced the concept of neutrosophic soft expert graph, strong neutrosophic soft expert graph, union and intersection of them has been explained with example which has wider application in the field of modern sciences and technology, especially in research areas of computer science including database theory, datamining, neural networks, expert systems, cluster analysis, control theory, andimage capturing. Using this concept we can extend our work in (1) Interval-valued neutrosophic soft expert graphs; (2) Bipolarneutrosophic soft expert regular graphs.

References

[1] Adam F. and N. Hassan, (2016) Multi Q-Fuzzy soft expert set and its applications, Journal of Intelligent and Fuzzy Systems 30(2), 943–950.

[2] Akram, M., &Shahzadi, S. Neutrosophic soft graphs with application. *Journal of Intelligent & Fuzzy Systems*, (Preprint), 1-18.

[3] Alkhazaleh, S. and A. R. Salleh, Soft expert sets, Advances in Decision Sciences (2011), Article ID 757868.1-12, http://dx.doi.org/10.1155/2011/757868

[4] Alkhazaleh, S. and Salleh, A.R. (2014) Fuzzy Soft Expert Set and Its Application. Applied Mathematics,5, 1349-1368. http://dx.doi.org/10.4236/am. 2014.59127
[5] Ayman A. Hazaymeh (2013). Fuzzy soft set and fuzzy soft expert set: some generalizations and hypothetical applications. PhD Thesis, Faculty of Science and Technology, UniversitiSains Islam Malaysia, Nilai, Negeri Sembilan.
[6] Alhazaymeh K. and N. Hassan, (2014) Mapping on generalized vague soft expert set, International Journal of Pure and Applied Mathematics 93(3), 369–376.
[7] Al-Quran A. and N. Hassan, (2016) Neutrosophic vague soft expert set theory, Journal of Intelligent and Fuzzy Systems 30(6), 3691–3702.
[8] Atanassov K. T., (1986)Intuitionistic Fuzzy Set. Fuzzy Sets and Systems, 20(1), 87-86.
[9] Broumi S. and F. Smarandache, (2013) Intuitionistic Neutrosophic Soft Set, Journal of Information and Computing Science, 8(2) (2013) 130-140.
[10] Broumi, S., Smarandache, F., Talea, M., &Bakali, A. (2016, February). An introduction to bipolar single valued neutrosophic graph theory. In *OPTIROB conference*.
[11]Broumi, S., Bakali, A., Talea, M., &Smarandache, F. (2016). Isolated Single Valued Neutrosophic Graphs. *Neutrosophic Sets and Systems*, 11,74-77.
[12] Broumi, S., Talea, M., Bakali, A., &Smarandache, F. (2016). On bipolar single valued neutrosophic graphs. *Journal of New Theory*, *11*, 84-102.
[13]Broumi, S., Talea, M., Bakali, A., &Smarandache, F. (2016). Interval valued neutrosophic graphs. *A Publication of Society for Mathematics of Uncertainty*, *10*, 1-35.
[14] Broumi, S., Talea, M., Smarandache, F., &Bakali, A. (2016). Single valued neutrosophic graphs: degree, order and size. In *IEEE world congress on computational intelligence*.
[15] Broumi, S., Talea, M., Bakali, A., &Smarandache, F. (2016). Single valued neutrosophic graphs. *Journal of New theory*, *10*, 86-101.
[16]Broumi, S., Talea, M., Bakali, A., &Smarandache, F. (2016). Decision-Making Method based on the Interval Valued Neutrosophic Graph. *FTC 2016 - Future Technologies Conference 2016 6-7 December 2016 | San Francisco, United States*
[17]Broumi S., M. Talea, A. Bakali and F. Smarandache, ON Strong Interval Valued Neutrosophic graphs, Critical Review, Vol. XII, 2016, pp. 49-71
[18] Basha, S. S., &Kartheek, E. (2015). Laplacian Energy of an Intuitionistic Fuzzy Graph. *Indian Journal of Science and Technology*, *8*(33).
[19] Deepa, G., Praba, B., &Chandrasekaran, V. M. (2016). A Study on Energy of an Intuitionistic Fuzzy Directed Graph. *Research Journal of Pharmacy and Technology*, *9*(2), 190-195.
[20] Deepa, G., Praba, B., &Chandrasekaran, V. M. (2015). Max-Min Intuitionistic Fuzzy matrix of an Intuitionistic Fuzzy Graph. *International Journal of Pure and Applied Mathematics*, *98*(3), 375-387.
[21] Gani, A. N., &Ahamed, M. B. (2003). Order and size in fuzzy graph. *Bulletin of Pure and Applied Sciences E*, *22*, 145-148.
[22] Gani, A. N., & Begum, S. S. (2010). Degree, order and size in intuitionistic fuzzy graphs.International journal of algorithms, Computing and Mathematics, 3(3), 11-16.
[23] Hassan N. and K. Alhazaymeh, (2013) Vague soft expert set theory, AIP Conf. Proc. 1522, 953–958.

[24]Mohinta, S., &Samanta, T. K. (2015). An Introduction to Fuzzy Soft Graph. *Mathematica Moravica*, *19*(2), 35-48.
[25]Muthuraj, R., &Sasireka, A. (2016). Fuzzy Dominator Coloring and Fuzzy Chromatic Number on Cartesian Product of Simple Fuzzy Graph. *Advances in Theoretical and Applied Mathematics*, *11*(3), 245-260.
[26]Mathew, S., &Sunitha, M. S. (2009). Types of arcs in a fuzzy graph. *Information Sciences*, *179*(11), 1760-1768.
[27]Mathew, S., &Sunitha, M. S. (2010). Node connectivity and arc connectivity of a fuzzy graph. *Information Sciences*, *180*(4), 519-531.
[28] Maji P.K., (2013) Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, 5/1, 157-168.
[29] Maji P. K., A. R. Roy and R. Biswas (2003) Soft set theory, Computers and Mathematics with Applications, 45(4-5): 555 – 562.
[30] Mathew, J. K., & Mathew, S. (2016). On strong extremal problems in fuzzy graphs. *Journal of Intelligent & Fuzzy Systems*, (Preprint)30, 2497–2503.
[31] Molodtsov D.A. (1999) Soft set theory-first results, Computers and Mathematics with Applications, 37, 19-31.
[32]Rashmanlou, H., Samanta, S., Pal, M., &Borzooei, R. A. (2016). A study on vague graphs. *SpringerPlus*, *5*(1), 1234.
[33] Shahzadi, S., &Akram, M. (2016). Intuitionistic fuzzy soft graphs with applications. *Journal of Applied Mathematics and Computing*, 1-24.
[34] Shah, N. (2016). Some Studies in Neutrosophic Graphs. *Neutrosophic Sets and Systems*,12, 54-64.
[35] Shah, N., & Hussain, A. (2016). Neutrosophic Soft Graphs. *Neutrosophic Sets and Systems*, *11*,31-43
[36] Sahin, M., S. Alkhazaleh and V. Ulucay, (2015) Neutrosophic soft expert sets, Applied Mathematics 6(1), 116–127.
[37] Sahoo, S., & Pal, M. (2015). Intuitionistic fuzzy competition graphs. *Journal of Applied Mathematics and Computing*, 1-21. doi:10.1007/s12190-015-0928-0
[38] Smarandache F., (2005) Neutrosophic set – A generalization of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics 24(3), 287–297.
[39] Smarandache F., (1998). A Unifying Field in Logics.Neutrosophy: NeutrosophicProbability,Set and Logic, Rehoboth: American Research Press.
[40] Singh, P. K. Three-way fuzzy concept lattice representation using neutrosophic set. *International Journal of Machine Learning and Cybernetics*, 1-11.DOI 10.1007/s13042-016-0585-0
[41] W. B. VasanthaKandasamy. and F. Smarandache ,Fuzzy Cognitive Maps and NeutrosophicCongtive Maps, Xiquan, Phoenix, AZ, USA, 2013, 213 p
[42] W. B. VasanthaKandasamy, K. Ilanthenral and F. Smarandache, Neutrosophic graphs: A New Dimension to Graph Theory. Kindle Edition.USA, 2015, 127p
[43] W.B. VasanthaKandasamy and F. Smarandache Analysis of social aspects of migrant laborers living with HIV/AIDS using Fuzzy Theory and Neutrosophic Cognitive Maps.Xiquan, Phoenix, 2004.
[44] Zadeh L. A., (1965) Fuzzy Sets, Inform. and Control, 8 : 338-353.