Continuous Two-Electron Theory of Electromagnetism

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ABSTRACT

This research programme continues with its fundamental analysis of the electromagnetic interaction. In contrast to the **continuous** charge model of electricity that is today used as the foundation for presentations of classical electromagnetism (CEM), this paper now analyzes the continuous interaction of pairs of charged **point** particles that better reflects the known basis of electricity – **electrons**. This analysis first demonstrates that all ***continuous*** theories of interaction between **point particles** that exhibit inertial resistance to changes in their motion are *inconsistent* with all asynchronous action-at-a-distance forms of interaction or equivalently, interactions limited to points on their 'light-cone'. This research programme is an extension of the Newtonian scheme of classical mechanics that represents the locations of point particles, not by standard algebraic vectors but by a more powerful, non-commutative complex algebra, based on Hamilton's quaternions, called here '***Natural Vectors***'. This NV representation is extended here from representing a single location (the 'field point') to representing the differences between pairs of point objects; this automatically advances the idea that even 'classical' electrons must be treated as 'fermions' (as this is an anti-symmetric algebraic representation). Adding the assumption of separability of the electromagnetic momentum to the previous single-time version now reproduces Planck's 1907 infamous proposal (not Einstein's) for defining relativistic forms of single particle momentum and energy but now in terms of EM electro-kinetic momentum between two particles, in contrast to Planck's original but unphysical assumption of a constant, mechanical force on a single particle that required the Lorentz transformation.

This paper also extends this new **two-electron** viewpoint to many-body situations involving myriads of pair-wise interactions by showing that classical electromagnetism is a consequence of the statistical effects of very many of these interactions arising from multiple, remote electrons moving within metallic conductors on one or many 'target' electrons. A new discrete, many-body approximation model ("**Mesoscopic Electrodynamics**") is developed here that is shown to be a covering theory for the standard (continuum) model of CEM. The emphasis here is shifted back from empty space to the actual experiments involving electrical currents in metallic wires that were the real foundation for CEM's integral and differential equations, which only summarized these effects mathematically but never provided any physical justification or insights. This theory now extends the rival, forgotten ('**continental**') approach to CEM to directly include **radiation**, as just a long-range **induction** effect, removing the only advantage previously associated with Maxwell's field theory. It also links directly to Newtonian mechanics to provide a seamless **unity** to all of classical physics. These results now demonstrate that, contrary to the orthodox consensus, Maxwell's Equations (as a field theory) are **not** a fundamental model for understanding the basic interaction between **any** types of elementary particles. This view challenges the last 150 years in theoretical physics that has been constructed only on the **mathematics** of continuous fields, leading to quantum field theories. This approach *eliminates* the force densities (electric and magnetic **fields**), the 'instant' Coulomb potential and the **single-time** ('God-like') view of nature that have dominated physics for 300 years, only because these ideas could use the simplified (but well-studied) mathematical representation of differential equations.

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 Version 2.180 29-04-2011 Version 1.0 21-10-2007 [123 pp;94Kw]

1. INTRODUCTION & OVERVIEW

1.1 INTRODUCTION

A major contention of this research programme is that the theoretical investigations into the foundations of the phenomenon of electromagnetism ended prematurely due to the exciting discoveries of the electron and atomic physics in the early part of the 20th Century. As a result, physics has relied excessively on the mathematical formulation of electromagnetism, known as Maxwell's Equations, without realizing that these equations have **no** foundations in the modern understanding of the basis of electricity, namely, the electron: or the view that electricity exists as point charges and **not** as continuous charge densities. The previous paper critically revisited Maxwell's EM theory, beginning with his model of the æther and ending with the model of continuous electric charge density that is the basis for today's view of classical EM. This paper moves through the era when the electronic model of electricity appeared around 1900 and then merged by Lorentz into a dualistic continuum-and-particle model of EM. Subsequent papers will focus exclusively on the electron as the basis of EM.

The authors of a widely read graduate text [64] on CEM begin their penultimate chapter with the important statement that: "*The fundamental problem of EM theory deals with the motion of charged particles interacting via the EM forces. The field concept is not necessary in developing equations for such motion; formulation in terms of action-at-a-distance is possible in a relativistically covariant way. … Separation of the problem into 'radiation' (production of a field by a charged system) and 'motion of a charged particle in a field' is in a certain sense artificial and introduces some difficulties*." This is a rather obtuse admission that Maxwell's Equations are incompatible with the concept of a point particle. In fact, all field theories are incompatible with this fundamental point-particle model of matter and such theories have led to the spurious concept of 'renormalization'. This should be acutely embarrassing for CEM, as the point-nature of electrons has been demonstrated repeatedly to the highest energy levels experimentally available, while CEM itself was built on Maxwell's rejected model of the continuous æther. The present theory now returns to the '**Continentalist**' tradition of CEM that was the principal rival to Maxwell's field theory in the Nineteenth Century but is now largely forgotten. This 'old' theory was an action-at-a-distance, mathematical model based on the intuitively correct proposal that electricity occurred in the form of point particles (now known as electrons). The present theory adds a physical model of **conduction** in metallic wires, as these were central to all experimental studies in CEM. The present theory shows that a relativistically invariant form of the 'mathematics of natural vectors' is the optimum representation to construct a theory of EM that is comprehensive and consistent at all levels of interaction: from two electrons up to mesoscopic aggregations that are actually the reality underlying CEM.

1.1.1 RESEARCH PROGRAMME

This paper is the third in the series documenting a new research programme that challenges some of the major **assumptions** of the last 150 years of theoretical physics. In addition to returning to the original approach to undertaking investigations in natural philosophy, emphasizing the value of philosophy and the history of science, this programme challenges the universal "**Continuum Hypothesis**". This is the (now implicit) metaphysical view that reality is fundamentally continuous. However, since 1900 physicists have reluctantly come to the realization that electricity is particulate (electrons) and matter **is** atomic. This has meant that only space itself (the 'æther') is left to carry the burden of physical continuity. This research programme takes an aggressively discrete position: its introduction was announced with the description of a new form of algebra [01] suitable for representing asynchronous action-at-a-distance. The goal is to describe the discrete nature of reality using only non-continuous concepts and discrete forms of mathematics to represent these concepts. Continuum mathematics will be reserved only for aggregating from the discrete microscopic world-view to the apparently continuous view of the macro world: the approach pioneered by Newton himself. In 1912, J. Henri Poincaré (1854-1912), recognizing the potentially disruptive impact of Planck's quantum of action, wrote that ever since Newton, the laws of physics were assumed to be inseparable from differential equations but the introduction of discontinuities associated with the new quantum theory now seemed to demand another form of mathematics [02]. The present research programme shares this viewpoint and this was one of the principal motivations for the creation of a new algebra ("Natural Vectors") for the investigation of all forms of electromagnetism that can be readily formulated as either continuous or discrete interactions.

1.1.2 OBJECTIVES

This paper is deeply indebted to the major efforts of one of the giants of science: James Clerk-Maxwell (1831-1879). In this light, an author can do no better than repeat one of Maxwell's recent biographers [03] when he quoted (with approval) how Maxwell wrote about scientific innovation that should "*jolt the reader into new patterns of thought … especially those readers who have become accustomed to mathematical forms of thought*."

1.1.2.1 Need for a New EM Theory

The principal objective of this paper is to demonstrate that Maxwell's theory is not a suitable model for the foundations of EM as electricity is now known to be grounded in the motion of inertial particles with finite electrical charge. Continuous EM interactions between pairs of charged particles "on the light-cone" will be shown to be incompatible with the inertial motion of electrons. Modern theories of EM based on quantizing either Maxwell's force field intensities would then seem to be 'building on sand'. A new, microscopic theory of EM is developed later in this paper paper.

1.1.2.2 Continuum Hypothesis

The second objective of this paper is to demonstrate that this failure has deep, historic roots, so deep that it has proven very difficult for modern scholars to move beyond the "Continuum Hypothesis", which is the foundation for all field theories. In keeping within the stated methodology of this research programme, many relevant summaries of the historical contributions of major physicists and philosophers will continue to be re-introduced herein since another lesson learned from the history of science is to challenge the mistaken belief that fundamental science evolves incrementally, with the present theory of classical EM forming a smooth continuum with the innovative theories of Maxwell and his followers. Whenever a major contributor is mentioned, his life-dates will be included as these temporal milestones indicate how scientific progress has been built on a network of innovation: one more reflecting a Hegelian dialectic, than a Darwinian evolutionary progression.

1.1.2.3 The Need for Philosophy

As this paper is the third in the series documenting a new research programme, it also tries to meet another of this research programme's objectives: namely, to emphasize the importance of philosophy in the study of nature – a viewpoint that only seems 'odd' by the recent standards of late twentieth century physics. It is another contention here that new **concepts** are more important than mathematical innovation; powerful symbolic manipulation makes for elegant mathematics but rarely provides much insight into the deeper mysteries of nature. This programme deliberately goes beyond phenomenology.

1.1.2.4 A Covering Theory of Classical EM

A further objective of the present paper is to show how this new theory can be a covering theory for the standard model of classical EM. This means that the detailed model based on the interactions between electrons can be systematically aggregated to a sufficient level where continuum results form an adequate approximation of classical electromagnetism. It was the aggregate or macroscopic level of electricity that formed the firm experimental basis for the later theories of EM that were developed to explain these phenomena. It is often forgotten that many alternative explanations can be created to explain any area of nature. The EM theory-battles of the 19th Century well illustrate these conflicts – they will be revisited here. What is usually not realized is that these alternative explanations fundamentally disagreed in their metaphysical assumptions, which were defended with almost religious zeal, even by the most 'objective' scientist. The present analysis will demonstrate that only one of Maxwell's foundational concepts (electro-kinetic momentum) need be retained in a modern theory of CEM that will be shown to rest far more on the particulate action-at-a-distance ideas of Maxwell's 'continental' rivals.

1.1.2.5 History & Philosophy of Science

As always throughout this series, two of the underlying objectives will continue to be the restoration of philosophy and the history of science to the forefront of fundamental research to regenerate their key role as engines of theoretical physics. As in each of the papers reporting on this new research programme, a broad historical perspective is deliberately introduced. This is to re-establish the context from which the new ideas are developed; these are often contrasted with the assumptions that usually drove the historical development of physics at the time. In other words, in order to understand the significance of these new ideas, it is important to recall the alternatives that were considered in the past. In particular, many of the most severe criticisms of what became the canonical evolution of physics were never answered – just ignored. This was the case in the area of studies of EM in the 19th Century, when the academic mathematicians, who were investigating the differential equations found with electric currents, excluded supporters of the alternative Newtonian-like, point-particle theories. A similar contentious metaphysical war was fought about the nature of light – this will be covered in another later paper.

1.2 OVERVIEW

In this overview, the contents of this paper are summarized, by including a brief description of each section and the major reasons the particular material has been included. The paper ends with a Summary and Conclusions that focuses on the implications of the material covered and the new results obtained, along with brief reviews of future papers in this series.

1.2.1 METAPHYSICS OF CLASSICAL EM

1.2.2.1 The Necessity of Metaphysics

This research programme is founded on the contention that theoretical physics is natural philosophy; without any philosophy physics then becomes indistinguishable from applied mathematics. The central metaphysical concept of all Maxwellian-type theories of EM, namely the **æther**, went into rapid decline soon after the 20th Century began. This has left the theory of EM without the philosophical foundations from which it developed. The mathematics of classical EM (summarized in the term: *Maxwell's Equations*) has been saved by adopting Helmholtz's fluid analogy from hydrodynamics. This step introduced the concept of continuous electric charge densities and this is how the subject is still taught today to undergraduate physicists. One of the major methodological principles that has guided this research programme is the idea that concepts that have proven successful in physics should only be modified rather than replaced. Accordingly, this programme is grounded in Newton's metaphysics and has attempted to maintain as much as possible of Newton's dynamical framework. This has meant retaining the 'passive' view of space that underlies the *Principia* and attention is focused on the concepts associated with the idea of point particles rather than "*the space in between*". Instead of following the near-universal adoption of the field concept that was introduced by Maxwell, this programme also returns to Newton's original action-at-a-distance ideas.

In reviewing modern texts on CEM, a student would never guess that there were ever any alternatives to Maxwell's theory. The most powerful alternatives centered on extending Newton's idea of instantaneous action-at-a-distance to one where a finite time must occur for any two charges that are interacting across a finite spatial separation. Most such attempts have retained only the traditional 'retarded' perspective wherein only the past effects the present; this programme adopts the more general symmetric viewpoint, where the future may also effect the present – hence the key phrase "asynchronous action at a distance" (or AAAD). The first successful 'retarded' attempt was due to L. V. Lorenz whose 'no-field' AAAD theory had produced all of the results that Maxwell's theory had achieved only two years earlier in 1865 based on his ideas of the scalar and vector potentials. This programme wishes to restore the reputation of major contributors to EM theory, such as Lorenz and Voigt, who are presently almost forgotten. Indeed, Voigt-like potentials assume a central role in the present theory.

Since electromagnetism in this research programme is viewed as not only fundamental but as the very foundation for all of physics it is necessary to investigate the actual foundations of physics itself; or in terms of philosophy, **ontology** – the study of existence. Unlike modern physics, this programme does not require an "*infamous boundary*" [04] when the very nature of the world changes, as the physical separation between objects shrinks from the macroscopic scale of everyday, human interactions to the microscopic world of the 'quantum'.

1.2.2.2 Classical Electron Theory

The ontological focus in this programme therefore centers on the principal point-particle of modern physics: the electron. Rather than follow Lorentz and produce a dualistic and asymmetric theory of electrons and electromagnetic force fields, this programme needs only to use electrons and their action-at-a-distance interactions. Modern physics has adopted a monistic, and almost a completely totalitarian theoretical perspective, by viewing all particles as fields but still fails to explain how the spatially diffuse concept of the field is able to manifest itself experimentally as point objects. The second section continues with its historical review emphasizing the struggle that the point concept in EM has had to overcome compared with the academically more popular continuum concepts. The seductive power of mathematics has meant that theoretical physicists have been most reluctant to give up their 300 year-old affaire with the simple mathematics of the continuum, particularly the **calculus**, even when irrefutable experiments have demonstrated repeatedly that the nature of the world is **discrete**. This is described here when physics around 1900 was faced with the realization that the electron was both a point particle and was the actual exemplar of electricity itself. Perhaps recognizing instinctively that Maxwell's continuum views of electricity and the point nature of the electron were incompatible, several attempts were first made at this time to produce finite models of the electron filled with continuous charge, almost always with bizarre results.

1.2.2 CLASSICAL ELECTRON THEORY

The second paper in this series began with an analysis of the modern view of classical EM, which is usually presented as if electricity exists in the form of a continuous fluid (charge density). This standard approach has the benefit that it preserves Maxwell's results while ignoring the metaphysical basis (the æther) that was used by Maxwell to derive the resulting EM equations. The modern approach also builds on the insights of the (continuum) vector calculus, especially as it was used by Helmholtz to generate a hydro-dynamical model of EM. This paper traces the historical introduction of the particulate view of electricity that was combined with the implicit continuum model of the EM interaction to arrive at the electron-æther theories of Lorentz and Abraham. These theories are still viewed today as justifying the retention of Maxwell's Equations as a detailed microscopic theory of the EM interaction. The present paper will demonstrate the inadequacy of this view.

1.2.3 BRIEF HISTORY OF TWO-ELECTRON DYNAMICS

Since this paper presents a new approach to the analysis of the continuous dynamics generated by the exclusive interactions between two electrons it is necessary to establish the historical context for this area of physics as all earlier efforts have, surprisingly, **failed** to make much progress with this 'simple' model, even though electrons and their interactions have been investigated theoretically for over 100 years and the classical two-body problem was the archetype of classical mechanics. The various investigations into this fundamental area of EM can be grouped into two major research areas. Each area has focused on one of the key characteristics that distinguishes EM from the simpler 'mechanical' forces described earlier in classical dynamics. These factors center firstly on the asynchronous nature of the EM interaction (instead of traditional instantaneous interactions) and secondly on the need to satisfy the Lorentz invariance constraints (versus the earlier Galilean invariance) that were implied by the need to preserve the **form** of the force intensity equations. The present paper provides an explanation for why all of these historical investigations in this area have inevitably failed to reach fruition.

1.2.3.1 Fokker Lagrangians

The asynchronous nature of the interaction between two remote electrons means that one electron's participation in the EM interaction occurs at an earlier time than the other electron's participation. This implies that both retarded and advanced solutions are equally valid mathematical solutions to the interaction. Conventionally, only the 'retarded' solution, where one electron emits an impulse **prior** to its later absorption by the second electron, has been retained as this seems to comply with the macroscopic behavior of human memory and agrees with an ancient preference for an asymmetric view of causality: only the past impacts the present, not the future. Fokker initiated one research programme by proposing a Lagrangian point particle model of EM that explicitly included both retarded and 'advanced' modes. This style of symmetric EM interaction did eliminate the self-interaction that had plagued finite models of the electron; so these views of temporal symmetry and the point nature of the electron are also adopted here, reflecting the temporal symmetries at the foundational levels of nature.

1.2.3.2 Relativistic Two-Electron Models

Since conventional physics decided that the acceptance of Maxwell's theory of EM required a relativistic formulation of mechanics there have been several attempts in the last 100 years to solve the 'simple' problem of two charged particles both moving under their mutual influence in a rigorous relativistic manner. Most of these comprehensive attempts were, at best, only 'solved' numerically (over a limited range of constraints) and could not be solved analytically; as a result they could not provide any deep insights into their own failures. Some of these well-known attempts are briefly reviewed here.

1.2.4 THE NEW ELECTRON INTERACTION

The common feature of the research programmes that have investigated the two-electron problem have implicitly included the assumption that the EM interaction occurs continuously between the two electrons. Since this assumption had always worked successfully for classical mechanics (two bodies) it never appeared necessary to challenge this "Continuum Hypothesis". Furthermore, continuous interactions have defeated all attempts to extend any successful two-body approach to three or more interacting objects – a massive challenge known as the intractable "**Three-Body** Problem".

One of the basic perspectives of this research programme is that the microscopic world is fundamentally discrete and as a result this discontinuous view is here extended to the nature of the EM interaction itself. In this paper it is shown that the continuous interaction model is **incompatible** with the two requirements that electrons are inertial particles and they only interact when they are "on each other's **light-cones**". Section four introduces the major features of the EM interaction that the present research programme proposes as the central characteristics of this basic interaction between the fundamental existents of the world.

1.2.4.1 Discrete EM

The present research programme continues to adopt many of Newton's philosophical preferences. In particular, space itself is viewed here as passive, defined only by the separation between electrons, in contrast to modern field theories that have space itself now becoming the principal actor in Nature's drama. In fact, the present programme puts **time** at the center of the stage, with space only playing a purely subsidiary role. This new view of time defines how each electron **may** interact with any of the other electrons in the universe. The central importance of the electron justifies a brief 'biographical' review of this fundamental particle, so this is summarized in section four of this paper along with a metaphysical justification for choosing electrons as the principal ontological components of this theory. This approach is contrasted with the continuous charge or "electric fluid" model that is today widely used to justify the mathematical form of classical electromagnetism.

1.2.4.2 Electron Characteristics

This research programme's own methodology demands that focusing on the electron (as the central ontological object) requires that its major characteristics be stated explicitly. The properties that define how electrons interact with each other become the principal hypotheses of this theory. This paper introduces some of these features informally; in a later paper the complete set of defining properties will be gathered together and formally re-introduced as a set of dynamical axioms. This sub-section firmly 'plants its flag' in the philosophical tradition known as "common-sense realism" (the world exists totally independent of humans), a view that has been dismissed by sophisticates in the 20th Century as naïve and simplistic.

1.2.4.3 EM Interaction

Ever since Newton, physics has split the interaction between two particles into the production of a third entity that has been identified with the concept of force. The source of this force was then usually dropped and the focus thereafter was on the force itself and its effect on one of the particles. This research programme rejects this reification of any such third-party 'objects' (even when treated as force-carrying 'particles', such as photons or gluons) and returns to the two electrons that are now treated equally and the interaction itself, which is now viewed as an intrinsic characteristic of such particles when they interact. This focus on the interaction is extended here to the concept of asynchronous action-at-a-distance (**AAAD**) as one of the most important differences between the EM interaction and the simpler interactions covered by classical mechanics, such as Newton's gravitational model that introduced the idea of instantaneous action-at-a-distance.

1.2.5 CNV TWO-ELECTRON ELECTROMAGNETISM

Section five introduces the universal, scalar constant *c* (popularly known to all physicists today as 'the speed of light') as the principal characteristic of the EM interaction between two electrons. It is defined in terms of the "**light-cone**" condition that defines when two electrons may interact based on their relative separations in space and time. As this theory is Newtonian, it is an action-at-a-distance theory so it does not imply that there is an entity that exists during the time of the interaction that carries energy or momentum between the electrons. The previous paper applied the new Natural Vector (**NV**) mathematics to situations involving the continuous, incompressible 'electric-fluid' model of electromagnetism to rapidly re-derive all of the results associated with the standard classical theory of EM. In this paper, the focus switches to the more realistic view that electricity ultimately consists of interactions between discrete electrons, not some unreal continuous fluid.

1.2.5.1 Two Particle Natural Vectors

The asynchronous viewpoint requires that two times must be considered for any single interaction. This forces the extension of single variable mathematics to an algebra and calculus that processes pairs of variables at once. Since further extensions to the real discrete situations in physics are anticipated then this modified new mathematics also includes the manipulation of discontinuous functions, specifically the definition of *Total Two-Time-Difference Differential*s. After recalling the key features of single-point Continuous Natural Vectors (**CNV**s) these two new mathematical areas are merged to define *Two-Particle Difference CNVs*. The most important feature of these new mathematical objects appears when they are separable into their individual components, as this immediately demonstrates their anti-symmetric properties, leading to the view that electrons are always *fermions*, even when their interactions are not quantized.

1.2.5.2 Two Particle CNV EM

The application of these ideas to the intrinsic *Separation* CNV of two electrons generates the *Two-Electron CNV Difference Velocity*. This new, central CNV becomes the basis for defining the two-electron current and momentum CNVs. A similar approach is then used to define the *Two-Electron CNV Potential*, in terms of a scalar two-electron potential. These simple extensions show that there is a direct, formal analogy between the single-point form of EM based on a continuous model of 'fluid electricity' and the two-electron, difference CNVs defined in this section. This analogy provides an explanation of the apparent paradox exposed in the previous paper that the electric current was an invariant constant, moving at light-speed.

1.2.5.3 Two Particle Special Relativity

In the final part of section five the standard particle results of the theory of special relativity (SR) are directly derived from the CNV single-point theory of EM by extending "Planck's Proposal" for relativistic momentum when subject to a fictitious mechanical force to EM interactions. It is shown that the assumption of separability of the electromagnetic momentum must be added for the single-time, single electron version of special relativity to be extended to the two-electron CNV theory.

1.2.6 TWO ELECTRON CONTINUOUS DYNAMICS

Section VI is the heart of this paper as it brings all of the perspectives introduced up to this point to contribute to an analysis of the dynamics of two electrons, defined as truly mathematical point particles, that are interacting here continuously.

1.2.6.1 Continuous Change

The interaction between two spatially separated electrons implies that two distinct time variables must be considered on an equal footing rather than the traditional single-time analyses. Fortunately, it is possible to consider the dynamics here from the perspective of a single time difference between the two 'ends' of a single interaction.

1.2.6.2 Symmetric Inertial Reference Frames

This research programme continues to follow Newton in his views on the inertial motion of material objects, in this case, defined down to the level of individual electrons. Subsequent papers will propose small modifications of Newton's Laws so that they can be used to describe the quantized dynamics generated by the interaction between electrons. Newton's proposed scheme for measuring the motion of objects implied that a theoretical 'device', known as an inertial frame of reference, could be constructed (in our imaginations) to assign numerical values to each location of an object in space at any instant of time. In theory, these reference numbers could be developed to an unlimited degree of accuracy relative to arbitrary intervals of space and time (the unit measures). The resulting numbers are usually represented as infinitely repeating decimal fractions or so-called "real numbers". The arbitrary nature of the unit measures and 'location' of the spatial and temporal origins indicates that the resulting dynamics must be independent of these types of human decisions. Poincare proposed in 1899 that these guidelines [05] should be raised to a fundamental principle of physics (known as the Principle of Relativity). Appeal is made here to "Ockham's Razor" to simplify the nature of the EM interaction between two electrons to propose a pair-wise symmetry over time so that a complete 'single' interaction consists of two symmetrical 'semi-interactions'. The mathematical analysis of electron interactions is simplified when viewed from an inertial reference-frame that incorporates the maximum degree of symmetry in both space and time; this is called the "Symmetric Inertial Reference Frame" (or **SIRF**). Within this frame, the total (combined) velocity of the two electrons, at any one time, is always zero.

1.2.6.3 Parallel Interactions

Section 6.3 investigates multiple interactions between two electrons, demonstrating that all consecutive interactions between any two electrons always occur in parallel, across space. This analysis is conducted in terms of asynchronous point-to-point impulses that obey Newton's Third Law of Motion, extended to finite differences in time. Each local EM impulse alters the corresponding electron's momentum according to Newton's Second Law of Motion (in its original form).

1.2.6.4 Space-Time Integrity

The final sub-section here extends Newton's three Laws of Motion to asynchronous interactions between pairs of electrons where each electron moves inertially between each interaction node. This now generates the central result of this paper that demonstrates that such inertial motion between interaction nodes that are on each electron's light-cone is incompatible with the assumption that the time intervals between interactions can all smoothly go to zero. When **finite** intervals between each node are defined then these combined criteria result in an expression, referred to here as the Space-Time Integrity Condition.

1.2.7 MESOSCOPIC ELECTRODYNAMICS

Section VII introduces an alternative approach to Classical Electromagnetism (CEM) that avoids all use of field concepts: as such, it eliminates the ideas of continuous force densities, like the electric and magnetic field vectors. Indeed, this separation is viewed here as only a mathematical convenience that introduces an artificial (non-physical) distinction between the unitary interaction between electrons into components that vary with the relative motion of the inertial frame of reference. This view returns to the empirical basis of CEM that was based on experiments between electrical currents in metallic conductors. This 'new' theory is actually an extension of the far-action (Continental) tradition in CEM that began with Ampere and ended with Weber due to the theorists' preference for the mathematical elegance of James Clerk-Maxwell's theory of EM fields. Such theorists ignored the discovery of the discrete carrier of electricity – the electron – and continued to build on the continuum model of the æther that had been the fashion in physics for almost 100 years.

The Platonic mathematicians were still comfortable with Maxwell's focus on the new concept of 'energy' that had emerged in the 1850s. Following Helmholtz in his flawed critique (based on instantaneous interactions) of far-action, most physicists seemed unaware that Maxwell and Faraday were both highly motivated in their views of nature by deep **religious** convictions that required their Christian god be imminent everywhere and at all times throughout the universe – a feature shared by the view that fields were real entities. In developing his field theory, Maxwell moved the focus away from the material reality of metallic conductors and filled the empty spaces between them with invisible, propagating 'energy'. The common point of contact between the new approach and Maxwell's EM theory occurs at and near the surfaces of the current carrying conductors, so the focus now needs to be returned to the massive numbers of electrons moving dynamically near the surface of metallic **conductors**. This requires introducing an explicit statistical model of interactions between large numbers of electrons as they readily move through a solid lattice of metallic ions. This is presented here as the **mesoscopic** model of multi-electron interactions. This is a conscious attempt to avoid the zero (i.e. continuum) spatial separation limit. The focus here is on re-introducing Maxwell's innovative concept of electro-kinetic momentum using the present theory's fixed exchange of momentum between each pair of electrons whenever they interact, in effect, the basic EM 'remote' impulse ΔI_0. Magnetism is viewed here simply as an epiphenomenon – the extra effects of relative motion between remote interactions between many electrons. Aggregates of millions of electrons are introduced that hide the detailed individual electron interactions and allow measurable averages to emerge from the collective effects of stable ensembles of very many electrons and positive ions moving around fixed lattice positions in solids. These aggregates are the reality defined by the concept of (continuous) electrical charge density. This is the key requirement that defines the mesoscopic approach where variations from statistical averages derived from sufficient numbers of electrons in a 'cell' of appropriate size are not significant within a cell but become important **between** cells (or layers of cells).

1.2.7.1 Electrodynamics – the Lost History

This major section begins with a reprieve of the major alternative to Maxwell's field theory of electrodynamics. This is included now as the present theory incorporates key parts of this venerable research into a new theory of electron dynamics. This programme rejects all field concepts and views magnetism simply as a macroscopic epiphenomenon associated with the interactions between electrons when they are moving relative to each other, particularly when large numbers are tightly constrained in a closed-circuit conductor. Ampère's dynamical experiments were the root of the 'Continentalist' tradition in viewing these new phenomena of electrodynamics as due to action-at-a-distance between point-like electrical charges. This section shows how the principal space-time parameter *c* (referred to as 'light-speed') entered physics through this approach rather than Maxwell's field theory, as is so often believed.

1.2.7.2 Mesoscopic Averages

Section 7.2 develops an electron-only alternative to EM field theory. This new theory is centered on the motion of excess electrons that are highly localized in current-carrying conductors – this contrasts with the mathematical idea of fields that are defined everywhere throughout space (even to the edge of the universe) but are not present within the conductors themselves. This explicitly statistical theory recovers the results of CEM, demonstrating that the traditional model is **not** fundamental but only valid over 'long' time averages for groups (or cells) of many (million plus) electrons that interact collectively. This **many-body** approach is sufficient to minimize the effects of fluctuations when smaller numbers of objects interact together. It is this localized view, centered on the point-particle model (rather than electrons as waves) that distinguishes this approach from Mead's "collective electrodynamics" – a model of electricity that remains much admired in this programme.

1.2.7.3 Electrodynamic Potential

A brief section is included next to refocus attention on the central concept of electrical potential, which was the original locus of Maxwell's EM mathematical research before his work was transformed into a field theory by Heaviside. A few physicists (like Feynman, Dirac, etc) have realized, it is the EM vector potential that plays the key role in linking electrical interactions to the idea of discrete (or particulate) momentum. This is the minority viewpoint adopted here: EM fields are viewed only as calculational (continuum) devices. As scalars, potentials have the gigantic simplicity of arithmetical summation. This is the mathematical requirement for computing both temporal and spatial averages, which are exploited in the mesoscopic model. Simple models are proposed here for each of the elements traditionally associated with studies of electrical current, so that a model can be developed based on the remote exchange of electron momentum as a replacement for Maxwell's Equations or their modern equivalent (Classical EM). This will be used later to show that there are physical explanations for optical effects that do not have to rely simply on the purely mathematical approach of these older, Nineteenth century EM 'field' theories.

1.2.7.4 Metallic Conduction

The core of this new physical approach is a fresh analysis on the phenomenon of electrical flow in metallic conductors as this lies at the heart of all the experiments that provided the factual basis for CEM in the 19th Century. Maxwell did **not** produce a model for **Ohm**'s Law; he simply adopted it as another empirical equation to his set of electric and magnetic field equations. Although superficially similar to the modern form of Drude's model of electron-lattice scattering, the new theory proposes an independent 'cohort' model of electron packets slowly moving through the conductor, being repeatedly accelerated only by their closest followers and then immediately losing this kinetic energy to the lattice; the whole process repeating many times as the conduction electrons move through the length of the wire. Unlike orthodox quantum models of metallic conduction, this approach rejects the idea that the mobile electrons move through the lattice like a 'free' electron gas, so that all 'Fermi-surface' ideas are bypassed. This model is shown in this section to relate to the Maxwell Equations through the earlier CNV mathematical model of EM; all infinitesimal limits are kept finite by identifying the finite number of conduction electrons per cell with the traditional continuum definition of density of 'electrical fluid'.

This model is then extended to very low temperatures (isolated metals) for a new explanation of zero electrical resistance or **superconductivity**. Unlike the standard BCS model, superconductivity is here explained without introducing quantum energy states. The mechanism proposed here is one of long-range 'hopping' of electrons between distant ions in the lattice that are correlated in their movement by the extended (low temperature) lattice displacements; it is only at very low temperatures that motion can be synchronized across pairs of electrons. It is **not** the quantization of magnetic flux that explains the Meissner effect as Faraday's law of EM induction predicts zero electromotive forces needed to generate an electrical current under steady-state conditions. Rather, it is the quantization of EM **action** between integral numbers of pairs of electrons, interacting via the intervening lattice and this theory's "Saturation Hypothesis" that explains this effect.

1.2.7.5 Electronic Equilibrium

Situations of spatial symmetry are analyzed next to provide a physical mesoscopic model of electron capacity without using any continuum calculus. The intent here is to abolish the idea that electricity is "just like water" (flowing through 'pipes' from 'pot to pot'). The long-range EM effects and the powerful current-current (**vector potential**) effects are dramatically different from the short-range, mechanical 'jostling' that characterizes the mass movement of neutral molecules in a liquid. This sub-section is introduced to show that the fundamental idea of a static, electrical force density ('the E field') is not even required for electrostatic calculations, such as capacitance, which is analyzed here as a stable, dynamic multi-electron effect.

1.2.7.6 Electronic Inertia

The next major sub-section returns to Henry's revolutionary experiments on current induction but now viewed as dynamic examples of many electron interactions constrained by both the inertia of the electrons themselves and their supportive lattice ions. The motivation here is again to eliminate imaginary, intermediate concepts from the role of foundational, ontological entities in EM theory: the 'magnetic' field is here shown repeatedly to be no more than a useful mathematical adjunct that can sometimes simplify a calculation in theoretical EM models. EM vector fields (represented by field lines) are seen as an abstraction from the interaction of real electrons; they compute the hypothetical (statistical) possibility of how a real electron would move **if** placed in a given situation. This sub-section includes a brief summary of the traditional approach to 'deriving' the magnetic field so that this can be later contrasted with the new External Conduction theory. The emphasis on electronic inertia is reinforced by returning to Maxwell's original adoption of the vector potential A (from Lorenz and Neumann) or as he referred to it – the density of electro-kinetic momentum; here always used with the explicit dimensionality factor: A/c. It is the central importance of **induction** in CEM that forces the attention here back to a deeper analysis of current induction.

1.2.7.7 External Conduction Interactions

In order to emphasize the real source of EM interactions, the present theory refers to its central mechanism as the External Conduction Interaction (or **XCI**) that manifests its effects in the short-range as induction and electro-dynamic forces and in long-range interactions as EM **radiation**. Only students of the history of CEM seem aware today that Ampère's force is not synonymous with the Lorentz force; this key difference is explored at the start of this sub-section. When an external set of charges interact with a current carrying conductor, some of the internal interactions must cease and the interactions switched from internal to external (according to the present Saturation Hypothesis, where an electron can only interact with **one** other electron at any specific instant); the XCI explores this ***net*** interaction with the external charges from a statistical viewpoint. In contrast to all earlier electrodynamical models, this theory proposes that the target-set of charges interacts with many of the charges in the source over a small but finite time duration and the interactions, even at the target, do **not** occur all at the same time. This is a rejection of both 'instantaneous' and continuum physics; hence its description as an interaction and not as a force.

This is the section where the central player in this new drama is introduced – the vector potential, or as Maxwell referred to it – the "electro-kinetic momentum". The term 'electro-kinetic' is retained here as the unifying concept of both camps in 'the EM wars' of the 19th Century with a bias shown explicitly to the 'Continentalists' for their commitment to action-at-a-distance between point-particles. The concept of vector potential is powerful enough to uniquely define the electro-kinetic energy, force and action as well as momentum in all EM situations: no other mathematical object is needed in this theory, which directly links classical EM concepts to the foundational concepts of Newtonian mechanics. The new CNV algebra is shown here to be the most direct (singular) representation of the vector (and scalar) potential view of EM interactions based on the relative separation and change of relative separation (velocity) between the pair of interacting objects, including one or two mesoscopic cells of conduction electrons. The heart of the CEM problem (and the CNV model) is the widespread failure to realize that all 'point' equations are not instantaneous relationships but that they are 'long-term' statistical time averages involving very many electric charges.

1.2.7.8 Near Conduction Interactions

This large sub-section examines the External Conduction Interaction (**XCI**) model in those situations involving a limited number of macroscopic conductors that are relatively close to each other. It begins with a review of the universal 'dipole' approximation, which can be used wherever there are two sources of activity that are close together compared to the distant point where their combined effects are being evaluated: a situation that occurs frequently with the mesoscopic model. The current in a metallic conductor is known to be neither a perfect fluid (Helmholtz) nor a set of free particles (Lorentz) but a dynamic, complex, collective system that self-adjusts its total behavior everywhere (globally) to minimize its overall action. A detailed analysis here shows that the mesoscopic model is consistent with the CEM definition of the retarded Neumann (or vector) potential. The ratio of the external induced change in momentum to the internal electrostatic impulses producing the current in the source wire is shown to be related to Weber's experimental ratio for the diminished interaction of charges in motion (induction due to conduction or current measured in EMUs) compared with the large effects they produce when there is no relative motion; i.e. the maximal electrostatic effects when measured in ESUs. This key ratio is the factor that appears everywhere in the Lorentz transformation and all relativistic theories – here it is shown to be proportional to the **probability** of a remote interaction occurring relative to the probability of a local (source) interaction. This sub-section also provides a detailed (mesoscopic) derivation of the Amperian forces between moving conductors: these differ radically from the Lorentz force when the conductors are aligned in the same straight line (longitudinal forces), agreeing with experiment. Two famous examples of the power of using the vector potential are used to show that 'magnetic' effects produced by steady currents in fixed and circular conductors are only approximations when based on traditional methods, like the Biot-Savart law. These results are then used to show that Faraday 'magnetic' induction as just an implicit example of Henry current induction. This approach removes the 'duplicity' of explanations in the Maxwell-Lorentz theory.

1.2.7.9 Far Conduction Interactions

This section examines situations where the spatial separations are large between the conductors, where temporal variations in one conductor can effect the electro-dynamic behavior of a remote ('far') conductor via the vector potential: a phenomenon known as EM radiation and believed to be only explainable by Maxwell's field theory. The focus here is on Hertz's famous experiments. Both straight ('electric') and circular ('magnetic') transmitting **antenna** arrangements are analyzed. Once again, the XCI theory is shown to be capable of providing a mesoscopic explanation for effects that had previously been believed to require some form of field theory for an explanation.

2. CLASSICAL ELECTRON THEORY

2.1 POINT PARTICLES

It is the central ontological hypothesis of this research programme that the fundamental substance of reality consists **only** of electrons that are viewed here as existing at only a finite number of distinct points in space at any instant in time. There is a much shorter history of this viewpoint than the rival claims of the continuum as is illustrated in the following excerpts. Before the 16th Century, the ideas of atomism and mechanical explanations found little support, primarily because this approach was not supported by Aristotle; while the original Greek proponents, such as Democritus (c. 460-370 BC), Epicurus (c. 341-270 BC) and Lucretius (c. 95-52 BC), were viewed as atheists by the religious academics that dominated European medieval intellectual life. In the *Principia* [06], Newton revived the particulate view, as the mathematics of a point-particle was simpler than any other model of matter. It was only in 1736 with *Mechanica* that Euler was able to describe mathematically the dynamics of rigid bodies. With his subsequent treatise in 1766 on fluid mechanics, *Opera Omnia*, Euler was able to present a mathematical description of ideal, incompressible liquids, as well as introduce the first mathematical field theory. It was not until 1845 that G. G. Stokes provided a mathematical model for continuous, solid material (a necessary step for describing transverse oscillations); this started a 60 year search [07] for a mechanical model of a solid, elastic medium to support transverse EM vibrations – the luminiferous æther.

Newton's most recent principal biographer, Richard Westfall, has described Newton's attitude to the concept of energy [08] discussed in Proposition XXXIX of the *Principia* where the quantity mv^2 appeared under the name *vis viva* or energy. This quantity became central to Leibniz's own theory of dynamics with its own ontological significance. However, for Newton 'force' was the major item in his own ontology while mv^2 was merely a mathematical expression that appeared in a given problem. This is also the viewpoint adopted in this research programme: energy is not an ontological existent but here it is viewed as only a dependent property of a set of electrons – the real existents. Only the point model of matter proposed by Roger Joseph Boscovich (1711-1787) in his Newtonian inspired magnum opus, *Theoria Philosophiae Naturalis*, resolves the ultimate regress of any finite particulate model as to the ultimate constituent of matter, since all points do not have to consist of anything else – a viewpoint supported by Immanuel Kant (1724-1804), one of Europe's greatest philosophers since the Enlightenment, and a vigorous supporter of the Newtonian approach to the world.

W. R. Hamilton proposed [09] a logically consistent definition for the 'atom of matter', which he viewed as a set of powers (or properties) associated with certain points in space that moved coherently through space over time. He viewed motion of an atom as occurring when this set of properties is subsequently associated with a neighboring point in space and no longer with the earlier point. This is the view supported by this programme but with the electron taking on this fundamental role.

From the very earliest studies of electricity, there has always been a vast underestimation of the complexity occurring in real conductors (wires) supporting an electric current, with the commonest view that it was "just like water in a pipe". This very simplistic view continued right up to the discovery of the electronic structure of matter. This simplification was exemplified by FitzGerald's emphasis on the surrounding EM 'field' of empty space, where he said: "*there is a great deal more going on outside the conductor than inside it*". Like Maxwell, he too [10] dismissed the idea of anything real moving bodily through the interior of a conductor. The idea of real electric charge awaited the discovery of the electron. Even then, the academic 'continuists' still tried to preserve the ideas of charge density as a model of electricity by inventing suitable limit definitions of clusters of electrons, always ignoring the massive repulsive effects caused by the close proximity of similar point charges. Since our experiments show that particles do occupy finite regions of space (e.g. tracks of electrons in cloud chambers) then those field theorists who doubt the reality of particles, such as Halvorson [11], are reduced to describing "these experiences as illusory". This is not the position of this research programme that responds that it is the fields that are only mathematical.

2.2 CONTINUUM THEORIES OF ELECTRICITY

2.2.1 MAXWELL'S DYNAMICAL FIELD THEORY

As Andrew Warwick [12] wrote recently: "*Maxwell's theory was centrally concerned with the electromagnetic effects of relative motion between fields and conductors. ... In Maxwell's scheme, an electric current was not thought as a flow of electrical substance but as a breaking down of the electric displacement, which, by some unknown mechanism, converted the EM energy stored in the æther into heat energy in the conductor. ... Likewise, the effects produced by the motion of a charged body were something of a side-issue, not perceived as having any real relevance to the central problem of conduction*."

2.2.1.1 Maxwell 'still used' after 150 Years

Undergraduates in physics still study Maxwell's EM theory as the prototypical model of a successful field theory. Today, Classical Electromagnetism (CEM) is usually presented as a blend of ideas that does **not** reflect the history of the evolution of this area of physics. An example of this modern approach was presented by Pauli's in his final lectures [26] on CEM. This new blended approach both eliminates Maxwell's æther and merges in Helmholtz's earlier electric fluid model as the source of electrical fluctuations – this after-the-facts, artificial (mathematical) model was critically reviewed [54] in UET2. Charge density is never directly measured (or measurable) but only aggregated into macroscopic 'fluxes' (electric currents) while the EM 'forces' remain totally invisible. All of this metaphysical 'baggage' has been swept under the rug of history (to minimize physicists' embarrassment) leaving only the mathematics behind for the modern presentation of CEM.

2.2.1.2 Maxwell's Dynamical Theory

Until the final versions of his approach to EM, Maxwell always tried to begin each of his earlier attempts at an EM theory with a concrete, visualizable model that he could convert into a set of mathematical equations. These earlier models were critical for Maxwell in identifying the key equations that he wished to retain in his own final EM theory. This technique for developing mathematical models of EM was described in paper II [54]. Maxwell's final views on EM began to emerge with his famous 1865 '*Dynamical*' paper [50, 51] that was examined more extensively in paper II; it was here that he explicitly dropped all references to characteristics of the æther – a step that left his contemporaries and admirers greatly puzzled. However, in the introductory remarks to this paper, Maxwell wrote that he had called it a "Dynamical Theory" because the theory assumes that in the space around electric bodies there is another form of matter in motion (i.e. the æther) by which the observed EM phenomena are produced. A little further, he writes "*there is an ætherial medium filling space and permeating bodies, capable of being set in motion and of transmitting that motion from one part to another, and of communicating that motion to gross matter*". The nature of the linkage between this luminiferous æther and normal (gross) matter was never explained. One of the leading Continental physicists of the later 19th Century, Ludwig Boltzmann was very sympathetic to Maxwell's EM theory (surprisingly, as he had studied with Helmholtz) and also believed, like MacCullagh, that the EM energy in empty space was due to the twisting of the æther in each spatial cell, a view shared by Joseph Larmor (1857-1942), the leading theoretical physicist at Cambridge at that time.

2.2.2 MAXWELL-HEAVISIDE MATH MODEL

2.2.2.1 Treatise on Electricity

By the time Maxwell completed his *Treatise* [98] he had developed two very different but complementary views of EM behavior. One focused on remote region-to-region, asynchronous distant action (ADA) that involved scalar and vector potentials (ϕ, A) and the other involving local spatial derivatives of these two potentials at a single 'field point'. However, Maxwell was forced to construct his equations to eliminate all references to the timing of the sources of these EM effects, reducing the action to a **single-time** theory. This simplification enabled Maxwell to use the mathematical techniques of Lagrangian mechanics to produce a local field theory. The penalty for this approach was the necessity for the resulting fields to pervade **all** of space. Any local field reduces electricity to a single, simple (field) point that still lacks the rotational features so characteristic of the phenomenon of magnetism, as MacCullagh had realized earlier, hence the need for 2 fields.

2.2.2.2 Heaviside's Vector Fields

The chief theoretician responsible for the major developments of Maxwell's EM theory was undoubtedly the English hermit and autodidact, Oliver Heaviside. He recast Maxwell's original twenty equations (in Cartesian form) into the now famous four vector and two scalar equations in 1885, using a new "vector" algebra that he independently pioneered, based on the innovative quaternions of William Rowan Hamilton [09]. Although Heaviside always saw himself as a natural philosopher, his experience with telegraphy (his first job) made a permanent impression on his approach to EM propagation. In contrast, Heinrich Hertz was a much better experimentalist than he was a theoretician. None-the-less, after he simply modified the Maxwell field equations to use full time differentials (without justification), Einstein was prepared to refer to this combined new set of mathematical equations as the Maxwell-Hertz equations, obviously unaware of Heaviside's major contributions. Worse, for simply metaphysical reasons, he joined Heaviside in rejecting Maxwell's potentials for the apparently much more 'measurable' fields [19]. This programme joins a tiny fraction of theoretical physicists that have decided that the physics of EM is more closely related to the potentials than to their spatial derivatives (the fields). Paper II was written around a critical analysis of the Maxwell-Heaviside Equations [54] and their immediate and direct simplification when these equations were rewritten using this programme's continuous natural vectors or CNVs.

2.2.2.3 The Heaviside (Lorentz) Force

The discovery of the electron forced the theoreticians of EM around 1900 to finally acknowledge that the physical basis of electricity was particulate and not "stresses in the æther" (the model used by Maxwell). However, loyalty to the Maxwell EM theory had become so strong that it was not discarded but integrated with the fact of the electron [10]. The key idea here was that the mysterious link between the æther and gross matter was now replaced by Lorentz with a mechanical equation of motion that accelerated a point electric charge when subject to Maxwellian EM force fields. However, this so-called 'Lorentz' force was first investigated [19] by Oliver Heaviside, who realized that, in contrast to CM which analyzed instantaneous forces between two particles, the EM force was defined between the line-of-centers of the target point charge and the **earlier** (retarded) location of the source point charge; i.e. $\underline{R}_1[t] - \underline{R}_2[t - R_{12}/c]$. In other words, the direction of the Heaviside electric force $\underline{E}_H[t - R_{12}/c]$ is based on where the two electrons are located at the **two** times involved in the interaction: in effect, this introduces a velocity-sensitive component to the force. In contrast, the Lorentz force $\underline{F}_L$ on a single point charge is based only on the current local values of the electric and magnetic fields.

$$\underline{F}_L[t] = \underline{E}[t] + \underline{v}[t] \wedge \underline{B}[t] / c = \underline{E}_H[t - R_{12}/c].$$

2.2.2.4 Canonical Maxwellian Electrodynamics

By 1905, Maxwell's theory was the principal explanation for EM and the phenomena of light. This theory was grounded in the concept of a stationary medium (the æther) that did not impede material bodies. The fact that this was a medium theory implied that the velocity of excitations through this medium (i.e. light) was independent of the speed of the source emitter relative to this medium: the central idea retained by Einstein in creating his special relativity theory. It is ironic that this key parameter of physics (the speed-of-light, *c*) was discovered first by Maxwell's German anti-ætherist rivals several years before the same number appeared in Maxwell's own theory. The breakthrough came in 1857 when Kirchoff proposed that voltage and current oscillations propagate along circuit wires with the speed of light. These effects result from the multiple inductance and capacitance **far** actions of vast numbers of current and conductor elements. Effectively, Kirchoff was the first to derive delays in the transmission of electrical disturbances along conductors, using a many-body interaction model.

Maxwell's differential equations must always be mathematically consistent with the macroscopic physical facts as these continuum equations were derived directly from the integral equations that summarized the experimental facts of electromagnetism: differentiation and integration are simply reciprocal mathematical operations. Since these basic experiments are now seen to have been undertaken on gigantic collections of electrons over significant time-scales (at the electronic level), it is an unwarranted assumption that these æther-based microscopic equations reflect the actual physical interactions between electrons, although this is exactly the path that the historical evolution of physics has followed. Maxwell's Equations are now viewed as fundamental rather than simply statistically consistent with macroscopic reality. It is usually forgotten that the mathematical operation of averaging does not provide any insight into the range of transactions that are aggregated together into a single number. Viewed from this perspective, the scalar and vector EM potentials are a mathematical technique for calculating statistical results when an electron is interacting with many other remote electrons.

As Jefimenko has emphasized, the Maxwell-Heaviside Equations are not locally causal but correlative: spatial and temporal changes are locally correlated because all EM changes on the target electron at a given point are due to common variations originating with the unique set of source charges. Many scientists today are proud of their 'positivist', anti-philosophical stance: "if it can't be measured, it's not real". However, Maxwell's EM theory fails this test. In empty space (div $\underline{E} = 0$), the equations generate wave-like solutions in the electric and magnetic fields, which are defined as force densities at a point. However, at **all** these vacuum points there is nothing electrical to detect the presence of these fields.

Maxwell's EM theory was formulated in an explicitly asymmetric manner. Specifying the force-densities as position-sensitive and velocity-sensitive fields ($\underline{E}$ and $\underline{B}$) at an arbitrary 'field point', this theory focuses on only **one** side of the phenomenon. This is particularly distracting when this target point is in empty space containing no matter at all. When two electrons are involved, CEM fails to focus on the symmetric nature of the interaction and the two times that arise naturally from this viewpoint; these obviously lead to the symmetric retarded and advanced EM field solutions, as neither the sending electron's interaction time nor the receiving electron's interaction time has any priority. This elimination of the source has meant that not even CEM has ever been able to solve even the two-body EM problem. Maxwell's EM field theory can now be seen as an attempt to describe, using continuum mathematics, the interactions between point electrons. In its modern formulation (CEM) this theory assumes continuity in both electric charge and in the interactions themselves over time resulting in an infinity of interactions even between two electrons in any finite time interval compounded by another infinity of interactions with each other electron in its historic light-cone.

2.3 LORENZ & LORENTZ : THE ELECTRON & THE ÆTHER

2.3.1 LORENZ & HELMHOLTZ EM MODELS

Paper II included an extensive, historical review, which covered EM alternatives to Maxwell's theory, especially the theories of Weber and Lorenz, as elements of these two theories have contributed major components to the present theory. That paper also included a brief commentary on Helmholz's electric-fluid model.

2.3.1.1 Lorenz's 1867 Paper on EM

The almost forgotten Danish physicist Ludwig Valentin Lorenz (1829-1891) proposed in 1867 an alternative theory [65] that extended Kirchhoff's current-induction model introduced ten years earlier. Lorenz had rejected the then popular concept of the luminiferous æther as the physical basis of light as "*it is scarcely possible to imagine a medium in which a wave motion could travel without a trace of longitudinal vibrations*". He was strongly motivated by "the idea of the unity of force" and so he rejected all physical hypotheses as being too premature but based on these analytical results he confidently identified light with transverse variations in electrical current. Lorenz assumed that Kirchhoff's empirical micro-equation (describing the mutual current inductance in conductors via reciprocal, remote current densities) was just the first term in a series expansion of a new, simpler equation. This paper was the first to publish the idea that local EM effects were due to remote source effects occurring **earlier** in time and not simultaneously. Most importantly, he introduced the retarded scalar and vector potentials based on his own earlier published solution to the inhomogenous wave equation. He also realized that the phase velocity of the propagating current fluctuations in a poorly conducting medium approaches that of light; in other words, he could view the vacuum as just a very poor conductor. If Lorenz had defined the "magnetic" field (like Maxwell) as the curl of the vector potential then Lorenz would have derived ALL of Maxwell's æther-based equations of EM.

2.3.1.2 Helmholtz's Electric Fluid Theory

Classical EM in the 19th Century is usually presented as a rivalry between two major approaches: the field theory of Clerk Maxwell and the action-at-a-distance theory of the 'Continentalists', exemplified by Wilhelm Weber. In fact, there was a third school headed by Herman von Helmholtz, who, in German speaking countries, was an even stronger rival of Weber than Maxwell. Helmholtz was the first to pioneer the fluid or hydrodynamic model of electricity and, as a study of modern textbooks shows, this actually was how Maxwell's ætherial field approach was transformed into the modern view of CEM, with Weber's EM theory becoming almost forgotten. The success of this merger is a direct reflection of the mathematical isomorphism of Helmholtz's fluid equations and Maxwell's EM equations – the underlying physical models were widely divergent, with each world-class physicist strongly criticizing the other natural philosopher's physical assumptions.

2.3.2 LORENTZ ELECTRON-ÆTHERTHEORY

From his doctoral thesis in 1875 onwards, Hendrik A. Lorentz based his own approach to electrodynamics on Helmholtz's instantaneous action-at-a-distance reformulation of Maxwell's EM theory in 1870 using his generalized potentials (not the earlier fluid-electricity model). Always convinced of the continuum nature of the æther as the medium 'carrying' the EM interaction, Lorentz combined the ideas of the action-at-a-distance 'EM Continentalists' in 1892 with Maxwell's æther theory by adding the hypothesis that the sources of the EM disturbances were microscopic charged particles that moved in the all-pervasive æther that was absolutely at rest. He also proposed that bulk matter was composed of molecules that each contained a particle of mass m and charge density ρ. These new, charged particles (or 'ions' as he first called them) were bound to their molecules but each could move harmonically within its own molecule. Lorentz then simply assumed that the æther was described only by the $\underline{E}$ and $\underline{B}$ fields of Heaviside's version of Maxwell's Equations in a vacuum, dropping Hertz's $\underline{D}$ and $\underline{H}$ fields, but adding Rowland's convective current moving with velocity $\underline{w}$ relative to the æther. Finally, Lorentz most importantly assumed that the total force $\underline{F}_L$ exerted by the æther on these new, charged discrete particles, each with **total** charge q, was given by '**his**' force law; so the basic 'Maxwell-Lorentz' EM equations were now:

$$\text{curl } \underline{B} = \partial \underline{E}/\partial t + 4\pi \rho \underline{w} \quad \& \quad \text{div } \underline{E} = 4\pi \rho \quad \& \quad \underline{F}_L = q\,\underline{E} + q\,\underline{w} \wedge \underline{B}$$

These equations were defined relative to a reference frame '**fixed**' in the æther. These equations were sufficient for Lorentz to obtain the EM and optical properties of bulk matter by suitably averaging over the EM fields arising from these charged vibrating microscopic particles. It is important to notice that Lorentz's theory is inherently asymmetric. The source of the EM phenomenon is still derived from a continuous charge density and the 'field' effects propagate through an ætherial medium but the measurable physics occurs at the conveniently discrete point charges (the 'electrons').

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2.3.4 LORENTZ THEORY VIOLATES NEWTON'S THIRD LAW

It is difficult to believe today, but in 1902 in his best-selling book *Science and Hypothesis*, France's leading scientist, Henri Poincaré declared that Lorentz's theory "was the most satisfactory of all the current microscopic theories of EM and matter because it best explains the known facts"; although Poincaré in 1900 had pointed out that Lorentz's model of the fixed EM æther violated Newton's Third Law of Motion. Even though Poincaré was able to soon "patch up" this problem by adding further hypotheses, Lorentz himself was still unconvinced by all this 'help', responding with the comment that "in an æther-based theory the action of an emitter of unidirectional radiation could not be simultaneously compensated for by the reaction of the absorber and Poincaré had 'dodged' this point." Furthermore, Lorentz also showed that whenever Poynting's energy current varied with time then the æther was no longer in equilibrium. In 1904, Poincaré finally admitted defeat [14] in this area, acknowledging Lorentz's observation of the non-simultaneity of action and reaction in any EM field theory. For most of his life, Boltzmann argued for the priority of particle mechanics as the foundation of physics, when 'atomism' was out of fashion. Just before his suicide in 1906, he updated this position [15] by emphasizing that *Electron Theory* should be seen as the future foundation for physics. This research programme is based on this radical viewpoint.

2.4 OTHER EM THEORIES OF THE ELECTRON

2.4.1 THE LARMOR ELECTRON

As Maxwell realized, the power of the Lagrangian method is that everything is focused on a single energy function and the system's internal structure can be ignored, encouraging a phenomenological approach to theories of physics. In 1894, these techniques were used very effectively by the Anglo-Irish physicist, Joseph Larmor (1857-1942) when he constructed his field theory of the electron [16], as the flow of a finite number of real, but specialized, locations in the æther. He proposed that this flow was proportional to the magnetic intensity (H); this implied that the æther must possess MacCullagh's rotational elasticity to generate the corresponding electric displacement (D). But in 1895, Larmor abandoned this 'vortex' model in favor of electric currents, conceived of as flows of electrically charged particles (e.g. electrons), as did Lorentz in the same year. These were the first formal breaks with the continuum model of the æther that had been the central foundation of Maxwell's EM theory. The Lagrangian approach always separates energy into energy of motion (kinetic) and energy of position (potential), where the system's state is totally determined by the positions and velocities of its 'parts'. Thus, Maxwell's assumption of a Lagrangian model of the luminiferous æther immediately implied oscillations in this elastic medium i.e. 'light'; a result that Maxwell was always trying to establish. In the 19th Century, many dynamical models of EM were being built by treating H as the medium's velocity (i.e. kinetic energy) and D as the curl of the corresponding mechanical displacement of the medium (i.e. potential energy). Although never acknowledged, Maxwell's dynamical theory of 1865 was based on a property of vortex motion in incompressible fluids that Helmholtz had deduced in 1858. So, for example "*if current is supposed to be proportional to the vorticity in an incompressible liquid, then the magnetic field has the same analytic form as the velocity field but unlike currents in closed circuits, closed vortices cannot exist without undergoing translational motion, even if no other vortices are present*." It is one of history's ironies that Larmor is almost forgotten today. As the chronicler of mathematical physics at Cambridge University [17], Andrew Warwick writes: "*Joseph Larmor was by far the most important contributor to the development of EM theory in Cambridge during the 1890s*."

This world-class physicist, Lucasian Professor from 1903 until 1932 and the immediate predecessor of Paul Dirac in the same professorship made famous by Newton, wrote his scientific best-seller *Æther & Matter* that has almost disappeared today while his *Collected Papers* are nowhere to be found across the Internet.

As FitzGerald wrote [18] to Heaviside: "*The electric displacement at a point is represented by a vector but it is very unlikely that it is really a simple displacement of the point; it is more likely to be a complex change in the structure of the æther at this point and this too can be represented by a vector. Accordingly, Larmor separates the electric force, which acts on the æther and produces the electric displacement, from the force on a moving electron due to its motion across a magnetic field*." Buchwald also writes: "*During 1894-1897 the basic principles of Maxwellian theory were abandoned and the entire subject reconstructed on a new foundation – the electron – by Joseph Larmor, in consultation with FitzGerald.*" The later followers of Maxwell had founded their dynamical theory of EM on two assumptions: 1) the EM energy must be localized in each volume element of the medium as in a material continuum, where it is localized by its kinetic and potential densities; 2) all systems are governed by Hamilton's Principle.

Larmor proposed [19] that his 'electrons' (which were conceived as stable and permanent singularities in the EM æther) had the following universal and invariant properties: 1) constant strength (charge), 2) fixed, maximum amplitude of internal radial vibration, 3) an identical phase in this vibration; in effect, a universal vibration rate that is always in phase. He viewed these singularities as universal constituents of gross matter based on remote spectroscopic observations confirming the same fixed number of elements and the independence of the gravitational constant across different elements. Most of these features are adopted in the present theory of the electron.

2.4.2 THE EM WORLD VIEW

Wilhelm Wien (1864-1928) first suggested in 1900, what later became known as the 'EM World View', that the mechanics of gross matter could be derived from Lorentz's EM theory of æther and ions. This suggestion was diametrically opposed to Hertz's earlier proposal in 1894 of a 'mechanical world view' that would derive the phenomena of EM from an ontology that only required Newtonian-like particles. Wien's proposal was an extension of Thomson's 1881 suggestion that inertial mass had its origins in the EM field based on his analogy with hydrodynamic drag, although Thomson was not suggesting that all mechanics could be reduced to EM. Wien quoted Searle's 1897 analysis of a moving rigid sphere covered with a uniform surface electrical charge where its inertia increased due to the self-induced drag of its own induced magnetic field. This inertial increase was thought to be confirmed by Lenard's recent (but very limited) measurements of the e/m ratio for cathode rays accelerated to about 0.3*c*. Kauffman summarized the new excitement surrounding the idea of the electron in 1900 when he declared [20] at the Annual Meeting of German Scientists "*Everywhere, the electron plays an important part in electrical and optical phenomena ... electrons are the long-sought-for primordial atoms*."

2.4.3 THE ABRAHAM EM ELECTRON

In 1903, Max Abraham (1875-1922) totally adopted Wien's suggestion when he proposed that all of the electron's inertial mass was EM in origin due to its own self-induced EM fields. This theory was based on assuming the electron to be a rigid sphere with either a uniform volume or surface charge distribution. The sphere had to be rigid (without any proposal for maintaining this rigidity) so that the electron would not explode under its own repulsive forces. Along with six (6) other major hypotheses, this electrical density was further assumed to be an invariant, fundamental constant of nature. The EM effects were assumed to be completely describable by the "Maxwell-Lorentz" EM field equations. Unfortunately, as often happens too frequently, the resulting equations developed by Abraham were unsolvable, as (in this case) the retarded nature of the EM interaction required complete knowledge of the entire history of the electron's motion. Using a 'quasi-stationary' approximation, wherein the electron does not radiate when it is subject to 'slow' external accelerations, Abraham was able to calculate the diameter D of a spherical electron with a uniform surface distribution as $D = e^2/mc^2$ (the so-called 'classical electron diameter', about 10^{-13} cm) with correction formulae for the electron's "longitudinal" and "transverse" masses, in the low velocity limit ($v << c$). In the light of these results, Kaufmann [21] re-analyzed all his own data and decided they now confirmed Abraham's model of the electron, at least for the transverse mass being purely EM; historical re-examination of Kaufmann's own photo's seem to belie any accuracy sufficient therein. The EM theory of matter analyzed mobile charge in terms of a rigid or deformable sphere containing either continuous electrical charge throughout its interior or restricted to its surface. The resulting analyses focused either on the magnetic field produced by the sphere's motion or on how this 'sphere' was acted upon by an external magnetic field. Maxwell's EM theory was also often applied to calculate the particle's "electromagnetic mass". As well as increasing its kinetic energy, any external force that accelerates this charge must also supply extra energy into the EM field, since a moving charge generates a magnetic field. The finite sphere's motion defined an inner surface of integration as well as the outer boundary at infinity. Most calculations followed Maxwell and assumed the use of the Coulomb gauge (div $\underline{A} = 0$) which resulted in the Laplacian form ($\nabla^2\phi = 0$), so that the scalar potential (ϕ) is not propagated across space.

The Maxwellians interpreted this result as the incompressibility of the electric 'quantity' so that the electric 'pressure' (ϕ) must be transmitted instantaneously through the medium; in other words [22], the longitudinal EM stresses were communicated instantaneously but displacements in the medium propagated laterally at finite speeds.

2.4.4 ABRAHAM-LORENTZ ELECTRON DIFFERENCES

Both Lorentz's and Abraham's models of the electron agreed on the calculation of the electron's "transverse" mass and this was the only component that was subject to experimental verification in Kaufmann's experiments. Both theories assumed that all of the electron's energy and momentum were completely EM. Abraham's own theory assumed a "rigid sphere which underwent an imaginary dilation in an imaginary co-ordinate system", while Lorentz's "deformable electron underwent a real contraction in the direction of motion but maintains its spherical shape in an imaginary co-ordinate frame". It was these 'metaphysical' distinctions [23] that lead to different predictions for the unmeasurable "longitudinal" electron mass.

Classical electrodynamics is asymmetric in distinguishing the empty field point ($\underline{x}$, t) from the historical, source point $\underline{x}'$ at another time t', where earlier there existed an element of electrical charge. This asymmetry is also reflected in the *acausal* pre-acceleration of the Abraham-Lorentz electron, which results from the finite size of the electron sphere (covered with electrical fluid) that moves as a rigid body. In other words, some micro-volumes (cells) of electrical fluid effect other cells in the sphere at a later time but these other parts react back instantly (since this is a mechanically rigid body) on the source cells. This asymmetry of action and reaction between the different cells in the electron contradicts Newton's Third Law of Mechanics, which declares these should be equal and opposite, guaranteeing total stability (and momentum) across the universe.

2.4.5 SOMMERFELD'S ELECTRON

Both the Lorentz and Abraham models of the electron began with structures of a finite size (that then required non-EM "cohesive" forces to maintain the rigid forms) as both scientists were reacting to Heaviside's 1889 analysis of Maxwell's theory that predicted a mathematical singularity at light-speed *c*, when an electrified point charge freely moved at this speed relative to the æther. In two papers in 1904 and 1905 [24] Arnold Sommerfeld (1868-1951) proved that a rigid spherical electron with a uniform volume electric charge distribution could undergo force-free motion both at and beyond light-speed, whereas if the charge was distributed uniformly over its surface then [25] it could not.

As Wolfgang Pauli (1900-1958) has emphasized [26], all finite-size electron models had to assume a rigid structure for the electron, so that their proponents could separate out a 'velocity' for the whole electron; in other words, they assumed that the electric charge distribution ρ, is rigid and moves only with translational motion. Mathematically, they wished to use the standard point-form for the current density, $\underline{J}[\underline{x},t] = \rho[\underline{x},t]\ \underline{v}[t]$ instead of the more general form $\text{div}\ \underline{J}[\underline{x},t] = -\,d/dt\ \rho[\underline{x},t]$ or even the more limited view of $\underline{v}[\underline{x}, t]$. Of course, no attempt was made to explain how all this mutually repulsive electrical fluid continued to flow without rapid dispersal: the mathematical equations (as usual) were their own self-justification.

2.4.6 WHEELER-FEYNMAN REMOTE ABSORBERS

Wheeler and Feynman in 1945 showed that classical electromagnetism (CEM) could be restated using Gauss Asynchronous Action-at-a-Distance between continuous charges. They showed that when the accelerated charge emitted a symmetric combination of both retarded and advanced waves towards the set of absorbers (that completely surrounded the charge) this would induce advanced waves far in the future to move backwards in time to the original charge. This would exactly cancel the original advanced wave of the accelerated charge and coherently reinforce the original half-retarded wave bringing it to full strength while slowing down ('radiation reaction') of the accelerated charge. This model produced the surprising result that **all** other electric charges throughout space and time (both past and future) determine the behavior of any specific charge at a given point in space and time. Some of these radical concepts have been incorporated into the present EM theory.

2.4.7 MEAD'S COLLECTIVE ELECTRON MODEL

There are few contemporary physicists that have thought as long and hard about EM as Carver Mead. The focus of Mead's final views (in this central area of physics) were the real charges that when in motion directly generated both the real electric currents and the vector potential. He rejected both the 'magnetic field' and the 'displacement current'. Very clearly, Mead describes the EM potentials [68] are not independent degrees of freedom but the potential for interaction. "They represent the net effect of all charges in the system on an infinitesimal charge at a given point." This is exactly the position taken in this EM research programme with respect to Maxwell's Equations and the key role of the vector potential in CEM.

Carver Mead, who studied EM with Feynman, developed an approach he calls *Collective Electrodynamics* in which all electromagnetic effects, including quantized energy transfer, result from the interactions of the **wave** functions of electrons behaving collectively. Mead rejects the traditional Maxwell-Heaviside EM fields and, like Dirac and Feynman as well as Jefimenko, focuses on the vector potential and real current density based on the electrons as the only real EM sources. In Mead's theory, Planck's energy–frequency relationship comes from the interactions of electron eigenstates, while no use is made of the photon concept. This approach is related to Cramer's transactional interpretation of quantum mechanics and to the related Wheeler-Feynman absorber theory of electrodynamics. Mead's and Cramer's theories are original approaches that have both been fruitful sources of concepts that have been incorporated into the present EM theory.

2.5 THE MODERN ELECTRON

Two modern historians of the electron [27] have written: "*Planck equated 'æther' with the 'vacuum' and ascribed no physical properties to the absolute vacuum*." As a result, the German physicist, Emil Cohn was the first to develop an electron theory where the vacuum played no role whatsoever (i.e. treated the æther as non-existent). The Russian, Yakov Frenkel was the first to dismiss theorizing about the internal EM structure of electrons, when in 1925 he proposed a relativistic theory of point electrons with no geometric extensions in space at all. Both Pauli and Dirac also believed that only point models of electrons were legitimate – the extensionality position taken by this research programme.

Buchwald and Warwick [27] have also made a very important point: "*The persistent disagreement about the basic nature of electricity has not inhibited electrical engineers and physicists from effectively harnessing it to develop the EM technologies of global telegraphy, power generation, light, traction and wireless transmission. ... Indeed, it was theoretical physics that followed from the results of physics experiments using new technologies that evolved independent of theory. The arrival of the first material artifacts of 20th Century electronics (e.g. Fleming's thermionic rectifying valve in 1904 and Braun's cathode ray oscilloscope in 1902) soon after the development of theories of electrons is thus little more than a coincidence*." So, it can only be hoped that more study of the history of theoretical physics would, perhaps, introduce a little more humility into the minds of theoreticians. We are far more followers and explainers than innovators in the benefits of science.

3. A BRIEF HISTORY OF TWO-ELECTRON DYNAMICS

3.1 FOKKER LAGRANGIANS

In 1922, H. Tetrode proposed [28] that the EM interaction consisted only of the exchange of photons between pairs of electrified particles. He pointed out that since light (the interaction) has a finite speed then the emitting charged particle must 'know' about the future location of the receiving particle, which also 'knows' where the emitted photon came from; in other words, the EM interaction is **not** a 'broadcast' theory (spherical EM waves) but a 'point-to-point' interaction that must be ray-like and symmetric in time – the basic viewpoint adopted by the present theory.

Due to severe logical problems in classical EM theory, such as the pre-acceleration of a point particle in 'response' to the arrival of a pulsed electric wave, Adrien Fokker in 1929, building on Tetrode's earlier suggestion, proposed [29] a point form Lagrangian where an accelerated electric point charge produces both advanced as well as retarded fields.

In 1945, Richard Feynman (1918-1979), along with John Wheeler, his supervisor at Princeton, published a joint paper [30] based on his PhD research demonstrating that the combination of half-retarded plus half-advanced EM waves emitted from an accelerated charge could generate only net retarded solutions. The advanced waves, originating later in the set of total absorbers surrounding the charge, completely cancelled the advanced wave of the accelerated charge while coherently reinforcing the half-retarded field to full strength and slowing down the accelerated charge ('radiation reaction').

Fokker-type action integral theories involve only asynchronous direct interactions at a distance without introducing explicit fields so that the particles involved do not experience any 'self-force'. Since these theories use Lagrangians describing only pair-wise line-of-centre interactions involving no radiation external to the charges in the system, it is not at all obvious that this approach is equivalent to Maxwell's EM theory where radiation is emitted spherically from all the continuous charge densities. Wheeler and Feynman [30] showed this equivalence only when all the emitted radiation from the system charges is ultimately absorbed (by an infinite number of remote, spherically-distributed charges) but this cannot be reconciled with even spherical emissions from a finite (particularly, small) number of particles.

Fritz Rohrlich also attempted to eliminate the infinities of classical electrodynamics in 1964 by proposing [31] a new action integral for a closed system of only one point charge interacting with a radiation field, such that this particle is 'free' (in uniform motion) asymptotically as time approaches plus or minus infinity. He separated the EM field into 'free' (radiation) and 'bound' (Coulomb) parts. The application of Hamilton's principle to this action integral resulted in the Lorentz-Dirac equation of motion for the electron. He then extended this equation to n-particles by adding the Coulomb interaction term between the particles – this resulted in a generalization of the Fokker action integral with extra EM field terms. This gave a set of coupled equations that again could not be solved: another mathematical example of 'formalized physics'.

Van Dam and Wigner in two papers [32] also proposed a Fokker-type asymptotic collision theory of multiple particles in terms of local time derivatives of each particle that conserved total energy and linear angular momentum across the collision when there are no bound-states involved and the 4-force exerted by one particle on another particle is anti-symmetric.

3.2 RELATIVISTIC TWO-ELECTRON MODELS

Although H. A. Lorentz merged his theory of the electron with Maxwell's theory of the æther, it is most surprising that neither Lorentz, nor anyone else for about 50 years, tried to investigate the interaction between two of Lorentz's electrons, each interacting through the intermediary Maxwellian EM æther, probably because most EM theorists continued to follow Lorentz in viewing the electron as the object (or target) of the EM fields rather than symmetrically as the prime source of these fields. The dropping of the æther concept in the early part of the 20th Century left Maxwell's EM theory without its philosophical roots but these were soon replaced by the continuum model of electrical fluid interacting with the ætherless vacuum of space itself. Since conventional physics has decided that the acceptance of Maxwell's theory of EM justifies the necessity for using relativistic formulations of mechanics, there have been several attempts in the last 100 years to solve the 'simple' problem of two charged particles moving under their mutual influence in a rigorous relativistic manner. All these attempts have failed, as the following excerpts will demonstrate.

In 1972, Anderson and von Baeyer used [33] the time-symmetric (i.e. retarded plus advanced) Lienard-Wiechart potentials in a Fokker-like action integral for just two identical charged particles initially moving towards each other along a straight line, so that the direct motion eliminated any 'magnetic' effects. They first assumed simple world-lines for each particle that

were used to numerically integrate the equations of motion to generate modified world-lines, this technique was then again repeated iteratively until convergence was achieved. The authors found no convergence for opposite charges in very nearly circular orbits, even for very low velocities, but did find convergence for the linear repulsive model when the limiting speed of each particle remained below 95% of light-speed but not at higher speeds. At intermediate energies the two approaching electrons reversed their motion at nuclear distances of the order of the 'classical electron radius', $r_0 = e^2 / mc^2 \cong 3 \times 10^{-13}$ cm.

Schild has written [34] in the introduction to his 1963 paper addressing the EM two-particle problem that: "*Unfortunately we do not know how to attack a highly relativistic two-body problem within the framework of quantum electrodynamics*." Schild first tried to solve a classical two electron ('positronium') problem by assuming that one electron circulates smoothly around the other, which is treated as a classical magnetic dipole at rest generating a static magnetic field. He equated the Lorentz force with the relativistic circular acceleration. He also assumed the relativistic mass formula for both the electron's total energy and the mechanical angular momentum, which he assumed is still given by Bohr's major quantum rule ($n\ h/2\pi$). Schild found the resulting motion highly relativistic (in contrast to the Bohr model of the hydrogen atom) with sub-nuclear orbitals, $r_n = r_0 / 2n$ and gigantic total energies, $E_n = 2\ mc^2\ n^2 / \alpha \cong 140\ n^2$ MeV. Schild interpreted this result to mean that the kinetic energy vastly exceeded the potential energy or, in classical terms, the 'magnetic' energy was very much more important in this interaction than the static Coulomb energy. Since Schild also knew that stationary circular motion of two point charges was not possible using purely retarded interactions he wrote "it is necessary to assume that the EM interaction is time-symmetric, i.e. half-retarded and half-advanced". This Fokker technique avoids any reference to the EM field variables, including any radiation reaction force, and reduces the problem to two coupled equations of relativistic motion involving each particle's local 4-velocity and the other particle's electromagnetic 4-momentum. The EM field at one particle arises only from the two points where each set of light cones intersect, i.e. the retarded and advanced points on the other's world-line. However, Schild then only used the electrostatic Coulomb interaction to 'prove' that the resulting orbits are the usual non-relativistic Bohr orbits (equivalently, $v_n = \alpha c/n$) with just small corrections for the time differences across the orbits and other relativistic effects. He failed to comment that this latter (Coulomb) approach contradicted his earlier 'quasi-classical' analysis that emphasized the importance of the 'magnetic' energy!

In 1963, Currie et al demonstrated [35] the impossibility of establishing a relativistic mechanics within the framework of the canonical representations of the Lorentz group, in other words, there can be no Hamiltonian formulations involving Dirac-like electrons. The authors stated (without proof) that their proof could only be circumvented if the existence of world-lines was given up (i.e. the concept of trajectories generated by the motion of point particles).

This problem was again addressed [36] by Bruhns in 1973 when he solved the two-body Fokker action integral in one spatial dimension by assuming that one electric charge experiences the retarded field of the second charge, which only experiences the advanced field of the first. Unfortunately, the initial conditions that he also assumed involved zero separation when their velocities were zero, ignoring the resulting infinite Coulomb repulsion. He declined to attempt any solution for the time-symmetric problem on the grounds that "*this problem has so far defied general analysis, since its solution requires knowledge of an infinite set of position-velocity data for each charge*."

4. THE NEW ELECTRON INTERACTION

4.1 DISCRETE ELECTROMAGNETISM

This research programme views electricity simply as collections of discrete point particles, which have, for over 100 years, been experimentally identified as electrons. These electrons are viewed as interacting cyclically only over a universal, non-zero time-rate in pair-wise combinations. Both electrons, in every individual interaction, are treated symmetrically across space at two distinct points in time, each with a history (and future) and each mutually influencing the other asynchronously. In contrast, orthodox physics has rejected any model of classical electricity in favor of simply a set of differential equations.

4.1.1 REJECTION OF THE CONTINUUM IN EM

Newton had a very skeptical view of DesCartes' æther model. When Newton introduced his concept of inertial mass, in Definition I on page one of the *Principia* he explicitly excluded any pervasive æther that might penetrate all matter with the comment "I have no regard in this place for a medium, if any such there is, that freely pervades the interstices between the parts of bodies" [37]. As Thayer writes "Newton's æther is similar to air, but far rarer, subtler and more strongly elastic. The æther is a vibrating medium, only the vibrations are far more swift and minuter than air … yet all are equally swift." But Newton did "not view light as either æther nor its vibrating medium", rather he viewed rays of light as "small bodies emitted every way from shining substances and when they impinge on any refracting or reflecting surfaces must necessarily excite vibrations in the æther." [38] This particulate view of light was a revolutionary rejection of the Cartesian consensus.

This research programme follows Newton in most of his metaphysical preferences. In particular, space is not viewed as an active component of nature but only as a passive background – the 'stage' on which the drama of the real world unfolds over all time as myriads of electrons weave their complex trajectories. In particular, since discrete electrons are known to be the ontological foundation of electricity then all models based on a continuum view of electricity are explicitly rejected; this not only includes any æther models but also continuous charge models that treat electricity as if it were an "electrical fluid".

Further, the idea of reference-frames is viewed here only as a mathematical convenience without any physical significance; only in our own imaginations can such frames be shifted, rotated or accelerated. In the opening remarks in his 1905 paper on electrodynamics [39], Albert Einstein (1879-1955) wrote "… all electrodynamics is based on kinematics of the rigid body, since the assertions of any such theory concern the relationships between rigid bodies (co-ordinate systems), clocks and electromagnetic processes." In contrast, this research programme rejects the idealization of 'rigid bodies' or time-keeping devices ('clocks') as useful metaphysical foundational concepts. In the final part of this paper, the concept of the basic 'unit of time' is proposed as the intrinsic keystone of nature, with the idea of space taking a secondary role to time through the concept of asynchronous temporal interactions between pairs of electrons defining spatial separations.

Only a few of the 'giants of physics', like Wolfgang Pauli, have even expressed concern that the continuum basis of EM field theory conflicts with the discrete nature of reality (e.g. e and m) [26]. This deep ontological awareness has also been reflected by a few modern authors, who all try to work around this central obstacle by defining special mathematical limit procedures for concepts like charge. This definitional conflict at the heart of classical EM is sharpened by the experimental fact that any attempt to measure the EM 'fields' can only be accomplished using corpuscular (discrete) test charges.

Field theories are also filled with 'plane waves' that pervade the whole of space, from one end to another, usually across all of time. These are a great mathematical simplification as they enable Fourier transforms to be taken of any phenomena that vary in space and/or time. But it takes a 'giant', like Pauli [40] to point out the physical paradox their usage generates, when he acknowledges "a plane wave is a mathematical idealization as it is infinite in extent and has, therefore, infinite energy." This does not bother mathematicians even though, if the energy of the wave is anywhere, then it must be everywhere.

4.1.2 ELECTRICITY IS DISCRETE

Since the electron is the 'star of our show', or the central metaphysical subject of this research programme, and 'history is always important' (section 1.1.2) then it is appropriate to provide some biographical history of this key concept. Indeed, in contrast to the other 'stable' components of matter (the proton and neutron) the electron has remained throughout its entire history as a truly elementary particle; its universality confirmed by the discovery of beta decay emerging from the nucleus. It was the Irish scientist, George Johnstone Stoney (1826-1911) who invented the term "**electron**" in 1874 to refer to a possible fundamental unit of electrical charge (positive or negative), based on Faraday's laws of electrolysis [41]. He then extended this new term in 1891 to include the idea of a tiny, charged particle (i.e. with mass) that emitted radiation as it rotated around, inside an atom, to which it was inseparably bound (therefore, having no independent existence).

Stoney introduced this extension to account for the double lines in gas spectra. Soon after, in 1894, both Lorentz and Larmor independently appropriated this term into their own "theories of the electron" (see later).

The story of the electron has been well described by science historians, Buchwald and Warwick [42]. In 1858, cathode rays were discovered by Julius Plucker (1801-1868) using the mercury vapor pump, invented by Heinrich Geisler just three years earlier. In 1879, William Crookes reported on several major characteristics of these rays in experiments involving gas filled electrical discharge tubes but it was J. J. Thomson (1856-1940) who was the first to propose the hypothesis that these rays were universal particles emitted from the metallic cathode and whose nature was independent of the type of gas or cathode. He was also alone in proposing that these charged particles were the universal constituents of all atoms when he wrote: "it is the fundamental substance in terms of which all electrical processes can be expressed". Contrary to modern mythology, this 'discovery' was not the consensus in 1906 when Thomson received the Nobel Prize for his "research on gases". Historians have subsequently confirmed that this 'discovery' was later promoted by some students of "J J", such as Paul Langevin and Ernest Rutherford (and his students, like Chadwick and Blackett), who came to dominate the early research on sub-atomic physics. Ironically, Thomson never referred to 'his corpuscles' as "electrons" until 1923 but he always remained totally convinced of their central importance to 20th century physics.

The early 20th century Cambridge school of atomic research was the first group to view the charge and mass characteristics as a particle's unique 'signature' [43]. In the case of the electron, the best values today are taken to be:

$$e = 4.774 \times 10^{-10} \text{ esu} = 1.602 \times 10^{-19} \text{ coulombs}; \qquad m = 9.10939 \times 10^{-28} \text{ grams}.$$

Pauli was always systematic in making his assumptions explicit, so that when he began his lecture course on electro-dynamics [44] he wrote "*It is often more convenient to think of charge distributions as being continuous, instead of consisting of a set of point charges*." It is with this 'small' step of mathematical idealization that field theory is smuggled into the modern view of the microphysics of charged particles. Pauli recognized the significance of this key assumption and added: "*As long as one remains in the domain of macroscopic physics then the actual atomic structure of electricity can be neglected*." Of course, Maxwell was personally unaware of the existence of electrons in his lifetime but almost all 20th Century mathematicians who write on EM have long ignored this physical caveat. Pauli carefully introduced a mathematical limit process for any volume of space such that the number of charges becomes infinite at the same rate that the magnitude of each electrical charge goes to zero while the total charge enclosed within the spatial volume remains **constant**.

4.1.3 EM INTERACTION IS DISCRETE

The Hamiltonian approach is only appropriate for describing the dynamics of systems when a system's interactions all occur at the same time; so it is appropriate for describing situations involving point collisions, instantaneous forces (e.g. gravity), forces that are derived from purely spatial potentials and localized interactions (the field model). If this technique is used for any other situation then it will only be an approximation that can be suitable if the time for the interaction to cross the whole system is much less than the 'natural' cycle times of the system oscillations.

Theories of light, beginning with Newton, continuing with Huygens, central to Maxwell and integral to QED, have all failed to produce a convincing, realist description of this key aspect of nature, whether based on particulate, wave or field models. This programme will offer a visualizable model of this phenomenon in terms of repetitive electron-electron interactions.

4.1.4 MOMENTUM EXCHANGE IS DISCRETE

Prior to 1870, physicists did not conceive of **energy** as an existential object ('real thing') so it could not have a location but when it was conceived of as a fluid then it could 'flow' as Poynting proposed in 1884. Heaviside, in 1891, recognized the distinction between conservation of energy over time versus across space: "If (energy) possessed continuity in time only, it might go out of existence at one place and come into existence simultaneously at another. This view does not recommend itself. The alternative is to assert continuity of existence in space, i.e. it traverses the intermediate space." [45] Heaviside ignored the other alternative (because he opposed action-at-a-distance) that the re-appearance occurred at a later time.

From 1907, when giving the Adamson lecture at the University of Manchester, J. J. Thomson considered the **æther** as an "invisible universe" (hidden dimensions?) where energy and momentum were temporarily stored when they appeared to be lost within EM interactions. This æther could therefore possess inertia and was the source of all electrical and magnetic forces. Thomson viewed this æther like "a bank in which energy may be deposited and withdrawn." Like Larmor, he

viewed this æther forming vortex rings that persisted over time; he maintained this view from his 1882 Adams Prize essay until his final book in 1936 [46], where he still insisted that all mass was electrical and resided in the electron. [47]

4.2 ELECTRON CHARACTERISTICS

In accordance with our stated methodological preferences, it is only appropriate that after introducing our central ontological subject (the electron) that the associated metaphysical assumptions that form the rest of this theory should be made explicit.

4.2.1 REAL PARTICLES

This research programme totally agrees with the fundamental axiom of the 18th Century Scottish School of 'Common Sense Philosophy', which believed that "**the world really does exist**". However, like Maxwell, this programme also recognizes that it is necessary to generate fruitful metaphysical hypotheses in order to expand our scientific knowledge. [48]

Aristotle, as a realist in contrast to Plato's universal idealism, began his book *Physics* by emphasizing the central importance of ontology and the need to focus on the ultimate objects of reality that exist independently in space and may change their relative positions over time. This emphasis on the study of real entities has distinguished physics from mathematics with its timeless concepts of geometry or logic from the very beginning of Western civilization. This distinction has been more than blurred with the developments of theoretical physics in the 20th Century; the results, in our view, have been disastrous.

Any fundamental theory of nature must be grounded in empirical experiments. Modern physics has demonstrated that the electron is incredibly stable (it has never, ever been seen to decay into anything else) and the electron is always indivisible, even under extreme duress (e.g. electron collisions at energies exceeding 90 GeV). These are very desirable properties for any form of foundational building material when constructing 'our castles in the air'.

The energetic 'disappearance' of an electron with its own 'anti-particle', the positron will be thought by many physicists to be a counter-example to the idea of the fundamental stability of the electron. This phenomenon will be addressed explicitly in a subsequent paper that introduces the positron into this theory. Time is the secret.

4.2.2 WHAT ARE ELECTRONS?

Every answer to the reductionist's question: "What is X made of?" will always generate the repetition of the same question. Like all such examples of recursion, there must be an exit to the loop to avoid infinite regress. Since this programme views electrons as the ultimate 'stuff' of reality then the answer to the question "What are electrons?" must be the self-reflexive response "They are what exists". In other words, *electrons are what everything else in the universe is made from*. Today's 'Standard Model' of Elementary Particles, while recognizing that electrons are one of the fundamental ingredients of nature, is not prepared to make this uniquely aggressive claim for the priority of electrons: it is one of the goals of this research programme to prove this claim for nature of the 'real stuff'. Subsequent papers in this programme will present all-electron models for all of the so-called 'elementary particles'. When anticipating the metaphysical question of his skeptical contemporaries to explain "what is gravity?" Newton gave the famous response "*hypotheses non fingo"* (I propose no hypotheses) [06]. This was his refusal to invent any new substance or fundamental entity with its own independent, ontological status, which he correctly saw would have been an empty verbal reply, supplying no further insight than he had already provided with his mathematical descriptions. He realized that gravity was an intrinsic property of the interaction between particles with inertial mass. Similarly, this research programme views all the so-called 'four fundamental forces' to be different aspects of the EM interaction between electrons and, like Newton, rejects the popular approach (exemplified by the 'Standard Model') that invents new entities as a mode of 'explanation'.

4.2.3 POINT PARTICLES

In his 1893 treatise on physics, Helmholtz, as a natural philosopher, developed his dynamics by first focusing on physical concepts, emphasizing the major feature of the "mass-point" concept, namely its continuous existence across time over space [49]: "the position of the point must be defined by a continuous and differentiable function of time; otherwise the point could be in two places at the same time, disappearing here and instantly re-appearing there. Such a discontinuity would violate the identity of the mass-point since matter cannot be created or destroyed." In contrast, the velocities and limit-centers of continuum "mass-cells" can be differentiated with respect to the spatial co-ordinates; this distinguishes the continuum model of matter from the discrete model, where time is the only primitive variable.
Furthermore, only systems of mass-points can undergo disordered motion where each point can move independently of its neighbors – this is not possible with continuously distributed matter. In this research programme the electron is assumed to be an actual point particle, that is, it has no finite spatial extent. This avoids the Classical Electron problems of the shape of the electron, the distribution of its mass and charge densities and the infinite electrical repulsion arising from the

infinitesimal proximity of its 'electric paint'. It should also be pointed out that this is not the "smallness assumption", where the spatial dimensions of the electron are simply assumed to be much smaller than the relative separation between distinct electrons; this assumption does not solve the problem of infinite 'self-energy'.

One of the most famous gatherings of international physicists was the Fifth Solvay Congress in 1927; this was the first congress held after The Great War that was attended by German scientists and was the venue for the famous Bohr-Einstein 'debates' on quantum mechanics. As President of the Congress, H. A. Lorentz summarized the discussions at the end and rejected the new direction of atomic physics as the success of abstract mathematics over the cost of intelligibility – a price he viewed as excessive. In particular, he still viewed an electron as "a corpuscle that at a given instant of time is located at a definite point in space and later when located elsewhere is connected continuously by its trajectory, which is a line in space. It does not appear to me as an *a priori* impossibility." The standard view today is that this classical conception is wrong and has been obsoleted by quantum theory, especially quantum field theory. This is not a view shared by this programme, which accepts Lorentz's 1927 challenge and explicitly returns to the Newtonian conception of a point particle; it will be shown later that a revision of this older view is 100% compatible with all the experimental evidence centered on the electron.

In summary, the class of real objects, known today as electrons, is considered here to be the basis (ontological primitives) of the present theory of fundamental physics. The investigation of this proposition forms the heart of this research programme. However, unlike Democritus and the ideas of classical materialism, this programme views electrons as having no geometric properties – as true point particles they have no shape nor any finite size. In fact, they have no significance outside of their ability to interact with one another. All of their properties are only defined relative to this inherent EM interaction.

4.3 THE ELECTROMAGNETIC INTERACTION

Maxwell began his major paper [50] with the words "*The most obvious mechanical phenomenon in electrical and magnetic experiments is the mutual action by which bodies in certain states set each other in motion while still at sensible distances from each other.*" [51] Although this resembled the physics of gravity, which Newton modeled as action-at-a-distance, Maxwell adopted a continuum approach, motivated primarily by personal religious and metaphysical reasons. [03]

In his lectures on electrodynamics [52], Pauli concludes with the following discussion: "*If light is emitted and then absorbed then the conservation laws for kinetic energy and mechanical momentum are valid **after** the absorption. Without associating energy and momentum with the electromagnetic field, these conservation laws cannot obtain at each instant of time between emission and absorption. However, it seems unnatural to eliminate the field since it is not apparent why more reality should be ascribed to the material particles than the field*." But Pauli then goes on to say: "*that the equations for a perfect vacuum (where $\rho = 0$ and $\underline{J} = 0$) are only an idealization, since electromagnetic fields can be produced and detected only with the use of mechanically describable particles that carry charge.*" Pauli describes this EM paradox with the comment: "*a noteworthy duality*". As a man of his times, and an excellent mathematician, Pauli's commitment to the concept of the field was too great to challenge.

Newton's most imaginative innovation in his theory of gravitation was the idea of action-at-a-distance (**AAD**); this was so revolutionary that even he had difficulty in believing in it, going as far as describing it as "a great absurdity". This concept was strongly rejected by most of his famous contemporaries, such as Huygens and Leibniz. Earlier, DesCartes had rejected, on religious grounds, Roberval's theory of universal gravitation just because it was based on AAD as "the material particles would need to be endowed with knowledge of a truly divine sort, so that they may know, without any medium, what takes place at very great distances and act accordingly." [53] DesCartes was not too concerned with the divine power of space.

In their attempt to reduce the problem of action-at-a-distance between remote charges to 'contact' forces between the differential cells that are in contact (the model behind all uses of the gradient operator ∇ in its various guises e.g. curl) field theorists introduced the spurious effects of an infinite number of third-party empty differential cells. Worse, this technique generated bizarre 'interactions' between two empty locations in vacuo, neither occupied by an electron. Of course, this situation cannot be directly verified without bringing in real electrons to check out these effects at these 'empty' locations. Similarly, the effects on iron filings around a magnet do not prove the existence of magnetic fields nor that there are any effects at these same locations when the iron filings are not present. It is not surprising that these attempts to solve one simple problem subsequently introduced spurious mathematical infinities – 'a cure worse than the disease'.

Both Newton's gravitational theory and Maxwell's theory of electromagnetism separate out the interaction between two remote bodies into a third participant – the 'force', which is given equal physical significance, especially an independent **existence**. This separation into an explicit intermediary (that certainly simplifies the mathematics) has diverted attention

away from the interaction and focused the analysis onto the mechanics of single points – particles for Newton and field points for Maxwell. This programme returns to the interaction itself and dispenses with mathematical 'third parties'.

There is no experimental evidence that there are **any** instantaneous effects in physics. Newton's gravity works as a 'single-time' theory because the changes at astronomical distances are too small to notice (e.g. the 8 minute delay from the Sun to the Earth compared with the 365 days to circulate once). It is the universal phenomenon of electromagnetism that has forced physicists to recognize the critical importance of asynchronous changes between interacting electrical objects.

With the introduction of finite delays across time for interactions between pairs of electrons, Kant's **causal** principle can no longer be limited to the idea that every event is preceded by a cause. Since the interaction is symmetric there will always be one event in the future of the other and to pick one event as the 'cause' of the other is simply anthropocentric – our animal memories only record the past and only macroscopic life forms can be said to intentionally (with respect to their own future) 'cause' another future event. Accordingly, this research programme explicitly assumes that the electromagnetic interaction between pairs of interacting electrons is symmetric across time. This idea is expanded further in the next paper in this series.

The point nature of the electron presents many difficult challenges to the continuum models of EM. For example, if all the other possible 'absorbing' electrons are outside of the maximum range of the two-electron interaction (see later) then the outward energy and momentum taken away by the outbound spherical EM waves during the approach phase of the two electrons' motion must be exactly reversed, in empty space, to recover the exact energy and momentum regained during the outbound phase when the electrons are receding away from each other. Such exact co-ordination across a wide area of space would be, to say the least, 'impressive' – as a mechanism, it would also be more than a little bit 'mysterious'.

This research programme views the "light-cone" condition (see later) as central to the electromagnetic interaction. This key constraint results from the necessary coordination across space and time for one electron to be exactly in the right place after 'sending' an impulse to the other electron and then receiving the responding impulse from the other electron. In other words, the 'round-trip' is the micro time average across the symmetric interaction; this is in contrast to the macroscopic reflections from mirrors and beam 'splitters' in arrangements like the famous Michelson-Morley experiment. Advanced effects are an integral part of the present view of the electron-electron interaction whereas either Maxwell's one-way (retarded) electromagnetic theory or the Wheeler-Feynman theory require a statistically improbable correlation between the advanced waves of all the absorbers at many remote points in space and many different future times of re-emission back to the present.

The present theory may be summarized as NOT being a field theory because in this theory, interactions:

1) are not seen as being everywhere throughout space and time but limited to only a finite number of electrons,
2) are not seen as occurring continuously but aperiodically at only finite time intervals,
3) are not viewed as independent particles ('bosons') but seen as the fundamental property of the electrons,
4) are not interpreted as a 'force', nor are they seen as derived from some timeless mathematical potential.

In short, this research programme presents a mechanical model of the electromagnetic interaction in so far as it is described only in terms of its explicit parts (electrons) and their locations and movements through space over time (i.e. their relative positions and velocities). This theory is not a mechanism in the Cartesian sense of contact forces; neither is this theory a mechanical theory in the Maxwellian sense. This EM theory is a discrete point-particle extension of Newtonian mechanics.

Mathematical Fields

It was emphasized that Maxwell developed his theory around innovative ideas on magnetism and simply imported the Coulomb field as a model of electrical interaction. The physicist's concept of the field reflects the conscious assumption that an effect can be separated from the dynamical interaction of its contributing entities. It is a deliberate attempt to simplify the complex world into a **single-time** model that then becomes much more amenable to mathematical manipulation and when viewed as continuously variable leads very easily to familiar differential equations. All attempts to reduce the physical world to field equations will always result in single point (space & time) formulations, whether the 'target' point is occupied by a particle or not. All the myriads of possible interactions with the rest of the universe are then replaced by a statistical approach based on functions of "real" numbers (each with an infinite number of decimal places) defined everywhere.

Fields in physics were only considered real prior to 1900 when the world was considered to consist of various continuous media (solids or Aethers). The experimental confirmation of the atomic and discrete nature of the world around that time

means that the concept of field must return to the world of abstract mathematics, useful when the discreteness of the world can be averaged over large enough space and time regions. Thus, real waves are always discontinuous at the real molecular level; examples are: water height, air pressure, solid distortions etc. Each of these is only meaningful as a conceptual model of a macroscopic measurement. Thus, the concept of a field as a mathematical intermediary remains perfectly valid – as a computational device but not as a direct representation of part of reality itself. The appearance of harmonic oscillators in all field theories is not a physical mechanism (made of what?) but the mathematical introduction of Fourier transforms (FTs). Since there are **no** physical scales of time or length in any interacting field theory (all interactions between fields occur at a point) then the appearance of an infinite number of terms in the FT series or infinite integrals) becomes inevitable.

Field theories absorb all the retarded effects of the remote sources by substituting mathematical variations everywhere at the present time: these are then integrated over all space for measurable results. This is a single-time technique to eliminate the multiple earlier times arising from the asynchronous interaction. Walter Ritz took an alternative, mathematical approach. He produced an expansion series in terms of the spatial locations and velocity vectors of the sources but now at the current time of the target particle; in effect, he was introducing an equivalent, 'instantaneous' force at the field point (but not everywhere) from the finite number of sources. Field theories make a major mistake when they invoke the partial time derivative ($\partial /\partial t$) instead of the total time derivative ($d/dt = \partial /\partial t + \underline{v} \bullet \nabla$) in systems where all the interactions between source and target time take a **finite** time, particularly when the source object is not 'fixed' and can react to the interaction (as it must to preserve global momentum). The use of the total time derivative is valid whenever the interaction is instantaneous or when the fluid media is 'unreactive' (ideal but not realistic models). It was this subtle view that prompted Phipps to point out that Maxwell's EM theory requires that the field detectors used by inertial observers must be stationary at the field point in their own reference frame even though the source currents must be moving. Phipps then used this viewpoint to promote Hertz's rival EM theory, which is a mathematical extension of Maxwell's theory but substitutes **total** time derivatives (d/dt) everywhere for Maxwell's restricted usage of **partial** time derivatives ($\partial/\partial t$); this change identifies the motion of the test particle with the resulting 'convective velocity'. This one simple replacement then makes the revised EM equations now Galilean invariant, eliminating the need for Lorentz invariance and Special Relativity. Phipps has traced this problem to Faraday's Law, which requires a total time derivative of the magnetic flux ($d\Phi/dt$) reflecting the **instantaneous**, change in the circuit's **complete** shape, invoking relative accelerations between different parts of the circuit. He goes on to emphasize that Hertz's source current density $\underline{J}_H$ is measured with a point detector moving with velocity $\underline{V}_d[t]$ in the observer's inertial reference frame so that the Maxwell standard current density $\underline{J}_M$ must be corrected; i.e. $\underline{J}_H[t] = \underline{J}_M[t] - \rho\, \underline{V}_d[t]$; this step inevitably introduces the ill-defined concept of current density ρ that must then be subject to the Lorentz transform in relativity treatments of Maxwell's EM theory to preserve the connection to the invariant charge of every electron.

The contemporary view that the ultimate basis of physical reality is defined in terms of fields is the unspoken regression from phenomenology to the Platonic metaphysical conjecture. This has been updated with the most recent form of 'infinite' mathematics (calculus), which we can readily manipulate after 200 years of development – the great danger always arises when the only tool the craftsman has is a screwdriver (linear, differentiable mathematics) so that every problem has to be screwed. The concept of a field mediating the influences between any collection of interacting particles is a microscopic model of the brokerage solution in human business affairs. When very many (N_1) customers need to deal with many (N_2) suppliers (such as insurance companies), they can either deal directly ($N_1 * N_2$ possible interactions) or go through a shared intermediary or broker (reducing the number of interactions to N_1+N_2). Similarly, the introduction of the electric field, via the linearly additive electric potential, and then computing the effect of this field on the electrons (as in plasma simulations) is the standard mathematical 'trick' to simplifying an intrinsically many-body problem (N^2) to a linear calculation (2N).

Relativistic Transform

The Lorentz transform is a necessary requirement that applies to all local field theories involving effects that propagate at finite speeds; it is the direct consequence of removing all the source and target particles that are the foundation of the basic interaction. This leaves objectless variations at a single point ('fields') that are computable everywhere throughout space at a single time: these values will vary with the mathematical frame of reference used to calculate their values. Any action-at-a-distance theory, whether finite or instantaneous, always computes the same relative spatial and temporal separations between the interacting pair of particles, so the relative velocity or acceleration of any third-party (observer) is totally irrelevant to the physics of this type of interaction: this is one of the great attractions of the far-action approach.

Galilean invariance is only valid for systems where the interactions are independent of velocity. By introducing an explicit velocity into 'his' EM force law on a point charge, Lorentz was implicitly introducing a special inertial frame of reference, namely the one where $\underline{H}$ is defined for the moving charge. Electromagnetic theories that include the 'Lorentz' force law

must inevitably fail this class of transformations. The inclusion of this component (even without any ætherial medium) compels EM theories to comply with the Relativistic transformation. In other words introducing 'invisible intermediaries', like inertia-less force fields, generates relativity. Einstein wrote in his famous 1905 paper that (in his view) electrodynamics was based on the Maxwell-Lorentz EM theory. This addition of Lorentz reflected the additional hypothesis that the force on an electrical particle was specified by what is now called the 'Lorentz Force Law' that aggregates all the charge on a particle to a single, scalar value, no matter what form the particle or electrical-fluid distribution was assumed. This addition was sufficient for Lorentz to link Maxwell's fields at a fixed point in the observer's frame to the inertial motion of the 'target' electron. This blend of fixed and variable motion, at one instant of time at one location in space, ultimately required the introduction of the Relativistic transformation to compensate for the implicit omission of the relative motion between the sources of the EM fields and the effects on the observable detector, represented by the motion of real, inertial electrons.

Finite propagation field theories all suffer from the relative nature of space and time – the base co-ordinate values inevitably change with the relative velocity of the 'observer'. The observed constancy of the phase velocity of the interaction, as in the MMX, means that no such field theory can satisfy the classical Galilean transform, which assumes an invariant rate of time evolution, independent of all relative motion. The additional requirement of 'source-sink' symmetry will inevitably compel the use of the Relativistic transformation for the fundamental space and time parameters in different frames of reference. If these parameters are interpreted as corresponding directly to reality then this Procrustean 'solution' must revolutionize the meaning of the most fundamental concepts of all human thinking, as Einstein realized, **but** this 'solution' is **not** necessary.

Einstein continued the field tradition of focusing only on a single point in isolation ("4D event"). In his final, semi-popular, exposition on special relativity, Einstein proposed that any singular event can be observed (how?) from the origins of two reference frames in relative rectilinear motion, with each frame "constructed" (some how?) from 'rigid' bodies and clocks. Throughout his life, Einstein failed in each of his attempts to prove 'his' famous equation: $E = mc^2$. His final effort in 1954 used von Laue's 1911 approach (without attribution) using Maxwell's EM theory involving continuous charge density, with an invariant total charge per 3D volume elements, to construct a **finite** electric charged body, which experiences the Lorentz force **density** per unit charge, integrated over a finite 4D 'volume' while assuming that the integrals transform like their corresponding differentials. This result was presented without any reference to von Laue's original conclusion that: "the mechanics of continuous media could not be deduced from the mechanics of point particles." Ohanian, who has studied all of Einstein's works extensively, claimed that von Laue's was the first general proof of the famous mass-energy formula for any kind of system with a **static** distribution of energy while ignoring the actual dynamic origin of all EM phenomena – moving electrons. Jefimenko has described this approach (trick?) in great detail [122].

Maxwell was a great student of Newtonian mechanics and always wished to produce a theory of EM that was consistent with classical mechanics: a major objective shared by the present research programme. However, Maxwell was astute enough to recognize that the most important distinction between inertial matter and electronic matter was that gravitational interactions occurred instantaneously between mass particles, no matter how large were the spatial separation between them, while every form of the EM interactions took a finite time to cross the space between. Maxwell's greatest innovation was the concept of a vector potential (or electro-kinetic momentum) to add directionality to the EM interaction that could never be represented by a scalar potential, as had worked for gravity. Maxwell's use of mathematical fields was a legitimate attempt to represent his continuous model of the luminiferous Aether. The discovery of the discrete nature of electricity around 1900 should have forced physicists to rethink their basic approach to EM but the continuous model of matter and the elegance of the vector differential form of Maxwell's Equations were too strong to give up what had become the new, central dogma of theoretical physics.

A desperate attempt was immediately made to justify retaining this 50-year investment in mathematics. The result became Lorentz's electron theory, which was then merged with Helmholtz's fluid model of electricity to create a new amalgam, still describable by Maxwell's Equations that is now taught to physics students as CEM. The vector potential implied that the recognition of electric currents had to be a central component of this theory with fresh attention showed to the velocity of the "field point". Unfortunately, Maxwell had created his field model upon the foundation of a fixed Aether: an idea that could no longer be justified. Worse, even the form of Maxwell's Equations became unrecognizable when they were restated in any inertial reference frame moving at a constant relative velocity. This ultimately led to the necessity of modifying the basic parameters (time and position) of these mathematical representations. Einstein formalized this approach in 1905 with a dramatic redefinition of time to arrive at the now required Lorentz transformations that must be used in subsequent local field theories, which are all ultimately modeled on Maxwell's mathematics. This long, sad saga illustrates the danger of creating a mathematics-first phenomenology instead of the physics-first approach using visualizable models that can then be described by the appropriate mathematics.

These attempts to use continuum mathematics to represent the discrete nature of the world are doomed to failure – they are the modern equivalent of the ancient, unsuccessful attempt “to square the circle”. The continuum hypothesis has run its course, it has served physics well but we now know that, at best, it is an approximation. Future progress will need to explicitly build the foundations of physics on discrete principles hence the use of discrete physical and mathematics underlying the present research programme.

5. CNV TWO-ELECTRON ELECTROMAGNETISM

5.1 TWO PARTICLE NATURAL VECTORS

5.1.1 TWO ELECTRON INTERACTIONS

The brief historical review above has shown that Maxwell's metaphysical conception of a continuous medium pervading all of space has become the *de facto* model of classical electromagnetism. Even when Maxwell's theory of electricity (stress in the medium) was eventually replaced with substance models of electricity, such as Helmholtz's electric-fluid model or H. A. Lorentz's Electron Theory (see 2.3.2), this only replaced the sources and focus of electrical variation – the medium was still retained to 'carry' the electromagnetic (EM) effects, such as energy and momentum, between the two points of interaction. Although the concept of 'æther' has been formally dropped from the 'fashion runway' of modern physics, its role has been absorbed into the modern view of the very nature of space itself: the E and B fields now just 'are', without any need for any supporting medium. This dualistic model of EM continued to haunt Einstein until his death; it was always his unitary dream that particles would be shown to be constructed from the fields themselves. This research programme might be viewed as 'Einstein's Nightmare' as it explicitly rejects all field concepts, not only metaphysically but also mathematically, and returns to the Newtonian view that all of physics can be modeled from a purely discrete viewpoint involving **only** point particles. This programme follows Gauss's suggestion that Newton's instantaneous action-at-a-distance model should be extended to include **asynchronous** interactions between pairs of point particles that interact at **different** times.

The hypothesis of *Standard Separation* is assumed for all interactions between pairs of particles. This is sometimes referred to as the "*Light-Cone*" condition (see I-7.4.5), where an interaction between two point particles may only occur when their absolute spatial separation, Δx divided by their absolute time separation, Δt is exactly equal to a universal constant, c that has the dimensions of speed and has come to be known as the "speed of light" with a value of about 3×10^{10} cm/sec.

$$\text{Light-Cone Condition:} \quad \Delta x / \Delta t = c \qquad \text{or} \qquad (\underline{x}_1 - \underline{x}_2) \bullet (\underline{x}_1 - \underline{x}_2) = c^2 (t_1 - t_2)^2$$

The previous paper [54] applied the new NV mathematics to situations involving the continuous, incompressible 'electric-fluid' model of electromagnetism to rapidly re-derive all of the results associated with the standard classical theory of EM. In this paper, the focus will switch to the joint kinematics of two point particles and then the extension to the dynamics of two interacting electrically charged point particles or 'classical electrons' (the quantum behavior will be reported later).

5.1.2 TWO-PARTICLE TWO-TIME DERIVATIVES

The definition of the total change of a function of the time-difference, $T = t_1 - t_2$ as the differential time-difference changes, is ambiguous; in order to distinguish from the local total time derivative (d/dt) we will use the notation $\mathscr{D}/\mathscr{D}T$. In terms of any continuous function F of a single parameter, ξ subject to a small (positive) change Δ, it is always possible to define a 'limit from above', F^+ and a 'limit from below', F^- for any specific value of the argument, ξ.

$$\textit{Above.Limit}: F^+[\xi] \equiv \lim_{\Delta \to 0} [\, F[\xi + \Delta] \,] \quad \& \quad \textit{Below.Limit}: F^-[\xi] \equiv \lim_{\Delta \to 0} [\, F[\xi - \Delta] \,]$$

In standard calculus, the Continuum Assumption is almost always made; namely, for **all** values of the function's argument that the 'limit from above' is equal to the 'limit from below', or the function is "smooth"; in other words:

$$\textit{Continuum Assumption}: \quad F^+[\xi] = F^-[\xi] = F[\xi]$$

In the case of each electron's position, $\underline{x}[t]$ this assumption appears valid for all points in time (see above) and this has also always been assumed to be equally true in conventional physics for the electron's velocity $\underline{v}[t]$. It will be demonstrated later that this is only a valid assumption whenever an electron is NOT interacting with any other electron (Newton's First Law). We will show that in order to explain the microscopic behavior of electrons we cannot make this assumption at any instant for the electron's velocity when the electron interacts with another electron – this is the basis of all quantum effects.

N.B. Functions use square brackets to delimit their arguments versus parentheses for algebraic disambiguation.

Now following the standard calculus approach and define limiting (or differential) differences, from 'above' and 'below':

$$\textit{Above.Diff}: \mathscr{D}^+F[\xi] / \mathscr{D}\xi \equiv \lim_{\Delta \to 0} [(\, F[\xi + \Delta] - F[\xi] \,) / \Delta] \quad \& \quad \textit{Below.Diff}: \mathscr{D}^-F[\xi] / \mathscr{D}\xi \equiv \lim_{\Delta \to 0} [(F[\xi] - F[\xi - \Delta]) / \Delta]$$

The average value of even discontinuous functions, at any value, can be defined unambiguously and this average (denoted by angular brackets) is "smooth":

$$\textit{Average.Differential}: \; < \mathscr{D}F[\xi] / \mathscr{D}\xi > \equiv \mathscr{D}F[\xi] / \mathscr{D}\xi \equiv \tfrac{1}{2} (\mathscr{D}^{+}F[\xi] / \mathscr{D}\xi + \mathscr{D}^{-}F[\xi] / \mathscr{D}\xi)$$

$$\textit{Total Two-Time-Difference Differential}: \mathscr{D}F[T] / \mathscr{D}T = \lim_{\Delta \to 0} [(F[T + \Delta] - F[T - \Delta]) / \mathbf{2}\Delta]$$

Since this is an average, the factor of '**2**' is important; we will use this definition for both the average velocity of each electron, $\underline{v}_j[t]$ and the average velocity of two interacting electrons $\underline{V}_{jk}[t_j - t_k]$. The problem now is that any small positive change Δ in the time difference between the electrons, labeled #1 & #2, i.e. $T = t_1 - t_2$ can be 'shared' unequally, like $(t_1 + \Delta)$ or $(t_2 - \Delta)$ or equally between the two times, as $(t_1 + \Delta/2)$ and $(t_2 - \Delta/2)$. Thus, the increase in the difference $(T + \Delta)$ is due to either a 'half increase' in t_1 and a 'half decrease' in t_2 or a 'half decrease' in t_1 and a 'half increase' in t_2. This ambiguity can be eliminated by only choosing the 'equal contribution' solution. In other words, the changes in time at the two electrons occur in <u>opposite</u> directions. This divergence in time differences at these two times is particularly important at "interaction times" when the change in velocity occurring at each electron as it participates in the mutual interaction is examined. This gives for the difference in locations of the two electrons:

$$\underline{X}_{12}[T + \Delta] - \underline{X}_{12}[T] = \{\underline{x}_1[t_1 + \Delta/2] - \underline{x}_1[t_1]\} + \{\underline{x}_2[t_2] - \underline{x}_2[t_2 - \Delta/2]\}$$

$$\underline{X}_{12}[T] - \underline{X}_{12}[T - \Delta] = \{\underline{x}_1[t_1] - \underline{x}_1[t_1 - \Delta/2]\} + \{\underline{x}_2[t_2 + \Delta/2] - \underline{x}_2[t_2]\}$$

Then, using the substitution $\delta = \Delta/2$ we can define the 'above' and 'below' two-electron difference velocities:

$$\underline{V}^{+}{}_{12}[T] \equiv \mathscr{D}^{+}\underline{X}_{12}[T] / \mathscr{D}T = \tfrac{1}{2} (\underline{v}^{+}{}_1[t_1] + \underline{v}^{-}{}_2[t_2])$$

$$\underline{V}^{-}{}_{12}[T] \equiv \mathscr{D}^{-}\underline{X}_{12}[T] / \mathscr{D}T = \tfrac{1}{2} (\underline{v}^{-}{}_1[t_1] + \underline{v}^{+}{}_2[t_2])$$

These two forms of $\underline{V}^{\lambda}{}_{12}[T]$ where $\lambda = \pm$, both satisfy the 'mixing' of changes in t_1 and t_2 that one might expect from an interaction occurring between the two electrons at t_1 and t_2. The final definition of *Total Two-Time-Difference Positional Differential*, or joint-velocity, can now be introduced, as:

$$\underline{V}_{12}[t] \equiv \mathscr{D}\underline{X}_{12}[T] / \mathscr{D}T \equiv < \mathscr{D}\underline{X}_{12}[T] / \mathscr{D}T > = \tfrac{1}{2} (\mathscr{D}^{+}\underline{X}_{12}[T] / \mathscr{D}T + \mathscr{D}^{-}\underline{X}_{12}[T] / \mathscr{D}T)$$

Thus,

$$<\underline{V}_{12}[t_1 - t_2]> = \tfrac{1}{2} (<\underline{v}_1[t_1]> + <\underline{v}_2[t_2]>) \quad \text{or} \quad \underline{V}_{12}[t_1 - t_2] = \tfrac{1}{2} (\underline{v}_1[t_1] + \underline{v}_2[t_2])$$

Although $\underline{X}_{12}[T]$ is the <u>difference</u> in the positions of the two electrons at two <u>different</u> times, the total differential of this positional difference with respect to this difference in times has the dimensions of a velocity, this definition is NOT the <u>difference</u> in the relative velocities at these two times $(\underline{v}_1[t_1] - \underline{v}_2[t_2])$ but the AVERAGE of these two velocities. This is viewed as the natural extension of the average of the definition of the SINGLE time average of the two velocities. All of these definitions are self-consistent and generate the full 'separability' of the individual velocities.

These results for the position and velocity differentials for two particles can be generalized to all anti-symmetric, <u>separable</u> functions F_{12} that describe the ***joint*** properties of associated pairs of electrons at two different locations in space $\underline{x}_1$ & $\underline{x}_2$ at two different times t_1 and t_2, when each particle's related (individual) property is characterized by $f_j(t_j; \underline{x}_j)$. In the following, the variable ξ denotes any one of the four fundamental dimensions $\{t ; x, y, z\}$.

$$\textit{Separable Two-Particle Property}: \; F_{12}[\xi_1, \xi_2] \equiv f_1[\xi_1] - f_2[\xi_2] = F_{12}[\xi_1 - \xi_2] = F_{12}[\xi]$$

The anti-symmetry of these separable functions determines that electrons are fermions, as $F_{12}[0] = 0$. As was shown for the velocity above, when discontinuous changes occur then the change in these two-electron functions with respect to any of the space-time variables is ambiguous, which will now be removed in a similar manner. The pre- and post-derivatives must first be defined for any scalar function of a single variable, $\psi[\xi]$ in terms of the differential limits of the pre- and post-differences.

$$\Delta^{\lambda}\psi[\xi] \equiv \lambda (\psi[\xi + \lambda \Delta] - \psi[\xi]) \quad \text{where} \quad \lambda = \pm 1 \text{ or } \pm$$

$$\partial^{\lambda}\psi[\xi] / \partial\xi \;\equiv\; \underset{\Delta \to 0}{\text{Limit}} \{ \Delta^{\lambda}\psi[\xi] / \Delta \}$$

A similar manipulation for deriving $\underline{V}^{\lambda}{}_{12}[T]$ generates the results: $\partial^{\lambda}F_{12}[\xi] / \partial\xi \;=\; ½ (\partial^{\lambda}f_1[\xi_1]/\partial\xi_1 \;+\; \partial^{-\lambda}f_2[\xi_2]/\partial\xi_2)$

Average values for all functions can be defined at any point or value ξ whether the function is discontinuous or not.

$$< F[\xi] > \;\equiv\; ½ (F^{+}[\xi] + F^{-}[\xi]) \;=\; \underset{\Delta \to 0}{\text{Limit}} \{ ½ (F[\xi + \Delta] - F[\xi - \Delta]) \} \;=\; F[\xi]$$

Thus,

$$\partial F_{12}[\xi]/\partial\xi \;=\; ½ (\partial f_1[\xi_1]/\partial\xi_1 \;+\; \partial f_2[\xi_2]/\partial\xi_2) \qquad \text{N.B. brackets often omitted}$$

So, the partial difference-derivative of an anti-symmetric, separable function $F_{12}[\xi]$, with respect to its single difference argument ξ is equal to the average value of the sum of the single-value derivatives $\partial f_j[\xi_j]/\partial\xi_j$ of its functional decomposition. These extended 'difference' definitions of partial difference-derivatives allow two-electron definitions to be formulated that bear marked resemblances to single value functions of simple vector calculus; for example, introducing the two-electron (spatial) difference-gradient operator, $\underline{\nabla}_{12}$.

$$\underline{\nabla}_{12}F_{12}[\underline{x}_1 - \underline{x}_2] \;=\; ½ (\underline{\nabla}_1 f_1[\underline{x}_1] \;+\; \underline{\nabla}_2 f_2[\underline{x}_2]) \qquad \text{where} \qquad \underline{\nabla}_1 f_1[\underline{x}_1] \;\equiv\; \hat{\mathbf{e}}_x \partial_x f_1[x] + \hat{\mathbf{e}}_y \partial_y f_1[y] + \hat{\mathbf{e}}_z \partial_z f_1[z]$$

$$\underline{\nabla}_{12} \bullet \underline{F}_{12}[\underline{x}_1 - \underline{x}_2] \;=\; ½ (\underline{\nabla}_1 \bullet \underline{f}_1[\underline{x}_1] \;+\; \underline{\nabla}_2 \bullet \underline{f}_2[\underline{x}_2]) \text{ and } \underline{\nabla}_{12} \wedge \underline{F}_{12}[\underline{x}_1 - \underline{x}_2] \;=\; ½ (\underline{\nabla}_1 \wedge \underline{f}_1[\underline{x}_1] \;+\; \underline{\nabla}_2 \wedge \underline{f}_2[\underline{x}_2])$$

Before extending these definitions to two-electron difference Continuous Natural Vectors (CNVs), we will first recall some of the major results already established in the first paper in this series [01] that described CNVs for single electrons.

5.1.3 SUMMARY OF SINGLE-POINT CNVs

The definition of a Natural Vector (see I-4.4) is viewed as an *imaginary, scalar quaternion*, symbolized by $\mathbf{Q}$.

$$\textbf{Natural Vector}: \qquad \mathbf{Q} \;\equiv\; i\,\mathbf{I_0}\,q_0 + \mathbf{I_1}\,q_1 + \mathbf{I_2}\,q_2 + \mathbf{I_3}q_3 \;\equiv\; \mathbf{Q}_0 + \textstyle\sum_j \mathbf{Q_j} \equiv \{ i\,q_0 \;;\; \underline{q} \} \;=\; i\,q_0\,\mathbf{I_0} + \textstyle\sum_j q_j\,\mathbf{I_j}$$

Here the $\mathbf{I_\mu}$ are defined in terms of real 4x4 matrices and $\mathbf{I_0}$ is isomorphic with the unit number, while the $\{\mathbf{I_1}, \mathbf{I_2}, \mathbf{I_3}\}$ are isomorphic with Hamilton's three linearly independent imaginary quantities $\{\mathbf{i, j, k}\}$. So these bases satisfy the group multiplication rules, using the indices j, k, l = 1, 2, 3 and μ = 0, j :

$$\mathbf{I_0}\,\mathbf{I_\mu} \;=\; \mathbf{I_\mu}\,\mathbf{I_0} \;=\; \mathbf{I_\mu} \qquad\qquad \mathbf{I_j}\,\mathbf{I_k} \;=\; -\,\delta_{ij}\,\mathbf{I_0} \;+ \varepsilon_{jkl}\,\mathbf{I_l}$$

Here, δ_{ij} is the Kronecker delta symbol with value +1 when both indices are equal or zero otherwise and ε_{jkl} is the cyclic permutation tensor whose value is zero unless all three indices are different when its value is +1 if the indices are cyclic (even permutation of 1,2,3) or –1 if anti-cyclic (odd permutation). The rules for (conjugate) addition and multiplication of two Natural Vectors $\mathbf{A}^*$ and $\mathbf{B}$ become:

Addition: $\qquad \mathbf{A}^* + \mathbf{B} \;=\; -\,i\,\mathbf{I_0}\,(a_0 - b_0) \;+\; (\underline{a} + \underline{b}) \bullet \underline{\mathbf{I}}$

Multiplication: $\qquad \mathbf{A}^*\mathbf{B} \;=\; \mathbf{I_0}\,(a_0\,b_0 - \underline{a} \bullet \underline{b}) \;+\; i\,\underline{\mathbf{I}} \bullet (b_0\,\underline{a} - a_0\,\underline{b}) \;+\; (\underline{a} \wedge \underline{b}) \bullet \underline{\mathbf{I}}$

Since electrons are each considered both unique and eternal they can be 'labeled' by a unique positive integer identifier 'k'. In the next section (§5.1.4) it is shown that electrons are fermions, so each electron will have a unique position in space ($\underline{x}$) at any time, t. It is the fundamental hypothesis of this research programme that these two parameters of every electron can be mapped into their own individual Natural Vector, $\mathbf{X}(k)$ or $\mathbf{X}_k$, rather than two separate traditional algebraic variables.

$$\text{Hypothesis:} \quad \{ \underline{x}(k: t_k) \} \;\cong\; \mathbf{X}_k \;\equiv\; i\,c\,t_k\,\mathbf{I_0} \;+\; \underline{x}_k \bullet \underline{\mathbf{I}}$$

The square (or 'norm') of this 'positional' NV is: $\qquad \mathbf{X}(t)^*\,\mathbf{X}[t] \;=\; (c^2 t^2 - x^2)\,\mathbf{I_0}$

Similarly,

$$\textbf{Velocity}: \ \mathbf{V}[t] \equiv \underset{\delta t \to 0}{\text{Limit}} \{ (\mathbf{X}[t+\delta t] - \mathbf{X}[t]) / \delta t \} = d\mathbf{X}[t]/dt = i\, c\, \mathbf{I_0} + \underline{v} \bullet \underline{\mathbf{I}}$$

The "norm" of this velocity NV is: $\mathbf{V}[t]^* \mathbf{V}[t] = (c^2 - v^2)\, \mathbf{I_0}$

We follow Hamilton and extend his 'nabla' (or 'gradient') 3D space operator ($\underline{\nabla}$) to the Natural Vector ***Gradient*** applied to any scalar function ψ that is continuous in the four space-time variables $\{t ; \underline{x}\}$:

$$\textbf{Gradient}: \ \nabla\psi[t ; \underline{x}] \equiv i\, \mathbf{I_0}\, \partial_0\psi[t ; \underline{x}] + \underline{\mathbf{I}} \bullet \underline{\nabla}\psi[t ; \underline{x}] \ \ \& \ \ \underline{\nabla} \equiv \hat{\mathbf{e}}_1\partial_1 + \hat{\mathbf{e}}_2\,\partial_2 + \hat{\mathbf{e}}_3\partial_3 \quad \partial_0 \equiv \partial/\, c\, \partial t \quad \partial_1 \equiv \partial/\, \partial x_1 \text{ etc}$$

The conjugate of the NV gradient operator can be applied to any continuous natural vector (CNV) function, $\mathbf{Q}[t ; \underline{x}]$:

$$\nabla^*\mathbf{Q}[t ; \underline{x}] = \mathbf{I_0}\, (\partial_0 q_0 - \underline{\nabla} \bullet \underline{q}) + i\, \underline{\mathbf{I}} \bullet (\underline{\nabla} q_0 - \partial_0 \underline{q}) + \underline{\mathbf{I}} \bullet (\underline{\nabla} \wedge \underline{q})$$

If $\nabla^*\mathbf{Q} = 0$ then: 1) $\underline{\nabla} \bullet \underline{q} = \partial_0 q_0$ 2) $\underline{\nabla} q_0 = \partial_0 \underline{q}$ 3) $\underline{\nabla} \wedge \underline{q} = 0$

We defined the "***Zero Condition***" CNV, $\mathbf{Z}$ as: $\mathbf{Z}\psi \equiv i\, \underline{\mathbf{I}} \bullet (c\, \underline{\nabla} + \underline{v}\, \partial_0)\psi - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\psi)$

The CNV equivalent of the total-time differential: $\mathbf{I_0}\, d\psi/dt = -\mathbf{V}^*\nabla^*\psi$ if $\mathbf{Z}\psi = 0$

The so-called conditional "***Flow***" equation can be derived for any continuous CNV $\mathbf{Q}$:

$$d\mathbf{Q}/dt + \mathbf{V}^*\nabla^*\mathbf{Q} = 0 \qquad \text{if} \qquad \mathbf{Z}\mathbf{Q} = 0$$

For any continuous function, $\alpha[t; \underline{x}]$ we can define generic *Voigt Vectors*, as these prove central in this programme.

$$\textbf{Voigt Vector}: \ \mathfrak{V} \equiv -i\, c\, \alpha[t; \underline{x}]\, \mathbf{I_0} + \alpha[t; \underline{x}]\, \underline{v}[t] \bullet \underline{\mathbf{I}} = \alpha[t; \underline{x}]\, \mathbf{V}^* = \{ i\, \mathfrak{V}_0; \underline{\mathfrak{V}} \}$$

The scalar and vector components of a *Voigt Vector* satisfy the following equation, named in honor of the pioneer of asynchronous EM - Ludvig V. Lorenz; this equation will also re-appear in many forms throughout this programme.

$$\text{The } \textbf{\textit{Lorenz Equation}}: \quad c\, \underline{\mathfrak{V}} + \underline{v}\, \mathfrak{V}_0 = 0$$

These CNVs satisfy several useful identities, presented next; again Q_0 and α are scalar functions and $\underline{Q}$ is a vector function.

1. $\nabla^*\mathbf{X} = -2\, \mathbf{I_0}$ 2. $\nabla^*\mathbf{V} = -i\, \underline{\mathbf{I}} \bullet \partial_0 \underline{v}$ 3. $\nabla^*\alpha = -i\, \mathbf{I_0}\, \partial_0\alpha + \underline{\mathbf{I}} \bullet \underline{\nabla}\alpha$

4. $\nabla^*(\alpha\, \mathbf{Q}) = \alpha\, (\nabla^*\mathbf{Q}) + (\nabla^*\alpha)\mathbf{Q}$ 5. $\nabla^*\nabla\alpha = \nabla\, \nabla^*\, \alpha = \mathbf{I_0}\, (\partial_0^2 - \underline{\nabla}^2)\, \alpha \equiv -\mathbf{I_0}\, \Box\alpha$

6. $\mathbf{V}^*\nabla^*\alpha = -\mathbf{I_0}\, (c\, \partial_0 + \underline{v} \bullet \underline{\nabla})\alpha - i\, \underline{\mathbf{I}} \bullet (c\, \underline{\nabla} + \underline{v}\, \partial_0)\, \alpha + \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\alpha)$

7. $\mathbf{Q}\, \nabla^*\alpha = \mathbf{I_0}\, (Q_0\, \partial_0\alpha - \underline{Q} \bullet \underline{\nabla}\alpha) + i\, \underline{\mathbf{I}} \bullet (Q_0\, \underline{\nabla}\alpha - \underline{Q}\, \partial_0\alpha) + \underline{\mathbf{I}} \bullet (\underline{Q} \wedge \underline{\nabla}\alpha)$

8. $\nabla^*(\alpha\, \mathbf{V}) = \mathbf{I_0}\, (\, c\, \partial_0\alpha - \underline{v} \bullet \underline{\nabla}\alpha) + i\, \underline{\mathbf{I}} \bullet (\, c\, \underline{\nabla}\alpha - \underline{v}\, \partial_0\alpha - \alpha\, \partial_0 \underline{v}) - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\alpha)$

5.1.4 TWO PARTICLE DIFFERENCE CNVs

The mathematics of continuous natural vectors (CNVs) of a single particle at a single location or 4D point in space and time can be extended to the situation of two point particles (arbitrarily labeled #1 & #2) at two locations in space $\underline{x}_k$ at times t_k, where k = 1 & 2, so as to define (see paper I-7.4.1) the two-particle *Difference* CNV, $\mathbf{X}_{12}$.

$$\textbf{\textit{Difference Vector}}: \qquad \mathbf{X}_{12} \equiv \mathbf{X}_1 - \mathbf{X}_2$$

The spatial and temporal differences can also be defined: $\underline{X} \equiv \underline{x}_1 - \underline{x}_2$ and $T \equiv t_1 - t_2$ so $\mathbf{X}_{12}[T; \underline{X}] = \mathbf{X}_{12}[t_1 - t_2; \underline{x}_1 - \underline{x}_2]$

It can be readily seen that $\mathbf{X}_{12}[T; \underline{X}]$ fulfills the principal requirement of Poincaré's 1899 'Principle of Relativity' [05], namely that physics should never depend on the choice of location of the spatial or temporal origins of any inertial reference frame, i.e. all equations should only depend on the absolute differences between the co-ordinates of the pair of particles. Further, two-particle separable NVs (see I-7.4.1) are inherently anti-symmetric with respect to the exchange of the labeling of their space and time arguments:

$$\mathbf{X}_{12}[-T; -\underline{X}] = -\mathbf{X}_{12}[T; \underline{X}]$$

Obviously, separable difference NVs are zero when evaluated at the same time ($t_1 = t_2$): $\mathbf{X}_{12}[0; \underline{0}] = 0$

This result can be interpreted, in a realistic philosophy, as each electron retaining its identity at all times (hence, eternal) as no two electrons can occupy the same position in space at the same time, i.e. their trajectories never cross. These are all different formulations of the definition of a *fermion* point particle, especially in quantum mechanics where two-electron wave functions $\psi(1,2)$ must be anti-symmetric and electrons must obey Fermi-Dirac statistics.

5.1.5 TWO PARTICLE CNV DERIVATIVES

Since natural vectors are linear functions of each electron's local parameters then NVs of each electron can be combined anti-symmetrically into separable natural vectors, when all the results of section 5.1.2 may be used. In particular, the NV definition of a single electron's velocity can be extended to a definition of the CNV of the joint-velocity of two electrons.

$$\mathbf{V}_k[t] = d\mathbf{X}_k(t_k)/dt_k = i\,c\,\mathbf{I_0} + \underline{v}_k \bullet \underline{\mathbf{I}} \quad \text{and} \quad \mathbf{V}^{\lambda}_{12}[T] = \mathscr{D}^{\lambda}\mathbf{X}_{12}[T]/\mathscr{D}T = i\,c\,\mathbf{I_0} + \underline{V}^{\lambda}_{12} \bullet \underline{\mathbf{I}}$$

$$\therefore \mathbf{V}_{12}[T] = i\,c\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{V}_{12} = i\,c\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{V}_{12}[t_1 - t_2] = i\,c\,\mathbf{I_0} + \tfrac{1}{2}(\underline{v}_1[t_1] + \underline{v}_2[t_2]) \bullet \underline{\mathbf{I}}$$

Two-Electron Difference Velocity CNV: $\mathbf{V}_{12}[t_1 - t_2] = \tfrac{1}{2}(\mathbf{V}_1[t_1] + \mathbf{V}_2[t_2]) = \mathscr{D}\mathbf{X}_{12}[T]/\mathscr{D}T$

The definition of averages of pre- and post-components, results in the decomposition of this average two-electron velocity:

$$\langle \mathbf{V}_{12}[t_1 - t_2] \rangle = \tfrac{1}{2} \langle (\mathbf{V}^{+}_{12}[t_1 - t_2] + \mathbf{V}^{-}_{12}[t_1 - t_2]) \rangle$$

This last identity is consistent with the definition of single-particle velocity invariants: $\mathbf{V}^{*}_{12}\,\mathbf{V}_{12} = (c^2 - \underline{V}_{12} \bullet \underline{V}_{12})\,\mathbf{I_0}$

These results can be generalized to any CNV representing separable two-electron properties, like $\mathbf{Q}_k$.

$$\mathbf{Q}_{12}[t_1 - t_2] = \tfrac{1}{2}(\mathbf{Q}_1[t_1] + \mathbf{Q}_2[t_2]) \quad \text{and} \quad \langle \mathbf{Q}_{12}[t_1 - t_2] \rangle = \tfrac{1}{2} \langle (\mathbf{Q}^{+}_{12}[t_1] + \mathbf{Q}^{-}_{12}[t_2]) \rangle$$

These results allow us to define the total time-difference differential, D/DT using the two-electron difference velocity.

Total Time-Difference Differential: $D\psi[T; \underline{X}]/DT \equiv (\mathscr{D}/\mathscr{D}T + \underline{V}_{12} \bullet \underline{\nabla}_{12})\,\psi[T; \underline{X}]$

The comparable one-electron vector identities are still valid with these extended two-electron difference operators.

Most importantly: a) $\mathscr{D}X_j/\mathscr{D}X_k = \delta_{jk}$ b) $\mathscr{D}X_j/\mathscr{D}T = 0$ c) $\underline{\nabla}_{12}\,\underline{V}_{12} = 0$ d) $\underline{\nabla}_{12} \bullet \underline{V}_{12} = 0$ e) $\underline{\nabla}_{12} \wedge \underline{V}_{12} = 0$

Finally, these definitions can be extended to the two-electron CNV difference gradient operator, defining $\mathscr{D}_0 \equiv \mathscr{D}/c\mathscr{D}T$

Two-Electron Difference Gradient CNV: $\boldsymbol{\nabla}_{12}\psi[T; \underline{X}] \equiv (i\,\mathbf{I}_0\,\mathscr{D}_0 + \underline{\mathbf{I}} \bullet \underline{\nabla}_{12})\,\psi[T; \underline{X}]$

$$\boldsymbol{\nabla}_{12}{}^{*}\mathbf{Q}_{12}[T; \underline{X}] = \mathbf{I_0}(\mathscr{D}_0 Q^0{}_{12} - \underline{\nabla}_{12} \bullet \underline{Q}_{12}) + i\,\underline{\mathbf{I}} \bullet (\underline{\nabla}_{12} Q^0{}_{12} - \mathscr{D}_0 \underline{Q}_{12}) + \underline{\mathbf{I}} \bullet (\underline{\nabla}_{12} \wedge \underline{Q}_{12})$$

If $\boldsymbol{\nabla}_{12}{}^{*}\mathbf{Q}_{12} = 0$ then: 1) $\underline{\nabla}_{12} \bullet \underline{Q}_{12} = \mathscr{D}_0 Q^0{}_{12}$ 2) $\underline{\nabla}_{12} Q^0{}_{12} = \mathscr{D}_0 \underline{Q}_{12}$ 3) $\underline{\nabla}_{12} \wedge \underline{Q}_{12} = 0$

So, all the results for one-electron CNV *Voigt Vectors* can be extended to two-electron difference Voigt vectors:

Two-Electron Voigt Vector: $\mathfrak{V}_{12} \equiv -i\,c\,\alpha_{12}[T; \underline{X}]\,\mathbf{I}_0 + \alpha_{12}[T; \underline{X}]\,\underline{V}_{12}[T] \bullet \underline{\mathbf{I}} = \alpha_{12}[T; \underline{X}]\,\mathbf{V}_{12}{}^{*} = \{ i\,\mathfrak{V}^0{}_{12}; \underline{\mathfrak{V}}_{12} \}$

The components of these two-electron *Voigt Vectors* also satisfy their own Lorenz equation: $c\,\underline{\mathfrak{V}}_{12} + \underline{V}_{12}\,\mathfrak{V}^0{}_{12} = 0$

All of the single-electron Voigt results can now be reproduced with these comparable two-electron CNVs and operators. In particular, they satisfy similar definitions of Gauge vectors, Flow vectors and the Wave equation. For example, if there exists a scalar, separable function $\psi_{12}(T; \underline{X})$ associated with the two electrons and if it is 'harmonic' so that it satisfies the extended two-electron Wave equation:

Double-Difference Wave Equation: $\Box_{12}\psi_{12}[T; \underline{X}] = 0$ where $\boldsymbol{\nabla}_{12}{}^{*}\boldsymbol{\nabla}_{12} \equiv -\mathbf{I_0}\,\Box_{12}$ and $\Box_{12} = \underline{\nabla}_{12} \bullet \underline{\nabla}_{12} - \mathscr{D}_0\,\mathscr{D}_0$

Similarly, since: $\mathbf{V}_{12}{}^{*}\boldsymbol{\nabla}_{12}{}^{*}\psi_{12}[T; \underline{X}] = -\mathbf{I_0}\,(\,\mathscr{D} / \mathscr{D}T + \underline{V}_{12} \bullet \underline{\nabla}_{12}\,)\,\psi - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla}_{12} + \underline{V}_{12}\mathscr{D}_0)\,\psi + \underline{\mathbf{I}} \bullet (\underline{V}_{12} \wedge \underline{\nabla}_{12})\,\psi$

And defining the Extended (Double-Difference) ***Zero*** CNV: $\mathbf{Z}_{12} \equiv i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla}_{12} + \underline{V}_{12}\mathscr{D}_0) - \underline{\mathbf{I}} \bullet (\underline{V}_{12} \wedge \underline{\nabla}_{12})$

This gives the Extended CNV ***Flow Equation***: $D\,\mathbf{Q}_{12} / DT + \mathbf{V}_{12}{}^{*}\boldsymbol{\nabla}_{12}{}^{*}\,\mathbf{Q}_{12} = 0$ whenever $\mathbf{Z}_{12}{}^{*}\,\mathbf{Q}_{12} = 0$

Again, by analogy, defining the Double-Difference Acceleration as the extended time-difference of the velocity:

Two-Electron Difference Acceleration CNV: $\mathcal{A}_{12}[t_1 - t_2] = \tfrac{1}{2}\,(\,\mathcal{A}_1[t_1] + \mathcal{A}_2[t_2]\,) = \mathscr{D}\mathbf{V}_{12}[T] / \mathscr{D}T$

Then the Two-Electron CNV Difference Velocity, $\mathbf{V}_{12}$ is a 'Flow Vector' ($\alpha_{12} = 1$) when its 'Zero Conditions' result in:

ii) $\underline{V}_{12} \bullet \underline{\mathcal{A}}_{12} = 0$ iv) $\underline{V}_{12} \wedge \underline{\mathcal{A}}_{12} = 0$

The previous paper (see II-5.1) indicated that $\mathbf{V}_{12}[T]$ is a constant of the motion, i.e. $D\mathbf{V}_{12}[T]/DT = 0$, or equivalently, when the Joint Current is considered to be 'harmonic' so that: $\Box_{12}\underline{V}_{12} = 0$ or equivalently: $\boldsymbol{\nabla}_{12}{}^{*}\mathbf{V}_{12} = 0$, so:

1) $\underline{\nabla}_{12} \bullet \underline{V}_{12} = 0$ 2) $\mathscr{D}_0\,\underline{V}_{12} = 0$ 3) $\underline{\nabla}_{12} \wedge \underline{V}_{12} = 0$

All these equations imply that any change in one electron's motion, at its time of interaction, is compensated in the other electron's reaction at its later time of interaction. In other words, the total (two-electron) momentum and total current of these two electrons at the two (different) times of their mutual interaction are conserved (see section 5.2.3).

Each harmonic function has its own corresponding ***Associate*** CNV, defined as: $\boldsymbol{\psi}'_{12} \equiv \boldsymbol{\nabla}_{12}\psi_{12}[T; \underline{X}]$ NB not conjugate. The addition of any Associate Natural Vector to a Voigt Vector, $\boldsymbol{\mathfrak{v}}$ defines its corresponding ***Gauge Vector***, $\boldsymbol{\mathfrak{v}}'$.

Definition: ***Gauge Transform*** $\boldsymbol{\mathfrak{v}}_{12}' \equiv \boldsymbol{\mathfrak{v}}_{12} + \boldsymbol{\nabla}_{12}\psi_{12}$ where $\Box_{12}\psi_{12} = 0$

The conjugate gradient of this Associate CNV is zero, since: $\boldsymbol{\nabla}_{12}{}^{*}\,\boldsymbol{\psi}'_{12} = -\mathbf{I_0}\,\Box_{12}\psi_{12} = 0 \;\therefore\; \boldsymbol{\nabla}_{12}{}^{*}\boldsymbol{\mathfrak{v}}_{12}' = \boldsymbol{\nabla}_{12}{}^{*}\boldsymbol{\mathfrak{v}}_{12}$

5.2 TWO PARTICLE CNV ELECTROMAGNETISM

The second paper in this series [54] applied the mathematics of single-time continuous natural vectors (CNVs) to a 'toy' model of classical electromagnetism (CEM) that represented electricity by a continuous, incompressible fluid (the so-called EM Helmholtz flow model). All of the standard results of CEM were very quickly recovered by assuming that the three central quantities in CEM (current, potential and electro-kinetic momentum) were simply components of comparable CNVs.

This latter paper focused, like Lorenz [55], on the interactions between remote regions of charge; this directly resulted in the standard retarded scalar and vector potentials propagating between the regions at 'light-speed'. Surprisingly, this type of result also appeared for the propagation of charge densities – this seemed to imply that electric currents were traveling at light-speed: an anomalous result, to say the least. This section will now provide an interesting explanation for this apparent mathematical paradox by replacing this continuous model of electricity with one that much closer reflects the physics of EM interaction; here idealized as the continuous interaction between two electrons remotely separated across space.

5.2.1 TWO PARTICLE CNV CURRENT

This research programme views electrons as the fundamental existents of reality with invariant and universal properties, which (by tradition) are associated with the electrons themselves. The two basic (and complementary properties) include their unit electric charge and individual inertial mass. The *electric charge* causes changes in the relative motion with the

other electrons with which they are interacting and is represented by a scalar quantity that is usually given the algebraic symbol 'q'. For unfortunate historical reasons the standard electron's charge was assigned a negative value ($q = -e$). In all EM interactions, it is the product of the charges ($q_1 * q_2$) that always determines the pair-wise behavior of the two interacting particles. The *inertial mass* causes each electron to resist changes caused by the interaction and is represented by a positive scalar quantity that is given the standard algebraic symbol '*m*'. It was shown in the first paper (see I-7.4.4) that these single electron quantities must be universal scalars for joint properties of pairs of electrons, like total momentum, to be conserved quantities. The other two universal parameters are best seen as characteristics of the interaction itself; the universal 'speed' factor (denoted by '*c*') scales the space-time separation parameters or relative speed between the electrons (as will be shown later), while the unit of change-in-action is denoted by Planck's constant ('*h*'). This last parameter will be introduced in the fifth paper. Since each electron is assumed here to occupy a unique position in space at all times (see §5.1.4) there is NO quantity in this theory that corresponds to the basic concept of 'charge density' in every continuum theory of EM, neither is there here any comparable concept of 'current density' but the concept of individual electron current is well-defined, created by the motion of each electron (identified as # 'k'). NB in this paper, no mention is made of positively charged electrons.

Single Electron Current : $\underline{\mathcal{J}}_k[t] \equiv q_k \underline{v}_k[t] = -e \underline{v}_k[t]$

The concept of the two-electron two-time 'joint velocity' (see §5.1.2) can now be introduced to represent the situation where two remote electrons (labeled #1 & #2) are interacting at different instants of time. This leads to the definition of the joint current (average two-time current) arising from the combined contributions of both electrons.

Joint Electric Current : $\underline{\mathcal{J}}_{12}[t_1 - t_2] \equiv -e \underline{V}_{12}[t_1 - t_2] = -e \tfrac{1}{2} (\underline{v}_1[t_1] + \underline{v}_2[t_2]) = \tfrac{1}{2} (\underline{\mathcal{J}}_1[t_1] + \underline{\mathcal{J}}_2[t_2])$

These definitions naturally lead into the definitions of their comparable single and double CNVs, $\boldsymbol{\mathcal{J}}_k$ and $\boldsymbol{\mathcal{J}}_{12}$ (see 5.1.5).

$$\boldsymbol{\mathcal{J}}_k[t] \equiv -e \mathbf{V}_k^*[t] = -e (-i c \mathbf{I_0} + \underline{v}_k \bullet \underline{\mathbf{I}}) \quad \& \quad \boldsymbol{\mathcal{J}}_{12}[t_1 - t_2] \equiv -e \mathbf{V}_{12}^*[t_1 - t_2] = -e (-i c \mathbf{I_0} + \underline{V}_{12} \bullet \underline{\mathbf{I}})$$

So, $\boldsymbol{\mathcal{J}}_{12}[t_1 - t_2] = \tfrac{1}{2} (\boldsymbol{\mathcal{J}}_1[t_1] + \boldsymbol{\mathcal{J}}_2[t_2])$ The joint-current CNV 'norm' is: $\boldsymbol{\mathcal{J}}_{12}[T]^* \boldsymbol{\mathcal{J}}_{12}[T] = e^2 (c^2 - \underline{V}_{12} \bullet \underline{V}_{12}) \mathbf{I_0}$

Section 5.1.5 showed that $\mathbf{V}_{12}[T]$ is a constant of the motion, so then $\boldsymbol{\mathcal{J}}_{12}[T]$ is also a constant of the motion, so the total current is conserved across the interaction.

5.2.2 TWO PARTICLE CNV POTENTIAL

This research programme returns to Maxwell's original EM focus [50] – his concept of the electro-kinetic momentum $\underline{A}$, (that later evolved into its modern term 'electromagnetic vector potential') as the function that characterizes the exchange of EM momentum between remote electrical charges. Soon after (and independent of Maxwell), L. V. Lorenz in 1867 analyzed the asynchronous EM interactions between remote regions of charge in terms of a scalar potential (ϕ) and a vector potential ($\underline{A}$) at the induction point due to the earlier (retarded) remote charge and current distributions [55]. However, in contrast to Maxwell who defined all his EM variables, including potentials, at ALL points throughout the universe (i.e. the EM æther), this theory defines the potentials only at the locations of the two electrons. Lorenz, like all other earlier electromagnetic and other 'force' theories, assumed spherical symmetry around the electrical charges, so that when Lorenz's source charges are reduced to a single electron (labeled #2) then his potentials at the (target) charge or electron #1 reduce simply to:

$$\phi_1 = \phi[t_1 ; \underline{x}_1] = + e / R \quad \& \qquad \underline{A}_1 = \underline{A}[t_1 ; \underline{x}_1] = - e \, \underline{v}[t_2 ; \underline{x}_2] \, / \, c \, R = - e \, \underline{v}_2 / c \, R$$

$$\text{Where} \quad \underline{R} = \underline{x}_1[t_1] - \underline{x}_2[t_2] \quad \& \quad t_2 = t_1 - R / c \quad \text{(the 'retarded' source time)}$$

The relationship between these two potentials always satisfy: $c \, \underline{A}_1 + \phi_1 \, \underline{v}_2 = 0$. This resembles the NV Lorenz Equation. Based on the results derived in the previous paper [54], this suggests the definition of a two-electron electromagnetic CNV potential, $\mathbf{A}_{12}$ as a two-electron *Voigt* vector, where the defining Voigt parameter α_{12} is identified with a two-electron scalar EM potential Φ_{12} (divided by c), so that:

$$\textbf{\textit{Two-Electron EM Potential}} : \; c \, \mathbf{A}_{12}[T; \underline{X}] \equiv \Phi_{12}[T; \underline{X}] \; \mathbf{V}^*_{12}[T; \underline{X}] = c \, \{ i \, a_{12} ; \underline{A}_{12} \}$$

Thus, $a_{12}[T; \underline{X}] = - \Phi_{12}[T; \underline{X}]$ & *Lorenz Equation* : $c \, \underline{A}_{12}[T; \underline{X}] = \Phi_{12}[T; \underline{X}] \; \underline{V}_{12}[T; \underline{X}]$

Since $\mathbf{V}^*_{12}$ is the prototypical two-electron Voigt vector, its total time-difference derivative is zero, so: $D\underline{V}_{12} / DT = 0$

Applying the total time-difference operator to the *Lorenz* equation: $c \, D\underline{A}_{12} / DT = \underline{V}_{12} \, D\Phi_{12} / DT$

Finally taking the 'divergence' of the Lorenz equation leads to: $\underline{\nabla}_{12} \bullet \underline{A}_{12} + \mathscr{D}_0 \, \Phi_{12} = D_0 \, \Phi_{12}$ where $D_0 \equiv D / c \, DT$

The equivalent assumption to the single-point Lorentz Gauge condition is that Φ_{12} is a total time-difference invariant, or:

$$D_0 \, \Phi_{12} = 0 \qquad \underline{\nabla}_{12} \bullet \underline{A}_{12} + \mathscr{D}_0 \, \Phi_{12} = 0 \qquad \text{This is equivalent to:} \qquad D_0 \, \mathbf{A}_{12}[T; \underline{X}] = 0$$

This also indicates that $\mathbf{A}_{12}$ is a two-electron *Flow Vector*, implying potential 'waves', thus: $\Box_{12} \, \mathbf{A}_{12}[T; \underline{X}] = 0$

5.2.3 TWO PARTICLE EM FORCES

This further suggests the definition of the two-electron gradient, $\mathbf{G}_{12}$ of $\mathbf{A}_{12}$ through the analogous definition (I-6.5):

$$\textbf{\textit{Two-Electron EM Gradient CNV}} : \quad i \, \mathbf{G}_{12} \equiv \nabla_{12}{}^* \mathbf{A}_{12} \qquad \text{or} \qquad i \, D\mathbf{A}_{12} = D\mathbf{X}^*_{12} \, \mathbf{G}_{12}$$

Using the standard notation for separating the real component ($\mathscr{R}e$) from the imaginary component ($\mathscr{I}m$) of any complex function, we can repeat the earlier definitions [54] of the electric ($\underline{E}$) and magnetic ($\underline{B}$) force vectors (per unit charge) as:

$$\textbf{\textit{Electric Force per Charge}} : \quad \underline{E}_{12} \equiv \mathscr{R}e \, (\underline{G}_{12}) \qquad \textbf{\textit{Magnetic Force per Charge}} : \underline{B}_{12} \equiv \mathscr{I}m \, (\underline{G}_{12})$$

$$\text{So,} \quad \mathbf{G}_{12} = - i \, \nabla_{12}{}^* \mathbf{A}_{12} = \{ i \, g_{12} ; (\underline{E}_{12} - i \, \underline{B}_{12}) \} \quad \& \quad \textbf{\textit{Two-Electron Force}} : \; \mathbf{F}_{12} \equiv - e \, \mathbf{G}_{12}$$

Producing similar EM force equations: 1) $g_{12} = D_0 \, \Phi_{12}$ 2) $\underline{E}_{12} = - \underline{\nabla}_{12} \, \Phi_{12} + \mathscr{D} \underline{A}_{12} / \mathscr{D} T$ 3) $\underline{B}_{12} = \underline{\nabla}_{12} \wedge \underline{A}_{12}$

Furthermore, if $\mathbf{A}_{12}$ is a two-electron *Flow Vector*, then: $D\mathbf{A}_{12} / DT = - \mathbf{V}^*_{12} \, \nabla_{12}{}^* \mathbf{A}_{12} = - i \, \mathbf{V}_{12}{}^* \mathbf{G}_{12} = 0$

The four components of this equation are: 1) $\underline{V}_{12} \bullet \underline{E}_{12} = 0$ 2) $\underline{V}_{12} \bullet \underline{B}_{12} = 0$ 3) $c \, \underline{E}_{12} + \underline{V}_{12} \wedge \underline{B}_{12} = 0$ 4) $c \, \underline{B}_{12} = \underline{V}_{12} \wedge \underline{E}_{12}$

Solving equations 3) & 4) gives: $c^2\,\underline{B}_{12} = \underline{V}_{12} \wedge c\,\underline{E}_{12} = -\,\underline{V}_{12} \wedge (\underline{V}_{12} \wedge \underline{B}_{12}) = (\underline{V}_{12} \bullet \underline{V}_{12})\,\underline{B}_{12} - \underline{V}_{12}\,(\underline{V}_{12} \bullet \underline{B}_{12}) = (V_{12})^2\,\underline{B}_{12}$

This has the important solution : $V_{12} = c$ thus, **the Two-Electron CNV Difference Velocity is always at 'light-speed'**.

Defining the two-time unit spatial vector by : $X\,\hat{\underline{e}}_{12} \equiv \underline{X} = \underline{x}_1 - \underline{x}_2$ and $\underline{c} \equiv c\,\hat{\underline{e}}_{12}$ with $\hat{\underline{e}}_{12} \bullet \hat{\underline{e}}_{12} = 1 \;\therefore\; \underline{V}_{12} = \underline{c}$

$$\therefore\; \mathbf{V}_{12}[T; \underline{X}] = c\,(i\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \hat{\underline{e}}_{12}) \quad \therefore\; \mathbf{V}^*_{12}[T; \underline{X}]\;\mathbf{V}_{12}[T; \underline{X}] = 0 \quad \text{and} \quad \tfrac{1}{2} < (\underline{v}_1[t_1] + \underline{v}_2[t_2]) > = c\,\hat{\underline{e}}_{12}$$

Since Φ_{12} is the unit energy per unit charge the EM energy for the two electrons is defined as: $\mathcal{U}_{12}[T; \underline{X}] \equiv e\,\Phi_{12}[T; X]$

As with the continuum model of electricity we will here assume that $\mathcal{U}_{12}$ is the temporal component of the two-electron EM Energy CNV, $\mathbf{U}_{12}$ associated with the EM two-electron potential, $\mathbf{A}_{12}$:

$$\textit{Two-Electron EM Energy CNV}: \; c\,\mathbf{U}_{12} \equiv -e\,\Phi_{12}\,\mathbf{V}^*_{12} = \Phi_{12}\,\mathbf{J}_{12} \quad \text{so,} \quad c\,\underline{U}_{12} = -e\,\Phi_{12}\,\underline{V}_{12} = \Phi_{12}\,\underline{J}_{12}$$

$$\therefore\; \mathbf{U}_{12}[T; \underline{X}] = -e\,\mathbf{A}_{12}[T; \underline{X}] \;\therefore\; D\mathbf{U}_{12}/DT = 0 \quad \text{so } \mathbf{U}_{12} \text{ is conserved across the interaction.}$$

5.2.4 TWO PARTICLE TOTAL ENERGY

Classically, a single particle (say, labeled #1) acted upon by a local, conservative force, $\underline{F}_1$ is equivalent to a particle moving in a time-independent potential U_1, whose magnitude varies across space but is constant in time, so: $\partial U_1/\partial t = 0$ & $\underline{a} = d\underline{v}/dt$

Mechanical force is defined as the negative gradient of the potential, $\underline{F}_1 \equiv -\underline{\nabla} U_1$ and $dU_1/dt = \underline{v}_1 \bullet \underline{\nabla} U_1 \;\therefore\; dU_1/dt = -\,\underline{v}_1 \bullet \underline{F}_1$

For a particle with invariant mass, m its kinetic energy is $K_1 = \tfrac{1}{2}\,m\,v_1^2 = \tfrac{1}{2}\,\underline{v}_1 \bullet \underline{p}_1 \;\therefore\; dK_1/dt = m\,\underline{v}_1 \bullet \underline{a}_1 = \underline{v}_1 \bullet \underline{F}_1 = -dU_1/dt$

This suggests the definition of the Total Energy, E_j as the sum of the Kinetic and Potential Energies: $E_j \equiv K_j + U_j$

So, for any single electron (labeled #j) $dE_j / dt = 0$ or $E(j: t; \underline{x}) = E^0(j)$ an invariant quantity or constant of the motion.

These simple classical (one-time) concepts can be extended to the CNV model of the EM interaction between two electrons by defining the Natural Vector equivalent of the mechanical (or kinetic) energy of the two electrons, each with inertial mass.

$$\textit{Two-Electron Kinetic Energy CNV}: \; \mathbf{K}_{12} \equiv i\;m\,\mathbf{V}^*_{12}\,\mathbf{V}_{12} = i\,m\,(c^2 - V_{12}^2)\,\mathbf{I_0} = \{\,i\,\mathcal{K}_{12}\,;\,0\}$$

Note that as this is a scalar invariant it is purely imaginary, so that $\mathcal{K}_{12}$ is real. Now in the first paper on NVs, several two-electron invariants were identified (see paper I-7.4.4). The two-electron total linear momentum, $\underline{P}_{12}$ is also defined in terms of $\underline{V}_{12}$ just like the two-electron current $\underline{J}_{12}$. These extensions lead to the definitions of their comparable CNVs, $\mathbf{P}_k$ and $\mathbf{P}_{12}$.

$$\mathbf{P}_k[t] \equiv m\,\mathbf{V}_k^*[t] = m\,(-i\,c\,\mathbf{I_0} + \underline{v}_k \bullet \underline{\mathbf{I}}) \quad \& \quad \mathbf{P}_{12}[t_1 - t_2] \equiv m\,\mathbf{V}_{12}^*[t_1 - t_2] = m\,(-i\,c\,\mathbf{I_0} + \underline{V}_{12} \bullet \underline{\mathbf{I}})$$

So, $\mathbf{P}_{12}[t_1 - t_2] = \tfrac{1}{2}\,(\mathbf{P}_1[t_1] + \mathbf{P}_2[t_2])$ The joint-momentum CNV 'norm' is: $\mathbf{P}_{12}[T]^*\,\mathbf{P}_{12}[T] = m^2\,(c^2 - \underline{V}_{12} \bullet \underline{V}_{12})\,\mathbf{I_0}$

Therefore, $\mathbf{K}_{12} = i\,\mathbf{P}_{12}\,\mathbf{V}_{12} = i\,\mathbf{V}_{12}\,\mathbf{P}_{12}$ another invariant since $D_0\,\mathbf{K}_{12}[T; \underline{X}] = 0$

We can now define the total two-electron Total Energy CNV, $\mathbf{E}_{12}$ for two electrons interacting electromagnetically:

$$\textit{Two-Electron Total Energy CNV}: \; \mathbf{E}_{12} \equiv \mathbf{K}_{12} + \mathbf{U}_{12} = \{\,i\,\mathcal{E}_{12}\,;\,\underline{E}_{12}\,\}$$

So, $\mathcal{E}_{12} = \mathcal{K}_{12} + \mathcal{U}_{12} = m\,(c^2 - V_{12}^2) - e\,\Phi_{12}$ & $\underline{E}_{12} = \underline{U}_{12} = \Phi_{12}\,\underline{J}_{12}\,/\,c$

This results in: $\mathbf{E}_{12} = \mathbf{V}^*_{12}\,(i\;m\,\mathbf{V}_{12} - e/c\,\Phi_{12}\,\mathbf{I_0}) = i\,\mathbf{V}^*_{12}\,(i\,(m\,c + e/c\,\Phi_{12})\,\mathbf{I_0} + m\,\underline{V}_{12} \bullet \underline{\mathbf{I}})$ & $D_0\mathbf{E}_{12} = 0$

This suggests a new CNV, the total system (two-electron) momentum, $\mathbf{\Pi}_{12}$ defined by the analog: $\mathbf{E}_{12} = i\,\mathbf{V}^*_{12}\mathbf{\Pi}_{12}$

Two-Electron Total Momentum CNV : $\boldsymbol{\Pi}_{12} \equiv i\,(m\,c + e/c\,\Phi_{12})\,\mathbf{I_0} + m\,\underline{V}_{12} \bullet \underline{\mathbf{I}} = \{\, i\,\Pi_0\,;\,\underline{\Pi}\}$

This has scalar and vector components: $\Pi_0 = (m\,c + e/c\,\Phi_{12})$ & $\underline{\Pi} = \underline{P}_{12}$ or $\mathcal{U}_{12}\,\mathbf{I_0} = c\,(\mathbf{P}_{12}^* - \boldsymbol{\Pi}_{12})$

Since $D_0\mathbf{K}_{12} = D_0(\mathbf{V}_{12}^*\mathbf{P}_{12}^*) = \mathbf{V}_{12}^*\,D_0(\mathbf{P}_{12}^*) = -D_0\mathbf{U}_{12} = c\,\mathbf{V}_{12}^*\,\boldsymbol{\nabla}_{12}^*\,\mathbf{U}_{12} = i\,c\,\mathbf{V}_{12}^*\,\mathbf{F}_{12}$

This results in the two-electron "Equation of Motion" : $\mathbf{F}_{12}^* = i\,D\mathbf{P}_{12}/DT$ or $e^2/m\,\mathbf{G}_{12}^* = -\,i\,D\mathbf{J}_{12}/DT$

Here, the generalization of the EM "Force per unit Charge" is the complex, spatial CNV, $\mathbf{G}_{12} = \{\,0\,;\,\underline{E}_{12} - i\,\underline{B}_{12}\,\}$

This has the 'norm' : $\mathbf{G}_{12}^*\mathbf{G}_{12} = -(\underline{E}_{12} \bullet \underline{E}_{12} + \underline{B}_{12} \bullet \underline{B}_{12})\,\mathbf{I_0} - 2\,i\,\underline{E}_{12} \wedge \underline{B}_{12} \bullet \underline{\mathbf{I}}$

We can generalize the EM Field Energy Density & Current : $\mathcal{W}_{12} = (\underline{E}_{12} \bullet \underline{E}_{12} + \underline{B}_{12} \bullet \underline{B}_{12})/8\pi$ & $\underline{\mathcal{W}}_{12} = (\underline{E}_{12} \wedge \underline{B}_{12})/4\pi$

The generalization of the two-electron Poynting Vector is $\underline{\mathcal{N}}_{12} = c\,\underline{\mathcal{W}}_{12}$ so $\mathbf{G}_{12}^*\mathbf{G}_{12} = -8\pi\,(\mathcal{W}_{12}\,\mathbf{I_0} + i\,\underline{\mathbf{I}} \bullet \underline{\mathcal{N}}_{12}/c)$

Using the mappings: $\mathcal{W}_{12} \Leftrightarrow -e\,\Phi_{12}$ & $\underline{\mathcal{N}}_{12} \Leftrightarrow -e\,\Phi_{12}\,\underline{V}_{12}$ then the final identity is achieved: $\mathbf{U}_{12} = \mathbf{G}_{12}^*\mathbf{G}_{12}/8\pi$

5.2.5 COVARIANT CONTINUOUS ELECTROMAGNETISM

The similarity of the forms of the equations of electromagnetism, when formulated as Continuous Natural Vectors (CNVs), as either single point equations [54] or as two-electron difference functions (this section) strongly suggests that it is the form of these equations (or covariance) that is the key to the mathematical description of continuous EM, as we can see.

Single Point	Two Electron
$\mathbf{J}[t] = \rho\,\mathbf{V}^*(t)$	$\mathbf{J}_{12}[t_1 - t_2] = -e\,\mathbf{V}_{12}^*[t_1 - t_2]$
$c\,\mathbf{A}[t] = \phi\,\mathbf{V}^*[t]$	$c\,\mathbf{A}_{12}[t_1 - t_2] = \Phi_{12}\,\mathbf{V}_{12}^*[t_1 - t_2]$
$d\mathbf{A}[t]/dt = 0$	$D\mathbf{A}_{12}[T]/D\,T = 0$
$\Box\mathbf{A}[t\,;\,\underline{x}] = 0$	$\Box_{12}\,\mathbf{A}_{12}[T;\,\underline{X}] = 0$
$i\,\mathbf{G}(t) = \boldsymbol{\nabla}^*\mathbf{A}[t]$	$i\,\mathbf{G}_{12}[t_1 - t_2] = \boldsymbol{\nabla}^*_{12}\,\mathbf{A}_{12}[t_1 - t_2]$
$c\,\mathbf{U}[t] = \rho\,\phi\,\mathbf{V}^*[t] = \phi\,\mathbf{J}[t]$	$c\,\mathbf{U}_{12}[t_1 - t_2] = -e\,\Phi_{12}\,\mathbf{V}_{12}^*[t_1 - t_2] = \Phi_{12}\,\mathbf{J}_{12}[t_1 - t_2]$

This 'similarity of form' strongly suggests that results derived from 'Single-Point' equations can be carried over directly to the 'Two-Electron' equations, using the above equivalence formulations. However, although the forms of these equations are comparable, it is critical to acknowledge that they reflect very different physical situations. The single-point equations were constructed from the assumption that electricity can be represented by a continuous charge density ρ, which does not reflect the experimental reality that electricity occurs in the form of finite, point charges, hence the two-electron model of EM presented in this paper. In this view, it is the long-standing attempt to conserve charge across differential cells in constant relative motion that results in the EM basis for the special theory of relativity: this will be discussed in detail in the next paper. It is also important to notice that it was the introduction by Lorentz of 'his' force equation on a point-charge particle q (and not logically on the charge density ρ) that enabled the Maxwell theory to be linked to real micro experiments.

5.3 TWO PARTICLE SPECIAL RELATIVITY

In this section, the standard results of the theory of special relativity (SR) will be directly derived from the CNV single-point theory of EM. It will be shown that the assumption of separability of the electromagnetic momentum must be added for the single-time version of special relativity to be extended to the two-electron CNV theory.

5.3.1 LORENTZIAN VECTORS

Generic *Voigt Vectors* were defined in section six of the first paper on natural vectors [55] and reviewed here in §5.1.3.

$$\textit{Voigt Vector}: \quad \mathfrak{V} = \alpha\, \mathbf{V}^* = -i\, c\, \alpha\, \mathbf{I_0} + \alpha\, \underline{v}[t] \bullet \underline{\mathbf{I}} = \{\, i\, \mathfrak{v}\, ;\, \underline{\mathfrak{V}}\, \} \qquad \text{So,} \quad \mathfrak{v} = -c\,\alpha \quad \& \quad \underline{\mathfrak{V}} = \alpha\, \underline{v}$$

We will denote the 'norm' of a Voigt vector by $\mathcal{N}^2$, so $\mathfrak{V}^*\, \mathfrak{V} = (\mathfrak{V}_0\, \mathfrak{V}_0 - \underline{\mathfrak{V}} \bullet \underline{\mathfrak{V}})\, \mathbf{I_0} \equiv \mathcal{N}^2\, \mathbf{I_0} \;\therefore\; c^2 \mathcal{N}^2 = (c^2 - v^2)\, \mathfrak{v}^2$

$$\therefore\; c\, d[\mathcal{N}^2] / dt = 2\, \mathfrak{v}\, (\, c\, d\mathfrak{v} / dt + \underline{v} \bullet d\underline{\mathfrak{V}} / dt\,)$$

If a Voigt vector's 'norm' is independent of time (constant of the motion) then $\mathcal{N}^2$ is constant, so: $c\, d\mathfrak{v} + \underline{v} \bullet d\underline{\mathfrak{V}} = 0$

The Lorenz equation is: $c\, \mathfrak{V} + \underline{v}\, \mathfrak{v} = 0$ and taking total time differentials gives: $c\, d\underline{\mathfrak{V}} + \underline{v}\, d\mathfrak{v} + \mathfrak{v}\, d\underline{v} = 0$

Taking the velocity vector product of this last equation gives: $c\, (\underline{v} \bullet d\underline{\mathfrak{V}}) + (\underline{v} \bullet \underline{v})\, d\mathfrak{v} + \mathfrak{v}\, (\underline{v} \bullet d\underline{v}) = 0$

$$\mathfrak{v}\, (\underline{v} \bullet d\underline{v}) + (\underline{v} \bullet \underline{v})\, d\mathfrak{v} = -c\, (\underline{v} \bullet d\underline{\mathfrak{V}}) = c^2\, d\mathfrak{v} \;\therefore\; (c^2 - v^2)\, d\mathfrak{v} = \mathfrak{v}\, (\underline{v} \bullet d\underline{v}) = \tfrac{1}{2}\, \mathfrak{v}\, d(v^2) \;\therefore\; 2(c^2 - v^2)\, d\mathfrak{v} = \mathfrak{v}\, d(v^2)$$

So, the scalar component, $\mathfrak{v}$ is only a function of v; the integral solution is: $\mathfrak{v}[v]^2 = c^2\, \mathfrak{v}[0]^2 / (c^2 - v^2) = c^2 \mathcal{N}^2 / (c^2 - v^2)$

So, $\mathfrak{v}[v] = -\mathcal{L}[v]\, \mathcal{N} = -c\, \alpha[v] \;\therefore\; \alpha[v] = \mathcal{L}[v]\, \mathcal{N} / c$ or $\mathcal{N} = c\, \alpha[0]$ where $\mathcal{L}[v] = 1/\sqrt{(1 - v^2/c^2)}$ the 'Lorentz' factor.

Therefore, when a Voigt Vector has an invariant 'norm' it is a 'Lorentzian' vector, $\mathfrak{V}_L$ (see I-7.6.3). It was clearly shown in the first paper [54] in section I-7.7, that the single-particle CNV could immediately reproduce Planck's famous derivation of relativistic mechanics [56] by simply assuming that the single particle momentum is represented by a Lorentz vector. It was also pointed out that this historical dynamical derivation requires the introduction of a fictitious, mechanical force that acts *instantly and constantly* on the particle throughout its acceleration – this is NOT a characteristic of EM.

5.3.2 MAXWELL'S ELECTRO-KINETIC VECTOR

In 1865, Maxwell introduced his central concept of the electro-kinetic momentum $\underline{A}$ in his "Dynamical Theory" paper [50]. The present research programme accepts the significance of this concept and simply proposes that this is the vector part of a Voigt natural vector $\mathbf{A}$ (see §5.2.2). The single-time approach will first be reprised as this leads directly to special relativity. It is always assumed that the particle's velocity is assumed to be a function only of time so functions of time become only functions of the scalar speed variable (v):

$$c\, \mathbf{A}[v] = \phi[v]\, \mathbf{V}^*[v] \;\therefore\; \mathbf{A}[v] = -i\, \phi[v]\, \mathbf{I_0} + \phi[v]\, \underline{\mathbf{I}} \bullet \underline{v} / c \quad \text{Comparing:} \quad \mathfrak{v}[v] = -\phi[v] \quad \underline{\mathfrak{V}}[v] = \underline{A}[v]$$

Defining $\phi_0 \equiv \phi[0]$ and since $\mathcal{N} = -\phi[0]$ then $\phi[v] = \mathcal{L}[v]\, \phi_0$ & $\underline{A}[v] = \underline{v}\, \mathcal{L}[v]\, \phi_0 / c$ while $\mathbf{A}^*[v]\, \mathbf{A}[v] = (\phi_0)^2\, \mathbf{I_0}$

This confirms that the potential's norm ($\mathcal{N}^2$) is independent of v or t and $\underline{A}[0] = 0$; for total-time differentials $\underline{v} \bullet d\underline{A} = c\, d\phi$

Translating Maxwell's hypothesis into a mechanical impulse for a particle with charge q: $d\underline{P} = q/c\; d\underline{A}$

The next step is the crucial assumption – this assumes that the interaction between two electrons ($q_k = -e$) can be separated into the sum of two independent and separable components, centered on each electron:

$$\textbf{\textit{Key Assumption}}: \quad \mathbf{A}_{12} = \mathbf{A}_1 + \mathbf{A}_2$$

So, for each electron (k = 1 or 2) separately: $c\, d\underline{P}_k = q_k\, d\underline{A}_k$ or $c\, d\underline{P}_k[v] = -e\, d\underline{A}_k[v]$ with $c\, \underline{P}_k[0] = -e\, \underline{A}_k[0] = 0$

This differential is now universally assumed to be integratable over a continuous change in v giving: $c\, \underline{P}_k[v_k] = -e\, \underline{A}_k[v_k]$

If, like Planck, it is further assumed [56] that each electron's total momentum $\underline{P}_k[v_k]$ is linearly related to its velocity, $\underline{v}_k$ by a mass-like, scalar function of its own speed (v_k), here denoted by $M_k[v_k]$, then:

$$\textbf{\textit{Planck's Proposal}}: \quad \underline{P}_k[v_k] \equiv M_k[v_k]\, \underline{v}_k$$

In the symmetrical inertial reference frame, the total momentum of the two electrons is zero: $\underline{P}_1[t] + \underline{P}_2[t] = 0$

Arbitrarily, taking electron #1 to define the positive velocity, then: $\underline{v}_1 = -\underline{v}_2 = \underline{v} \;\; \therefore M_1[v_1] = M_2[v_2] = M[v]$

This produces the desired result: $\underline{P}_k[v] = M[v]\,\underline{v}_k$ writing $m_0 \equiv M[0]$

Comparing this with the potential: $c^2 M_k[v] = q_k \phi_k[v]$ so $q_k \phi_0 = m_0 c^2$ or $M[v] = \mathcal{L}[v]\, m_0$ & $q_k \phi[v] = \mathcal{L}[v]\, m_0 c^2$

If each electron's kinetic energy is represented by the scalar function K_k, then Thomson & Tait's differential definition [57] for the change in a particle's kinetic energy (see §6.4) is related to the scalar product of its velocity and impulse ($d\underline{P}_k$):

$$dK_k = \underline{v}_k \bullet d\underline{P}_k = q_k (\underline{v}_k \bullet d\underline{A}_k) / c = (\underline{J}_k \bullet d\underline{A}_k) / c = q_k\, d\phi_k = -dU_k \quad \text{since } U_k = -q_k \phi_k$$

We can now define the total energy, E_k as the sum of the (mechanical) kinetic energy and the (electromagnetic) potential energy.

Total Energy: $E_k \equiv K_k + U_k \;\; \therefore \;\; dE_k = dK_k + dU_k = 0$ so the particle's total energy is constant.

If the particle's kinetic energy is zero when its potential energy is ϕ_0; integrating gives: $K_k[v] = q_k(\phi_k[v] - \phi_0) = (M - m_0)c^2$

Since $U_k = -q_k \phi_k = -\mathcal{L}[v]\, m_0 c^2$ then $U_k^2 = m_0^2 c^4 + P_k^2$ Note this involves U_k not E_k.

If one assumes a Coulomb style of potential between the electrons, the minimal spatial separation occurs at a distance Λ, so

$\phi_0 = e / \Lambda$. This results in the 'nuclear' (or classical electron) radius: $\Lambda = e^2 / m_0 c^2$

All of these definitions result in Lorenz type equations, such as: $c\,\underline{P}_k + U_k \underline{v}_k / c = 0$ or $c^2 \underline{P}_k + U_k \underline{v}_k = 0$

This shows that both $\underline{P}_k$ and $\underline{U}_k$ form the vector components of simple Voigt vectors: $c\,\mathbf{P}_k = i\, U_k \mathbf{I_0} + c\,\underline{\mathbf{I}} \bullet \underline{P}_s = \mathbf{U}_k = q_k \mathbf{A}_k$

5.3.3 THE SEPARABILITY ASSUMPTION

It has been demonstrated above that to extend the results of special relativity from single-time continuous charge models of EM to the situation of discrete electricity (such as point charges) involving even only two such electrons it is necessary to not only assume that the 'norm' of the EM electro-kinetic energy is independent of time but that its two-electron difference CNV is separable; it was also necessary to define each electron's momentum as a function of its speed: 'Planck's Proposal'. The best two examples of kinematic two-electron difference CNVs correspond to the 4D space-time *Difference* CNV, $\mathbf{X}_{12}$ and its 'derivative', the two-electron *Difference-Velocity* CNV, $\mathbf{V}_{12}$. The forms of their separability were examined in §5.1.4 and §5.1.5 where their decomposition was described in terms of single-particle quantities that could be determined independently.

$$\mathbf{X}_{12}[T; \underline{X}] = \mathbf{X}_{12}[t_1 - t_2; \underline{x}_1 - \underline{x}_2] = \mathbf{X}_1[t_1; \underline{x}_1] - \mathbf{X}_2[t_2; \underline{x}_2]$$

$$\mathbf{V}_{12}[T; \underline{X}] = \mathbf{V}_{12}[t_1 - t_2; \underline{x}_1 - \underline{x}_2] = \tfrac{1}{2} (\mathbf{V}_1[t_1; \underline{x}_1] + \mathbf{V}_2[t_2; \underline{x}_2])$$

But when the two-electron CNV Potential, $\mathbf{A}_{12}$ is examined (§5.2.2) it can be readily seen that although it involves the two-electron difference-velocity it does NOT simply decompose into a symmetric two component CNV, as is needed, *unless the scalar EM potential Φ_{12} is a constant* - this is NOT the standard assumption of a simple Coulomb potential that varies with distance.

$$c\,\mathbf{A}_{12}[t_1 - t_2 ; \underline{x}_1 - \underline{x}_2] = \tfrac{1}{2} \Phi_{12}[t_1 - t_2 ; \underline{x}_1 - \underline{x}_2] (\mathbf{V}_1^*[t_1 ; \underline{x}_1] + \mathbf{V}_2^*[t_2 ; \underline{x}_2])$$

So, $\mathbf{A}_{12}[t_1 - t_2 ; \underline{x}_1 - \underline{x}_2] = \tfrac{1}{2} (\mathbf{A}_1[t_1 ; \underline{x}_1] + \mathbf{A}_2[t_2 ; \underline{x}_2])$ if and only if:

$$\Phi_{12}[t_1 - t_2 ; \underline{x}_1 - \underline{x}_2] = \phi_0 \quad \& \quad c\,\mathbf{A}_k[t_k ; \underline{x}_k] = \tfrac{1}{2} \phi_0 \mathbf{V}_k^*[t_k ; \underline{x}_k] \quad \text{Thus: } q_k \mathbf{A}_k[t_k ; \underline{x}_k] = m_0 c\, \mathbf{V}_k^*[t_k ; \underline{x}_k]$$

This result is a foreshadowing of Einstein's 'photon' hypothesis, as will be shown in a later paper.

The only alternative to the maximum potential, ϕ_0 is the frame invariant form: $\Phi_{12}[t_1 - t_2 ; \underline{x}_1 - \underline{x}_2] = N_{12}\, e / R$

This scales with the separation R = c T, to reduce the maximum potential, so that: $\Phi_{12}[T ; R] = N_{12}\, (\Lambda / R)\, \phi_0$

5.3.4 EM ENERGY DENSITY

The total EM energy density $\mathbf{W}$ was defined (as a pseudo CNV) at the field point in (II §6.5):

$$\mathbf{W} \equiv \mathbf{J}^* \mathbf{A} / c = i \rho (\phi - \underline{w} \bullet \underline{A} / c)\, \mathbf{I}_0 = \{ i \mathcal{W}_0 ; \underline{\mathcal{W}} \} \qquad \therefore \ \underline{\mathcal{W}} = 0 \ \text{ and } \ \mathcal{W}_0 = -(U_S + \mathcal{K}_M)$$

Here, the static energy density $U_S = -\rho \phi$ & the 'magnetic' (or kinetic) energy density $\mathcal{K}_M = \rho\, \underline{w} \bullet \underline{A} / c = \underline{\mathcal{J}}_T \bullet \underline{A} / c$

In the case of conductors, the kinetic energy density is shared between the EM momentum of the mobile electrons (the current with density $\underline{\mathcal{J}}$ and drift velocity $\underline{\mathcal{U}}$) and the mechanical momentum of the ions (lattice), moving with velocity $\underline{v}$.

$$\therefore \ \mathcal{K}_M = \rho\, (\underline{v} + \underline{\mathcal{U}}) \bullet \underline{A} / c = \rho\, \underline{v} \bullet \underline{A} / c + \underline{\mathcal{J}} \bullet \underline{A} / c \qquad \text{since } \ \underline{\mathcal{J}} = \rho\, \underline{\mathcal{U}}$$

If the conductors are fixed (or very heavy) the lattice movement is negligible (v = 0) leaving only currents: $\mathcal{K}_M = \underline{\mathcal{J}} \bullet \underline{A} / c$.

Since the both the static potential ϕ and the vector potential $\underline{A}$ at the local cells ζ in the target conductor are the direct consequence of the retarded static and moving charge densities (ρ' and $\underline{\mathcal{J}}'$) in the source conductor that vary inversely with their spatial separation r then a small change in this separation will exchange a small change in potential energy for a small change in kinetic energy.

$$\Delta \mathcal{W}_0 = 0 \qquad \therefore \ \Delta\phi = \Delta[\, \underline{w} \bullet \underline{A} / c\,] = \underline{w} \bullet \Delta \underline{A} / c$$

This spatial variation is equivalent to the application of a force $\Delta F_1 = -\rho\, \Delta\phi / \Delta x$. This is maximized by use of the gradient operator; i.e. $\Delta \underline{F}_1 = -\rho\, \underline{\nabla}\phi$. This total force is both internal (electro-kinetic force density $\underline{E}_M$) and lattice-mechanical $\Delta \underline{F}_L$.

$$\therefore \ \Delta \underline{F}_1 = -\rho\, \underline{\nabla}[\, \underline{w} \bullet \underline{A} / c\,] = -\rho\, (\underline{v} + \underline{\mathcal{U}}) \bullet \underline{\nabla}[\, \underline{A}\,] / c = -(\rho\, \underline{v} + \underline{\mathcal{J}}) \bullet \sum\nolimits_k \underline{\mathcal{I}}'\, [\zeta', t_k]\, \Delta x_0\, \underline{\nabla}[\, 1 / r\,] / c^2$$

$$\therefore \ \Delta \underline{F}_1 = (\rho\, \underline{v} + \underline{\mathcal{J}}) \bullet \sum\nolimits_k \underline{\mathcal{I}}'\, [\zeta', t_k]\, \Delta x_0\, \underline{r} / (c^2\, r^3) = (\rho\, \underline{v} + \underline{\mathcal{J}}) \bullet \underline{\mathcal{I}}' \oint d\zeta'\, \underline{r} / (c^2\, r^3) = \Delta \underline{F}_L + \rho\, \underline{E}_M$$

6. TWO-ELECTRON CONTINUOUS ELECTRODYNAMICS

6.1 CONTINUOUS CHANGE

The anti-symmetric nature of two-electron functions means that no two electrons can ever occupy the same location in space at the same time. Further the EM interaction proposed here is between two electrons, so there is no 'self-interaction'; this has been both the problem generated by finite models of the electron 'filled' with continuous electrical 'fluid' (e.g. Lorentz) and the source of modern EM field theories where the interaction always occurs at points of zero extent. Furthermore, due to the anti-symmetric nature of the electron-pair functions, there is never an interaction between two electrons at zero spatial separation: the infinite 'curse' of point-particle models of EM. Since pairs of electrons only interact in this theory whenever there is a finite spatial separation and when both electrons are 'on each other's light-cone' then there is **no** significant single time that characterizes the interactions between any two electrons. Each electron (labeled #k) is characterized by both its local position in space ($\underline{x}_k$) at its own 'local' time (t_k), so the symbol T is reserved here to denote the differences between the times when the two electrons interact, i.e. $T \equiv t_1 - t_2$.

In subsequent papers that explore the interactions involving three or more electrons, the "saturation hypothesis" will be proposed that limits, at any single local interaction time, the interaction of any one electron to only one other electron. This choice of "saturated interaction" is a conscious rejection of the "superposition hypothesis" that has always been universally assumed in both mechanics and EM since Newton. Accordingly, if two electrons begin interacting together when electron #1's local time is t_b and continues to interact exclusively with electron #2 until a later time (for electron #1) of t_a, when it either ceases to interact with electron #2 or switches its interaction 'partner' to another electron, then the duration of this interaction is referred to as its mutual "interaction-period" which is denoted by $\mathscr{T}_{ab}(1) \equiv t_a - t_b$. It is important to realize that while any two electrons are interacting continuously, no other electron can interact with either of these two electrons and so they are effectively unobservable to all other electrons in the universe during each electron's interaction-period. However, since this research programme is based on a 'realistic' and not a 'positivistic' philosophy then it remains metaphysically consistent to view the two electrons as continuing to exist at all times. However, the saturation hypothesis will remain mute here and in any other idealized investigation of the interactions between only two electrons.

For two negatively charged electrons, which repulse each other, there will always be a minimum spatial separation d_0 that occurs during the interaction-period and will depend on the initial conditions when the interactions began. Since these interaction events must be on their light-cone this will occur at a minimum time separation of T_0, both defined by:

$$\text{Minima:} \qquad \underline{d}_0 \equiv \underline{x}[1: t_{10}] - \underline{x}[2: t_{20}] = \underline{x}_{10} - \underline{x}_{20} \quad \text{and} \quad T_0 \equiv t_{10} - t_{20}$$

Similarly, there will always occur a maximum spatial separation d_* and maximum temporal separation T_* between these two electrons during their interaction-period:

$$\text{Maxima:} \qquad \underline{d}_* \equiv \underline{x}[1: t_{1*}] - \underline{x}[2: t_{2*}] = \underline{x}_{1*} - \underline{x}_{2*} \quad \text{and} \quad T_* \equiv t_{1*} - t_{2*}$$

6.2 SYMMETRIC INERTIAL REFERENCE FRAMES

Using only "simple clocks" to define temporal durations plus line-of-sight observations and common-sense methods of defining spatial separations and directions, it is an observable fact that under many circumstances, local material objects appear to travel at constant speeds in straight lines relative to the distant stars for finite time durations. Classical mechanics was the first successful scheme to describe the motions of material bodies in numerical terms. Newton first introduced the concept of *inertial mass* to describe how bodies move when they remain cohesive by retaining their straight-line motion between 'significant' local events (like collisions or separations). In a complementary manner, he introduced the concept of *impulse* to describe how bodies change their uniform motion when subject to an impulse event at a point in time. When one pair of impulses is exchanged between two interacting bodies, their total inertial momentum remains unchanged. These new ideas were organized into a set of three inter-dependent hypotheses that have become known as Newton's Laws of Motion.

It might be thought unusual that the concept of 'impulse' is used here in reference to Newton's Laws when the standard view today is that Newton wrote about 'forces' not 'impulse'. The root of this misunderstanding is that Newton wrote in Latin and used the word '*vis*', which most English translators since Motte have usually translated as 'force'. Earlier in *De Motu* [58] Newton had moved between discrete force (impulse), continuous force and inertial force.

However, Bernard Cohen (the founder of Harvard's History of Science Department and editor of Newton's papers) has pointed out (in a small supplementary note) in his *Birth of a New Physics* [59] that Newton presented two forms of the Second Law in the original *Principia* but gave primacy to an instantaneous force or impact (impulse) in formulating his Second Law: "The change in motion is proportional to 'the motive force impressed'. Newton conceived that F = ma is a limiting case of the impact law, the situation when the time between successive impacts decreases indefinitely, so that the force ultimately achieves the limiting condition of acting continuously. The law F = ma was thus considered by Newton as derived from the impact law." Since this paper demonstrates that for asynchronous interactions on the light-cone this limit is never reached it is important always to remember Newton's original impulse formulation of the dynamics of inertial particles.

It is an experimental fact that at any location only three spatial directions at 90° to each other need be defined to establish the angular separation between any two other points in space relative to the location position using line-of-sight techniques. These 3 directions are referred to as orthogonal axes when equal units of distance can be determined along each direction. When a 'master' clock is placed at the origin, which is considered stationary or moving at a constant velocity relative to the distant stars, then other standard clocks, (consistent with the methods of speed measurements) may be placed at any spatial location, all changing at the same rate and all synchronized to indicate the same duration of time as the master-clock then the total assembly is referred to as a 4-dimensional inertial reference frame. This was Newton's implied space-time scheme for defining the basis for comparing theoretical (or calculated) results with experimental observations in the real world. Any inertial frame of reference could be constructed (if only in our imaginations) to assign numerical values to each location of an object in space at any instant of time. In theory, these reference numbers could be developed to an unlimited degree of accuracy relative to arbitrary intervals of space and time (the unit measures). The resulting numbers are usually represented as endless repeating decimal fractions (Newton's terminology) or so-called "real numbers". This scheme is also considered suitable for the current research programme at all scales of the universe from the closest inter-electronic separations to the largest separations between the galaxies. In the present theory, space itself has no significance beyond the equal-time separation of pairs of electrons. In this scheme, reference-frames cannot be shifted, rotated or accelerated as they have no material reality – they are simply mathematical abstractions that support experimental observations. There are no actual 'rigid rods' except in the minds of idealists.

Since both the location of the origin of the reference frame and its orientation (relative to distant stars) is quite arbitrary then these settings must **not** appear in any formulation of any theory of electron dynamics. Similarly, the value chosen for when the master-clock is initialized should not be relevant. This universal Relativity Principle (first proposed by Poincare in 1899 [05]) implies that only differences of spatial and temporal separations should appear in any mathematical formulations, since activity in the world is independent of how **we** describe or measure it. These requirements are usually known as invariant co-ordinate transforms. This research programme will show that Newton's scheme is actually a self-consistent approach for representing the real-world dynamics of all objects constructed from electrons. It will also be demonstrated that the two-way optical scheme proposed by Einstein in 1905 [39] is incorrect. This latter 'thought-experiment' for defining kinematics was the basis for defining Lorentz transformations of the spatial and temporal co-ordinates themselves relative to moving sets of inertial/optical reference frames. As in classical mechanics, it will be shown that the Galilean transforms are sufficient in this programme to describe all such relative motion. It will be seen that the Lorentz transforms are only a requirement for field theories that assume continuous interactions and ignore the interactions between sources and 'targets'.

One could imagine that the fundamental interaction between two electrons is totally asymmetric; in other words it requires four distinct and different event times, this is illustrated below:

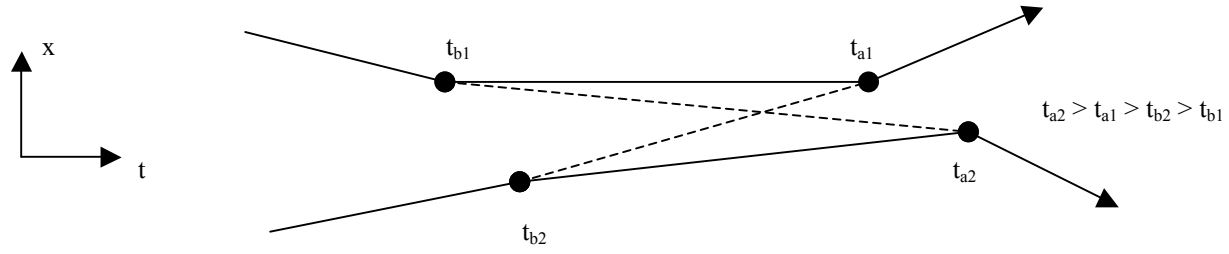

Fig. 1 Totally Asymmetric Interaction

This type of interaction would require very specific co-ordination and so an appeal is made to Ockham's Razor that nature prefers a simpler situation that might be called the "Principle of Maximum Temporal Symmetry", which effectively requires that the fundamental interaction should remain independent of which electron is labeled #1 or #2. This is equivalent to

assuming: $t_{a1} = t_{a2} = t_a$ and $t_{b1} = t_{b2} = t_b$. Additionally, the interaction should not change sign if $t_b > t_a$, so the interaction should only depend on $(t_a - t_b)^2$, in effect, the value of the time parameter could be reversed at this fundamental level and the basic interaction would remain unchanged. So, this theory assumes that, at the most fundamental level, maximal temporal and spatial symmetry exists when only one interaction occurs. Hence, one single interaction always involves both electrons in a symmetric exchange of energy and momentum. This is illustrated below, where the origin O' is offset from the center.

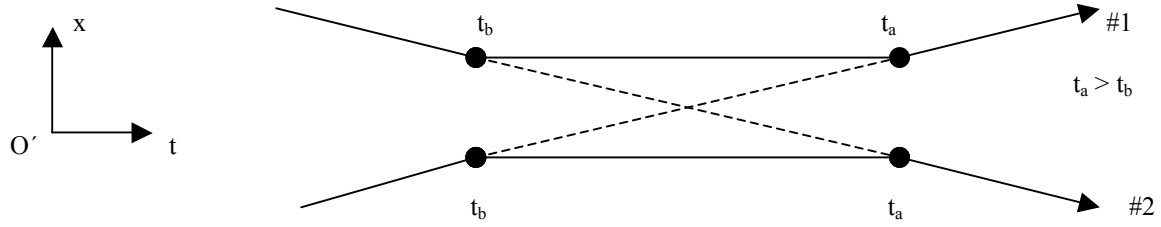

Fig. 2 Totally Symmetric Interaction

The mathematical analysis of electron interactions is simplified when viewed from an inertial reference-frame that itself incorporates the maximum degree of symmetry in both space and time; this is called the "**S**ymmetric **I**nertial **R**eference **F**rame" (or **SIRF**). Within this frame, the total (combined) velocity of the two electrons, $\underline{V}$ at any one time t, is always zero. Since every electron has the same, universal inertial mass parameter, m then this immediately implies that the SIRF is equal to the conventional Center-of-Momentum reference frame; as momentum is Newton's "quantity of motion", then $\underline{p}_k = m\ \underline{v}_k$. So, in the SIRF:

$$\underline{V}(t) = \underline{v}_1[t] + \underline{v}_2[t] = 0 \quad \text{and} \quad \underline{P}(t) = \underline{p}_1[t] + \underline{p}_2[t] = 0$$

Here, the origin (O) of the SIRF is located at the center of the interaction (hence the nomenclature).

NB A single interaction involves a double impulse between both electrons but this is NOT **two** separate interactions.

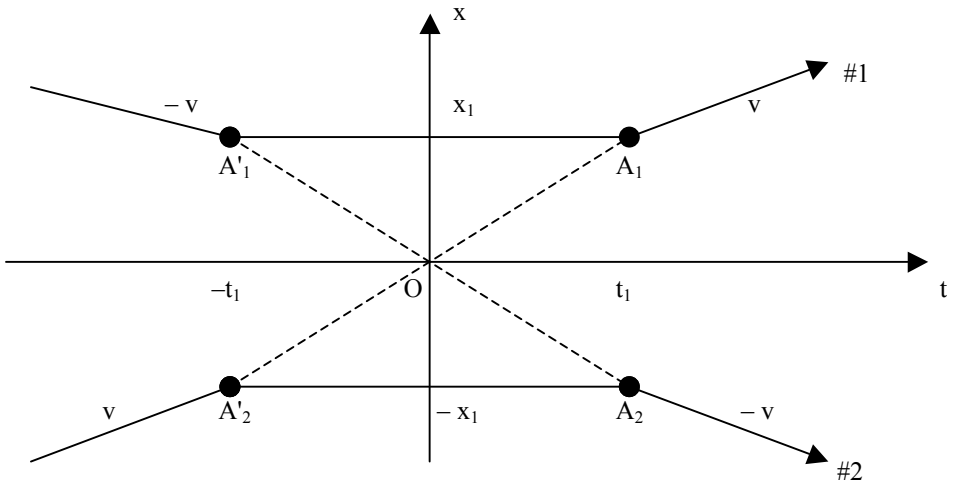

Fig. 3 Interactions in the SIRF

At this point, a useful labeling convention for the interaction nodes will be introduced when there occur several consecutive interactions. Letters are assigned alphabetically beginning with the interaction nodes closest to the temporal origin; those occurring before time zero will be labeled with dashes; for example:

$$A_k \Leftrightarrow \underline{x}[k: t_1]\ , \quad A'_k \Leftrightarrow \underline{x}[k: -t_1]\ , \quad B_k \Leftrightarrow \underline{x}[k: t_2]\ , \quad B'_k \Leftrightarrow \underline{x}[k: -t_2]\ , \text{ etc}$$

The notation $\langle a'_j : b_k \rangle$ will be used to refer to the impulse (or semi-interaction) that occurs between electron #j at node a' and electron #k at node b. It should be noted that for each electron to lose energy during the convergence phase ($t < 0$) and

subsequently regain energy during the divergence phase (t > 0), it is necessary that every impulse begin and end on a node. At first glance, the "double-convergent" interaction (fig. 4b) would appear to be the simpler double interaction as it involves just adding the extra impulse pair $\langle B'_2 : B_1 \rangle$ & $\langle B'_1 : B_2 \rangle$ to the single interaction represented by the pair $\langle A'_2 : A_1 \rangle$ and $\langle A'_1 : A_2 \rangle$. The "double-parallel" interaction is more complex: this involves 'unlinking' the first impulse $\langle A'_1 : A_2 \rangle$ and 'relinking' them to both $\langle A'_1 : B_2 \rangle$ & $\langle B'_1 : A_2 \rangle$; similarly for the other half of the interaction $\langle A'_2 : A_1 \rangle$ is 'cross-linked' to both $\langle A'_2 : B_1 \rangle$ & $\langle B'_2 : A_1 \rangle$. Unfortunately, the requirement that all impulses occur on the light-cone compel the choice of the more complex "double-parallel" as nature's solution to the double interaction hence eliminating the possibility of simple, additive interactions, as will now be demonstrated by showing that the alternative possibility leads to super-luminal speeds. There are two possibilities linking the interaction nodes of electron #1 and electron #2 depending on whether the links are parallel or convergent.

Case "Double-Convergent":

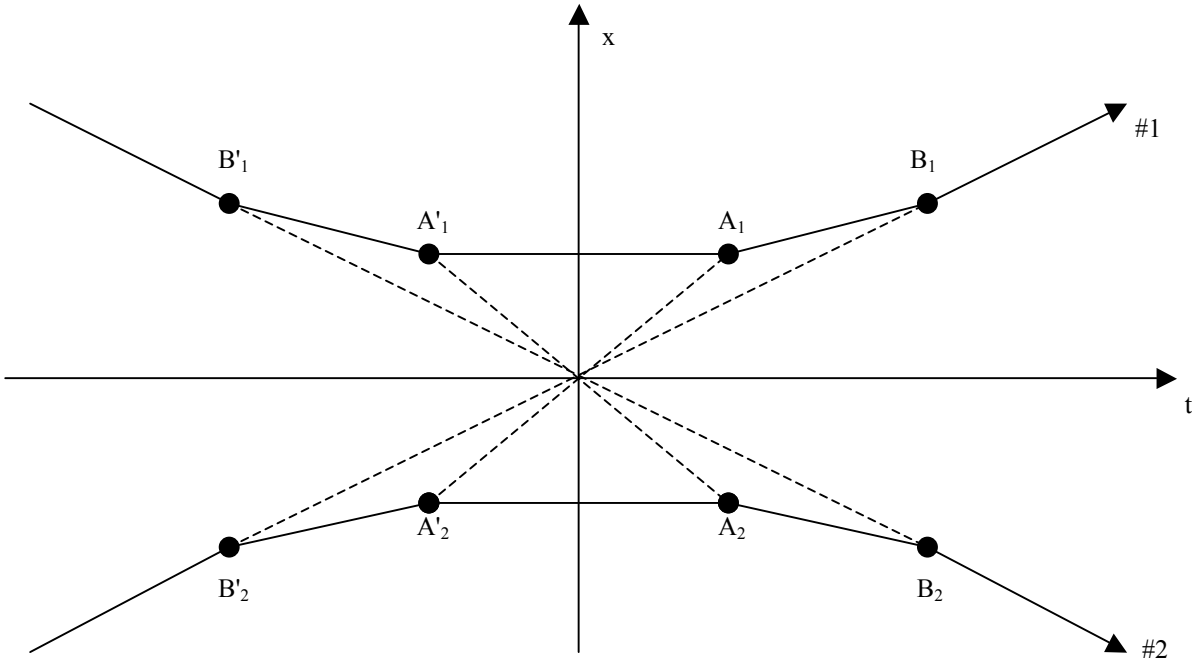

Fig. 4a The Double-Convergent Interactions

The impulse $\langle A'_2 : A_1 \rangle$ needs to communicate between node $A'_2 \Leftrightarrow (-\underline{x}_{10} , -t_{10})$ & $A_1 \Leftrightarrow (\underline{x}_{10} , t_{10})$. The complementary impulse $\langle A'_1 : A_2 \rangle$ communicates between $A'_1 \Leftrightarrow (\underline{x}_{10} , -t_{10})$ & $A_2 \Leftrightarrow (-\underline{x}_{20} , t_{10})$. In either case, the spatial separation D_1 is equal to 2 x_{10} and the temporal separation T_1 is equal to 2 t_{10} ; as these nodes are on the light-cone $D_1 = c\ T_1$ or $x_{10} = c\ t_{10}$. For the impulse $\langle B'_2 : B_1 \rangle$ needs to communicate between node $B'_2 \Leftrightarrow (-\underline{x}_{20} , -t_{20})$ & $B_1 \Leftrightarrow (\underline{x}_{20} , t_{20})$; its complementary impulse $\langle B'_1 : B_2 \rangle$ communicates between $B'_1 \Leftrightarrow (\underline{x}_{20} , -t_{20})$ & $B_2 \Leftrightarrow (-\underline{x}_{20} , t_{20})$; therefore $x_{20} = c\ t_{20}$. The *inter-nodal* velocities $\underline{v}_k$ are defined in terms of the speed of electron #1 going from the node at t_{k+1} to t_{k+2} :

$$\underline{v}_k \equiv (\underline{x}(k: t_{k+2}) - \underline{x}(k: t_{k+1})) / (t_{k+2} - t_{k+1})$$

Thus, v_0 is the speed of electron #1 going from node A_1 to node B_1 (k = 0). For simplicity, this can be reduced to motion in one direction only (say $\underline{z}$); then:

$$x_{20} = x_{10} + v_0 (t_2 - t_1) = c\ t_1 + v_0 (t_2 - t_1) = v_0\ t_2 + (c - v_0)\ t_1 = c\ t_2 \quad \text{thus} \quad (c - v_0)\ t_1 = (c - v_0)\ t_2$$

Therefore, if $v_0 \neq c$, especially when $v_0 << c$ then $t_2 = t_1$, which indicates there can only be **one** possible interaction producing a maximum change in speed, namely c. Alternatively, if we assume that $t_2 \neq t_1$ then $v_0 = c$; this first speed is c. This means that a single impulse, say $\langle A'_2 : A_1 \rangle$ accelerates the electron from rest ('traveling' from $A'_1 = \underline{x}_{10}$ to $A_1 = \underline{x}_{10}$) to its maximum speed, c. This means that a second impulse, say $\langle B'_2 : B_1 \rangle$ would further accelerate it to a speed beyond c. But this is excluded from the earlier analysis of the EM interaction that indicated that no electron can be accelerated above

light-speed by any one other electron. This is a logical contradiction implying that this type of double interaction does **not** occur.

Case "Double-Parallel":

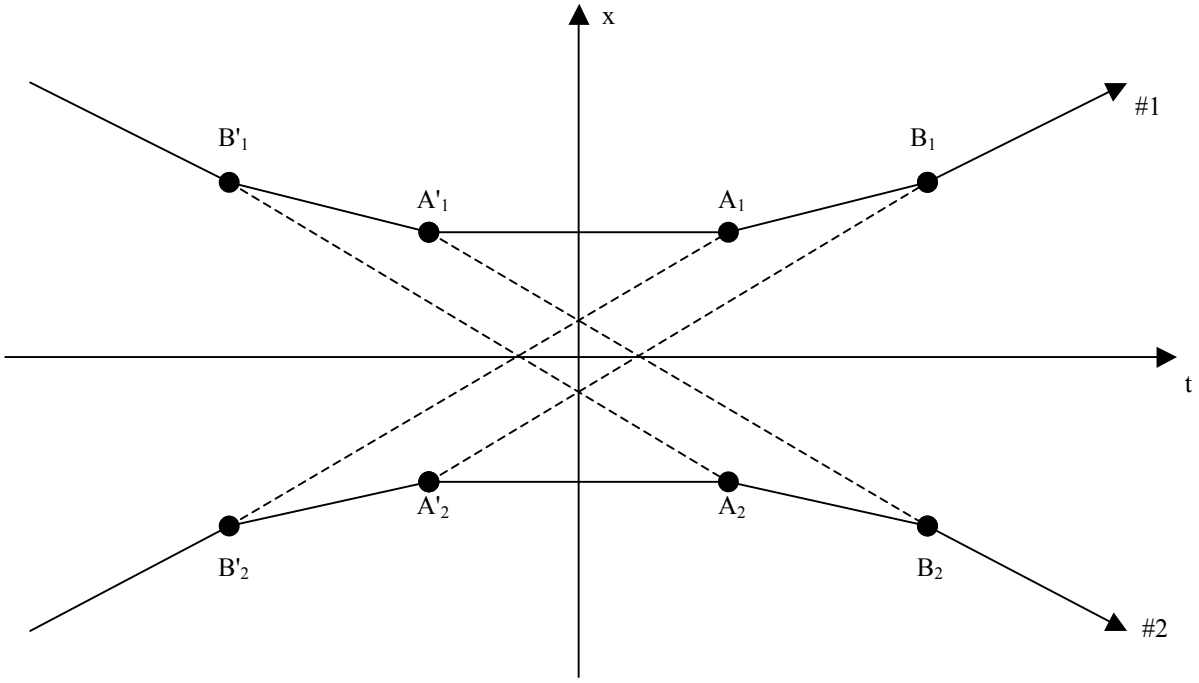

Fig 4b The Double-Parallel Interactions

In this situation all four impulses communicate across the same distance D_2 in the same time T_2 ; so consider the impulse $\langle B'_2 : A_1 \rangle$ involving the nodes $B'_2 \Leftrightarrow (-\underline{x}_{20} , -t_{20})$ and $A_1 \Leftrightarrow (\underline{x}_{10} , t_{10})$ thus $D_2 = x_{10} + x_{20}$ and $T_2 = t_{10} + t_{20}$
But these are on the light-cone, so $x_{10} + x_{20} = c (t_{10} + t_{20})$ but by definition $x_{20} - x_{10} = v_0 (t_{20} - t_{10})$

Introducing the first time and space differences, $\Delta t_1 \equiv t_{20} - t_{10}$ & $\Delta x_1 \equiv x_{20} - x_{10}$ then $\Delta x_1 = v_0 \Delta t_1$

Since $x_{20} = c\, t_{10} + \frac{1}{2} (c + v_0) \Delta t_1$ and $x_{10} = c\, t_{10} + \frac{1}{2} (c - v_0) \Delta t_1$ then for $v_0 << c$ $x_{10} \cong x_{20} \cong c (t_{10} + \frac{1}{2} \Delta t_1)$

Thus, this solution permits <u>any</u> value for v_0, from near zero to almost c, for various changes in final speed. A similar analysis can be conducted for three and four consecutive interactions. In every case, whenever two impulses originating with the <u>same</u> electron 'cross' then the result is that the electrons reach light-speed during an interim transition from one node to the next. This leads to the conclusion that **only** '*parallel-interactions*' provide a valid mechanism for accelerating each electron during the divergence phase and decelerating during the convergence phase. The corresponding space-time diagrams are referred to as "**Canonical Interaction Diagrams**" (or CIDs) and are 'constructed' according to the following rules.

1) All 'links' (impulses) correspond to light-speed (i.e. at 45º).
2) The innermost node of one electron 'links' to the outermost node of the other electron.
3) All 'links' (or 'rays') are parallel with incremental (but not equal) separation.

These rules will be illustrated in the next section that explores the consequence of two electrons interacting via "parallel-interactions".

6.3 PARALLEL INTERACTIONS

This section will investigate the interactions between two electrons in much greater detail. It will demonstrate that all consecutive interactions between any two electrons always occur in parallel, across space. Only three consecutive inter-electron interactions need be considered, further interactions follow the same argument recursively. The primary focus will be on the 'outbound' phase of one of the electrons (arbitrarily labeled here #1) when it is receding from their closest spatial separation from the symmetric origin (the analysis is conducted in the SIRF, see section 6.2). In this phase, electron #1 is accelerating as it is receiving a series of impulses from electron #2 that is in its 'inbound' phase and decelerating, from the reaction to each impulse. The middle node of these three semi-interactions will be labeled t_1 for electron #1 and labeled t_2 for electron #2. The immediately prior nodes will be labeled t_1' and t_2' respectively, while the immediately successor nodes will be labeled t_1'' and t_2''. A similar notation will be used to label their corresponding spatial locations, e.g. $\underline{x}_1'' \equiv \underline{x}(1: t_1'')$. Additionally, the following convention will be used for labeling differences between all node variables.

$$\Delta A_n \equiv A_n'' - A_n \quad \& \quad \Delta A_n' \equiv A_n - A_n' \qquad \text{e.g.} \qquad \Delta t_n \equiv t_n'' - t_n \quad \& \quad \Delta \underline{x}_n' \equiv \underline{x}_n - \underline{x}_n'$$

Each inter-nodal velocity is defined using the finite differences in space and time: $\underline{v}_n \equiv \Delta \underline{x}_n / \Delta t_n \quad \& \quad \underline{v}_n' \equiv \Delta \underline{x}_n' / \Delta t_n'$ This situation is illustrated below.

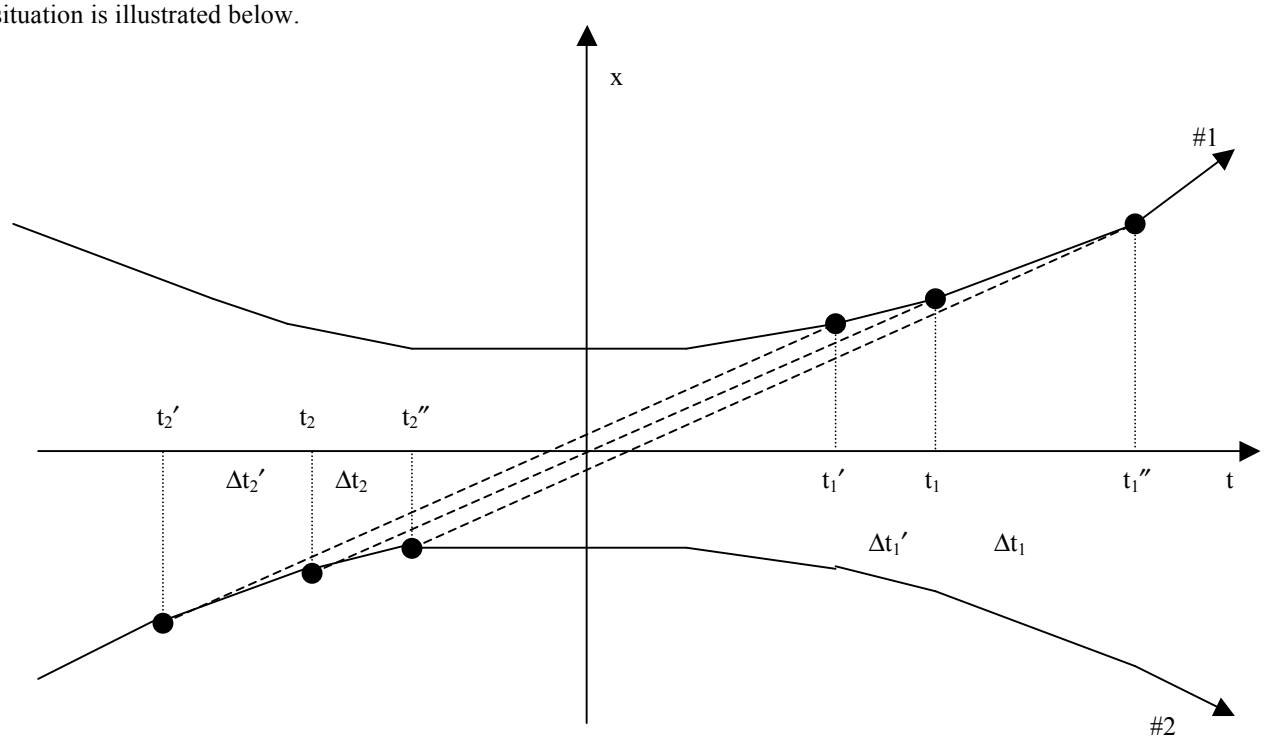

Fig. 5 Three Interactions in the SIRF

It will prove useful to define the "spatial separation vector", $\underline{\mathcal{S}}$ and the "temporal separation interval", $\mathcal{T}$ for each node pair.

$$\underline{\mathcal{S}} \equiv \underline{x}_1 - \underline{x}_2 \;, \quad \underline{\mathcal{S}}' \equiv \underline{x}_1' - \underline{x}_2' \;, \quad \underline{\mathcal{S}}'' \equiv \underline{x}_1'' - \underline{x}_2'' \;; \quad \mathcal{T} \equiv t_1 - t_2 \;, \quad \mathcal{T}' \equiv t_1' - t_2' \;, \quad \mathcal{T}'' \equiv t_1'' - t_2''$$

For this set of interactions, $t_1 > t_2$ so all the $\mathcal{T}$'s are positive while the differences are positive or zero: $\Delta t_n \geq 0 \;\; \& \;\; \Delta t_n' \geq 0$. By convention, the positive spatial directions are chosen as parallel to the motion of electron #1 during its 'outbound' phase. Equivalently, the speed of electron #1 in its outbound phase is positive. The closest inter-electron separation ($2\, d_0$) occurs at $t = 0$ when the electrons have no individual motion in this reference frame in one particular spatial direction which is taken to be the z-direction. In the SIRF the inbound and outbound motion is symmetrical, i.e. $\underline{x}[2: t] = -\underline{x}[1: t]$.

$$\therefore \quad \underline{\mathcal{S}}'' - \underline{\mathcal{S}}' = \underline{v}_1 \Delta t_1 - \underline{v}_2 \Delta t_2 + \underline{v}_1' \Delta t_1' - \underline{v}_2' \Delta t_2' = (\underline{\mathcal{S}}'' - \underline{\mathcal{S}}) + (\underline{\mathcal{S}} - \underline{\mathcal{S}}') = \Delta \underline{\mathcal{S}} + \Delta \underline{\mathcal{S}}'$$

The single semi-interaction between t_1 and t_2 involves an impulse $\Delta\underline{\mathbf{I}}_{21}$ from electron #2 to electron #1 changing the velocity of electron #1 from $\underline{v}_1'$ to $\underline{v}_1$ or in terms of its linear momentum (defined as Newton's quantity of motion) $\underline{p}_k[t] \equiv m\,\underline{v}_k[t]$:

$$\textit{Interaction Impulse}: \quad \Delta\underline{\mathbf{I}}_{21}[t_2\,,\,t_1] \;\equiv\; \Delta\underline{p}[1{:}\,t_1] \;=\; \underline{p}_1[t_1] \;-\; \underline{p}_1'[t_1]$$

This is simply Newton's Second Law of Motion (defining the concept of 'impulse') applied to the EM electron interaction. In contrast to all 'virtual particle' models, there is **no** assumption here that a 'virtual particle', such as a 'photon', is emitted from electron #2 and this 'photon' travels to electron #1 where it is absorbed. Here the one single (shared) semi-interaction is viewed as occurring asynchronously between the pair of electrons at two different times; it is manifest as a local impulse, $\Delta\underline{\mathbf{I}}_{21}$ at the later event (at electron #1) and as a reactive-impulse, $\Delta\underline{\mathbf{I}}_{12}$ at the earlier event (at electron #2).

$$\textit{Reaction Impulse}: \quad \Delta\underline{\mathbf{I}}_{12}[t_1\,,\,t_2] \;\equiv\; \Delta\underline{p}[2{:}\,t_2] \;=\; \underline{p}_2[t_2] \;-\; \underline{p}_2'[t_2]$$

Following the Newtonian programme, the Third Law of motion is also accepted here as always applying to the inter-electron interaction; namely: action equals reaction (even when the interaction occurs asynchronously at <u>different</u> times).

$$\textit{Asynchronous Conservation}: \quad \Delta\underline{\mathbf{I}}_{12}[t_1\,,\,t_2] \;=\; -\;\Delta\underline{\mathbf{I}}_{21}[t_2\,,\,t_1]$$

Immediately, this gives: $\underline{p}_1[t_1] + \underline{p}_2[t_2] = \underline{p}_1'[t_1] + \underline{p}_2'[t_2]$ or $\underline{v}_1[t_1] + \underline{v}_2[t_2] = \underline{v}_1'[t_1] + \underline{v}_2'[t_2]$

Therefore, Newton's Third Law, <u>extended</u> to the EM electron interaction, continues to conserve Total Linear Momentum across the interaction; alternatively stated: the Total Electron Velocity <u>before</u> the interaction equals the Total Electron Velocity <u>after</u> the interaction – a result that agrees with the two-particle Natural Vector analysis established earlier.

Also, using $\mathcal{T}'' = \Delta t_1 - \Delta t_2 + \mathcal{T}$ & $\mathcal{T}' = -\Delta t_1' + \Delta t_2' + \mathcal{T}$ and that all nodes are on the light-cones, $\mathcal{S} = c\,\mathcal{T}$ etc.

Introducing unit vectors, $\underline{s}$ for the spatial separation vectors: $\underline{\mathcal{S}} = \mathcal{S}\,\underline{s}$ and using the earlier results for the other separations.

$\therefore \quad \underline{\mathcal{S}}'' = c\,\mathcal{T}''\,\underline{s}'' = c\,(\Delta t_1 - \Delta t_2 + \mathcal{T})\,\underline{s}'' = \underline{\mathcal{S}} + \Delta\underline{\mathcal{S}} = c\,\mathcal{T}\,\underline{s} + \Delta\underline{\mathcal{S}} \quad \therefore \underline{s}'' = \underline{s} \;\&\; \Delta\underline{\mathcal{S}} = c\,(\Delta t_1 - \Delta t_2)\,\underline{s}''$
and
$\therefore \quad \underline{\mathcal{S}}' = c\,\mathcal{T}'\,\underline{s}' = c\,(-\Delta t_1' + \Delta t_2' + \mathcal{T})\,\underline{s}' = \underline{\mathcal{S}} - \Delta\underline{\mathcal{S}}' = c\,\mathcal{T}\,\underline{s} - \Delta\underline{\mathcal{S}}' \quad \therefore \underline{s}' = \underline{s} \;\&\; \Delta\underline{\mathcal{S}}' = c\,(\Delta t_1' - \Delta t_2')\,\underline{s}'$

These vector decompositions are justified on the basis that the interval $\mathcal{T}$ is arbitrary, thus: $\underline{s}'' = \underline{s}' = \underline{s}$ Q.E.D.
This demonstrates that *all impulses act in parallel across space between the two electrons*. It is useful to define the interaction-velocity, in terms of the universal speed parameter, c.

$$\textit{Interaction-Velocity}: \quad \underline{c} \equiv c\,\underline{s} \qquad \text{where } \underline{s} = \text{unit vector } \{\,\underline{x}_1[t_1] - \underline{x}_2[t_2]\,\} \text{ i.e. } \underline{\text{Any}} \text{ interaction pair.}$$

Now, this result imposes several constraints on the changes in each electron's velocity:

$$\therefore\; c\,(\Delta t_1 - \Delta t_2)\,\underline{s} = \underline{v}_1\,\Delta t_1 - \underline{v}_2\,\Delta t_2 \quad \&\quad c\,(\Delta t_1' - \Delta t_2')\,\underline{s} = \underline{v}_1'\,\Delta t_1' - \underline{v}_2'\,\Delta t_2'$$

Most importantly, if it is assumed that these time intervals are equal (anticipating the limit of all $\Delta t \to 0$) then:
When: $\Delta t_1 = \Delta t_2$ & $\Delta t_1' = \Delta t_2'$ then: $(\underline{v}_1 - \underline{v}_2)\,\Delta t_1 = 0$ & $(\underline{v}_1' - \underline{v}_2')\,\Delta t_1' = 0$

But, in general, $\underline{v}_1 \neq \underline{v}_2$ & $\underline{v}_1' \neq \underline{v}_2'$ so $\Delta t_1 = 0$ & $\Delta t_1' = 0$ $\therefore t_1'' = t_1' = t_1$ Similarly: $t_2'' = t_2' = t_2$

So, if the time-intervals are <u>equal</u> between the three successive interactions then they can only occur <u>simultaneously</u>.
Alternatively, if the interaction intervals are non-zero (i.e. not continuous, $\Delta t_1 \neq 0$ & $\Delta t_1' \neq 0$) then $\underline{v}_1 = \underline{v}_2$ & $\underline{v}_1' = \underline{v}_2'$.
In this case, the impulse results in no change in speed – not a very impressive form of interaction.

Moreover, if: $\Delta t_1 = \Delta t_2$ & $\Delta t_1' = \Delta t_2'$ then: $\Delta t_2 + \Delta t_2' = \Delta t_1 + \Delta t_1'$ But $\Delta t_n + \Delta t_n' = t_n'' - t_n'$ $\therefore t_2'' - t_2' = t_1'' - t_1'$

$$\therefore \quad \underline{\mathcal{S}}'' - \underline{\mathcal{S}}' = c\,(\,\Delta t_1 + \Delta t_1' - \Delta t_2 - \Delta t_2'\,)\,\underline{s} = c\,(\,(t_1'' - t_1') - (t_2'' - t_2')\,)\,\underline{s} = 0 \quad \therefore \quad \underline{\mathcal{S}}'' = \underline{\mathcal{S}}' \;\&\; \mathcal{T}'' = \mathcal{T}'$$

Thus, with 'matched' time differentials (i.e. $\Delta t_1 = \Delta t_2$ & $\Delta t_1' = \Delta t_2'$) there will be **NO** net change in the spatial separation from the first of the three interactions (t_1' & t_2') to the final interaction (t_1'' & t_2''). In effect, this assumption results only in the exchange of velocities between the pair of interacting electrons. In summary, ***there are NO continuous interactions***.

6.4 SPACE-TIME INTEGRITY

In the present theory, all electrons are considered as real entities; in other words, each example of this class of objects is considered to have a permanent and unconditional existence, independent of the existence or activities of any other object in the universe. Mathematically, since all electrons are also considered here to be 'point' objects with zero spatial extent then their existence is reflected as continuous trajectories across space throughout all of time. As was discussed in section 5.1.2, this means that the *location* of each unique electron (labeled #n) at any time, 't' can be represented by a unique combination of three real numbers, $x_j(n: t)$ that may be combined in a given reference frame (like the SIRF described in section 6.2) into a 3D vector, $\underline{x}(n: t)$. In terms of the concepts of 'above' and 'below' limits, this position vector is defined for all times, 't' so when subject to a small change Δ, it is always possible to define a 'limit from above', $\underline{x}^+$ and a 'limit from below', $\underline{x}^-$ for any specific value of the time argument, 't'.

$$\textit{Above.Limit}: \ \underline{x}^+[n: t] \equiv \lim_{\Delta \to 0} [\ \underline{x}[n: t+\Delta]\ ] \ \ \& \ \ \textit{Below.Limit}: \ \underline{x}^-[n: t] \equiv \lim_{\Delta \to 0} [\ \underline{x}[n: t-\Delta]\ ]$$

Here, as in standard calculus, the Continuum Assumption is always made for each electron's location representation; namely for all values of the location function's argument then the 'limit from above' is equal to the 'limit from below', or every electron's location function is "smooth" with respect to time; in other words:

$$\textit{Continuum Assumption}: \ \underline{x}^+[n: t] = \underline{x}^-[n: t] = \underline{x}[t]$$

The following analysis is based on extending Newton's three Laws of Motion to asynchronous interactions between pairs of electrons described in section 6.3 where each electron moves inertially between each interaction node, as in figure 5. Using the same notation as in section 6.3 and focusing only on the motion of the two electrons around the interaction node n, then:

$$\underline{x}_n'' = \underline{x}_n + \Delta\underline{x}_n = \underline{x}_n + \underline{v}_n \Delta t_n \quad \& \quad \underline{\mathcal{S}}'' = \underline{c}\, \mathcal{T}'' \quad \text{or} \quad \underline{x}_1'' - \underline{x}_2'' = \underline{c}\ (t_1'' - t_2'')$$

$$\therefore \quad \underline{x}_2 = \underline{x}_2'' - \underline{v}_2 \Delta t_2 = \underline{x}_1'' - \underline{c}\ (t_1'' - t_2'') - \underline{v}_2 \Delta t_2 \quad \text{but} \quad \underline{x}_1 - \underline{x}_2 = \underline{c}\ (t_1 - t_2)$$

$$\therefore \quad \underline{x}_1'' = \underline{x}_1 + \underline{v}_1 \Delta t_1 = \underline{x}_2 + \underline{c}\ (t_1 - t_2) + \underline{v}_1 \Delta t_1 = \underline{x}_1'' - \underline{c}\ (t_1'' - t_2'') - \underline{v}_2 \Delta t_2 + \underline{c}\ (t_1 - t_2) + \underline{v}_1 \Delta t_1$$

$$\therefore \quad \underline{v}_1 \Delta t_1 - \underline{v}_2 \Delta t_2 = \underline{c}\,(t_1'' - t_2'' - t_1 + t_2) = \underline{c}\,(t_1'' - t_1 - t_2'' + t_2) = \underline{c}\,(\Delta t_1 - \Delta t_2)$$

This results in the Before Space-Time Integrity Condition : $(\underline{c} - \underline{v}_1)\,\Delta t_1 = (\underline{c} - \underline{v}_2)\,\Delta t_2$

Similarly, considering the semi-interaction between nodes at t_n and t_n' , then : $(\underline{c} - \underline{v}_1')\,\Delta t_1' = (\underline{c} - \underline{v}_2')\,\Delta t_2'$

We will now introduce the 'before & after' notation around node, n using the convention: $A_n^+ \equiv A_n$ & $A_n^- \equiv A_n'$

So, both integrity conditions may be combined using the '<u>before & after</u>' variable, $\lambda = \pm$, to produce:

$$\textbf{\textit{The Space-Time Integrity Condition}}: \quad \mathbf{(\underline{c} - \underline{v}_1^{\lambda}[t_1])\,\Delta t_1^{\lambda} = (\underline{c} - \underline{v}_2^{\lambda}[t_2])\,\Delta t_2^{\lambda}}$$

This is a necessary and sufficient condition for two electrons to interact at times t_{1n} and t_{2n} in terms of their before and after velocities, $\underline{v}^{\lambda}(k: t_{kn})$. This enables the earlier results in section 6.3 to be quickly recovered, thus:

A) Assuming : $\Delta t_2 = \Delta t_1 = \Delta$ & $\Delta t_2' = \Delta t_1' = \Delta'$ but not necessarily $\Delta = \Delta'$

then: $\Delta\underline{x}_2 = \Delta\underline{x}_1$ & $\Delta\underline{x}_2' = \Delta\underline{x}_1'$ so $\underline{v}_2 = \underline{v}_1$ & $\underline{v}_2' = \underline{v}_1'$

Therefore, with these assumptions, the two electrons continue to <u>move exactly in parallel</u> to each other across these arbitrary interacting times, even in the continuous interaction limit $\Delta \to 0$ and $\Delta' \to 0$. This prediction does not reflect the reality of

the mutual repulsion of two electrons when interacting, so these assumptions must be rejected, implying that $\Delta t_2 \neq \Delta t_1$ and $\Delta t_2' \neq \Delta t_1'$.

B) Assuming : $\Delta t_1' = \Delta t_1 = \Delta_1$ & $\Delta t_2' = \Delta t_2 = \Delta_2$ but not necessarily $\Delta_1 = \Delta_2$

then either 1): $\Delta \underline{x}_1' = \Delta \underline{x}_1$ & $\Delta \underline{x}_2' = \Delta \underline{x}_2$ so $\underline{v}_1' = \underline{v}_1$ & $\underline{v}_2' = \underline{v}_2$

or 2): $\Delta \underline{x}_2 = \Delta \underline{x}_1$ & $\Delta \underline{x}_2' = \Delta \underline{x}_1'$ so $\underline{v}_1 \Delta_1 = \underline{v}_2 \Delta_2$ & $\underline{v}_2' \Delta_2 = \underline{v}_1' \Delta_1$

Solution (B1) can be rejected since this leaves each electron unchanged ($\underline{v}_n' = \underline{v}_n$) : the 'boring' non-interaction.

While solution (B2) in the continuum limit of both $\Delta_1 \rightarrow 0$ and $\Delta_2 \rightarrow 0$ implies that in the limit $\Delta_1 \rightarrow \Delta_2$ so $\underline{v}_2 = \underline{v}_1$ and $\underline{v}_2' = \underline{v}_1'$; but this is the same result as assumption (A), which was rejected leading to the principal conclusion of this paper that there is *NO CONTINUOUS EM interaction possible between two identical, inertial repulsive particles*. Thus, any EM interaction with asynchronous temporal delays, characterized by a finite space-time propagation parameter, like c, can only interact <u>discontinuously</u> over time. The remainder of this research programme will therefore be based on the necessary assumption that all electromagnetic interactions between electrons occur at an exact, integral multiple (N) of a universal time duration. This new universal constant will be referred to as the (electron) ***chronon*** and will be denoted by τ.

$$\textit{Chronon} : \quad \Delta_1 \equiv N_1 \tau \quad \& \quad \Delta_2 \equiv N_2 \tau$$

The Space-Time Integrity Condition now becomes an integer constraint: $(\underline{c} - \underline{v}_1^{\lambda}[t_1]) N_1^{\lambda} = (\underline{c} - \underline{v}_2^{\lambda}[t_2]) N_2^{\lambda}$

A re-examination of the trajectories of the two electrons in figure 5 now clearly shows the physics of the EM interaction. By focusing on one single semi-interaction, say the n^{th} consecutive interaction between these two electrons, then electron #1 is at the P_1 or (t_1 ; $\underline{x}_1$) when it 'receives' the impulse from electron #2 that 'emitted' this same impulse when it was at P_2 or (t_2 ; $\underline{x}_2$). Traditionally, in EM theory electron #1 is usually referred to as the 'target' electron as it is located at the 'field point' while electron #2 is referred to as the 'source' electron. Colloquially, we might say that electron #1 is at the "here-and-now" while electron #2 was "there-and-then". The next interaction occurs for electron #1 after a time interval Δt_1 when it has traveled a distance $\Delta \underline{x}_1$ at speed $\underline{v}_1$ when it has reached P_1'' or (t_1'' ; $\underline{x}_1''$); this next impulse was 'emitted' by electron #2 when it was at P_2'' or (t_2'' ; $\underline{x}_2''$), where t_2'' occurs after a time interval Δt_2 after time t_2 and $\underline{x}_2''$ is a distance $\Delta \underline{x}_2$ away from $\underline{x}_2$ and was reached at speed $\underline{v}_2$. Again, we might freely refer to P_1'' as the "here-after" and the point P_2'' as the "there-after". The conclusion of the above analysis is that *it is <u>impossible</u> for the target electron to reach the "here-after" from the "here-and-now" in the same time interval as the source-electron reaches the "there-after" from the "there-and-then"* (in other words, it is <u>never possible</u> that $\Delta t_1 = \Delta t_2$). The **finite** speed (*c*) of the EM interaction requires that ($\Delta t_1 \neq \Delta t_2$).

This section concludes with the re-establishment of a formula for the changes in kinetic energy of point particles due to finite impulses first established by W. Thomson (1824-1907) and P. G. Tait (1831-1901), two giants of 19^{th} Century mechanics, who directly inspired Clerk-Maxwell when they systematically reformulated Newton's classical mechanics in terms of discrete changes [57]. This result will be used many times in most of the following papers in this programme.

Since $K_j = \frac{1}{2} m (\underline{v}_j \bullet \underline{v}_j)$ then $\Delta K_j[t_{jn}] = K_j^{+}[t_{jn}] - K_j^{-}[t_{jn}] = \frac{1}{2} m (\underline{v}_j^{+} \bullet \underline{v}_j^{+} - \underline{v}_j^{-} \bullet \underline{v}_j^{-}) = \frac{1}{2} m (\underline{v}_j^{+} + \underline{v}_j^{-}) \bullet (\underline{v}_j^{+} - \underline{v}_j^{-})$

Remembering that $< \underline{v}_j > = \frac{1}{2} (\underline{v}_j^{+} + \underline{v}_j^{-})$ and $\Delta \underline{\mathbf{I}}_j = \Delta \underline{p}_j = \underline{p}_j^{+} - \underline{p}_j^{-} = m (\underline{v}_j^{+} - \underline{v}_j^{-})$ then $\Delta K_j[t_{jn}] = < \underline{v}_j > \bullet \Delta \underline{\mathbf{I}}_j$

The new results established in this paper, especially the Space-Time Integrity Condition and the finite time interval (or chronon) will be explored in much more detail in the remaining papers in this series, including the hypothesis that electrons have an intrinsic (but very short-range) **local motion** that is actually the 'cause' of the phenomenon of 'spin'.

7. MESOSCOPIC ELECTRODYNAMICS

7.1 ELECTRODYNAMICS – THE LOST HISTORY

This sub-section will summarize the alternative developments of 19th Century "Continental" electrodynamics that are almost never discussed or, at best, when they are briefly reviewed, are dismissed as a mistaken path before James Clerk Maxwell *successfully discovered* EM field theory. This almost lost history [60] is included now as the present theory incorporates key parts of this earlier research into a new theory of electron dynamics: both in its classical and quantum forms. Much of the story of the research into electrical effects involves the interwoven effects of magnetism – a macroscopic phenomenon that has been investigated for hundreds of years. It is to be expected that when a concept in natural philosophy, like magnetism, has an extensive ancestry it will be assumed to be a natural part of reality. Indeed, in the 18th Century fluid models were all the rage, so scientists viewed magnetism as the result of a separation (or polarization) of two distinct magnetic fluids, known as "boreal" (northern) or "austral" (southern), within any magnetic body: these ideas readily evolved into field theory.

This programme rejects all field concepts and therefore views magnetism only as a macroscopic **epiphenomenon** due to the interactions between electrons when they are moving relative to each other in all three spatial dimensions. This section will set the stage for the subsequent development of a mesoscopic theory of electrodynamics that can also generate oscillatory interactions between charge-densities in remote conductors without introducing a metaphysical energy-carrier ("light") that requires an imaginary set of force-densities. Extending this theory to oscillations in atomic systems (in a subsequent paper) will lead to a new theory of emission and absorption between remote atoms without any recourse to quantum field concepts.

It was Joseph Priestley who first proposed [61] that the force between electrical bodies probably resembled Newton's law of gravitation; that is, its strength diminished with the square of the distance between them, while Henry Cavendish had shown (but not published) that this force also varied with the products of the electrical charges (Q_1 and Q_2) on the two bodies. Electrical studies were first given a public mathematical formulation in 1783 by Charles Augustin de Coulomb (1736-1806). It also seems that in deriving the force between two point particles, Coulomb relied more on analogies with Newton's law of gravitation than his doubtful measurements with a torsion balance, which could not be reproduced in recent experiments [62]. He claimed that the (static) force $\underline{F}_1$ on particle #1 with an electrical charge Q_1 due to the presence of a second with charge Q_2, separated by a spatial distance r depends on the inverse square of the distance:

$$\underline{F}_1 = Q_1 Q_2 \, \underline{r} \, / \, r^3 \qquad \text{where} \quad \underline{r} = \underline{x}_1 - \underline{x}_2$$

Early 19th Century electrical studies focused on the flow of electricity through circuits involving metallic wires and sources of dynamic electricity, following the invention in 1800 of the electrical 'pile' (or battery) by Alessandro Volta (1745-1827). It was inevitable that after the ready availability of sources of current electricity, an experimentalist like the Danish scientist, Hans Christian Oersted (1777-1851) would soon discover (in 1820) that an electric current in a metallic wire had an effect on a nearby magnet, inventing the new science of "electro-magnetism". Immediately on reading of this discovery, André-Marie Ampère (1775-1836) saw this as a powerful tool to investigate the microscopic depths of matter, which he intensely believed was atomic in nature, rather than fluidic or continuous. He set himself the goal of demonstrating that magnetism was the result of permanent electrical currents surrounding the molecules of matter. His first experiments demonstrated that two parallel conducting wires attract or repel each other depending on whether the two currents flow in similar or opposite directions. He next created an equivalent bar magnet by passing an electric current through a helically coiled wire: a device he called a "solenoid". In 1826, he named these force effects of electric currents or electricity in motion "***electrodynamics***". He summarized these results (now known as "Ampère's force law") in terms of the forces between any two current-carrying elements; these were based on the distance between them (like Newton and Coulomb) but now also based on the relative directions of the current elements. Few of his contemporaries were prepared to test (or even accept) this microscopic view of reality, as it was too radical a change from their earlier beliefs. A major problem here was that Ampère first proposed his force law in terms of (un-measurable) forces between infinitesimal components (current elements) of the complete circuits. In modern (vector) notation, the fractional electric force $d^2\underline{F}$ on the current element $\mathcal{I}\, d\underline{x}$ in the (target) conductor (when this element is located at $\underline{x}$) created by the current element $\mathcal{I}'d\underline{x}'$ in the nearby source circuit (located at $\underline{x}'$) carrying a steady current $\mathcal{I}'$ (all components measured at the same time, or valid for stationary currents) can be written as [63]:

$$d^2\mathcal{F} = -\mathcal{I}[\underline{x}]\,\mathcal{I}'[\underline{x}']\,\{\,2(d\underline{x}\bullet d\underline{x}') - 3(\underline{r}\bullet d\underline{x})(\underline{r}\bullet d\underline{x}')/r^2\,\}\,\underline{r}\,/\,r^3 \qquad \text{where} \quad \underline{r} = \underline{x} - \underline{x}'$$

As with the other early force laws, the electrical product factors were not determined but assumed by analogy with Newton. Modern experiments have actually confirmed a similar type of law for the forces between complete (closed) circuits [64]. In modern (vector) notation, the total electric force $\underline{F}$ on the closed, target circuit carrying a steady current $\mathcal{I}$ created by a nearby closed, source circuit carrying a steady current $\mathcal{I}'$ can be written as:

$$\mathcal{F} = -\oint \oint (d\underline{x} \bullet d\underline{x}') \mathcal{I}[\underline{x}] \mathcal{I}'[\underline{x}'] \underline{r} / r^3 \quad \text{where} \quad \underline{r} = \underline{x} - \underline{x}'$$

Carl Friedrich Gauss (1777-1855) was one of the few 19th Century scientists to recognize the revolutionary significance of Ampère's work, so that in 1845 he encouraged his friend (and ex-assistant) Wilhelm Eduard Weber (1804-1891) to retain Ampère's force formula (then just revised by Neumann). Gauss reminded Weber of his own idea that had obsessed him: the truly unique and original idea that ***electrodynamic action is not simultaneous across space but needs a finite time***. This concept was first picked up by Gauss's most famous student, Bernhard Riemann (1826-1866), who proposed in 1855 that the EM retarded potential propagated across space at the speed of light – an idea that was only advanced ten years later by L. V. Lorenz [65] and then expanded upon later by Carl Neumann (the son of Franz). Maxwell was aware of this research.

Weber did extend Ampère's results for fixed conductors to moving conductors by adopting his professor's (Gustav Fechner) suggestion of <u>two</u> sets of equal and oppositely charged electric particles, all moving through the conductor with equal speed but moving in opposite directions (the first nearly-correct model of electric current in a wire) and the true model of magnetic effects. Using this model of interactions between remote point charges, Weber proposed his force law (on Q_1) in 1846 [60].

$$\underline{F}_1 = Q_1 Q_2 \{1 - v^2 / c^2 + 2r\, a / c^2\} \underline{r} / r^3 \quad \text{where} \quad \underline{r}[t] = \underline{x}_1[t] - \underline{x}_2[t] \;\; ; \;\; \underline{v}[t] = d/dt [\underline{r}] \;\; ; \;\; \underline{a}[t] = d/dt [\underline{v}]$$

Weber's force law depends only on point charges and is fully symmetrical and fully relational; i.e. it is independent of the co-ordinates or the motion of observers. It must be pointed out that Weber, like all scientists of his time (and later!) was a "continuist": he simply assumed that the interaction between two charged bodies occurred continuously throughout time (i.e. forces, not impulses). He ignored Gauss and also assumed that the far interactions were instantaneous, so all his results are at a single time (e.g. $\underline{F}_1[t] = \underline{F}_2[t]$) and all his derivatives assume an equal small time difference shrinking equally fast to zero when computing derivatives for either of both particles. None of these assumptions will be followed here in this programme.

In 1855, Weber and his friend Rudolf Kohlrausch published their experimentally determined value of the ED constant *c* as 3.10×10^{10} cm/sec [66]. Riemann, observing this experiment, was the **first** to point out that that this value was very close to Fizeau's experimental determination of the velocity of light (§2.2.4). Weber identified *c* as the relative velocity at which two electric charges can no longer influence each other ("the ultimate speed limit"): this is the viewpoint adopted by this programme. At that time (before the discovery of beta rays), the absence of high-speed ($v \approx c$) electric charges prevented Weber from investigating these effects further. In 1871, Weber rebutted Helmholtz, who had claimed that Weber's force law violated the principle of conservation of energy as Helmholtz had conducted his analysis at one single time across all space so that he had implicitly assumed that electric charges could interact at all speeds, including infinitely fast; i.e. using instantaneous forces. In this rebuttal, Weber also identified the minimum separation that must occur between two charges – this was the origin of the electron's "classical radius". In the same memoir, Weber also introduced the universal concept of charge-to-mass ratio (*e*/*m*) and equivalently, the proton-electron mass ratio (*M*/*m*). Unfortunately, Weber never pursued Gauss's asynchronous ED action concept and so ended up with only an approximate version of the ultimate force law: an objective of the present paper. In contrast, Clerk-Maxwell offered his own phenomenological set of differential field equations, incorporating the 'reality' of magnetism and building on the 19th Century obsession with the Aether concept.

Ampere's original form of his differential force law only captured the radial force between two current elements. Lucas has shown that adding a conservative term, which integrates to zero around a closed source, adds an extra tangential component leading to an additional torque that characterizes magnetic effects [67]. This leads to the now standard '**triple cross**' form of Ampere's differential force between current elements (as well as complete circuits); this is the <u>critical</u> step in defining the **B** field. However, this also <u>removes</u> the critical longitudinal force component, which appears experimentally (see §7.7.1 later). Ampère's circuital form can be transformed when the conductors are uniform (e.g. wires) so that $\mathcal{I}[\underline{x}] = \mathcal{I}$ and $\mathcal{I}'[\underline{x}'] = \mathcal{I}'$.

Since: $\underline{r} / r^3 = -\nabla[1/r] \;\; \therefore \oint d\underline{x} \bullet \underline{r} / r^3 = -\oint d[\nabla[1/r]] = 0$ and $\underline{A} \wedge (\underline{B} \wedge \underline{C}) = (\underline{A} \bullet \underline{C})\, \underline{B} - (\underline{A} \bullet \underline{B})\, \underline{C}$

$$\therefore \quad \mathcal{F} = \mathcal{I}\mathcal{I}' \oint \oint \{(d\underline{x} \bullet \underline{r})\, d\underline{x}' - (d\underline{x} \bullet d\underline{x}')\, \underline{r}\} / r^3 \quad \therefore \quad \mathcal{F} = \mathcal{I}\mathcal{I}' \oint \oint d\underline{x} \wedge (d\underline{x}' \wedge \underline{r}) / r^3$$

$$d^2\mathcal{F} = \mathcal{I}\mathcal{I}'\, d\underline{x} \wedge (d\underline{x}' \wedge \underline{r}) / r^3 = \mathcal{I}\, d\underline{x} \wedge d\underline{B}' \qquad \text{where} \quad d\underline{B}' = \mathcal{I}'\, d\underline{x}' \wedge \underline{r} / r^3$$

Electron Units

Electrodynamics has always been cursed by a profusion of conflicting units. This is particularly true with respect to electrical charge and current, where ESU was used for static electric charge and EMU for currents. Since this programme views the micro-world exclusively in terms of electrons, it proposes to cut this Gordian knot by always viewing electric charge only as the number of electrons, each with charge e (in ESU) and by viewing electric current as the number of electrons N_A passing orthogonally through an area per second. This approach will now be used to unify Ampère's force law, which was always defined in electromagnetic units (EMU or amps), with the idea of flow of electrons. This is readily accomplished by using conventional dimensional analysis, with [Q] used to represent the dimension of electric charge and using Coulomb's static force law where the force F between two electrified bodies is found to vary inversely with the square of their separation.

$\therefore$ $[F] = [Q]^2 [L]^{-2}$ Newton's Law requires dimensionally: $[F] = [M] [a] = [M] [L] [T]^{-2}$ $\therefore$ $[Q]^2 = [M] [L]^3 [T]^{-2}$

There is no point in evaluating the dimensionality of an isolated charge in this theory as it **always** appears as a product. This dimensional argument can now be applied to Ampère's differential force using Grassmann's substitution: $\mathcal{I}\, d\underline{x} = \underline{u}\, dQ$. The revised form of the force law must therefore incorporate a dimensional constant β, whose value is to be determined.

$$d^2\underline{F} = \beta\, \mathcal{I}\mathcal{I}'\, d\underline{x} \wedge (d\underline{x}' \wedge \underline{r}) / r^3 = \beta\, dQ\, dQ'\, \underline{u} \wedge (\underline{u}' \wedge \underline{r}) / r^3 \;\therefore\; [d^2\underline{F}] = [\beta] [Q]^2 [L/T]^2 [L]^{-2} = [Q]^2 [L]^{-2} \;\therefore\; [\beta] = [L/T]^{-2}$$

If this is to be viewed as a universal constant with the dimensions of the inverse square of a velocity then it strongly suggests that this parameter is related to the speed of 'light' c. The hypothesis then becomes: $\beta = 1 / c^2$. The new form of Ampère's force law, written in terms of electrical currents measured in ESU per second (or electrons/sec), is:

$$d^2\underline{F} = \mathcal{I}\mathcal{I}'\, d\underline{x} \wedge (d\underline{x}' \wedge \underline{r}) / c^2 r^3 = \mathcal{I}\, d\underline{x} \wedge d\underline{\mathbf{B}} / c \qquad \text{where} \quad d\underline{\mathbf{B}} = \mathcal{I}'\, d\underline{x}' \wedge \underline{r} / c\, r^3$$

Alternatively, the force on a group of $d\mathcal{N}[\underline{x},t]$ electrons traveling at an average speed $u[\underline{x},t]$ through a metallic conductor, produced by another a group of $d\mathcal{N}'[\underline{x}',t]$ electrons traveling at an average speed $u'[\underline{x}',t]$ through another conductor, is:

$$d^2\underline{F} = d\mathcal{N}\, d\mathcal{N}'\, \underline{u} \wedge (\underline{u}' \wedge \underline{r})\, e^2 / c^2 r^3$$

This introduces the principal space-time parameter c into the foundational current-current ED interaction; here it scales each of the velocities $\underline{u}$ and $\underline{u}'$, a scaling which always characterizes relativistic EM. When this c is combined with the universal electric charge e^2 on the electrons moving through the conductors, this also suggestively hints at the quantization of action h that must occur at the extreme micro-level of electron-electron interactions, characterized by Sommerfeld's universal Fine Structure Constant $\alpha \equiv 2\pi e^2 / h c$. Constant currents in a circuit are only possible if there is no accumulation (or loss) of charge anywhere in the conductor **and** if external sources of electrical energy exist generating electrical potential differences within the circuit that is, electromotive force (EMF). The local effects of these sources cannot conserve energy so they cannot be represented mathematically by the gradient of a scalar potential, $\phi[\underline{x}, t]$; i.e. $\underline{\mathbf{E}}_S = -\underline{\nabla}[\phi]$ which implies that the equivalent, local force density $\underline{\mathbf{E}}_m$ is rotational (a non-zero curl) – a condition satisfied by the vector potential $\underline{A}$. This speed corresponded to Weber's estimate that 30 billion ESU units of charge had to flow through a current element to generate as much force 'inductively' as could be experienced directly by a static charge at the same spatial separation. This offers a major clue to 'magnetic' induction: ***the statistical interaction between several electrical charges in relative motion***.

It must be pointed out that although the differential form of Ampère's force law appears like an inverse square law, when the integrations are carried out around both complete circuits, the induced voltage varies with the inverse distance between the centers of the two circuits. The focus on induced voltage emphasizes that these forces act on the currents (electrons) and not mechanically on the conductors themselves (unlike Ampère's force law). Even Maxwell had to admit in his *Treatise* that he did not understand the relationship between mechanical and electrical forces, which were both produced by electrical currents; worse, Ampère's force disappears as soon as one current stops flowing even though all the charges are still present in both conductors. Again, in contrast to timeless mechanical forces, induction laws always involve variations in time.

The modern view that electrical phenomena are ultimately due to electrons means that Coulomb's static (timeless) summary of forces between aggregates of massive numbers of charges **cannot** be the basis for a fundamental theory of universal EM. Individual electrons can never be "pinned" – they are always in motion, if not before an interaction then certainly afterwards. This implies that electrodynamics should be a more fruitful source for a foundational theory at all scales of aggregation.

7.2 MESOSCOPIC AVERAGES

This section develops an electron-only alternative to EM field theory. Maxwell's EM theory was a linear, analytic theory involving differentials of space and time that could be integrated over all of simultaneous space up to the macroscopic reality of electric wires and magnets, which were the subject of the supportive experimental laws (e.g. Oersted, Faraday, etc); at best, Maxwell's EM is a theory of mesoscopic space and time averages (millions of electrons) but not a fundamental, microscopic theory of dynamical reality (individual electrons). It makes the simplest assumption: that every microscopic interaction is identical and equal to the totality divided by the total number; i.e. a simple average. Since all macroscopic measurements involve myriads of electrons, the present theory explicitly introduces a mesoscopic-averaging scheme that deliberately minimizes statistical fluctuations while recognizing that this is only an averaging process at the atomic scale. This technique introduces the idea of the mesoscopic cell (the 'cell') that contains sufficient electrons that contribute to any measurement of any property *f* that may vary from electron to electron or vary over time with every electron when viewed over time (like position). Large numbers of electrons contribute collectively to generate an aggregate macroscopic value F. In explicit contrast to the differential calculus, the mesoscopic approach never takes the infinitesimal limit, where the cell shrinks to zero size or even to sub-atomic sizes. Since these cells are defined in terms of large numbers of electrons, this approach is limited to real, material media, which are assumed to be homogenous, unless stated otherwise: the metallic conductor will be the prototypical example. In such a cubic cell of linear size Δx_0 the 3D volume has a size $(\Delta x_0)^3$; let each such cell contain (as a spatial or temporal average) ΔN_0 electrons. When many (K) cells are compared, each centered at a location $\underline{X}_j$ and therefore denoted as C_j, the actual number of electrons in each cell, at any given time t_0, is ΔN_j, so the average fluctuation in the electron count per cell across this set is defined by its root mean square (RMS) variation:

$$RMS[\Delta N] \;=\; SQRT[\; 1/K \textstyle\sum_j (\Delta N_j - \Delta N_0)^2 \;]$$

The average number of electrons in each cell is defined by the requirement that the variation in number is less than 0.1%.

$$RMS[\Delta N] \,/\, \Delta N_0 \;\leq\; 1 \,/\, 1000$$

It is a well-known statistical result that when there are a large number of components, the RMS (root mean square) variation approximates very well to the square root of the average.

$$RMS[\Delta N] \;\cong \sqrt{(\Delta N_0)} \qquad \therefore \;\; \sqrt{(\Delta N_0)} \;\cong\; 1000 \qquad \therefore \;\; \Delta N_0 \cong 10^6$$

Thus, each cell must contain at least one million electrons contributing to the macroscopic measurement. The number of atoms (*n*) in a cubic centimeter of material is approximately equal to Avogadro's Number, which is 6×10^{23}, so each cell's volume should be about 10^{-18} cm^3 or equivalently, about 10^{18} cells per cubic centimeter. This means that each cell's linear dimension is about 10^{-6} cm, so that the mesoscopic cell's linear size will be defined as exactly 10^{-6} cm with a spatial volume of exactly 10^{-18} cm^3 containing approximately about 10^5 electrons. The time for any EM action to 'cross' each such cell is about 10^{-17} seconds. For visible light, where there about 10^{14} variations per second, each electron in any such cell would experience only about 0.1% of the variation per cycle; in effect, optical effects appear as an almost constant effect across each cell with a small (1 to 2%) variation from one cell to the next adjacent longitudinal cell. If the time for an external EM effect to reach the middle of the k^{th} cell is t_k then every electron within this cell is influenced by this 'passing' effect within a time of $(t_k \pm 10^{-17})$ seconds, so it again an acceptable approximation (within 0.1%) to treat all the electrons in this cell as reacting at the same time t_k. Consider a specific homogenous material medium, containing *n* active electrons in every cubic centimeter (i.e. its active electron density is *n*) then each cell in this medium contains $\Delta\mathcal{N}_0$ 'active' electrons.

$$\Delta\mathcal{N}_0 \;=\; n\,(\Delta x_0)^3 \qquad\qquad \{\; \text{If } n \;=\; 10^{23}/\,\text{cm}^3 \;\text{ then: }\; \Delta\mathcal{N}_0 \;=\; 10^5 \;\}$$

The electrons in the k^{th} material cell can be identified (or 'labeled') by a double index from {k,1} to {k, $\Delta\mathcal{N}_0$} , so that all these electrons in this cell contribute to the measured quantity F an amount ΔF_k at an average time t_k ; defined by:

$$\Delta F_k[t] \;=\; \textstyle\sum_i f[\;\underline{x}_i\,,\,t\;] \;=\; \Delta\mathcal{N}_0\, f_k[t]$$

In the case of electrical charge, where every electron contributes an identical charge *e*, the total charge is the same in each cell, an amount ΔQ_0, where: $\Delta Q_0 \;=\; e\,\Delta\mathcal{N}_0 \;=\; n\,e\,\Delta^3 x_0$

This approach agrees with Mead's view of 'collective electrodynamics', where "the properties of **each** electron depend on the state of the entire ensemble." [68]

7.3 ELECTRODYNAMIC POTENTIAL

The concept of electrical potential is central in electricity and deserves its own brief discussion. External (real) work $\mathcal{W}$ is required to overcome the inherent attraction between positive and negative electric charges. When this work is performed by chemical or mechanical means across a metallic conductor it is referred to as electromotive force or "EMF": a confusing term (introduced by Volta in analogy with water flowing through a tube), as this is not actually a force but potential energy. It was conceived as electrical 'height', in analogy with a gravitational potential that causes objects to move 'down-hill'. All these electrical energy sources, when attached to the two ends of a conductor, create an electrical potential energy difference per unit charge $\mathcal{V}$ that generates electrical currents in the conductor; this universal capability is measured in ***volts*** or joules per coulomb, as heat is created while the connection persists. Galvanic cells (e.g. lead acid) produce about 2.1 volts per cell. In a neutral metallic conductor, there must be an equal quantity of both positive and negative charges, $\pm Q$.

$$\therefore \quad \mathcal{W} \equiv Q\,\mathcal{V}$$

One of the great confusions surrounding the historical study of classical electromagnetism involves the idea of potentials. When Maxwell originally developed his dynamic theory of EM in 1864 [69] he introduced his greatest physical innovation: the electro-kinetic momentum (denoted A); this new EM concept was specifically designed to play the same foundational role as Newton's concept of momentum in classical mechanics. Like particle momentum, this too was a vector quantity but Maxwell picked up this idea from Franz Ernst Neumann's 1845 concept of an EM potential due to steady electric currents, that is to say, an EM form of *momentum per charge* generated by remote charges in motion; i.e. a **vector potential** [70]. Unfortunately, Gauss's idea of dimensional analysis was not yet widely adopted, so Maxwell always failed to distinguish his electro-kinetic symbol's role in equations as momentum or energy. This is critical because the relationship between them involves the 'speed of light' parameter *c*. This error persists today but is usually hidden by the use of SI units, which hide this key dimensionality factor in spurious parameters of the vacuum. This is why this research programme remains firmly committed to the cm/gm/sec convention, which should have been named the **C**arl **G**auss **S**ystem for its brilliant introduction of implicit dimensional analysis in every equation. This loss of physical dimensionality information still persists today with mathematical physicists blithely using "relativistic units", where *c* is set to unity; this is extremely dangerous, as Maxwell demonstrated. In fact, this explicit use of vacuum parameters is re-establishing the Aether model as the implicit conceptual foundation of CEM, forcing students to continue learning about the nonsense of the "displacement current" in empty space.

This problem is avoided in the present approach, which uses *natural vectors* to represent its major physical variables; these mathematical objects are always dimensionally homogenous in their four components (space and time) [71]. In the present theory, the symbol A is reserved exclusively in EM for the vector potential energy per unit charge at a point; to complement the static (or scalar) potential energy per unit charge at a point, denoted by the symbol ϕ. Maxwell's original electro-kinetic momentum will always be introduced explicitly as A /*c*. It is this 'velocity' factor which is the reason [72] why the 'photon' model relates its energy (transferred) $\Delta\mathcal{E}$ to its momentum (transferred) $\Delta\mathcal{P}$ through the necessary relationship: $\Delta\mathcal{E} = c\,\Delta\mathcal{P}$.

7.4 METALLIC CONDUCTION

Early 19th Century electrical studies focused on the flow of electricity through circuits involving metallic wires and sources of dynamic electricity, following the invention in 1800 of the electrical 'pile' (or battery) by Alessandro Volta (1745-1827). Techniques were soon established to measure the quantity of electricity flowing steadily per unit of time: a measure referred to as current with units of amperes (or amps). These effects included the gross mutual influences between electrical currents and permanent iron magnets. As early as 1827, Georg Ohm (1789-1854) published his quantitative researches on electricity flowing through certain metallic conductors [73]. He showed experimentally that there exists a linear relationship for many metals between the magnitude of the current in a conductor and the potential difference across the circuit ("Ohm's Law").

Since 1900, electricity has been recognized as the macroscopic effects of vast numbers of electrons; for example, a cubic centimeter of copper contains about 10^{23} atoms (*n*) each contributing one electron to possible conduction effects. In an electrically neutral copper block, there are always as many mobile electrons per cell as there are copper atoms, in the form of positive ions, so although the mobile electrons can move between atoms - they do so in an apparently random manner (at least to us). Over any measurable time interval, as many mobile electrons will move into a cell as move out the opposite side; in this one sense, these electrons appear to behave like an incompressible fluid. At all the edges of the metallic block, the net attraction of the fixed atoms is strong enough to retain these electrons from leaving the metal so their movement is constrained by the outer edges of the block. Even though this is a dynamic situation, the average values (such as the number density *n* and charge density ρ_e) remain constant as long as the external conditions (context) remain fixed.

7.4.1 METAL MODELS

Free Electron Gas

The principal conceptual problem in the modern theory of metals is the universal assumption that the valence electrons behave like a non-interacting perfect gas; that is, they behave like 'free' electrons and may therefore, be represented in QM by plane wave functions. This is the origin of the mathematical concept of the 'Fermi Sea'. This foundational idea is given no physical justification, particularly as both the positive ions and the electrons themselves are interacting closely together (at least in conventional electrodynamics) through the powerful Coulomb potential. Authors of two respected modern texts on the theory of solids give the following cursory 'justifications' before developing several hundred pages of mathematics based on this assumption. Cusack writes: "It will be readily admitted that metals contain free electrons. … Without further argument, we shall therefore take it that metals contain considerable numbers of electrons free to move about the lattice. This collection will be called a 'free electron gas'." [74] Ziman also writes in his chapter on Electron States: "We shall use the one-electron model, where we ignore any interactions between the valence electrons due to their Coulomb repulsion." This is almost immediately followed by simplifying the spatial potential function to a constant throughout the whole metal, ignoring the huge variations introduced by the positive ions forming the lattice, so that he can say: "The electrons then fall automatically into plane-wave states." [75] As a typical phenomenological theory, agreement with experiment is given as sufficient justification for this very non-physical theory; that this math does not map to the physics is never discussed.

These free electrons are then assigned quantized wavelengths in a cube of dimensions L cm according to the periodic rule:

$$P_x = \eta_x h / L \ , \ P_y = \eta_y h / L \ , \ P_z = \eta_z h / L \quad \text{where} \quad \eta_x , \eta_y , \eta_z = \pm 1, \pm 2, \pm 3, \ \ldots$$

When the metal is in equilibrium and totally isolated from all external influences (i.e. the metal is considered to be thermally at the absolute zero of temperature 0° K) then the electrons are assumed to be distributed in the lowest energy configuration. This involves all the points in momentum space centered on the zero kinetic energy origin around cells of size $(h / L)^3$, so the number of states with momentum less than or equal to a value P is:

$$4/3 \ \pi \ (P \ L / h)^3$$

Thus, the number of states with momentum between P and (P + ΔP) per unit volume of metal is: $4\pi \ P^2 \ \Delta P / h^3$
This is multiplied by a factor of 2 (to allow for electron 'spin'). The energy density of states $\mathbf{G}[\varepsilon]$ is defined such that the number of possible states of one electron with kinetic energy between ε and (ε + Δε) is $\mathbf{G}[\varepsilon]\Delta\varepsilon$, where $\varepsilon = P^2 / 2m$.

$$\therefore \ \mathbf{G}[\varepsilon]\Delta\varepsilon = 2 * 4\pi \ P^2 \ \Delta P / h^3 = 8\pi \ P \ (P \ \Delta P) / h^3 = 8\pi \ (m \ v) \ (m \ \Delta\varepsilon) / h^3 \ \therefore \ \mathbf{G}[\varepsilon] = 4\pi \ (2m / h^2)^{3/2} \ \varepsilon^{½}$$

Temperature is introduced through the Fermi-Dirac distribution function $\mathbf{F}[\varepsilon]$ that defines the probability that an electron will have energy ε in an ensemble of electrons in thermal equilibrium at temperature T. Standard texts[76] derive the result:

$$\mathbf{F}[\varepsilon] = 1 / (1 + \exp[\ (\varepsilon - \varepsilon_F) / kT])$$

Here, ε_F is known as the Fermi energy level, which is the maximum electron energy at absolute zero. For pure copper, the calculation finds that this has the value 11 x 10^{-12} erg or 7.0 eV. However, the thermal energy kT is only 1.38 x 10^{-13} erg or about 0.1 eV even for temperatures around T = 1000° K. [77]

Calculations involving this free electron gas model always use the function $\mathbf{N}[\varepsilon]$, where the product $\mathbf{N}[\varepsilon] \ \Delta\varepsilon$ represents the number of electrons per unit volume, in thermal equilibrium, occupying energy levels between ε and (ε + Δε); this equals the number of levels in this range multiplied by the probability that an electron will in fact occupy a level of energy ε.

$$\therefore \ \mathbf{N}[\varepsilon] = \mathbf{G}[\varepsilon] \ \mathbf{F}[\varepsilon]$$

These formulae can then be used to calculate the average kinetic energy of an electron in copper at absolute zero of 4.1 eV, which would require a temperature of about 30,000° K if classical statistics were to be used. Most real metallic electrical properties are determined by electrons with kinetic energy very near to ε_F but Ziman confesses that the free-electron model is contradicted by the fact that "these electrons are in states of wavelength comparable with the lattice spacing." [78] Any waves with this size of wavelength would be heavily scattered (diffracted) by the lattice.

New Metal Model

The new theory of metals presented here is focused back on the atomic lattice rather than the valence electrons. It is the collective motion of the ionic cores (the nuclei with their associated, tightly-bound electrons) that determines almost all the characteristics of metals, even the electrical properties (as will be shown below during the relaxation phase in the Impulse Conduction Model). As the metal lattice balances its energy exchanges with its surroundings (local and remote), it is the extent of its own random vibrations that reflect its thermal equilibrium: these are characterized by the Boltzmann energy kT, where T is the absolute temperature. The more readily the lattice vibrates, the more the lattice interferes with the movement of the mobile electrons and thereby increasing the metal's resistivity. The non-local (or mobile) electrons (for simplicity, one per ion) move between ions under the direct exchange of EM impulses either from other mobile electrons **or** from the positive proton in the nucleus. It is this theory's Saturation Hypothesis that implies that all the other ionic electrons are totally engaged with their nuclear partners with the result that the majority of the electrons and protons within each ion play no external role in the interactions involving mobile electrons. Without the Saturation Hypothesis, the CEM model of 'fluid charge density' must be invoked to produce Coulomb screening [79], which is a bizarre blend of discrete and continuum models of electricity and usually re-appears when QM calculations fail and classical ideas have to brought back on board.

The interactions involving mobile electrons are extremely complex *in toto* but will always involve exchanges of momentum along inter-lattice positions, thus justifying Bloch's theorem [80]. Furthermore, this motion is highly correlated: when an electron is expelled from a neutral atom the ion left behind can then become a possible host for another mobile electron, drawn in by the unbalanced positive charge. Most electrons will 'hop' to a nearest neighbor; this will minimize the total action, as it will take the least time but this does require that the new host ion has previously lost its own mobile electron. Sometimes, this action will be reciprocal when two valence electrons on neighboring atoms approach each other closer than the radial distance to their own nucleus when both electrons will be expelled. Quantizing these local actions means that this will occur with the maximum speed, this will correlate with the valence electron's own orbital velocity. When metallic equilibrium situations are considered then the mobile electrons must distribute their motions together in a time-independent manner. It is this physical model that accounts for the utility of the mathematical notion of Fermi energy plus the fact that it is these highest-energy electrons ('short-hops') that determine most of the physics. This new model of metals will be given a more extensive mathematical formulation in a later paper discussing the dynamics of many-electron systems.

7.4.2 ELECTRON CONDUCTION

Resistivity

In honor of two of the original French investigators of current electricity, the Standard International Unit of electrical charge is denoted as one ***coulomb*** and defined as: 6.24×10^{18} electrons, while the flow of this number of electrons moving across a fixed boundary in one second defines one *ampere* (or ***amp***). In its modern form, Ohm's Law defines the resistance $\mathcal{R}$ (in units of ***ohms***) of a conductor when subject to a constant potential difference of $\mathcal{V}$ volts producing a current of $\mathcal{I}$ amps, as:

$$\mathcal{V} \equiv \mathcal{I}\,\mathcal{R}$$

This can be reformulated in terms of the electric force density $\underline{E}$ and the current density $\underline{\mathcal{J}}$, defined in a uniform, rectangular conductor of length L, cross-sectional area S and unit resistivity ρ_r (or conductivity $\sigma \equiv 1 / \rho_r$).

$$E \equiv \mathcal{V} / L \qquad \mathcal{J} \equiv \mathcal{I} / S \qquad \therefore\ E = (\mathcal{R} / L)\, S\, (\mathcal{I} / S) \equiv \rho_r \mathcal{J} \quad \therefore\ \mathcal{R} = \rho_r L / S$$

It should be emphasized that Maxwell did **not** produce a model for Ohm's Law, not least, because he rejected the idea of localized electric charge. His field theory deliberately avoided discussing the interior of conductors and focused exclusively on the space around them. In his theory, he simply added (but never used) the empirical equation [81]: $\underline{E} = \rho_r \underline{\mathcal{J}}$. This key equation is simply repeated, without comment, in most texts on classical EM expounding Maxwell's Equations. Ironically, Maxwell's EM theory derived from experiments with conducting circuits, facts he wanted to leave behind for empty space.

The Drude Model

Paul Drude developed a classical theory of metallic conduction [82] in 1900, involving inelastic collisions between mobile electrons (moving at an average 'drift' velocity $\mathcal{U}$) and fixed positive ions, both treated as spheres of finite size to calculate the average time τ_D between collisions; this was sufficient to give a semi-quantitative explanation of Ohm's experimental law. Drude 'solved' this problem by introducing the phenomenological electron ('friction') equation:

$$P = e\,E\,\tau_D = m\,\mathcal{U} \quad \therefore \quad E = m\,\mathcal{U} / e\,\tau_D = (m / n\,e^2\,\tau_D)\,n\,e\,\mathcal{U} = (m / n\,e^2\,\tau_D)\,\mathcal{J} \quad \therefore \quad \rho_D = m / (n\,e^2\,\tau_D)$$

Calculations indicate that these conduction electrons have a long 'mean free path'; at normal temperatures (300° K), only one scattering interaction (on average) occurs with the ionic matrix as an electron passes about 100 ionic centers. In marked contrast, at similar temperatures, water molecules collide with each other about fifty times more frequently. These mobile electrons move at relatively high speeds (about $c/200$). Electrical conduction is only macroscopically analogous to water flowing in a pipe. The drift velocity (in copper at room temperature) is calculated to be about 4 mm per second when all the mobile electrons are assumed to contribute to the observed current: this is incredibly slow motion, in comparison.

Impulse Conduction Model

The present mesoscopic model of conduction is based on the following considerations. The nuclei and electrons in a metal form a dynamic system – everything is in motion. Each ion is a local dynamic sub-system of one neutral atom less its mobile electron. The bound electrons in each ion are also moving at very high speed (approximately $c/1000$) localized around each nucleus by the EM attraction between the positive nuclear protons and the negative electrons. Each atom is a conservative system that does not exchange energy with its neighbors in its lowest energy state. Each ion interacts with its neighboring ions as they all vibrate around their average lattice position. The mobile electrons move between the ions but not necessarily just between adjacent ions. These electrons both interact with each other and the totality of the ions in a complex but stable configuration. The complete system is in thermal equilibrium with its surroundings when the block receives as much energy from the outside as it transmits to its external environment. The totality of all N mobile electrons form their own sub-system (collectivity) with its own velocity distribution $F[v_j]$, where j is a monotonically increasing integer set, such that:

$$v_{j+1} \geq v_j \quad \text{where} \quad j = 1, 2, \ldots, N$$

This allows a long-term (high) average speed to be defined: $< v > \equiv 1/N \sum_I F[v_I]\,v_I \approx c / 200$ at 300° K

As the temperature (T) of the block rises, the ions oscillate around their equilibrium lattice positions with larger and larger amplitudes so the mobile electrons are more likely to interact with the vibrating ions and lose some of their kinetic energy to the ionic sub-system. If the mobile electrons are accelerated by the nearby presence of 'external' electrons they will more frequently interact with these ions, which appears as heating of the metallic block with a corresponding rise in temperature. The equip-partition of thermal energy means that these electron-ion interactions are limited to a maximum energy inter-change of kT (much smaller than the average kinetic energy of the mobile electrons) so these interactions do not alter the electron velocity distribution very much, even with a small change in temperature ΔT; i.e. $F[v_I, T + \Delta T] \approx F[v_I, T]$.

The present theory of multiple-electron interactions follows Mead's proposal [83] that "the behavior of collective electron systems is dominated by the interaction of **each** element with **all** the others." Although Mead focused on superconductivity, this view is here considered appropriate for all many-electron systems irrespective of temperature or context since Mead defined any 'collective' situation, as one where "the properties of each electron depend on the state of the entire ensemble." When excess electrons are brought close to one end of a copper wire by connecting to a battery, the localization of the EM interaction means that it is the nearby electrons that repel each other most actively, pushing nearby electrons both outwards to the edge of the conductor and away from the higher concentration. The surface distribution of mobile electrons rapidly stabilizes while the mobile electrons further down the wire (the "longitudinal electrons") try to move even further away. The repulsive impulses from the electrons in the cells with excess concentration increase the average longitudinal velocity but this is countered by the decreases induced by the electrons in the 'fixed' atoms of the metal (scattering). In this model, the conduction electrons have relatively few interactions with the core ionic lattice but many interactions with other nearby electrons moving in the same direction; the net motion of these mobile electrons away from the 'negative' end towards the 'positive' end of the block constitute the observed electric current.

In the equilibrium state, as many electrons enter the wire from the negative battery connection as flow out at the positive connection. These much **fewer**, extra-longitudinal (the real 'conduction') electrons compose all of the observed current, whereas in the Drude model **all** the mobile electrons, moving in all directions at very high (Fermi) speeds and scattering much more frequently, contribute (longitudinally) to the current.

When the external battery is connected across a conductor and the situation is in equilibrium (independent of time) then this model assumes that in a time Δt_0 seconds, an extra $\Delta N'$ electrons will enter the negative end-face of the conductor and fill one cell layer in this time, raising the electron density from n to n', where $n' = n + \Delta n$. The original density n of electrons is just sufficient to keep the cell electrically neutral but while the extra electrons are in a cell, it takes on (temporally) a small extra negative charge ΔQ^* but this charge is not observable externally due to the Saturation Hypothesis such that these extra electrons interact internally with their local neighbors in the adjacent cells. This behaves like a small pulse of extra charge moving from one cell layer to the next (longitudinal) layer in a time Δt_0 at an average speed $\mathcal{U}$ that constitutes the current $\mathcal{I}$.

$$\therefore \quad \mathcal{U} = \Delta x_0 / \Delta t_0 \qquad\qquad \mathcal{I} = e\, \Delta N' / \Delta t_0.$$

In the following theoretical model, a metallic (wire) conductor is viewed as a rectangular block of length L cm and width w cm (L >> w) with a square cross-section of area $S = w^2$. The mobile electrons are assumed to exist uniformly throughout the conductor with a density of n mobile electrons per cm^3 so the total number of mobile electrons in this block is $N = n\, S\, L$. The number of mesoscopic cells along any longitudinal edge is $N_L = L / \Delta x_0$. The number of cells in each layer across any cross-section is $N_S = S / (\Delta x_0)^2$. Let the first ('negative') cross-section (or layer) of cells be identified as #1 with the next cross-section labeled #2, etc through to the 'positive' end-face cells in the layer labeled #N_L.

$$\therefore \quad \Delta N' = \Delta n\, S\, \Delta x_0 \qquad \therefore \quad \mathcal{I} = e\, \Delta n\, S\, \mathcal{U} = \mathcal{J}\, S \qquad \therefore \quad \mathcal{J} = e\, \Delta n\, \mathcal{U} = \rho_c\, \mathcal{U} \qquad \text{where} \quad \rho_c \equiv e\, \Delta n$$

The total extra charge in each layer (say, #l) of the block: $Q_l = e\, \Delta N' = e\, \Delta n\, S\, \Delta x_0 \equiv N_S\, \Delta Q^* \quad \therefore \quad \Delta Q^* = \rho_c\, (\Delta x_0)^3$

The equilibrium flow between cells is idealized here as a two-phase process occurring over a cycle time of Δt_0 seconds; for simplicity, each phase is assumed to be of equal duration (i.e. a time $\frac{1}{2}\, \Delta t_0$ seconds). The first phase is referred to as the acceleration phase and the second as the relaxation phase. In each acceleration phase the conduction electrons in a specific layer (say, #l) interact repeatedly with the conduction electrons in the previous adjacent layer, say #(l – 1), with the result that the conduction electrons in layer #l increase their net longitudinal speed from zero to $2\mathcal{U}$, producing an average speed $\mathcal{U}$ for all the electrons in the cell. In the relaxation phase, this group of slowly moving conduction electrons in this layer interact only with the ionic lattice in their path at a higher than thermal equilibrium rate losing their extra kinetic energy to the lattice and restoring their average longitudinal speed back to zero. Since these extra electrons from the #l layer have now moved into the #(l + 1) layer the process can repeat until this cohort of extra conduction electrons leave the block at the positive connection. Specifically, as a direct result of the Saturation Hypothesis and the Localization Principle, during the acceleration phase all the electrons in any one layer only interact locally with electrons in the two adjacent layers. Similarly, during the relaxation phase, the conduction electrons in any one layer only scatter and lose energy with some of the ions in that layer and the next. This gain and loss of kinetic energy occurs repeatedly while the conduction electrons 'drift' across from the negative to the positive end of the block at a net average speed of $\mathcal{U}$ cm/sec; it is the locality of both of these interactions that means the current-voltage results are independent of the spatial shape or twisting of the wire.
Let ΔN^* represent the average number of conduction electrons in any cell. $\therefore \quad \Delta N^* = \Delta n\, (\Delta x_0)^3 \quad \& \quad \Delta Q^* = e\, \Delta N^*$

In the **relaxation phase**, each 'packet' of conduction electron, in any one cell, loses its extra kinetic energy as it crosses one cell by scattering against the ionic lattice in a time of $\frac{1}{2}\Delta t_0$ seconds while it reduces its average speed from $\mathcal{U}$ to zero. This requires the lattice absorb $\Delta \mathcal{W}$ of energy from each conduction packet by exchanging f_L impulses, each of magnitude $\Delta \mathbf{I}_L$, between the lattice and each of these accelerated electron-packets. Let each of these lattice impulses occur over an average ('relaxation') time of δt seconds, while the packet covers an average longitudinal distance ('mean free path') of δx.

$$\therefore \quad \tfrac{1}{2}\, \Delta t_0 = f_L\, \delta t \qquad \& \qquad \tfrac{1}{2} \Delta x_0 = f_L\, \delta x \qquad \& \qquad \mathcal{U} = \Delta x_0 / \Delta t_0 = \delta x / \delta t$$

This is equivalent to a continuous frictional force $\mathcal{F}_L$ slowing the electrons across a finite distance δx, so: $\Delta \mathbf{I}_L = \mathcal{F}_L\, \delta t$.

$$\therefore \quad \Delta \mathcal{W} = f_L\, \delta \mathcal{W} = f_L\, (\mathcal{F}_L\, \delta x) = f_L\, (\mathcal{F}_L\, \delta t)\, (\delta x / \delta t) = \mathcal{U}\, f_L\, \Delta \mathbf{I}_L$$

These f_L lattice impulses reduce the speed of the accelerated electrons from a maximum longitudinal speed of $2\mathcal{U}$ to zero.

$$F_L \, \Delta \mathbf{I}_L = (m \, \Delta N^*) \, 2\mathcal{U} = 2 \, m \, \Delta N^* \, (\Delta x_0 / \Delta t_0) \qquad \therefore \quad \Delta \mathcal{W} = 2 \, m \, \mathcal{U} \, \Delta N^* \, (\Delta x_0 / \Delta t_0)$$

However, this is just the change in potential energy for the packet crossing each cell. $\Delta \mathcal{W} = \Delta Q^* \, \Delta \mathcal{V} = e \, \Delta N^* \mathcal{V} / N_L$

$$\therefore \quad \mathcal{V} = 2 \, m \, \mathcal{U} \, (N_L \, \Delta x_0) / (e \, \Delta t_0) = 2 \, m \, L \, (e \, \mathcal{U} \, S \, \Delta n) / (S \, \Delta n \, e^2 \, \Delta t_0) = 2 \, m \, (L / S) \, \mathcal{I} / (\Delta n \, e^2 \, \Delta t_0)$$

$$\therefore \quad \mathcal{V} = \mathcal{I} \, \mathcal{R} = \mathcal{I} \, \rho_L \, (L / S) \qquad\qquad \therefore \quad \rho_L = m / (e^2 \, \Delta n \, ½ \, \Delta t_0)$$

This can be compared with the classical (Drude) result, where the resistivity is: $\rho_D = m / (e^2 \, n \, \tau_D)$

Since this is a measured quantity, these two values must be equivalent but now the symbols taken on a quite different interpretation. In the Drude model, each electron scatters randomly in 3 dimensions and all mobile electrons in the neutral metal (with a density of n electrons per cm^3) participate in both the current and the scattering with a relaxation time of τ_D. In the present theory, only the much smaller number of conduction electrons, 'injected' by the battery (with a density of Δn electrons per cm^3) constitutes the conduction current entering each mesoscopic layer in a time Δt_0. These models can be compared more closely by writing the present theory's 'longitudinal packet relaxation' time τ_L for δt and writing $\mathcal{U}_L$ for the 'drift' velocity of the electron-package here to distinguish it from Drude's total electron drift velocity $\mathcal{U}_D$, which can then be compared through their role in the measurable current $\mathcal{I}$.

$$\therefore \quad \mathcal{I}_L = \Delta n \, e \, \mathcal{U}_L \, S \quad \& \quad \mathcal{I}_D = n \, e \, \mathcal{U}_D \, S \qquad\qquad \therefore \quad \mathcal{U}_L = (n / \Delta n) \, \mathcal{U}_D \quad \& \quad \tau_L = (n / f_L \, \Delta n) \, \tau_D$$

If these two relaxation times are assumed to be equal then: $n = f_L \, \Delta n \qquad \& \qquad \mathcal{U}_L = f_L \, \mathcal{U}_D$

Thus, the present model predicts a somewhat larger value (approximately $f_L \approx 5$ times larger) for the drift velocity.

The Drude model also assumes a constant velocity approach with the internal electric field balanced by the frictional force: this would only be so in the longitudinal direction, where the field parallels the shape of the extended metallic wire. The present model offers a more dynamic description of the **acceleration phase**, which is described next.

The acceleration of all the conduction electrons in a particular cell layer (say, #l) is due to all the (collective) interactions with all the conduction electrons in <u>all</u> the cells in the #(l – 1) layer. Let ΔP_l represent the total increase in the longitudinal momentum of all the extra conduction electrons in one layer after they have been repulsed by all the conduction electrons in the previous layer in any single acceleration phase.

$$\therefore \quad \Delta P_l = 2 \, m \mathcal{U} \, \Delta N'$$

$$\therefore \quad e \, \Delta P_l = 2 \, m \, (\Delta n \, e \, \mathcal{U} \, S) = 2 \, m \, \Delta x_0 \, \mathcal{I} = 2 \, m \, \Delta x_0 \mathcal{V} / \mathcal{R} = 2 \, m \, \Delta x_0 \mathcal{V} \, S / (L \, \rho_L) = 2 \, m \, S \mathcal{V} / (N_L \, \rho_L)$$

$$= 2 \, m \, S \, \Delta \mathcal{V} / \rho_L = 2 \, m \, S \, \Delta \mathcal{V} \, (\Delta n \, e^2 \, \Delta t_0) / (2 \, m) = e^2 \, \Delta \mathcal{V} \, (\Delta n \, S \, \Delta x_0) \Delta t_0 / \Delta x_0 = e^2 \, \Delta \mathcal{V} \, \Delta N' / \mathcal{U}$$

$$\therefore \quad \mathcal{U} \, \Delta P_l = e \, \Delta N' \, \Delta \mathcal{V} = Q_l \, \Delta \mathcal{V} = \Delta \mathcal{W}_l$$

Thus, the external work $\Delta \mathcal{W}_l$ performed by the battery (or other external source of EMF) to accelerate all the conduction electrons in layer #l is equal to the increase in their total mechanical momentum ($2m\mathcal{U} \, \Delta N'$) multiplied by their average speed $\mathcal{U}$. This can also be compared with the increase in their total kinetic energy $\Delta \mathcal{K}_l$.

$$\therefore \quad \Delta \mathcal{K}_l = \Delta N' \, \Delta [½ \, m \mathcal{U}^2] = \Delta N' \, (m \mathcal{U} \, \Delta \mathcal{U}) = \Delta N' \, (2 \, m \mathcal{U}^2) = \mathcal{U} \, \Delta P_l \qquad\qquad \therefore \quad \Delta \mathcal{W}_l = \Delta \mathcal{K}_l$$

This is the standard result from the classical mechanics of particles that equates the change in external work with the change in kinetic energy but now extended to the EM interactions between conduction electrons in metals.

The CEM Potential Model

The total change in momentum per unit charge, induced in each electron across one cell in layer #*l* by all the electrons in the previous layer during the acceleration period ½Δt_0 seconds is the definition of Maxwell's electro-kinetic momentum $\Delta A_l / c$, where $\Delta A[t; \underline{x}_l]$ is the change in the vector potential between the centers of two cells in adjacent layers in the longitudinal direction (§5.3.2). In the equilibrium condition of steady current, this is independent of location; so, the net change in EM momentum of <u>all</u> the accelerated electrons originating with electrons in the previous layer during one half-cycle is ΔP_l.

$$\therefore \quad \Delta P_l = Q_l \Delta A_l / c \qquad \therefore \quad 2 m \mathcal{U} \Delta N' = e \Delta N' \Delta A_l / c \qquad \therefore \quad e \Delta A_l = 2 m \mathcal{U} c$$

This shows each conduction electron's **change** in momentum ($2m\mathcal{U}$) is being 'moved along' at the 'speed of light' c. [84] These results can be combined to evaluate the change in the EM energy per unit charge $\Delta\phi_l$ between two layers of the block.

$$Q_l \Delta \mathcal{V} = \mathcal{U} \Delta P_l = \mathcal{U} Q_l \Delta A_l / c \qquad \therefore \quad c \Delta \mathcal{V}_l = \mathcal{U} \Delta A_l \qquad \therefore \quad e \Delta \mathcal{V}_l = 2 m \mathcal{U}^2$$

$$\therefore \quad c \Delta A_l = \mathcal{U} \Delta \phi_l \qquad \therefore \quad e c^2 \Delta \mathcal{V}_l = e c \mathcal{U} \Delta A_l = e \mathcal{U}^2 \Delta \phi_l \qquad \therefore \quad \Delta \mathcal{V}_l = (\mathcal{U} / c)^2 \Delta \phi_l \qquad \therefore \quad e \Delta \phi_l = 2 m c^2$$

$$\text{Since} \quad m c^2 \cong 0.511 \text{ eV} \qquad \therefore \quad \Delta \phi_l \cong 1{,}000{,}000 \text{ volts}$$

At first sight, this seems an astonishingly large difference across each layer of electrons but remember this is a **calculated** <u>field</u> quantity ($\Delta\phi_l$), whereas for real drift velocities in the range of 3 cm/sec the factor $(\mathcal{U}/c)^2$ reduces the difference in the **real** potential difference ($\Delta\mathcal{V}_l$) by 10^{-20} to about 10^{-14} volts across a single cell layer of magnitude 10^{-6} cm. Since here, all conditions are assumed uniform throughout the conductor the energy difference <u>across a cell</u> is directly related to the total energy difference across the block in the longitudinal direction.

$$\Delta \mathcal{V} = (\Delta x_0 / L) \mathcal{V} = \mathcal{V} / N_L = (\mathcal{V} / L) \Delta x_0 \equiv E \Delta x_0$$

This defines the (uniform) electric force density per unit charge E in the longitudinal direction, as the potential gradient.

$$E \equiv \Delta \mathcal{V} / \Delta x_0 \cong 10^{-8} \text{ volts per cm}$$

So, the work done in moving the active electrons in one cell (say, #k) in the longitudinal direction against their attractive ions in the lattice is $\Delta \mathcal{W}_k$. This can be expressed in terms of a force density per cubic centimeter, $\mathcal{G}_k \equiv F_k / \Delta^3 x_0$.

$$\therefore \quad \Delta \mathcal{W}_k = \Delta Q_k \Delta \mathcal{V}_k = \Delta Q_k \Delta \mathcal{V} = \Delta Q_k E \Delta x_0 \equiv F_k \Delta x_0 \qquad \therefore \quad F_k = E \Delta Q_k \qquad \therefore \quad \mathcal{G}_k = \rho_c E$$

These results may also be compared with the Drude model, again using the subscript L to distinguish the present ('**L**attice') theory and a D subscript for the **D**rude model, focusing now on a single electron.

$$\Delta p_k / \Delta t_0 = e E_L = e \Delta \mathcal{V} / \Delta x_0 = 2 m \mathcal{U}^2 / \Delta x_0 = 2 m \mathcal{U}_L / \Delta t_0 = m \mathcal{U}_L / (f_L \delta t) = m \mathcal{U}_L / (f_L \tau_L)$$

But Drude's force equation was: $e E_D = m \mathcal{U}_D / \tau_D \qquad \therefore \quad E_L / E_D = (\mathcal{U}_L / f_L \mathcal{U}_D) (\tau_D / \tau_L)$

Thus, as in the comparison of resistivity calculations, these two models can be made consistent ($E_L = E_D$) by assuming that they share the same value for the lattice-relaxation time ($\tau_L = \tau_D$) and the drift velocities are in constant ratio ($\mathcal{U}_L = f_L \mathcal{U}_D$).

Although the results of the present theory of conduction resemble those of the Drude theory there are significant physical differences centered on the proposal that the relatively **few** conduction electrons originate within the voltage source (battery) and flow through the equilibrium collection of lattice-bound ions and mobile electrons that have configured themselves into a stable system. The conduction electrons here only flow longitudinally at a slow, average (drift) speed being pushed along by the next cohort of similar electrons, whereas in the modern version of the Drude model, it is **all** the mobile electrons that bounce around in all directions at the high (Fermi) speed while making very many collisions with the ionic lattice.

7.4.3 SUPERCONDUCTIVITY

Overview

The electrical resistivity of all metallic conductors decreases gradually as the temperature is lowered and with ordinary conductors, such as copper, this decrease is limited by impurities and other defects; even near absolute zero, samples of copper show some resistance. Despite these imperfections, in some materials (known as superconductors) the resistance drops abruptly to zero when the material is cooled below its critical temperature (T_c). Any electric current flowing in a loop of superconducting wire can even persist indefinitely with no external power source: an impossibility with finite resistivity.

The phenomenon of superconductivity [85] was discovered in 1911 by the Dutch physicist Kamerlingh Omnes [1853-1926]. This unexpected electrical effect was complemented in 1933 by an equally unexpected magnetic effect [86]found by Walther Meissner and Robert Ochsenfeld, who discovered that the strength of observed magnetic effects around samples of tin and lead suddenly increased as the samples were cooled through their transition temperatures when they became superconducting. This was interpreted as the metals becoming perfectly diamagnetic with the exclusion of all weak magnetic fields from inside the samples – this is now referred to as the Meissner effect. These effects are regarded as purely quantum mechanical effects.

In a superconductor, the electrical current is only found very near the surface with unusual characteristics that result in both persistent electrical currents (zero electrical resistance) and the absence of interior magnetic effects. This new discovery led in 1935 to the first theory of superconductivity by the London brothers, Fritz and Heinz [87]. Although their simple theory could explain and predict macroscopic observations (i.e. it was a phenomenological theory), it could not provide a more detailed, microscopic explanation. A major insight was provided by the experimental observations in 1950 that the critical temperature decreases as the mass of the metallic isotope increases. In contrast to almost all other electrical properties that are independent of the isotope involved, this indicated the key role that the lattice plays in the dynamics of superconductors. This insight was incorporated into the first microscopic theory of superconductivity that was proposed in 1957 by Bardeen, Cooper and Schrieffer (and always referred to as BCS) [88,89]. Even this theory, which focused on zero resistivity, also failed to provide a physical explanation of the Meissner effect that could arise under steady-state conditions where Faraday's law of EM induction predicts zero electromotive forces needed to generate an electrical current. The only attempt to date is the transient super-current theory by Kozynchenko [90] but this has mainly been ignored as it involved extending the canonical Maxwell-Lorentz equations.

Most of the physical properties of superconductors vary from material to material (such as the heat capacity) and the critical temperature, critical magnetic field strength and critical current density at which superconductivity is destroyed. There are now many exotic alloys that demonstrate superconductivity but all examples are metallic, with the best known being lead (T_c = 7.2° K), mercury (4.2° K) and tin (3.7° K). The best conductors (such as silver, copper, gold) are not superconducting as their crystalline characteristics are viewed as "too tight" to readily generate lattice vibrations at near zero temperatures. The obvious presence of impurities also interferes with the long-range effects (coherence) associated with lattice vibrations that appear necessary for superconductivity. All of these clues point to the key significance of the lattice vibrations.

However, there is a class of properties that are independent of the underlying material. For instance, all superconductors have *exactly* zero resistance to low applied currents when there is no magnetic field present or when the applied magnetic field does not exceed a critical value (H_c). When persistent currents are observed in toroidal rings of superconducting metal, it is also observed that the enclosed magnetic flux is quantized in units of $ch/2e$ (or 2.07×10^{-7} gauss / cm^2). Since this flux is produced by both the external EM sources and the surface currents in the superconductor, it implies that these surface currents are themselves quantized.

The existence of these universal properties has been interpreted to imply that superconductivity is a thermodynamic phase and as such possesses certain distinguishing properties, which are largely independent of microscopic details. The present theory acknowledges these universal characteristics but instead attributes them to the universal nature of the electron-electron interaction when mediated by a suitable ionic metallic lattice; this new theory of superconductivity incorporates some of the features from both the London and BCS models, which will first be summarized for later comparison.

The London Model

Inspired by the recent discovery of the Meissner effect, the Londons extended Drude's classical theory of conduction into the realm of superconductivity by defining a superconducting current density $\underline{\mathcal{J}}_s$ in terms of a free parameter n_s (somehow related to the material's electron density) and the electron current velocity $\underline{v}$. $\underline{\mathcal{J}}_s \equiv n_s e \underline{v}$

Classical electrodynamics had found [91] that the momentum $\underline{P}$ of a particle, like an electron with charge e and mass m, is related at every point $\{t; \underline{x}\}$ to its velocity $\underline{v}$ when influenced by an EM field, characterized by its vector potential $\underline{A}$, by:

$$m\, \underline{v}[t; \underline{x}] = \underline{P}[t; \underline{x}] - e\, \underline{A}[t; \underline{x}] / c$$

It was then assumed that the superconducting state of a system is the ground state and therefore each electron's canonical momentum must be zero; i.e. $\underline{P}[t; \underline{x}] = 0$

$$\therefore \quad \underline{v}[t; \underline{x}] = -(e / m c)\, \underline{A}[t; \underline{x}] \qquad \therefore \quad \underline{\mathcal{J}}_s[t; \underline{x}] = -(n_s e^2 / m c)\, \underline{A}[t; \underline{x}]$$

The EM force densities (electric $\underline{E}$ and magnetic $\underline{B}$) are defined by: $\underline{E} = -\nabla\phi - 1/c\, \partial/\partial t\, \underline{A}$ & $\underline{B} = \nabla \wedge \underline{A}$

This gives the two London superconducting field equations: $\partial \underline{\mathcal{J}}_s / \partial t = (n_s e^2 / m)\, \underline{E}$ & $\nabla \wedge \underline{\mathcal{J}}_s = -(n_s e^2 / mc)\, \underline{B}$

If Ampere's law is also assumed to hold inside a superconductor for the super-current; i.e. $\nabla \wedge \underline{B} = (4\pi / c)\, \underline{\mathcal{J}}_s$

Using the vector identity: $\nabla \wedge (\nabla \wedge \underline{F}) = \nabla (\nabla \bullet \underline{F}) - \nabla^2 \underline{F}$ with $\nabla \bullet \underline{B} = 0$ & $\nabla \bullet \underline{\mathcal{J}}_s = 0$ if $\partial \rho_s / \partial t = 0$

This gives the two (similar) equations:

$$\nabla^2 \underline{\mathcal{J}}_s = \underline{\mathcal{J}}_s / \lambda^2 \quad \& \quad \nabla^2 \underline{B} = \underline{B} / \lambda^2 \quad \text{where} \quad \lambda^2 = mc^2 / (4\pi n_s e^2) = 1 / (4\pi n_s \Lambda_0)$$

These are the London 'depth' equations that introduce the characteristic length parameter λ while $\Lambda_0 = e^2 / mc^2$: the classical electron radius (2.1×10^{-13} cm). The meaning of λ can be seen by considering a wide slab of superconducting material oriented along the (large) longitudinal z-direction with the x-direction defined inwards into its interior. Both the motion of the electrons (velocity) and the vector potential are oriented along the longitudinal direction; i.e. V_z and A_z. Therefore, a constant external magnetic field $\underline{B}$ is parallel to the y-direction at the surface ($B_y[0] = B_0$) then its value at a depth x below the surface is given by the solution:

$$B_y[x] = B_0 \exp[-x / \lambda]$$

This shows that the magnetic field is exponentially suppressed with depth, justifying the naming of λ as the 'skin depth' or 'penetration' parameter. Similarly, for the current density: $\mathcal{J}_z[x] = \mathcal{J}_0 \exp[-x / \lambda]$. This shows that the current has also a maximum value at the surface (J_0) and falls off exponentially below the penetration depth. Experiments indicate that this value is about 5×10^{-6} cm so the superconducting electron density n_s is about 10^{22} per cm^3, which is actually very close to the conduction electron density in the normal metal. The first London equation (in this geometry) converts to:

$$\exp[-x / \lambda]\, \partial \mathcal{J}_0[t] / \partial t = (n_s e^2 / m)\, E[t; \underline{x}] = (c^2 / 4\pi\lambda^2)\, E[t; \underline{x}]$$

This has the solutions: $E_z[t; \underline{x}] = E_0 \exp[-x / \lambda]$ & $\mathcal{J}_0[t] = (c^2 / 4\pi\lambda^2)\, E_0\, t$ $\therefore$ $\mathcal{J}_z[t; \underline{x}] = (c^2 / 4\pi\lambda^2)\, E_0\, t \exp[-x / \lambda]$

This requires the vector potential take the form: $A_z[t; \underline{x}] = -c\, t\, E_z[t; \underline{x}] = -c\, t\, E_0 \exp[-x / \lambda]$

Thus, it can be seen that the London solution is unstable, indicating a linear growth in the super-current with time (uniform acceleration). Thus, the London theory could explain the Meissner effect but not the persistence of current when no voltage is applied. A better solution required the BCS quantum mechanical approach to lattice oscillations.

The BCS Model

The BCS model extends the idea that the mobile electrons in a conductor behave as individual 'free' particles ('Fermi sea') by assuming that pairs of electrons near the Fermi surface tightly correlate their motion within the total collectivity. The standard interpretation here is that this pairing (known as 'Cooper pairs') is caused by an attractive force between electrons, mediated by their individual interactions with well-defined vibrations of the ionic lattice ('phonons'). Further, the new wave functions of these pairs overlap very strongly forming an even tighter sub-set or sub-ensemble "condensate" [75]. However, most importantly, uncoupling one Cooper pair requires changing the motion (and energy) of all the very many other pairs. The quantum mechanical calculation of such an idealized system of very many electron-phonon-electron **triplets** shows that the first excited state of this many-body system is separated by a finite amount from the ground state and this energy gap is larger (2½ to 3½) than the thermal energy exchangeable with the rest of the vibrating lattice. This implies that the totality of 'electronic fluid' is not scattered thermally by the lattice so that it can flow unimpeded without losing any further energy thus providing a quantum mechanical explanation for zero resistivity in the superconducting state.

Ziman pointed out [92] that only the highest frequency lattice vibrations (at the Debye limit f_D) play any significant role in the BCS theory of superconductivity. It is these highest energy ionic oscillations in position that interact with the relatively small number of 'fast' electrons moving near the Fermi velocity limit V_F (about $c/200$). These Debye vibrations occur at the limit of shortest wavelength corresponding to single ionic separations and provide exchange energies of about $h f_D = k \Theta_D$ or about 6×10^{-3} eV (where Θ_D is the Debye temperature, which is much larger than the environmental, thermal energy kT or nearly 4×10^{-4} eV) and large exchanges of momentum between the electrons in the Cooper pair. At absolute zero, the BCS theory predicts that 100% of the conduction electrons are paired into the superconducting ground state.

The BCS formalism is based on a 'reduced' or attractive potential between pairs of electrons near the Fermi energy limit E_F that overcomes their 'free' motion in the normal metal. This approach used a 'good guess' (or "variational ansatz") for the proposed two-electron wave function. The BCS theory was set on a firmer footing in 1958, when Bogoliubov showed that the BCS wave function could be obtained using a canonical transformation of the electronic Hamiltonian. [93]

Problems with the BCS Model

There are several major problems with the BCS model that are rarely discussed (primarily because it was the first and still remains the only micro-model of superconductivity). Unfortunately, like too much of contemporary theoretical physics, there is a great deal of difficult mathematics but not much physics. The mathematics requires that the interaction between the electrons and the phonon introduce an attractive force or potential (the 'Cooper mechanism') but again little attention is spent on this central physical requirement, particularly as the mathematics is developed in the QM 'momentum space' rather than in the real world of locations and events. The most popular mechanism appears to be that the movement of the valence electrons distort the lattice, thus Ziman writes: "In a lattice an electron tends to pull the positive ions towards itself, so that it is surrounded by a region where the lattice is slightly denser than usual. The two electrons can so to speak gain by sitting close together in the same depression in the mattress." [94] Apart from this weak justification by analogy (what is the mattress?), there is still no actual discussion of the mechanism by which the electrons interact with the phonon. The much larger mass of the lattice ions compared with the electron mass should mean that it is the electrons that are pulled much more out of their 'free' trajectories than the ions forming the phonons. This is not reflected in the mathematics.

Worse, the so-called 'coherence' range or separation ξ_0 (viewed as the minimum real distance between the electrons when their motion is correlated into Cooper pairs) has been calculated to be about 10^{-5} cm, which is hundreds of ionic separations across the lattice, so that superconductivity is very much a non-local effect, while the illustrations of the BCS model show the two electrons moving closely in parallel sharing the same 'dip in the mattress' about one ionic separation apart.

Even worse, the correlated electron pairs are assumed to have opposite momenta ($\pm$ P) so they should rapidly be separating or converging but not traveling in parallel as is usually illustrated. This parallel motion of electrons must result in slower electrons being overtaken by faster ones moving in the same direction: this should result in massive repulsive energies at such short range. Apart from Kadin, who has been mostly ignored [95], there have been very few attempts to address this problem and even he posits **local** distortions in the electron conduction charge density as the main physical mechanism. This miasma of conflicting physical concepts is another example of today's technique of first guessing at a mathematical solution before conceptualizing a physical model and once again results in the lack of real conceptual understanding of this very important physical phenomenon.

Impulse Superconductivity Model

This section will extend the new model of metals, briefly described above, to very low temperatures near absolute zero. It must be remembered that the idea of absolute zero is an asymptotic concept. The Kelvin scale of temperatures was invented to describe the amount of energy that was being exchanged between a material system and its environment. In reality, no object can be completely isolated from the rest of the world so that there will always be some small measure of exchange. In a metal at extremely low temperatures, the ions will almost all be located at their perfect lattice positions and this will result in almost every conduction electron 'hopping' from one ion to its nearest neighbor. At equilibrium, all electrons will have coordinated their movements such that over any finite time interval (even one as small as 10^{-15} seconds) almost every single mesoscopic cell will appear electrically neutral. Occasionally, at these extremely low temperatures, an interaction will occur with an external electron and the ion involved (probably near the surface) will move a little away from its neutral position and in the direction of the external impulse. Information on this ion displacement will be conveyed to the next ion in this direction and a small extended oscillation of multiple ions will be created that will move off through the lattice. The lower the temperature, the longer this perturbation will persist. This extended, coherent movement of many ions is known as a phonon as the stimulating energy exchanged is always quantized but it this now distributed across all the ions involved. This fixed amount of vibrational energy associated with a particular phonon means that the number of ions involved rapidly stabilizes and the whole collection moves across the total crystal until it reaches an ionic position which has been displaced by another phonon at that instance creating an example of phonon-phonon scattering. This rare possibility increases with temperature. The movement of the valence electrons is fast enough that they will stay correlated with this spatial variation at very low temperatures. However, it is the motion of the lattice that is viewed here as critical to superconductivity. The extended vibrations across space act to correlate the motions of electrons in time along the direction of the phonon wave. As this new theory always views electrons as strongly interacting point particles, it rejects the non-interacting 'Fermi Sea' hypothesis. The new model is based on the correlated motion of only a small subset of all the possible valence electrons.

Persistent Current Model

In an elongated sample of conducting metal, the majority of these phonons will move backwards and forwards along the crystal particularly if the external perturbation is initiated along the longitudinal (z) axis. If a source of external electrons is established at or near one end, the extra electrons will enter the conductor under the influence of the constant external EMF and will start to interact primarily with the conductor's own valence electrons since almost all the ions are still involved with all the other electrons (the Saturation Hypothesis). This will cause some of the nearby valence electrons to move away from the new electrons until a stable situation is achieved where all the new (or excited) electrons are moving along the surface layers to the opposite end of the conductor, where they will re-enter the external (driving) circuit. The penetration depth of about 4×10^{-6} cm is about 200 lattice separations (approximately 2×10^{-8} cm apart) or about two mesoscopic cell widths. If two other detection points are established between the external emitter and collection points ('4-point probe') then a voltage can be measured between the two new points. If the voltage is equal at these two intermediate points but there still exists a steady current in the metal, there must be no resistance and so superconductivity is determined to be present in the setup.

A similar analysis will be conducted as in the earlier Impulse Conduction Model. In the following theoretical model, the wire conductor is treated as a rectangular block of length L cm and width w cm with a square cross-section of area $S = w^2$.
In order to achieve exact synchronization of the lattice and electron motions, it will prove useful to adjust the mesoscopic cell size Δx_0 so that it is an integer number N_D of ion separations D_0 ; i.e. $\Delta x_0 \equiv N_D D_0$ (typically $N_D \cong 50$). The number of surface cells in each perimeter around a cross-section is $N_s = 4w / \Delta x_0$; the number of cells along any longitudinal edge $N_L = L / \Delta x_0$. Since $L >> w >> \Delta x_0$ then $N_L >> N_s >> 1$. The total number of valence electrons in this block, when there is no external current source is $N = n S L$. Let the first ('negative') cross-section (or layer) of cells be identified as #1 with the next cross-section labeled #2, etc through to the 'positive' end-face cells in the layer labeled #N_L. When an external current source is connected across both ends of this conductor and the situation is in equilibrium (independent of time) then this model assumes that in a time Δt_S seconds, an extra ΔN_S electrons will enter the negative end-face of the conductor and fill one mesosopic (cross-sectional) cell layer in this time, raising the electron density from n to n_S, where $n_S = n + \Delta n_S$. The original density n of electrons is still sufficient to keep these cells electrically neutral but while the extra electrons are in a cell, it takes on (temporally) a small extra negative charge ΔQ_S but this charge is not usually observable externally due to the new Saturation Hypothesis such that these extra electrons interact mainly internally with their very close local neighbors.

However, when these injected electrons are involved in a direct current in the longitudinal direction they are concentrated (to a good approximation) only in the surface cells with cross-section S and with density of Δn_S electrons per surface cell.

$$\therefore \quad S = S - (w - 2\,\Delta x_0)^2 = 4\,\Delta x_0\,(w - \Delta x_0) \cong 4w\,\Delta x_0 \qquad \therefore \quad S / S = 4\,\Delta x_0 / w << 1$$

$$\therefore \quad \Delta N_S = \Delta n_S\, S\, \Delta x_0 \qquad \therefore \quad \mathcal{I}_S = e\,\Delta n_S\, S\, \mathcal{U}_S = J_S\, S \quad \therefore \quad J_S = e\,\Delta n_S\, \mathcal{U}_S = \rho_S \mathcal{U}_S \qquad \text{where} \quad \rho_S \equiv e\,\Delta n_S$$

Since these electrons are constrained to the surface layers there are far fewer injected electrons needed to form a current with the same density as in the normal metallic state. However, the current source can still supply the same number of electrons per second in both cases, so if: $\Delta N_S = \Delta N'$ then:

$$\therefore \quad \Delta n_S = (S / S)\,\Delta n = (w / 4\Delta x_0)\,\Delta n \qquad \text{or} \qquad \rho_S = (w / 4\Delta x_0)\,\rho_c$$

This increases the density of super-conduction electrons (Δn_S) by a factor of $w/(4\Delta x_0)$ in the current-carrying surface cells; for example, for a wire of width 0.04 mm this factor is 1000, so instead of about 10^5 electrons per surface cell there will be about 10^8 electrons. This very dense 'cloud' of electrons in each surface cell now has a powerful, net repulsion between the electrons in each cell that readily overwhelms the attractive interactions of the 10^5 ions in the lattice. This increase not only generates a set of intra-electron impulses pushing the electrons away from each other down the wire but also creates a much greater interaction between the electrons and the lattice vibrations, so much so, that these inter-cellular interactions now can potentially span several cellular layers as highly accelerated electrons are pushed quickly past many intermediate layers.

The total <u>extra</u> charge in each layer (say, #l) of the block: $Q_l = e\,\Delta N_S = e\,\Delta n_S\, S\, \Delta x_0 \equiv N_S\, \Delta Q_S \quad \therefore \quad \Delta Q_S = \rho_S\,(\Delta x_0)^3$

Let the surface cells at a given layer be labeled sequentially, say in a clockwise direction viewed along the z-axis, (from an arbitrary origin) from #1 to #N_S. Any surface cell in the block can be referred to by a unique pair of discrete indices {k,j}, where k labels the cell's circumferential position and j labels the cell's longitudinal layer; i.e. k = 1, … N_S ; j = 1, … N_L.

Eventually, this repulsive pulse will progress from layer #1 to layer #N_L. In each case, electrons will predominantly move only in a longitudinal 'pipe' through the metallic block. The equilibrium flow may be approximated by electrons associated with ions in transverse cell #k in layer #l moving freely to another ion in another layer #l' but with the same transverse cell #k. Thus, hereafter, the transverse cell index will remain implied, as all the injected electrons will retain their original cell#.

Unlike the standard model of conduction described above, where electrons in one cell only influence (that is, interact) with electrons in adjacent layers and each cohort trudges slowly along the wire; in the superconducting state, a mobile electron can jump across many intermediate layers to arrive at an ion that is exactly correlated with this type of movement by these lattice vibrations and the fact that many of the intermediate ionic sites are electrically neutral with their full complement of electrons. These 'long-jumps' are possible because the motion of these mobile electrons is correlated between themselves to minimize their total kinetic action. Above the critical temperature, there are no long-term stable 'landing-zones' that can be expected to be free at the right spatial separation, at the expected arrival time of the electron, due to random movements of the lattice induced by thermal variations. In fact, there can be many electrons 'hopping' across the lattice, all of them mutually correlated and each correlating with one single phonon. At very low temperatures, these phonons will be only longitudinal and will be taken to be only one ion wide. This is in contrast to higher temperatures, where all directions are possible and the vibrations can correlate across all three spatial directions. At extremely low temperatures (say, much less than 1° K) then all the mobile electrons can be correlated with discrete lattice vibrations. The phonons are assumed to travel at a slow speed of S cm/sec while vibrating at the Debye frequency f_D (or period $\mathcal{T}_D$) and spanning the Debye wavelength, which is taken to be the mean inter-ionic separation of the lattice in the longitudinal direction D_0.

$$\therefore \quad S = D_0 / \mathcal{T}_D = D_0 f_D = k\,\Theta_D\, D_0 / h$$

This is also the (longitudinal) speed of sound (hence acoustic phonons). In typical superconducting metals, such as mercury or lead, this has the value of about 1.5 to 2.0 x 10^5 cm/sec, which is much slower than intra-atomic electron velocities. This is equivalent to a Debye frequency of about 10^{13} cycles/sec; i.e. $\mathcal{T}_D \cong 10^{-13}$ seconds.

The motion in the superconducting state is also similar to the normal conductor where alternating acceleration and relaxation phases were proposed but now, at very low temperatures, the phonons travel in parallel directions for extended periods with minimal loss of energy. So, in the relaxation phase the electrons 'merge' elastically with the ions and temporarily exchange kinetic energy with their correlating phonon. As a direct result of the Saturation Hypothesis and the Localization Principle, during the acceleration phase all the mobile electrons in any one layer overwhelmingly interact locally with mobile electrons in the **same** layer. This conservative inter-change of kinetic energy between the electrons and the ions occurs repeatedly while the conduction electrons 'hop' across from the negative to the positive end of the block at a net average speed of $\mathcal{U}_S$ cm/sec; again, it is the locality of <u>all</u> of these interactions that means that the constant current results of superconductivity are independent of the spatial shape or twisting of the wire. Now, during each acceleration phase, all the mobile electrons in each of the cells in each layer are given various mutual impulses over a 'launch' time of ½ Δt_S seconds. After receiving these impulses, each mobile electron will then travel down through the lattice with a velocity V_ζ in the z-direction and due to the correlation with its guiding phonon it will 'land' at a suitable ion (and take up a new stable orbit). It will be assumed that the 'landing' process also takes a finite time of ½ Δt_S seconds. Other electrons (with different speeds) will arrive in this same cell during this period before 'hopping' onwards to their destination in the final cell layer (i.e. $\#N_L$). The electron may simply move to the next cell in one 'period' (i.e. in the fundamental superconducting cycle time Δt_S), in which case it will only have moved an average distance Δx_0 giving it an average speed of S. Otherwise, the electron will have moved across a total of ζ intermediate cells so that the distance traveled 'per hop' Δz_ζ is a multiple ζ of the inter-cellular distance Δx_0. In order to maintain temporal commensurability (i.e. phase coherence) it is then necessary to equate the time to travel 'per hop' Δt_ζ (that is, excluding the acceleration and deceleration time intervals) with the cycle time Δt_S.

The key here to understanding superconductivity is synchronization. It is necessary that the separation between the 'launch' ion's location and the 'landing' ion's location remain exquisitely synchronized with the timing of the electron as it moves between them. In other words, the motion of the electron must remain exactly in phase with the motion of the vibrating ionic lattice if no loss of energy is to occur between the moving electrons and the vibrating lattice; this time difference must equal an integer number of complete lattice vibrational cycles. Since there are N_D ions along each edge of a mesoscopic cell, then:

$$\Delta t_S = N_D \mathcal{T}_D = \Delta x_0 \mathcal{T}_D / D_0 = \Delta x_0 / S \qquad \therefore \quad S = \Delta x_0 / \Delta t_S$$

So, for $\zeta > 0$

$$\therefore \quad \Delta z_\zeta = \zeta \Delta x_0 = V_\zeta \Delta t_\zeta \quad \& \quad \Delta t_\zeta = \Delta t_S \qquad \therefore \quad V_\zeta = \zeta S$$

When an electron traveling at speed V_ζ leaves an ion in the $\#l$ layer it must land on an ion in the $\#l'$ layer exactly Δt_S seconds later (if adjacent: $l' = l + 1$) or exactly $2\Delta t_S$ seconds later (if not adjacent: $l' = l + 1 + \zeta$ where $\zeta > 0$). A critical feature of this model of superconductivity is that by 'landing' on intermediate ions, the mobile electron can next 'hop' at a different speed than the speed that it arrived at the ion. In effect, the phonon allows the electron to break its journey into a series of distinct segments; in effect, it 'forgets' what the initial speed it had when it left the first cell <u>and</u> in each previous segment – the only constraint is that it must finally arrive at the last (collection) cell.

If the superconducting current is to remain constant, there can be no 'bunching up' of electrons in any cell layer over time. This requires that the number of mobile electrons ΔN_l entering or leaving any cell in each period Δt_S at the $\#l$ layer must be constant; in particular, this must be equal to the number reaching the collector at layer $\#N_L$ and being emitted at layer #1.

$$\therefore \quad \Delta N_l = Q_l / e = \Delta N_S = \mathcal{I}_S \Delta t_S / e$$

When at equilibrium, let $\eta_{ll'}$ represent the number of mobile electrons that leave cell l and 'hop' to cell l'; this is equivalent to determining how many electrons leave cell $\#l$ per period with velocity V_ζ. The electrons that leave cell $\#l$ could next go to any cell in the remaining 'downstream' section of the slab; that is: $l' = l+1, l+2, \ldots, N_L$. The electrons that arrive at cell $\# l'$ could have just landed from a hop 'upstream' from any cell, where: $l = l' - 1, l' - 2, \ldots, 3, 2, 1$.

$$\therefore \quad \Delta N_l = \sum_l \eta_{ll'} = \sum_{l'} \eta_{ll'} = \Delta N_S$$

From symmetry considerations one would expect that: $\eta_{ll'} = \eta_{l'l}$ and $\eta_{ll} = 0$

However, examining cell #2 first, shows that the electrons arriving there can come only from cell #1.

$$\therefore \quad \Delta N_2 = \eta_{12} = \Delta N_S \qquad \therefore \quad \eta_{1k} = 0 \quad \text{if } k > 2 \quad \text{in particular} \quad \eta_{13} = 0$$

So, examining cell #3, the only electrons arriving must come from cell #2, so again:

$$\therefore \quad \Delta N_3 = \eta_{23} = \Delta N_S \qquad \therefore \quad \eta_{2k} = 0 \quad \text{if } k > 3 \quad \text{in particular} \quad \eta_{24} = 0$$

This process can be repeated recursively, showing that only previous cells supply electrons.

$$\therefore \quad \Delta N_l = \eta_{l\text{-}1\,l} = \Delta N_S \qquad \therefore \quad \eta_{lk} = 0 \quad \text{if } k > l + 1$$

This proves that even in the superconducting case, the mobile electron movement is limited to moving between adjacent mesoscopic cells (i.e. $\zeta = 0$) but now at the speed of sound of the metallic lattice S at very close to absolute zero. So, again this appears like a small pulse of extra charge moving slowly from one surface cell layer to the next (longitudinal) surface layer in a time Δt_S at an average speed S that constitutes the superconducting current I_S.

$$\therefore \quad I_S = e\,\Delta N_S / \Delta t_S = e\,\Delta n_S\, S\,\Delta x_0 / \Delta t_S = e\,\Delta n_S\, 4w\,\Delta x_0\, S = 4\,e\,\Delta n_S\, S^2\, N_D\, \mathcal{T}_D\, w$$

The density of mobile electrons, Δn_S is determined by the external source of these electrons, while the three parameters S, N_D and $\mathcal{T}_D$ are properties of the metal, leaving the current directly proportional to the width w of the sample. Since there is current without energy loss, there is now a superconducting current present.

It can now be seen that the one-dimensional, low-energy, long-range lattice vibrations (phonons) play the central role in the phenomenon of superconductivity. Each phonon synchronizes many electrons (not just pairs) as they move longitudinally in the surface cells of the metal, from the external electron source to its distant collector. Each electron helps maintain this stable lattice motion while the disruption of this phonon would impact very many electrons in their motion and would therefore require significantly more disruptive energy than is usually exchanged with other external energy sources at this temperature. It is destroying this collective co-operation that invokes the phenomenon of a macroscopic phase transition.

As the intensity of the electron source is increased, more and more of the mobile electrons will not be able to remain exactly synchronized with the ionic lattice movements, so their longitudinal progress will disrupt the phonon synchronization of all the other electrons, to the point where a critical current value will destroy all superconductivity in the sample. Similarly, an increase in the temperature of the sample will correspond to more random lattice vibrations, again destroying all these superconducting effects above a critical temperature.

The Meissner effect is first explained by the recognition that magnetic effects are simply the subset of electron interactions that vary with the relative velocity of the interacting electrons, particularly when very many source electrons are moving in a synchronized manner and can collectively affect a few 'target' electrons. In other words, the concept of a magnetic field is viewed here purely as an aid to calculations: it is an existential fiction just like its twin, the motion-insensitive, electric field. When a weak source of 'magnetic effects' is brought near a superconducting sample, the electrons in the 'magnetic source' will be ignored by all the electrons in the sample. Any electrons outside the sample will be the ones that are even more affected by the 'magnetic' electrons – this has been interpreted as an expulsion of the magnetic field from the region of the superconductor. The key mobile electrons moving near the surface of the superconducting sample are busy interacting only with their local neighbors and the lattice ions in the surface (Saturation Hypothesis) while the valence electrons below the surface layer are simply more affected by their own local interactions. However, as more and more moving electrons in the 'magnetic source' are brought into the game (e.g. by increasing the electromagnetic current strengths) they can be ignored less and less by the electrons in the superconducting sample, where their original, locally optimized internal motions, will be finally disrupted by these stronger external interruptions that will eventually destroy the superconducting synchronization at a given value of the magnetic field intensity (H_c) as the mobile electrons are moved laterally by the 'Lorentz' force.

It is obvious that if the sample is bent around on itself (i.e. $\#N_L = \#1$) then any longitudinal surface currents will persist in the superconductor, once established, if the shape distortion is low enough not to disrupt the propagation of the lattice vibrations.

7.5 ELECTRONIC EQUILIBRIUM

7.5.1 ELECTRON CAPACITY

Overview

This sub-section continues to eliminate imaginary, intermediate concepts from the role of foundational, ontological entities in EM theory. One of the key steps in the history of CEM was the introduction of the idea of the **electric field**; this was used to first smuggle in the time-independent concept of electrostatic potential that continues to play a key role in modern theory. The electric field concept played a central role in Coulomb's electrical analogy with gravitational forces and in the associated idea of electrical capacity, where the electrical 'fluid' flowed through 'pipes' or conductors into electrical 'pots' or capacitors (a watery analogy that is still used today to great effect by electrical 'plumbers' or electricians).

Isolated metallic conductors are electrically neutral overall on a long-time average (longer than 10^{-16} sec). This view can be extended down to each mesoscopic cell (§7.2) where, on average, the number of mobile electrons (each with a unit negative charge - *e*) will equal (for simplicity) the number of lattice ions (each with a unit positive charge +*e*). Obviously, the new model of metals developed above (§7.4.1) must cover the case where there are excess electrons in each cell but there is **no** net motion in one direction, as was described in the impulse conduction model (§7.4.2). These excess electrons are typically introduced by connecting to a source of external EMF. Once again, the collective repulsion between these excess ions and the mobile electrons, now all moving together co-operatively through the lattice to minimize total action, defines the long-term average configuration. This equilibrium configuration both conserves energy (i.e. no net energy is exchanged with the conductor's environment) and maintains all cells at the same level of potential $\mathcal{V}$; since there is no net electric current in any direction the vector potential A must be zero. The total net electric charge in each cell Q will determine this potential based on the configuration of the conductor and its proximate surroundings, especially the presence of any other conductors; this ratio defines the **capacitance** of the macroscopic configuration denoted by the symbol *C*, measured in coulombs per volt.

$$C \equiv Q / \mathcal{V} \quad \text{(in cm)}$$

Let Δn represent the average number of excess electrons in each mesoscopic cell when the potential is $\mathcal{V}[\Delta n]$. $\therefore Q = e\,\Delta n$

Let an external source of electrons increase each cell by one extra electron. This will require the external source to expend energy to overcome the net repulsion from all the electrons already present in the cell; i.e. to do extra work $\Delta\mathcal{W}[\Delta n]$.

$$\therefore \quad \Delta\mathcal{W}[\Delta n] = e\mathcal{V}[\Delta n] = e\,Q[\Delta n] / C = e^2\,\Delta n / C$$

$$\therefore \quad \mathcal{W}[N] = \Delta\mathcal{W}[1] + \Delta\mathcal{W}[2] + \ldots + \Delta\mathcal{W}[N] = (1 + 2 + \ldots + N)\,e^2 / C = N\,(N + 1)\,e^2 / 2C \approx N^2\,e^2 / 2C$$

$$\therefore \quad \mathcal{W}[N] = Q^2[N] / 2C = (C\,\mathcal{V}[N])^2 / 2C \qquad \therefore \quad \mathcal{W} = \tfrac{1}{2}\,C\,\mathcal{V}^2 = \tfrac{1}{2}\,Q\,\mathcal{V} = \tfrac{1}{2}\,Q^2 / C$$

Thus, the electrical potential energy in each cell $\mathcal{W}$ is determined by the **square** of the number of excess electrons; this is a key characteristic of all collective systems [96]. In contrast, with field theory that views this electrical energy being carried (stored) by the electric field throughout **all** space, the present theory views this energy as a property of the spatial and temporal configuration of **all** of the excess electrons in each cell; i.e. their relative positions and motions of these electrons.

Configurations Overview

The total capacitance *C* is a property of the conductor as a whole and its context; it is therefore determined by its shape in 3D space and its immediate surroundings – both non-conductors and other conductors. These factors will establish which mesoscopic cells have an excess or deficit of electrons (relative to the number of ions or mobile electrons) as the electrons move around the conductor to minimize the total energy and establish a common EM potential. The greater the number of excess electrons that can be injected into an isolated conductor the greater the capacitance; alternatively, the capacitance is increased when a given number of electrons can be stored at a lower potential. Obviously, electrons cannot be repulsed further than the boundaries of the conductor, so that cells at and near the surface will most likely retain the most charge.

Even a single solid block of metal has a finite capacitance. Its capacitance can be maximized by maximizing its surface area and the separation between similarly charged cells. The presence (or deficit) of other nearby electrons will alter the detailed distribution of charges on the target conductor (hence its capacitance) while still respecting the mobility constraint that each cell must remain at the same average potential – an effect readily demonstrated in the interior of a Faraday cage.

7.5.2 CAPACITANCE EXAMPLES

Isolated Conducting Sphere

Consider a metallic sphere of radius a located a great distance from any other matter. Assume that this sphere had been previously in contact with a source of electrons so that overall it now contains an excess of N electrons. The totality of these excess electrons constitutes a (negative) charge –Q on the sphere. These electrons will rapidly distribute themselves in cells all over the surface of the sphere (whether it is solid or hollow) as this configuration will minimize the repulsion between all of them. Now consider the work done $\mathcal{W}[r]$ in bringing one extra electron from a great distance L >> a (from the center of the sphere assumed to be at the origin of a 3D co-ordinate system) up to a distance r from the center (this might be achieved by an electron being projected towards the sphere at an initially very high speed). During its journey towards the sphere, the electron is subject to a vast number of interactions with all the excess electrons on the sphere, which may (statistically) be described by the collective average of the Coulomb electrostatic force F acting radially. The incremental work done (energy loss of the electron) at a separation r as it narrows the separation by a further distance Δr is $\Delta\mathcal{W}[r]$.

$$Q = -N e \quad F[r] = Ne^2 / r^2 \qquad \therefore \quad \Delta\mathcal{W}[r] = F[r]\,\Delta r = -e\,\Delta\mathcal{V}[r] \qquad \therefore \quad \Delta\mathcal{V}[r] = -N e\,\Delta r / r^2$$

$$\therefore \quad \mathcal{V}[a] = \sum_{k=1}^{k=n} \Delta\mathcal{V}[r_k] \approx -N e \int_L^a dr / r^2 = Q \left[1 / r \right]_L^a = Q\,(1/a - 1/L) \approx Q / a = Q / C \qquad \therefore C = a$$

Conducting Sphere

Jefimenko presents electrostatics as the study of equilibrium charge distributions in his magnificent text on CEM [97]. He makes the useful distinction between the calculation of potentials **of** charged conductors and the calculation of voltages **between** charged conductors, especially capacitors: viewed as a pair of conductors carrying equal amounts of opposite charge. This allows the definition of the capacitance of conducting systems to be generalized to:

$$C \equiv Q / \varphi$$

Here φ is defined as the electrical potential $\mathcal{V}$ of an isolated conductor or the difference in potential (voltage V) between the two conductors. $\varphi = \mathcal{V}$ (conductor) or $\varphi = \mathcal{V}_1 - \mathcal{V}_2 = V$ (capacitor).

This can now be used to calculate the capacitance of a spherical capacitor consisting of an inner metallic sphere holding a charge of N excess electrons surrounded by a larger metallic sphere. The mobile electrons on the outer sphere are repelled by the excess electrons on the inner sphere and establish an equilibrium configuration where the inner surface of the larger sphere suffers from a deficiency of N electrons while its outer surface has an excess N of electrons. The capacitance is calculated between the outer surface of the inner sphere (radius a) and the outer surface (radius b) of the larger sphere.

$$Q = N e \quad \therefore \mathcal{V}_a = Q / a \quad \& \quad \mathcal{V}_b = Q / b \quad \therefore V_{ab} = \mathcal{V}_a - \mathcal{V}_b = Q\,(1 / a - 1 / b) = Q / C \quad \therefore C = a\,b / (b - a)$$

Parallel Plates

When an excess of electrons arises on a metallic plate, its capacitance can be enhanced by placing an identical plate parallel to the first. The excess electrons on the first plate repel the mobile electrons on the second plate to the far surface leaving a net deficiency of electrons on the closer inner surface. This attracts more of the excess electrons on the first plate to its own inner surface leaving its own outer surface with a net electron deficiency ('positive charge'). The equilibrium configuration (no electric currents) results in an equal number of excess electrons (amount of charge) being found in each surface cell; this leads to an average charge density per cm^2 $\sigma = Q/A$ where A is the area of each plate and $Q = N e$. The potential $\mathcal{V}$ is found to be equal everywhere between the two plates (except close to the edges) and reflects the (constant) work needed to move one electron anywhere across the separation **a** between the plates.

$$\therefore \mathcal{W} = e\mathcal{V} = (e\,\sigma)\,a \qquad \therefore \mathcal{V} = \sigma a = Q\,a / A = Q / C \qquad \therefore C = A / a$$

The general equation for the potential $\mathcal{V}$ due to a static continuous charge with density ρ distributed over a closed volume is:

$$\mathcal{V}[\underline{x}] = \sum_{k=1}^{k=n} \Delta\mathcal{V}[\underline{x} - \underline{x}_k] = \sum_{k=1}^{k=n} \Delta Q_k / |\underline{x} - \underline{x}_k| \approx \iiint d^3x'\, \rho[\underline{x}'] / |\underline{x} - \underline{x}'|$$

7.6 ELECTRONIC INERTIA

7.6.1 ELECTRON CURRENTS

Historical Overview

This sub-section continues to eliminate imaginary, intermediate concepts from the role of foundational, ontological entities in EM theory. This sub-section will show that the concept of a ***magnetic field*** is purely a mathematical adjunct and plays no role in the reality of EM phenomena. Only one year after Oersted's discovery of electro-magnetism in 1820, another first-rate experimentalist, Michael Faraday (1791-1867) invented the first homo-polar electric motor and soon realized (by 1831) that moving magnets could influence the flow of electrical current. Both Oersted and Faraday were strongly influenced in their research connecting electricity and magnetism by the Kantian idea of the unity of nature. As an autodidact, Faraday's knowledge of mathematics was limited to simple algebra but he had a powerful, visual imagination, which he used to help him understand the EM phenomena he was investigating. Faraday's concept of lines of flux emanating from charged bodies and magnets provided him a way to visualize the EM forces he assumed to exist between conductors across space (that for religious reasons, he knew could not be empty). This concept of electric and magnetic **fields** (or lines of force) was rejected by his fellow scientists until Maxwell provided a mathematical formulation that provided a deeper Platonic justification. It was Maxwell's view that these forces were states of stress and strain in a real (but invisible) Aether but, by then, many 19th Century scientists agreed with Faraday and now viewed these 'lines' as real, going so far as claiming that "only forces are real" – an interesting parallel to the modern position that "only energy is real". Today, the reality of electric and magnetic fields is still a widely held idea, as can be judged by the modern (Wikipedia) view of Oersted's and Faraday's contributions. "Today, Oersted is credited with observing that electric currents induce magnetic *fields*. … He (Faraday) demonstrated that a changing magnetic *field* produces an electric *field*." These are perfect examples of interpolating theoretical interpretations into experimental discoveries as a justification for a theoretical (metaphysical) set of assumptions – our modern theology!

Although the present programme is monistic (following Leibniz), it disagrees with Faraday in viewing force as "the only substance": instead, giving this fundamental status to the electron but it does view EM as the only form of fundamental interaction in nature. The EM field model is viewed here as only appropriate when statistical averages can be modeled in terms of mathematical intermediaries that replace asynchronous action-at-a-distance interactions with a set of single-time partial differential equations across all of space. Field-lines are viewed as only providing a great help (like Faraday) with visualization. Vector fields (represented by field lines) are seen as an abstraction from the interaction of real electrons; they represent the hypothetical (statistical) possibility of how a real electron would move **if** placed in such a situation. Like lines of latitude and longitude on a globe, field lines are not physical lines but, as Kuhn pointed out, "aids to our imagination".

Magnetic Fields

Although the idea of the magnetic induction field arose with Faraday's magnetic experiments inducing electrical currents, some modern, graduate texts on EM prefer to introduce the idea of a magnetic field $\underline{\mathbf{B}}[\underline{x}, t]$ by following the actual historical sequence of the experimental discoveries that finally resulted in the theory of classical electromagnetism (CEM). This is the approach followed here but it is very important to point out that a "switch" usually occurs during this process that gets little or no attention. This key switch results from first viewing the $\underline{\mathbf{B}}$ field as an intermediary in the calculation of the mechanical force **on** a complete conducting circuit that is carrying an electric current moved by a similar conductor (Ampère) and then viewing it as the **cause** of an electromotive force **within** a conductor that moves only the charges that constitute the current **in** the target conductor (Faraday). In reality, this allows approximations to be made in the calculation of the intermediate $\underline{\mathbf{B}}$ quantity (or its associated flux) that simplifies the overall, exact calculation that is often too intractable.

A well-known exposition of this approach [64] begins with the 'triple-cross' form of Ampère's circuital force law rather than Ampère's original differential form involving two circuit elements. This is done to separate the interaction expression into a magnetic field produced by the electrical current in the source circuit and a **mechanical** force exerted by this field on the target circuit. All of this despite the fact that these two forms are only equivalent under special circumstances (§7.1): the target circuit must be closed (to drop the total derivative), the wires must have fixed cross-sections so that the currents can be moved outside the integrals) and the movement is slow enough to preserve rigidity (one time). Even Maxwell himself in his *Treatise* referred to Ampère's original formulation as "perfect in form and unassailable in accuracy … the cardinal formula of electrodynamics." Indeed, Maxwell always thought of Ampère as the "Newton of electricity". [98]

$$\mathcal{F}[t] \equiv \mathcal{I} \oint d\underline{x} \wedge \underline{\mathbf{B}}[\underline{x}, t] / c \qquad \therefore \quad \underline{\mathbf{B}}[\underline{x}, t] = -\mathcal{I}' \oint d\underline{x}' \wedge \underline{\nabla}[1/r] / c$$

This 'generative' definition of the magnetic induction field $\underline{\mathbf{B}}$ is an extension of the Biot-Savart law. In terms of the current density vector $\underline{\mathbf{J}}$ at each location in the uniform wire, these equations can be restated as volume integrals over each wire.

$$\mathcal{F}[\,t\,] = \iiint d^3x\, \underline{\mathbf{J}}[\underline{x}, t] \wedge \underline{\mathbf{B}}[\underline{x}, t] / c \qquad \therefore \quad \underline{\mathbf{B}}[\underline{x}, t] = -\iiint d^3x'\, \underline{\mathbf{J}}'[\underline{x}', t] \wedge \underline{\nabla}[1/r] / c$$

$$\text{Since } \underline{\nabla} \wedge \underline{\nabla}[\psi] = 0 \quad \therefore \quad \underline{\nabla} \bullet \underline{\mathbf{B}}[\underline{x}, t] = -\iiint d^3x'\, \underline{\mathbf{J}}'[\underline{x}', t] \bullet \underline{\nabla} \wedge \underline{\nabla}[1/c\,r] \quad \therefore \quad \underline{\nabla} \bullet \underline{\mathbf{B}}[\underline{x}, t] = 0$$

When the current is confined to a filament-like region of space where the constant cross-section dS' is much smaller than the distance r from all the points of this region to the point of observation $\underline{x}$, then the variation over dS' may be neglected. Since the current in the filament must be directed along the direction of the filament then: $\underline{\mathbf{J}}'\, d^3x' = \underline{\mathbf{J}}'\, dS'\, d\underline{l}' = \mathbf{J}'\, dS'\, d\underline{l}'$.

$$\therefore \quad \underline{\mathbf{B}} = \iiint d^3x'\, \underline{\mathbf{J}}' \wedge \underline{r} / cr^3 = \iint dS'\, \underline{\mathbf{J}}' \oint d\underline{l}' \wedge \underline{r} / cr^3 = \mathcal{I}' \oint d\underline{l}' \wedge \underline{r} / cr^3$$

Helmholtz first proved that the 3D current-density integral form of the definition of the induction field vector $\underline{\mathbf{B}}$ is consistent with Ampère's Law by using **Poisson's Vector Field theorem** which states that **any** vector function defined across **all** of 3D space (and vanishing at great distances) can be expressed in terms of its divergence and curl [99].

$$4\pi\, \underline{\mathbf{B}}[\underline{x}] = -\iiint_{all} d^3x' \{ \nabla' [\, \nabla' \bullet \underline{\mathbf{B}}[\underline{x}'] \,] - \nabla' \wedge [\nabla' \wedge \underline{\mathbf{B}}[\underline{x}'] \,] \} / |\, \underline{x} - \underline{x}' \,| \qquad \text{but} \quad \nabla' \bullet \underline{\mathbf{B}}[\underline{x}'] = 0 \quad \text{with } \underline{r} = \underline{x} - \underline{x}'$$

$$\therefore \quad 4\pi\, \underline{\mathbf{B}}[\underline{x}] = \iiint_{all} d^3x'\, \nabla' \wedge [\nabla' \wedge \underline{\mathbf{B}}'] \,] / r = \iiint_{all} d^3x' \{ (\nabla' \wedge \underline{\mathbf{B}}') \wedge \underline{r} / r^3 + \nabla' \wedge [\nabla' \wedge \underline{\mathbf{B}}' / r \,] \} \quad \text{with } \underline{\mathbf{B}}' = \underline{\mathbf{B}}[\underline{x}']$$

$$= -\iiint_{all} d^3x'\, (\nabla' \wedge \underline{\mathbf{B}}') \wedge \nabla [1/r] + \iint d\underline{S}' \bullet [\nabla' \wedge \underline{\mathbf{B}}' / r \,] = -\iiint_{all} d^3x'\, (\nabla' \wedge \underline{\mathbf{B}}') \wedge \nabla [1/r]$$

$$= -4\pi \iiint_{all} d^3x'\, \underline{\mathbf{J}}'[\underline{x}'] \wedge \underline{\nabla}[1/r] / c \qquad \therefore \quad \nabla' \wedge \underline{\mathbf{B}}' = 4\pi\, \underline{\mathbf{J}}' / c \qquad \therefore \quad \nabla \wedge \underline{\mathbf{B}}[\underline{x}, t] = 4\pi\, \underline{\mathbf{J}}[\underline{x}, t] / c$$

$$\therefore \quad 4\pi \iint d\underline{S} \bullet \underline{\mathbf{J}} / c = \iint d\underline{S} \bullet \nabla \wedge \underline{\mathbf{B}} = \oint d\underline{l} \bullet \underline{\mathbf{B}} \qquad \therefore \quad \oint_{\pi} dl \bullet \underline{\mathbf{B}}[\underline{x}, t] = 4\pi / c \iint_{\Sigma} ds \bullet \underline{\mathbf{J}}[\underline{x}, t]$$

The enclosed current $\mathcal{I}'$ passing through the closed path π is equal to the current density flowing through **all** of the closed surface Σ; this leads to the more common or circuital form of Ampère's law that is used to calculate $\underline{\mathrm{B}}$ in many situations.

$$\therefore \quad \oint_{\pi} dl \bullet \underline{\mathbf{B}}[\underline{x}, t] = 4\pi\, \mathcal{I}' / c$$

7.6.2 MAGNETIC INDUCTION

Magnetic Induction

Faraday first noticed that the rate at which changes in the magnetic environment of a conductor determined the amount of current induced in the conductor. He determined, in his model, that the amount of induced voltage across the ends of the conductor was proportional to the rate of change of the magnetic flux lines flowing through the circuit. Faraday's famous 'magnetic' linkage of two conducting circuits via a 'magnetic bridge' (such as an iron ring) was, in reality, a successful attempt to align permanent electronic orbitals in the iron to communicate the electric effects from the first circuit to the second circuit. All the intermediate and adjacent orbitals then cancel out their contributions leaving only the unbalanced ones at the edge of the iron. The view that changing the magnetic environment generates an electrical effect is known as magnetic induction – it lies at the heart of Maxwell's theory of EM: it is the origin of the idea of the magnetic field $\underline{\mathbf{B}}$.

When a **thin** conductor (defines a closed boundary π defining a one-sided surface Σ) is exposed to the influences of a nearby magnet, it is possible to define a total magnetic flux Φ through the circuit in terms of the magnetic field everywhere inside:

$$\Phi[t] \equiv \iint_{\Sigma} ds \bullet \underline{\mathbf{B}}[\underline{x}, t]$$

Faraday's ***flux rule*** relates the electromotive force (or EMF) $\mathcal{E}$ in a closed, electrically conducting circuit to the rate of change

of the magnetic flux Φ through the circuit caused by **both** the motion of the circuit **and** by the time variation of the magnetic field $\underline{\mathbf{B}}[t]$. This absolute rate of change requires the **total** time derivative. For circuits in which a **thin** conductor defines a closed boundary π defining a one-sided surface Σ, the Faraday (filamentary circuit) flux rule can be expressed as:

$$\mathcal{E}[t] \;=\; -\,\mathrm{d}/\mathrm{dt}\,[\Phi[t] / c] \;\equiv\; \oint_{\pi} dl \bullet \underline{E}_m[\underline{x}, t]$$

This formulation is universal, if it is recognized that the location and/or shape of the circuit $\pi[t]$ may vary with time or that the bounded surface $\Sigma[t]$ may also vary over time. The use of CGS units also explicitly introduces the 'velocity of light' c that is needed for correct dimensionality of these equations and is hidden in the pseudo parameters of the vacuum in SI units. It is these subtleties that are often not discussed with the mathematical formulation of Faraday's brilliant physical intuition. This form also emphasizes that only the magnetic part of the electric field $\underline{\mathbf{E}}_m$ contributes to the circulatory EMF; this allows an electrostatic potential ϕ also to be introduced, where: $\underline{\mathbf{E}}_s \;=\; -\,\underline{\nabla}\phi$ as this integrates to zero around any closed path.

Electro-kinetic Momentum

The next step here is to return to Maxwell's adoption of the vector potential $\underline{A}$ or as he referred to it – the density of electro-kinetic momentum [100]; here always used with the explicit dimensionality factor $\underline{A} / c$ (§7.3). This was also at the heart of L. V. Lorenz's EM theory [65] although Maxwell never acknowledged this overlap in key concepts; perhaps because Lorenz never needed to define another mathematical variable: the curl of this vector, or as it is better known, the magnetic force (or flux) density $\underline{\mathbf{B}}$. Like Lorenz, this theory too will usually avoid introducing this unnecessary mathematical intermediary.

$$\underline{\mathbf{B}}[\underline{x}, t] \;\equiv\; \underline{\nabla} \wedge \underline{A}[\underline{x}, t] \quad \therefore\; \Phi[t] \;=\; \oint_{\pi} dl \bullet \underline{A}[\underline{x}, t] \quad \therefore\; \mathcal{E}[t] = -\,\mathrm{d}/\mathrm{dt} \oint_{\pi} dl \bullet \underline{A}[\underline{x}, t] / c \quad \therefore\; \underline{\mathbf{E}}_m[\underline{x},t] = -\,\mathrm{d}/\mathrm{dt}\; \underline{A}[\underline{x}, t] / c$$

This form returns to Maxwell's starting point in his study of EM before he developed his partial differential equations. The circuit π will always be restricted to paths **within** a conductor where real charges exist so that the work done $\mathcal{W}$ in moving a real charge Q around the path will equal the charge times the potential drop $\mathcal{V}$ from the highest to lowest point of potential. This programme expands on Feynman's position [101] that "only the $\underline{A}$ field is real" (since here $\underline{A}$ only needs to be defined within the conductors and the $\underline{B}$ field is viewed just as a mathematical manipulation of the vector potential). This view is confirmed by the real Aharonov-Bohm effect [102], which has demonstrated finite, measurable values of $\underline{A}$, while the $\underline{\mathbf{B}}$ field is everywhere and always zero. The circulatory integral (not quite complete) shows that the total flux is just another way of looking at the electro-kinetic momentum when it is evaluated all around the conduction circuit.

$$\therefore\; \underline{\mathbf{B}}[\underline{x}, t] \;= \underline{\nabla} \wedge \underline{A}[\underline{x}, t] \;= \underline{\nabla} \wedge \iiint \mathrm{d}^3x'\; \underline{\mathcal{J}}'[\underline{x}', t] / c\, r \quad \therefore\; \underline{A}[\underline{x}, t] = \iiint \mathrm{d}^3x'\; \underline{\mathcal{J}}'[\underline{x}', t] / c\, r \;= \oint \mathrm{d}\underline{x}'\; \mathcal{I}'[\underline{x}', t] / c\, r$$

Franz Ernst Neumann (1798-1895) rejected Faraday's lines-of-force model of induction and returned to Ampère's force law as the source of this line integral that inspired Maxwell to focus on the vector potential. This form will be used here later in its expanded form, reflecting the finite time delays that occur when electrons interact remotely across space; i.e. including Gauss's suggestion of asynchronous, remote (far) action. The final form must also be fully-relational – that is, it must only depend on the differences in locations of the interacting objects, both locally and globally.

The final step is to express the vector potential $\underline{A}$ at a point $\underline{x}$ in the target circuit created by each of the k meso-segments in the source circuit, centered at the location $\underline{x}'_k$ and moving at a relative velocity $\underline{u}'_k$, carrying $\Delta\mathcal{N}'_k$ electrons.

$$\therefore\; \Delta\underline{B} \;=\; e\,\Delta\mathcal{N}'\,\underline{u}' \wedge \underline{r}\; / c\, r^3 \;=\; e\,\Delta\mathcal{N}'\,\{\,\underline{\nabla}' \wedge (\underline{u}' / r) \;-\; 1 / r\;\; \underline{\nabla}' \wedge \underline{u}'\} / c \;=\; e\,\Delta\mathcal{N}'\;\; \underline{\nabla}' \wedge (\underline{u}' / c\, r) \quad \text{when } \mathcal{I}' \text{ constant.}$$

$$\text{But } \Delta\underline{\mathbf{B}}[\underline{x}, t] = \underline{\nabla} \wedge \Delta\underline{A}[\underline{x}, t] \quad \therefore\; \Delta\underline{A} \;=\; e\,\underline{u}'\,\Delta\mathcal{N}' / c\, r \quad \therefore\; \underline{A}[\underline{x}, t] \;=\; e \sum_k \underline{u}'_k\,\Delta\mathcal{N}'_k / c\, r_k \quad \&\quad \underline{r}_k \;=\; \underline{x} - \underline{x}'_k$$

The present micro-theory of EM interactions between pairs of electrons views the mutual interaction as always occurring on their mutual light-cones; this is equivalent to using retarded ($\underline{A}^-$) and advanced ($\underline{A}^+$) potentials when extended to groups of electrons operating collectively in the mesoscopic model (see §7.6.2). These are defined [103] as:

$$\underline{A}^{\lambda}[\underline{x}_j, t] \;=\; e \sum_k \underline{u}'\,[\underline{x}'_k, t_\lambda]\;\; \Delta\mathcal{N}'\,[\underline{x}'_k, t_\lambda] / c\, r_{jk} \quad \&\quad \underline{r}_{jk} \;=\; \underline{x}_j - \underline{x}'_k \;\;;\;\; t_\lambda \;=\; t + \lambda\, r_{jk} / c$$

Problems with Magnetic Induction

The contributions to the net magnetic EMF associated with the motion of conductors are often referred to as motional EMFs and those associated with the time variation of the magnetic field are referred to as induced EMFs (field changes). Even Maxwell drew attention to this duality in his 1861 paper *On the Physical Lines of Force* [104]. Feynman later pointed out that he knew no other place in physics where two different phenomena had to be used to gain a real understanding [122]. Worse, this apparent dichotomy threw Einstein off into his bizarre restatement of reality (space and time) when he discussed this 'weird' situation in the introduction to his famous relativity paper in 1905 [39].

Maxwell ignored all these subtleties when he mathematically transformed Faraday's intuition into a differential equation. He simply used Stokes' theorem to transform a fixed surface integral, over **all** of space at **one** instant of time, into a line integral around another (finite) closed but fixed boundary. In effect, he moved the *complete* time differential operator d/dt inside the integral and effectively converted it into an explicit or *partial* time differential operator $\partial/\partial t$ (although this vital distinction was not explicitly made in his time, nor was it important with Maxwell's assumption of a fixed Aether). The result is known as the **Maxwell-Faraday** equation (assumed valid even for closed paths where no conductors are present):

$$\underline{\nabla} \wedge \underline{E}_m[\underline{x}, t] = -\partial/\partial t\, [\, \underline{\mathbf{B}}[\underline{x}, t]\,] / c$$

Since: $\underline{\nabla} \wedge [\underline{u} \wedge \underline{Q}] = \underline{u}\,(\underline{\nabla} \bullet \underline{Q}) - (\underline{u} \bullet \underline{\nabla})\,\underline{Q}$ and the 'convective' or complete time derivative D/Dt is defined as [106]:

$$D\psi/Dt \equiv \partial\psi/\partial t + (\underline{u} \bullet \underline{\nabla})\psi \quad \therefore \quad D/Dt[Q] = \partial/\partial t\,[Q] + \underline{u}\,(\underline{\nabla} \bullet \underline{Q}) - \underline{\nabla} \wedge [\underline{u} \wedge \underline{Q}]$$

Where $\underline{u}[\underline{x},t]$ is the global velocity of the circuit element: $\underline{u}[\underline{x},t] \equiv d\underline{l}\,[\underline{x},t]\,/dt$ and always: $\underline{\nabla} \bullet \underline{B} = 0$
Thus, the correct formulation of the Maxwell-Faraday equation, incorporating moving reference frames, should be:

$$\underline{\nabla} \wedge \underline{\mathbf{E}}_m[\underline{x}, t] = -D/Dt\,[\underline{\mathbf{B}}[\underline{x}, t] / c]$$

It was this subtle mistake that led Lorentz to introduce 'his' force law as a field/velocity *compensation* into electrodynamics:

$$\underline{\mathbf{E}}_L[\underline{x}, t] = \underline{u}[\underline{x},t] \wedge \underline{\mathbf{B}}[\underline{x}, t] / c$$

$$\mathcal{E}[t] = \oint_\pi dl \bullet \{\, \underline{\mathbf{E}}_s[\underline{x}, t] + \underline{\mathbf{E}}_L[\underline{x}, t]\, \} = \oint_\pi dl \bullet \{\, \underline{\mathbf{E}}_s[\underline{x}, t] + \underline{u}[\underline{x},t] \wedge \underline{\mathbf{B}}[\underline{x}, t] / c\, \}$$

This is the reason why so many expositions of Faraday's law are forced to discuss both motional (Lorentz) and field (Maxwell) components of the effects of relative electrical and magnetic motion. This is a direct consequence of separating out the unitary interaction between source and target currents and charges into simultaneous mathematical intermediaries that need to be evaluated across all of space. Maxwell needed to introduce the partial time derivative because he wanted to end up with the wave equation for both his electric and magnetic field vectors: preserving these leads to the Lorentz transform.

A major problem with the Lorentz force is that it was first introduced (by Maxwell and then next by Heaviside) to handle real forces on macro-currents (that is, the movement of conductors) while Lorentz wanted it only for his theory of electrons. A single velocity parameter $\underline{V}$ can only be used for motional analyses if rigid circuits are being analyzed (i.e. $\underline{V} = \underline{u}[\underline{x},t]$ for all $\underline{x}$ & t) but this is only an approximation for actual conductors in the real world. As has been shown above (§7.4.1) there is a massive level of aggregation needed between these two levels (electrons and conductors) that requires a detailed theory, which has to explain, at least, how impulses between remote electrons get converted to momentum changes in the lattice for macro movements of the whole conductor (or equivalently: how electromotive forces relate to mechanical forces). A theory will be presented next to describe remote EM interactions between conductors without using electric or magnetic fields.

This approach, so far, is purely mathematical in its definitions and manipulations. It does not address the central problem, ignored by Lorentz, that although an electron beam in *vacuo* behaves **like** an electric current, this does not mean that the electric current in a metallic conductor **is** simply the unimpeded flow of electrons through the lattice. The ions, although relatively fixed, still contribute to the overall dynamics of the current in the conductor and even the motion of the whole conductor itself. Lorentz was able to force a marriage between the continuum views of Maxwell (now reduced to a mere skeleton of abstract mathematical equations) and the new, recently discovered discrete, particulate model of electricity – the electron. CEM disguises the widely divergent models of electricity resulting from this forced merger by re-introducing the fluid (continuous) model of electricity first proposed by Helmholtz and always vigorously rejected by Maxwell. [107]

7.6.3 ELECTRON INDUCTION

Current Induction and Inductance

The history of science has failed to credit the true originality of Joseph Henry's (1797-1878) research on current induction. Textbook authors, in their rush to reify the magnetic field and ignore the remote-action contributions of the 19th century '**continental**' physicists, have simply subsumed Henry's contributions in this area as 'just' sources of the magnetic field that Faraday (at the same time) had demonstrated will generate induction. This fails to make the real distinction that Henry was investigating mutual induction **between** remote currents, while Faraday was investigating the effects of nearby permanent magnets to induce currents. It is only the modern theory (and Ampère's guess) that permanent magnets can be viewed as lossless micro-currents that now allow us to see the connection. However, it is **Henry**'s research, rather than Faraday's, that uncovers the more fundamental physics of the phenomenon of induction. This will be the path followed in this programme.

The idea of inductance entered EM theory as the electrical analog of inertial mass, where currents will resist both accelerative changes to the flow of electric charge and changes to steady electric current. This electrical reluctance to change is universal, whether in a single closed circuit or in proximate circuits (open or closed). It will be shown that this is directly related to the fact that all electrons are inertial particles (in the Newtonian sense) and resist changes in their motion when interacting. This universal electrical behavior is known as EM **induction** but when restricted to closed electrically conducting circuits is called **inductance**, where its value varies with the relative spatial configuration and materials used in the circuit. Self-inductance of a circuit $\mathcal{L}$ is a measure of the reluctance of a single electrical closed circuit to change the value of its current with time. Two or more circuits can exhibit both this individual behavior and can also exhibit mutual reluctance to changes in each other's currents; this cross-linkage is known as mutual inductance $\mathcal{M}$ and is key to much of today's electrical technology. In CEM today, all of these effects are attributed to changes in the magnetic field produced by one part of the circuit that then reacts with other parts of the circuit; the present theory dispenses with this redundant, invisible intermediary concept.
The electromotive force (or EMF) $\mathcal{E}$ induced in a circuit when changes occur in its current $\mathcal{I}$ was found by Henry to be:

$$\mathcal{E}[t] \equiv -\mathcal{L}\, d/dt\, [\mathcal{I}[t]]$$

In the case of two linked circuits, a change in the current in the second (source) circuit $\mathcal{I}'$ will induce an EMF in the first.

$$\mathcal{E}[t] \equiv -\mathcal{M}\, d/dt\, [\mathcal{I}'[t]]$$

This approach can be related to Faraday's view of magnetic induction through his flux rule (§7.6.2) generating the EMF $\mathcal{E}$.

$$\therefore\ \mathcal{E}[t] = -\mathcal{L}\, d/dt\, [\mathcal{I}[t]] = -d/dt\, [\Phi[t] / c] \quad \therefore\ d\Phi[t] = c\,\mathcal{L}\, d\mathcal{I}[t] \quad \therefore\ \Phi[t] = c\,\mathcal{L}\,\mathcal{I}[t] \ (\text{or } c\,\mathcal{M}\,\mathcal{I}[t])$$

Henry's law of self-induction may also be restated in integral form:

$$\mathcal{L}\,\mathcal{I}[t] = -\int dt'\ \mathcal{V}[t'] = \Phi[t] / c$$

An important step now is to eliminate all references to magnetism in the phenomenon of induction by returning to Henry's law for mutual induction, discussed above and eliminating even the intermediate role of the vector potential, $\underline{A}$.

$$\mathcal{E}[t] = -d/dt \oint d\underline{x} \bullet \underline{A}[\underline{x}, t] / c = -d/dt \oint d\underline{x} \bullet \oint d\underline{x}'\, \mathcal{I}'[\underline{x}', t] / c^2\, r \quad \therefore\ \mathcal{M} = \oint \oint d\underline{x} \bullet d\underline{x}' / c^2\, r \quad (\text{in sec}^2/\text{cm})$$

This is Neumann's purely geometrical formula for the calculation of the mutual inductance between two interacting circuits.

Self-Induction

The phenomenon of self-induction is simpler than interactions between two circuits and offers some useful insights. When a switch to a source of external EMF is closed, the current does not immediately jump to its maximum constant value but rises (relatively) gradually from zero. Maxwell's EM field theory claims that this transient effect is wholly due to the back EMF in the complete circuit that takes on instantly the full external value of the battery but in the opposing direction [108]. Since all induction effects are conceived of as a result of interactions between all parts of the circuit this would be an example of instantaneous far action, which contradicts Maxwell's model of finite propagation of local actions through all media. It was Neumann's original view that self-induction must result from simultaneous far-actions. However, the mesoscopic model in this theory views this effect as an example of vector potential effects propagated at light-speed through the whole conductor; that is, as exchanges of EM momentum between interacting electrons at remote distances ("asynchronous far-action").

In the new model of impulse conduction developed above (§7.4.2), the galvanic electric current in a wire (considered as a thin, metallic conductor of uniform cross-section) is generated by very many excess electrons being injected into the wire by an external battery. In contrast to steady currents, increasing currents correspond to the rate of electron injection increasing with time until the maximum current is reached, when stable long-term equilibrium conditions are re-established. Some of the excess electrons will join the cells in a given layer while others will move (under net repulsion) faster than the average speed of the mobile electrons in the cell into the next layer; this will be repeated (very rapidly) until a new average current is stabilized throughout the whole wire. Let each quasi-equilibrium state correspond to (on average) one extra electron per cell; at time t, there will be Δn **excess** electrons per cell generating a current $\mathcal{I}[\Delta n]$ with a voltage drop across the wire of $\mathcal{V}[\Delta n]$.

Once again, the total repulsion between these excess electrons and the mobile electrons, now all moving together collectively through the lattice to minimize total action, defines **all** the long-term (longer than 10^{-16} sec) average configurations. This will require the external source to expend energy to overcome the net repulsion from all the electrons already present in the cell **and** to re-arrange the motion of all the excess electrons on this cell's light-cone. This extra work is reflected in the voltage drop, which moves **all** the mobile electrons to a new higher speed; i.e. to do extra work $\Delta\mathcal{W}[\Delta n]$ in an interval $\Delta t[\Delta n]$.

$$\text{Now } \mathcal{V}[\Delta n] = \mathcal{L}\,\Delta\mathcal{I}[\Delta n]/\Delta t \quad \therefore\ \Delta\mathcal{W}[\Delta n] = e\,\Delta n\,\mathcal{V}[\Delta n] = \mathcal{L}\,\Delta\mathcal{I}[\Delta n]\,(e\,\Delta n/\Delta t) = \mathcal{L}\,\Delta\mathcal{I}[\Delta n]\,\mathcal{I}[\Delta n] = \tfrac{1}{2}\,\mathcal{L}\,\Delta[\mathcal{I}^2[\Delta n]]$$

$$\therefore\ \mathcal{W}[N] = \Delta\mathcal{W}[1] + \Delta\mathcal{W}[2] + \ldots + \Delta\mathcal{W}[N] = \tfrac{1}{2}\,\mathcal{L}\,(\Delta\mathcal{I}^2[1] + \ldots + \Delta\mathcal{I}^2[N]) = \tfrac{1}{2}\,\mathcal{L}\,(\mathcal{I}^2[N] - \mathcal{I}^2[1]) \approx \tfrac{1}{2}\,\mathcal{L}\,\mathcal{I}^2[N]$$

This result is valid for all numbers of excess electrons per cell, which corresponds to the time t when this number is reached; in other words, there is a direct correspondence between the number of excess electrons N and the time t (N ⇒ t).

$$\therefore\quad \mathcal{W}[t] = \tfrac{1}{2}\,\mathcal{L}\,\mathcal{I}^2[t]$$

Thus, the total electrical energy stored in a complete circuit carrying a current $\mathcal{I}$ (corresponding to the electrical potential energy in each cell $\mathcal{W}$) is determined by the square of the current or the square of number of excess electrons moving across any cross-section per unit time; again, this is a key characteristic of all collective systems [109]. In contrast, with field theory that views this electrical energy being carried (stored) by the electric field throughout **all** space, the present theory views this energy as a property of the spatial and temporal configuration of **all** of the excess electrons in **each** cell; i.e. their relative positions and motions of these electrons. The finite changes in electric energy $\Delta\mathcal{W}$ may be written in several equivalent ways.

$$\therefore\quad \Delta\mathcal{W} = \mathcal{V}\,\Delta Q = \mathcal{V}\,\mathcal{I}\,\Delta t = \mathcal{L}\,\mathcal{I}\,\Delta\mathcal{I} = \mathcal{I}\,\Delta\Phi/c = (\underline{\mathcal{I}}\cdot\underline{A})\,\Delta x/c = (\underline{\mathcal{J}}\cdot\underline{A})\,\Delta^3 x/c$$

Induction Field of a Straight Wire

The value of introducing the idea of the magnetic field as a mathematical intermediary is well illustrated in problems that involve calculating the effects of induction; this useful approximation demonstrates why Faraday's simplifying flux ideas overshadowed those of the more rigorous model proposed by the Amperian school. This approach often involves the filamentary approximation described by the Biot-Savart law (see §7.6.1), written here as:

$$\underline{\mathbf{B}}[\underline{x}] = \mathcal{I}' \oint d\underline{z}' \wedge \underline{r}/cr^3$$

Here, the requirement is to calculate the $\underline{\mathbf{B}}$ field 'generated' by a straight segment of a thin, long wire carrying a steady current $\mathcal{I}'$ at a point that is a vertical distance R from the wire. Symmetry suggests using cylindrical coordinates {R, θ, z'}, where the origin is placed directly below the observation point on the wire. Let the target point $\underline{x}$ subtend an angle ψ from the vertical relative to the source element at z' of length dz', so that it makes an angle b with the far end of the wire and an angle –a, with the near-end of the wire. The value $d\underline{z}' \wedge \underline{r} = r\cos\psi\, dz'\,\hat{\underline{e}}$, where $\hat{\underline{e}}$ is a unit vector in the circular direction normal to the wire and forming a right-handed system with the current $\mathcal{I}'$ directed along the z-axis.

Since $R = r\cos\psi$ and $z' = R\tan\psi$ $\quad\therefore\quad dz' = R\,d\psi/\cos^2\psi \quad\therefore\quad d\underline{z}' \wedge \underline{r}/r^3 = \cos\psi\, d\psi/R\,\hat{\underline{e}}$

Writing $\underline{\mathbf{B}}[\underline{x}] = B[R]\,\hat{\underline{e}}$ $\quad\therefore\ B[R] = \mathcal{I}'\int_{-a}^{b} d\psi\cos\psi/cR = \mathcal{I}'(\sin a + \sin b)/cR \quad\therefore\ B[R] \approx 2\mathcal{I}'/cR$

The direct application of Ampere's circuital law gives: $4\pi\,\mathcal{I}'/c = \oint_{\pi} dl \bullet \underline{\mathbf{B}}[\underline{x}] = \int d\theta\, R\, B = 2\pi R B \quad\therefore\ B[R] = 2\mathcal{I}'/cR$

7.6.4 INDUCTANCE EXAMPLES

Self-Induction: Straight Wires

The calculation of the self-inductance in a long straight wire (diameter much smaller than its length) illustrates several key points. Although transverse motion plays a major role in understanding Ampère's force law, it can be ignored in most cases of self-induction, where longitudinal motion (along) the wire is the dominant effect. Consider a straight wire of length a that is divided into n equal segments of size Δx ($a = n\,\Delta x$). Let each segment be numbered from the origin ($x = 0$) so that they are identified by the distance of their centers from the origin; for the k^{th} segment: $x_k = (k - \tfrac{1}{2})\,\Delta x$ for $k = 1, 2, \ldots, n$. Neumann's integral self-induction formula can be converted to discrete finite differences.

$$\therefore \quad \mathcal{L} = \oint \oint d\underline{x} \bullet d\underline{x}' / c^2 r \qquad \therefore \quad c^2\mathcal{L} = \sum_{j=1}^{j=n} {\sum_{k=1}^{k=n}}' \Delta x_j\, \Delta x_k / |\, x_j - x_k\,| = \Delta x \sum_{j=1}^{j=n} {\sum_{k=1}^{k=n}}' 1 / |\, j - k\,|$$

Obviously, the physics of the interaction excludes the "self-term" ($k = j$) signified by the dashed summation).

$$\text{Let} \quad S_j \equiv {\sum_{k=1}^{k=n}}' 1 / |\, j - k\,| = \sum_{k=1}^{k=j-1} 1 / (j - k) + \sum_{k=j+1}^{k=n} 1 / (k - j) = \sum_{m=1}^{m=j-1} 1 / m + \sum_{m=1}^{m=n-j} 1 / m = H_{j-1} + H_{n-j} \quad \& \quad H_n = \sum_{k=1}^{k=n} 1 / k$$

Here, H_n is the 'Harmonic' function or n^{th} harmonic number, where: $H_n = \gamma + \ln[n] + 1/2n - 1/12n^2 + O[1/n^3]$ with the Euler-Mascharoni constant $\gamma \approx 0.577\ldots$ N.B. H_n is not defined for $n = 0$ but can be extended to this value with: $H_0 = 0$.

$$\text{Let} \quad K_n \equiv \tfrac{1}{2} \sum_{j=1}^{n} S_j = \tfrac{1}{2} \sum_{j=1}^{n} (H_{j-1} + H_{n-j}) = \sum_{j=1}^{n-1} H_j = n + (n-1)/2 + (n-2)/3 + (n-3)/4 + \ldots + 1/(n-1) + (1-1)$$

$$\therefore \quad K_n = n\,(1 + 1/2 + 1/3 + \ldots + 1/(n-1)) + 1 - \sum_{j=1}^{n-1} (k-1)/k = n\,H_{n-1} + 1 - n + H_{n-1} = n\,H_{n-1} - n + 2$$

For very large $n >> 1$, $K_n = (n-1)\,H_{n-1} \approx n\,H_n \approx n \ln[n] \qquad \therefore \quad \mathcal{L} = (2\,a / c^2) \ln [a / \Delta x]$

Thus, the self-inductance per unit length (using $\Delta x \approx 10^{-6}$ cm) gives a value of about $25 / c^2$ or 3×10^{-20} (sec^2/cm^2).

This value can be seen to be both very small and independent of the characteristics of the metallic conductor (i.e. universal) as the only physics here is in the electrical resistance of the metal. An identical result can be achieved in the continuum limit as long as the separation distance is always positive and the self-interaction of a segment is eliminated – this is achieved by respecting a finite gap ($D = \Delta x$) between the two parts of the integrals evaluated symmetrically over $2b = a$.

$$c^2\mathcal{L} = \int_{-b}^{b} dx \int_{-b}^{b} dx' / |\, x' - x\,| = \int_{-b}^{b} dx \left(\int_{x+D}^{b} dx' / (x' - x) + \int_{-b}^{x-D} dx' / (x - x') \right) = \int_{-b}^{b} dx \left(\int_{D}^{b-x} dy / y + \int_{D}^{b+x} dy / y \right)$$

$$= \int_{-b}^{b} dx\ (\ln[b - x] - \ln[D] + \ln[b + x] - \ln[D]) = 2 \int_{0}^{2b} dy / y - 4b \ln[D] = 2a\,(\ln[a / D] - 1) \approx 2a \ln[a / \Delta x]$$

In most calculations, it is sufficient to replace the cell parameter Δx with the diameter of the wire d as the logarithmic form implies that this just adds a small constant to the final value; thus:

$$\mathcal{L} = (2a / c^2) \ln [a / d]$$

For large n ($n >> 1$) Stirling's approximation gives : $K_n \approx n \ln[n] \approx \ln[\,n!\,]$; this result occurs in statistical mechanics when all possible interactions can occur in any time interval between n interacting objects.

For an insulated double-straight wire that is joined ('shorted') at its far end, each segment dx' on the outward 'leg' is in the opposite direction from the matching segment on the inward leg; the sum of these two contributions ($d\underline{x} \bullet d\underline{x}'$) almost cancel out completely for very thin wires, so the self-inductance of this configuration is almost exactly zero for all lengths.

Mutual-Induction: Parallel Wires

The simplifying step of using "magnetic fields" can be readily seen in the calculation of the mutual inductance $\mathcal{M}$ between two thin, very long parallel wires (each of radius a) whose centers are a distance d apart and carrying equal currents $\mathcal{I}$ in opposite directions. Since the currents are in opposite directions, the magnetic fields of each wire combine additively. The total induction field $\underline{B}$ crosses the plane between the two wires in a vertical direction and its value varies with the distance x from the center of one of the wires.

$$\therefore \quad B[x] \approx 2\mathcal{I} \{ 1 / x + 1 / (d - x) \} / c$$

The total magnetic flux Φ passing between a length L of these wires passes through the rectangle of length L and width defined from $x = a$ to $x = d - a$ in strips of area L dx is:

$$\therefore \quad \Phi = \int dS\, B[x] = 2\,\mathcal{I} / c \int_{a}^{d-a} \{ 1 / x + 1 / (d - x) \}\, L\, dx = 4\, L\, \mathcal{I} / c \ln [(d - a) / a] = c\, \mathcal{M}\, \mathcal{I}$$

$$\therefore \quad \mathcal{M} \approx (4 L / c^2) \ln [\, d / a \,] \qquad \text{when } d >> a$$

Self-Induction: Short Solenoids

The calculation of the self-inductance in a long wire (diameter much smaller than its length) coiled into a helix illustrates several key points about the powerful effects of currents acting upon themselves. The complete use of Neumann's formula for circular wires involves elliptical integrals that can only be evaluated numerically or approximately in certain limits. A very useful approximation gives similar results for solenoids of large radius R made from insulated thin wires of very small radius a ($R >> a$). This approximation reduces the mutual effect of circuit elements to the small, parallel segments of length Δx ($\Delta x = R\, \Delta\theta$) acting only on other parallel segments; in other words, the inverse distance factor ($1/ r$) dominates when the parallel wires are close together. If the linear extent of the solenoid is b cm and there are n turns in the solenoid: $b = n\, 2a$. In this approximation, there are very many turns in the solenoid ($n >> 1$ or $b >> a$). The total length of wire in the whole solenoid is therefore $n 2\pi R$. Let each turn be numbered from the origin ($z = 0$) so that they are identified by the distance of their centers from the origin; for the k^{th} segment: $z_k = (k - \tfrac{1}{2})\, 2a$ for $k = 1, 2, \ldots, n$. Neumann's integral formula can now be converted to discrete finite differences and $r_{kj} = |k - j|\, 2a$. If the segment $d\underline{x}$ at centered at $\underline{x}$ makes an angle θ with the (arbitrary) x-axis then the only segments $d\underline{x}'$ in the other turns that make any major contribution to the self-inductance are those that also make an angle θ with the x-axis. For simplicity, assume: $\Delta x = 2a$.

$$\therefore\ c^2 \mathcal{L} = \oint \oint d\underline{x} \bullet d\underline{x}' / r \approx \oint d\underline{x}_j \bullet d\underline{x}_k / r_{kj} \approx R \oint d\theta \sum_{j=1}^{j=n} {\sum_{k=1}^{k=n}}' \Delta x_k / (\,| x_j - x_k |\,) = n\, 2\pi R \sum_{j=1}^{j=n} {\sum_{k=1}^{k=n}}' 1 / |\, j - k \,|$$

Obviously, the physics of the interaction **excludes** the "self-term" ($k = j$) signified by the dashed summation). However, this was the double 'harmonic' sum that appeared in the calculation of the self-inductance of a straight wire, approximating to the value n ln [n] for large values of n.

$$\therefore \quad \mathcal{L} = 4\pi\, R\, n^2 \ln[n] / c^2$$

A more accurate calculation replaces the ln[n] factor by unity but since n = b/2a this is a close enough rough approximation.

7.7 EXTERNAL CONDUCTION INTERACTIONS

7.7.1 LONGITUDINAL INTERACTIONS

Although the application of Ampère's and Lorentz's force laws agree in their predictions for the gross motion of conductors when they carry currents flowing in closed circuits, they differ on the force distributions around the circuits and differ even more fundamentally when applied to open circuits like radio antennas. Specifically, Ampère's forces within the same conductor do not contribute to the action/reaction forces between two remote circuits while Lorentz's force cannot exist within any conductor below the surface depth. By design, Ampèrian forces are instantaneous and obey Newton's Third Law equating action and reaction; this is in marked contrast to Lorentz's force which carries energy and momentum between the conductors at light-speed and needs the introduction of Planck's ad-hoc definition of relativistic momentum to maintain the conservation of energy and momentum. Lorentz redefined the **meaning** of Ampère's current element from a moving section **of** the conductor itself (filled with electrical fluid) to the group of electrons moving **in** this section of conductor. This required inventing a new force that combined the longitudinal effects on the electrons (EMF) and a perpendicular component that moved the conductor transversely. Lorentz's marriage of continuum mathematics (field theory) and the actual discovery of real electrons led quickly to the new orthodoxy that a final theory of EM had been created, so that by the early 1920s, Ampère's forces were not even being discussed in academic texts on electricity and magnetism [110]. Perhaps this was because the Lorentz force law disagreed in certain areas with the demonstrated predictions of the alternatives. [111]

A central question that has still to be answered is the role of longitudinal forces in conductors. This is not just a theoretical question but one that traces its roots to the experimental work of Carl Hering (1860-1926) who drove large currents through baths of liquid mercury and demonstrated the existence of major forces acting in the direction of the current flow [112]. This was a crucial experiment (it remains the basis for all modern liquid metal pumps), as the Lorentz force law predicts no such longitudinal effects in liquid conductors; indeed, it denies any effects between two aligned straight sections of conductors with a finite separation as there is no magnetic field in the longitudinal direction (see §7.8.4). These experiments were repeated recently [113] with very dramatic results but have also (like Hering's work) been ignored nonetheless by the mainstream physics community as they threaten the conventional basis of all of EM theory – both classical and quantum field theory.

7.7.2 REMOTE MESOSCOPIC INTERACTIONS

So Near, So Far?

The lost history of electrodynamics throws up some important differences in the treatment of forces between the 'far-action' theories of the 19th Century 'Continentalist' tradition (exemplified by Ampere, Weber and Neumann) and the modern 'local-action' (or field) theory of CEM based on the Maxwell-Heaviside equations and the critical Lorentz force equation (a mix that is sometimes referred to as "relativistic electromagnetism"). These distinctions can be seen in the following table.

FORCE THEORY	AMPERE	WEBER	NEUMANN	LORENTZ
Applicability	Metallic conductors	Electric charges	Closed circuits	Conductors, elect. beam
Source Element	Current element	Point charge	Current elements	Magnetic field
Target Element	Current element	Point charge	Electric fluid	Point charge
Constraints	Constant currents	None	Rigid conductors	Velocity < Light-speed
Force Type	Mechanical	Mech. & Electrical	Electrical	Mech. & Electrical
Disappearance	Either current stops	No relative motion	Source current stops	Mag. field disappears
Action = Reaction?	Yes	Yes	No	No
Instant Action?	Yes	Yes	Yes	No
Time Dependence	Static	Rel. velocity & accel.	Source current accel.	Target part. velocity
Spatial Separation	Far	Far	Far	Local
Distance Effect	$1 / r^2$	$1 / r^2$	$1 / r$	$1 / r$
Longitudinal Force?	Yes	Yes	Yes	No

Here, mechanical forces act **on** the conductor itself, while electrical forces act only on the current moving **in** the conductor. The non-instant action for the Lorentz force refers to the time delay in changes at the source of the magnetic field reaching the test particle. The spatial separation refers to the distance between the source and target elements at the time of the inter-action (not ultimate sources, as in the magnetic field, which is always treated as local to the target particle).

It may be seen from this table that a comprehensive theory should describe mechanical forces on conductors and electrical forces (EMF) on currents, including isolated electrons. The new interaction must be asynchronous, reflecting the finite time differences characteristic of all EM interactions; it should respect Newton's Third Law of Motion (action equals reaction) which conserves momentum (but not necessarily locally); it should be fully relational – depending only on the difference between the elements' parameters at the two times of the interaction; it should vary inversely with distance and should also include a longitudinal component. None of the existing force laws meets all of these requirements.

It must also be made explicit that since neither forces nor potentials are given any existential status here then any criticism against asynchronous interactions (e.g. retarded far-action) is totally rejected [114] – nothing **physical** is emitted here from any source that is received at a different time by the target element. Statements such as "emitted impulse" or "absorption of the impulse" are to be interpreted as "aids to thinking" - visualizations of local changes in momentum that are synchronized across both 'ends' of a unitary interaction: mathematical separations to aid in calculations, without any correspondence with reality. Furthermore, although this theory occasionally introduces potential functions, these are also to be understood only as "aids to calculation" and must not be considered as corresponding with any aspects of reality. The Graneaus also rejected advanced interactions [115], in the Wheeler and Feynman model of CEM [116], on the grounds that this would require the magical power of "clairvoyance" on the part of the electrons. This attempt to dismiss symmetric-causality at the micro level of the world on the *a priori* assumption that everything at the macroscopic (human) level applies at the scale of fundamental interactions is not an acceptable view of how theoretical physics is to advance. Hypotheses (like symmetric-causality or the existence of an EM Aether) are proposed and then their consequences are investigated and should only be dismissed if the resulting predictions "fail to measure up". The instantaneous action of gravitation across the universe has never presented insuperable difficulties to students of physics, even though some major philosophers (like Leibniz) objected mightily.

Mesoscopic Current Elements

The Graneaus view the Amperian current element as residing entirely in the conductor's ions [117] while Lorentz viewed the Amperian current element as consisting entirely of the conductor's electrons, as he was unaware of the yet to be discovered nucleus and the resulting ionic structures. Here, both conductors and electrons (carriers of the current within conductors) must be considered, so it seems reasonable to extend the mesoscopic theory of current within a single conductor (§7.4.1) to interactions between conductors themselves. The Principle of Simplicity (Ockham's razor) suggests that the phenomena of induction be merged with the phenomena of EM radiation as both result in EM actions at a distance. However, in order to accommodate mutual induction this must then include situations where the current is initially only present in one of the two conductors (the source) as well as the basic Amperian situation where both conductors are independently transporting current. Thus, in this extended theory, the ***current element*** will be defined as a **finite** length of the conductor (along its longitudinal dimension) containing **both** a fixed number of ions and all the mobile electrons in this piece at an instant of time. The size of this current element is defined as equal to the full width of the wire with its length equal to the mesoscopic cell's linear size Δx_0. In both cases, where there is no current or when the current is steady, then the average number (but not identity) of the mobile electrons in a given current element is fixed. The reason for emphasizing the finite length of the current element is it has always been assumed that there exists a continuous force exchange between elements so that in the infinitesimal limit of continuous time (i.e. $\Delta t \rightarrow 0$) there is no difference between the interactions at the two ends of the current element (thus no torque) and that the force can then be assumed to apply at the center of the element. This has been disguised by introducing the 'triple-cross' form Ampère's law (see §7.1 and §7.6.1), which actually substitutes the Biot-Savart complete circuit form necessary for replacing part of the multiplicative product with the definition of the magnetic flux vector: this is actually how the mathematics of field theory entered into the foundations of electrodynamics. The other important fact that is needed in this new approach is to acknowledge explicitly that in perfect conductors there are only surface currents (disguised again by referring only to conductors as 'wires') so interactions between separate conductors mostly occur on their nearest surfaces.

Mesoscopic Amperian Impulses

The simplest situation occurs (as Ampère realized) when the electric current is constant in both conductors and when the two conductors are parallel to each other. When the currents flow in the same direction, the two wires attract one another but when the currents flow in the opposite directions, the two wires repel one another. First of all, consider the situation when there is no current in either wire. The mobile electrons move around all the mesoscopic cells in each conductor in an apparently random manner (at least to us), with no direction having any long-term preference. As electrons reach the surface in an outward direction, their motion is reversed by the attraction (momentum exchange) from positive ions in the surface cells.

In terms of the simple mesoscopic theory of a single conductor introduced earlier (§7.4.1), the existence of a galvanic current results from an excess of mobile electrons in every surface part of the wire. These additional electrons in both wires would occasionally interact with one another and if, on average, they were not moving down the wire would behave like a capacitor (§7.5.1) but only if the surface charge density were different (usually the case); so in the static case (no currents) there would always be a net attraction between the two parallel wires, independent of the relative current directions.

7.7.3 EXTERNAL CONDUCTION FORCES

The External Conduction Interaction

The remainder of this section will explore the electrodynamics **created** by a single, metallic conductor, in the form of a closed circuit, carrying various electrical currents produced by external sources of EMF; such a circuit will be referred to as the 'source'. This investigation will focus on situations where there is a single set of remote charges (the 'target') that is external to the circuit but are still interacting with some of the electrical charges in the source circuit. When a conductor, such as the source, is carrying a current then most of the electrons and ions in the surface mesoscopic cells are interacting with other electrons and ions in the same conductor, as described earlier (§7.4.1). However, when an external set of charges interact with the conductor then some of the internal interactions must cease and the interactions switched from internal to external (according to the present Saturation Hypothesis, where an electron can only interact with one other electron at any specific instant). The present theory, known as the *External Conduction Interaction* (XCI) explores this ***net*** interaction with these external charges from a statistical viewpoint. Obviously, this is not a fundamental theory of electromagnetism but it will be shown that this corresponds to the level of aggregation demonstrated in Maxwell's theory of classical EM and will prove sufficient to explain both Henry's discoveries in current induction and Hertz's discovery of remote EM oscillations.

Target Charges

Since this is not a field theory, the nature and behavior of the target-set of charges play key roles in this theory. The target-set may be only a single charged particle (like an electron or proton) or even a tight aggregate of such particles that behave in a unitary manner over the time frames under investigation; these will be referred to collectively as a 'charged particle'. The target-set may also be a finite length of another metallic conductor, in which case, the mobile electrons and ions in its surface cells constitute the set of target charges. If this conductor can support an electric current then it must also form a closed circuit; if this current exists prior to the interaction with the source then it must have its own source of EMF. The target-set may be stationary or moving relative to the source prior to the interaction between them. In all cases, both the source and target conductors are assumed to have an equal number of mobile (or conduction) electrons per cell as the number of positive (lattice) ions in each cell when there is no current present. Current interactions must involve either two events or two charges at two different times – this leads to the idea of the dipole approximation (see later – §7.8.1).

Finite Space & Time Averaging

In contrast to all earlier electrodynamical models, this theory proposes that the target-set interacts with many of the charges in the source over a small but finite time duration ΔT and the interactions, even at the target, do **not** occur all at the same time. This is a rejection of both 'instantaneous' and continuum physics; hence the description as interaction and not force. This target interaction duration ΔT will vary with the situation and nature of the target-set: only in deriving the link with CEM will the infinitesimal limit of continuous interaction time (i.e. $\Delta T \rightarrow 0$) be taken. In the case of a single target electron, this interaction duration must be sufficient to describe a stable (repeatable, measurable) response; in other words, because of the Saturation Hypothesis it must span a duration wherein very many interactions can occur serially between the target electron and a significant fraction of all the charges in the source conductor. Since this theory is deliberately excluding all possible electro-static interactions, there must be an equal number of interactions with the negatively charged mobile electrons and the positive ions in both the source and target-sets. Steady source currents only produce relatively short-range static effects (so-called 'magneto-statics') therefore the focus here will be on source currents that either vary in time (a fixed source conductor) or on steady source currents in conductors that are being moved (externally) with relative acceleration. Amperian forces will only be considered when one or both conductors may move while carrying fixed currents. When the target is a conductor then the averaging calculation over the interaction duration will be taken over a single mesoscopic cell in the target metal and over a time scale where the relative change is not too great (the prior definition of mesoscopic). For example, if the source EMF is oscillating in a time period of T_s seconds (i.e. with a frequency $\nu_s = 1/T_s$) then $\Delta T << T_s$ (or equivalently, $\nu_s \Delta T << 1$).

Retarded Interactions

Ultimately, this theory offers a time-symmetric view of the fundamental interaction between two electrons due to the finite time duration between the pair of events that defines each micro-interaction; neither electron can stake a claim to precedence

for being the present time, each defines its own (local) time. However, in the external conduction interaction an asymmetric viewpoint is adopted: the target electron is chosen as the measurable locale, so the center of its interaction duration defines the experimenter's "now" (denoted by the usual symbol t). All the many times in the past of this 'now', where each micro-emission event occurs become its own source time (each denoted by t′), are then averaged in their contribution to the net vector potential < A[t] >. The time of each of these source events has been the standard definition of *retarded time* in all EM texts. In order to comply with (and demonstrate the connection to) earlier research this will be the choice made herein.

7.7.4 CEM MODELS – Fields, Potentials & Particles

The Mathematics & Physics of Fields

All of the historical approaches just described continue the traditional separation of the force density F at the target-set into two distinct force field densities: the electric E-field and the magnetic B-field. Apart from the historic sense of trying to explain 'magnetism' with the B-field, this separation is a **mathematical** consequence of Helmholtz's realization that every continuous (regular) **vector** field in three dimensions F can be described everywhere by the sum of a conservative field V (defined by a scalar gradient) and a rotational vector field W (defined by the curl of another vector function); that is:

$$\underline{F} = \underline{V} + \underline{W} \qquad \text{where} \quad \underline{V} = -\,q\,\{\,\nabla\phi + \partial/\partial t\,\underline{A}\,/\,c\,\} \qquad \text{and} \quad \underline{W} = q\,\{\,\underline{u} \wedge \underline{\nabla} \wedge \underline{A}\,\}\,/\,c$$

Here, ϕ is a scalar function and A is a vector function that are both normalized per unit charge q into potential functions; the factor u is a local velocity function defined relative to the co-ordinate reference frame while *c* is a universal space-time ratio with the dimension of a velocity that results in similar homogeneous units being used for space and time components. The partial time derivative is added whenever there is the possibility of a vector field changing its value explicitly with time. If the target-set is a single charged particle then u is usually interpreted as the instantaneous velocity of the particle. This is a convenient separation of the target interaction into an instantaneous force component that is velocity-dependent and a part that is independent of velocity. Obviously, only the component parallel to the velocity can contribute to a change in the particle's kinetic energy (speed) while the part transverse to the velocity only changes the direction of the particle's motion.

Field theory was introduced by mathematicians for the study of continuous functions in two or more independent variables. Maxwell was one of the first physicists to realize that this new area of mathematics could be used to map reality, particularly when the independent variables were associated with locations in three-dimensional space. This could be made even more powerful when a real, continuous medium was posited as existing throughout all (or a macroscopic region) of space. This led to the study of elastic solids, which were soon found to support transverse vibrations that were considered a critical part of the phenomena of light required to explain polarization effects. This was the origin of Maxwell's Aether model of light.

The key to understanding this mapping between fields and physical reality is to view the continuous geometry of space itself as the justification for the use of continuous mathematical functions. Space has never been considered to have any 'holes' and real objects have always been viewed as **existing** continuously in space across a period of time. There is thus a widespread tendency to view all the possible points **of** space x {x, y, z} as all the possible locations **in** space that a particle **might** exist. This is also the origin of the concept of *potential* where events **might** occur if a particle just happened to exist at that point. Mathematically, fields involve functions F[x] that can be defined at all nearby points through another function G[x] such that: F[x+dx] = G[x] F[x], where x is a vector (3D set) in a Euclidean space defined relative to a fixed origin x = 0 and dx is an infinitesimal displacement from the set x in any direction {x+dx, y+dy, z+dz}. When using the mathematics of fields in physics, theoreticians can simply view them as useful calculational tools or, as is more often the case, representing a new set of physical **entities**, such as the electric and magnetic fields. The present theory rejects this latter position as ontologically redundant and fundamentally in conflict with the experimental evidence that the real world is discrete. Electrons have been demonstrated to exhibit point-like characteristics that exist in space; this theory adopts the traditional model that at any time t an individual electron exists at only one specific point in space, referred to as x[t]. The impossibility of measuring the point locations of a specific electron at two different times t and (t+Δt) as $\Delta t \rightarrow 0$ without impacting its velocity (quantum mechanics) is not viewed here as a sufficient justification for rejecting this powerful model of material reality (i.e. "point particles") that has served physics well since Newton and is consistent with macro objects. At an empty field point, it is always mathematically meaningful to compute the changes (in either potentials or fields) in time at the same point in space; it is also meaningful to calculate the field variations at nearby locations in space at the same time; this gives real meaning to the differential local space operator: del or ∇. This is why all field theories use the **partial** time derivative (∂/∂t).

However, when real charged particles are considered as components of the target-set, the operation ∂/∂t is physically problematic: this is even more so, when the interaction is discrete in time (impulses, not forces), as in the present physical theory. The inertia of real material particles means that only certain locations in the particle's trajectory are significant, only

one of which will be the location of the target's next interaction. This is why Newtonian particle physics always uses **full** time derivatives (d/dt) to compute changes in momentum. Since Maxwell was creating a field theory, he was always using the **partial** time derivative (although implicitly, due to his weak notation). This usage implies that this derivative limit is being evaluated at the exact same location in space: the field point. However, during this very brief (but finite) time interval not only would the spatial co-ordinates of the source charges have changed but so would the exact location of the real target particle that is interacting with the sources [118]. Thus, partial derivatives are a direct consequence of using mathematical field theory and this is only valid for time-independent interactions (continuous spatial potentials) and **fails** to represent the reality of any particle-to-particle interaction that varies over time, like EM induction and radiation.
This is the key reason for introducing non-physical concepts such as fields and potentials rather than locations of real particles, as in the present theory; it is also the principal reason why Lorentz invariance **must** always be invoked when considering **local field** theories. In contrast, all physical theories constructed around the concept of real, point particles use full time-derivatives when describing continuous changes that effect the ever-changing locations of real particles over time. This is also the subtle reason that Faraday's "flux rule" (§7.6.2) uses full time derivatives since he was manipulating real, macroscopic objects in his experiments (both at the source and at the target), not microscopic, imaginary fields.

Fields cannot represent Point Particles

The Lorentz 'force' law is only valid for target particles **if** the EM field carries energy and transports momentum (defined by the Poynting vector) since it results not only from using local magnetic fields but also from the explicit use of the **partial** time derivative to define conservation everywhere in an elastic medium. Even here, the assumption (rarely made explicit) is that there are 'actors' that move all the EM sources via 'external forces' [119], which leads to the bogus idea of 'radiation reaction' (another source of infinities in CEM). Important statements like the following [120] are blithely made without any form of justification: "If an electron radiates by virtue of an acceleration produced by an external force, the external force must supply both the energy and the momentum required by the change in the fields. This can only be done by means of a reaction force, produced by action of the radiation field on the electron itself." The image here is of a macroscopic charged object that can be mechanically moved around – this is totally impossible in practice since electrons can only be moved by their explicit interactions with other microscopic electric charges. The imagery used here by these authors is a projection from macroscopic experiments down to sub-microscopic distances that is totally unwarranted. Ironically, these authors find that this 'inertial reaction force' is an "exact analogy of the back EMF of an electrical circuit"; in other words, Jefimenko's electro-kinetic force and find that the classical electron radius Λ_0 (§7.4.3) must be introduced to avoid an infinite reaction. Since these authors, like Maxwell, do not develop any microscopic model of conduction and also base their potential theory on Coulomb's law, they can only state that Λ_0 is the minimum value "at which the vector potential breaks down owing to the structure of the electron." Recognizing that instantaneous forces are impossible, the calculation of reaction forces can only be calculated "over a sufficiently long period of time." All of these types of calculation have some parts of the "electric fluid" of the finite electron interacting with other parts of the same electron: all such "self-interactions" arise when merging the particle model of electricity (experimentally confirmed) with the mathematical theory of localized fields (CEM). All of these deep mathematical difficulties disappear when a purely particle model of EM is adopted as in the present case.

7.7.5 ELECTRO-KINETIC MOMENTUM

The EM interaction is sometimes viewed as originating with electric activity in a conductor and a potential being 'emitted' from the source in all directions and traveled across the universe until different parts in **all** the various directions encountered different charged target-sets, re-absorbing the emitted EM energy [116]. The EM interaction is not seen as completed until all absorptions everywhere of the original emission have eventually absorbed all the emitted energy. This was the implicit physics of the remote EM phenomena but the focus is always on the mathematics at an arbitrary, intermediate field point. In case it is thought the XCI theory is simply quantizing the electric fluid in the traditional CEM approach, it is important to emphasize several **physical** distinctions. Firstly, the charge density in the present theory is inherently discrete as it reflects the point-particle nature of electrons (and very nearly so of the ionic nuclei): two facts that were unknown in the days of Helmholtz. Secondly, this is a "two-fluid" model, like Weber, where the positive ions play an active role in the metallic conductors, not least, preventing the negative charge density of the electrons blowing themselves apart when they approach each other too closely – a problem still facing physicists in Lorentz's time when electron models needed mysterious 'glue' to avoid this problem.
The ions are also critical in reducing the net electro-static effects at the target-set to (almost) zero, leaving the XCI to be defined overwhelmingly by the point-to-point nature of the vector potential at the target-set: this re-emphasizes the dynamical nature of this interaction. Thirdly, this is not a field theory: only locations in space where point charges are found have any significance here; integrations over all space or integration paths in empty space have no physical role here, they are merely short-hand for locations where charges exist. In order for Helmholtz's fluid model of real charge density to be merged with

Maxwell's field theory of forces defined outside conductors ("the space between"), it is always necessary to restrict CEM to the only common locations – the surface cells of the conductors: the only stage whereon the action of the XCI is played out. Even Huygen's 'secondary wavelets' are only significant in the XCI theory as secondary re-emissions (further interactions) from actual charges: these secondary re-emissions do not exist everywhere throughout space.

The mathematical focus of the XCI theory is the EM vector potential. It is surely not a coincidence that this was Maxwell's original starting point in 1864 with his *Dynamical Theory* (§7.3) – this is where the interactions occur at the target-set: in effect, "it's where the action is". As described before [121], Maxwell re-interpreted Neumann's vector potential $\underline{A}$ at his field point $\{\underline{x}; t\}$ as the total **impulse** of the electromotive force $\underline{E}_m$ generated by the removal of all source magnets and currents.

$$\underline{E}_m[t; \underline{x}] = -\partial/\partial t\, [\, \underline{A}[t; \underline{x}]\,] / c$$

This is also the focus in Jefimenko's groundbreaking study of retardation in EM [122]. Jefimenko begins with the four Maxwell-Heaviside partial differential equations for the electric and magnetic force densities in empty space and quickly transforms them into the general inhomogeneous vector wave equations for each of these vector fields. Using Helmholtz's inversion theorem these are then transformed into integral equations defined in terms of the source charge density and current density distributions over all space but now defined at the retarded times $t' = t - r / c$; thus (with $\underline{r} = \underline{x} - \underline{x}'$):

$$\mathbf{\underline{E}}[t; \underline{x}] = -\iiint dx'\, \{\, \nabla'\rho'[t'; \underline{x}'] + \partial/\partial t\, [\underline{\mathcal{J}}'[t'; \underline{x}']\,] / c^2\, \} / r \quad \& \quad \mathbf{\underline{B}}[t; \underline{x}] = \iiint dx'\, \nabla' \wedge \underline{\mathcal{J}}'[t'; \underline{x}'] / c\, r$$

As Jefimenko points out, these equations **correlate** the electric field ($\underline{E}$) at the target with the local gradient (∇') of the source charge distribution $\rho'[t'; \underline{x}']$ and correlate the target induced magnetic field ($\underline{B}$) with the local curl ($\nabla' \wedge$) of the source current distribution $\underline{\mathcal{J}}'[t'; \underline{x}']$ and not with the source distributions directly. In effect, it is the inhomogeneities of these distributions that are the **cause** of the remote interaction – the view shared by the XCI theory. This is in contrast to the widely held view that rotational changes in the remote $\underline{E}$ field are caused by local time changes in the remote $\underline{B}$ field and vice-versa. He also transforms these generic volume integrals to the interiors and surfaces of conductors where abrupt changes occur. The final step was to move the partial time derivative outside the integral and re-introduce the scalar and vector potentials explicitly.

$$\underline{E} = -\nabla\phi - \partial/\partial t\, \underline{A} / c = \underline{E}_S + \underline{E}_M \qquad \phi[t; \underline{x}] = \iiint dx'\, \rho'[t'; \underline{x}'] / r \qquad \underline{A}[t; \underline{x}] = \iiint dx'\, \underline{\mathcal{J}}'[t'; \underline{x}'] / c\, r$$

These equations were first derived by Lorenz in 1867 [55] and, it may be noticed, do not require the $\underline{B}$ field; these retarded potentials satisfy the Lorenz (gauge) condition (which is a direct consequence of charge conservation everywhere [123]):

$$\nabla \bullet \underline{A} + \partial/\partial t\, (\phi / c) = 0 \qquad \text{or} \qquad d/dt[\phi] = 0$$

Carver Mead also begins his study of collective electrodynamics [124] with the definition of the vector potential, noting that the 1/r factor under the integral results from solving the spherical wave equations for the potentials using Green's functions. In the XCI, the 'twisting' effects (represented by the differential curl operator) result from adding multiple impulses from many different locations across two or three dimensions, as the source wire must turn back on itself in a closed circuit.

Since the present theory is first and foremost a physical theory based on the existence of real, point particles (and only then a mathematical representation of these physical concepts) it represents real changes over time by using **full time-derivatives** when describing continuous changes that effect the ever-changing locations of real particles over time. This is also the subtle reason that Faraday's "flux rule" (§7.6.2) uses full time derivatives since he was manipulating real, macroscopic objects in his experiments, not microscopic, imaginary fields.

As a result, the fundamental force per unit target point charge $\underline{E}_m$ due to **all** remote source charges in **m**otion (separated in space by a distance r at the earlier source-time t') is **defined** here as:

$$\mathbf{\underline{E}}_m[\underline{x}, t] = -d/dt\, \underline{A}[\underline{x}, t] / c \quad \text{where:} \quad \underline{A}[\underline{x}, t] = \oiiint d^3x'\, \mathbf{\underline{J}}'[\underline{x}', t - r / c] / c\, r = \oint d\underline{x}'\, \underline{\mathcal{I}}'[\underline{x}', t - r / c] / c\, r$$

When these forces are experienced by mobile electrons in the target conductor, they experience an EMF $\mathcal{E}$, defined as:

$$\mathcal{E}[t] \;=\; -\,d/dt\,[\Phi[t]\,/\,c] \;=\; -\,d/dt\,[\oint_{\pi} dl \bullet \underline{A}[\underline{x},\,t]\,/\,c] \;\equiv\; \oint_{\pi} dl \bullet \underline{E}_m[\underline{x},\,t]$$

This is the foundational definition of Jefimenko's electro-kinetic force, $\underline{E}_m$ but now using the **full** time derivative. It is not a coincidence that this was the starting point of Maxwell's dynamical theory of EM based on potentials, not fields (§7.6.2).

Rather than talking about "magnetic flux", it must be noted that the quantity Φ / c has the dimensions of **action** per unit charge. In fact, this is the central link between Newtonian mechanics and classical electro-dynamics: when a circuit is conveying a charge Q the electro-kinetic action is:

$$\mathfrak{A} \;=\; Q\,\Phi\,/\,c$$

7.7.6 THE CNV CEM MODEL

In a similar sense that special relativity theory 'covers' classical mechanics in the limit of low speeds, or wave or quantum mechanics 'covers' statistical mechanics when the action becomes continuous, the present theory is a 'covering-theory' of classical electromagnetism (CEM). In particular, the degree of electron aggregation, posited here at the mesoscopic level, offers a more realistic explanation of metallic conduction than provided by CEM. Maxwell's field theory of EM was supplemented **inside** conductors by the experimental relationship (Ohm's Law), in its (hypothetical) continuum field form:

$$\underline{\mathcal{J}}\,[t;\,\underline{x}] \;=\; \sigma\,\underline{E}\,[t;\,\underline{x}]$$

Maxwell's Aether theory not only had **no** explanation for this relationship but he always refused to view electric charge as a property of ordinary ('ponderable') matter, viewing it exclusively as a stressed state of the real Aether itself. Thus, his theory was developed **between** matter, in the empty space all around the locations of real matter, with no further explanation for the interaction between the lumeniferous Aether and matter, itself. It was this philosophical "mish-mash" of physical and mathematical concepts that finally forced Maxwell to adopt a purely phenomenological model of EM in his *Treatise* where the Aether only played the role of a continuous, solid-like medium (capable of transverse vibrations) that could be subjected to a Lagrangian type of rigorous mathematical analysis. This confusing state of affairs was 'swept under the rug' after the electron was discovered when a new amalgam of 'electric fluid' ideas from Helmholtz were merged with the now canonical equations of Maxwell-Heaviside to produce what is today taught to physics undergraduates as CEM [125]. One of the major problems with this 'shotgun marriage' is that the dynamics of even two charged point particles subject only to CEM has resisted all attempts at analytical solutions (more details can be found in §3.2). Nonetheless, modern physics has not hesitated to adopt the mathematics of classical Maxwellian field theory as its model for all contemporary forces between even very small numbers of fundamental particles at the most foundational level of reality.

When the remote target set is simply one electron a potential energy can be defined at every location at an instant of time when the target electron at that location is stationary or in motion. In this formulation, the continuous potential function only acquires reality when a location in space $\{t;\,\underline{x}\}$ is actually occupied by an electrically charged particle; in other words, there is the possibility or *potential* for change available at that spot. This allows the definition of both a scalar potential ϕ and a ("magnetic") vector potential $\underline{A}$, where the $\phi[t;\,\underline{x}]$ represents the EM *energy of position* per **stationary** unit charge and $\underline{A}[t;\,\underline{x}]$ represents the EM *energy of motion* per unit charge should the charged particle be **moving** (relative to the reference frame) with instantaneous velocity $\underline{v}[t;\,\underline{x}]$. The present theory offers a direct link to CEM, using both a mesoscopic model of electron aggregate interactions and CNVs. Section 6.2 of paper II defined the continuous natural vector (CNV) potential $\mathbf{A}[t;\,\underline{x}]$ at any point in space $\underline{x}$ and time t, as the unified combination of the scalar potential ϕ and the (magnetic) vector potential $\underline{A}$. This section also showed that only the Lorenz gauge is electro-dynamically consistent. The Lorenz gauge condition relates the electromagnetic vector potential $\underline{A}$ to the velocity $\underline{v}$ of the electric charges in motion that would create the static EM potential ϕ at every point in space and time. The CNV potential is defined in terms of the CNV velocity $\mathbf{V}$ or the four real quaternion bases $\mathbf{I}_\mu$ that form the foundation of all natural vectors.

$$\mathbf{A} \;\equiv\; \phi\,\mathbf{V}^*\,/\,c \;=\; \{\,-\,i\,\phi\,\mathbf{I}_0 \;+\; \underline{A} \bullet \underline{\mathbf{I}}\,\} \quad \text{where } c\,\underline{A} \;=\; \underline{v}\,\phi \quad \text{(the Lorenz Equation)}$$

The electric charge density $\rho[t;\,\underline{x}]$ is a continuous function of space and time defining the number of charges in any cell; it scales the potentials to the actual energy densities U_0 and $\underline{U}$ that are combined into the Energy-Density vector $\mathbf{U}$, where:

$$\mathbf{U} \;\equiv\; \rho\,\mathbf{A} \;=\; \{\,i\,U_0\,\mathbf{I}_0 + \underline{U} \bullet \underline{\mathbf{I}}\,\} \;=\; \{\,-\,i\,\rho\,\phi\,\mathbf{I}_0 \;+\; \rho\,\underline{A} \bullet \underline{\mathbf{I}}\,\} \;=\; \rho\,\phi\,\mathbf{V}^*\,/\,c \qquad \therefore \quad c\,\underline{U} \;=\; \rho\,\phi\,\underline{v}$$

Both source currents and target sets of charges in motion can be described in terms of the CNV current density $\mathbf{J}$ [126].

$$\mathbf{J} \equiv \rho\, \mathbf{V}^* = \{ -i\,c\,\rho\, \mathbf{I}_0 + \underline{J} \bullet \underline{\mathbf{I}} \} \quad \text{where } \underline{J} = \underline{v}\,\rho \quad \therefore\ c\,\mathbf{U} = \phi\,\mathbf{J}$$

Paper II also introduced CNV equivalents of all the 3D mathematical definitions used in CEM: these included the EM Force Density (per unit charge) CNV $\mathbf{G}$ and the EM Force CNV $\mathbf{F}$. These are inter-related as can be seen below.

$$\mathbf{G} \equiv -i\,\nabla^*\,\mathbf{A} = \{ i\,G_0\,\mathbf{I}_0 + \underline{G} \bullet \underline{\mathbf{I}} \} = i/c\ \mathbf{I}_0\, d\phi/dt - (\underline{\nabla}\phi + \partial/\partial t\, \underline{A}/c) \bullet \underline{\mathbf{I}} - i\,\underline{\mathbf{I}} \bullet \underline{\nabla} \wedge \underline{A} \quad \text{where } \underline{G} \equiv \underline{E} - i\,\underline{B}$$

$$\mathbf{F} \equiv \rho\,\mathbf{G} = \{ i\,F_0\,\mathbf{I}_0 + \underline{F} \bullet \underline{\mathbf{I}} \} = -i\,\nabla^*\,\mathbf{U} = -i\,\nabla^*\,[\rho\,\phi\,\mathbf{V}^*]/c$$

These 4-dimensional definitions allow linkages to purely scalar or vector EM quantities (not true CNVs), such as the Coulomb energy density $\mathbf{W}$, the Searle potential Φ and the Heaviside (Lorentz) force density $\mathbf{R}$; thus:

$$\mathbf{W} \equiv \mathbf{J}^*\,\mathbf{A}/c = i\,\rho\,\Phi\,\mathbf{I}_0 \quad \text{where } \Phi = \phi - \underline{v} \bullet \underline{A}/c = (1 - v^2/c^2)\,\phi$$

$$\mathbf{R} \equiv i\,\mathbf{V}^*\,\mathbf{F}/c = \rho\,\underline{f} \bullet \underline{\mathbf{I}} \quad \text{where } \underline{f} = \underline{E} + \underline{v} \wedge \underline{B}/c \quad \therefore\ c\,\mathbf{R} = -d\mathbf{U}/dt$$

If the scalar potential is assumed to be spherically symmetric, then: $-\square\,\phi = 4\,\pi\,\rho$ where $\square = -(\underline{\nabla} \bullet \underline{\nabla} - 1/c^2\,\partial^2/\partial t^2)$

This is equivalent to Maxwell's (inhomogeneous) Equations: $i\,\nabla\mathbf{G} = 4\,\pi\,\mathbf{J}/c$ or $\square\,\mathbf{A} = -4\,\pi\,\mathbf{J}/c$

This latter form has the retarded solutions: $\mathbf{A}[t; \underline{x}] = \iiint_{all} d^3\underline{x}'\ \mathbf{J}[t - r/c\ ;\ \underline{x}']/c\,r$

Since the velocity CNV is a Voigt vector then: $d/dt\ \mathbf{V}^* = 0$ and resetting the baseline (or constant value ϕ_0) for these two potentials was shown to be equivalent to the Lorenz gauge condition: $\underline{\nabla} \bullet \underline{A} + 1/c\ \partial\phi/\partial t = 0$ leading to: $d\phi/dt = 0$.

$\therefore$ $d/dt\ \mathbf{A} = 0$ so that $\mathbf{A}$ is a Flow vector with the property that: $\square\,\mathbf{A} = 0$ ('EM waves') wherever $\mathbf{J} = 0$.

When the charge-density is invariant: $d/dt\ \rho = 0$ then so is the CNV current; i.e. $d/dt\ \mathbf{J} = 0$ leading to: $\square\,\mathbf{G} = 0$

Interpreting the CNV Mathematical Model

As paper II concluded, there are several surprising results arising from the mathematics of the CNV EM model, not the least of which are that the CNV potential and CNV current are invariant in time; i.e. $d/dt\ \mathbf{A} = 0$ and $d/dt\ \mathbf{J} = 0$. Furthermore, the equivalent condition (namely: $\mathbf{V}^*\mathbf{G} = 0$) leads to four component ("EM Wave-Solution") equations:

1) $\underline{v} \bullet \underline{E} = 0$ 2) $\underline{v} \bullet \underline{B} = 0$ 3) $c\,\underline{E} + \underline{v} \wedge \underline{B} = 0$ 4) $c\,\underline{B} = \underline{v} \wedge \underline{E}$

This set not only shows that $\underline{E}$ and $\underline{B}$ are mutually orthogonal and orthogonal to $\underline{v}$ but the only non-trivial solution is $v = c$.

The solution to these apparent paradoxes is that these are **not** partial differential equations about EM variations **at** the single field point $\{t\ ;\ \underline{x}\}$ but that they are average relationships **between** the statistical EM target set (maybe, just one electron) that is centered at $\{t\ ;\ \underline{x}\}$ and the statistical EM sources centered at $\{t'\ ;\ \underline{x}'\}$. This is reflected in the CNV separation $\mathbf{X}$, which is separable in both the space and time differences, $\underline{X}$ and T.

$$\mathbf{X}[t, t'; \underline{x}, \underline{x}'] \equiv \{ i\,c\,(t - t')\mathbf{I}_0 + (\underline{x} - \underline{x}') \bullet \underline{\mathbf{I}} \} = \mathbf{X}[t - t'; \underline{x} - \underline{x}'] = \mathbf{X}[T; \underline{X}] \quad \text{where } T = t - t' \ \&\ \underline{X} = \underline{x} - \underline{x}'$$

$$\therefore\ \mathbf{X}[t - t'; \underline{x} - \underline{x}'] = \mathbf{X}[t\ ;\ \underline{x}] - \mathbf{X}'[t'; \underline{x}']$$

Viewed in this light: $\mathbf{V}[t, t'; \underline{x}, \underline{x}'] \equiv d\mathbf{X}[T; \underline{X}]/dT = \{ i\,c\,\mathbf{I}_0 + \underline{V} \bullet \underline{\mathbf{I}} \}$ where $\underline{V} = d\,\underline{X}/dT$

So that: $v = c$ is actually the CNV identity: $\mathbf{V} = \mathbf{C} = c\,\{ i\,\mathbf{I}_0 + \underline{e} \bullet \underline{\mathbf{I}} \} = \{ i\,c\,\mathbf{I}_0 + \underline{C} \bullet \underline{\mathbf{I}} \}$ where $\underline{e} = (\underline{x} - \underline{x}')/X$

In other words, $\underline{C}$ is the 'light-vector' that connects the source to the target as a direct vector; i.e. $\underline{C} \bullet \underline{C} = (\underline{X} \bullet \underline{X})/T^2 = c^2$

This is the traditional 'light-cone' when the point {t ; $\underline{x}$} is allowed to vary spherically around {t′ ; $\underline{x}'$}; i.e. $\mathbf{C}^*\mathbf{C} = 0$

The idea of separability of the separation CNV $\mathbf{X}$ and its two-time difference differential (or velocity CNV $\mathbf{V}$) was briefly mentioned in paper II (§7.7) but was more thoroughly investigated here in part §5.2 of this paper, which focused on the interaction between only two electrons acting as point charged-particles. When the CNV variable is a two-particle Voigt vector, like $\mathbf{V}$:

$$\mathbf{V}[t - t'; \underline{x} - \underline{x}'] = \mathbf{V}[t ; \underline{x}] + \mathbf{V}'[t'; \underline{x}']$$

Whenever a CNV variable $\mathbf{Q}_0$ is both separable and (two-time) temporally invariant (i.e. d/dT $\mathbf{Q}_0[T; \underline{X}] = 0$) then its local derivative is exchangeable **between the two electrons** so that its total value before the exchange is equal to the total value after the exchange (i.e. after the interaction is complete). For the important Voigt vectors, like current $\mathbf{J}$ and potential $\mathbf{A}$:

$$\text{Since: } d\mathbf{J}/dT = 0 \quad \text{then:} \quad \mathbf{J}[T; \underline{X}] = \mathbf{J}[t ; \underline{x}] + \mathbf{J}'[t'; \underline{x}'] = \mathbf{J}_0 \quad \therefore \; d/dt\, \mathbf{J}[t ; \underline{x}] = - d/dt'\, \mathbf{J}'[t'; \underline{x}']$$

This is the EM asynchronous action-at-a-distance physics behind the 4-vector particle invariants that are more popularly known as relativistic mechanics, which relied on Planck's arbitrary (and mysterious) relativistic force mechanism. In the present mesoscopic theory, these are extended to the following two location (source cell and target set) CNV variables:

1) Velocity $\mathbf{V}^*$ 2) Momentum $\mathbf{P} = m\mathbf{V}^*$ 3) Current Density $\mathbf{J} = \rho\mathbf{V}^*$

4) EM Potential $\mathbf{A} = \phi\mathbf{V}^* / c$ 4) EM Energy $\mathbf{U} = \rho\mathbf{A} = \phi\mathbf{J} / c$ 5) Mechanical Energy $\mathbf{K} = \mathbf{PV}$

6) EM Force Density $\mathbf{G} = -i\nabla^*\mathbf{A}$ 7) Rotational Momentum $\mathbf{L} = \mathbf{X}^*\mathbf{P} - \mathbf{P}^*\mathbf{X}$

The most significant conserved scalar EM quantities are: ρ, ϕ, U_0 ($\rho\phi$), $\mathcal{M}_0$ ($\rho\phi/c^2$) and $\underline{\mathcal{P}}_0$ ($\mathcal{M}_0\,\underline{c}$).

These results re-interpret the Maxwellian EM field theory or "EM Wave" equations, with $\mathbf{G}_0 = 0$ or $\mathbf{G}_1 = -\mathbf{G}_2$

1) $\underline{e} \bullet \underline{E}_1 = 0$ 2) $\underline{e} \bullet \underline{B}_1 = 0$ 3) $\underline{E}_1 + \underline{e} \wedge \underline{B}_1 = 0$ 4) $\underline{B}_1 = \underline{e} \wedge \underline{E}_1$

It must be emphasized (see paper I) that it is the unexpected properties of Voigt Vectors ($\mathfrak{V}[t; \underline{x}] \equiv \alpha[t; \underline{x}]\mathbf{V}^*[t; \underline{x}]$) that make them so suitable for representing the asynchronous interactions between charged point particles. In particular, the ability of the Voigt Vector parameter α to satisfy the so-called "Zero Condition" allows its total time derivative to equate to the "flow gradient" (i.e. d/dt $\mathfrak{V} = -\mathbf{V}^*\nabla^*\,\mathfrak{V}$); it is this key property that results in temporal invariance of important two-electrons physical quantities. The definitions of the CNV EM quantities $\mathbf{A}$, $\mathbf{J}$ and $\mathbf{G}$ show that they are all simple Voigt Vectors but this is not true for the representation of the EM Energy Density CNV $\mathbf{U}$ where the velocity multiplier α is now not a single function but a product of two functions of the space and time parameters (i.e. $\alpha[t; \underline{x}] = \rho[t; \underline{x}]\,\phi[t; \underline{x}]$) and this <u>product</u> does not satisfy the Zero Condition nor does it permit a well-defined limit process in its physical definition as each component needs to its own separate limit function, as its surrounding space and time regions shrink to zero. Furthermore, problems arise with the two ancillary 'classical' quantities representing the Coulomb energy CNV $\mathbf{W}$ and Heaviside force density $\mathbf{R}$; their explicit expansion shows that these are truncated CNVs without the full components expected of CNVs.

7.7.7 CONTEXTUAL INTERACTIONS

Overview

The CNV EM mathematical model would appear to describe a world where many independent pairs of electrons (some in the source interacting with an equal number in the target set) are simply aggregated (**added**) together. This view totally ignores all the interactions between the charges **within** each set. In CEM, every charged particle interacts continuously with every other charged particle in the universe – this is *interaction over-kill*, a frenetic level of interaction that would blow apart the electric charge density everywhere and instantaneously. This is obviously not the case in the real world and ignores the experimental roots of CEM, which were constructed around steady and variable electrical currents in finite metallic circuits.

Implicit in the potential approach is the idea of a single time that spans all space. This was vital to classical physics as it simplified all the continuum mathematics first invented by Newton. It was fortunate that the gravitational force is so weak that the rotational periods of planetary-sized bodies are long enough that the approximation of instantaneous forces was sufficiently accurate to allow for agreements between the new mathematical predictions and the observed measurements. This major approximation will not work for EM dynamics where the electrical forces are vastly stronger (10^{40}) than comparable gravitational forces but it is adequate for so-called electro-static and magneto-static situations. It is also sufficient for 'steady' currents where single-time averaging works quite well. It is **never** appropriate in so-called 'radiation' phenomena (such as light) where retarded (and advanced) solutions are required for the wave equations with finite (c) velocities. It is also this domain that led to the development of special relativity that **chose** to retain the single-time perspective instead of the simpler, physical but not mathematical conception of interactions between particles at two distinct times, as proposed here.

Average Charge Density

Unlike CEM, the present definitions remain finite with several thousand electrons aggregated into a mesoscopic cell, where both intra and inter-cellular distances remain finite, avoiding the infinite repulsion that hides behind the definition of charge density $\rho[t; \underline{x}]$ that is continuous in CEM. Furthermore, the mesoscopic cells also **explicitly** contain positive ions forming the lattice so that this collectivity is genuinely electrically neutral without having to invoke additional, mysterious forces to counteract the powerful, repulsive Coulomb forces between the always electrically negative 'electric fluid'. In the present theory, charge density is a discrete quantity whose time-average ρ_k is defined at the center of each cell $\{k\}$.

$$< \rho[t; \underline{x}_k] > = \rho_k = e\,\Delta n[t] \quad \& \quad < \underline{\mathcal{J}}[t; \underline{x}_k] > = \underline{\mathcal{J}}_k = e\,\underline{\mathcal{U}}_L\,\Delta n[t] = \rho_k\,\underline{\mathcal{U}}_L \qquad \text{while} \quad \Delta x_k = \Delta x_0 \neq 0$$

$$\text{For time-averages:} \qquad < d/dt\ \rho[t; \underline{x}_k] > \approx < d/dt < \rho[t; \underline{x}_k] >> \approx d/dt < \rho[t; \underline{x}_k] >$$

The present theory of EM is ultimately a microscopic model of interactions between **pairs** of electrons. The mesoscopic model allows these individual interactions to be statistically aggregated to a level where the net density of mobile electrons and ions in a metal can be approximated by the central continuous function of space and time: the electric charge density ρ. The unit-charge here is always the electronic charge e; while at the final continuum limit ($\Delta n \rightarrow 1$), this is incompressible.

Mechanism of the External Conduction Interaction (XCI)

However, the essence of the **e**xternal **c**onduction **i**nteraction (**XCI**) is that the nature of the target-set determines **how** the source-set of charges interact with the target-set. The interaction perspective does not propose that the source spontaneously changes momentum and later other material objects (the targets) just happen to complete the interaction. This separation of the interaction into a remote force (at the targets) and a local reaction (at the source) has been the approach used ever since it was introduced by Newton. Even in the present theory, the division of the unitary interaction is still (mathematically) split into an emitted impulse at the source and an impulse absorbed later at the receiver. However, this is only for computational purposes, the **interaction is unitary** and the emission is fully determined by the eventual absorption. Although this model is deterministic (at the individual interaction level), it is **not** determinable (ever) by human beings: the initial macroscopic configurations can never be exactly reset by humans – only approximately so, hence it is a mesoscopic statistical theory. When an electron (or ion) in the source conductor 'chooses' to interact with a remote electron (or ion), this action is at the expense of an action with a local electron (or ion) somewhere else in the source conductor. When a source electron (or ion) loses (or gains) a quantum of momentum ($\Delta p_0 = m\,b$) locally, this is gained (or lost) by the remote charge involved. When this source charge is a mobile electron, the lost (gained) momentum is reflected in a decrease (increase) in part of the source electron current (electromotive force).

When the interaction source charge is a lattice ion, the lost (gained) momentum is reflected in a decrease (increase) in the overall momentum of the source conductor – if free to move (ponderomotive force) or a change in the internal momentum of the lattice vibrations (phonons) when the conductor is fixed (see §7.8.4 later).

When electric current in a fixed wire is viewed in terms of electrons as particles then the current at any point is simply the net number of mobile electrons per second crossing a single layer of cells in the wire (at that distance along the circuit). If the current is steady then this number is very nearly constant over macroscopic time-scales. However, when a change in the external EM source is introduced this then produces a change in the mobile electron density ρ and/or the speed $\mathcal{U}$ that these mobile electrons are 'drifting' through the wire. In effect, this is a change in the mechanical momentum $\underline{P}$ of these electrons as they move along the wire. This is reduced by the need to also change the motion of all the mobile electrons in the wire that are ultimately linked to the set of source electrons in the accelerated mesoscopic layer. This linkage is affected by the change in the internal EM momentum or vector potential (i.e. $\underline{A}'_{int} / c$). So, in a mesoscopic cell at a distance ζ′ along a wire of cross-section S:

$$\underline{A}'_{int}[t'; \zeta'] = \int d\zeta'' \, \underline{I}[t'' ; \zeta''] / c\, r \quad \text{where } r = \zeta' - \zeta'' \quad \& \quad t'' = t' - r / c \quad \& \quad \min[r] = \Delta x_0$$

However: $$\underline{\mathcal{J}}' = \rho' \underline{\mathcal{U}}' = e \Delta N' \underline{\mathcal{U}}' = (e / m)\, m \Delta N' \underline{\mathcal{U}}' = (e / m)\, \underline{P}' \quad \therefore \quad d\underline{\mathcal{J}}' / dt' = (e / m)\, d\underline{P}' / dt'$$

Therefore, the remote conservation of total current (XCI) is simply the remote conservation of mechanical momentum between the charges in the source mesoscopic cell and the charges in the (remote) target set as they interact asynchronously.

$$\therefore \quad d/dt\, \mathbf{P}[t ; \underline{x}] = - d/dt'\, \mathbf{P}'[t'; \underline{x}'] \quad \Rightarrow \quad d/dt\, \boldsymbol{\mathcal{J}}[t ; \underline{x}] = - d/dt'\, \boldsymbol{\mathcal{J}}'[t'; \underline{x}']$$

In the XCI, the remote charges experience a net impulse from net changes that occur in the source situation; even when the source current is steady there are still changes occurring in the source conductor but they are representative of changes that are occurring throughout the source conductor. Only if these 'global changes' undergo localized changes in time does the XCI occur with remote charges. In other words, every electric charge distribution always involves the movement of point electric charges. In 'free' space, electrons will always repel each other, thereby altering each other's mutual motion. Within metallic conductors, 'local' electrons move around their own nearby ions and, on average, therefore result in a small net positive electric charge at the long-term average location of each ion. The mobile electrons move between these ionic centers and exhibit no net charge or no net direction at the mesoscopic level when they are not influenced by external charges. If this 'random' movement of mobile electrons is subject to a net external influence then just a **few** of these electrons react with the external charges instead of with all the internal charges they had been interacting. This means that some of these electrons change their relative locations within the original local charge density distribution in a mesoscopic cell (i.e. ∂ρ/∂t) and/or they move differently between mesoscopic cells contributing to a net change in the average current density ($\partial\underline{\mathcal{J}}/\partial t$). The fact that electrons are real, material particles means that when they are in motion their Newtonian inertia will overwhelmingly determine where they will move in the next small time-interval (in contrast with the arbitrary displacement of a field point). This 'inertial' property was also implicit in the Helmholtzian fluid model that underlies the mathematics of modern CEM.

Changes in Average Vector Potentials

CEM and the CNV EM theory will be examined first as both are continuum mathematical models. The interpretation of the CNV model above shows that the unitary interaction can be analyzed in terms of its two components at the source (S) and target (T) rather than from the singular perspective of the field point alone. Thus, the total, two-time potential CNV $\mathbf{A}$ can be viewed at the target at time t and the source at time t′; following the conventional approach, the time-at-target is seen as occurring later than the time-at-source. This means that the target potential $\mathbf{A}^-$ is the retarded EM response to changes at the source at the earlier time t′. However, since this interaction is unitary between the target and source then the source potential $\mathbf{A}^+$ is the advanced reaction at the source to the necessary future changes at the target at the later time t.

$$\mathbf{A}[t - t'; \underline{x} - \underline{x}'] \equiv \mathbf{A}^-[t ; \underline{x}] + \mathbf{A}^+[t'; \underline{x}'] \qquad \text{with } r = |\, \underline{x} - \underline{x}' \,| = c\, |\, t - t' \,|$$

Where:

$$\mathbf{A}^-[t ; \underline{x}] = \iiint d^3\underline{x}'\, \boldsymbol{\mathcal{J}}'[t - r / c ; \underline{x}'] / c\, r \quad \text{and} \quad \mathbf{A}^+[t'; \underline{x}'] = \iiint d^3\underline{x}\, \boldsymbol{\mathcal{J}}[t' + r / c ; \underline{x}] / c\, r$$

This unitary view illustrates why Feynman and Wheeler had to introduce symmetric combinations of half-retarded and half-advanced fields into their classical theory of EM [116]. The difference here is that there are no self-interactions as r is finite.

$$\therefore \quad \mathbf{G}[t - t'; \underline{x} - \underline{x}'] = \mathbf{G}^{-}[t ; \underline{x}] + \mathbf{G}^{+}[t'; \underline{x}'] \qquad \text{since} \quad \mathbf{G} \equiv -i \nabla^{*} \mathbf{A}$$

then:

$$\mathbf{G}^{-}[t ; \underline{x}] = -i \nabla^{*} \iiint d^3\underline{x}' \, \mathbf{J}'[t - r / c ; \underline{x}'] / c\, r \quad \text{and} \quad \mathbf{G}^{+}[t'; \underline{x}'] = -i \nabla'^{*} \iiint d^3\underline{x} \, \mathbf{J}[t' + r / c ; \underline{x}] / c\, r$$

Since EM energy and momentum $\mathbf{U}$ must also be conserved across this unitary interaction, while $d\phi/dt = 0$ & $\underline{\nabla}\phi = 0$

$$\therefore \quad \underline{F}_m[t] = -\, d/dt \, [\rho[t] \iiint dz' \, \underline{J}'[t - r / c ; z'] / c^2 r \,] = -\, \underline{F}'_m[t'] = d/dt'[\rho'[t'] \iiint dz \, \underline{J}[t' + r / c ; z'] / c^2 r \,]$$

The heart of the CEM problem (and the CNV model) is the widespread failure to realize that all these 'point' equations are not instantaneous relationships but that they are 'long-term' statistical time averages involving very many electric charges. Unfortunately, these instantaneous relationships have then been assumed to be the basis for the fundamental EM interaction. It is only by invoking a restriction principle, like the Saturation Hypothesis, that limits each fundamental EM interaction to a pair-wise exchange can energy and momentum be fully conserved at the (delayed) completion of the exchange. Thus, the average CNV potential in the target mesoscopic cell #j is the result of a series of micro-interactions $\Delta\mathbf{A}^{-}_k[t'' ; \underline{x}_j]$, each with a **single** electron (or ion) in the source mesoscopic cell #k.

$$\therefore \quad <\mathbf{A}^{-}[t ; \underline{x}_j]> \equiv 1/N \sum_k^N \Delta\mathbf{A}^{-}_k[t'' ; \underline{x}_j] \quad \text{where } t'' \text{ is defined 'around' t: i.e. } t - N\tau/2 < t'' < t + N\tau/2$$

$$\text{where:} \qquad \Delta\mathbf{A}^{-}_k[t'' ; \underline{x}_j] = e\, \mathbf{V}'^{*}_k / c\, r_{jk} \qquad \text{and} \qquad r_{jk} = |\, \underline{x}_j - \underline{x}_k' \,|$$

Single Target Electron

This section will anticipate the introduction in the next paper of the Discrete Temporal Hypothesis where each electron does not interact continuously over time (the classical hypothesis) but may only interact with another electron on a periodic time cycle, characterized by a new universal, fundamental constant: the quantum of time or **chronon**, designated by the symbol τ.

The simplest situation to analyze occurs when the target-set consists only of a single ('free') electron as **all** the remote source impulses are absorbed by this one particle. Consider a single electron at a distance R from the center of a small, straight wire (length 2L) whose orientation defines the z-direction and its center define the spatial origin. At time zero, a source of EMF is connected to both ends of this wire that injects $\Delta N'$ extra electrons into the wire at z = + L over a time duration of ΔT_0. This appears like a pulse of current that travels down the wire at a speed $\mathcal{U}$ according to the impulse conduction model described in §7.4.2. The pulse will complete its journey across the wire in a time duration T_L ($2L/\mathcal{U}$), which will be much longer than ΔT_0. The target interaction duration ΔT must be comparable to T_L so that the effects from interactions with many locations spanning the whole wire can be averaged together, so that it seems reasonable to set them equal; i.e. $\Delta T = 2L/\mathcal{U}$. In contrast to the isolated conductor, where the accelerated mobile electrons only interact with the lattice ions and other mobile electrons, now at each chronon cycle τ <u>one</u> of the extra 'pulse' electrons interacts with the remote electron, which gains Δp_0 in momentum in a direction <u>parallel</u> to the wire. This is because this is an XCI interaction and not a Coulomb repulsion as the positive ions in the conductor 'mask' (or compensate) for the Coulomb forces from the 'pulse' electrons. This means, to conserve linear momentum, the pulse electron 'failed' to contribute to the acceleration of its cohort of mobile electrons, so their actual drift speed is slightly less than $\mathcal{U}$ by a tiny amount $\Delta\mathcal{U}$; since $\mathcal{U} >> \Delta\mathcal{U}$ then T_L is effectively unchanged.

The total change in momentum per unit charge, induced in each electron across one cell in layer #*l* by all the electrons in the previous layer during the acceleration period $\frac{1}{2}\Delta t_0$ seconds is the definition of Maxwell's electro-kinetic momentum $\Delta A_l / c$, where $\Delta A[t; \underline{x}_l]$ is the change in the vector potential between the centers of two cells in adjacent layers in the longitudinal direction. In order to simplify the calculation, this will be assumed constant across the whole wire; so, the net change in EM momentum of <u>all</u> the accelerated mobile electrons originating with mobile electrons in the previous layer during one half-cycle is ΔP_l.

$$\therefore \quad \Delta P_l = Q_l \, \Delta A_l / c \qquad \therefore \quad 2\, m\mathcal{U}\, \Delta N' = e\, \Delta N'\, \Delta A_l / c \qquad \therefore \quad e\, \Delta A_l = 2\, m\mathcal{U}\, c$$

The total change in the electro-kinetic vector across the whole wire (without the external electron) is $A = (2L / \Delta x_0)\, \Delta A_l$, where Δx_0 is the linear dimension of a mesoscopic cell. When the external electron is present and interacting with the wire during this acceleration period then the total reduced change in the electro-kinetic vector across the whole wire will be A′.

$$\Delta A = A' - A = 4\, m\, c\, L\, \Delta \mathcal{U} / e\, \Delta x_0$$

During each acceleration period (of duration $\frac{1}{2}\Delta t_0$), as the pulse passes through any one mesoscopic cell of size Δx_0 the external electron will participate in η interactions with mobile electrons in this cell, each persisting over one chronon τ.

$$\therefore \quad \tfrac{1}{2}\, \Delta t_0 = \eta\, \tau \quad \text{and} \quad \mathcal{U} = \Delta x_0 / \Delta t_0 \quad \therefore \quad \eta\, \tau = \Delta x_0 / 2\, \mathcal{U}$$

This series of η impulses during each acceleration period is equivalent to a constant force F acting across this period and causing an increase in the external electrons speed by $\Delta v = \eta\, b$. $\therefore$ $(\frac{1}{2}\Delta t_0)\, F = \eta\, \Delta p_0 = \eta\, m\, b$ $\therefore$ $F = m\, b / \tau$

This is just the XCI force density E_M that was introduced above, acting on the electron. $F = -\, e\, E_M = (e/c)\, \Delta A / (\frac{1}{2}\, \Delta t_0)$

$$\therefore \quad \eta\, m\, b\, c = e\, \Delta A = 4\, m\, c\, L\, \Delta \mathcal{U} / \Delta x_0 = 2\, m\, c\, L\, \Delta \mathcal{U} / (\mathcal{U}\, \eta\, \tau) \quad \therefore \quad \Delta \mathcal{U} = \eta^2\, (\tau / \Delta T)\, b = \eta\, (\tau / \Delta T)\, \Delta v$$

This defines the quantum of velocity-change b in terms of the change in conduction drift speed $\Delta \mathcal{U}$: $b = 4L\, \Delta \mathcal{U} / (\eta\, \Delta x_0)$
Now $N = L / \Delta x_0$ the total number of (linear) cells along the length of the wire, so during the full time ΔT of all the remote interactions between the external electron and all the cells in the wire the electron increases its speed from zero to $V = N\, \Delta v$.

$$\therefore \quad V = 4\, N^2\, \Delta \mathcal{U}$$

Since typical drift velocities in metals ($\mathcal{U}$) are about 3 cm/sec and the mesoscopic length $\Delta x_0 \approx 10^{-6}$ cm then the number of interactions per linear cell length η is approximately 10^{17} but this originates from a finite width w of wire of about 1 mm diameter so this would consist of about 10^5 transverse cells across the surface, so each cell would contribute about 10^{12} interactions as a pulse moves across it.

7.8 NEAR CONDUCTION INTERACTIONS

This section will now examine the External Conduction Interaction (XCI) model in greater detail. The focus here will be on those situations involving a limited number of macroscopic conductors that are relatively close to each other: that is, their spatial separations are comparable to the macroscopic dimensions of the conductors themselves. Before presenting detailed models of EM forces **in** conductors and EM forces **on** conductors, this section will first review a mathematical approximation that is used extensively in many EM models, including the mesoscopic model used here. This approximation is simply a geometric approximation involving very acute triangles, where one side of the triangle is very much smaller than the other two sides. It is useful in physics wherever there are two sources of activity that are close together compared to the distant point where their combined effects are being evaluated. Jefimenko has provided a very detailed exposition of the effects of retardation on moving "packets" of electric charge [127]. He demonstrates how the varying distances from differing parts of the moving package appear to distort the shape of the retarded package and therefore the effective amount of charge that is seen by a fixed observer (the "field point"). When charges at only two source points are considered this is referred to as the *dipole model* due to the analogies with electric and magnetic dipoles.

7.8.1 THE DIPOLE APPROXIMATION

Consider two source points Q_1 and Q_2 at $\underline{x}_1$ and $\underline{x}_2$ respectively in a conductor occupied by charges at times t_1 and t_2. Let the interactions with these two charges occur at a remote (target) point P at $\underline{x}$ at time t (by convention: $t_1 < t_2 < t$).

Define: $\underline{r}_1 \equiv \underline{x} - \underline{x}_1$ and $\underline{r}_2 \equiv \underline{x} - \underline{x}_2$; the mid-point Q at $< \underline{x}' >$ is defined between Q_1 and Q_2; $< \underline{x}' > \equiv (\underline{x}_1 + \underline{x}_2)/2$

Introducing two other vectors $\underline{QP}$ and $\underline{Q_1Q_2}$: $\underline{r} \equiv \underline{x} - < \underline{x}' >$ and $2\,\underline{L} \equiv \underline{x}_2 - \underline{x}_1$. The objective is to calculate: $(r_1 - r_2)$

The angle θ defines the angle made between the two source points and the line between the midpoint Q and the target P.

$$\therefore \quad \underline{r} \bullet \underline{L} = r\,L\cos\theta \qquad \text{The } \textbf{Dipole Approximation} \text{ occurs whenever: } r >> L.$$

Let H_1 be defined as the exterior perpendicular point on the vector $\underline{PQ}$ from the point Q_1 and H_2 be defined as the interior perpendicular point on the vector $\underline{PQ}$ from the point Q_2. This permits two new lengths g and h to be defined.

$$\text{Define: } g = Q_1H_1 = Q_2H_2 = L\sin\theta \quad \& \quad h = QH_1 = QH_2 = L\cos\theta \quad \therefore\ r >> g\ \&\ r >> h$$

$$\therefore\ r_1^2 = (r+h)^2 + g^2 = r^2 + g^2 + h^2 + 2\,r\,h = r^2 + L^2 + 2\,r\,h \quad \therefore\ r_1 = r\,\{1 + (L^2 + 2\,r\,h)/r^2\}^{½} \approx r + (L^2 + 2\,r\,h)/2r$$

$$\therefore\ r_2^2 = (r-h)^2 + g^2 = r^2 + g^2 + h^2 - 2\,r\,h = r^2 + L^2 - 2\,r\,h \quad \therefore\ r_2 = r\,\{1 + (L^2 - 2\,r\,h)/r^2\}^{½} \approx r + (L^2 - 2\,r\,h)/2r$$

$$\therefore\ r_1 - r_2 \approx 2\,h \qquad \therefore\ r_1 - r_2 \approx 2L\cos\theta \quad \text{when } r >> L$$

This is the dipole approximation: the difference in distance between any two fixed points and a third point is independent of the distance from their mid-point r and only depends on the distance between the two points L and the cosine of the angle θ to the third point when their average distance to the third point is much greater than their separation. Although the three points (Q_1, Q_2 and P) define a common 2D plane, this result is still valid when the source points (Q_1, Q_2) are displaced out of this plane into the full 3D space by an equal amount αL but in <u>opposite</u> directions, as long as this extra displacement is comparable to the original separation ($\alpha \approx 1$). Let the center point Q define the 3D origin and now let Q_2Q_1 define the y-axis and the target point P be set in the x-y plane; i.e. Q_1 is located at {0, L, 0} and Q_2 is located at {0, –L, 0}, so that P is located at {r sin θ, –r cos θ, 0}. The two new displacements define the 3D source points Q'_1 and Q'_2, which are located at {0, L, αL} and {0, –L, –αL}. The two new sources are separated in space by a distance: $L' = L\sqrt{(1+\alpha^2)}$. The point P remains in the x-y plane so that the two new distances from the 3D source points are r'_1 and r'_2 respectively, so that the new difference: $r'_1 - r'_2 \approx 2L'\cos\theta$. This result will prove useful when considering circular ('magnetic') dipoles.
In terms of the distance R from a fixed Q_2 and the angle Θ defined at Q_2 (instead of θ at Q): $\underline{R} \bullet \underline{L} = R\,L\cos\Theta$

$$\therefore\ g = 2L\sin\Theta\ \&\ h = 2L\cos\Theta \quad \therefore\ r_1^2 = R^2 + 4L^2 + 2Rh \quad \therefore\ r_1 - R \approx 2L\cos\Theta \quad \text{when } R >> L$$

$$r_1 = c\,(t - t_1)\ \&\ r_2 = c\,(t - t_2)\ \therefore\ t = t_1 + r_1/c = t_2 + r_2/c\ \ \therefore\ \Delta t = t_2 - t_1 = (r_1 - r_2)/c\ \ \therefore\ \Delta t \approx 2L\cos\theta/c$$

Single Source Charge

Consider a single electron that moves directly from Q_1 to Q_2 (a distance $2L = \Delta x$) in a time Δt with a speed $v = \Delta x / \Delta t$.

$$\therefore \quad v \approx 2L\, c / (2L \cos\theta) \qquad \therefore \quad v \approx c / \cos\theta$$

When $\theta = 0$ (longitudinal) then $v = c$ but if $\theta > 0$ then $v > c$ including $\theta = \pi/2$ when $\Delta t = 0$. This means that a single, mobile electron in the source conductor **cannot** interact twice consecutively with the same remote target electron except in the bizarre circumstance when the remote charge is in a direct longitudinal direction with the source current direction that is also moving at light-speed. This is because *c* is defined as the maximum speed between two interacting electrons.

This illustrates the need for a **set** of source charges interacting with a single target electron or if the target set consists of many charges then it is still possible for a single source charge to interact repeatedly with **different** charges in the target set.

2D Electric Dipole

When two opposite charges retain their relative position and interact with a third remote charge they form an electrostatic dipole; this takes on an interesting perspective when retardation is introduced (Coulomb forces are never instantaneous). Consider a single negative charge ($-e$) at Q_1 and a single positive charge ($+e$) at Q_2 separated by a distance $l = 2L$. The standard approach in EM is to invoke the Superposition Principle [128] where all simultaneous effects are simply additive. The electrostatic (Coulomb) potential from a single charge q at a separation r is $\phi[q, r] = -q / r$. Thus, the electrostatic field $\underline{E}_S$ and the E/S potential due to the two charges in the dipole (when $r >> l$) is $\phi[P]$:

$$\phi[P] = \phi[-e, r_1] + \phi[+e, r_2] = e / r_1 - e / r_2 = e\,(r_2 - r_1) / r_1 r_2 \qquad \therefore \quad \phi[r, \theta] \approx -e\, l \cos\theta / r^2 = -e\, \underline{r} \bullet \underline{l} / r^3$$

$$\underline{E}_S[r, \theta] = -\underline{\nabla}\, \phi[r, \theta] \approx e\, l\, \{\hat{\underline{e}}_r\, \partial/\partial r + \hat{\underline{e}}_\theta / r\ \partial/\partial\theta\}[\cos\theta / r^2] \qquad \therefore \quad \underline{E}_S[r, \theta] \approx -e\, l\, \{2\cos\theta\, \hat{\underline{e}}_r + \sin\theta\, \hat{\underline{e}}_\theta\} / r^3$$

These are the standard results for the electrostatic dipole but are only valid "on average" for far distances. If the interactions from each of the two source charges are synchronized during the emission process then they must both be emitted at the same time ($t_2 = t_1$), in which case, they will only be received simultaneously at remote points that are located on circles centered at the mid-point of the dipole and oriented along their axis; i.e. if $\theta = \pi/2$ but then $\cos \pi/2 = 0$ and $\sin \pi/2 = 1$; so:

$$\underline{E}_S[r, \pi/2] \approx -e\, l\, \hat{\underline{e}}_\theta / r^3$$

Dynamic effects can be introduced into this model by allowing each charge to oscillate harmonically from one extreme to the other along a one-dimensional path, taken to be the x-axis centered at the origin. At time zero, let the negative charge (#1) be at the origin but moving along the positive x-axis (with unit vector $\hat{\underline{e}}_x$) with initial speed u_0 reaching the location $x = L$ at time $t = T_0/4$ when it stops and reverses its motion. The complementary, positive charge (#2) also starts at the origin at time zero but is first moving in the negative x-direction with the same speed u_0. Since the particles are moving backwards and forwards harmonically along the x-axis, their motion is characterized by the circular frequency $\omega = 2\pi / T_0$. The two locations are:

$$\underline{x}_1[t] = \hat{\underline{e}}_x\, L \sin\omega t \quad \& \quad \underline{x}_2[t] = -\underline{x}_1[t] = -\hat{\underline{e}}_x\, L \sin\omega t \qquad \therefore \quad \underline{l}\,[t] = \underline{x}_1[t] - \underline{x}_2[t] = \hat{\underline{e}}_x\, 2L \sin\omega t$$

Thus the varying Coulomb force experienced by an electron a 'far' distance R along the z-axis is: $\underline{E}_C[R,t] \approx e\, l\,[t]\, \hat{\underline{e}}_x / R^3$

$$\therefore \quad \underline{E}_C[R,t] = \hat{\underline{e}}_x\, E_C[R] \sin\omega t \qquad \text{where:} \quad E_C[R] \approx 2\, e\, L / R^3 \quad \text{when} \quad R >> L$$

$$\therefore \quad \underline{u}_1[t] = d/dt[\, \underline{x}_1[t]\,] = \hat{\underline{e}}_x\, u_0 \cos\omega t = -\underline{u}_2[t] \qquad \therefore \quad u_0 = \omega L = 2\pi L / T_0$$

$$\underline{I}_1[t] = -e\, \underline{u}_1[t] \quad \& \quad \underline{I}_2[t] = +e\, \underline{u}_2[t] \qquad \therefore \quad \underline{I}\,'[t] = \underline{I}_1[t] + \underline{I}_2[t] = -2\, e\, \underline{u}_1[t] = -\hat{\underline{e}}_x\, 2\, e\, u_0 \cos\omega t$$

$$\therefore \quad \underline{\mathbf{E}}_m[\underline{x},t] = -d/dt[\, \underline{A}[\underline{x}, t]\,] / c = -d/dt\, [\oint d\underline{x}'\, \underline{I}\,'[\underline{x}', t'] / c^2 r\,] \approx 2\, e\, u_0 / (c^2\, R) d/dt\, [\cos\omega\,(t - R/c)] = E_D[R, t']\, \hat{\underline{e}}_x$$

$$\therefore \quad E_D[R, t] = E_D[R] \sin\omega t \quad \text{where:} \quad E_D[R] \approx 2\, e\, L\, \omega^2 / c^2\, R \quad \text{when} \quad R >> L \quad \therefore \quad E_D[R] / E_C[R] \approx (u_0/c)^2\, (R/2\pi L)^2$$

Rotating Electric (magnetic) Dipole

Consider, next an electron rotating around a heavy ('fixed') positive ion, both interacting with a stationary, remote electron. Let the fixed ion define the origin $\underline{x}_2 = \{0, 0, 0\}$ and let the remote electron define the positive z-axis; i.e. $\underline{x} = \{0, 0, R\}$. Let the position of the moving electron at $t_1 = 0$ be $\underline{x}_1 = \{l, 0, 0\}$ and allow it to rotate around the ion in an anti-clockwise direction in a period T_0 at a constant angular speed $\omega = 2\pi/T_0$; at time t_1 the dipole axis Q_1Q_2 makes an angle $\psi[t_1] = \omega\, t_1$ with the x-axis. The projection of the dipole on the x-axis defines a length x and the projection of the dipole on the y-axis defines a length y, both varying harmonically with time. $x = l \cos\psi$ & $y = l \sin\psi$. Note: $r_2 = R$ (fixed).

Let $R_1[t_1]$ represent the distance at time t_1 between the rotating electron's location $\{x, y, 0\}$ and the remote electron at P. In the x-y plane the distance between the x-projection of the rotating electron and P is still r_1 where: $r_1 - R \approx l \cos\theta$ and θ is measured from the negative x-axis at the origin (i.e. cylindrical co-ordinates).

Since $R_1^2 = r_1^2 + y^2 \;\; \therefore \; R_1 \approx r_1 + y^2 / 2r_1 \;\; \therefore \; R_1 - R \approx l\{\cos\theta + (l/2R)\sin^2\psi\} \;\; \therefore \; R_1 - R \approx l\cos\theta$ when $R >> l$

Thus, this rotating model of the 'heavy' dipole still appears like a simple, fixed dipole at far distances, independent of time. The combined electrostatic potential of the ion and the rotating electron at the position P of the remote electron is $\phi[P]$.

$$\therefore \; \phi[P] = \phi[-e, R_1] + \phi[+e, R] = e / R_1 - e / R = e\,(R - R_1) / RR_1 \;\; \therefore \;\; \phi[R, \theta; t] \approx -e\, l \cos\theta / R^2 = e\, \underline{R} \bullet \underline{l} / R^3$$

$$\underline{E}_S[R, \theta] = -\underline{\nabla}\,\phi[R, \theta] \approx -e\, l\, \{\hat{\underline{e}}_r\, \partial/\partial r + \hat{\underline{e}}_\theta / r \;\partial/\partial\theta\}[\cos\theta / r^2] \;\; \therefore \;\; \underline{E}_S[R, \theta; t] \approx -e\, l\, \{2\cos\theta\, \hat{\underline{e}}_x + \sin\theta\, \hat{\underline{e}}_\theta\} / R^3$$

Although the source electron is now rotating around the 'fixed' positive ion, a remote electron still experiences a constant ('static') electric force, which (along the z-axis) varies with the inverse cube of the separation (z^3), directed along the x-axis. This model is explicitly dynamic as the source electron moves continuously around the 'fixed' central ion with a constant tangential speed $u_0 = \omega\, l$. The location of this electron at time t is $\underline{x}_1$ and its linear velocity is $\underline{u}_1$, where:

$$\underline{x}_1[t] = \hat{\underline{e}}_x\, l \cos\omega t + \hat{\underline{e}}_y\, l \sin\omega t \;\; \therefore \;\; \underline{u}_1[t] = d/dt[\,\underline{x}_1[t]\,] = -\hat{\underline{e}}_x\, u_0 \sin\omega t + \hat{\underline{e}}_y\, u_0 \cos\omega t$$

The source current $\underline{\mathcal{I}}'[t]$ is localized completely at the position of the rotating electron. $\therefore \; \underline{\mathcal{I}}'[t] = \underline{\mathcal{I}}_1[t] = -e\, \underline{u}_1[t]$

$$\therefore \; \underline{\mathbf{E}}_R[\underline{x}, t] = -d/dt[\,\underline{A}[\underline{x}, t]\,] / c = -d/dt\,[\oint d\underline{x}_1\, \mathcal{I}'[\underline{x}_1, t_1] / c^2 r\,] \approx e / (c^2 R)\; d/dt\,[\,\underline{u}_1[t - R/c]]$$

$$= e\, u_0 / (c^2 R)\; d/dt\,[-\hat{\underline{e}}_x \sin\omega t + \hat{\underline{e}}_y \cos\omega t\,] = -e\,\omega\, u_0 / (c^2 R)\;\{\hat{\underline{e}}_x \cos\omega t + \hat{\underline{e}}_y \sin\omega t\}$$

$$\therefore \; \underline{\mathbf{E}}_R[\underline{x}, t] = -E_D[R]\, \underline{x}_1[t - R/c] / l \qquad \text{where:} \;\; E_D[R] \approx 2\, e\, L\, \omega^2 / c^2 R \;\text{ when }\; R >> L$$

$$\therefore \; E_D[R] / E_C[R] = (\omega R / c)^2 = (u_0 / c)^2\, (R/2L)^2$$

This is very similar to the result for the one-dimensional symmetric dipole that is oscillating harmonically along its axis. Thus, in both dipole models, the dynamical effects dominate the static effects at 'radiation' distances: $R >> (u_0 / c)\, L$. This requirement can be restated in two other forms. Mathematically, whenever a harmonic spatial oscillation occurs it is always possible to define a 'wavelength' λ using the fundamental definition: $\lambda \equiv c / \nu$. This leads to: $E_D / E_C = (2\pi R / \lambda)^2$ that is much greater than unity whenever $R >> \lambda$; this is the conventional '**far-radiation**' requirement. Alternatively, one can always define the 'delay' or 'transmission' time $T_C \equiv R / c$ between the cause of the EM effects at the source and their far action at the target. This leads to: $E_D / E_C = (2\pi\, T_C / T_0)^2$ that is much greater than unity whenever $T_C >> T_0$; this is the statistical 'far-radiation' requirement, where changes at the source occur much faster than the time to create remote effects. These dipole models are purely mathematical; in reality, even if the positive charge were a heavy ion the negative charge would still be a mobile electron that would react to its interaction with the remote (target) charge – an effect that is always ignored in all field based EM theories, where only the empty field-point is considered ($q \to 0$) and momentum exchange is ignored.

Lienard-Wiechert Potentials

Consider a mesoscopic packet of electric charge of length $\Delta\zeta'$ near the location ζ' at time t' moving at an average speed $\mathcal{U}$ (over a brief time $\Delta t'$) in the longitudinal direction in a metallic conductor of uniform cross-section S that interacts over a small time Δt) with a test charge at a remote (distant external) target point O at $\{\underline{x};\, t\}$. These interactions involve **all** the excess electrons between the rear of the packet at $\underline{x}'_1$ at t'_1 and the forward edge of the packet at $\underline{x}'_2$ at t'_2. Let the distance between the target point at O and $\underline{x}'_1$ be r_1 and the distance between the target point at O and $\underline{x}'_2$ be r_2. Assume that the packet is moving towards the observation point (i.e. $r_1 > r_2$) then the interactions at the leading edge must occur later than those involving the rest of the packet, including the trailing edge, if they are all to arrive "simultaneously" at O; that is, within the small temporal arrival window Δt (i.e. $t > t'_2 > t'_1$). This results in a difference in transmission time at O of ΔT.

$$\therefore\ \Delta T = t'_2 - t'_1 = (t - t'_1) - (t - t'_2) = r_1 / c - r_2 / c = (r_1 - r_2) / c$$

During this period ΔT the front of the packet moves a distance $\Delta R = \mathcal{U}\, \Delta T$. So, when in motion, the distance $\Delta\zeta^*$ between the front of this packet and the trailing edge that are <u>both</u> interacting with the same remote target point (in space and time) is:

$$\Delta\zeta^* = \Delta\zeta' + \Delta R = \Delta\zeta' + (r_1 - r_2)\, \mathcal{U} / c$$

Whenever the target point is external to the source conductor then: $r_1 >> \Delta\zeta^*$ and $r_2 >> \Delta\zeta^*$ so the dipole approximation is appropriate.

$$\therefore\ (r_1 - r_2) \approx \Delta\zeta^* \cos\theta = \Delta\zeta^* (\underline{r} \bullet \underline{\mathcal{U}}) / r\, \mathcal{U} \qquad \therefore\ \Delta\zeta^* \approx \Delta\zeta' / \{1 - (\underline{r} \bullet \underline{\mathcal{U}}) / r\, c\}$$

In this approximation, the distance r is the separation between the target and the midpoint of the 'optical' separation $\Delta\zeta^*$. This result explicitly ignores all reactions of the source charges or acceleration of the target charge caused by all these EM interactions, which are absorbed into the average (local) velocity $\mathcal{U}$ of the source packet. This will be small unless the whole source conductor is moving at very high speeds relative to the (fixed) target point.

Thus, the region of space occupied by the charge in a moving conductor that affects the EM response at a remote target point is **different** from the region of space actually occupied (instantaneously) by the charges if they were **not** in motion. This leads to the concept of 'optical' (or retarded) volume $\Delta^3x^* = S\, \Delta\zeta^*$ compared with the stationary volume $\Delta^3x' = S\, \Delta\zeta'$.

When the electric charge is assumed to occur in the form of a uniform charge density ρ so that: $\Delta Q' = \rho\, \Delta^3x'$ then the amount of electric charge as 'seen' in the retarded volume is: $\Delta Q^* = \rho^*\, \Delta^3x^*$. This was (and is) the standard derivation of the retarded potentials from remote <u>point</u> charges in motion by allowing these spatial volumes to become infinitesimals.

$$\therefore\ \phi[\underline{x};\, t] = \Delta Q^*[\underline{x};\, t] / r \approx \Delta Q'[\underline{x}';\, t'] / r\{1 - (\underline{r} \bullet \underline{\mathcal{U}}) / r\, c\} \ \&\ \underline{A}[\underline{x};\, t] = \Delta\underline{J}^*[\underline{x};\, t] / cr \approx \underline{\mathcal{U}}\, \Delta Q'[\underline{x}';\, t'] / cr\{1 - (\underline{r} \bullet \underline{\mathcal{U}}) / r\, c\}$$

These are the Lienard-Wiechert (L-W) potentials that were introduced into EM theory in 1898 and 1900 respectively. This approach to the L-W potentials has been acutely criticized by Jefimenko in his major text on retardation in EM [129], he also emphasizes the approximations involved in the definitions of a "point charge", particularly that this concept reflects the attitude of the observer towards an electric source distribution and provides no knowledge of the structure or constitution of the source; in other words, this approximation can be applied to any **finite** charge distribution **viewed** from a distance large compared with the linear dimensions of the distribution. Jefimenko was always completely consistent in approaching CEM in terms of continuous charge distributions and always rejected the idea of actual point charges. The standard derivation of the L-W potentials begins with a **single** finite charge distribution and finally takes the limit of an infinitesimal length while ignoring the simultaneous requirement to define the independent limit process involved in the definition of the velocity of a point charge. This approach also treats the source charge as a "rigid" body - a concept that contradicts the key assumptions underlying Special Relativity Theory (SRT) even though this L-W approach is still used to introduce relativistic concepts into Lorentz-invariant formulations of CEM by acknowledging the universally invariant charge *e* of the electron; naively:

$$\Delta Q^* = \Delta Q' = e \qquad \therefore\ \rho^* = \{1 - (\underline{r} \bullet \underline{\mathcal{U}}) / r\, c\}\, \rho$$

The actual L-T also chose to transform the time intervals, so that (eventually): $\rho^\dagger = \{1 - (\underline{r} \bullet \underline{\mathcal{U}}) / r\, c\}^{1/2}\, \rho$
This approach was central to the evolution of SRT but the point of this analysis here is to emphasize that these results are only **statistically** valid when the sources involve macro conductors, **not** single, high-speed electrons.

7.8.2 MESOSCOPIC EXCHANGES

Current-Current Interactions

Interactions based on charges moving in conductors are always collective (many-to-many) dynamical responses between stable ion/electron flows through space. When the target is also a conductor, this is **not** the same situation as simply summing up the effects on a **single** target electron or test-charge (many-to-one), so the simple potential model cannot be expected to apply except as a first approximation. The discussion above showed that even when a single electron is involved, interactions involving two macroscopic sized conductors are always examples of 'far action' as mesoscopic range is limited to within a single conductor's cell; even adjacent conductors are separated by greater than mesoscopic distances.

Differential Momentum Exchanges

Jefimenko is one of the few authors of texts on CEM that emphasizes that **induction** arises from the same **conduction**-based retarded interaction that characterizes all EM forces relating to temporal changes in source currents [130], rejecting the usual (mathematical) explanation that "a changing magnetic field produces an electric field ('Faraday induction') and a changing electric field produces an magnetic field ('Maxwell induction'). Jefimenko calls Maxwell's "magnetic" part of the electric density field vector ($\underline{E}_M$) the *electro-kinetic* field to emphasize its **causative** origins in the movement of the electrical charges (i.e. source currents). He points out that $\underline{E}_M$ *is always parallel to the* ***change*** *in the (retarded) current* $\underline{J}'$ in contrast to the retarded location direction $\underline{R}$ that is a characteristic of static interactions and this type of interaction exists only as long as the source current is changing in time. Again, unlike the electrostatic force, which is always a direct attraction or repulsion between charges, the electro-kinetic force is a "dragging" force that causes electric charges to move parallel (or anti-parallel) relative to the direction of the current. The minus sign in the definition of $\underline{E}_M$ is the explanation of Lenz's law of induction where the current induced in the target conductor opposes the current generating this effect (in parallel conductors) when the inducing current is increasing and in the same direction when the source current is decreasing. This also explains why the strongest induced current is produced between parallel conductors whereas no induction takes place between conductors at right angles to each other [131]. Jefimenko also emphasizes that this type of force is always much smaller in strength than the electrostatic force on free charges because of the c^2 term in the denominator; this can only be compensated for when the source current changes extremely rapidly. The 1/r term means that when current changes are not too rapid this effect can still create observable effects in conductors especially when the conductors are parallel and close together as in a solenoid.

7.8.3 MESOSCOPIC INDUCTION

A New Induction Model

This integral view will be the model of interacting conductors that is developed next. The objective will be to integrate the global dynamics between two conductors from a macroscopic perspective with the internal motion of the electrons within each of the individual conductors. At this stage, the model will only be presented at the intermediate level (mesoscopic) where aggregation reflects stable, time-averaged behavior but even here, the effects of retardation are important to the physics. The new theory of induction between conductors presented here is based on a fusion of several key ideas from the history of electrodynamics that have been summarized so far. They include Maxwell's concept of electro-kinetic momentum, Henry's idea of remote current induction, Gauss's deep intuition of asynchronous remote action and the new theory of mesoscopic metallic conduction. This new "mix" will be related back to Ampère's original idea of mechanical forces between current elements. Faraday's changing flux requirement is just a mathematical technique that draws attention **away** from the actual locus of the moving electric charges in the source conducting circuit to the rest of the space around the conductor. Faraday's induction model also fails to calculate the induced voltage in **open** circuits where the flux linkage is undefined. This means that time variations of remote magnetic fields must be invoked to explain the action of open antennas in Maxwell's theory. The reality here is that **remote** external electrical sources must create spatial differences in the local induced forces **within** the target conductor to drive (negative) electric charges from locations of lower potential to locations of higher potential. The emphasis here will be on **motional induction**, not a theory of mathematical "magnetic" induction.

Like Weber, the present theory follows in Ampère's tradition of remote action between current elements in relative motion but now recognizes that the interaction is not instantaneous but asynchronous (on the light-cone of each pair of interacting charges) and the positive charges form a semi-rigid lattice due to their much greater inertial mass than the electrons.
The new theory will respect Henry's induction findings; namely, that the appearance of an electrical current in a closed circuit will appear when there is relative motion between it and another interacting conductor both carrying steady currents

or when there is a change in the intensity of current in this or another interacting conductor. Induction will explain EM "radiation". The new theory of induction presented here is focused both on the atomic lattice **and** on the valence electrons in conductors. It is the collective motion of **both** the ionic cores (the nuclei with their associated, tightly-bound electrons) and the loosely bound valence electrons that determines all the characteristics of metals in motion. The reason that Faraday's notion of lines of force (especially magnetic lines) works so well for his model of induction is that it implicitly brings in the interactions between **all** the charges that are moving in each remote conductor (or source of 'magnetic' effects).

The valence electrons and ions in a metallic conductor will rapidly produce the lowest energy solution (thus the smallest action). Once this stable state is rapidly achieved, the **totality** of all the valence electrons (and the ions) will resist external disturbances; any such external disturbance will then only be absorbed by achieving another, stable global minimum energy solution. An isolated conductor in its normal state will have no net charge overall or in each of its mesoscopic cells as there will be as many valence (mobile) electrons as there are net positive (quantized) charges from the ionic lattice. An isolated conductor will have no preferred direction in space so there is no net momentum for the overall conductor or even internally in its (logical) sub-systems (i.e. valence electrons or electrical current, ionic motion or phonons or in their combination or electron-phonon mix). However, when the internal components of a conductor do acquire net momentum (usually in the longitudinal direction, in the case of a wire) this momentum is much more than just the arithmetic summation of the valence electrons but now the totality of all the component sub-systems and this can be very large ("electromagnetic momentum"). Similarly, when the whole ionic system acquires a net momentum then the whole lattice must move with constant relative velocity if the conductor is to retain its overall macroscopic shape ('quasi-rigidity'); this is the source of Amperian forces.

The long-range (far) effects are due to the directed nature of the sources (changes in current or its equivalent electro-kinetic momentum – the vector potential). These effects are directional (i.e. vectorial) in contrast to the scalar potential, which is symmetrical in all spatial directions. When this EM-directed effect is combined with the electron Saturation Hypothesis, the result is a remote exchange of momentum, between electrons or conductors, without any energy loss over **all** distances. Any energy losses will be due to electron-ion 'collisions' in the normal conduction state; i.e. resistance (see §7.4.2). Like every potential (or even force), the vector potential does **not** carry energy (or momentum) between remote locations. Their spatial variation is a mathematical prediction of what might occur **if** a charge were to be found there. Real variations in the source currents will then be reflected in variations at remote distances after time delays based on 'light-speed' calculations but this does **not** mean that any physical effects are being transported across the intervening space by any form of *energy carrier*.

Conduction-Induction

Consider a homogenous metallic conductor of finite length $\mathtt{L}$ and uniform cross-section $\mathtt{S}$ where its length is much greater than its width $\mathtt{w}$ ($\mathtt{L} >> \mathtt{w}$), i.e. a "wire" connected to an external supply of electrons (each with an electric charge – *e*) that are forcibly injected into one end of the wire and collected at the far end. This movement of electrons through the wire requires electrical work $\mathcal{W}$ to be done to overcome the resistance of the existing electrons in the wire; this external work per unit charge conveyed through the wire is the electromotive force $\mathcal{E}$. When $\mathtt{N}_i$ electrons are injected into the wire per second then this corresponds to an electrical charge transfer rate or current $\mathcal{I}$.

$$\mathcal{I} = dQ/dt = -e\,\mathtt{N}_i \quad \& \quad \mathcal{W} = Q\,\mathcal{E} \quad \therefore \; d\mathcal{W}/dt = dQ/dt\,\mathcal{E} = \mathcal{I}\,\mathcal{E} \quad \therefore \; \mathcal{W}[t] = \int_0^t dt'\,\mathcal{I}[t']\,\mathcal{E}[t']$$

This analysis assumes quasi-equilibrium conditions; i.e. there is no accumulation of electrons anywhere along the wire – as many electrons emerge per second at the far end of the wire as are injected at the entry end. This analysis can be extended to time-dependent situations where there are variations both in time and in space at each point along the wire. The mesoscopic approximation is still used: this assumes that the conditions are uniform across the wire at every point ζ, measured from the injection point. In addition, it assumes that the number of excess electrons $\Delta\mathcal{N}_k[t]$ in any mesoscopic cell (labeled #k) are sufficient to produce an average smooth flow of electrons longitudinally down the wire, so the average 'drift' velocity $\mathcal{U}$ is only a function of this single longitudinal position parameter ζ and, perhaps, time. The total number of excess electrons in a unit mesoscopic cross-section of the wire (of length Δx_0 and volume $\mathtt{S}\,\Delta x_0$) centered at the position ζ is defined as:

$$\Delta\mathcal{N}[\zeta, t] = \sum\nolimits_k^* \Delta\mathcal{N}_k[t]$$

Here, $\sum_k^*$ indicates a sum over all cells k that are **in** the same unit cross-section; i.e. at the same distance ζ.

Let $\Delta Q_k[t]$ represent the excess electric charge in cell #k at time t while $\Delta Q[\zeta, t]$ represents the total excess electric charge in the unit mesoscopic cross-section at the distance ζ at time t.

$$\therefore \quad \Delta Q_k[t] = -e\,\Delta\mathcal{N}_k[t] \qquad\qquad \therefore \quad \Delta Q[\zeta, t] = \sum\nolimits_k^* \Delta Q_k[t] = -e \sum\nolimits_k^* \Delta\mathcal{N}_k[t]$$

Since these excess electrons are all drifting down the wire (under their mutual repulsion, see §7.6.2) they constitute the macro (observable) electric current $\mathcal{I}$; so, by applying dimensional analysis to a unit mesoscopic volume:

$$\therefore \quad \mathcal{I}[\zeta, t]\,\Delta x_0 = \Delta Q[\zeta, t]\,\mathcal{U}[\zeta, t] = -e\,\mathcal{U}[\zeta, t] \sum\nolimits_k^* \Delta\mathcal{N}_k[t]$$

Since each electron is also characterized by an intrinsic and invariant inertial mass m that resists its change in motion, the net motion of all these excess electrons also contributes a mass-flow current or mechanical momentum $\underline{\mathcal{P}}\,[\zeta, t]$ at each position along the wire. The total mass $\mathcal{M}$ of these $\Delta\mathcal{N}$ excess electrons at time t and at this position ζ along the wire is:

$$\mathcal{M}[\zeta, t] = m\,\Delta\mathcal{N}[\zeta, t] = m \sum\nolimits_k^* \Delta\mathcal{N}_k[t] \qquad\qquad \therefore \quad \mathcal{P}[\zeta, t] = \mathcal{M}[\zeta; t]\,\mathcal{U}[\zeta, t] = m\,\mathcal{U}[\zeta; t] \sum\nolimits_k^* \Delta\mathcal{N}_k[t]$$

$$\therefore \quad e\,\mathcal{P}[\zeta; t] = -m\,\mathcal{I}[\zeta; t]\,\Delta x_0$$

The electrostatic potential energy at any point $\{\underline{x};\, t\}$ due to **all** the excess electrons in cell #k centered at $\underline{x}_k$ is $\Delta\phi_k[\underline{x};\, t]$, where the separation vector $\underline{r} = \underline{x} - \underline{x}_k$. Statistically, this is defined by the long-time experimental average or retarded Coulomb potential. $\therefore \quad \Delta\phi_k[\underline{x};\, t] = \Delta Q_k[\underline{x};\, t_k] / r \qquad$ where $\quad t_k = t - r / c$

If the target charge $\mathtt{q}$ consists of $\mathtt{n}$ electrons ($\mathtt{n} \geq 1$), localized around the point $\underline{x}$ at time t and moving with an average velocity $\underline{w}[\underline{x};\, t]$ at that time, while the source charges in cell #k are moving with an average velocity $\underline{u}_k[t_k]$ then their average, "two-time" velocity (see §5.1.2) is $\underline{V}_{12}(\underline{x} - \underline{x}_k\,;\, t - t_k)$, where:

$$\underline{V}_{12}[\underline{X};\, T] = \tfrac{1}{2}\{\,\underline{w}[\underline{x};\, t] + \underline{u}_k[t_k]\,\} \quad \text{with:} \quad \underline{X} = \underline{x} - \underline{x}_k = \underline{r} > 0 \quad \& \quad T = t - t_k = r / c$$

This is the velocity used in the definition of the two-particle CNV velocity $\mathbf{V}_{12}$ used to define the interaction between two electrons but now extended to average interactions between two mesoscopic sets of electrons. This leads directly to the "joint" CNV electric current (see §5.2.1) $\mathbf{J}_{12}$ for the "mesoscopic interaction".

$$\mathbf{J}_{12}[\underline{X};\, T] = \mathtt{q}\,\mathbf{V}_{12}^*[\underline{X};\, T] = -\mathtt{n}\,e\,\{-i\,c\,\mathbf{I}_0 + \underline{V}_{12}[\underline{X};\, T] \bullet \underline{\mathbf{I}}\,\} = \tfrac{1}{2}\,\{\,\mathbf{J}[\underline{x};\, t] + \mathbf{J}_k[\underline{x}_k;\, t_k]\,\}$$

Where: $\qquad \mathbf{J}[t] = \mathtt{q}\,\{-i\,c\,\mathbf{I}_0 + \underline{w}[t] \bullet \underline{\mathbf{I}}\,\}$ and $\mathbf{J}_k[t_k] = \mathtt{q}\,\{-i\,c\,\mathbf{I}_0 + \underline{u}_k[t_k] \bullet \underline{\mathbf{I}}\,\}$

This reflects the Saturation Hypothesis again, where only $\mathtt{n}$ electrons in the source set interact with the target electrons. These ideas can also be extended from the two-particle CNV potential (see §5.2.2) to the two mesoscopic sets of charges.

$$c\,\mathbf{A}_{12}[\underline{X};\, T] = \Phi_{12}[\underline{X};\, T]\;\mathbf{V}_{12}^*[\underline{X};\, T]$$

As was seen earlier, the two-particle CNV potential $\mathbf{A}_{12}$ is only separable if the two-particle scalar potential Φ_{12} is either a constant ϕ_0 or a simple function of the difference in the two-electron space-time parameters, such as:

$$\phi_0 = \mathtt{n}\,e / \Lambda \qquad\qquad \text{or} \qquad\qquad \Phi_{12}[\underline{X};\, T] = \mathtt{n}\,e / R$$

Here, Λ is the classical electron radius, reflecting the closest separation that two electrons can ever achieve while R is the actual (two-time) spatial separation $R = c\,T$. It is important to emphasize that $\mathtt{n}$ is not the total number of excess electrons in the source cell unless this is smaller than the number of excess electrons in the target mesoscopic cell as the Saturation Hypothesis limits the interactions to a finite number of pair-wise interactions within the interaction (or relaxation) time Δt_0.

Since the CNV potential is separable, the target charge experiences the separable potential $\Delta\mathbf{A}$ while the source charges in cell #k experience the (reaction) potential $\Delta\mathbf{A}_k$ arising from the target charges.

$$\Delta\mathbf{A}_{12}[\underline{X};\, T] = \tfrac{1}{2}\,\{\Delta_k\mathbf{A}[\underline{x};\, t] + \Delta\mathbf{A}_k[t_k]\} \quad \text{with:} \quad c\,\Delta_k\mathbf{A}[\underline{x};\, t] = \Delta\phi_k[\underline{x};\, t]\,\{-i\,c\,\mathbf{I}_0 + \underline{u}_k[t_k] \bullet \underline{\mathbf{I}}\,\}$$

This latter form illustrates the Lorenz equation at the target point due to the k-th source cell: $c\,\Delta_k\underline{A}[\underline{x};\,t] = \underline{u}_k[t_k]\,\Delta\phi_k[\underline{x};\,t]$

This result can be used with the previous equation relating to the current in a unit mesoscopic section of the source wire.

$$\therefore\quad \mathcal{I}[\zeta, t_k]\,\Delta x_0 = -e\,\mathcal{U}[\zeta, t_k]\sum\nolimits_k^{*}\Delta\mathcal{N}_k[t_k] = \sum\nolimits_k^{*}\mathcal{U}[\zeta, t_k]\,\Delta Q_k[t_k] = \mathfrak{r}\sum\nolimits_k^{*}\mathcal{U}[\zeta, t_k]\,\Delta\phi_k[\underline{x};\,t] = c\,\mathfrak{r}\sum\nolimits_k^{*}\Delta_k A[\underline{x};\,t]$$

But the vector potential at the target charge is due to potential interactions with **all** the source segments averaged over a small (but finite) interaction time, such as 10^{-15} seconds as long as these segments are still on the target light-cone at t.

$$\therefore\quad \underline{A}[\underline{x};\,t] = \sum\nolimits_k \Delta_k\underline{A}[\underline{x};\,t] = \sum\nolimits_k \underline{\mathcal{I}}[\zeta, t_k]\,\Delta x_0 / c\,\mathfrak{r}\quad\therefore\quad \underline{A}[\underline{x};\,t] = \oint d\zeta\,\underline{\mathcal{I}}[\zeta,\, t - \mathfrak{r}/c] / c\,\mathfrak{r}$$

Thus, the mesoscopic model is consistent with the CEM definition of the retarded Neumann (or vector) potential.
The current in the source wire is caused by local accelerations in the mesoscopic cell as mobile electrons are repulsed by the electrostatic impulses of the other excess electrons in the prior cell. These electrostatic impulses correspond mathematically to the continuous action of a Coulomb-like force F_S acting constantly over the small relaxation time Δt_0 over the length Δx_0.

$$\Delta P_S = \Delta\mathbf{I}_S = F_S\,\Delta t_0 = (\mathfrak{q}\,\Delta\phi_k / \Delta x_0)\,\Delta t_0 = \mathfrak{q}\,\Delta\phi_k / (\Delta x_0 / \Delta t_0) = \mathfrak{q}\,\Delta\phi_k / \mathcal{U}\quad\therefore\quad \Delta\mathcal{W}_{Sk} = \mathfrak{q}\,\Delta\phi_k = \mathcal{U}\,\Delta P_S$$

The induced change in momentum $\Delta\underline{P}_I$ is caused by an induction impulse; i.e. a change in the vector potential $\Delta\underline{A}_k$.

$$\Delta P_I = F_I\,\Delta t_0 = \Delta\mathbf{I}_I = \mathfrak{q}\,\Delta A_k / c = \mathfrak{q}\,\mathcal{U}\,\Delta\phi_k / c^2 = (\mathcal{U} / c)^2\,\Delta P_S$$

The change in kinetic energy caused by this induction impulse $\Delta\mathcal{K}_{Ik}$ is given by classical Thomson-Tait formula [57].

$$\Delta\mathcal{W}_{Ik} = \Delta\mathcal{K}_{Ik} = \mathcal{U}\,\Delta\mathbf{I}_I = \mathfrak{q}\,\Delta A_k\,\mathcal{U} / c = (\mathcal{U} / c)^2\,\mathcal{U}\,\Delta P_S = (\mathcal{U} / c)^2\,\Delta\mathcal{W}_{Sk}$$

$$\therefore\quad \Delta P_I / \Delta P_S = F_I / F_S = (\mathcal{U} / c)^2\qquad\&\qquad \Delta\mathcal{W}_{Ik} / \Delta\mathcal{W}_{Sk} = (\mathcal{U} / c)^2 \equiv \gamma$$

γ is Weber's experimental ratio for the diminished interaction of charges in motion (induction due to conduction or current measured in EMUs) compared with the large effects they produce when there is no relative motion; i.e. the maximal electrostatic effects when measured in ESUs. This also agrees with the final form of Weber's electro-dynamic force between two charges ($\mathfrak{q}$ and $\mathfrak{q}'$) when they are in relative motion with the (instantaneous) velocity $\mathcal{U}$ and separation r [132]:

$$F_I = \mathfrak{q}\,\mathfrak{q}'\{\,1 - \mathcal{U}^2 / c^2\,\} / \mathfrak{r}^2$$

As Weber pointed out in his 1856 memoir with his colleague, Kohlrausch it is the extremely large value of the fundamental parameter *c* that makes the electro-dynamic (i.e. induction) effects seem almost infinitesimal in comparison with electrostatic effects and so only appear (as in situations involving galvanic currents) when the electrostatic interactions cancel each other completely in virtue of the neutralization of positive and negative electricity. This induction result can readily be given a statistical interpretation: let $\mathbb{N}_k[t]$ represent the number of interactions per second that occur in the mesoscopic cell #k in the source when the target wire is **not** present (i.e. only local electrostatic repulsions are generating the isolated current) and $\mathbb{N}_{Sk}[t]$ be this rate when the target wire is present that diverts the number $\mathbb{N}_{Ik}[t]$ interactions per second. These rates are proportional to the forces involved as they reflect the number of appropriate impulses per second or probability of an interaction $\mathscr{P}$.

$$\therefore\quad \mathbb{N}_k[t] = \mathbb{N}_{Sk}[t] + \mathbb{N}_{Ik}[t]\quad\text{but}\quad \mathbb{N}_{Ik}[t] = \gamma\,\mathbb{N}_k[t]\quad\therefore\quad \mathbb{N}_{Sk}[t] = (1 - \gamma)\,\mathbb{N}_k[t]$$

$$\therefore\ \mathscr{P}_{Ik} = (\mathcal{U}_k^2 / c^2)\,\mathscr{P}_k\qquad\&\qquad \mathscr{P}'_{Sk} = (1 - \mathcal{U}_k^2 / c^2)\,\mathscr{P}_k$$

It is this frequency reduction that is the explanation for the so-called relativistic mass effect that was measured by Kaufmann in 1903; this led directly to Planck's erroneous idea in 1907 that it was the electron's mass the increased rather than the EM force that diminished with increases in relative velocity. This was the principal experimental justification of this error that resulted in Einstein's need to redefine space and time through the Lorentz transform to preserve Maxwell's EM theory.

7.8.4 MOVING CONDUCTORS - FORCES

Mesoscopic Mechanism

Conduction is a holistic (i.e. collective) many-body phenomenon. The current I (or current density $\mathbf{J}$) is the result of a stable number of excess mobile electrons drifting along the wire in response to their equilibrium rate of injection $\mathbb{N}_i$ by the external source of EMF. The average longitudinal drift velocity u in any mesoscopic cell is due to the net interactions of all the other mobile electrons in the wire outside this cell. All of this dynamic activity along the wire is summarized by the average value of the vector potential $\underline{A}[\zeta]$ at a distance ζ from the injection point. This stable collective situation in a single current carrying wire is changed by the presence of other conductors when they are themselves carrying current and/or moving relative to the 'source' wire. Any attempt to reduce these dynamic, collective effects to an analytic theory (i.e. an additive summation of independent parts, especially infinitesimal elements) is at best an approximation – this is the case with CEM.

All interactions are simply transfers of momentum between the 'sender' and the 'receiver', even though there will always be a finite time difference (asynchronicity) between these two EM events, the total momentum is conserved across the interaction. In contrast to potential models, momentum is only exchanged between the source and the target charges **when** an interaction actually occurs; this fact is hidden in all time-averaged experiments and in theories such as Ampère's dynamical work with steady currents. This momentum exchange always requires a finite time to complete as the two components of the exchange are separated across a finite distance. This finite time delay was unknown (and unobservable) to early experimental scientists, such as Ampère, who therefore formulated his force law as a Newtonian instantaneous interaction. Although Weber had been informed by Gauss of asynchronicity, he too chose to adopt a Newtonian-style instantaneous interaction (perhaps, because the resulting mathematics is much simpler). In reality, this means that both Ampère's and Weber's microscopic force laws are, at best, good approximations but cannot be viewed as definitive. In contrast, the circuital form (integrated around the two full conducting circuits) has been confirmed to great accuracy and must be viewed as an accurate representation of reality when the circuit sizes and separations are below experimental time discrimination. In other words, when experimental distances are of the order of millimeters then sub-circuits will only begin to appear when time discrimination is smaller than 10^{-11} seconds.

The present Impulse model of electrical conduction (§7.4.2), involving a repetitive cycle of an accelerative phase followed by a deceleration phase, will now be extended further beyond EM induction to explain Ampère's seminal observation of forces between current-carrying circuits by following Weber and focusing on the role of the positive ions that form the semi-rigid structure of every metallic lattice. In the case of induction, the principal mechanism was the remote interaction between a few mobile electrons in a mesoscopic cell in one circuit and a few mobile electrons in a mesoscopic cell in the other circuit; these interactions occurring in the first phase of the conduction cycle. As Weber pointed out, these effects are very small unless the direct (Coulomb) interactions effectively cancel out because of compensating positive and negative charges. The explanation for Amperian forces needs to explicitly invoke the combined role of the positive (semi-rigid) lattice ions and the mobile electrons that interact with them: both locally and now remotely. Unlike induction with its changing source current, the Amperian situation can occur with steady electrical currents in both circuits. When the currents in two parallel circuits interact, the total momentum (both mechanical and electrical) must always be preserved across the complete interaction. If the two circuits were not in relative motion before the interaction then the momentum exchange must produce an absolute, transverse motion between them in opposite directions – the real electrodynamic effect discovered by Ampère in 1820. This extra mechanical motion requires kinetic energy that must originate from the electrical energy of both circuits and ultimately arises from the external sources of EMF. As they are metallic, the semi-rigid approximation assumes that this new motion occurs uniformly throughout both of the conductors, particularly when the two interacting segments are straight. When the second wire is close, a few (say, γ) of the accelerating impulses per cell in each half cycle no longer involve just internal electrons in this wire but now involve some of the mobile electrons in the other wire. Similarly, a small fraction γ of the decelerating impulses will be redirected to ions in the first wire. The first (electron-electron) redirection when the source current is changing accounts for mutual induction while the second will now be shown to be the cause of Amperian circuit forces even when both of the currents are 'constant'. Although the two electrical currents in both circuits of Ampère's experiment are 'steady', the present theory interprets the physics here in terms involving these tiny, undetectable fluctuations in current intensity of the Impulse Conduction model as the microscopic mechanism of the new mutual induction model. In effect, tiny amounts of momentum are being exchanged backwards and forwards between the two circuits that are still measurable in the aggregate as forces on the circuits at the scale of the laboratory, even though there appears to be no time average variation in either current. There is no net longitudinal change in electron momentum (the average current is unchanged) but there is a net transverse change. The focus now shifts to the second-half or decelerating phase of this model.

It will prove helpful to remember two central features of the Impulse Conduction model in a single conductor that impose major constraints on the possible micro-interactions between two current carrying conductors. In the acceleration (first) phase, the excess electrons behind (i.e. closer to their EMF source) the mobile electron repel the electron by increasing its momentum in the forward (anti-parallel) direction through a series of mutually repulsive impulses. In order to restore this electron to the long-term average velocity, this electron must lose this newly acquired excess momentum in the decelerating (second) phase of this cycle, interacting attractively with some of the positive ions of the lattice. In order for the electron to lose momentum in this secondary interaction, it must already be further forward along the conductor in the direction of the net electron flow (i.e. past the ion) when the attractive impulse is experienced. The narrow shape of the conductor defines the average charge distribution and this will constrain the mobile electron to move only longitudinally down the wire. In the decelerating phase, each of the faster electrons interacts with a positive ion (located behind the direction of flow) and effectively 'tugs' on the ion thereby increasing the velocity of the ion (relative to the lattice) at the expense of its own forward momentum. A series of this type of electron-ion interaction will dissipate the electron's net extra speed of $\mathcal{U}$ while increasing the energy of the lattice ions. This is equivalent to an exchange of momentum Δp between the mobile electrons and the lattice, for if the ionic system has an effective mass of $\mathcal{M}$ then its increase in speed would be V, where: $\mathcal{M}\ V = \Delta p$. In effect, some of the external (driving) energy or EMF is transferred from electrical conduction **in** the wires into mechanical (or kinetic) energy **of** the wires through a redirection of momentum from electrons to the ions in the <u>other</u> lattice.

Ampère's Experiments

In Ampère's original experiments, a steady electrical current $\mathcal{I}'$ throughout one circuit (the source) interacted with all of the current $\mathcal{I}$ throughout the other circuit (the target). The present EM theory returns to Ampère's imaginative atomic model of electricity wherein he viewed "atoms of electric charge" flowing through all circuits that were carrying electrical current. His original explanation for these observed forces (§7.1) involved very short circuit lengths (*current elements*) that interacted with each other. In order to comply with Newton's Third Law of (mechanical) Motion, this was seen as a reciprocal interaction in order to preserve the total mechanical momentum of both current elements and therefore the total mechanical momentum of both circuits. Since electrons were assigned a negative charge (for historical reasons) but are now known to be the mobile carriers of electrical currents it becomes necessary to draw the distinction between the direction of conventional current flow and the actual movement of electrons through metallic circuits, which are always anti-parallel to the nominal current. In this model, the emphasis will always be on the reality of the mobile electrons (the anti-current) and not the conventional current. The mobile electrons in a surface cell in the target circuit are overwhelmingly influenced at any time t by the excess of other, nearby mobile electrons. It is this internal set of interactions, both intra-electron and electron-ion, that are responsible for the steady current in each wire. As was shown earlier (§7.4.2), the change in momentum, induced in each electron across one cell in layer #l (at location ζ) by all the electrons in the previous layer during the acceleration period $\frac{1}{2}\Delta t_0$ seconds is the definition of Maxwell's electro-kinetic momentum $e\ \Delta A_l / c$, where the quantity $\Delta A[\zeta,t]$ is the change in the vector potential between the centers of two cells in adjacent layers in the positive longitudinal direction $+\zeta$. For a single, isolated wire, in the equilibrium condition of steady current, this is independent of location; so the net change in EM momentum of <u>all</u> the accelerated electrons in layer #l originating with electrons in the previous layer during one half-cycle is ΔP_l.

$$\therefore\ \ \Delta P_l = Q_l\,\Delta A_l / c \qquad \therefore\ \ 2\,m\mathcal{U}\,\Delta N' = e\,\Delta N'\,\Delta A_l / c \qquad \therefore\ \ \Delta p = \Delta P_l / \Delta N' = e\,\Delta A_l / c = 2\,m\mathcal{U}$$

In terms of the Impulse Conduction model, a very small fraction γ of the electrons in any source cell (say, #k) of mesoscopic length Δx_0 may interact with a small fraction of the electrons in any remote target cell (say, #j), also of length Δx_0, in each acceleration /deceleration period Δt_0, as described above in the conduction model of mutual induction. In terms of idealized uniform wires, let $\Delta\mathcal{N}_k[t]$ and $\Delta\mathcal{N}'_k[t']$ represent the number of excess electrons in each mesoscopic cell at the interaction times t and t' moving with relative drift velocities $\mathcal{U}$ and $\mathcal{U}'$ through their respective circuits, so that each unit mesoscopic cross-section of each wire (of length Δx_0) centered at positions ζ and ζ' now represent Ampère's original current elements. Again, the sums over all cells k and k' that are **in** the same unit cross-sections (i.e. at the same distances ζ and ζ' along their respective wires from their EMF sources with a 'constant' separation distance r as long as the separation between the wires is much greater than their diameters) define the total excess electric charges $\Delta Q[\zeta, t]$ and $\Delta Q'[\zeta', t']$ in the unit mesoscopic current elements. The fractional remote interaction between these unit current elements (while maintaining total momentum) is the mechanism behind the relative vector potentials $\Delta\underline{A}[\zeta, t]$ and $\Delta\underline{A}'[\zeta', t']$ that are produced by the relative changes in the motion of their own excess electric charges $\Delta Q'[\zeta', t']$ and $\Delta Q[\zeta, t]$ respectively.
The changes in these local vector potentials are manifested as net momentum transferred to each set of remote electrons $\Delta\underline{\mathbf{P}}[\zeta, t]$ via an equivalent set of pairs of impulses $\Delta\mathbf{I}$ and $\Delta\mathbf{I}'$.

$$\Delta\underline{\mathbf{P}}[\zeta, t] = \Delta Q[\zeta, t]\, \Delta\underline{A}[\zeta, t] / c = \Delta\underline{\mathbf{I}}[\zeta, t] \quad \& \quad \Delta\underline{\mathbf{P}}'[\zeta', t'] = \Delta Q'[\zeta', t']\, \Delta\underline{A}'[\zeta', t'] / c = \Delta\underline{\mathbf{I}}'[\zeta', t']$$

This separation of the unitary interactions into a "transmitted impulse" $\Delta\underline{\mathbf{I}}[\zeta, t]$ and a (temporal) disjoint reaction $\Delta\underline{\mathbf{I}}'[\zeta', t']$ is only a calculational technique and is **not** to be viewed as having existential significance. The core of the two-time electron model is the asynchronous conservation of momentum (§5.2.5).

$$\Delta\underline{\mathbf{P}}_{12}[\zeta - \zeta' ; t - t'] = 0 \qquad \therefore \quad \Delta\underline{\mathbf{P}}[\zeta, t] + \Delta\underline{\mathbf{P}}'[\zeta', t'] = 0 \qquad \therefore \quad \Delta\underline{\mathbf{I}}'[\zeta', t'] = -\Delta\underline{\mathbf{I}}[\zeta, t]$$

$$\therefore \qquad \Delta Q[\zeta, t]\, \Delta\underline{A}[\zeta, t] = -\Delta Q'[\zeta', t']\, \Delta\underline{A}'[\zeta', t'] = c\, \mathbb{N}_{12}\, \Delta\underline{\mathbf{I}}_0 = \mathbb{N}_{12}\, m\, b\, c = \text{constant}$$

Here, b is the universal, smallest quantum of change in speed (where, $\Delta\mathbf{I}_0 = m\, b$, see paper V) and $\mathbb{N}_{12}$ is a constant of the experimental configuration, including the geometry and nature of the conductors and external EMF sources.

In order to simplify the mathematics, it is easier to imagine that both conductors are identical in material and in cross-section so that the mass per unit length of each wire is the same. In this case, both wires will move with the same speed v as a result of the EM interaction between their currents. Let wire #1 move with velocity $\underline{v}$ and wire #2 with velocity $\underline{v}'$. Since there was no net motion of the two wires when no currents were present there can be no net momentum of the two wires after the (longitudinal) currents are established. $\therefore \quad \underline{v} + \underline{v}' = 0$ Thus, the wires always move relatively anti-parallel.

Within wire #1 the average electron drift speed is $\mathcal{U}$ correlated with the current $\mathcal{I}$ while in wire #2 the corresponding values are $\mathcal{U}'$ and $\mathcal{I}'$ respectively. Let the average velocities of mesoscopic cell clusters in wire #1 and #2 be $\underline{w}$ and $\underline{w}'$ respectively, defined relative to the inertial reference frame defined above; here $\underline{v}$ is the average velocity of the lattice while $\underline{\mathcal{U}}$ is defined relative to the lattice itself. In the two-dimensional model with parallel currents, there is no loss in generality in assuming that $\underline{v}$ is orthogonal to the current.

$$\underline{w} = \underline{v} + \underline{\mathcal{U}} \quad \& \quad \underline{v} \bullet \underline{\mathcal{U}} = 0$$

The total change in momentum on one of these target layers, say $\Delta\underline{\mathbf{P}}[\zeta, t]$, over one acceleration /deceleration period Δt_0 is defined by the change in the vector potential $\Delta Q[\zeta, t]\, \Delta\underline{A}[\zeta, t]$ 'generated' by changes in the 'source' cell, including changes in current $\Delta\underline{\mathcal{J}}'$ and changes in relative separation $\Delta\underline{r}$.

The rate of impulses 'received' by the target layer at ζ is equivalent to a mechanical force: $\Delta\underline{F}_1[\zeta, t] = \Delta\underline{\mathbf{I}}[\zeta, t] / \Delta t$. This is close enough to the continuum limit ($\Delta t \to 0$) or total time differential, so the calculus approximation can still be used.

$$\therefore \quad \Delta\underline{F}_1[\zeta, t] = D/Dt\, [\Delta Q[\zeta]\, \underline{A}[\zeta, t] / c\,] = \Delta Q[\zeta]\, \{\, \partial/\partial t + \underline{w} \bullet \underline{\nabla}\, \}\, \underline{A}[\zeta, t] / c$$

The mechanical work done $\Delta\mathcal{W}$ by this remote force $\Delta\underline{F}_1$ (originating with circuit #2) on the target current element as it is moved bodily through a distance $\Delta\underline{x}$. $\therefore \quad \Delta\mathcal{W} = \Delta\underline{F}_1 \bullet \Delta\underline{x}$.

In the Amperian experiment, both currents are steady so: $< \underline{A}[\zeta, t] > = \underline{A}[\zeta]$ and the electro-kinetic term $<\partial/\partial t\, \underline{A}[\zeta]> = 0$. This force on the target layer is both internal (electro-static force density $\Delta\underline{F}_S$) and lattice-mechanical $\Delta\underline{F}_L$.

$$\therefore \quad \underline{w} \bullet \underline{\nabla}[\, \underline{A}\,] = \underline{\nabla}[\, \underline{w} \bullet \underline{A}\,] - \underline{w} \wedge (\underline{\nabla} \wedge \underline{A}) = \underline{\nabla}[\, \underline{w} \bullet \underline{w}\, \phi / c\,] - \underline{w} \wedge (\underline{\nabla} \wedge \underline{A}) = w^2\, \underline{\nabla}[\, \phi\,] / c - \underline{w} \wedge (\underline{\nabla} \wedge \underline{A})$$

Here the Lorenz condition ($c\, \underline{A} = \underline{w}\, \phi$) has been used and w^2 is independent of location separation r to order $(w/c)^2$.

$$\therefore \quad \Delta\underline{F}_1[\zeta] = < \Delta\underline{F}_1[\zeta, t] > = \Delta Q[\zeta]\, \{\, w^2\, \underline{\nabla}\phi[\zeta] / c^2 - \underline{w} \wedge (\underline{\nabla} \wedge \underline{A}[\zeta]) / c\, \} = \Delta\underline{F}_S + \Delta\underline{F}_L$$

The lattice part was used by Lorentz with the 'magnetic' field ($\underline{B} = \underline{\nabla} \wedge \underline{A}$), for 'his' force; $\Delta\underline{F}_L = -\Delta Q\, \underline{w} \wedge \underline{B} / c$

For 'steady' currents through fixed diameter wires (§7.8.3): $\Delta Q[\zeta]\, \mathcal{U}[\zeta] = \mathcal{I}\, \Delta x_0$

The total force $\underline{F}_1$ on the complete target wire (circuit #1) is the sum of all these force elements $\Delta\underline{F}_1[\zeta_k]$ for all locations ζ_k around the complete circuit.

$$\therefore \quad \underline{F}_1 = \sum_k \Delta\underline{F}_1[\zeta_k] = \underline{F}_S + \underline{F}_L \quad \text{where:} \quad \underline{F}_S = \sum_k \Delta\underline{F}_S[\zeta_k] = \mathrm{w}\, I \oint d\zeta\, \underline{\nabla}\phi\,[\zeta] / c^2 = \mathrm{w}\, I \oint d\,[\underline{\nabla}\phi] / c^2 = 0$$

$$\therefore \quad \underline{F}_L = -\oint d\zeta\, \underline{I}\,[\zeta] \wedge (\underline{\nabla} \wedge \underline{A}[\zeta]) / c = -\oint d\zeta\, \underline{I}\,[\zeta] \wedge \oint d\zeta'\, \underline{\nabla} \wedge \underline{I}\,'[\zeta'] / c^2\, \mathrm{r} = \oint d\zeta \oint d\zeta'\, \underline{I}\,[\zeta] \wedge \underline{I}\,'[\zeta'] \wedge \underline{\mathrm{r}} / (c^2\, \mathrm{r}^3)$$

Alternatively: $\Delta\underline{F}_1[\zeta] = \Delta Q\, \underline{\mathrm{w}} \bullet \underline{\nabla}[\,\underline{A}\,] / c = \underline{I}\, \Delta x_0 \bullet \underline{\nabla}[\,\underline{A}\,] / c = \underline{I}\, \Delta x_0 \bullet \oint d\zeta'\, \underline{\nabla}\, [\, \underline{I}\,'[\zeta'] / c^2\, \mathrm{r}]$

$$\therefore \quad \Delta\underline{F}_1[\zeta] = -\Delta x_0 \oint d\zeta'\, \underline{I} \bullet \underline{I}\,'\, \underline{\mathrm{r}} / (c^2\, \mathrm{r}^3) \qquad \therefore \quad \underline{F}_1 = -\oint d\zeta \oint d\zeta'\, \underline{I}\,[\zeta] \bullet \underline{I}\,'[\zeta']\, \underline{\mathrm{r}} / (c^2\, \mathrm{r}^3)$$

Parallel Currents

Consider an idealized model of Ampère's experimental arrangement consisting of two equal straight lengths of metallic conductor (each of length 2L) pinned to rigid frames (to measure the forces) with the rest of their circuits shielded from the exposed sections that are separated by a distance D at their centers. The exposed lengths will be restricted to a 2D (x-y) plane. Let the center of circuit #2 (the "source") define the co-ordinate origins and let its straight section define the x-axis, while its (positive) current I' defines the positive direction of this axis. Let the center of circuit #1 (the "target") be located a distance D above the y-axis at {0, D} and its length be oriented (anti-clockwise) at an angle ψ to the x-axis. The currents are parallel when the (positive) current I in the target circuit also flows in the positive x-direction. In order for the two circuits not to touch, it is necessary to enforce the constraint: $L \sin\psi < D$. The wires must also be 'thin'; i.e. $D >> \mathrm{w}$.

Let cell #j be a (signed) distance ζ from the center of wire #1 and a distance r from remote 'partner' cell #k at a distance ζ' from its own center; in other words: $\{x', y'\} = \{0, \zeta'\}$ & $\{x, y\} = \{\zeta \cos\psi, D + \zeta \sin\psi\}$. The constraint defining which circuit elements (or mesoscopic cells) interact remotely is the Light-Cone Condition that reflects the finite time delay of the interaction.

$$r^2 = (x - x')^2 + (y - y')^2 = c^2 (t - t')^2 = D^2 + \zeta^2 + \zeta'^2 + 2\zeta (D \sin\psi - \zeta' \cos\psi)$$

The closest #2 cell (at ζ'_0) to a #1 cell (at ζ) occurs vertically below ζ at $\zeta'_0 = \zeta \cos\psi$. However, the closest #1 cell (at ζ_0) to a #2 cell (at ζ') occurs at an angle ψ to the left of ζ' at $\zeta_0 = \zeta' \cos\psi - D \sin\psi$. This is asymmetric ($\zeta'_0 \neq \zeta_0$) except when $\psi = 0$; i.e. parallel. Thus, the parallel case actually hides the fact that when the mutual interaction is not instantaneous but asynchronous (i.e. it takes a finite time for the action at ζ' to manifest its effect at ζ) it is asymmetric pairs of cells in each wire, like [ζ & ζ'_0] and [ζ_0 & ζ'], that have the dominant effect on each other, and not those equally distant from their own EMFs. Therefore, Ampère was mistaken in seeking an analytic differential force between all pairs of current elements (d^2F_{12}). The maximum exchange of momentum experienced by a target cell at the longitudinal location ζ therefore originates from the vector potential at the source cells at the longitudinal location ζ'_0 and their nearest neighbors along the source wire. In any case, now that the physics has been exposed, the mathematics is simplified by examining only parallel wires. This Amperian total force formula on wire #1 can be evaluated in the completely parallel case as just described above where the currents are independent of location $\underline{I}\,[\zeta] = \underline{I}$. The total force $\underline{F}$ on the complete target wire (circuit #1) is the sum of all these force increments $\Delta\underline{F}_1[\zeta]$ for all ***exposed*** locations ζ around circuit #1 (i.e. from x = – L to + L) that could interact with the all ***exposed*** (unshielded) locations ζ' around circuit #1 (i.e. from x' = – L to + L): symmetry can simplify the integrals.

$$\therefore \quad \underline{F} = -\iiint d^3x\, \underline{J}\,[\zeta] \bullet \oint d\zeta'\, \underline{I}\,'[\zeta']\, \underline{\mathrm{r}} / (c^2\, \mathrm{r}^3) = -\oint d\zeta \oint d\zeta'\, \underline{I}\,[\zeta] \bullet \underline{I}\,'[\zeta']\, \underline{\mathrm{r}} / (c^2\, \mathrm{r}^3) = F_x\, \hat{\underline{e}}_x + F_y\, \hat{\underline{e}}_y$$

Let $\underline{F} = \int_{-L}^{L} dx\, \underline{F}\,[x]$ $\quad \therefore \quad \underline{F}\,[x] = -(\underline{I} \bullet \underline{I}\,')\, \underline{G}[x] / c^2 = \underline{F}_x[x]\, \hat{\underline{e}}_x + \underline{F}_y[x]\, \hat{\underline{e}}_y$ and $\underline{G}[x] = \int_{-L}^{L} dx'\, \underline{\mathrm{r}} / \mathrm{r}^3$

Where: $\underline{\mathrm{r}} = \underline{x} - \underline{x}' = (x - x')\, \hat{\underline{e}}_x + D\, \hat{\underline{e}}_y$ and $\cos\theta = (x - x') / D$ & $\sin\theta = D / r$

$$\therefore \quad \underline{G}[x] = \int_{-L}^{L} dx'\, \underline{\mathrm{r}} / \mathrm{r}^3 = \int_{x-L}^{x+L} d\xi\, (\xi\, \hat{\underline{e}}_x + D\, \hat{\underline{e}}_y) / (D^2 + \xi^2)^{3/2} = G_x[x]\, \hat{\underline{e}}_x + D\, G_y[x]\, \hat{\underline{e}}_y$$

$$\therefore \quad G_x[x] = 1 / \sqrt{(D^2 + (x - L)^2)} - 1 / \sqrt{(D^2 + (x + L)^2)}$$

$$\therefore \quad G_y[x] \;=\; (L + x) / D^2 \sqrt{(\, D^2 + (L + x)^2 \,)} \;+\; (L - x) / D^2 \sqrt{(\, D^2 + (L - x)^2 \,)}$$

$$\therefore \quad F_x \;=\; \int_{-L}^{L} dx\, G_x[x] \;=\; \int_{0}^{2L} dz \,/\, \sqrt{(\, D^2 + z^2 \,)} - \int_{0}^{2L} dz \,/\, \sqrt{(\, D^2 + z^2 \,)} \;=\; 0$$

$$\therefore \quad F_y \;=\; D \int_{-L}^{L} dx\, G_y[x] \;=\; \int_{0}^{2L} dz\; z \,/\, D \sqrt{(\, D^2 + z^2 \,)} + \int_{0}^{2L} dz\, z \,/\, D \sqrt{(\, D^2 + z^2 \,)} \;=\; 2 \,\{\sqrt{(D^2 + 4L^2)} - D \,\} \,/\, D$$

$$\therefore \quad \underline{F}_P \;=\; -\; 2\, (\, \underline{I} \bullet \underline{I}' \,)\, \{\sqrt{(D^2 + 4L^2)} - D \,\} \,/\, (c^2\, D)\, \hat{\underline{e}}_y$$

When the two currents are orthogonal ($\underline{I} \bullet \underline{I}' = 0$) the electro-dynamic force between them disappears. When the currents are parallel ($\underline{I} \bullet \underline{I}' = I\, I'$) the total force on this part of circuit #1 is in the negative y direction, that is, towards circuit #2 while if the currents are anti-parallel then ($\underline{I} \bullet \underline{I}' = -\, I\, I'$) so the total force on this part of circuit #1 is then directed in the positive y direction; that is, away from circuit #2. These are all consistent with Ampere's qualitative results.

If the two parallel wires are close together (i.e. $L >> D$) $\quad \therefore \quad \underline{F}_P \approx -\; 4\, (\underline{I} \bullet \underline{I}')\; L \,/\, (c^2\, D)\, \hat{\underline{e}}_y$ (linear in L).

If the two parallel wires are far apart (i.e. $D >> L$) $\quad \therefore \quad \underline{F}_P \approx -\; 4\, (\underline{I} \bullet \underline{I}')\; L^2 \,/\, (c^2\, D^2)\, \hat{\underline{e}}_y$ (inverse in D^2).

Longitudinal Currents

Now consider a modified version of Ampère's experimental arrangement of two equal straight lengths of metallic conductor (each of length 2L) pinned to rigid frames (to measure the forces), with the rest of their circuits shielded from the exposed sections but now both circuits are reduced to a single dimension (x-axis). Again, the co-ordinate system is chosen as the middle of circuit #2 (the "source") with the center of circuit #1 (the "target") located a distance D along the x-axis. In order for the two circuits not to touch, it is necessary to enforce the constraint: $D > 2L$. The total force on wire #1 is $\underline{F}$.

$$\therefore \quad \underline{F} \;=\; -\oint d\zeta \oint d\zeta'\; \underline{I}\,[\zeta] \bullet \underline{I}'\,[\zeta']\; \underline{r} \,/\, (c^2\, r^3) \;=\; -\,(\underline{I} \bullet \underline{I}') \,/\, c^2 \int_{-L}^{L} d\zeta \int_{-L}^{L} d\zeta'\, \underline{r} \,/\, r^3 \;=\; \int_{-L}^{L} d\zeta\, \underline{F}\,[\zeta] \;=\; F_x\, \hat{\underline{e}}_x$$

Here: $\underline{r} \;=\; \underline{x} - \underline{x}' \;=\; (x - x')\, \hat{\underline{e}}_x \quad$ and $\quad x \;=\; D + \zeta \quad \&\quad x' \;=\; \zeta'$

$$\therefore \quad \underline{F}\,[x] \;=\; -\,(\underline{I} \bullet \underline{I}')\, \underline{G}[x] \,/\, c^2 \;=\; \underline{F}_x[x]\, \hat{\underline{e}}_x \qquad \text{and} \qquad \underline{G}[x] \;=\; \int_{-L}^{L} dx'\, \underline{r} \,/\, r^3 \;=\; G[x]\, \hat{\underline{e}}_x$$

$$\therefore \quad G[x] \;=\; \int_{-L}^{L} dx' \,/\, (x - x')^2 \;=\; 1 \,/\, (x - L) + 1 \,/\, (\, x + L)$$

$$\therefore \quad F_x \;=\; -\,(\underline{I} \bullet \underline{I}') \,/\, c^2 \int_{D-L}^{D+L} dx\, G[x] \qquad \therefore \quad \underline{F}_L \;=\; -\,(\underline{I} \bullet \underline{I}') \,/\, c^2 \ln[\, (D + 2L) \,/\, (D - 2L) \,]\, \hat{\underline{e}}_x$$

Note, there is now a force between two circuits that are aligned in a single direction, **contrary** to the Lorentz force prediction. Again, there is no force when the two currents are orthogonal; when the two currents are parallel the total force on this part of circuit #1 is in the negative x direction, that is, towards circuit #2 while if the currents are anti-parallel the total force on this part of circuit #1 is then directed in the positive x direction; that is, away from circuit #2. Let $d = D - 2L$.

If the two wires are close together (i.e. $L >> d$) $\quad \therefore \quad \underline{F}_L \approx -\,(\underline{I} \bullet \underline{I}') \ln[4L \,/\, d] \,/\, c^2\; \hat{\underline{e}}_y$ (log in L).

If the two wires are far apart (i.e. $D >> L$) $\quad \therefore \quad \underline{F}_L \approx -\; 4\, (\underline{I} \bullet \underline{I}')\; (L \,/\, D) \,/\, c^2\; \hat{\underline{e}}_y$ (inverse in D).

When the wires are far apart, the ratio of longitudinal to parallel forces is: $F_L \,/\, F_P \;=\; L \,/\, D << 1$ (but NOT zero).

7.8.5 MOVING CONDUCTORS - INDUCTION

Vector Potential of a Fixed Circular Wire

Only a small minority of physicists has focused on the vector potential as the central concept in their approach to CEM; even fewer theoreticians of electrical engineering have given much attention to this "remote corner" of CEM – C. John Carpenter spent many years writing papers on this subject [133] but unfortunately these were usually ignored. This section will show that, even in the traditional area of magnetostatics, the vector potential provides a much deeper physical insight into the phenomena of magnetism than the traditional approaches using magnetic potentials or Ampere's equation. Two paradigmatic situations will be examined: current loops and straight conductors. Experiments with insulated coils of thin conducting wire connected to voltaic cells were central to most of the early experiments involving current electricity and magnetism. The calculation of the magnetic field along the axis of such a coil is a famous undergraduate challenge.

One of the most popular derivations in CEM [134] is the calculation of a magnetic field due to constant current carried in a thin, circular wire loop. The usual approach is to begin with the Biot-Savart law for the magnetic field around a wire and sum up the effects of the whole loop. Since the source current $\mathcal{I}'$ is constant, all time effects can be ignored including all retardation delays. Although this is a motional effect, there are no changes in motion involved, hence "magnetostatics".

In this case, consider the wire to be of total length 2L formed into a circle of radius a ($L = \pi a$), carrying a constant current This approach involves calculating the vector potential $\underline{A}$ at any location $\underline{x}$ on a target loop of wire of radius R. Symmetry suggests using cylindrical coordinates {R, θ, z}, where the origin is at the center of the source wire with the z-axis defined by the axis of the current direction. There is no loss in overall generality in aligning the two loops and setting the target (or 'field') point $\underline{x}$ in the x-z plane (i.e. θ = 0), so that: $\underline{x} = \{x, y, z\}$ $x = R \cos\theta = R$ and $y = R \sin\theta = 0$. The source element at $\underline{x}'$ will be integrated all around the source wire, where: $\underline{x}' = \{x', y', 0\}$ $x' = a \cos\theta'$ & $y' = a \sin\theta'$

Separation: $\underline{r} = \underline{x} - \underline{x}' = \{R - a\cos\theta', -a\sin\theta', z\}$ $\therefore\ r^2 = a^2 + z^2 + R^2 - 2aR\cos\theta'$ where: $0 \le \theta' \le 2\pi$

$$\underline{\mathbf{A}}[\underline{x}, t] = \oint dx' \underline{\mathcal{I}}'[\underline{x}', t - r/c]/c\,r \qquad \therefore\ \underline{A}[R, z] = \mathcal{I}' \oint d\underline{x}'/c\,r$$

This integral can be more easily evaluated by selecting 4 symmetry points in each of the four quadrants around the circle. Let these points be defined by a new angle φ, such that: $\theta_1' = \varphi$, $\theta_2' = \pi - \varphi$, $\theta_3' = \pi + \varphi$, $\theta_4' = 2\pi - \varphi$; $0 \le \varphi \le \pi/2$. Each of these points is at the center of a small segment of the circle of length $\delta x' = a\,\delta\varphi$. The source loop lies in the x-y plane so the current element at $\underline{x}'$ is actually a directed (vector) segment $\delta\underline{x}'$, which can be split into its x and y components.

$\therefore\ \delta\underline{x}' = -\delta x' \sin\theta' \hat{\underline{e}}_x + \delta x' \cos\theta' \hat{\underline{e}}_y$ $\therefore\ \delta\underline{x}_1' = -\delta x' \sin\varphi \hat{\underline{e}}_x + \delta x' \cos\varphi \hat{\underline{e}}_y$ & $\delta\underline{x}_4' = \delta x' \sin\varphi \hat{\underline{e}}_x + \delta x' \cos\varphi \hat{\underline{e}}_y$

Similarly, for the lower quadrant points: $\therefore\ \delta\underline{x}_2' = -\delta x' \sin\varphi \hat{\underline{e}}_x - \delta x' \cos\varphi \hat{\underline{e}}_y$ & $\delta\underline{x}_3' = \delta x' \sin\varphi \hat{\underline{e}}_x - \delta x' \cos\varphi \hat{\underline{e}}_y$

Now, points $\underline{x}_1'$ and $\underline{x}_4'$ are the same distance r_1 from the target point at $\underline{x}$. $\therefore\ r_1 = \sqrt{(a^2 + z^2 + R^2 - 2aR\cos\varphi)}$

and points $\underline{x}_2'$ and $\underline{x}_3'$ are the same distance r_2 from the target point at $\underline{x}$. $\therefore\ r_2 = \sqrt{(a^2 + z^2 + R^2 + 2aR\cos\varphi)}$

$$\therefore\ \{\delta\underline{x}_1'/r_1 + \delta\underline{x}_2'/r_2 + \delta\underline{x}_3'/r_3 + \delta\underline{x}_4'/r_4\} = 2\cos\varphi\,\delta x'(1/r_1 - 1/r_2)\hat{\underline{e}}_y = 2a\cos\varphi\,\delta\varphi(1/r_1 - 1/r_2)\hat{\underline{e}}_y$$

$$\text{Let} \quad \underline{A}[R, z] = G[R, z]\,2a\,\mathcal{I}'/c\ \hat{\underline{e}}_y \qquad \therefore\ G[R, z] = \int_0^{\pi/2} d\varphi \cos\varphi\,(1/r_1 - 1/r_2)$$

The most interesting situations occur near the axis of the source wire circle, that is where R << a where:

$$(1/r_1 - 1/r_2) \approx -2aR\cos\varphi/(a^2 + z^2)^{3/2} \ \therefore\ G[\delta R, z] \approx -2a\,\delta R/(a^2 + z^2)^{3/2} \int_0^{\pi/2} d\varphi \cos^2\varphi = -\pi a\,\delta R/2(a^2 + z^2)^{3/2}$$

$$\therefore\ \underline{A}[\delta R, z] \approx -\pi a^2 \mathcal{I}'\,\delta R/(c\,(a^2 + z^2)^{3/2})\,\hat{\underline{e}}_\theta \qquad \text{for } \delta R << a$$

Since this expression is independent of the cylindrical angle, θ it describes a vector function that parallels the source current and has a constant magnitude at a given distance (|z|) from the center of the source loop and at a constant distance δR around this axis, whenever this displacement is much smaller than the size of the source loop (δR << a). It should be noted that actually on the axis itself (δR = 0) its value is zero, showing that it is the **difference** in distances from the target (or 'field' point) to different points around the current loop that generate 'magnetic' effects. In reviewing this last derivation, it can be seen that the integral is very different than the one that would have resulted from a static electric charge distributed equally around the loop. It is the vector nature of the integral that contributes critical features to the result. Relative to a 'vertical' displacement of the target point (e.g. along the x-axis) the 'left-right' symmetry (± y offsets, like the $\underline{x}_1'$ and $\underline{x}_4'$ points) cancels out the contributions in the vertical direction. The 'up-down' symmetry (± x offsets, like the $\underline{x}_1'$ and $\underline{x}_2'$ points) then introduces differences in the distances (r_1 and r_2) from points in the upper and lower halves of the circle. It is these differences that lead to an equivalent dipole-like final result; this is not obvious from any of the standard approaches.

When the source wire is lengthened and looped N times, close together, then the $\underline{A}$ integral must also be lengthened N times.

$$\therefore \quad \underline{A}[\delta R, z] \approx -\pi a^2 N \mathcal{I}' \delta R / (c (a^2 + z^2)^{3/2}) \hat{\underline{e}}_\theta \qquad \text{for } \delta R << a$$

This is an *almost* **exact** solution for the effect of the current flowing through a thin, circular wire near the axis of the source loop. All solutions are anti-parallel to the current and proportional to its magnitude; there are two useful approximations.

a) Near the center of the wire (|δz| & δR << a): $\underline{A}[\delta R, \delta z] \approx -\underline{\mathcal{I}}' \pi N \delta R / c a \qquad \text{for } \delta R > 0$

b) Long distances from the wire (|z| >> a & δR): $\underline{A}[\delta R, z] \approx -\underline{\mathcal{I}}' \pi N \delta R a^2 / c | z |^3 = -\underline{\mathcal{I}}' N L \delta R a / c | z |^3$

The induction field $\underline{B}$ is defined everywhere as the curl of the vector potential $\underline{A}$: $\underline{\mathbf{B}}[\underline{x}, t] = \nabla \wedge \underline{\mathbf{A}}[\underline{x}, t]$.

Any vector function $\underline{F}$ takes the following form in cylindrical co-ordinates: $\underline{F} = F_R \hat{\underline{e}}_R + F_\theta \hat{\underline{e}}_\theta + F_Z \hat{\underline{e}}_Z$

Thus, the vector potential due to a circular wire, near its axis, is: $A_\theta = -\pi a^2 \mathcal{I}' N R / (c (a^2 + z^2)^{3/2})$ and $A_R = 0$ & $A_Z = 0$

The curl of any vector function $\underline{F}$ takes the following form in cylindrical co-ordinates:

$$\underline{\nabla} \wedge \underline{F} = \{ 1/R \, \partial F_Z / \partial\theta - \partial F_\theta / \partial z \} \hat{\underline{e}}_R + \{ \partial F_R / \partial z - \partial F_Z / \partial R \} \hat{\underline{e}}_\theta + 1/R \{ \partial(RF_\theta) / \partial R - \partial F_R / \partial\theta \} \hat{\underline{e}}_Z$$

$$\therefore \quad \underline{\mathbf{B}} = \nabla \wedge \underline{\mathbf{A}} = -\partial A_\theta / \partial z \, \hat{\underline{e}}_R + 1/R \, \partial(RA_\theta) / \partial R \, \hat{\underline{e}}_Z = -\pi a^2 N \mathcal{I}' \{ 2 \hat{\underline{e}}_Z + 3 R z / (a^2 + z^2) \hat{\underline{e}}_\theta \} / (c (a^2 + z^2)^{3/2})$$

$$\therefore \quad \underline{\mathbf{B}}[\delta R, z] \approx -2\pi a^2 N \mathcal{I}' / (c(a^2 + z^2)^{3/2}) \hat{\underline{e}}_Z \qquad \text{for } \delta R << a$$

However, the distance from the axial point z to any point on the current loop is ρ, where $\rho^2 = (a^2 + z^2)$ and this axial point makes an angle Θ (relative to the z-axis) with every point on the loop, so that: $\sin\Theta = a / \rho$.

$$\therefore \quad \underline{\mathbf{B}}[\delta R, z] \approx -2\pi N \mathcal{I}' a^2 / (c \rho^3) \hat{\underline{e}}_Z = -2\pi N \mathcal{I}' \sin^3\Theta / (c a) \hat{\underline{e}}_Z \qquad \text{for } \delta R << a$$

a) Near the center of the wire (|δz| & δR << a; Θ ≈ π/2): $\underline{B}[\delta R, \delta z] \approx -2\pi N \mathcal{I}' / (c a)$

b) Long distances along the axis (|z| >> a >> δR; Θ ≈ 0): $\underline{B}[\delta R, z] \approx -2\pi N \mathcal{I}' a^2 / (c z^3)$

The center-of-the-loop result is the standard Biot-Savart approximation, which does **not** make explicit the dropping of the small transverse ($\hat{\underline{e}}_\theta$) component. The long-distance result recovers the dipole-like (inverse-cube) approximation when two small negative charges are circulating around a fixed, massive object with a similar-sized positive charge (see §7.8.1). Both results illustrate that the Amperian (and Maxwellian) models are both only approximations to the physics of the vector potential.

Vector Potential of a Fixed Straight Wire

One of the most useful formulas in XCI physics is the value of the vector potential $\underline{A}[\underline{x}]$ at any location around a section of a straight wire of length 2L carrying a constant current $\mathcal{I}'$. Since the source current is constant, all time effects can be ignored including all retardation delays. Symmetry suggests using cylindrical coordinates {R, θ, z}, where the origin is at the mid-point of the wire with the z-axis defined by the current direction; there is no loss in overall generality in setting the target (or 'field') point in the x-z plane (i.e. θ = 0), so that: $x = R \cos\theta = R$ and $y = R \sin\theta = 0$.

Target: $\underline{x} = \{R, 0, z\}$ Source: $\underline{x}' = \{0, 0, \zeta\}$ Separation: $\underline{r} = \underline{x} - \underline{x}' = \{R, 0, z - \zeta\}$ where: $-L \le \zeta \le L$

$$\mathbf{A}[\underline{x}, t] = \oint dx' \underline{\mathcal{I}}'[\underline{x}', t - r/c]/c\, r \qquad \therefore \quad \underline{A}[\underline{x}] = \underline{\mathcal{I}}' \oint dx'/c\, r = A[\underline{x}]\, \hat{\underline{e}}_z$$

$$\text{Let} \quad A[\underline{x}] = G[\underline{x}]\, \mathcal{I}'/c \qquad \therefore \quad G[\underline{x}] = \int_{-L}^{L} d\zeta / r = \int_{-L}^{L} d\zeta / \sqrt{(R^2 + (\zeta - z)^2)}$$

Substituting: $s = z - \zeta \quad ds = -d\zeta \quad \therefore \quad G[\underline{x}] = \int_{z-L}^{z+L} ds / \sqrt{(R^2 + s^2)} = \ln[s + \sqrt{(R^2 + s^2)}]\,|_{z-L}^{z+L}$

$$\therefore \quad G[\underline{x}] = \ln[z + L + \sqrt{(R^2 + (z + L)^2)}] - \ln[z - L + \sqrt{(R^2 + (z - L)^2)}]$$

$$\therefore \quad \underline{A}[R, z] = 1/c\, \underline{\mathcal{I}}' \ln |(\sqrt{(R^2 + (z + L)^2)} + (z + L))/(\sqrt{(R^2 + (z - L)^2)} + (z - L))| \quad \text{not } R = 0 \;\&\; |z| = L$$

This is an **exact** solution for the effect of the current flowing through a thin, straight wire at all points {x, y, z} around the wire. All solutions are parallel to the current and proportional to its magnitude; there are three very useful approximations.

a) Near the surface of the wire ($\delta R \ll L$): $\underline{A}[\delta R, z] \approx \underline{\mathcal{I}}' \ln |(z + L)/(z - L)|/c$ for $|z| \neq L$

b) Near the center of the wire ($|\delta z|$ & $\delta R \ll L$): $\underline{A}[\delta R, \delta z] \approx \underline{\mathcal{I}}' 2 \ln(2L/\delta R)/c$ for $\delta R > 0$

c) Long distances from the wire ($R \gg L + z$): $\underline{A}[R, y, z] \approx \underline{\mathcal{I}}' 2L/c\, R$

Near the surface of the wire (a) the vector potential due to its current is independent of the perpendicular distance R and only varies logarithmically with z; in particular, near the center of the wire (b) the vector potential is independent of z but varies inversely logarithmically with R; while for long distances from the wire (c) the vector potential only varies inversely with the perpendicular distance R and is independent of the angular orientation (i.e. independent of z).

The induction field $\underline{B}$ is defined everywhere as the curl of the vector potential $\underline{A}$: $\mathbf{B}[\underline{x}, t] = \nabla \wedge \mathbf{A}[\underline{x}, t]$.

When the vector potential is produced by a thin wire, as above, there is no variation in the y-direction, so in this case.

$$\underline{B}[x, y, z] = -1/c\, \mathcal{I}' \partial/\partial x\, [G[x, y, z]]\, \hat{\underline{e}}_y = B[R, z]\, \hat{\underline{e}}_y$$

a) Near the surface of the wire ($\delta R \ll L$): $\underline{B}[\delta R, z] \approx 0$

b) Near the center of the wire ($|\delta z|$ & $\delta R \ll L$): $\underline{B}[\delta R, \delta z] \approx 2\mathcal{I}'/(c\, \delta R)$ (Biot-Savart approx. see §7.6.3).

c) Long distances from the wire ($R \gg L + z$): $\underline{B}[R, z] \approx 2\mathcal{I}'(L/c\, R^2)$ (the inverse-square approx.)

This illustrates that the Amperian (and Maxwellian) models are again both only approximations to the physics of the vector potential.

Faraday Induction of a Moving Straight Wire

The vector potential results derived above will now be applied to understand the physics of Faraday 'magnetic' induction as just an implicit example of Henry current induction by making the source of the 'magnetic field' completely explicit.

Consider four (4) straight conductors (in the form of rigid wires) arranged in a two-dimensional structure (in the x-z plane) according to the following scheme. A long rigid wire W_S of length $2L$ is connected to an external source of EMF with its fixed center defining the co-ordinate origin and its steady current $\underline{I}'$ defining the z-axis. Two shorter wires, W and W′, both of length d, are aligned parallel to each other in the x-direction at a separation l fixed symmetrically around the x-axis at a distance b ($l = 2b$); the near ends of these two wires are fixed at a distance a above the source wire W_S while the far ends, (fixed even farther above the source wire at a distance $a + d$ and labeled β and β′ respectively) are connected by a voltmeter. The fourth (or 'target') wire W_T lies across these two wires but can move freely up and down the x-axis. The target wire W_T is moved by an external mechanical force at a constant speed v in the positive x-direction. The whole of the target wire is always parallel to the source wire; the left-end of W_T is labeled α and the right-end α′: at time t the target wire is located at a distance x above W_S. The closed conduction path Π is formed from the voltmeter VM connected to the far-end of W, down this wire to its near-end, across the target wire to the near-end of W′, back up to its far-end and finally back to the voltmeter.

$$\Pi \equiv \{VM \to \beta \to \alpha \to \alpha' \to a' \to VM\} \qquad \text{Scale: } L >> a \;\; \& \;\; L >> b$$

$$\therefore \; \alpha[t + \Delta t] = \alpha[t] + \Delta x \quad \therefore \; \Delta x = \alpha[t + \Delta t] - \alpha[t] = \Delta\alpha[t] = v\,\Delta t$$

Henry found that this configuration would produce a voltage $\mathcal{V}$ or EMF $\mathcal{E}$ in the circuit Π when there is **relative motion** between this closed circuit, particularly the target wire W_T and the source current in the source wire W_S. Since Π is a closed conducting circuit then a current will be induced in this circuit such that when a charge Q is moved completely around this circuit then a quantity of work $\mathcal{W}$ is accomplished, where $\mathcal{W} = Q\,\mathcal{V}$ and $\mathcal{V} = \mathcal{E}$. Now, the electro-kinetic momentum per unit charge $\underline{\mathbf{P}}[\underline{x}, t]$ at every point around the circuit Π[t] is directly related to the vector potential $\underline{\mathbf{A}}[\underline{x}, t]$ experienced at each point in the circuit; this is sufficient to define the electro-kinetic action $\mathfrak{A}$ in this circuit.

$$\underline{\mathbf{P}}[\underline{x}, t] = \underline{\mathbf{A}}[\underline{x}, t] / c \qquad \mathfrak{A}[t] = Q \oint d\underline{x} \cdot \underline{\mathbf{P}}[\underline{x}, t] \qquad \text{Also:} \quad \Delta\mathfrak{A} = -\mathcal{W}\,\Delta t = -Q\,\mathcal{V}\,\Delta t$$

$$\therefore \quad \mathcal{E}[t] = -\Delta\Phi[t] / c\,\Delta t \qquad \text{where:} \quad \Phi[t] = \oint d\underline{x} \cdot \underline{\mathbf{A}}[\underline{x}, t]$$

Since $\underline{\mathbf{A}}[\underline{x}, t]$ is always parallel to the z-axis, only the section of Π through the wire W_T contributes to the integral $\Phi[t]$.

$$\therefore \quad \Phi[t] = \int_{-b}^{b} dz\, \mathbf{A}_Z[x, z; t] = \int_{-b}^{b} dz\, \mathbf{A}_Z[x, t] = 2b\, \mathbf{A}_Z[x, t] = l\, \mathbf{A}_Z[x, t] \quad \therefore \quad \Phi[t + \Delta t] = l\, \mathbf{A}_Z[x + \Delta x, t + \Delta t]$$

$$\therefore \; \Delta\Phi[t] = \Phi[t + \Delta t] - \Phi[t] = l\{\mathbf{A}_Z[x + \Delta x, t + \Delta t] - \mathbf{A}_Z[x, t]\} = l\,\Delta_t[\mathbf{A}_Z[x, t]] \quad \therefore \quad \Delta\Phi[t] / \Delta t = l\,(\Delta x / \Delta t)\,\Delta_x[\mathbf{A}_Z[x, t]]$$

In the infinitesimal limit ($\Delta t \to 0$): $\mathcal{E}[t] = -l\,v / c\; d/dx\,[\,\mathbf{A}_Z[x, t]\,]$ Now, $\mathbf{A}_Z[x, t] = -I'\,2 \ln(x / 2L) / c$

$$\therefore \; d/dx\,[\,\mathbf{A}_Z[x, t]\,] = -2I' / c\,x = -\mathbf{B}_y[x, t] \qquad \therefore \quad \mathcal{E}[t] = 2\,l\,v\,I' / (c^2\,x) = l\,(v / c)\,\mathbf{B}_y[x, t]$$

This is the same result when using the "Lorentz force" law: $\mathcal{E} = l\,\underline{v} \wedge \underline{\mathbf{B}}[x,t] / c$ but here $\mathbf{B}_y$ is just a computed term.

This would be the exact same result if the Π circuit is static (i.e. the target wire W_T is fixed relative to its support wires W and W′) and the source wire W_S rigidly moves away at the same speed v but in the negative x-direction. There are **not** two causes of induced EMF in this theory, unlike the Maxwell-Lorentz theory, which needs both the Lorentz equation for the motional EMF effect of a magnetic field on a moving (target) wire and the Maxwellian equation (curl $\underline{\mathbf{E}}$) for transformer EMF effects of a time-varying magnetic field: a duplicity first pointed out by Maxwell himself [98] and famously repeated by Feynman [105]. This 'dichotomy' was also one of the principal motivations for Einstein to develop his Special Theory of relativity [39] that can now be seen as an unnecessary attempt to save what is actually only a statistical theory of large numbers of electrons interacting remotely in conducting circuits. Indeed, the above results are fully Galilean invariant as they are totally independent of the steady motion of any third-party observers. This illustrates the danger of trying to enforce covariance (form invariance) for what are no more than intermediate mathematical terms, such as electric and magnetic fields.

7.9 FAR CONDUCTION INTERACTIONS (Radiation)

This section will address situations where the spatial separations are large between the conductors, where temporal variations in one conductor can effect the electro-dynamic behavior of a remote ('far') conductor: a phenomenon known as EM radiation.

7.9.1 MESOSCOPIC RADIATION

Hertzian Radiation

In 1886, the young and ambitious German physicist Heinrich Hertz (1857–1894) developed a complementary pair of dipole transmitter and receiver devices, which he used to investigate remote EM coupling, hoping to find "a new scientific effect". The transmitter was built around a Ruhmkorff coil-driven spark gap connected at the center of a one meter wire pair with capacity spheres at each end for circuit resonance adjustments: this circuit acted as a radiator forming a center-fed driven element that is not electrically grounded for its operation. His receiver (in a darkened room) consisted of a simple half-wave dipole antenna connected to a coil with a spark gap, whereupon an induced spark would be seen once the transmitter, some distance away, was activated. A glass panel placed between the source of EM waves and the receiver absorbed ultraviolet radiation that Hertz showed assisted in increasing the sparking across the gap (the photoelectric effect). By adjusting the frequency and distance between the two antennas, he was able to demonstrate resonance conditions, reflection and measure the speed of transmission between them. With these devices, Hertz was able to demonstrate in 1887 the existence of remote EM induction (asynchronous action-at-a-distance) in the receiving circuit. Since he was aware of Maxwell's EM theory, he then interpreted these results in terms of the production and reception of EM waves of very high frequency ('radio waves') as both Weber's and Helmholtz's electrical theories failed to predict EM waves. It was an irony of history that Hertz did not live to see the practical importance of his experiments, viewing them as a purely scientific investigation.

Hertz's Explanation

Hertz realized, after exploring various circuit configurations and without any theoretical calculations, that his transmitter was generating an oscillation in its current at a vastly higher frequency (about 150 MHz) than in the driving coil (20 Hz). As Jed Buchwald (the eminent historian of 19th Century physics) wrote [135] in his in-depth review of these experiments: "Hertz did not discover the efficacy of the discharge circuit from theory; he was surprised by it." Hertz decided that the originating spark was generating effects in this circuit that could not have been anticipated from any of the existing EM theories and the best existing circuits at that time were only capable of generating frequencies no higher than about 1 MHz. There was nothing about high-frequency oscillations to suggest that they might be useful in producing strong inductive effects. At first, Hertz believed that the spark was only creating a single pulse per driving cycle but manipulation of the capacitance capabilities at each end of his transmitting dipole convinced him that he had achieved spontaneous, very high frequency oscillations lasting thousands of cycles with the spark acting as a fast-acting switch. The linear dimensions of his dipole leads were each about 150 cm, so that finite time delays were needed for charges to move from one end of the open linear circuit to the other. The great surprise in all these experiments was that the linear part of the receiver could be many centimeters away from the parallel transmitter, without any explicit capacitive or inductive circuit coupling. Hertz enhanced the receptivity of his receiver by tuning it to resonate with the transmitter – when acting in this receiving mode he referred to it as a 'resonator'. It is now known that these are the simplest practical antennas possible, from a theoretical point of view. One of Hertz's key observations was that the sparking behavior of the resonator was governed by the force that acted on the part opposite its gap and this was strongest when this part of the receiver was parallel to the body of the transmitter. Hertz was convinced that he was investigating a wave phenomenon as he found periodic spatial variations with separation.

Open Circuits

The external conduction theory so far has only covered closed source circuits. This was always the focus of EM experimental research in the 19th Century. It was probably why Hertz's discovery of remote EM oscillations came as such a huge surprise – certainly to the major followers of Maxwell, who did not believe in electrical current as arising from the flow of an electrical substance. However, there is nothing in the present theory developed so far that limits the external conduction interaction to closed circuits – the only requirement is that the source conductor sustain currents in a direction long enough for the averaging that defines this interaction to occur with remote target charges. This is the case with the Hertzian dipole where electrons flow out from the EMF source, through the linear section of the antenna and rebound back towards the EMF source (preferably with a capacitor at the end of each linear section which will slow down the overall cycle time as the moving charges accumulate before returning back down the linear section).

7.9.2 LINEAR FAR INTERACTIONS (LFI)

LFI Overview

The original experiments by Hertz in 1888 (see §7.9.1) showed that a remote receiving circuit could have electric currents induced in it by current oscillations in an open (non-closed) conducting arrangement consisting of two linear sections, fed by an oscillating EMF source. The receiver was maximally effected when it contained a conducting section that was positioned parallel to the linear sections of the transmitter (or source). The EMF source must oscillate fast enough (Hertz operated at about 10^8 cycles/sec) so that XCI effects dominate over Coulomb or conduction effects. This means that the internal electric effects within the conductor of the transmitter can reach the far end of the linear section very much sooner than the actual arrival of the excess local density of the mobile electrons. This allows the transmitter to effectively behave as if the current density is constant along the whole length of its linear sections during any small, time interval.

LFI Model

Consider the source to consist of a straight length of thin wire of total length L cut exactly in half (defining the spatial origin) with the direction of the wire defining the z-axis. The two halves of the wire are connected to a shielded source of EMF that generates an alternating current with a cycle time of T_0; in other words, it supplies a sinusoidal current of maximum strength I_0 at a linear frequency ν (or circular frequency ω), where: $\nu = 1 / T_0 = \omega / 2\pi$. The EM internal effects will synchronize all the mobile electrons in the source conductor at 'light-speed' c; that is, the EMF may be viewed as generating sine waves of current along the z-axis for mesoscopic cells at a distance z from the origin such that: $I[t;z] = I_0 \sin[2\pi (c\, t - z) / cT_0]$. The Hertzian approximation occurs when the temporal variations completely dominate the spatial variations all along the antenna; this requires that in the time for the current to fluctuate by one half-cycle ($t = T_0 / 2$) there are minimal spatial variations, even at the end of the linear section ($z = L / 2$). $\therefore\ c\,(T_0 / 2) >> L / 2$ or $\lambda >> L$ (where the wavelength λ is defined as cT_0).

$$\therefore \quad I[t; z] \approx I_0 \sin[\, \omega t \,] \quad \text{for } 0 < z < L$$

EM 'far radiation' effects are always evaluated at great distances from the source, so that if the center of the receiver is located at a distance R from the center of the transmitter then this condition is equivalent to the approximation: $R >> \lambda$. If the receiver consists of a fixed wire circuit with an exposed length l then this will produce the maximum induced current in the receiver when this exposed length is parallel to the linear section of the source as this will maximize the exchange of momentum between the mobile electrons in the two conductors. As variations occur in the current strength of the transmitter then corresponding variations will occur in the receiving circuit, so a varying current will be induced in the receiver with the same temporal characteristics; i.e. the same cycle time T_0. Similarly, if the wavelength is much greater than the dimensions of the receiver (i.e. $\lambda >> l$) then the induced current I_R will only vary with time over the whole of the receiving circuit. Since the receiver is fixed, all of the interactions will exchange momentum only with the conduction electrons in the receiver; this will be interpreted as the creation of an EMF in the receiver or as an induced voltage $\mathcal{V}[t]$ as the Coulomb forces cancel.

$$\mathcal{V}[t] = l\, E_m[t] = -l\ d/dt \left[\iiint dz'\, J'[\, t - R / c\, ;\, z'] \,/\, c^2\, r \,\right] = -l\, L / (R\, c^2)\ d/dt \left[\, I_0 \sin[\, \omega\, (\, t - R / c)\,]\, \right]$$

$$\therefore \quad \mathcal{V}[t] = -\mathcal{V}_L \cos[\, \omega\, (t - R / c)\,] \quad \text{where} \quad \mathcal{V}_L = l\, L\, \omega\, I_0 / R\, c^2$$

Thus, the induced voltage varies directly with the frequency and strength of the current in the source transmitter (antenna) and inversely (not inverse square) with the distance separating the receiver from the transmitter. Most importantly, it varies with the product of the length of the receiver (l) and the size of the transmitter (L). The receiver's net voltage and current are one quarter-cycle out of phase with the instant periodic variations of EMF in the source transmitter. When the straight section of the receiving conductor makes a non-zero angle (say θ) with the direction of the antenna (relative to the z-axis) then the relative momentum is diminished by a factor of $\cos\theta$ so that $\mathcal{V}_\theta = \mathcal{V}_R \cos\theta$. If the location of the receiver makes an angle θ with the x-axis then the maximum current is induced when the straight section of the receiving conductor is now orthogonal to the relative position vector between the centers of the receiver and the transmitter, which therefore reduces the induced voltage by the same $\cos\theta$ factor.

7.9.3 CIRCULAR FAR INTERACTIONS (CFI)

CFI Overview

Although Hertz first investigated remote EM oscillations using a linear transmitter, investigations soon showed that similar effects could be achieved by modifying the transmitter into a circular loop. Since currents flowing through such loops have long been interpreted as the source of magnetic fields, this type of arrangement is often referred to as a "magnetic dipole antenna". Ironically, it is the remote electric effects that are much stronger than the magnetic effects and it is this class of interactions that produce the induced electric currents in the surface of remote metallic receiving circuits. When the source circuit is in the form of a circular loop then it is found that there are no induced effects along the longitudinal axis, while the maximal effects occur in the plane of the loop. This is understandable since for any point along the longitudinal axis pairs of the current elements at opposite points around the circle contribute an equal contribution but in exactly opposite directions. Along the circumference of the circle, the effects of the current elements nearest to the remote receiver are stronger (1/r) than those on the far side of the loop, so the net induced force is parallel to the nearest elements. Again, the EMF source must oscillate fast enough (at least at megacycle frequencies) so that XCI effects dominate over Coulomb or conduction effects. Physically, this means that the internal electric effects within the conductor of the transmitter can effect mobile electrons all around the transmitter and reach the section furthest from the source leads very much sooner than the actual arrival of the excess local density of mobile electrons. This allows the transmitter again to effectively behave as if the current density is constant along the whole length around the loop during any small, time interval.

CFI Model

Consider the source to consist of a piece of thin wire of total length L formed into a circle of radius a ($L = 2\pi a$) with its center defining the spatial origin and lying in the x-y plane with the direction of its longitudinal axis defining the z-axis. The two ends of the wire are connected to a shielded source of EMF that generates an alternating current with a cycle time of T_0; in other words, the EMF generates a sinusoidal current of maximum strength I_0 at a frequency ν (or circular frequency ω), where: $\nu = 1 / T_0 = \omega / 2\pi$. Over mesoscopic time intervals, the EM internal effects of localized excess mobile electron density concentrations will synchronize all the mobile electrons in the source conductor at 'light-speed' c; that is, the EMF may be viewed as generating sine waves of current along the circumference of the loop such that the current at time t in all mesoscopic surface cells at a distance ζ measured from the x-axis is: $I[t; \zeta] = I_0 \sin[2\pi (c\, t - \zeta) / cT_0]$. Again, the Hertzian approximation occurs when the temporal variations completely dominate the spatial variations, all along the antenna; this requires that in the time for the current to fluctuate by one half-cycle ($t = T_0 / 2$) there are minimal spatial variations, even at the far side of the loop section ($\zeta = L / 2$). $\therefore\ c\,(T_0 / 2) >> L / 2$ or $\lambda >> L$ (where the wavelength λ is defined as cT_0).

$$\therefore\quad I[t; \zeta] \approx I_0 \sin[\,\omega\, t\,] \quad \text{for } 0 < \zeta < 2\pi a$$

EM 'far radiation' effects are always evaluated at great distances from the source, so that if the center of the receiver is located on the x-axis at a distance R from the center of the transmitter then this condition is equivalent to: $R >> \lambda$.
Again, let the receiver consists of a small, fixed wire circuit with an exposed length l. As variations occur periodically in the current strength of the transmitter then corresponding variations will occur in the receiving circuit, so a varying current will be induced in the receiver with the same temporal characteristics; i.e. the same cycle time T_0. Similarly, if the wavelength is much greater than the dimensions of the receiver (i.e. $\lambda >> l$) then the induced current I_R will only vary with time over the whole of the receiving circuit. Since the receiver is fixed, all of the interactions will exchange momentum only with the conduction electrons in the receiver; this will be interpreted as the creation of an EMF in the receiver or as an induced voltage $\mathcal{V}[t]$ as the Coulomb forces from the source ions and mobile electrons cancel. Let $d\zeta'$ define the direction of the current.

$$\underline{E}_m[t\,;\,\underline{x}] = -\,d/dt\,[\iiint d\zeta'\, J'[\,t - r/c\,;\,\zeta']\ /\,c^2\, r\,] = -\,d/dt\,[\oint d\zeta'\, I'[\,t - r/c\,;\,\zeta']\ /\,c^2\, r\,]$$

$$\therefore\quad \underline{E}_m[t\,;\,\underline{x}] = -\,\omega\, I_0 / c^2 \oint d\zeta' \cos[\,\omega\,(\,t - r/c)\,]\,/\,r \quad \text{where} \quad \underline{r} = \underline{x} - \underline{x}' = -\,\underline{r}'$$

This line integral can be transformed to a surface integral bounded by the loop using: $\oint d\underline{\zeta}\,\psi = \iint d\underline{\sigma} \wedge \underline{\nabla}\,\psi$

Also, the source gradient operator ($\underline{\nabla}'$) applied to a product gives: $\underline{\nabla}'\,[\psi / r\,] = \underline{\nabla}'\,[\psi]\,/\,r + \psi\,\underline{r}\,/\,r^3$

$$\therefore \quad \underline{E}_m[t\,;\underline{x}] = -\,\omega\, I_0 / c^2 \iint d\underline{\sigma}' \wedge \{\, \underline{\nabla}'[\cos[\,\omega\,(t - r/c)\,] + \underline{r}\cos[\,\omega\,(t - r/c)\,]\,/\,r^2\,\}\,/\,r$$

$$= -\,\omega\, I_0 / c^2 \iint d\underline{\sigma}' \wedge \{\, (\omega/c)\,\underline{r}'\sin[\,\omega\,(t - r/c)\,] + \underline{r}\cos[\,\omega\,(t - r/c)\,]\,/\,r\,\}\,/\,r^2$$

$$= -\,\omega\, I_0 / c^2 \iint d\underline{\sigma}' \wedge \underline{r}\,\{-(\omega/c)\sin[\,\omega\,(t - r/c)\,] + \cos[\,\omega\,(t - r/c)\,]\,/\,r\,\}\,/\,r^2$$

Now $\omega / c = 2\pi / (cT_0) = 2\pi / \lambda$ while at 'far' distances: $r \approx R >> \lambda$ so the second term is much smaller than the first. Since $R >> a$, the integrand is nearly constant over the surface area of the circular antenna (πa^2) and $d\underline{\sigma}'$ is parallel to z-axis.

$$\therefore \quad \underline{E}_m[t\,;R\,\hat{\underline{e}}_1] \approx \omega^2 I_0 / (R^2 c^3) \iint d\underline{\sigma}' \wedge \underline{R}\sin[\,\omega\,(t - R/c)\,] \approx \omega^2 I_0\ \pi a^2 \sin[\,\omega\,(t - R/c)\,]\ /\,(R\,c^3)\ \hat{\underline{e}}_2$$

If the whole length of the receiver is oriented along the y-axis ($\hat{\underline{e}}_2$), then the induced EMF is constant. $\mathcal{V}[t] = l\, E_m[t]$

$$\therefore \quad \mathcal{V}[t] = \mathcal{V}_C \sin[\,\omega\,(t - R/c)\,] \quad \text{where} \quad \mathcal{V}_C = l\,\omega^2 I_0\,\pi a^2 / R\,c^3$$

Thus, the induced voltage produced by the circular antenna varies directly with the square of the frequency and strength of the current in the source transmitter (antenna) and inversely (not inverse square) with the distance separating the receiver from the transmitter. . Most importantly, it varies with the length of the receiver (l) and the <u>area</u> enclosed by the transmitter. The receiver's induced voltage and current are exactly in phase with the periodic variations of EMF in the source transmitter. The maximum induced voltages in receivers of equal length and positioned at the same distance from linear and circular dipole antennas carrying the same maximum current may be compared directly.

$$\mathcal{V}_L / \mathcal{V}_C = L\,c / (\omega\,\pi\,a^2) = 2\,L\,\lambda / (2\pi\,a)^2 = 2\,\lambda / L >> 1$$

For the same length of wire used in the transmitter and for a given length of wire in the receiver, the linear antenna will induce a much greater voltage than the circular antenna; this is why most antennas used for commercial broadcasting are linear and oriented vertically.

These ideal calculations have demonstrated that the XCI fluctuations can explain how remote, oscillating currents can be induced in remote conductors without requiring the concept of intermediate "waves in the Aether" to connect receivers to transmitters. This illustrates how philosophers impose their own views of reality on the interpretation of their mathematics, which have been shown here to accommodate either far-action (as demonstrated here) or local-action, as demonstrated by Maxwell's field theory. As Maxwell admitted, when reviewing the far-action theories of Weber and Lorenz, both types of theory can **equally** describe the phenomena of electro-magnetism. The challenge is not to create mathematical summaries that describe the phenomena but to 'go under the surface' and produce realistic models that **explain** these effects.

7.9.4 SUMMARY OF THE EXTERNAL CONDUCTION-INDUCTION (XCI) INTERACTION

1. Vector Potential: $\underline{\mathbf{A}}[\underline{x}, t] = \oint dx'\, \underline{\mathcal{I}}[\underline{x}', t - r/c] / c\, r$ where: $\underline{r} = \underline{x} - \underline{x}'$ & $\min[r] = \Delta x_0$
2. Electro-kinetic Momentum: $\underline{\mathbf{P}}[\underline{x}, t] = \underline{\mathbf{A}}[\underline{x}, t] / c$ (E/M momentum per unit charge, q)
3. Electro-kinetic Force: $\underline{\mathbf{E}}_m[\underline{x}, t] = -\,d/dt\ \underline{\mathbf{P}}[\underline{x}, t]$ (Electric induction force per unit charge)
4. Electro-kinetic Energy: $\mathcal{E}[t] = \oint d\underline{x} \bullet \underline{\mathbf{E}}_m[\underline{x}, t]$ (E.M.F. = E/M energy per unit charge)
5. Electro-kinetic Action: $\mathfrak{A} = q \oint d\underline{x} \bullet \underline{\mathbf{P}}[\underline{x}, t]$ (Induction 'flux': $\Phi[t] = \oint d\underline{x} \bullet \underline{\mathbf{A}}[\underline{x}, t]$)
6. Electro-dynamic Force: $\underline{\mathbf{F}} = \oint d\zeta\, \nabla[\,\underline{\mathcal{I}}[\zeta] \bullet \underline{\mathbf{P}}[\zeta]\,] = -\,\underline{F}' = -\oint d\zeta \oint d\zeta'\ \underline{\mathcal{I}}[\zeta] \bullet \underline{\mathcal{I}}'[\zeta']\ \underline{r}\ /\,(c^2\, r^3)$
7. Induction ('magnetic') Field: $\underline{\mathbf{B}}[\underline{x}, t] = \nabla \wedge \underline{\mathbf{A}}[\underline{x}, t]$ (Source current spatial variations)

8. SUMMARY & CONCLUSIONS

In this final section, the results and conclusions from this paper will be briefly summarized in order to draw out the major implications from the material. The paper concludes with brief summaries of future papers in this programme. We will let the comparison of the analytic predictions of this theory with experiment be the justification for the assumptions made here.

8.1 OBJECTIVES

The objective of this paper was to demonstrate that after 150 years, Maxwell's theory of EM is **not** a suitable model for electricity, which is now known to be grounded in the motion of inertial particles with finite electrical charge. Classical EM is no longer based on Maxwell's theory of a universal æther but it is still taught as if electricity can be represented by a continuous form of electricity, now called electric charge density. Sometimes attempts are made to define continuum charge by introducing special mathematical limit processes that are then used to justify the development of Maxwell Equations but these techniques will never be appropriate to describe the interactions between a small number of electrons; none the less, Maxwell's Equations are still thought appropriate to form the basis of even quantum electro-dynamics (QED).

This research programme is based on the experimental observation that electricity consists of **electrons** that have been found to have no finite size; in other words, they can be viewed as ideal, mathematical points. But these fundamental particles do have a finite inertial mass that limits how they change their state of motion when involved together in EM interactions. As shown in section six, this constraint means that the asynchronous action-at-a-distance interaction, described colloquially as "on the light-cone", **forbids** this EM interaction from acting continuously. This also implies that no form of the differential calculus can be used to represent this basic interaction. Accordingly, any statistical summarization, like Maxwell's Field Equations, cannot be taken as a fundamental representation of this universal phenomenon. All field theories constructed upon similar assumptions of continuous interactions between inertial particles must also be abandoned. At the most fundamental level of physics, the mathematics used to represent reality must reflect the discreteness found in nature, not the seductive mathematics of the continuum that have been used for so long, while the world was thought to be continuous.

8.2 CLASSICAL ELECTRON THEORY

This research programme is positioned in the mainstream **Newtonian** tradition. The basic mathematical and metaphysical assumptions of this programme are grounded in experiments, which indicate that the real world exists in the form of discrete particles. In following Newton's metaphysics, this now means that in this programme reality is considered to consist only of electrons (viewed only as point particles) and the only interaction between them is electromagnetic. This approach avoids the problems of repulsive electric fluids, whether "flowing" in conductors forming "current densities", or as the "filler" in finite, rigid models of the electron that were popular around 1900, as proposed by such pioneers as Lorentz and Abraham. A brief historical summary of the state of knowledge when physics was first trying to understand the nature of the electron was presented in section two so that the critical transition from the continuum view of EM to one incorporating a more discrete viewpoint can be better appreciated. Larmor, in particular, proposed a model of the electron as a 'vortex' in the æther; although the present theory rejects all ideas of the æther, Larmor posited several characteristics of 'his' electron that reappear in this new view of the electron. It was shown that these 'atomic' ideas were gradually included in EM while still retaining the continuum mathematics implied by the use of partial differential equations. The ongoing view that the EM interaction itself was best described by the mathematics of 'fields' also retained the idea that the fundamental existents (the electric and magnetic fields) in this theory always existed everywhere throughout space and time – the ultimate expression of the plenum metaphysical view of reality. Finite models of the electron always produced equations that were unsolvable, as the retarded nature of the EM interaction required knowledge of the entire history of the electrons' motion. This situation continues today to confound classical solutions to the two-electron problem. These classical models also generated acausal pre-accelerations due to the necessity of treating finite spheres of the electron as rigid bodies. This asymmetry of action and reaction between the different elements in the finite electron contradicts Newton's Third Law of Mechanics, which declares these should be equal and opposite, guaranteeing total stability (and momentum) across the universe. These effects would **not** be present in a completely point model of electric charge: the view adopted here.

8.3 TWO-ELECTRON ELECTRON DYNAMICS

Section three described some of the published attempts to solve the model problem of two charged point particles moving under their mutual EM interactions without introducing any quantum restrictions. This review covered over 80 years of attempts to apply the concepts of classical EM to the dynamics of 'classical' electrons. The relativistic formulations used were effectively imposing the 'light-cone' condition on the interaction, while the use of differential operators reflected the assumption of continuous interactions between the two point charges. **All these attempts have failed**. The results of this paper (section six) now describe why all of these different approaches were doomed to failure. These major conclusions confirm the earlier results of the paper by Currie et al [35] that proved that it is impossible, within the framework of the canonical representations of the Lorentz group, to establish a relativistic mechanics for point particles using either standard Hamiltonian or Lagrangian techniques, as both of these mathematical approaches assume continuous forms of interaction and infinitesimal variations.

8.4 THE NEW ELECTRON INTERACTION

The fourth section focuses the study of EM back onto the interaction itself rather than the traditional single "field point" (or even a single electron) that is the locus of the EM field. This EM interaction is seen as the **only** defining property of every electron in the universe. This programme rejects the idea that a real particle can have any properties that are intrinsic; that is to say, all particle properties are a reflection of the interaction between the particles, even the so-called 'intrinsic' electron properties, such as inertial mass, *m* and electric charge, *e*. Both electrons, in every individual interaction, are treated totally symmetrically across space at two distinct points in time, each with a history (and future) and each mutually influencing the other asynchronously; this introduces the central idea that interactions are defined mathematically in terms of the differences in their times and locations of interactions. The interaction itself in this programme is **not** seen as a 'virtual' particle (as in modern field theories) even though finite amounts of momentum and energy are exchanged between the electrons each time they interact. Virtual particles would need to introduce another level of basic existence, confounding this ontological unity.

8.4.3 EM Interaction

It is one of the ironies in the history of physics that Maxwell first made his reputation by successfully developing 'atomistic' theories (the Rings of Saturn and the molecular model of gases) but for his theory of EM he was compelled to adopt the metaphysical programme of the 'energists', who viewed atoms as mathematical fictions, in contrast to their own view of the ontological primacy of energy. With the demise of the æther foundation of EM, all that was left of Maxwell's original EM conception were the 'fields of energy' ($\underline{E}$ and $\underline{B}$) and their purely mathematical descriptions. In contrast, the present theory is grounded on the experimental material facts of electrons and their intrinsic energy, inseparable from their movement and interactions. The concept of 'space' is not viewed as an ontological primitive but only as a conceptual 'short-hand' (like the idea of a hole in the ground – only the ground exists): as the relative separation between two electrons diminishes nothing real ('space') goes out of existence but the potential for dramatic action increases.

8.5 CNV TWO-ELECTRON ELECTROMAGNETISM

Section five established the mathematics used in this programme for describing continuous interactions between just two electrons. This was done by generalizing from the non-commutative algebra at a single point in space and time to a pair of such space-time points. In order to eventually describe discontinuous functions it was necessary to define two-particle, two-time derivatives in terms of the time-difference between the times at the two electrons when the single interaction occurs. The difference in the positions of the two electrons at two different times defines their mutual positional difference and the total differential of this positional difference with respect to their difference in times has the dimensions of a velocity but this definition is NOT the difference in the relative velocities at these two times ($\underline{v}_1(t_1) - \underline{v}_2(t_2)$) but the average of these two velocities. This is viewed as the natural two-time extension of the average of the definition of the SINGLE time average of the two velocities. All of these definitions are self-consistent and generate the full 'separability' of the individual velocities. These results for the position and velocity differentials for two particles, can be generalized to all anti-symmetric, separable functions F_{12} that describe the ***joint*** properties of associated pairs of electrons at two different locations in space $\underline{x}_1$ & $\underline{x}_2$ at two different times t_1 and t_2, when each particle's related (individual) property is characterized by $f_j[t_j; \underline{x}_j]$.

$$\textit{Separable Two-Particle Property}: \; F_{12}[\xi_1, \xi_2] \equiv f_1[\xi_1] - f_2[\xi_2] = F_{12}[\xi_1 - \xi_2] = F_{12}[\xi]$$

Here, the variable ξ denotes any one of the four fundamental dimensions $\{t ; x, y, z\}$. The anti-symmetry of these separable functions determines that electrons are fermions, as $F_{12}[0] = 0$. This technique was extended to 'difference' definitions of partial difference-derivatives allowing two-electron definitions to be formulated that bear marked resemblances to single value functions of standard vector calculus, including the two-electron (spatial) **difference-gradient** operator, $\underline{\nabla}_{12}$.

Since natural vectors (NVs) are linear functions of each electron's local parameters then the NVs of each electron can be combined anti-symmetrically into separable natural vectors, when all the earlier results for single-point CNVs may be used. The NV definition of a single electron's velocity was extended to form the definition of the CNV of the joint-velocity of two electrons. This became the basis for the extended definition of a two-electron Voigt vector. All of the single-electron Voigt vector results were reproduced but now in terms of these comparable two-electron CNVs and operators; in particular, the critical relationships now involve definitions of two-electron Gauge vectors, Flow vectors and the Wave equation.

The second paper in this series [54] applied the mathematics of single-time continuous natural vectors (CNVs) to a simple model of classical electromagnetism (CEM) that represented electricity by a continuous, incompressible fluid (the so-called Helmholtz fluid model). All of the standard results of CEM were very quickly recovered by assuming that the three central quantities in CEM (current, potential and electro-kinetic momentum) were simply components of comparable CNVs. Now, in section 5.2, comparable definitions involving just two electrons were developed that behaved functionally (covariant) like these earlier single point EM CNVs; these new results were summarized in section 5.2.5.

In section 5.3 the standard results of the theory of **special relativity** were directly derived from the CNV single-point theory of EM. It was shown that the assumption of separability of the electromagnetic momentum had to be added for the single-time version of special relativity to be extended to the two-electron CNV theory. This reproduced the results arising from the famous 'Planck Proposal' of 1907 for defining the relativistic forms of single particle momentum and energy but now in terms of the EM electro-kinetic momentum: in contrast to Planck's assumptions of constant, mechanical force on a particle. However, the present derivation implies that the two-electron scalar EM potential must now be a constant and this cannot be reconciled with the simplistic view that this is just the conventional Coulomb potential that varies with mutual distance. This apparent paradox will be resolved in the next paper, which will present a more realistic theory of classical electromagnetism.

8.6 TWO-ELECTRON CONTINUOUS ELECTRODYNAMICS

Section six is key to this paper where the dynamics of pairs of electrons moving under their mutual interaction is revealed. The new insights obtained from recognizing that applying the two-particle form of continuous natural vectors (developed in section five) to the locational and velocity differences of point particles centered on the idea that even simple classical (point) electrons must be treated as 'fermions'; in other words, their two-electron representations must be anti-symmetric. This view implied that their trajectories always remain distinct and there can be **no** self-interactions. Electromagnetism is here based on the proposition that interactions between electrons **may** only occur when they are "on each other's light-cone".

It was recognized in section 6.2 that the motion of a pair of interacting electrons is viewed optimally from a **s**ymmetric **i**nertial **r**eference **f**rame (or **SIRF**) centered on the set of interactions. This concept is essentially Newtonian in its views on the passive nature of space and time forming the framework for the inertial motion of each electron between interactions. Just as Newton introduced in his *Principia*, the concept of a particle's *inertia* to **resist** change in a particle's quantity of motion (or momentum) he simultaneously invented the concept of *impulse* to **generate** the change in the relative motion between particles. This programme adopts Newton's scheme for mechanics and extends it to the phenomenon of EM where interactions occur between electrons when they are separated across space and time ("asynchronous action-at-a-distance"). It is proposed that the fundamental EM interaction is symmetric in the SIRF so that the total (combined vector) velocity of the two electrons is always zero at any single instant of time. Since the inertial mass of each electron is an invariant in this theory then this proposition is just equivalent to the statement that the combined two-electron momentum is always zero, whenever the two electrons are committed to a series of consecutive interactions. In this model the basic EM interaction is always viewed as consisting of two perfectly correlated 'semi-interactions' so that the total interaction is independent of all 'labeling' conventions. The subsequent analysis demonstrated that these consecutive semi-interactions always occurred in parallel across space – any other combination leading to logical contradictions involving interactions 'off' the light-cone. The extension of Newton's Third Law of Motion to asynchronous interactions preserves the total symmetry of motion across the extended interaction thus agreeing with the analysis of the two-electron CNV velocity. This analysis leads to the **major** conclusion of this paper that continuous interactions on the light-cone are incompatible with the inertial property of real particles. An alternative mathematical formulation of this result is that the 'norm' of the CNV joint-velocity of any two interacting electrons is always zero.

8.7 MESOSCOPIC ELECTRODYNAMICS

Section seven is the largest part of this paper as it offers a new **physical** theory of classical electromagnetism (CEM) that is an alternative to both the original, aether-based theory of Maxwell and to the fluid-model of Helmholtz, which has now become the modern formulation of CEM in its charge-density manifestation. Maxwell's theory persists simply as a set of differential equations, a ghost-like remnant without its author's original ontological foundation. The Helmholtz model has undergone a similar ahistorical transformation, replacing Helmholtz's mathematics with those of his rival, Maxwell and absorbing the discovery around 1900 of the discrete electron into the non-physical amalgam of continuous electric charge. This new physical theory is a many-body model of *point-charges interacting in and between metallic conductors*: it forms the physical foundation of the new theory of CEM that complements the new mathematical representation (CNVs) that was used in the first part of the paper. This paper was written to demonstrate that the rival model (to EM field theory) developed between 1820 and 1880 by Ampère, Gauss, Weber, Neumann, Riemann and Lorenz (the '**continental**' tradition) and based on the metaphysics of Newton's mechanical philosophy of point-particles acting at a distance could be extended to a unified model of all CEM, including induction and radiation. It was the failure of this approach to account for radiation that allowed Maxwell's field theory to be viewed as the only explanation for 'light' and other remote EM interactions. This new theory incorporates key parts of this older research tradition into a new theory of multi-electron dynamics. This programme rejects all field concepts and views magnetism simply as a macroscopic epiphenomenon associated with the interactions between electrons when they are moving relative to each other, particularly when large numbers are tightly constrained in a closed-circuit conductor. Highly localized **particles**, not universal fields over all space, now become again the focus in physics.

8.7.1 MESOSCOPIC AVERAGES

The focus is now returned to the massive numbers of electrons moving dynamically near the surface of metallic conductors. This requires introducing an explicit statistical model of interactions between large numbers of electrons as they readily move through a fixed lattice of metallic ions. This is presented here as the **mesoscopic** model of multi-electron interactions. This is a conscious attempt to avoid the zero (i.e. continuum) spatial separation limit. The focus here is on re-introducing Maxwell's innovative concept of electro-kinetic momentum, using the present theory's exchange of real momentum between each pair of electrons whenever they interact. Magnetism is explicitly shown here to be simply an epiphenomenon – the extra effects of relative motion between remote interactions between many electrons. Aggregates of millions of electrons in each 'cell' hide the detailed individual pair-wise interactions and allow measurable averages to emerge from the collective effects of stable ensembles of very many electrons and positive ions moving around fixed lattice positions in solid metals. It is these complex aggregates that are the reality defined by the concept of (continuous) electrical charge density. It is the critical requirement that variations from statistical averages derived from sufficient numbers of electrons in a 'cell' of appropriate size are not significant *within* a cell but become important *between* cells (or layers of cells) that defines the mesoscopic approach here.

8.7.2 ELECTRODYNAMIC POTENTIAL

The use of mesoscopic cells retains the idea of localization from the single electron to several million of them **and** their lattice ions. This explicitly *statistical* theory recovers the results of CEM, demonstrating that the traditional model is **not** fundamental but only valid over '**long**' time averages for groups (or cells) of many (million plus) electrons that interact collectively. This averaging process leads naturally to the idea of the EM potentials, which sum up the effects from many different sources. It is the **vector potential** though that becomes the key mathematical concept in this approach, not the simple scalar (or Coulomb) potential, which usually dominates the stage in traditional presentations of CEM.

8.7.3 METALLIC CONDUCTION

Neither Maxwell nor his followers produced a theory of conduction because they did **not** believe in the concept of localized electrical charge. If physics is to remain a science then it must be grounded in the reality of experiments – not the elegance of mathematics. Soon after 1910, the rush to explore the consequences of Planck's quantization hypothesis meant that mainstream researchers in physics lost interest in developing a deeper understanding of metallic conduction; only Drude proposed a simple model to explain Ohm's Law of resistivity but this was not incorporated into the standard model of CEM, which soon took on a purely mathematical formulation. Metallic conduction was not revisited until the 'free-electron' quantum model of Fermi surfaces was used to explain semiconductors and insulators. The present model rejects this last approach as one that ignores the powerful forces present in metals: conduction electrons are **not** free particles: the resulting mathematical model should represent this fact. This new model of conductivity is extended to very low temperatures (isolated metals) for a different explanation of zero electrical resistance or **superconductivity**. Unlike the standard BCS model, superconductivity is here explained without introducing quantum energy states.

8.7.4 ELECTRONIC EQUILIBRIUM

This short section showed that even 'electro-static' situations, like the calculation of capacitance, do not even need the static electric field concept, as even these situations are actually examples of **dynamic** equilibrium. The mesoscopic approach showed that traditional continuum (calculus) techniques are not required but are simply mathematical 'conveniences'.

8.7.5 ELECTRONIC INERTIA

The concept of electronic inertia is shown here to be grounded in the mechanical inertia of the electrons themselves and their supportive lattice ions. Mathematically, this is summarized by the vector potential, which averages the delayed effects of the net electron currents in conductors on each other and other (even remote) electric charges in motion. This technique directly bypasses the traditional vector force densities (electric and magnetic fields) and calculates the statistical (time-averaged) possibility of how a real electron would move **if** placed in a given situation. Most physicists no longer know that Maxwell began his researches into EM by adopting the Neumann concept of the vector potential as his starting point before Maxwell eventually introduced the 'twisted spatial derivative' (curl) of this function to introduce the 'magnetic' field while dropping the sources of this velocity-based interaction. Since Maxwell had not fully appreciated Gauss's revolutionary dimensional analysis he failed to include the necessary **space-time conversion constant** c that is now called (unfortunately) 'the speed of light'. This was recognized by his rival Weber, who first estimated this quantity as involved in velocity effects in conductors. Maxwell referred to the vector potential (without attribution) when he introduced his density of electro-kinetic momentum, this is re-introduced again here but now with the explicit dimensionality factor A / c. This paper shows that **all** the dynamic effects of EM, from induction to radiation, can be calculated from the electro-kinetic momentum: no EM fields are needed.

8.7.6 EXTERNAL CONDUCTION INTERACTIONS

In order to emphasize the real source of EM interactions, the present theory refers to its central mechanism as the External Conduction Interaction (or **XCI**) that manifests its effects in the short-range as induction and electro-dynamic forces and in long-range interactions as EM radiation. Just as the magnetic field is a useful approximation in many situations, so is the so-called Lorentz force, which is defined in terms of the effects of a local magnetic field on a point charge in motion. In fact, it is Ampère's force that is more fundamental, as is shown here and recognized as such by the 'Continentalist' EM researchers. One of the key ideas in this research programme is that the 'Hyper-active' hypothesis should be replaced by the 'Saturation' hypothesis: this is the assumption that a given electron only interacts, at most, with one other electron at any one instant – **not** with all other charges at every instant (i.e. continuously): an assumption that is more convenient in simple mathematics. This new theory proposes that the target-set of electric charges interacts with many of the charges in the source over a small but finite time duration and the interactions, even at the target, do **not** occur all at the same time. This is a direct rejection of both 'instantaneous' and continuum physics; hence its description here as a discrete interaction and not as a 'force'.

The new CNV algebra is shown here to be the most direct (singular) representation of the vector (and scalar) potential views of EM interactions based on the relative separation and change of relative separation (velocity) between the pair of interacting objects, including one or two mesoscopic cells of very many conduction electrons. The concept of vector potential is shown to be sufficient to uniquely define the electro-kinetic energy, force and action as well as momentum in all EM situations: **no** other mathematical object is needed in this theory, which directly links classical EM concepts to the foundational concepts of Newtonian mechanics.

8.7.7 NEAR CONDUCTION INTERACTIONS

Several situations involving a limited number of macroscopic conductors that are relatively close to each other are examined in terms of the External Conduction Interaction (**XCI**) model. A detailed analysis here shows that the mesoscopic model is consistent with the CEM definition of the retarded Neumann (or vector) potential. The ratio of the external induced change in momentum to the internal electrostatic impulses producing the current in the source wire is here shown to be related to Weber's experimental ratio for the diminished effects of the interaction of the charges in motion (induction due to current) compared with the local, large effects they produce when there is no relative motion; i.e. the maximal electrostatic effects. It is not a coincidence that this key ratio is the factor that appears everywhere in the Lorentz transformation and all relativistic theories – here it is shown to be proportional to the probability of a remote interaction occurring relative to the probability of a local (source) interaction. The theory of Amperian forces between moving conductors is then derived using the XCI: this analysis confirms that this force differs radically from the standard Lorentz force when the conductors are aligned in the same straight line (longitudinal forces), agreeing with all experiments. Two examples using the vector potential are used to show that 'magnetic' effects produced by steady currents in fixed and circular conductors are only approximations when based on traditional methods, like the Biot-Savart law. These results show that Faraday 'magnetic' induction is just an implicit example of Henry's **current** induction. This removes the embarrassing 'duplicity' of explanations in CEM theory.

8.7.8 FAR CONDUCTION INTERACTIONS

This section examines situations where the spatial separations are large between the conductors, where temporal variations in one conductor can effect the electro-dynamic behavior of a remote ('far') conductor via the vector potential: a phenomenon known as EM radiation and believed to be only explainable by Maxwell's field theory. The focus here is on Hertz's famous experiments. Both straight ('electric') and circular ('magnetic') transmitting antenna arrangements are analyzed. Once again, the **XCI** theory is shown to be capable of providing a mesoscopic explanation for effects that had previously been believed to require some form of field theory for an explanation. It was this belief that the 'Continentalist' theory of EM failed to give any explanation that was used to justify dropping their Newtonian style approach and led to the notorious **schism** between mechanical and field-based theories: a split that propagated into optics and then into quantum theory. One of the primary motivations of the present programme is to return physics to a unified view of the world – both at the macroscopic and microscopic levels.

8.7.9 EXTERNAL CONDUCTION IMPLICATIONS

This paper has developed a comprehensive model of classical electromagnetism constructed on a new physical model of electrical conduction. This model proposes a *mesoscopic* view that explains the *macroscopic* experiments in terms of large numbers of mobile electrons moving as cohorts through the crystalline lattice of metallic solids. These 'cells' of electrons interact statistically like pairs of classical point-particles (the *microscopic* view); both levels are described mathematically by the use of Continuous Natural Vectors. This model is shown to be compatible with the vector potential concept of Neumann.

The analysis of the inertial motion of the two electrons between two interaction nodes leads to an equation referred to as the "Space-Time Integrity Condition" (§6.4). This equation defines the necessary and sufficient conditions for the 'before' and 'after' velocities of each electron relative to any particular interaction node and for the time intervals between each one of the successive interaction and is independent of the particular form of the interaction itself. This same equation immediately demonstrates again that *the continuous interaction solution is always forbidden*. The remainder of this research programme will therefore be based on the necessary assumption that all real electromagnetic interactions between electrons occur at an exact, integral multiple of a universal, time duration. This new universal constant will be referred to as the ***chronon***. It will be demonstrated that this proposition is sufficient to both eliminate the bizarre redefinitions of space and time intervals associated with the special theory of relativity and the baffling paradoxes that have been associated with the far too many interpretations of conventional quantum mechanics. Subsequent papers will demonstrate these claims in detail.

The standard approach to classical electromagnetism (CEM) has retained the mathematics of Maxwell's field equations and transposed these upon the non-physical charge-density model of Maxwell's rival, **Helmholtz**. In contrast, the successful use of the present model fully describes the macroscopic (experimental) phenomena of electromagnetism, both at the physical (conceptual) level and at the mathematical level in a single, integrated approach. This has the following major implications:

1. CEM is only a **mathematical** abstraction derived from experiments on electrical conduction in wires.
2. Electrical conduction is always a many-body (**collective**) system, too difficult to describe analytically.
3. The vector potential is sufficient to calculate **all** EM dynamic phenomena based on conduction sources.
4. A **mesoscopic** statistical theory of conduction can explain all the physics of induction and radiation.
5. Field theory is **not** appropriate as the foundational basis for describing any fundamental interactions.

8.8 CONCLUDING REMARKS

This paper has shown that the Natural Vector mathematics at the heart of this programme could be readily extended from representing single points to interactions between pairs of point objects, like electrons. The focus in this programme so far has been on analyzing continuous interactions, as this has always been central to physics ever since Newton. However, this paper has also shown that the concept of continuous asynchronous interactions is incompatible with **point** particles that have intrinsic inertial mass – the universal property of all real fundamental particles. Future papers will address this requirement.

The lack of awareness of the history of classical electromagnetism (CEM) has meant that modern students of physics (and their professors!) have only a mathematical view of the phenomena of EM: there is no sense of the actual micro-activity that is occurring in this area. In fact, there is almost no awareness that this subject is built on numerous experiments involving the interactions between metallic circuits (wires) carrying electrical currents. It was Oersted's discovery of magnetic effects created by galvanic currents almost 200 years ago that began the research that led to the science of "**electromagnetism**". At the heart of CEM is Faraday's intuitive idea of an all-pervasive 'magnetic field' that not only interacts with macroscopic magnets, but is also 'caused' to appear everywhere across all of space by both magnets and conduction currents. It was James Clerk-Maxwell who imported the new mathematics (field theory) used to describe the stresses and strains in a perfect, continuous medium to support his intuition that vibrations in such a 'luminiferous **Aether**' were the actual mechanism of light, itself. Maxwell's focus was always on local activity in this medium, manifest as electric and magnetic force densities and totally rejected the 'Continentalist' model of point, electric charges in motion. It was vital to Maxwell's model of EM that this Aether was completely stationary. This was necessary as the values of the electric and magnetic force density fields would vary when described by observers in different inertial reference frames; indeed, the magnetic field could even disappear from certain perspectives – this was an embarrassing characteristic for any entity that has to be viewed as fundamental. In fact, the magnetic field model is not even a physical explanation of EM unless this field is assumed to **exist**; i.e. a theory of reality can only be constructed on real entities that are presumed to have an independent existence. Maxwell had to invent his 'displacement current' as an extra, local term to describe mathematically the effects generated by the very many source electrons that were interacting with the target electron from many diverse locations (at different times). Eventually, Maxwell himself had to give up on imagining a mechanism for his medium, leaving him only with a mathematical set of equations. As Maxwell scholar, Torrance has written: "*There can be no doubt that Clerk Maxwell remained ill at ease with his own work right up to the end of his life, for somehow his theories did not satisfy his own standards of tenability*." [136] This is well illustrated by the CEM prediction that when a stationary charged particle is caused to move by a pulse-type force, the resulting velocity grows exponentially over time. [137] Obviously, this has never been measured and is usually dismissed with a wave of the hand that such pulses are not real. (Anomalies in science should always be investigated, not ignored.)

CEM is now taught as an abstract set of vector *differential* equations involving force-field densities in empty space, somehow self-generating out of nothing. The historical reality was that the results of many experimental observations on closed electrical circuits were summarized into a set of *integral* equations defined in and through these circuits. It was an act of pure imagination to then introduce physical intermediaries (the EM fields) that existed **everywhere** throughout space (but not **in** the wires) that are the cause of EM activity. The discovery of the existence of electrons as particles that carry quantized units of electrical charge around 1900 presented physics with the opportunity to return to the action-at-a-distance physics between point particles that had grounded classical mechanics and had been the mathematical basis for the Continentalist alternative to Maxwellian EM field theory. This step has not been taken due the elegance of Maxwell's (actually Heaviside's) EM Equations, which were greatly admired by the mathematicians in the twentieth century who have hijacked theoretical physics and who have only a symbolic interest in the world. Due to Maxwell's mistaken idea of a fixed Aether that supported static and dynamic stresses and strains, canonical CEM has adopted the view that these source-less force densities can be separated into those fields (electric) that only effect stationary charges from those fields (magnetic) that vary with the motion of the "test charge". Worse, in order to maintain this mathematical simplicity, the idea of a **single**, **universal** "time" parameter had to be retained, instead of building on the inherently two-time nature of the EM interaction between the remote charges (Gauss's *asynchronous action-at-a-distance*). This rejection of reality was accomplished by imposing the additional mathematical requirement of **locality**: a test charge is only affected by the fields at its own location at each instant of time. This major omission of the actual sources then led to the need for the Lorentz transformations to save the appearance of the EM equations, at **one** point in space and time, as seen by inertial observers moving at different fixed speeds. Einstein then was forced to redefine the foundational concepts of physics (space and time) to "derive" the Lorentz transformations from these newly modified mathematical parameters – physics has suffered ever since from this unnecessary introduction of illogicality and break with common experience.

The present theory rejects all of this mathematical superstructure that has now **no** physical correspondence and returns to the original action-at-a-distance concepts that had worked successfully for the first 200 years of physics. This theory is based on the modern view of matter, particularly metallic conductors, where electrons move through a 'fixed' crystalline lattice. The local repulsion of mobile electrons caused by local concentrations of excess electrons (introduced by the external sources of EMF) is balanced by attractive interactions with the positive ions forming the lattice. This is an extended form of Drude's conduction theory, described in terms of an average, statistical aggregation (the "mesoscopic conduction" model).

The heart of the new theory is centered on a deeper analysis of motional induction (or "magnetic" induction, as it is better known). Jefimenko was one of the few authors of CEM texts that recognized the centrality of this concept but restricted his analysis to mathematical current densities that are subject to generic time variations; he failed to introduce any microscopic model of electrical interactions, as is done in the present theory. This **physical view** of the statistical interactions between mesoscopic aggregations of electrons in conductors is sufficient to provide a single unified theory of induction and radiation: the only differences being the range of interaction and the rapidity in the changes in the source currents. The new mesoscopic model is shown here to be consistent with the CEM definition of the Neumann (or vector) potential that was central to the 19th century Continentalist model of EM and actually was the starting point for Maxwell's own EM ideas. The critical extension now is the incorporation of Gauss's radical suggestion that, unlike instantaneous gravitational interactions, the EM interaction is **asynchronous** – a finite time difference is required for EM interactions to occur between remote electrical objects. This is equivalent to the *Light-Cone* Condition and is manifest as delayed times appearing in the vector potential.

The major critique of this paper has been focused on the modern foundations of classical electromagnetism (CEM). Ever since Maxwell's original metaphysical model of the æther was discarded, CEM has been grounded in the concept of charge density and its corresponding current density as the sources of all electrical activity. However, this attempt to preserve the elegant continuous mathematics of partial differential equations as the description of CEM has failed to disguise the real nature of electricity, which occurs in the form of electrons. These discrete point particles always repel each other at all spatial separations so that the infinitesimal limit of the calculus eventually penetrates through all limit definitions of charge density as an adequate representation of a flux of real electrons. The inherent conflict of this foundational representation in CEM between these two limiting procedures restricts CEM to situations where the approximations of 'remote' interactions occur (relative to the extent of the charge sources) and/or macro collections of atomic objects of heavy positive charges together with electrons form the collective, net source of electrical effects. These CEM assumptions of continuity have also resulted in the need to invent the special theory of relativity to describe the electro-dynamical effects of charges in motion. A more realistic model of CEM will be presented in the next paper.

As a result of this analysis, this programme must now be extended to *non-continuous interactions* where the time intervals between consecutive interactions must remain **finite**. This will involve extending the mathematics of Natural Vectors to represent discrete (or discontinuous) variables. This will be the focus of the next paper in this CEM series that will feature Discrete Natural Vectors to form a DNV theory of interactions between pairs of classical point electrons. A later series of papers will report on the extension of these techniques to real electrons, whose interactions are themselves discrete in the amount of action that can be transferred on each occasion; these extensions will lead to a new theory of quantum electron mechanics (QEM). One of these later QEM papers will show that the introduction of interactions that are symmetric in time (i.e. the inclusion of 'advanced' effects) is the explanation for the Dirac relativistic theory of the electron and will show that the dual characteristic of transverse motion (i.e. two directions of circulation) is sufficient to explain the 'positron' – Dirac's famous positively charged 'anti-particle' of the electron. Indeed, this microscopic rotation around the electron's own axis of motion will be seen as the mechanical explanation of that QM 'mystery' – the **spin** of the electron. Subsequent papers will explore the resulting physics involving both positive and negative electrons at various scales of separation and quantity.

DEDICATION

This paper is dedicated to two independent researchers in physics: Oleg D. Jefimenko (1922-2009) and Carver A. Mead who both realized that the study of EM phenomena still needed more investigation. Each offered radical ideas (retardation and collective electrodynamics) to move forward the study of the interactions between large numbers of electrons. A further tribute is due to C. J. Carpenter, who also tried to focus the theoreticians of EM on the central importance of the vector potential in classical electrodynamics.

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