Continuous Natural Vector Theory of Electromagnetism

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II

ABSTRACT

A new algebraic representation is used to immediately recover all the major results of classical electromagnetism. This new representation ('**Natural Vectors**') is based on Hamilton's quaternions and completes the original attempt by Maxwell to use this powerful, non-commutative algebra in the final presentation of his theory in his *Treatise*. The foundational hypothesis here is that the principal electromagnetic variables are best represented by Natural Vectors, rather than the conventional 3D vectors defined by 'real numbers'. The present results avoid all use of the field concept and validate the retarded scalar and vector **potentials** approach first introduced by L. V. Lorenz, who combined Gauss's 1845 suggestion of the finite speed of interaction with Newton's action-at-a-distance model of physics into a charge-potential model of electromagnetism in 1867. This new approach demonstrates the primacy and physical significance of the 'Lorenz gauge'. Not withstanding Maxwell's **aether** theory, the present results are based on the continuous charge-density substance model of electricity that is used today to develop Maxwell's Equations for classical electromagnetism. The present analysis also demonstrates that Helmholtz's '**fluid**' model of electricity is one of the few that can result in an electromagnetic 'explanation' for the phenomenon of light. Unlike algebraic Minkowski 4-vectors, the more powerful 4-dimensional covariant Natural Vectors used here generate all the differential equations normally found in classical electromagnetism in an immediate and direct **algebraic** manner. This new theory focuses on the remote interaction between charges, which then appears both as variations in the charge-density and the potentials "traveling at light-speed across space". Surprisingly, this same result also appears for the current-density; this suggests that the conventional interpretation of this major symbol in Maxwell's Equations needs to be questioned further.

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Continuous Natural Vector Theory of Electromagnetism

ABSTRACT 1
1. INTRODUCTION & OVERVIEW 9
1.1 INTRODUCTION 9
1.1.1 RESEARCH PROGRAMME 9
A Major Paradigm Shift 9
Maxwell's Introductions 9
Einstein just wanted to understand Electrons 9
Newton's Introduction to Principia 9
The Continuum Hypothesis 10
The Continuum Fallacy 10
Physics: Two, One, Zero Objects 10
The Four Basic Concepts 10
Euclidean Theory 10
Relying on Visualization 10
1.1.2 OBJECTIVES 11
The Value of Natural Vectors in EM 11
The Value of Philosophy & History 11
Investigating the Charge-Potential EM Model 11
1.1.3 PHILOSOPY IS IMPORTANT 11
Modern Physics rejects Philosophy 11
Feynman View of Physics 11
Maxwell's View of Science 12
Defining Reality 12
'Schoolmen' prefer the Plenum 12
Aether, not Ether 12
Human Bias 12
1.1.4 HISTORY IS IMPORTANT 12
Nineteenth Century EM Mythology 12
History is always Important 13
A Brief History of Electricity 13
Maxwell's Treatise 13
Maxwell started with the Potentials 13
Not the Maxwell-Hertz Equations 13
1.1.5 THE RISE OF FIELD THEORY 14
Maxwell's Philosophical Introductions 14
Maxwell rejects Action-at-a-Distance 14
Infinite Decimal Numbers are not Real 14
Continuous Matter in 19th Century 14
Misdirection of Natural Philosophy 14
1.2 OVERVIEW 15
1.2.1 THEORETICAL PHYSICS IS METAPHYSICS 15
Philosophy must be Explicit 15
Maxwell's Natural Philosophy Questions 15
The Principle of 'Least Change' 15

1.2.2 METAPHYSICS OF ELECTROMAGNETISM 15
The Necessity of Metaphysics 15
Aetherial EM Metaphysics 16
Contemporary Views on Classical EM 16
EM Alternatives to Maxwell 16
1.2.3 MAXWELL-HEAVISIDE AETHER THEORY 16
1.2.4 MODERN CLASSICAL ELECTRODYNAMICS 17
1.2.5 SUMMARY OF CONTINUOUS NATURAL VECTORS 17
1.2.6 THE CNV ELECTROMAGNETIC MODEL 17
1.2.7 SUMMARY AND CONCLUSIONS 17
2. METAPHYSICS OF ELECTROMAGNETISM 18
2.1 THE NECESSITY OF METAPHYSICS 18
The Goal of Physics 18
2.1.1 NATURAL PHILOSOPHY 18
Philosophy as Latin Sloganeering 18
Natural Philosophy corrupted into Theology 18
2.1.1a PHILOSOPHY AS LINGUISTICS 18
The Primacy of Language 18
Philosophy as Verbal Maps 19
Need for Fundamental Definitions 19
To Be, to Have & to Do 19
Time is the Beginning 19
Impermanence in the World 19
2.1.1b ONTOLOGY: THE FOUNDATION 19
Greek Ontology 19
Real Language models the World 20
Vacuum is really Nothing 20
World based on 'Substance' 20
Non-interacting Ontology 20
Universe is 'All the Interactions' 20
What Is & What Happens 20
Entities & Processes 20
Qualities of Objects 20
Ockham rejects the 'Invisibles' 21
Accepting that Reality is Real 21
Abstractions are not Real 21
The What & the Why of Philosophy 21
2.1.2 PHILOSOPHY BEFORE MATHEMATICS 21
Geometry was first a Language Model 21
Philosophy, Logic and Math 21
Timeless Logic 22
Math maps Philosophy 22
Merton Verbal Math 22
Symbolic Representations 22
Physics adopts Mathematics 22
Bohr demands Physics before Math 23

Debate on Philosophy vs Mathematics 23
'Mathematics is Beautiful' 23
James Jeans as a Pythagorean 23
Pythagoras Triumphant 23
Today's Platonic Physicists 23
Re-ification of Math 23
Unreal Intermediate Symbols 24
Training Physicists like Mathematicians 24
The Cambridge Tripos 'Grind' 24
Quantum Mechanics needs more than Math 24
Science is not unsolvable Equations 24
The Mathematical Challenge 24
Aristotle: Philosophy has turned into Math 24
Summary: Not Pythagoreanism 25
Only Two Math-first Predictions 25
2.1.3 NEWTONIAN METAPHYSICS 25
Mechanical Metaphysics 25
Descartes' Natural Philosophy 25
Newton's Model of Physics 25
Newton rejects the Occult 25
Newton invents an Aether 25
Newton inserts Time into Geometry 26
Newton's Views on 'Mass' 26
Jammer's Views on 'Mass' 26
Momentum & Kinetic Energy for Mass 26
Not Newtonian Absolute Space & Time 26
Newton's Central Observer 26
Systems must exceed Three Particles 26
French Mathematicians transform Newton 27
Classical Mechanics' invalid Assumptions 27
2.1.4 MODERN METAPHYSICS OF SCIENCE 27
Kant's Ontology & Human Imagination 27
Fechner's Ontology 28
Turning Particles into Rigid Bodies 28
Theory's Philosophical Primitives 28
Energy is not an Existent 28
Work, not Energy 28
Spatial Potentials 28
No Conservative Forces 28
Least Action – across time 28
Planck rejects Mechanical Picture 29
Einstein seeks Unitary Basis 29
Ritz rejects Maxwell & Einstein 29
Entities vs Relationships 29
Cartwright: Models, not Reality 29
Theoretical Representations are not Unique 29

The Macrosphere 29
Students want only the 'Truth' 30
Meta-Principles of Science 30
Progressive Science 30
The Failure of Operationalism 30
Reductionism in Classical Physics 30
Reductionism as a Strategy 30
Physics is not just Analysis 31
Physics also needs Synthesis 31
2.1.5 VISUAL IMAGERY IN PHYSICS 31
Power of Visual Imagination 31
Newton's Visual Imagination 31
Faraday's Visual Imagination 31
Maxwell's Visual Imagination 31
Faraday's 'Lines-of-Force' 31
Maxwell's Philosophical Mentors 32
Einstein's Rhetoric 32
2.2 AETHERIAL EM METAPHYSICS 32
Metaphysical Rivalry 32
Maxwell demanded Metaphysics 32
2.2.1 THE ELECTROMAGNETIC AETHER 33
MacCullagh's EM Aether 33
Maxwell adopts Faraday's 'Lines of Force' 33
Maxwell assumes an Aether 33
Maxwell likes DesCartes' Aether 33
Maxwell's EM Aether similar to MacCullagh's 34
Models of the EM Aether 34
Boltzmann's View of Maxwell's Aether 34
The Aether results in Field Theories 34
Maxwellians need the Aether for Fields 34
Demise of the Aether in 1900 34
Aether Confusion in 1910 34
2.2.2 MAXWELL'S ELECTROMAGNETIC THEORY 35
Maxwell's Two earlier EM Papers 35
Maxwell's Schizoid 'Dynamical Theory' 35
Maxwell creates a Magnetic Theory 36
Maxwell adopts Electrostatics 36
Maxwell creates Two EM Theories 36
Wires, Fields & Potentials 36
Maxwell as 'Continuist' 36
Maxwell's Current as Aether Displacement 36
Discharging the Dielectric Condenser 37
Helmholtz's Microscopic EM Theory 37
Maxwell rejects the 'Electric Fluid' Model 37
Heaviside's Rewrite of Maxwell's EM Theory 37
Heaviside drops the Potentials 38

Experiments fail to detect dE/dt 38
2.2.3 ELECTROMAGNETIC WAVES 38
Maxwell'sTheory anticipates Light 38
Hertz proves Helmholtz, not Maxwell 38
Hertz re-derives Heaviside Equations 38
Hertz: Fluctuations, not Waves 38
2.2.4 CONTINUUM ELECTROMAGNETIC MATH 38
First Formal Physics 38
Field Theories involve Limits 39
Macroscopic to Microscopic 39
Instantaneous Surface Integrals 39
Maxwell's Mathematical Errors 39
Mesoscopic Theory 39
Maxwell's math 'Skeleton' 39
2.3 CONTEMPORARY VIEWS ON ELECTRODYNAMICS 40
2.3.1 COULOMB'S LAW 40
Theory only, Experiments inaccurate 40
Herzfeld's 'Derivation' of EM 40
2.3.2 'LORENTZ' FORCE 40
History of the 'Lorentz' Force 40
2.3.3 CLASSICAL ELECTRODYNAMICS TEXTS 40
Abraham & Becker's Text 41
Griffith's Text 41
Jackson's Text 41
EM Texts ignore History 42
2.3.4 QUANTUM ELECTRODYNAMICS 42
2.4 EM ALTERNATIVES TO MAXWELL 43
2.4.1 WEBER'S SINGLE-TIME PARTICLE THEORY 43
Weber's Emphasis on Interactions 43
Weber's Relative EM Force Law 43
Weber's Absolute Time 43
Franz Neumann's Vector Potential 43
Kirchhoff's Potentials 44
Maxwell uses Kirchhoff's Velocity 44
Carl Neumann's Electrical Conductors 44
Weber's FinalEM Theory 44
Helmholtz's Critique 44
C. Neumann defends Weber 44
Maxwell mystified by Weber's EM Theory 44
2.4.2 RETARDED ACTION-AT-A-DISTANCE 45
DesCartes' Contact Model 45
Medieval Resistance to AAD 45
Newton's views AAD as 'absurd' 45
Riemann follows up on Gauss's Suggestion 45
Gauss's Speed "c" Suggestion 45
Assis compromises Weber 46

Lorenz 1867 Paper on EM 46
Whitaker on Lorenz's 1867 Paper 46
Lorenz & Retardation 46
Jefimenko's causeless E&H 46
Borrowing Energy from the Future 46
3. MAXWELL-HEAVISIDE AETHER THEORY 47
3.1 EM FORCE FIELD EQUATIONS 47
3.2 MAXWELL'S DYNAMICAL THEORY 47
3.3 PROBLEMS WITH MAXWELL'S DYNAMICAL THEORY 49
3.4 EM ENERGY FLOW 50
4. MODERN CLASSICAL ELECTRODYNAMICS 51
4.1 HELMHOLTZ FLUID DYNAMICS EM MODEL 51
4.2 CHARGE-DENSITY ELECTROMAGNETISM 53
4.3 RECOVERING MAXWELL'S EQUATIONS 54
5. SUMMARY OF CONTINUOUS NATURAL VECTORS 55
5.1 DEFINITION OF SINGLE-POINT CNVs 55
5.2 SUMMARY OF CNV FLOW VECTORS 55
5.3 SUMMARY OF CNV VOIGT VECTORS 56
6. THE CNV ELECTROMAGNETIC MODEL 57
6.1 EM CURRENT 57
6.2 EM POTENTIALS 58
6.3 EM FORCES 58
6.4 EM GAUGE CONDITIONS 59
6.5 EM ENERGY 60
6.6 EM MOMENTUM 61
6.7 TEMPORAL INVARIANTS 62
7. SUMMARY & CONCLUSIONS 63
7.1 OBJECTIVES 63
The Study of History & Philosophy 63
7.2 METAPHYSICS OF EM 64
7.2.1 METAPHYSICS OF PHYSICS 64
Philosophy is Linguistic 64
Theoretical Physics is always Metaphysics 64
Theoretical Physics is now Pythagorean 64
Theoretical Physics is more than Equations 64
Maxwell calls for Interpretation of Math Symbols 64
Physics can progress from its Mistakes 64
Field Theory as Pure Imagination 64
The Epistemology of Science 64
Theory Refutation 65
Metaphysical Foundations – the Entity 65
Electrons are Fundamental 65
Entities, Relationships & Attributes 65
Realism as the Philosophy of Physics 65
Electrons are Real 65
Time is Universal 65

Time is Critical 65
7.2.2 AETHERIAL EM METAPHYSICS 66
Maxwell hijacked Newton in 1864 66
Maxwell's God fills the Void 66
Maxwell's Modeling Failure 66
Maxwell's Philosophical Errors 66
Maxwell's Failure in 1864 67
Maxwell's Theory same as Weber's Theory? 67
Classical EM is only an Approximation 67
7.2.3 CONTEMPORARY VIEWS OF ELECTRODYNAMICS 68
Electrostatic and Magnetic Forces 68
Modern Electrodynamic Texts 68
7.2.4 EM ALTERNATIVES TO MAXWELL 68
Maxwell acknowledges his Rivals 68
Weber's Contributions 68
Continental Contributions 68
7.3 MAXWELL-HEAVISIDE AETHER THEORY 69
7.4 MODERN CLASSICAL ELECTRODYNAMICS 69
7.5 SUMMARY OF CONTINUOUS NATURAL VECTORS 69
The Power of Non-Commutative Algebra in EM 69
7.6 THE CNV ELECTROMAGNETIC MODEL 70
7.7 CONCLUDING REMARKS 71
8. REFERENCES 72

1. INTRODUCTION & OVERVIEW

1.1 INTRODUCTION

This is the second paper reporting on a new theoretical research programme centered on the creation of a new physical and mathematical model of the microscopic activities of electrons, viewed here as the ontological foundation of all physical reality. For over 100 years now, physics has realized that the electron is the basis of all the phenomena associated with the concept of electricity. A major contention of this research is that the theoretical investigations into the actual foundations of electromagnetism (EM) ended prematurely due to the exciting discoveries of the electron and the new atomic physics in the early part of the 20th Century. As a result, physics has relied excessively on the mathematical formulation of EM, known as **Maxwell's Equations**, which are now developed through the artificial construct of 'charge-density'. Before presenting a more realistic theory of EM based purely on electrons, this paper demonstrates that the new algebraic representation, which forms the mathematical foundation of this programme, can used to **immediately** recover all the major results of classical electromagnetism (CEM). This new representation ('Natural Vectors') is based on Hamilton's quaternions and completes the original attempt by Maxwell to use this powerful, non-commutative algebra in the final presentation of his EM theory.

1.1.1 RESEARCH PROGRAMME

A Major Paradigm Shift

Physics, as a science, is divided into two very diverse parts: experimental and theoretical. Experimental physics discovers new phenomena that are reproducible by all investigators – as such its new findings may be astonishing but they are always irrefutable. The same cannot be said for the theoretical descriptions provided to explain these new phenomena. These are often contentious and, only in the case of major predictions that fail to match experiment, are not refutable. In these cases, sociological factors come into play to determine which theories flourish and which fade away. Once a theoretical paradigm becomes established it becomes very difficult for theoreticians to challenge the established orthodoxy of standard theory. This was the situation in the 19th century when two rival theories were competing to provide a comprehensive explanation for the phenomena of electromagnetism. The modern view awarded the victory to Maxwell's theory. The present research challenges this decision at the most fundamental levels. Accordingly, this paper must review both the historical context of this great debate and the metaphysical assumptions that underlay both positions. This review is necessary as the Maxwell 'victory' has defined the manner in which theoretical physics has now been conducted for almost 150 years. The present research programme contends that the paradoxes and lack of real progress in theoretical physics today is linked directly to this decision resulting in the present orthodoxy that 'field theory' is the only way for theoretical physics to be conducted.

Maxwell's Introduction

This research programme does not accept the widespread opinion today that EM is now a completed area of research. The prescient words used in 1873 by James Clerk Maxwell (1831-1879) in the Preface to his magnum opus *Treatise on Electricity & Magnetism* [1] still continue to be relevant (page x): "It appears to me that the study of electromagnetism has now become of the first importance as a means of promoting the progress of science." This is especially true in this research programme since it not only views EM as one of the four fundamental forces of nature but as the first and only form of interaction occurring in the real world, with electrons as the only fundamental existents.

Einstein just wanted to understand Electrons

This viewpoint leads to a very short statement from Einstein (quoted by Malcolm Mac Gregor) that summarizes the goal of the present research programme: "It would be sufficient to really understand the electron." [2]

Newton's Introduction to Principia

In introducing a radically different theory of physics it is impossible to resist re-using Newton's own introduction [3]:
"I design not to confute or convince you, but only to present and submit my thoughts to your consideration and judgement."

The Continuum Hypothesis

This paper forms the second in the CEM series documenting a new research programme that challenges some of the major assumptions of the last 150 years of theoretical physics. In addition to returning to the original approach to undertaking investigations in natural philosophy, emphasizing the value of philosophy and the history of science, this programme now challenges the foundational "Continuum Hypothesis". This is the universal (but implicit) metaphysical view that reality is fundamentally continuous so that it must be described by continuous mathematics. Since 1900, physicists have reluctantly come to the realization that electricity is only particulate (electrons) and matter is atomic. This has meant that only space and time are left to carry the burden of physical continuity. Accordingly, applied mathematicians, who now dominate all departments of theoretical physics, have focused on field theory as the last area where the mathematics of the continuum (differential calculus and continuous group theory) can still be used. This new research programme takes an aggressively discrete position: its introduction was announced with the description of a new form of algebra suitable for representing asynchronous action-at-a-distance [4]. The goal here is to describe the discrete nature of reality using only non-continuous concepts and ultimately only discrete forms of mathematics to represent these concepts. Continuum mathematics will be reserved in this research programme only for aggregating from the discrete microscopic world-view to the <u>apparently</u> continuous view of the macro world.

The Continuum Fallacy

Almost all physicists since Newton have assumed the validity of the Continuum Hypothesis when describing the dynamics of matter (the time interval between interactions smoothly goes to zero). This hypothesis is a very good approximation in many macro situations (e.g. planetary motion) and therefore is useful. But in two 'extreme' realms beyond the 'classical', where standard physics has continued to make this assumption, the resulting predictions appear bizarre. By emphasizing discrete mathematical representations, the goal here is to avoid the infinite (integral) divergences that have plagued major areas of fundamental physics for over 75 years.

Physics: Two, One, Zero Objects

The history of physics illustrates a very embarrassing reduction in capability in solving the exact dynamics of a limited number of interacting objects. Newton's theory of gravity could solve the problem of two attracting masses but has never been able to solve any problem involving three or more distinct masses - the infamous 'three-body problem'. Einstein's own theory of gravity could only solve problems involving a single body but not two or more. The 'best theory in physics', namely quantum electrodynamics (QED), cannot even solve the single electron problem since this EM theory views 'empty space' (no bodies) as infinitely divergent with space filled with an infinite number of virtual particles. A powerful indicator of progress in physics will be the exact solution of systems involving three or more interacting electrons.

The Four Basic Concepts

The present research programme is founded on four inter-related concepts, none of which can be defined or even understood without the others; these four foundational concepts are: 1) Time 2) Space 3) Electrons 4) Interactions (between electrons).

Euclidean Theory

Statements of the type: "Space (or nature) is Euclidean" assume that the world can be mapped by standard geometry, where the only basic elements are the straight-line and the point. This is the position taken here by this research programme as the metaphysical primitives of the real world are simply taken to be electrons and their interactions ('light'). Here electrons are assumed to be true point particles while their interactions are hypothesized to occur between only two electrons at only two points in time, involving the shortest spatial distance between them in space, so that 'light' is viewed here as infinitely 'thin' point-to-point lines, not as expanding spherical waves spreading throughout all of space for all of time.

Relying on Visualization

This programme was developed throughout with an explicit commitment to 'visualization', especially in the use of imagery that can be abstracted in the form of graphical models and diagrams [5]. This mode of thinking was not used by Dirac but was used extensively by Newton, Einstein and most 19th Century physicists; since the introduction of quantum mechanics in 1926 this technique has been heavily de-emphasized, except, perhaps by Feynman. As theoretical physics has now become almost totally mathematical physics it has focused nearly exclusively on analytic and algebraic methods. However this type of intellectual thinking fails to draw on the enormous areas of the mammalian brain that are dedicated to visual processing, including the human visual imagination.

1.1.2 OBJECTIVES

The Value of Natural Vectors in EM

The principal objective of this paper is to demonstrate that a particularization of Hamilton's quaternions, referred to here as 'Natural Vectors', is a better representation for describing the classical theory of electromagnetism (EM) than the traditional use of 3D vectors that have been used for studying this area of physics since Heaviside rewrote Maxwell's Equations over 100 years ago. Although this research programme is firmly committed to the idea of a discrete view of the world, it was deemed important that its mathematical foundations be seen as capable of describing classical electromagnetism (CEM) before being extended into discrete formulations that are more suitable for describing the discrete nature of electrons. A new theory of EM will be presented in subsequent papers that is inherently quantized but can be readily approximated back to traditional CEM.

The Value of Philosophy & History

The second objective of this paper is to demonstrate that Maxwell's theory is deeply flawed in its metaphysical foundations. This will be accomplished by conducting both historical and philosophical analyses of its wider context and interpretation. A review of the historical background to Maxwell's revolutionary theory will show that he was <u>reacting</u> to a much older tradition in natural philosophy that had extended Newton's classical approach to the study of EM. This 'Continental' EM programme was centered on the remote interactions between charged particles and not on the dynamics of an invisible and universal medium. It will be shown that Maxwell's rejection of this older approach was motivated by his philosophical preferences as he finally admitted that this rival 'German' approach was equally successful in describing EM phenomena. The new EM theory to be developed in this new research programme has a much greater affinity with this nearly forgotten alternative approach (exemplified by its principal proponent, Wilhelm Weber), so this will be given some visibility herein.

Investigating the Charge-Potential EM Model

Maxwell's views on the nature of electricity were key to the development of his EM theory but were subsequently found to be totally erroneous. Rather than 'stresses in the aether' manifesting electric charge, free electricity was subsequently seen as a special kind of continuous, perfect fluid now known as 'charge density'. This is the basis for all modern expositions of CEM. Mathematical 'gymnastics' were finally introduced when the electron was discovered and electricity was recognized to be particulate and not discrete: the goal was to preserve the mathematical form of Maxwell's Equations at all cost, as this set of vector differential equations had become one of the central foundations of modern physics. Indeed, these continuum equations were the ultimate justification for all 'field theoretic' approaches used throughout advanced quantum theory. This reinterpretation of Maxwell's equations in CEM is reviewed here to fulfill a further objective of this paper: namely exposing the broad range of mathematical assumptions that have had to be made that have minimal support in any view of reality.

1.1.3 PHILOSOPY IS IMPORTANT

As this paper is the second in the series documenting a new research programme, it continues to try to meet another of this research programme's objectives: namely, to emphasize the importance of philosophy in the study of nature – a viewpoint that only seems 'odd' by the recent standards of the late twentieth century physics. It is another contention here that new concepts are more important than mathematical innovation; powerful symbolic manipulations make for elegant mathematics but provide little insight into the deeper mysteries of nature. In this research, mathematics always plays a subsidiary role.

Modern Physics rejects Philosophy

The refusal to admit any explicit philosophical interpretation into modern physics, which is the default situation today, is itself an implicit philosophical position – the Trojan Horse of the mathematicians (Pythagoreanism) or the 'Common-Sense' of the experimentalists (Positivism). The confusion surrounding the foundations of physics today is viewed here as a direct consequence of this explicit rejection of philosophy – a result that our predecessors in natural philosophy would have easily predicted.

Feynman View of Physics

It was unfortunate that Richard P. Feynman (1918-1988), one of the most original, visual and intuitive physicists of the 20^{th} Century, was typical of the modern physicist in that he showed almost no interest in the history or philosophy of science.

Maxwell's View of Science

Science examines sets of similar real objects or events. If these phenomena recur then humans can either manipulate these objects directly through experimental science (e.g. electronics) or just observe these objects (e.g. astronomy). As Maxwell wrote "abstract thought needs to be continually checked through contact with reality, and it is more important that our basic thoughts should have a living root in experience than they should be perfectly self-consistent." As a major scholar of nature, Maxwell also wrote [6]: "As regards the most interesting of all subjects, the history of the development of scientific ideas."

Defining Reality

This programme defines 'reality' as that part of the world that interacts with human beings, whether we are conscious of this interaction or not. This reality is independent of any specific human being and cannot be ignored as our individual existence is contingent on our successful choices when dealing with this reality. Humans have developed quite successful mappings of this reality that can be communicated via symbols between individuals to assist in our choices and activities. All natural languages use declarative statements to describe these mappings. The concept of 'truth' is the measure used to indicate the accuracy of this mapping process, so truth will possess the same characteristics of objectivity, consistency, etc that a society ascribes to its view of reality. Some societies develop meta-processes for validating their language maps. Individual events are mapped successfully as 'facts', while recurring patterns for classes of similar events can become scientific 'laws'. When individuals deny the possibility of either reality or truth they are attempting to evade responsibility for their own choices and actions. In its extreme form, 'nihilism becomes the philosophy of the embittered and the failed'.

'Schoolmen' prefer the Plenum

Philosophy has proposed two opposing and complementary views of the foundation of the world – the *point* and the *plenum*: or reality viewed as constructed from particles or as an 'aether'. The all-pervasive aether has been the preferred world-view with academics ever since the medieval 'Schoolmen' revived Aristotle's "fifth element in the heavenly spheres". This is still the unspoken assumption today in physics although it is usually cast in its modern mathematical form – the "Continuum Hypothesis", as such it is rarely challenged; this research programme now directly takes up this ancient challenge.

Aether, not Ether

In spite of spell-checkers, this research programme uses the traditional spelling for the concept of an all-pervading medium: 'aether' as this was always the original Anglicization of the original German word Äther, where the umlaut a is replaced by the dual letters 'ae'. The word 'ether' will be reserved for that chemical compound that rapidly induces sleep in its victims.

Human Bias

Appealing to macroscopic human sensory experience as a method to judge the validity of a scientific proposition (e.g. the future as well as the past also influences the present) is to revert to pre-atomic or even classical arrogance when the idea of the atom was rejected because 'we cannot see them, so they cannot exist'. The present research programme will explicitly introduce a series of theoretical propositions that, at first glance, are "preposterous" by this standard of 'common sense'. As history has shown, it is only by challenging these unspoken, universal assumptions that progress can hope to be made.

1.1.4 HISTORY IS IMPORTANT

Information without context is not knowledge; it is therefore important that the modern student of physics re-establish his roots in the history of his subject. Without this historical perspective he will neither understand why his equations take the form that they do nor, indeed, what even these symbols represent; worse, he will not appreciate what physical and even metaphysical assumptions underlie these theories. This lack of historical context has impeded recent progress in physics.

Nineteenth Century EM Mythology

As physics is a science dedicated to the understanding of reality, this paper has deliberately included a summarized history of the developments in EM research in the 19th Century. The primary motivation for this inclusion is that, as shown here, the influential authors of some of the most popular textbooks on this subject have failed to tell this story – this omission has resulted in a false mythology of classical EM that is widely believed but bears little resemblance to the facts. Accordingly, if major revisions are to be admitted into this foundational area of physics then it is necessary to place any new theory in its correct historical context, so as to illustrate its continuity and innovations with respect to <u>all</u> prior investigations.

In keeping within the stated methodology of this research programme described in the first paper in this series [4], many relevant summaries of the historical contributions by major physicists and philosophers will continue to be re-introduced since another lesson learned from the history of science is that fundamental science does not evolve incrementally. This is illustrated by showing how not only Maxwell's own ideas and those of his followers in EM developed but also by showing how the present theory of classical EM developed discontinuously after Maxwell's views on the aether were discarded. Whenever a major contributor is first mentioned, his life-dates will be included, as these temporal milestones indicate how scientific progress has been built on a network of innovation: once more, reflecting a Hegelian dialectic rather than a slow Darwinian evolutionary (incremental) progression in fundamental thought.

History is always Important

As in all organic processes, whether for an individual, a society or a cultural activity like physics, what has gone before is the major contributor to what is the situation today. Ignoring history is a very dangerous strategy. It is this conviction that drives the current research programme but does not form any part of the education of contemporary physicists. Sadly, if the History of Science (or even Physics) is ever offered to today's students it is viewed, at best, as an amusing elective, and not critical to the understanding or professional success of the modern physicist. This programme will try to refute this view.

A Brief History of Electricity

Soon after Isaac Newton (1642-1727) introduced the geometric form of mathematical physics in his magisterial book, the *Principia* [7], the study of electricity began to intrigue natural philosophers. One of the earliest of these thinkers was the English philosopher Francis Hawksbee (1666-1713), who visualized electricity as "effluvia" (a cloud of subtle, Newtonian corpuscles) that were all invisible weightless and able to pass through certain, solid materials (like glass) without interaction. Electricity was recognized to be an important characteristic of the world [8], so that by the end of the 18th Century it formed a major component of every course on natural philosophy. It was Benjamin Franklin (1706-1790) who correctly guessed that rubbing glass does not produce electricity but either adds or removes it from the normal, equilibrium state of zero net electricity thus making bodies appear to be positively or negatively charged.

Maxwell's Treatise

Although today's physics students all acknowledge the pioneering contributions of James Clerk Maxwell for developing the classical theory of classical electromagnetism, nearly all students (and most of their teachers) do not realize that Maxwell's *Treatise on Electricity & Magnetism* [1] does not contain the four famous "Maxwell Equations", nor does it even suggest how EM waves might be produced or detected. In fact, this famous text does not even explain the simple reflection of light. When it was first published in 1873 the *Treatise* was read by only a few since it was seen as awkward, confusing and very difficult. Even at Maxwell's own university (Cambridge) it was seen primarily only as a source for questions on the final mathematical (and very tough) 'Tripos' examinations. Maxwell's tragic early death at age 49 from stomach cancer meant that the elaboration of this theory required the further theoretical and experimental investigations of three younger British physicists: George F. FitzGerald (1851-1901), Oliver Lodge (1851-1940) and Oliver Heaviside (1850-1925). It is their radically, revised version that history has remembered as Maxwell's theory of electromagnetism [9].

Maxwell started with the Potentials

As B. J. Hunt, the biographer of these three most famous "Maxwellians" wrote [10]: "few readers of 20th Century textbooks on electromagnetism would ever guess that Maxwell built his theory on the EM potentials ($\underline{A}$ and ϕ)". As will be shown here, the obsession with the *field* concept is so strong that these *potentials* are now viewed only as arbitrary 'conveniences'. This was the perspective that the second-generation 'Maxwellians', especially John Poynting and Joseph Larmor, brought to fruition before the fundamental flaw in this whole theory of the aether began to unravel with the discovery of the electron.

Not the Maxwell-Hertz Equations

It is one of the travesties of science that many scientists, including Albert Einstein (1879-1955) in his 1905 special relativity paper [11], referred to the most famous equations in 19th Century physics as the "Maxwell-Hertz Equations", although Heinrich R. Hertz (1857-1894) himself had already acknowledged in 1893 in his text *Principles of Mechanics* that he had been strongly influenced by Heaviside's vector field formulation. Furthermore, Hertz's infamous remark (now the modern viewpoint) that "Maxwell's theory is Maxwell's Equations" was itself a direct reference to the later Heaviside mathematical reformulation, since Hertz was opposed to Maxwell's foundational EM concept of 'displacement' in the free aether [12].

1.1.5 THE RISE OF FIELD THEORY

Maxwell's Philosophical Introductions

Maxwell introduced each of his three major papers on EM "with a longish philosophical introduction". As a mathematician and natural philosopher, Maxwell was aware of the dangers of complex mathematics: "… the mere retention of intricate mathematics in the memory materially interferes with further progress". He also cautioned against using just mathematical formulations for establishing the basis for new directions in science as "we can entirely lose sight of the phenomenon to be explained … never obtaining more extended views of the connexions of the subject". [13] – one of the ironies of history.

Maxwell rejects Action-at-a-Distance

Maxwell, with his extensive knowledge of the history of science [14], joined the majority of philosophers and still rejected Newton's revolutionary concept of 'Action-at-a-Distance' as the basis for EM in favor of a continuous medium (the aether) even though several major French and German 19th Century physicists, such as Ampere and Weber, had adopted Newton's idea for gravity as the model for EM. While Ampere and others expressed the effects of EM in terms of summations (or integrals) of flux, Maxwell re-expressed these effects only in differential form, where the force in any one element (or cell) depends only on the immediately adjacent neighboring elements (nearest six cells) with effects passing continuously (in time) from element to element across all of space. When these cells (of aether) are treated in the limit as vanishingly small (differentials) then this theory becomes an example of a field theory that is mapped by 'real' numbers. In this mathematical light, Maxwell can be rightly seen as the 'father of field theory'.

Infinite Decimal Numbers are not Real

As a move towards greater intellectual honesty, mathematicians would be well advised to return to Newton's original nomenclature of (infinite) "decimal numbers" rather than the modern usage of "real numbers", a term which is more than ontologically suspect and reflects the Pythagorean world viewpoint. Such powerful terminology can seduce the scientist into thinking that all representations of reality must involve this notation [15]. The so-called 'existence' of mathematical objects also (deliberately) clouds the ontological distinctions between physics and mathematics.

Continuous Matter in 19th Century

All field theories rely on variables that are continuous functions of position in space. Prior to the 20th Century, this was a consistent viewpoint because matter (the 'stuff' of the real world) was itself seen as continuous and could, in theory, be divided indefinitely without limit. The EM aether was viewed as just another form of matter (invisible and imponderable) that in itself was also continuous and infused 'gross' matter that was subject to the mutual effects of gravity (ponderable matter). These two forms of matter only interacted in their local neighborhoods and not by remote action at a distance.

Misdirection of Natural Philosophy

Although the Maxwellian programme was a magnificent exercise in conceptual and mathematical innovation, it is the central contention of this research programme that this introduced the most serious misdirection in the evolution of natural philosophy since its Newtonian origins – field theory has been a 150 year-old mistake. Maxwell's EM theory was a model of an elastic continuum focusing on both the kinetic energy of an aether cell and its energy of displacement. When the idea of the luminiferous aether was abolished then Maxwell's EM theory should have been replaced as well but scientists have a history of holding on to obsolete theories for a very long time. This programme will suggest a new, discrete alternative.

1.2 OVERVIEW

In this overview, the contents of each section are briefly described along with the major reasons that the particular material has been included. The paper ends with a Summary and Conclusions that focus on the implications of the material covered and the new results obtained, along with brief reviews of future papers in this series.

1.2.1 THEORETICAL PHYSICS IS METAPHYSICS

Philosophy must be Explicit

All fundamental theories in physics are logical sets of explicit hypotheses, usually presented in a mathematical framework. Historically, these hypotheses were usually accompanied by a metaphysical discussion describing the implications of these assumptions. Philosophers of modern science have pointed out the deep problems associated with developments in physics over the last hundred years but rather than address these serious issues, almost all contemporary physicists have rejected all philosophical aspects of their science, especially the metaphysical foundations, which have now gone underground while still remaining implicit behind the surface of advanced mathematics. It is the firm belief of this programme that progress in physics will only resume when physics faces up to these hidden problems and rebuilds on solid, philosophical foundations.

Maxwell's Natural Philosophy Questions

Any natural philosopher can take inspiration from one of its greatest practitioners, James Clerk Maxwell. This is illustrated by the principal questions he presented to his audience when he gave his Introductory Lecture on his appointment to the first Cavendish Professorship at Cambridge University. On that occasion he identified two major metaphysical questions: firstly, "was matter continuous or discontinuous?"; secondly, "should matter be described in probabilistic or deterministic terms?". It was this mode of thinking that meant that he could only conceive of a 'displacement current' in vacuo by assuming the existence of a material aether. This programme will follow the tradition of natural philosophy in its research into nature.

The Principle of 'Least Change'

This research programme has adopted the conservative principle of 'least change' in its approach to extending the science that has preceded it; in other words, concepts that have proven successful in physics should only be modified rather than replaced. Newtonian mechanics proved its value for over 150 years before Maxwell introduced his revolutionary changes associated with the use of 'field' concepts in physics. This programme has tried to retain as much as possible of Newton's metaphysics that provided the foundations for Classical Mechanics. This has meant retaining the 'passive' view of space that underlies the *Principia* and attention is focused on the concepts associated with the idea of point particles rather than the "space in between". In particular, the key concept of the mass-point particle is still the principal existential object in this research but with the electron now playing this central role. Rejecting the field concept, this programme also now returns to a modified version of Newton's original action-at-a-distance ideas. Newton's original impulse model of interaction is again resurrected but here with the new focus now equalized between the two basic particles interacting asynchronously at two different times; this extension is minimized by limiting each interaction to only two electrons in any one time cycle rather than the multi-source averaging over space and time that resulted in the traditional 'force' concept.

1.2.2 METAPHYSICS OF ELECTROMAGNETISM

The Necessity of Metaphysics

This research programme is founded on the contention that theoretical physics is natural philosophy; without any philosophy physics then becomes indistinguishable from applied mathematics. The first part of section two briefly argues for a return to this older perspective. Since electromagnetism in this research programme is viewed as not only fundamental but is the very foundation for all of physics it is necessary to investigate the actual foundations of physics itself; or in terms of philosophy, ontology – the study of existence. Unlike modern physics, this programme does not require an "infamous boundary" [16] when the very nature of the world changes, as the physical separation between objects shrinks from the macroscopic scale of everyday interactions to the microscopic world of the 'quantum'. Quantum effects will be introduced in a later paper.

Aetherial EM Metaphysics

The ontological focus in this programme therefore centers on the principal point-particle of modern physics: the electron. Rather than follow Lorentz and produce a dualistic theory of electrons and EM fields, this programme needs only to use electrons and their action-at-a-distance interactions. Modern physics has adopted a monistic theoretical perspective by viewing all particles as fields but still fails to explain how the spatially diffuse concept of the field is able to manifest itself experimentally as point objects. The second section continues with its historical review emphasizing the struggle that the point concept has had to overcome compared with the academically more popular continuum concepts. The very seductive power of mathematics has meant that theoretical physicists have been most reluctant to give up their 300 year-old affaire with the mathematics of the continuum, particularly the calculus, even when all irrefutable experiments have demonstrated repeatedly the discrete nature of the world. This is described here when physics around 1900 was faced with the realization that the electron was both a point particle and was the actual exemplar of electricity itself. Perhaps recognizing instinctively that Maxwell's continuum views of electricity and the point nature of the electron were incompatible, several attempts were made at this time to produce finite models of the electron, almost always with bizarre results.

Contemporary Views on Classical EM

Belief in the central metaphysical concept of all Maxwellian type theories of EM, namely the aether, went into rapid decline soon after the 20th Century began. Popular mythology has attributed the 'death of the aether' to the publication of Einstein's famous 1905 paper on electrodynamics [11], where he claimed: "The introduction of a 'luminiferous aether' will prove to be superfluous because the view developed here will introduce neither an 'absolutely resting space' provided with special properties, nor associate a velocity vector with a point in empty space in which electromagnetic processes occur." In reality, very much more significant in the demise of the aether were the several influential EM textbooks that took a Helmholtz-like approach of using continuous velocity fields in their exposition of electrodynamics. Three of these very influential texts are reviewed here to show how the transition from a physical to a mathematical viewpoint arose. These historical summaries set the scene for the development of the special theory of relativity that will become the focus of the fourth paper in this series.

EM Alternatives to Maxwell

In reviewing modern texts on EM a reader would never guess that there were ever any alternatives to Maxwell's EM theory. This sub-section briefly describes some of these alternatives as they have provided fruitful material for the present approach. The most powerful contributions center on extending Newton's idea of instantaneous action-at-a-distance to one where a finite time must occur for any two particles that are interacting across a finite spatial separation. Most such attempts have retained only the traditional 'retarded' perspective wherein only the past effects the present; this programme adopts the more general symmetric viewpoint, where the future may also effect the present – hence the new phrase "asynchronous action at a distance" (or AAAD). The first successful 'retarded' attempt was due to L. V. Lorenz [17] whose 'no-field' AAD theory, using his ideas of the scalar and vector potentials, had produced all of the results that Maxwell's theory had achieved only two years earlier in 1865. Like the references to Voigt, this programme wishes to help restore the reputation of other major contributors to EM theory such as Lorenz, who are presently almost forgotten. Indeed, so-called Voigt-vector potentials assume a central role in the present theory.

1.2.3 MAXWELL-HEAVISIDE AETHER THEORY

Since this programme rejects all of Maxwell's metaphysical assumptions underlying the final version of his EM theories only the final mathematical equations of his 'dynamical' theory of 1864 are summarized at the beginning of section three. This summary is presented in the modern notation introduced by Heaviside when he 'gutted' Hamilton's 4D quaternions to produce the 3D vector notation better suited for a purely geometric exposition. This paper reverses this process and shows that a much more powerful mathematics for representing EM has been overlooked for over 150 years. The next sub-section quickly recreates Maxwell's original derivation to illustrate how Maxwell finally thought about this area of physics that had obsessed him for most of his life. As primarily a theory of magnetism, Maxwell focused on the concept of electro-kinetic momentum (now called vector potential) viewed as the impulse of the electromotive force generated by the removal of all the source magnets and currents. Maxwell's most original contribution to EM theory was the addition of the 'displacement current' into Oersted's law so that the free-space versions of the resulting equations reduced to the standard wave equations. This recreation illustrates the numerous errors that Maxwell made in developing his original group of eight equations. The algebraic corrections made by Heaviside are then added that resulted in the famous four 'Maxwell Equations'. Fortunately, this 'cleanup' eliminated the controversial displacement current along with the scalar and vector potentials that Heaviside viewed (correctly) as implying 'spooky' action-at-a-distance, exemplified by Lorenz's 1867 theory.

1.2.4 MODERN CLASSICAL ELECTRODYNAMICS

Section four is included to show the evolution of the modern approach to CEM that is actually based on Helmholtz's failed hydro-dynamical model of EM that provided the mathematical isomorphism with parts of Maxwell's EM theory. The key feature of all of these 'continuum' models is their subtle inter-weaving of the point-particle viewpoint with the continuum vector field models defined in terms of calculus-like, limit definitions. The approach used here follows closely the lectures on electrodynamics given by Pauli who carefully introduced limit definitions for the charge-density and force intensities. None of the modern expositions points out the significance of assuming that these limit definitions are compatible with the limit definitions of velocity of the actual charges. The incompressibility of the continuous charge-density is later shown to be critical when the Natural Vector approach recreates all of the results derived in this section; in effect, this treats remote effects as instantaneous as all of these 'fluid' models transmit changes instantly from the remote source to the distant 'field point'. The resemblance to Coulomb's electrostatic law for the definition of irrotational fields is not a coincidence here but becomes the pivotal assumption in all versions of these CEM theories.

1.2.5 SUMMARY OF CONTINUOUS NATURAL VECTORS

In order to expedite the central component of this paper, the relevant details of the first paper in this series that introduced *Natural Vectors* are summarized here in section five for the particular case of *Continuous Natural Vectors*. The central hypothesis in this programme is that the location and local time of each electron are better represented by a Natural Vector than by a 3D vector representing a (globally) time dependent location. In the continuum version of Natural Vectors the four component version of Hamilton's 'nabla' or gradient operator plays a central role. Simple conjugate multiplication is quite sufficient to generate all the required formulae that describe relativistically covariant formulations of particle mechanics and classical EM. The key results obtained here use the mathematics of the major class of CNVs, referred to as *Voigt Vectors*, that are defined in terms of the basic CNV velocity. The temporal and spatial components of these Voigt Vectors are related through a simple equation, named here as the 'Lorenz Equation'. The conjugate gradient of a Voigt Vector defines both the electric and magnetic force intensities as the two components of a single force density, which is a complex 3D function. This section concludes with the definition of *Associate CNVs* that are intimately related to the class of harmonic functions.

1.2.6 THE CNV ELECTROMAGNETIC MODEL

The heart of this paper is section six where all the important results of CEM are rapidly obtained using the general formulas for Voigt Vectors and other features of the CNV formalism that were introduced in section five. The present continuum theory is based only on four (4) similar definitional hypotheses wherein the four principal EM concepts are each mapped into individual Voigt Vectors, rather than the regular mix of standard scalar and 3D vector variables. These key concepts include the EM current, EM potential, EM force and EM energy. Both the Lorenz gauge condition and the EM momentum emerge quite naturally in this approach. Unlike the conventional textbook approach (that is described in section four) that derives Maxwell's Equations from line and surface integral macroscopic laws, the present approach only needs to propose that the charge density and EM scalar potential are constant across universal time (temporal invariants).

1.2.7 SUMMARY AND CONCLUSIONS

The final section (seven) summarizes the major points brought out in the first sections of this paper and summarizes the major results obtained in section six, discussing their implications. This final section reprises the various criticisms raised throughout this paper of the standard approach now followed in presenting classical electromagnetism to today's students of physics. Most of these points are rarely, if ever, raised explicitly, giving these students the false impression that Maxwell's theory of EM is one of the best-established theories in modern physics. This is an important impression as CEM, presented as a successful field theory, has become the primary justification for the revolutionary turn away from Newtonian physics to field theories, which are now seen as the only acceptable form of theoretical physics: a position that is strongly contested by this research programme.

The paper ends with a more extended description of some of the remaining papers in this classical electromagnetism (CEM) and quantum electromagnetism (QEM) series that gives a brief preview of the main thrust of this new research programme.

2. METAPHYSICS OF ELECTROMAGNETISM

2.1 THE NECESSITY OF METAPHYSICS

This paper returns to the view of physics as Natural Philosophy that was universally held by students of nature prior to the 20th Century. In this section an argument is made for justifying this older perspective and, in particular, it argues for the methodological priority of philosophy over mathematics. A brief history of the role of metaphysics underpinning the core developments of theoretical physics is provided to remind the reader how important this view has been throughout physics. Since this programme views the fundamental flaws in the current views of electromagnetism as the direct result of mistakes made in the assumptions that have been made about the nature of electricity itself, it is then necessary to review the actual metaphysical foundations of this key area in physics. This style of analysis requires a broader examination of the role of philosophy in descriptions of nature as this is now an almost forgotten component of modern, scientific education.

The Goal of Physics

The goal of physics prior to the 20th Century was to develop a comprehensive, integrated <u>understanding</u> of physical phenomena; simply creating mathematical equations describing parts of the world (especially equations that cannot be solved) was never viewed as the final results of this foundational science – a view shared by this research programme.

2.1.1 NATURAL PHILOSOPHY

It is the central motivation of this research programme that there are deep, unresolved problems lying at the heart of modern physics, whose origins can be traced to basic, fundamental philosophical misjudgments when the original theories were first proposed. As this programme unfolds these major problems will be raised in the context of new, alternative formulations.

Philosophy as Latin Sloganeering

Since academic communications prior to the 19th Century were conducted in Latin, almost all the early philosophical disputes were characterized by sloganeering – usually in Latin. Newton's contemporary and rival, Gottfried W. Leibniz (1646-1716) stated: *Quod non agit, non existit* (whatever does not interact, does not exist) - this is one of the foundational axioms of the present research programme, even though Leibniz rejected the Newtonian concept of action-at-a-distance. In contrast, the Newtonian Roger J. Boscovich (1711-1787) reversed this viewpoint with: *Operari sequitur esse* (action needs or follows existence) [18]. It will be seen that this rhetorical technique for 'resolving' basic disputes in theoretical physics has continued up until the present day: only the language has been changed.

Natural Philosophy corrupted into Theology

Prior to the 20th Century, almost all European university education was conducted at religious institutions, such as Oxford and Cambridge in England; as a result most studies of natural philosophy were undertaken from a religious perspective. As an example, the view of Bishop George Berkley (1685-1753) that space must be filled with divine and animate matter ('the vital spirit of the world') was a response to Newton's heretical idea of particles attracting each other across empty space, since this would attribute spiritual power to inert matter. Even though these religious and metaphysical ideas directly influenced Faraday and Maxwell amongst others, this is not a sufficient reason today to reject the study of natural philosophy in modern physics; this would be just 'throwing the baby out with the bath water' [19].

2.1.1a PHILOSOPHY AS LINGUISTICS

The Primacy of Language

Humans, above all other animals, have created a social form of communication that is symbolic; today this is manifest as over 6,000 natural languages. These are the richest and most powerful forms we have devised to describe our natural world. It must always be recognized that this conscious form of internal, symbolic manipulation is only a tiny mapping of the vast range of interactions that each of us is exposed to every second [20] but it is the best we can do when we think about our situation. When these schemes provide accurate representations that prove useful in our lives then societies retain such descriptions as 'knowledge' that is retained and communicated through the generations. In this light, scientists do not 'discover the truth' (as many believe) but invent new descriptions that prove useful in the long-term development of our culture.

Philosophy as Verbal Maps

Human beings are capable of using their imagination to create linguistic models or 'maps'. When these maps refer to reality we call the results 'philosophy', if these maps transcend reality then the results become 'theology'. Non-linguistic models, formulated as visual imagery or symbolic subsets can result in geometry and mathematics. Theology is not considered here.

Need for Fundamental Definitions

Prior to the 20th Century, physics was explicitly viewed as natural philosophy, with mathematics as one its most important tools to be used for translating its conceptual innovations into formulae that, hopefully, could be used for calculations. This focus meant that understanding and analyzing the central concepts of science, such as 'force', was a necessary and fruitful activity. The present research programme returns to this historical approach by analyzing its foundational concepts, like 'time' and 'existence'. As Maxwell wrote in one of his last articles [21]: "Even in the most mature sciences, such as geometry and dynamics, the study of the definitions still leads original thinkers into new regions of investigation".

To Be, to Have & to Do

The ideas of space and time are at the deepest foundation layers of all Western thought. This view is reflected in the three most important verbs in European languages, which in English are 1) to BE (is) 2) to CONSIST of (has) 3) to DO (does). It is extremely difficult to analyze reality without using these verbs explicitly in any resulting discussion.

Time is the Beginning

In this research, the concept of 'time' is taken as basic (while undefined but intuitively appreciated by all life forms). An object is considered 'real' if it '*exists* in time', while an event is real if it '*occurs* in time'; a relationship is also considered real if it '*persists* in time'. As properties of time, as perceived by humans, the verbs 'exist', 'occur' and 'persist' are each considered metaphysical primitives that cannot be separated from our intuitions about time itself. Further, it is illogical to ask 'does time exist?' as time itself is the <u>basis</u> for determining the existence of all other objects but to deny the centrality of time is the height of intellectual folly. Furthermore, the infinitude of time and/or space is irrelevant to the logical or physical consistency of this programme: all humans are finite beings. These views may be contrasted with the total geometrization (spatialization) of physics by Albert Einstein in his theory of General Relativity and the elimination of time from this theory by Kurt Gödel (1906-1978). The seductive word '*infinity*' is not considered a useful concept in this research programme.

Impermanence in the World

Parmenides and Plato (c.428 – 347 BC) were mistaken in viewing 'being' as necessarily eternal: these were just further ancient attempts to pre-empt the 'divine' as the only true foundation of reality. It is certainly true that, except for all the fundamental entities, everything else in reality is transitory. All macroscopic objects (directly observable by humans) are composites and come into existence when their parts come together and continue to exist (or 'be') until they fall apart.

2.1.1b ONTOLOGY: THE FOUNDATION

Greek Ontology

Ontology is considered a major component of the study of metaphysics; it is centered on the deep concepts of 'being' or 'existence' and the categorical structure of reality. Aristotle (384-322BC) is credited with the idea that existing 'things' belong to different categories. A categorical scheme typically exhibits a hierarchical structure with 'being' or 'entity' (the technical terms for 'thing') as the topmost category, embracing every <u>thing</u> that exists in the world. It is now appreciated that the natural sciences embody implicit ontological schemes, which cannot be justified on purely empirical grounds. The history of ontological thought reflects many of the deepest concerns of natural philosophers. It was Leukippos, the teacher of Demokritos who anticipated our view of the ultimate ('uncuttable') form of matter that can be called 'what is', namely, the electron. These primary objects must be free to move through the void (which may be called 'what is not'); accordingly, 'space' is not an entity, like an electron, but a 'relationship' <u>between</u> electrons. Rene DesCartes (1596-1650), following the Stoics, denied the possibility of a void and so rejected atomism – a near universal view until the 20th Century. DesCartes also fell into the Ionian trap of focusing on fundamental substances (or objects) whose shapes and sizes, as well as motions, were to account for all the variety in the world [22].

Real Language models the World

All real nouns in modern languages map to real-world objects, not abstractions; all real verbs map to actual activities and relationships in the real world. In this programme all real nouns ultimately map to persistent collections of electrons while all real verbs eventually map down to interactions (no matter how complex) between electrons.

Vacuum is really Nothing

This research programme rejects all claims for the reality of the vacuum, here it is viewed as literally 'no thing': it is not viewed here as populated by 'virtual' particles or as a polarizable medium; all concepts of a continuum are rejected in this theory. Furthermore, all concepts associated with the ideas of the 'field' are dismissed as simply mathematical schemes.

World based on 'Substance'

The eminent historian of science, Max Jammer began his study of the concept of mass [18] with the deeper concept of 'substance' that was fundamental to Classical Greek philosophy: "as that which exists in such a manner that it requires no other thing for its existence." This view was first used by Aristotle in his apparently paradoxical statement that "the first matter is not body and has no extension". In fact, Aristotle was anticipating that the concept of a material point particle is the fundamental existent of physical reality – a view grounding the philosophy of this research programme.

Non-interacting Ontology

This traditional view of ontology falls into a logical error when it assumes that its 'unconditional existence' implies that a substance requires no kind of interaction (not just human interactions). This leads to the so-called paradoxes exemplified by "the unseen tree falling in the forest" and in its modern form manifested as QED infinities, where 'free' particles are deemed to still exist when they are distantly separated 'at infinity'. The solution here is that the class of existents does not require any other class of existents but each class could **not** be said to exist if its members did not even interact with other members.

Universe is 'All the Interactions'

The view of foundational interacting objects leads to a self-consistent definition of the Universe as the totality of existents; so the test of any object's existence is its ability to interact with other parts of the Universe and not, for example, whether the object has spatial extension (DesCartes' erroneous viewpoint).

What Is & What Happens

The objective world cannot be ignored – it will interact with each of us whether we choose to or not. As one of the 'giants' of physics, Wolfgang Pauli (1900-1958) has pointed out, European languages reflect this basic fact with two fundamental words, which in English are "reality" (from Latin 'res' or thing) and "actuality" (from 'agare' to do), in other words, 'what is and what happens' – the two core semantic verbs [23]. These remarks illustrate Pauli's deep interest in philosophy.

Entities & Processes

The very concept of an entity implies that the existence of every object of an entity-class is not contingent on the existence of any other object in the universe; thus entities must be eternal – they must exist forever: these are the fundamental objects of reality. A process may be defined as an example of reality that persists while its several component objects continue to interact amongst themselves. When any process is concentrated in space it may be referred to as a composite or complex object and while processes persist they may be viewed as unitary objects (holons), for example, atoms, molecules, cells, humans, etc. If the identity of a process can be maintained across extended time spans then the process may be treated like an entity; thus 'verbs' become 'nouns'. Note: all entities are objects but not all objects are entities.

Qualities of Objects

The quality of objects was first codified by Aristotle, who distinguished primary from secondary qualities. In contrast and as a modern Pythagorean, Galileo strongly preferred that object properties be restricted to only those that could be measured. J. B. Stallo and other Victorian philosophers viewed properties as characteristics of relationships between objects rather than intrinsic to the objects alone [24]. The 20th Century further dethroned the idea that an object's properties were invariant and central to its identity. Einstein's special theory of relativity emphasized that the values of an object's properties could vary by reference frame, while the early Positivists took Galileo's views to a further extreme and limited reality itself to only those properties that could be measured. Quantum mechanics challenged the whole notion that atomic scale objects even had any properties that could be assigned due to the disturbing effects of the act of measurement by macroscopic observers.

Ockham rejects the 'Invisibles'

The English philosopher William of Ockham (1285-1347) proposed a methodological 'principle of simplicity' for clarifying thinking about reality where he recommended that preference should be given to theories that involve the least number of new entities or hypotheses. Most physicists today seem unaware of one implication of "Ockham's razor" implying that in seeking the simplest explanation for any natural phenomenon it is usually advisable to 'dispense with what is not observed'. The application of this dictum eliminated both the concepts of phlogiston and the aether but today's physicists are quite happy to retain invisible EM fields, photons, quarks, strings, etc.

Accepting that Reality is Real

The present research programme agrees with the basic metaphysical position underlying Classical Mechanics, namely that "matter is real", down to the smallest representatives of existence. This 'Reality Hypothesis' proposes that all matter has substance, form and behavior, all independent of whether observed by humans or not; this matter exists in space at all times. The alternative metaphysical viewpoint has always viewed 'space' itself as the ultimate reality defining all of the Universe.

Abstractions are not Real

Since Newton, mathematical concepts, like 'force', have been given as much ontological reality as the particles on which they act – even though they have never been observed independently of the very particles upon which they act. This failure in linguistic analysis, not surprisingly, results in bizarre word-games and paradoxes. For example, it is meaningless to ask how long the love lasted between Abelard and Eloise after they had died. This failure to distinguish entities from processes and relationships persists today and even leads to contradictions in such tightly defined areas of human activity as the Law.

The What & the Why of Philosophy

Philosophy is the recursive analysis and synthesis of concepts to clarify our verbal maps of the world. One of the major pillars of this research programme is the firm belief in the primacy of philosophy ('Queen of the Sciences') while modern mathematics is viewed as a powerful but secondary tool ('Servant to the Queen'). The present programme follows in the Aristotelian tradition of realism, in contrast to Plato's idealism; this approach to Natural Philosophy is based on the English school of Newtonianism and the Scottish school of Common-Sense Philosophy. Physics as the study of reality is grounded in the area of philosophy known as metaphysics – those concepts posited to account for the foundations of reality. Indeed, Greek philosophy before Socrates was metaphysical and synonymous with physics, with statements like Thales' claim, that 'everything is made of water'. In contrast to his teacher Plato's theory of Eternal Forms, Aristotle viewed the world in terms of universals – features of how the world itself is organized. Some philosophers, like the Logical Positivists, have rejected all forms of metaphysics as meaningless, a position still popular with many modern physicists. However, if metaphysics is defined as those features of the world that are currently beyond the realm of technology then it allows humans to discuss the world as possibly involving such ideas that may eventually 'materialize' to our senses, such as the atomic view of the world.

2.1.2 PHILOSOPHY BEFORE MATHEMATICS

Geometry was first a Language Model

It should always be remembered that the first mathematical system, Greek geometry, was formulated entirely in natural language [25]. Euclid recognized the fundamental distinction between the results of the operation of counting (positive integers) and geometric magnitudes that only admit qualitative comparisons (equal, greater than, less than) between scale-less line segments. Franciscus Vieta (1540-1603) is considered to be the first European to use letters to denote arithmetic numbers (integers). Magnitudes were first extended to ratios (often to an arbitrary unit) by DesCartes in 1637 and mapped to decimal fractions. Following Vieta, DesCartes also used alphabetic letters to denote line segments; then using a model of triangular proportions he was able to give a non-rectangular example of 'length multiplication' and by also using Euclid, a geometric method for illustrating the square root of any positive 'real' number.

Philosophy, Logic and Math

In this programme the foundational terms must always be defined as clearly as possible. Accordingly, 'philosophy' was defined here as the verbal analysis of our social mapping of the world; 'logic' is defined here as the verbal and symbolic analysis of timeless relationships between several concepts, finally 'mathematics' as defined here means the study of symbolic relationships between the kinds of abstractions that are used in both logic and theoretical physics.

Timeless Logic

The use of logic is only valid for timeless sets, such as abstractions; this requirement positions logic itself at the center of language and allows it to be mapped into mathematics, which is also timeless. Symbolically, if M is any member of an abstract set S, defined through a relationship R, then at any instances of time T or T′ the nature of logic requires:

$$1.\ \ S(T) = S(T') = S \qquad 2.\ \ R(M; S, T) = R(M; S, T') = R(M; S)$$

Math maps Philosophy

Science is grounded on the two metaphysical concepts of 'existence' (ontology) and 'causality'. These two concepts are supplemented by several related (and usually unspoken) assumptions, including the presupposition that both concepts can be fully represented by mathematics. Arithmetic is assumed to map existence (e.g. two type-A objects exist : $2A = A + A$) and linear calculus is assumed to be able to represent causality (e.g. force causes a change in momentum : $F\ dt \Rightarrow dP$). Further, science always assumes a theory of measurement, such that human acts of measuring reality can be accurately represented by the mathematics of 'real' numbers, while the act of measurement itself does not alter the phenomenon.

Merton Verbal Math

Although our concepts are understood through the use of symbolic natural languages, the manipulation of some concepts can be greatly improved when they can be represented by single symbols; in other words, mathematical notation is vastly more condensed than the verbosity of natural languages. Goldman [26] describes the difficulties of the Mertonian School-men in 14th century Oxford who had discovered the mean-speed theorem for constant acceleration but could only describe it in natural language terms, as they did not possess any algebra to exploit this important result, which today would appear as the simple algebraic equation: $S = (U + V) * T / 2$.

Symbolic Representations

Since the publication of Newton's *Principia* [7], physics has been successful in representing certain aspects of reality by algebraic symbols – that is, these symbols (usually single alphabetic letters) can be combined arithmetically and can even be multiplied together. This algebraic representation was limited first to the use of 'real' numbers and then complex numbers, which are both commutative algebras, where the order of the symbols in multiplications is irrelevant. It was only with the invention of matrices by Arthur Cayley (1821-1895), the invention of quaternions by William Rowan Hamilton (1805-1865) and the introduction of higher algebra by William K. Clifford (1845-1879) in the mid-19th Century that non-commutative, multiplicative algebras became possible so that now the order of the symbols could represent distinct aspects of reality. These more powerful representations have been exploited in many areas of 20th century physics, especially quantum mechanics. This non-commutative area has become the foundational mathematics of this research programme.

Physics adopts Mathematics

The present programme begins with philosophy. This is where the underlying concepts are introduced and must be justified, all in terms of natural language, which remains our most powerful means to communicate and to reflect consciously. Once new concepts have been established as appropriate then a suitable form of symbolic representation can be investigated as this provides more ideas to be considered at the same time while, sometimes, permitting standard forms of manipulation that can generate new insights. Ideally a specific branch of established mathematics suggests itself for manipulating theoretical symbols, such as 'scalar algebra'; this permits a well-established set of rules to be brought to bear whose cohesion has been already demonstrated. This approach has been clearly demonstrated in the historical evolution of western physics, especially with the use of numbers to map time-varying phenomena. Around 1600, Galileo Galilei (1564-1642), using his pulse as a primitive clock, was able to (verbally) describe the behavior of a simple pendulum. Soon after, around 1620, the mathematician and scientist René DesCartes invented the basis of the modern algebraic power notation (e.g. ax^2) to describe geometry, which until then only had visual and verbal descriptions. The natural philosopher, Isaac Newton adopted this Cartesian geometry and added the concept of time generators to analyze static curves as analogues of time-varying motion; his revolutionary 'fluxions' became the basis of modern differential calculus. Phenomena that could be characterized by a single time value, such as center-of-mass position, were mapped to 'real' numbers using arbitrary scales of length and time. Newton used the new algebra to represent compound concepts as the product of real numbers; e.g. 'quantity of motion' (or 'momentum') was represented by the single symbol 'P' or the compound multiplication 'M * V' to generate algebraic definitions of mass 'times' velocity: $P = M\ V$.

Bohr demands Physics before Math

Perhaps as a result of a life-time of trying to understand the real meaning of quantum mechanics, Niels Bohr (1885-1962) believed that [27]: "a complete physical explanation should absolutely precede the mathematical formulation, no matter how rigorously the mathematical formalism is developed; in other words, mathematical clarity in itself has no virtue in physics".

Debate on Philosophy vs Mathematics

The 'giants' of modern physics could disagree fundamentally on the role of mathematics in the evolution of theoretical physics. Bohr believed that physical and philosophical considerations should be recognized as primary while Dirac and Einstein were always seeking new mathematics that could be applied to unlocking the secrets of nature.

'Mathematics is Beautiful'

Both Einstein and Dirac were seduced by the 'Goddess of Mathematical Beauty', each spending the last 50 years of their lives in a fruitless search for unified theories of physics by focusing exclusively on finding new forms of fundamental field equations. The 'Continuum Hypothesis' was never even once considered as a possible source of their major difficulties!

James Jeans as a Pythagorean

It was the followers of Pythagoras (c.550-c.500 BC) who introduced the idea into Western thought that [28]: "numbers are the first things in the whole of nature". This led them to believe that that the universe could be explained and understood purely in mathematical terms. Plato was strongly influenced by these metaphysical notions, especially the ideas of 'perfect' geometric forms and 'pure' numbers. Since there were no such examples in this world then these ideas must exist in some other 'perfect, changeless higher sphere'. 'Pythagoreanism' later became, in effect, a brand of Platonism, with emphasis on number theory and the more mystical aspects of his thought. It was Aristotle, who was Plato's best student, that challenged these abstractions and grounded his science in the empirical world of observation. This disagreement on the very nature of reality has persisted throughout the subsequent history of the Western intellectual tradition as exemplified by the English mathematical-physicist, Sir James H. Jeans (1877-1946), who claimed in 1930 that "the Great Architect of the Universe now begins to appear as a pure mathematician"; of course, this would limit the type of person who could then claim to be a High-Priest.

Pythagoras Triumphant

Although most physicists today would deny that they are followers of any philosophy, most are unconscious Platonists and adepts of Pythagoras. Like Plato and Planck, they believe in the superiority of the 'timeless' and the eternal, unchanging 'Laws of Nature'. The effective banishment of time from modern physics is more than acceptable to this older metaphysical viewpoint. The universal high regard for the primacy of measurements in physics would gratify any ancient Pythagorean, as all of them would agree that the results of measurements ('real numbers') are the very essence of reality.

Today's Platonic Physicists

The minimal exposure to philosophy means that many of today's theoretical physicists have unconsciously adopted the Platonic assumption that a formal identity between mathematical variables is synonymous with the identity of the physical entities represented. Worse, intermediate mathematical quantities introduced into the evaluation of such mathematical equations lead to the assumption that there exist corresponding physical entities in the real world.

Re-ification of Math

The most persistent error made by theoreticians throughout the history of science is the reification of their mathematical symbols, especially those that have been introduced simply as intermediate variables to simplify the calculations and are never themselves measured directly; for example: Euclid's straight-lines, Ptolemy's epicycles, Newton's forces, Maxwell's fields, Schrödinger's wave functions, etc. None of these concepts exist in the real world; just as no roads are ever painted red or yellow but they are on all road maps. Even the co-ordinates in the Lorentz transform are never measured, it is only the unchanging interference fringes in Michelson's experiments that are observed.

Unreal Intermediate Symbols

Physics often relies on the mathematical technique of summation and its inverse, decomposition to develop representational schemes of reality; very popular techniques illustrating this strategy include Fourier analysis and vector decomposition. It is important to realize that such decompositions do not automatically confer ontological significance except when the parts do have independent existence. The best ontological test occurs whenever there is only one possible, finite and independent decomposition into parts, where the physical absence of any one of the resulting parts changes the nature of the system. This is never the case when the decomposition is made into an infinite number of parts (such as 'fields') when any one mode could be removed with only vanishingly small consequences.

Training Physicists like Mathematicians

Lord Rayleigh predicted in 1894 (after reviewing the proposed changes in the physics curriculum at Cambridge University) that the consequences of basing mathematical physics training on 'pure' mathematics rather than applied mathematics would be the take-over of the university physics departments by pure mathematicians. In his view, physicists were driven by their intuitive understanding of physical entities and their processes, whereas pure mathematics is characterized by proof, rigor, and elegance, with its foundations in mathematical definitions, abstract symbols and logic [29]. In fact, pure mathematics can be best understood as the investigation of the logical implications of arbitrary definitions that arise from the imagination of very creative mathematicians.

The Cambridge Tripos 'Grind'

The 19th century Cambridge mathematical *Tripos* system demonstrated the failure of the 'grinding' approach to the creation of new theoretical knowledge. Generations of undergraduates were intellectually destroyed by this intensely competitive system. Only four men (Maxwell, Rayleigh, Stokes and Thomson) emerged in 100 years to make significant contributions to the progress of physics. Ironically, and with the same limited success, today's worldwide competitive approach to post-doctoral physics has produced thousands of professionally qualified individuals, with superb abilities to pass their exams, but it has similarly failed to contribute to the evolution of any significant theoretical physics since 1960 [30].

Quantum Mechanics needs more than Math

The math-first approach became the dominant approach to innovation in physics with the development of the modern version of quantum mechanics, beginning around 1925. The resulting confusions and paradoxes confounded the original developers of this mathematical theory that predicts the simpler parts of the micro-world. The failure to understand what the QM mathematics represented resulted in a general rejection of attempts to describe the world in natural language or simple visual imagery, hence the final rejection of philosophy. Even great theorists like Dirac and Feynman had highly developed physical intuition but both explicitly rejected the value of philosophy in generating new ideas, with tragic results.

Science is not unsolvable Equations

The new idea that the aim of science, especially theoretical physics, is to 'discover' (or invent) mathematical descriptions of reality that are presented only in the form of equations, that are too complicated to be solved, is an intellectual dead-end: particularly, if the mathematicians are unable to agree on the meaning or significance of the symbols employed.

The Mathematical Challenge

The modern viewpoint has been succinctly described by a contemporary quantum theorist, Chris Isham [31]: "The central challenge for a theoretical physicist is to find the types of mathematical structures that can successfully encode the significance and structures of an object." This is the mathematical challenge to physicists based on the ancient philosophy of realism, the idea that real objects exist and possess real properties that can be measured – hence, mathematical.

Aristotle: Philosophy has turned into Math

Stephen Toulmin, a senior historian of science, quotes Aristotle's response to Plato's love-affair with mathematics [32]: "Philosophy has turned into mathematics for present-day thinkers, despite their claim that mathematics is to be treated as a means to some other end." So, after 2500 years, *plus ca change …*

Summary: Not Pythagoreanism

In summary, this research programme rejects the Pythagorean Programme ("All is Math") in physics. The world is much more complicated than the simplifying assumptions needed by mathematics. Imagery, visualization and intuition precede equations or even the timeless world of geometry. Mathematics is "not the language of God", as one of the most famous followers of Pythagoras, Galileo has claimed but the invention of mathematicians, who are not gods, not even high-priests. Mathematical hypotheses are not the most useful starting point for developing new physics, especially when they result only in unsolvable equations or worse, equations that cannot be interpreted into the natural languages of all human discourse.

Only Two Math-first Predictions

It appears that there have only been two examples in the history of theoretical physics where a mathematical innovation has preceded and prompted a new physical discovery. The most famous must be the prediction by Paul A. M. Dirac (1902-1984) whose dual solutions of his relativistic equation of the electron in 1928 lead to his prediction of the positively charged electron (or positron) that was subsequently discovered by Carl Anderson in 1932. A much weaker prediction was that of Murray Gell-Mann, whose group-theoretical (SU3) approach to fundamental particles resulted in his prediction of the omega-minus resonance that was discovered some months later in 1964. Much more frequently, a mathematical description has been subsequently provided for an earlier physical discovery.

2.1.3 NEWTONIAN METAPHYSICS

Mechanical Metaphysics

The modern mechanical programme was initiated in 1621 by Johannes Kepler (1571-1630) when he replaced the concept of 'living energy' (*anima* or vitalism, which generated qualitative changes, even at a distance) with an inanimate, mechanical power (*vis*) that only produced quantitative changes in the world [33]. Ironically, it was the devout Catholic philosopher René DesCartes, who proposed the first mechanical theory of nature in his treatise on philosophy, where (in contrast with Galileo) he proposed that inertial motion was rectilinear. DesCartes' world-view pictured the universe as totally filled with inter-connected vortices, each containing rotating massive particles. All action was through direct, mechanical contact only.

Descartes' Natural Philosophy

DesCartes introduced the concept of inertia of matter in his *Principles of Philosophy*, published in 1644 just two years after Isaac Newton's birth. Newton later adopted this key concept into his own mechanics but his 'far-action' model was a direct rebuttal of DesCartes' contact ('near-action') or "mechanical" model of material action. Similarly, Newton's entity model of isolated point particles was also a successful and direct rejection of DesCartes model of vortices filling the plenum of space.

Newton's Model of Physics

David Hestenes has summarized Newton's metaphysical model into two propositions [34]: 1) every real object is no more than a composite of particles (each with a trajectory in space and time), and 2) each particle's behavior is caused only by the interaction with other particles. Newton's goal was in the tradition of William of Ockham, that is to explain the diverse properties of real objects in terms of as few kinds of particles and as few types of interaction as possible.

Newton rejects the Occult

Newton rejected speculative conjectures that were lacking experimental demonstration; he called these ideas 'hypotheses' but in modern terminology would be more accurately called 'speculations'. He was adamantly opposed to all Cartesian and Aristotelian 'occult' properties, since by definition, these properties are not directly available to sensory or experimental observation [35]. It is very likely that Newton would today reject those primary occult properties of space known as fields.

Newton invents an Aether

In spite of his own rules to reject hypotheses, Newton could not resist proposing a model for the aether in the second edition of his experimental masterpiece on *Opticks* in 1717. He proposed that this luminiferous medium was composed of particles that mutually repelled each other (Query 31) and were able to penetrate the empty spaces between the particles of ordinary matter ('subtle'), with no measurable weight ('imponderable') but attracted by the particles of ordinary matter. This was actually a very prescient 18th Century view of the modern electron [36].

Newton inserts Time into Geometry

Geometry has always been viewed as 'the science of space' in that its figures are unchanging and timeless. In developing the calculus Newton was able to insert time into geometry. Newton followed Torricelli and Roberval in treating curves as the paths of points in motion; in effect, a single, universal time defining the rate at which spatial curves 'grow'. This idea of continuous motion or flow replaced infinitesimals by instantaneous velocities or 'fluxions'. Newton generalized Wallis's method of squaring or quadratures (i.e. integration) by changing the fixed upper limit of the integral into a variable, thereby establishing the 'inverse' operation of differentiation. Unlike Leibniz, Newton never demonstrated the generalized case for differentiation as he had no adequate symbolism, his fluxions, å and ä etc were always rates of change with respect to time alone – fortunately for physics, this was quite enough [37].

Newton's Views on 'Mass'

One of the most important innovations in Newton's physics was his concept of 'mass'. From the very beginning, as in Definition III of his *Principia*, he viewed this concept as two-sided. The 'force of inactivity' (*vis inertiae*) is not only resistive to externally impressed changes to a body's state of motion (momentum) but its 'innate force of matter' (*vis insita*) is also an active impulse, in so far as the body endeavors to change the state of motion of another body [38].

Jammer's Views on 'Mass'

The historian of science, Max Jammer ends his 1961 treatise on the historical view of the concept of mass with an important admonition [39]: "The notion of mass seems to elude all attempts at a fully comprehensive elucidation and a logically, as well as scientifically unobjectionable definition. … The modern physicist should always be aware that our conceptual foundations, especially the concept of mass, are entangled with serious uncertainties and perplexing difficulties that have not yet been resolved." This text was awarded the 1960 Monograph Prize by the American Academy of Arts and Sciences and was the heart of his series of studies on space, mass, force and quantum mechanics.

Momentum & Kinetic Energy for Mass

It often seems to be forgotten that in theoretical physics the concepts of 'momentum' and 'energy' are both only <u>derived</u> definitions, both based on the unobservable concept of 'mass' – as a result, neither of these key quantities are ever measured directly but are calculated from their appropriate definitions, whether classical or relativistic. All measurements of particle activity are ultimately based on observations of spatial and temporal differences. As a result, the corresponding mechanical conservation laws for all of these basic concepts eventually reduce to accepted scientific conventions: <u>based on theory</u>.

Not Newtonian Absolute Space & Time

Although Newton presented his theory of mechanics in the *Principia* purely as a geometrical exposition [40] he was well aware that his metaphysics was grounded in the concepts of 'absolute space' and 'absolute time' so that he made this key position explicit in the *Scholium* (discussion section) immediately following his introductory eight definitions. But it was Leibniz who correctly pointed out that both of these concepts are relationally dependent on the existence of real objects, he defined 'space' as the order of co-existent phenomena and 'time' as the order of successive phenomena. This makes all motion relative: the viewpoint adopted by Assis in his text *Relational Mechanics* [41]. This is also the viewpoint adopted in this research programme.

Newton's Central Observer

Although Newton chose to view space and time as "God's sensorium" it is clear that his foundational algebraic parameters (t; x, y, z) are not defined relative to any 'absolute inertial frame' (or absolute time and space) but are ratios (with respect to some arbitrary units of temporal and spatial separation) relative to the arbitrary position of a fixed observer (at the 'origin') who is immobile relative to the 'fixed stars'.

Systems must exceed Three Particles

The challenge for all dynamical theories is to describe the analytic behavior of at least three (3) particles, since:
1) the concept of zero particles is meaningless as space is scale-less and without any means of distinguishing one location from another,
2) one particles theories cannot distinguish direction or time as there are no other reference points,
3) two particle theories are reducible to equivalent one-body problems when the forces are instantaneous, and
4) the three-body problem has defied analysis since Newton first described it.

French Mathematicians transform Newton

The great early-19th century French mathematicians transformed Newton's finite difference theory of impulses causing finite momentum changes between discrete particles into an analytical mathematical theory of continuum mechanics involving both partial differential equations and integral calculus. These major contributions created the foundations for the later development of field theory; these mathematicians and their major works included:

1)	Joseph Louis Lagrange	(1736-1813)	*Analytical Mechanics*	(1788)
2)	Augustin Louis Cauchy	(1789-1857)	*Course of Analysis*	(1821)
3)	Jean Baptiste Joseph Fourier	(1768-1830)	*Analytical Theory of Heat*	(1822)
4)	Pierre Simon Laplace	(1749-1827)	*Treatise on Celestial Mechanics*	(1825)
5)	Simon Denis Poisson	(1781-1840)	*Treatise on Mechanics*	(1833)

Classical Mechanics' invalid Assumptions

There is a need for a new science of mechanics as Classical Mechanics (CM) is based on a set of assumptions about the real world that have been determined by experiment to be invalid; these erroneous assumptions include:

1) One single time can characterize the whole system when all interactions propagate at finite speed.
2) All interactions are combined additively while Quantum Mechanics (QM) indicates interactions are 'saturated'.
3) Dynamic (time-varying) interactions operate continuously while QM indicates they are discontinuous.
4) Time-varying interactions can be averaged into spatially-varying effects i.e. potentials.
5) Matter is continuous when it exists in the form of atoms and electricity occurs as discrete electrons.
6) Measurements can be made vanishingly small when the 'external' measurement electrons are part of the system.
7) Universal reference frames can be constructed from 'rigid bodies' and 'spot clocks' – both only imaginary concepts.
8) Variations across space and time can be described by differential equations, which contradicts #3.

So, CM is represented by the mathematics of the differential calculus, partial differential equations and field concepts; in effect, it is limited to systems approximated by instantaneous interactions between two point particles (two-body problems). It is usually thought that QM overcomes these limitations but QM has been constructed from CM by adding unphysical mathematical transformations (disguised by advanced mathematics, such as 'Hilbert Spaces') that essentially, as in Wave Mechanics, replaces the standard algebraic representation of the momentum variables by an imaginary differential operator or in Dirac's formulation replaces the classical Poisson brackets with operator commutators. These techniques have allowed QM to extend the two-body problem down to atomic dimensions and predict the energy levels of the hydrogen atom.

2.1.4 MODERN METAPHYSICS OF SCIENCE

Kant's Ontology & Human Imagination

A leading philosopher (and physicist) of the Enlightenment, Immanuel Kant (1724-1804) described his own views on the foundations of reality [42]: "The elementary system of the moving forces of matter depends on the existence of a <u>substance</u> which is the basis (the primordially originating moving force) of all moving forces of matter; and of which it can be said as a postulate: there exists a universally distributed all-penetrating matter, which agitates itself uniformly in all its parts and endlessly persists in this motion." This is almost the view of this programme, except Kant appears to thinking of a Cartesian plenum, while this new research is based on a discrete model of the world consisting only of electrons. Kant first proposed that our ideas of space and time are the necessary forms of our sensible intuition. Conceptually, all humans will structure the awareness of our experience, whether private (subjective) or shareable (objective), in terms of our fundamental ideas of 'space' and 'time'. Our intuitions of 'space' center on our mode of perceiving the invariant relationships between objects. Our idea of 'time' reflects our mode of perceiving relationships between objects in space that vary across time [43]. Our geometry is the study of 2D and 3D invariant relationships between the idealized definitions of lines and point, while algebra is the study of sequences of relationships between objects (like 'photographs') across time. When 'skipping' is allowed across several intermediate sequences then the notion of *multiplication* is introduced that is seen as equivalent to *groups* of individual steps since *counting* is viewed as the fundamental analogue of 'next step'. The ideas of logic are thus grounded in the invariant sets of rules for any of our human *games* – a given logic defines how the symbols of a particular game may be legitimately manipulated. Each logic is a self-contained, self-consistent creation of human minds that may or may not refer to the real world. Science is a social activity that attempts to integrate logic and objective sensory data into a useful scheme to understand the world. *Understanding* then becomes the conscious awareness of the relationships between various parts of our mental map of the world while *meaning* refers to the mapping between our symbols and reality.

Fechner's Ontology

The German natural philosopher, Gustav T. Fechner (1801-1887) wrote in 1855 when he attempted to define the world of aggregated, macroscopic material reality [44]: "Matter is constructed from immaterial relations between atoms." This is almost an acceptable definition in the present theory, if the term 'atom' is replaced with electron and the word 'relations' is replaced with the more specific term 'interactions'.

Turning Particles into Rigid Bodies

Theoretical physics often idealizes the concept of a particle as an equivalent material point by introducing the particle as any material body whose internal dimensions are 'negligible' compared to the distances over which it is interacting. Now the term 'negligible' is not synonymous with zero so that all the parts of the particle must move instantly together. This type of 'reasonable' type of approximation is explicitly rejected in the present theory as this approach gently smuggles in the hidden assumptions of both a 'rigid body' and 'instantaneous action across space'. These two suspect concepts are viewed here as the source of many major problems in physics.

Theory's Philosophical Primitives

The German physicist Gustav R. Kirchhoff (1824-1887) viewed the concepts of mass and force as secondary concepts, that are found useful for simplifying various equations in the appropriate limit conditions – this view contrasted with Newtonian metaphysics were mass and impulse where introduced as basic hypothetical concepts. Later, in 1900, the German physical-chemist F. Wilhelm Ostwald (1853-1932) proposed 'energy' as the replacement foundational concept for both particles and their interactions [45]. This is not the position taken here, which remains basically Newtonian in its metaphysics.

Energy is not an Existent

One of the major philosophical errors committed over 100 years ago and allowed to persist due to the lack of interest in philosophy by modern physicists is the false idea that energy is an ontological primitive. This research programme returns to Newton's original idea that energy is only the property of particles, relative to each other. Kinetic energy is defined here only relative to the motions of the electrons that are interacting (including the 'measurement' electrons) while the interaction energy is not the 'potential energy' of some invisible field around the particles but is the energy <u>being transferred</u> between the pair of interacting electrons that is unavailable to the rest of the universe until the interaction is complete.

Work, not Energy

The focus by physicists on the concept of 'energy' will always prove to be insufficient as energy eventually ends up as random 3D molecular motion i.e. chaos. It is directed energy or 'work' that structures the universe into macroscopic forms and humans eventually need time-patterned descriptions of this work (i.e. information) to make sense of it all.

Spatial Potentials

Every physical model describable by 'conservative forces' is actually independent of time. These purely geometrical models will therefore introduce their force concept through a purely spatial potential energy function.

No Conservative Forces

It is important to remember that in reality there are no conservative forces, where the world remains unchanged after a particle is moved around the point source of such a force, described as a potential field, and returns to its original starting point because no such sources are actually immovable (finite mass) and the mobile particle cannot be 'gripped' (by some invisible hand and moved around some arbitrary trajectory: both particle and source will move only along definite fixed trajectories based on their starting conditions. Only if the mobile particle could move in a perfectly circular orbit would there be no transfer of energy between the two objects – this requires the Continuum Hypothesis to define the motion.

Least Action – across time

FitzGerald rejected formulations of physics based on Least-Action Principles as he believed that this made the present depend on the future. However, the power of this type of starting point for any theory of physics is that this viewpoint recognizes that action across <u>all</u> time is primary, not the view that any arbitrary time-point (the 'present') is special and time is asymmetric, seeing only history as significant [46].

Planck rejects Mechanical Picture

Since Newton, physics has been centered on mechanics or the study of the motion of matter. This continued to be the focus of many leading 19^{th} century physicists including Woldemar Voigt (1850-1919) and Paul Volkmann (1856-1938) who both published influential texts on physics emphasizing the epistemology of natural sciences. Almost forgotten, Voigt published more theoretical papers than any other physicist at this time and introduced the term 'tensor'; he was also the first to derive (and publish) the 'Lorentz' transform in 1887 [47]. Ludwig Boltzmann (1844-1906) also agreed with their perspective but all were strongly opposed by their principal rival in German physics, Max Planck (1858-1947), who, by 1910 was not only 'pope' of German physics but deeply objected to all mechanical pictures as having "only historical significance" [48].

Einstein seeks Unitary Basis

Einstein demonstrated his Platonic metaphysical position when he wrote: "What appears certain to me, as against Lorentz, is that in the foundation of any consistent field theory, there shall not be, in addition to the concept of the field, any concept concerning particles. The field, as represented by differential equations, takes the place of the concept of force." [49] .

Ritz rejects Maxwell & Einstein

Einstein's contemporary and rival, Walther Ritz (1878-1909) in his 1908 paper [50] on EM, criticized the existing state of electrodynamics while supporting the Principle of Relativity. He rejected Maxwell's aether-based model with its 'invisible' force fields and encouraged a return to Weber's remote-action theory but with the additional constraint that even the force between charges be replaced with just a description of their relative motion – these ideas foreshadowing the approach used in the current research programme. In effect, Ritz was supporting L. V. Lorenz's use of retarded potentials while rejecting advanced potentials on the grounds that the spherical convergence of radiation was never observed and this would violate energy conservation. A ray-like, point-to-point interaction is shown here to be more fundamental than spherical 'waves'.

Entities vs Relationships

In 1971, Karl Hubner [51] interpreted the difference between Bohr and Einstein's views on quantum mechanics as one where "for Einstein, relations are defined by substances while for Bohr, substances are defined by relations." In effect, Einstein viewed reality as consisting of entities that possess properties independent of their relationship to other entities, whereas Bohr (and this research programme) viewed reality as defined by the relationship <u>between</u> entities. Measurements were seen by Bohr as an example of relationships between micro entities (e.g. electrons) and macro entities, like humans. Since this is a disagreement in principle (i.e. between fundamental propositions, analogous to the famous disagreement on the foundations of natural philosophy between Newton and Leibniz), it is <u>not</u> reconcilable but is itself metaphysical.

Cartwright: Models, not Reality

Nancy Cartwright, an astute philosophical critic of theoretical physics, has pointed out that [52]: "mathematical physics is awash with non-physical idealizations" such as infinite potentials, zero time correlations, perfectly rigid bodies, frictionless surfaces, etc. She also claims that the fundamental theoretical laws of physics do <u>not</u> describe reality but only mathematical models, while numerical predictions usually rely on a long series of approximations. As a result, she believes that most experiments allow us, at best, to make inferences to the most probable cause, since in most situations we are usually limited to the measurement of effects.

Theoretical Representations are not Unique

Since all representations of the world are never unique all scientists must always acknowledge that the accuracy of their predictions as judged by experimental facts do not imply the validity of their corresponding theories and especially do not confirm the reality of any of their intermediate concepts. As will be shown here, Maxwell's Equations in their modern formulation are developed as differential equations that are consistent with macroscopic experiments involving electricity and magnetism. As such, one cannot then use these same experiments to 'prove' that this differential formulation is the one and only 'true' microscopic model of EM reality – this would be falling into the ancient fallacy first described by Aristotle of 'affirming the antecedent'. However, experiments can always refute any theory, as Maxwell implied: *reality rules*.

The Macrosphere

All macroscopic objects constitute the macroscopic world. Humans need technology to penetrate to greater level of detail. On a timescale less than 10^{-6} seconds or collections of atoms less than 10^{20} then we are examining the 'microscopic'.

Students want only the 'Truth'

The modern 'technocratic' approach to learning physics rejects teaching the historical evolution of knowledge. As early as 1954, Cal Tech physics professor Tom Lauritsen was upset with students who were "impatient with the historical approach and only want a clear statement of current knowledge to write down in their notebooks (even though) this attitude simply produces handbook engineers." The modern assumption is that the "current belief is the Truth; it represents the Facts." [53]

Meta-Principles of Science

When comparing one scientific theory with its competitors there are several meta-principles that most scientists would subscribe to in defending their preference for a particular theory. Firstly, most scientists will agree that they would prefer any theory for explaining specific phenomena over no theory at all. A theory has greater credence when it can explain a broader range of phenomena than any of its competitors. The principle of parsimony is much admired: a theory with the fewest number of independent assumptions will usually be preferred over others requiring more assumptions. Although mathematicians use the technique of logical contradiction to establish their proofs, physicists are prepared (albeit with a great deal of reluctance) to accept apparent paradoxes in a theory but not logical contradictions. Theories that can make predictions that are later experimentally confirmed are very much admired; retrodictive experimental confirmations are considered a good second-best. Recently, theory falsifiability, suggested by Karl Popper (1902-1994) involving the ability to conduct a crucial test that would invalidate a theory, is regarded as a *sine qua non* for being a scientific theory [54]. Finally, theories that can be given a mathematical formulation have always been much desired in physics, never more so than today; this is particularly true when the theory's mathematics is susceptible to exact, analytical solutions.

Progressive Science

Science can be viewed as progressive when scientific theories become more and more universal and when they require fewer (and simpler) hypotheses while predicting more accurate numeric agreement with experimental measurements.

The Failure of Operationalism

Operationalism continues the tradition of hubris in philosophy in so far as its proponents believe that only those elements of nature are real if they are measurable by human beings. Acknowledging that electrons cannot be 'tagged' and so <u>to us</u> are indistinguishable does not mean *ipso facto* that electrons have no individuality at the level of reality [55].

Reductionism in Classical Physics

Traditional physics is a "reductionist" approach to understanding the world. Newton first reduced the complexity of a real, macroscopic object to a single point in space and time (its center of mass). Each position along an arbitrary space dimension is mapped to a single, 'real' number [x] as is the mysterious dimension of time [t], both defined relative to an arbitrary but fixed 'origin' using arbitrary measures of spatial and temporal duration (e. g. unit measures like meter or second). Newton further reduced the analysis of motion of such an object to a limiting relative change in position or instantaneous velocity v, defined at every instant of time: $v(t) = dx(t)/dt$ along with a limiting relative change in velocity or instantaneous acceleration defined as $a(t) = dv(t)/dt$. Finally Newton reduced the complexity of interactions between bodies to the action of an external instantaneous cause or 'force' F on just one of the interacting bodies. The resistance of such a body to this external change was reduced to a single, 'real' number: the 'mass' of the body (again, measured in arbitrary units); this whole scheme finally resulting in a single 'Law of Motion' $F(t) = m\ a(t)$. Similarly, Maxwell had reduced the target object to an arbitrary, infinitesimal 'field point' in his EM theory, while reducing this force concept to the idea of a 'force-density' or 'field intensity' at a single point in space and time.

Reductionism as a Strategy

Physics has now exhausted the *Strategy of Reductionism* – the reduction of complexity to its simplest forms. After Isaac Newton reduced massive astronomical objects to an equivalent single point, Leonhard P. Euler (1707-1783) extended this approach to all of Classical Mechanics through the concept of a 'rigid body'. Again, interactions between separate objects were replaced with a single 'force' (abstracting away all the remote sources of this interaction) which then interacts with the single (target) object <u>at one place and one time</u>. Furthermore, two-body <u>instantaneous</u> interactions were further reduced to a single 'equivalent' centre-of-mass system; then dynamical, time-insensitive interactions were reduced to equivalent spatial forces through the concept of 'potential energy'. Time-delayed interactions between several bodies (e.g. EM) were reduced to local 'fields' acting at only <u>one</u> point in space and time. This strategy has led from Classical Mechanics to Maxwellian EM, through Special Relativity to Quantum Electro-Dynamics (QED): the ultimate theory of activity at a point in space.

Physics is not just Analysis

The search for the 'ultimate level of reality', exemplified by String Theory, is based on *the fallacy of analysis*, which is the proposition that analysis is the principal method of human epistemology. This continues the very long Western tradition of focusing on identifying the basic substance of the natural world. This approach minimizes the importance of understanding the interaction <u>between</u> objects at various levels of aggregation and fails to identify <u>holistic</u> properties that only emerge from new, nested levels of complexity.

Physics also needs Synthesis

The modern scientific search for the foundational (smallest) components of reality (e.g. atoms, quarks, strings, etc) is the direct descendent of the Classical Greek philosophical tradition of analysis. This quest is fundamentally flawed since even a complete understanding of the *parts*, without a comprehensive knowledge of <u>how</u> they are assembled into *wholes*, does not give any insight into their possible composites. Knowing that cathedrals are built entirely from stone does not help us distinguish them from simple stonewalls or pig sties.

2.1.5 VISUAL IMAGERY IN PHYSICS

Power of Visual Imagination

Visually based intuition is the most powerful method for developing new concepts or theories in physics since it is now known that by far the largest part of the human brain (35%) is devoted to visual processing over all other sensory modes and vastly more than is devoted to our logical processing areas. Furthermore, the multi-dimensional nature of visual imagery establishes multiple linkages between the image components providing more associative pathways through the conceptual network ('jumps') and also minimizes the devastating impact of a single error that can occur with linear, logical thinking.

Newton's Visual Imagination

Newton repeatedly demonstrated the power of visualization and analogy in his comparisons of aether and air, forces and strings, and gravity and fluids. His choice of Euclid as the method of presenting his physics in the form of geometric proofs was not an accident (but he did wish to keep his invention of the calculus a secret from his peers and competitors).

Faraday's Visual Imagination

Michael Faraday (1791-1867) was famous in his own time for his visual imagination [56]. The science writer, David Lindley has written in his biography of Lord Kelvin: "No scientist, not even Newton or Einstein, had a greater power of imagination than Faraday, who constructed his physics entirely in pictures without the aid of any mathematics, for the single reason that he knew no mathematics. … But it was Faraday more than anyone who originated the modern view of the EM field" [57]. Indeed, even though as a mathematician, Maxwell was aware of the major work being done by German scientists in this area, he admitted in the Preface to his *Treatise* that [58]: "Before I began the study of electricity I resolved to read no mathematics on the subject till I had first read through Faraday's *Experimental Researches in Electricity.*"

Maxwell's Visual Imagination

Maxwell's recent biographer reports that "Maxwell thought in very visual terms, using diagrams and models, geometrically and synthetically. But he subjected his visual ideas to rigorous analysis, philosophical and mathematical." He also writes that Peter Guthrie Tait also said of his friend and correspondent, Maxwell that [59]: "he always preferred to have before him a geometrical or physical representation of the problem in which he was engaged."

Faraday's 'Lines-of-Force'

Faraday had the image of continuous electric and magnetic effects pervading **all** of space through real "lines-of-force". Maxwell was completely convinced that Faraday's model of EM was an accurate description of reality that could be readily described in mathematical terms. As he also wrote in the Preface to his *Treatise* [60]*: "*Faraday saw lines-of-force crossing all space where the mathematicians saw centers of force acting at a distance; Faraday saw a medium where they saw nothing but distance: Faraday sought the seat of the phenomena in real action going on in the medium." Fluctuations traveling along these lines-of-force were the necessary image for Maxwell's mathematical model of light waves. In other words, Faraday believed (rather than demonstrated) that his lines-of-force remained in place around magnets even when the iron filings were removed. Even today, there is NO experimental evidence that an individual electron is surrounded everywhere by EM fields. All such claims are based on implicit theoretical and usually metaphysical assumptions.

Maxwell's Philosophical Mentors

While studying at the Edinburgh Academy (and for the rest of his life), Maxwell was strongly influenced [61] by two of his professors. James Forbes was Professor of Natural Philosophy, "who had an intense dislike of the mathematical physicist's game of inventing mathematical models because they were solvable rather than because they have any relevance to the physical problem in hand." Further, Maxwell's Professor of Philosophy, Sir William Hamilton (not the more famous Irish inventor of quaternions) "thought philosophy was an end it itself while mathematics is not adapted to the real problems of life, as mathematics is only solvable when the number of variables is small." Hamilton encouraged Maxwell's interest in timeless geometry over algebra or calculus since "in the algebraic approach an answer could often be derived by purely symbolic manipulation without the manipulator ever needing to understand (or visualize) what he is doing".

Einstein's Rhetoric

Einstein's use of the phrase 'thought experiment' was a successful appeal to the visual imagination of his audience as this technique immediately communicated the essence of his ideas without any need for logic or mathematics. But this was also a deliberate rhetorical attempt to associate the objective and scientific world of experiments and real measurements with the private world of the personal imagination. In the present programme, the use of visual imagery is always encouraged but it is never to be confused with the cold, hard facts of experiment.

2.2 AETHERIAL EM METAPHYSICS

It is a central proposition of the present paper that Clerk Maxwell's theory of electromagnetism is a direct consequence of his own metaphysical views on the nature of reality. History has rejected almost all of these 19th century assumptions yet modern physics has retained the mathematical results that are summarized by the Maxwell-Heaviside equations. This is intellectual hypocrisy – the present research programme believes that any fundamental theory of physics must be grounded in a metaphysical model that is consistent with the best empirical evidence: the alternative is mathematics, not physics.

Metaphysical Rivalry

Late 19th century theoretical physics, especially the investigation of EM, was the scene of an intense rivalry between two deeply opposed metaphysical traditions – the action-at-a-distance theories involving discrete particles propounded by scientists like Gauss, Weber etc versus the contact action of continuum (wave) physics of Maxwell, Hertz, etc. The triumph of the continuum model in 20th century physics was greatly assisted, especially in Germany, by the publication in the 1890s of several very influential texts on electrodynamics by eminent academics, such as Emil Cohn, Paul Drude, August Föppl, Max Abraham and Ludwig Boltzmann; all of these were based on a blend of Maxwell's and Hertz's EM theories. This powerful influence of advanced textbooks has continued throughout the 20th Century, as will be shown in section 2.3.3; the views (and prejudices) of these few authors substitute for the independent thought of most contemporary students of EM.

Maxwell demanded Metaphysics

As a sophisticated natural philosopher, Maxwell frequently objected to the theories of other mathematical physicists (like those of his friend, P. G. Tait) when they were based on weak metaphysical grounds. He was particularly riled that certain approaches to fundamental theories (including Newton's Laws of Motion) did not rest on solid metaphysical assumptions. In particular, he objected to using Newton's arguments of 'natural analogy', which would assume large-scale, observable properties (e.g. spatial extension) apply equally to unobservable microscopic equivalents. Ironically, Maxwell himself fell into the same trap when he extended the properties of macroscopic dielectric and magnetic media to the vacuum of space. He would be appalled today to discover that 'his' EM theory was now viewed simply as a set of mathematical equations: this lack of metaphysical propositions was his greatest objection to the action-at-a-distance theories of his Continental rivals.

2.2.1 THE ELECTROMAGNETIC AETHER

Although it is presently not fashionable to refer to the EM aether, it cannot be over-emphasized that this concept was at the very heart of Maxwell's theory. The resulting equations were derived as a result of the proposed properties of this medium. It was for this reason that all believers in the continuum model of light and electricity throughout the 19th Century (and into the 20th Century) rationally accepted the reality of this aether, as the following excerpts will demonstrate.

MacCullagh's EM Aether

The Irish physicist, James MacCullagh (1809-1847) proposed an important Lagrangian model of the luminiferous aether in 1839 when he hypothesized that the potential energy of an element of aether was proportional to the square of its absolute rotation (or 'curl') [62]. Astonishingly, this bizarre proposal of frictionless, rotational distortions pervading all of space was an excellent solution to the optical experiments involving light's reflection, refraction and polarization. In 1862, his fellow Irishman and Maxwell's colleague at Cambridge, George Gabriel Stokes (1819-1903) destroyed much of the interest in this original theory when he wrote that for an elastic, solid aether this model violated conservation of angular momentum and would need a complementary counter-acting torque – although this could have been readily fixed with linked pairs of cells.

Maxwell adopts Faraday's 'Lines of Force'

In his first paper in 1855, analyzing Faraday's 'lines-of-force' [63], Maxwell proposed a model of EM that was based on a mass-less, incompressible 'electric' fluid that emerged from source points and disappeared down corresponding sink-holes. In Maxwell's second (and most bizarre) paper on EM [64] in 1861 he pursued his own 'metaphysical demon' of trying to identify what really exists in nature (ontology) that appears as the phenomena of EM by inventing here a system of entities, present across all space, that had inertial mass but no weight (imponderable) so they obeyed Newton's Laws of Motion but not the Law of Gravitation. Magnetism was viewed as hexagonal (hence close-packing) vortices where their axes are now parallel to the lines-of-force. But this model needed "a layer of particles acting as idle wheels, interposed between each vortex and the next. So, these mass-less particles played the part of electricity, their motion of translation constituting an electric current. These phenomena are due to the existence of matter under certain conditions of motion in any part of the magnetic field and not to direct action at a distance between the magnets or currents." Maxwell viewed this as a form of matter because he needed it to carry large quantities of angular momentum and energy, yet still remain invisible. Therefore, Maxwell viewed this scheme as a background aether, coupled in some way with ordinary matter with which it can exchange mechanical energy. "In the transmission of motion from one vortex to another, there arises an electromotive force between the particles and the vortices, by which the particles are pressed in one direction and the vortices in the opposite direction. The reaction on the vortices is equal and opposite, so that the electromotive force cannot move any part of the medium as a whole, it can only produce currents." Maxwell introduced his 'displacement current' when electrical conductors enclosed non-conducting dielectric media, in analogy with molecular electric dipoles; he also imputed electricity to the medium itself when dielectrics recovered their configuration [65].

Maxwell assumes an Aether

As one of Maxwell's recent biographers has written "The 1865 paper [66] cheated; it got all the results with no mention of anything but the electric and magnetic fields, with nothing at all about the aether." It was this omission that generated huge conceptual difficulties for his friends and other contemporary physicists, like Lord Kelvin. However, in the introductory remarks to this paper, Maxwell wrote [67] that he had called it a "Dynamical Theory" because it assumes that in the space around electric bodies there is another form of matter in motion (i.e. the aether) by which the observed EM phenomena are produced. A little further he writes "there is an aetherial medium filling space and permeating bodies, capable of being set in motion and of transmitting that motion from one part to another, and of communicating that motion to gross matter".

Maxwell likes DesCartes' Aether

Maxwell viewed the EM aether as real: a necessary medium for Faraday's 'lines-of-force', a view shared by all of the believers in Maxwell's theory. This aether had many resemblances to DesCartes' optical model of the aether in that both models viewed this as a dense but subtle medium that existed everywhere throughout space while pervading every material solid body, gas or fluid. This consisted of the minutest globules spread out in a packed, continuous series; a motion that is propagated along a succession of these globules was the basis of light and color [68].

Maxwell's EM Aether similar to MacCullagh's

In 1878, FitzGerald recognized a direct parallel between MacCullagh's and Maxwell's theories of the aether by identifying the spatial displacement of the aether (R) with Maxwell's electro-kinetic momentum (or vector potential, A) but with the roles of potential and kinetic energy reversed. However, this identification implied the continual creation and annihilation of aether – this was completely unacceptable to all those who believed that the aether was a real substance. So FitzGerald then changed his identification to one where the velocity of the aether's displacement was identified with the magnetic field at every point so that magnetism became a closed flow of aether and this choice was quite acceptable to all the believers in the primacy of the aether. FitzGerald (like Maxwell) remained a firm believer in the reality of the aether until the end of his life [67]. It was this 'hydrodynamic' approach, first used by Helmholtz, that was the basis for the adoption of Maxwell's approach by the late 19th Century supporters as the mathematics was isomorphic between these two different models of EM.

Models of the EM Aether

In contrast to EM theories based on action-at-a-distance between electrical particles separated by empty space, there was an almost universal commitment by British scientists between 1880 and 1910 to the primacy and reality of the concept of the aether. Numerous analogies, or models, of this aether were constructed involving elastic solids, fluids, vortices and even wheels and pulleys. As Oliver Lodge asked rhetorically in 1889 "What is the aether? – This is the key question of the physical world at the present time." Much of this discussion can be traced back to the religious metaphysics of Bishop George Berkeley (1685-1753), who viewed space as filled with the immanence of the Christian God, a view later shared by both Faraday and Maxwell. This idealist philosophy regarded the natural world as a projection of 'the thoughts of God' [69].

Boltzmann's View of Maxwell's Aether

One of the leading Continental physicists of the 19th Century, Ludwig Boltzmann was very sympathetic to Maxwell's EM theory but believed [70] that the EM energy in empty space was due to the twisting of the aether in each spatial cell, a view shared by Joseph Larmor (1857-1942), the leading theoretical physicist at Cambridge at that time.

The Aether results in Field Theories

William Thomson, later Lord Kelvin (1824-1907), was seduced by Helmholtz's classic paper on the stability of rotational material, proposed his own vortex model of atoms in the aether, which at that time was "the philosopher's stone of Victorian physics" [71]. It was this failure to analyze the concept of the aether that lead to physics taking the wrong turn to field theories for the last 150 years.

Maxwellians need the Aether for Fields

Almost all physicists around 1900 thought that the concept of the aether was needed to explain the existence of electric and magnetic fields as well as being needed to explain how an EM wave could transport energy through an otherwise empty space. Surprisingly, this foundational view of reality is no longer acceptable to the present generation of physicists.

Demise of the Aether in 1900

Andrew Warwick, has summarized the state of EM theory at the end of 19th century [72], "Many of the mathematical techniques originated in the study of continuum mechanics; the aether concept lent plausibility to the application of these techniques to EM theory although by 1900 the aether concept itself no longer acted as a heuristic guide to the development of EM theory as it had between 1860 and 1890."

Aether Confusion in 1910

In his contribution to a recent 'histories of the electron', Helge Kragh, Professor of the History of Science, has written [73]: "In 1910, physicists could consistently deny the principle of relativity and the aether, as Cohn did; or deny the relativity principle and accept the aether, as Abraham did; or accept both the relativity principle and accept the aether, as Lorentz did; or accept the relativity principle and deny the aether, as Einstein did. No wonder physicists were confused."

The consequence of building fundamental theories in physics upon weak metaphysical foundations is well-illustrated by the ongoing controversy associated with Maxwell's "displacement current". As recently as 1998, Roche published a paper [74] in a leading physics journal that reviewed the extended history of this dispute into the nature of one of Maxwell's major theoretical innovations. This often-vehement disagreement centers on the question of whether this concept represents an electric current (in free space) in some real sense or not. In Roche's view, this issue reflects the contemporary response to unresolved interpretations of electromagnetism inherited directly from the 19th Century. L. V. Lorenz's EM paper [17] was so little known (although briefly acknowledged by Maxwell in his 1873 *Treatise*) that in 1894 Poincare had to re-establish the fact that calculations with retarded electric potentials, propagating at light-speed, only need reference the real conduction sources [75]. He compared this with Maxwell's EM field theory that used (incorrectly) the instantaneous (non-retarded) Coulomb potential plus the 'displacement current' to calculate the Maxwell vector potential. Unfortunately neither Poincare nor Lorentz could face up to the logic of their own analysis as they still retained their belief in the reality of the luminiferous aether with its time-varying stresses; in other words, they continued to believe in the reality of Maxwell's 'displacement current'. Although acknowledging a few EM theorists who had dismissed this aetherial concept, such as the paper [76] provocatively entitled *Displacement Current, a Useless Concept*, Roche also could not face up to the fury of Maxwell's vociferous defenders and support the Newtonian view that Lorenz's action-at-a-distance EM theory of retarded potentials had successfully replaced any need for mysterious, all-pervasive fields as early as 1867. The survival of this controversy to this day demonstrates that old ideas in physics die hard, especially when they are propagated by generations of textbook writers who continue to repeat what their own professors told them as undergraduates, without any real knowledge of the historical evolution of their subject.

2.2.2 MAXWELL'S ELECTROMAGNETIC THEORY

Maxwell's Two earlier EM Papers

Maxwell's second EM paper in 1861 developed an explicit mechanical model of the aether so that he could understand the electro-tonic state (i.e. the magnetic vector potential). This bizarre 'gears and wheels' model involved idling-wheels, which he identified with moving particles of aetherial electricity (i.e. the displacement current). Once again, Maxwell only viewed this model as an aid to visualization and not as a representation of reality. In his third and final EM paper completed in 1864 (but published in 1865) Maxwell retained both the necessary concepts of an elastic displacement current (even in a vacuum) and his vector potential function to derive a purely mathematical model of EM in the form of 20 differential equations. His assumption of an aether with intrinsic elasticity not surprisingly resulted in vibrations that propagated across space that he identified with light as the calculated speed of wave propagation matched recent experimental values determined for light.

Maxwell's Schizoid 'Dynamical Theory'

As Simpson concludes [77] in his detailed line-by-line review of Maxwell's 1861 EM paper [64] "Eight pages of difficult, physical argument, entangled in the physics of elastic media, are needed before Maxwell can make the major claim that light consists in the transverse undulations in the hypothetical medium, which he viewed as the cause of electrical and magnetic phenomena." Although the physical model of this second EM paper had to be rejected as too fantastical, it produced both the insights and mathematical results that Maxwell needed. So, in 1865 in his third, and most important EM paper [66], Maxwell was able to recover all his desired mathematical results but he was reduced to working with the kind of phenomenological model (using Hamiltonian mathematics) that he had long despised when he rejected Gauss and Weber's action-at-a-distance equations as lacking in physical insight. Maxwell was compelled in this third paper to view energy as an existential entity so that, as Simpson describes "we end up with a level of abstraction in which force, momentum and even mass itself become no more than mathematical metaphors." Maxwell oscillated between presenting this theory as only metaphor and as an ontological theory of reality. So Maxwell writes in section 73: "I wish to merely direct the mind of the reader to those mechanical phenomena which will assist the reader in understanding electrical ones. All such phrases in the present paper are to be considered as illustrative, not explanatory." As Simpson summarizes this section, "electromotive force is only a metaphorical force." But later in the paper when Maxwell introduces his EM field as "the space containing and surrounding the electric and magnetic bodies which contain matter in motion (aether) by which the observed EM phenomena are produced – even in a vacuum." Maxwell explicitly claims that EM involves "undulations of an aetherial substance, not of gross matter." He soon exposes his real beliefs when he writes that "The EM energy resides in the field in two equal but different forms, which may be described without hypothesis as magnetic polarization (kinetic energy) and electrical polarization (elastic, potential energy), or according to a very probable hypothesis, as the (magnetic) motion and (electric) strain of one and the same medium."

Maxwell creates a Magnetic Theory

Maxwell's 'Dynamical' theory was actually not a theory of EM but only a theory of magnetism. The emphasis throughout was on the 'electronic momentum' (now called the vector potential $\underline{A}$) and its associated magnetic force field ($\underline{B}$ = curl $\underline{A}$). The electric field $\underline{E}$ was only introduced through the instantaneous, static electric potential ϕ ($\underline{E}$ = – grad ϕ). Maxwell [78] "conceived the generation of light to be a mechanical rather than an EM process, a phenomenon of molecular motion in the aether."

Maxwell adopts Electrostatics

As a corollary of Maxwell's focus on the phenomenon of magnetism he was quite satisfied to simply adopt Coulomb's Law as the basis of the electric features in his theory. This was accomplished by simply using the static, instantaneous electric potential or equivalently, using what is now known as the Maxwell gauge (div $\underline{A} = 0$). However, by definition, electrostatics excludes time, in other words, this is at best a time-averaged description and ignores the finite time-delays of all changes of EM phenomenon.

Maxwell creates Two EM Theories

By the time Maxwell completed his *Treatise* [1] he had developed two very different but complementary views of EM behavior. One focused on delayed region-to-region, asynchronous distant action (ADA) that involved scalar and vector potentials (ϕ, $\underline{A}$) and the other involving local derivatives of these two potentials at a single 'field point'. Maxwell's early disciple, Oliver Heaviside was also philosophically opposed to the ADA view of the world and so welcomed the spark experiments of Heinrich R. Hertz (1857-1894) as a verification of the field viewpoint ("It killed those 'spooky' potentials."). But Maxwell needed to construct his equations to eliminate all references to the <u>timing</u> of the source of these EM effects, reducing the action to a single-time theory. This simplification enabled Maxwell to use the mathematical techniques of Lagrangian mechanics, producing a <u>local</u> field theory. The penalty for this approach was the necessity for the resulting fields to pervade all of space. Any local field reduces electricity to a single, simple point that still lacks the rotational features so characteristic of the phenomenon of magnetism, as MacCullagh had realized earlier, hence the need for 2 fields.

Wires, Fields & Potentials

By focusing only on a single, closed, current-carrying wire, Maxwell could describe the total kinetic energy, at any time t, as the total space integral of the product of the current density, $\underline{J}$ and the vector potential $\underline{A}$, in other words $\underline{J}(\underline{x},t)\bullet\underline{A}(\underline{x},t)$. This was an action-at-a-distance theory but Maxwell wanted to emphasize the kinetic energy in the space around and <u>outside</u> the wire (i.e. the field), especially in terms of the magnetic field $\underline{H}$, so he converts the integrand to a self-product $\underline{H}(\underline{x},t)\bullet\underline{H}(\underline{x},t)$. Similarly, he would view the total electrostatic (or potential) energy in terms of the total space integral of the product of the local charge density ρ in the wire and the static potential ϕ, in other words $\rho(\underline{x},t)\ \phi(\underline{x},t)$. But, again Maxwell wanted these results only in terms of the electric field intensity, $\underline{E}$ so this integrand too was converted to a self-product $\underline{E}(\underline{x},t)\bullet\underline{E}(\underline{x},t)$.

Maxwell as 'Continuist'

In addition to finite propagation delays, the tangential effects of the Heaviside interaction further distinguish the 'optical' effects of EM from the 'central forces' of Newton's mechanics. Furthermore, classical physics drew a sharp distinction between matter and radiation – the focus was on 'substance' rather than entities and interactions. As a 'continuist', Maxwell was profoundly opposed to the discrete view of electricity, he focused on closed-loop magnetic <u>and</u> electric currents, hence his invention of his 'displacement current'. His equations emphasize effects not sources, hence his erroneous conclusion [79]: "that light consists in the transverse undulations of the same <u>medium</u>, which is the <u>cause</u> of EM phenomena."

Maxwell's Current as Aether Displacement

Andrew Warwick, an expert in the history of 19^{th} century physics, wrote [29]: "Maxwell's theory was centrally concerned with the EM effects of relative motion between fields and conductors. In Maxwell's scheme an electric current was not thought of as a flow of electrical substance but as a breaking down of the electric displacement that, by some unknown mechanism, converted the EM energy stored in the aether into heat energy in the conductor. Likewise, the effect produced by the motion of a charged body was something of a side-issue, not perceived as having any real relevance to the central problem of conduction."

Discharging the Dielectric Condenser

Maxwell was focused on the short time (by human perceptions) that a dielectric condenser was being charged up after its 'exposure' to an influx of electricity from an external source. He concentrated on 'small' volumes within this dielectric with his 7-cell field model. Since he viewed matter and empty space (aether) as just different densities of 'dielectric', he could extend his analysis to fluctuations in the vacuum, i.e. 'light' – Maxwell's original objective. Throughout Maxwell's theory electric charge is not viewed as a *substance* (such as electrons) to be acted upon: it is just a *condition*, in and across space itself, that can be changed in time; this can be seen from his equation defining 'free' electric charge density, ρ in terms of the divergence in the displacement current density, $\underline{D}$ such that $\rho = \text{div}\ \underline{D}$. In this theory a finite quantity of charge is always the integration of spatial variations in displacement over a finite area or volume of space at any instant of time. This type of distinction was clearly pointed out by Heaviside, writing to FitzGerald in 1900 while emphasizing the kinds of errors that can result from thinking of electric displacement as movements of points in space. As Heaviside also wrote in 1889 [67]: "when a charge is conveyed into any region, an equal displacement simultaneously leaves it through its boundary." In effect, Heaviside inverted the modern approach demonstrating the displacement current must be part of curl $\underline{H}$.

Helmholtz's Microscopic EM Theory

At the end of Part II of his 1861 paper, Maxwell acknowledged the 1859 paper on Fluid Motion as a model of EM [80] by Hermann von Helmholtz (1821-1894). This paper proposed a complementary model to Maxwell's, where electric currents, rather than magnetic vortex lines, correspond to the axes of rotation of the fluid. Helmholtz had modified his microscopic version of Ampere's Law to accommodate his own theory of electrical conduction (Ohm's Law). This required the non-conducting part of the current to consist of the difference between the rate of change of displacement and the induced electric field. This totally conflicted with Maxwell's view that 'charge' is just an epi-phenomenon due to discontinuities in induction within the medium. Contrary to the views of most Continental physicists, who had followed Helmholtz in EM, Maxwell's EM theory is not a limiting case of Helmholtz's theory when the polarization equals the displacement ($\underline{P} = \underline{D}$). Helmholtz's EM theory could only introduce finitely propagated effects in *vacuo* (i.e. radiation) by further assuming that the aether is polarizable due to EM induction and by further assuming that the vacuum is identifiable with the aether. It appears that Hertz, who believed he was following both Helmholtz and Maxwell, clearly did not understand Maxwell's conceptual distinction between electric displacement ($\underline{D}$) and electric intensity ($\underline{E}$) – even though both have the same magnitude in vacuo. As a result, Hertz compounded the confusion between Maxwell and Helmholtz [81]; a confusion that persists even today when undergraduate physics students are taught Maxwell's EM theory as if it deals with continuous fluid electricity.

Maxwell rejects the 'Electric Fluid' Model

Although Maxwell had personally used the 'electric fluid' model in his 1855 paper and acknowledged Helmholtz's major use of this concept in 1861, Maxwell used the opportunity of writing Faraday's obituary in *Nature* in 1867 [82] to damn this concept (which ironically has now become the basis for teaching Maxwellian EM theory). Maxwell specifically praised Faraday for eliminating such concepts as 'electric fluids' from EM by contemptuously banishing this 'old and popular phrase into the region of newspaper paragraphs' (which also indicates Maxwell's regard for the news media of his time).

Heaviside's Rewrite of Maxwell's EM Theory

The chief theoretician responsible for the major developments of Maxwell's EM theory was undoubtedly the English 'hermit' and autodidact, Oliver Heaviside. He recast Maxwell's original 20 equations (in Cartesian form) into the now famous four vector and two scalar equations in 1885, using a vector algebra that he independently pioneered, based on the innovative quaternions of William Rowan Hamilton (1805-1865). Although Heaviside always viewed himself as a natural philosopher, his experience with telegraphy (his first job) made a permanent impression on his approach to EM propagation. His early 'career' was blocked by William Henry Pearce FRS, the Chief Electrician of the British Post Office and the man responsible for all telegraphic communications throughout the British Empire at that time. Heaviside's published output was prodigious (over 2,000 pages of original printed material) while his physical and mathematical insights (e.g. vector algebra and the 'Heaviside layer') were considered worthy enough by many to merit the Nobel Prize. Unfortunately, his lifelong non-academic status meant that many of his ground-breaking results (like the first derivation [83], in 1889, of the 'Lorentz' force) were not published in 'official' science journals and so were missed by most contemporary academic scientists [84].

Heaviside drops the Potentials

In rewriting Maxwell's EM theory, Heaviside not only simplified the mathematical presentation but he consciously dropped all explicit references to Maxwell's scalar and vector potentials (ϕ, $\underline{A}$). Although he subsequently claimed that his own rewrite caused "a heap of metaphysics to disappear" [85] he actually promoted the purely mathematical concept of force-density fields (i.e. $\underline{E}$ and $\underline{H}$) to a fundamental ontological status (a process referred to by philosophers as "reification" or making something real) whose ghostly existence continues to haunt all of modern field theory, both classical and quantum.

Experiments fail to detect dE/dt

In 1876 Nikolai Schiller conducted a series of experiments that tried to detect the electro-dynamic effects of rapidly time-varying electric fields ($\partial\underline{E}/\partial t$) similar to those produced by remote currents in real conductors ($\underline{J}$ and $\partial\underline{J}/\partial t$). His null results were then recognized as a major blow to Helmholtz's theory of electrodynamics but no one since seems to have commented that these experiments also deal an equally serious blow to the idea of Maxwell's 'displacement current' [86].

2.2.3 ELECTROMAGNETIC WAVES

Maxwell'sTheory anticipates Light

In his 1865 'Dynamical' paper, Maxwell did not prove that light was an electromagnetic vibration but assumed this result from the beginning by starting with an 'elastic' medium (the aether) pervading all of space that could support undulations by transmitting motion from one part to another with great, but still finite, velocity. Maxwell's extension of the ideas of electric polarization ($\underline{D}$) and induced magnetization ($\underline{B}$) from real, material media to the vacuum of empty space was not justified.

Hertz proves Helmholtz, not Maxwell

As a specialist in the study of Maxwell and 19th Century physics, Harman has written [87]: "The generation of EM waves was not a straight-forward deduction from Maxwell's concept of molecular motion in the aether. Nor did Heinrich Hertz deduce the possibility of generating EM waves from any simple interpretation of Maxwell's theory of the EM field. Hertz was working within the framework of Helmholtz's electrodynamics, a theory of physics, which in its essentials diverges radically from the suppositions of Maxwellian theory. Helmholtz's electrodynamics assumed all physical effects could be deduced from an interaction energy between physical bodies, while Maxwell's theory aimed to reduce effects to states of the mediating field or aether. Helmholtz had constructed his own theory around Riemann's 1858 conclusion that the action of electric masses on each other are propagated with the velocity of light."

Hertz re-derives Heaviside Equations

Ironically, in 1884, Hertz working from a generalized action-at-a-distance theory arrived at a similar set of free-space 'field' equations as Heaviside, illustrating the complementary nature of these two approaches. This new confusion was absorbed by Hendrik Antoon Lorentz (1853-1928) who, from his 1875 dissertation, strongly preferred the particulate view of matter so that he adopted Hertz's equations for a stationary EM aether and linked them to a discrete structure of matter (electrons) via 'his' Lorentz force in an unsymmetrical manner, where aether oscillations effected the electron but not vice-versa [88] .

Hertz: Fluctuations, not Waves

It is important to note that the famous 'spark' experiments of Hertz did not prove that EM situations were 'wavelike' but only that EM is a time-varying phenomenon that could undergo 'interference'. The wavelengths used by Hertz in 1888 (about 3m) were enormously longer than those known for visible light (about 10^{-6} m) [89].

2.2.4 CONTINUUM ELECTROMAGNETIC MATH

First Formal Physics

It is often claimed that Maxwell' Equations in his theory of EM are the last example of classical physics; rather they should be viewed as the first example of modern, 'formal' physics since all earlier mechanical inspirations had been discarded by 1864 for a purely Lagrangian generalized set of initial equations. In this type of formulation all interactions are implicitly assumed to be instantaneous (rather than retarded) so the model of rigid Hamiltonian mechanics is assumed (then forgotten).

Field Theories involve Limits

Field theories are always presented as point-value equations, only describing relationships at a single point in 3D space and 1D time (or at '4-vector' points). However all field theory variables (e.g. ρ, E) are actually 'limit-definitions'. Dynamical field theories always use differential equations, which implicitly assume the well-defined infinitesimal limits of dx and dt.

Macroscopic to Microscopic

Maxwell's theory assumes that macroscopically validated experiments (summarized as integral equations) can be extended physically to the microscopic level (represented by differential equations). This leap of the imagination assumes that these mathematical limit procedures (small quantities going to zero) and the boundary conditions (at infinite spatial distances) are both valid physically as well as mathematically. These metaphysical assumptions are needed to define the electric field intensity from the 'Lorentz' force equation as charge, velocity and interaction-time all go smoothly ('continuously') to zero.

Instantaneous Surface Integrals

Maxwell, like many field theorists after him, mistakenly converts volume integrals across space to surface integrals (usually at infinity), **all** at the same time. But these fields propagate at finite speed, so this mathematical technique is only physically valid for instantaneous effects, when the concept of a universal, simultaneous time is valid, as Newton knew long ago.

Maxwell's Mathematical Errors

Ronald Anderson, in his detailed historical review [90] shows how Maxwell tried to have his "continental cake" (potential theory) while "eating it" (focusing on the empty space between the material electric and magnetic substances). Anderson described how Maxwell identified his mathematical version of Faraday's electro-tonic state *existing everywhere* throughout space with the remote action-at-a-distance EM vector potential introduced in 1845 by F. E. Neumann that was *only* defined finitely throughout two interacting remote electrical circuits. Maxwell originally defined his magnetic vector function (B) in terms of his electro-tonic momentum A through the definition B = curl A. Initially, in 1865 [66] he emphatically viewed A as the fundamental quantity of his EM theory - he changed this to only having mathematical significance by 1873 when he published his *Treatise* [91] where he posited the derivatives of A (the B and E fields) as representing reality through the distortions in the medium of space itself (the "aether"). It is ironic that today's mathematical physicists continue to preserve the physical significance of Maxwell's B and E fields while rejecting (following Einstein) Maxwell's primary concept – the aetherial medium that alone established these fields. Anderson's careful analysis pointed out how Maxwell used integration of his differential equations across all space to match up with the integration over the finite locations of the real charges and currents of the Continental theorists, like Weber and Gauss, who focused on the particle view of electricity. Anderson also reminds the reader that in 1902, in his Cambridge University prize-winning essay, M. H. MacDonald noted that this "match-up" fails when radiation at large distances generates significant contributions to remote surface integrals, that cannot be ignored. Although this remote radiation was identified by Maxwell as 'light', he also ignored this effect when he evaluated his integrals. MacDonald's old essay also described how Maxwell also converted a volume integral, enclosing two electrically interacting bodies and the potentials between them, into a surface integral around only one of these bodies and then interpreting the resulting integrand as the medium's "stress" throughout all space (today's EM stress tensor). Maxwell's explicit criticism of remote action, that was favored by his Continental rivals, centered on the fact that this was present without offering any intervening medium to 'carry' the interaction – a viewpoint still shared by today's (local) field theorists, reflecting (perhaps unconsciously) their ancient Cartesian biases.

Mesoscopic Theory

Maxwell's EM theory was a linear analytic theory involving differentials of space and time that could be integrated over all of simultaneous space up to the macroscopic reality of electric wires and magnets that were the subject of the supportive experimental laws (e.g. Oersted, Faraday, etc); so that - at best - Maxwell's EM is a theory of mesoscopic space and time averages (hundreds of electrons) but not a fundamental, microscopic theory of dynamical reality (individual electrons).

Maxwell's math 'Skeleton'

It is ironic that Maxwell rejected the mathematically abstract Equations-of-Motion approach of Weber's and Thomson's Action-at-a-Distance models in favor of the 'reality' of the EM field when all that remains of Maxwell's own legacy today are his own mathematical equations of the EM field. Unfortunately, this style of providing simply mathematical 'skeletons' without any physical or philosophical 'clothing' is quite acceptable to today's Emperors of Mathematical Physics.

2.3 CONTEMPORARY VIEWS ON ELECTRODYNAMICS

2.3.1 COULOMB'S LAW

Theory only, Experiments inaccurate

It is very unlikely that Charles-Augustin de Coulomb (1736-1806) could have made the measurements that resulted in his famous Law of Electrostatic Force because of the limited accuracy of his equipment (using a torsion balance). Specifically, any measurement of the quantity of static electric charge was severely problematic at this time, while his measurements of experimental separations could only have been approximately correct. These points have been raised in a recent paper by Heering [92]. Further, the situation is actually dynamic at the microscopic level, not static, so that, at best, Coulomb could only have been working with macroscopic averages of charge, space and time. It is likely that he simply wanted a direct electrical analogue of Newton's Law of Gravitation. Unfortunately, this 'law' has taken on a central role in all subsequent studies in electromagnetism – the present research will challenge this universal assumption in later papers.

Herzfeld's 'Derivation' of EM

Karl F. Herzfeld, in 1932, deduced [93] Coulomb's Law of Electrostatics and Maxwell's laws of electrodynamics (both in vacuo) based on the axioms of electric charge continuity, force-density existence, continuity and homogeneity of microscopic forces, the existence and linear superposition of conservative potentials with spherical symmetry around point charges and defined completely by the total charge distribution. These nine independent (and metaphysical) assumptions imply partial differential field equations based on a static spherical potential with the generic form: $\phi(r) = \rho(r) \exp(\pm ir/\lambda)/r$. Finally, by further assuming 4D invariance (i.e. using the Minkowski imaginary form for the time dimension, $x_0 = i\, c\, t$) he proved that the potential must itself be a Minkowski 4-vector ($i\,\phi$, $\mathbf{A}$). This large set of assumptions still needs one special reference frame in which all charges are at rest (relative to each other) or the individual charges (density?) move at different, though constant speeds; these assumptions contradicting the well-known theorem [94] of Samuel Earnshaw (1805-1888).

2.3.2 'LORENTZ' FORCE

History of the 'Lorentz' Force

A. K. T. Assis has pointed out [95] that the magnetic force ($\underline{F}$) on a moving point charge Q (that is commonly referred to today as the 'Lorentz force') was first derived in 1881 (although not quite correctly) by J. J. Thomson, who used Lagrangian mechanics with Maxwell's aether theory to calculate the energy effects due to a uniformly charged sphere. Thomson was explicit in identifying the velocity $\underline{V}$ with that of the particle relative to the medium through which it is moving and not the aether, unless the medium was stationary with respect to the aether that is supporting the magnetic field B. In 1889, Oliver Heaviside included the convection current effect in a similar calculation [83] to derive the correct form ($\underline{F} = Q\, \underline{V} \wedge \underline{B} / c$). He too assumed that this velocity was also defined relative to the EM medium. It was not until 1895 that Lorentz presented this expression for this magnetic force based on the magnetic force equation of Hermann Grassmann (1809-1877) in 1845 (using $Q\underline{V}$ for $\underline{I}$ dl). Once again, Lorentz did not acknowledge this source of this equation, nor any of the earlier work done in this area. Not only did he not derive this equation but he did not specify the definition of the velocity involved. The final evolution of this equation occurred in 1905 when Einstein interpreted Lorentz's force as the velocity relative to any inertial observer in the inertial frame that 'measured' the $\underline{B}$ field in his famous paper on Special Relativity [11].

2.3.3 CLASSICAL ELECTRODYNAMICS TEXTS

Several factors converged at the beginning of the 20th Century to remove the concept of the aether from the foundations of EM theory. The rise of *Positivism* in philosophy meant that the focus for physicists increasingly centered on what could be measured or, at least, observed. The electron (and other 'fundamental' particles) moved to center stage as theoretical and experimental physics first became 'Atomic Physics' and soon after, 'Quantum Physics'. The increasing popularity of Albert Einstein's 1905 interpretation and analysis of the so-called Maxwell-Hertz equations of EM [11] that explicitly denied the need to invoke any physical properties of the "aether" resulted in space itself directly taking on the role of 'supporting' the electric and magnetic fields everywhere. The demise of the aether was soon reflected in the appearance of a new generation of textbooks on EM for physics students that began dropping the Maxwellian approach to EM with the result that today's students would scarcely know how this central theory of physics became simply a set of mathematical equations. A brief review of some of the most popular physics texts today illustrates a very telling tale of how 20th Century physics has adopted a quite different approach to the study of nature that is almost totally divorced from its own historical roots.

Abraham & Becker's Text

The famous textbook by Max Abraham & Richard Becker [96] was one of the most influential texts on EM for many years. The earliest version appeared in 1894 as *An Introduction to Maxwell's Theory* but was written by August Föppl. Its second edition was thoroughly revised and published in 1904 as the first volume of Max Abraham's *Theory of Electricity*. Seven editions appeared over the next 25 years before Becker, professor of physics at Göttingen, took over the authorship in 1930. It was this version that was first translated into English, appearing in 1932. The 14th German edition appeared in 1949 with its translation into English appearing first in 1950 and was reprinted several times thereafter. Ultimately, this text evolved one more time [97] when Becker became sole author and added a second volume on quantum theory and electron theory. In his first preface (1930) Becker emphasized the difference in principle between the physicist's and the engineer's views of Maxwell's theory, with the engineer holding onto Maxwell's original aether view, especially in a vacuum, with the electric and displacement vectors ($\underline{E}$ & $\underline{D}$) as quantities of quite different kinds, "related to one another, more or less like tension and extension in the theory of elasticity. From this point of view it must seem questionable to put the factor of proportionality K, in the equation $\underline{D} = K\,\underline{E}$, equal to unity for empty space. But the distinction between $\underline{D}$ and $\underline{E}$, which is closely connected with the mechanical theory of the aether, has been absolutely abandoned in modern physics, the EM conditions at any point in empty space being now regarded as completely defined by $\underline{E}$ and $\underline{B}$. In the Gaussian system of units, the numerical identity of $\underline{E}$ and $\underline{D}$ (for empty space) reflects the fact that for the physicist $\underline{E}$ and $\underline{D}$ are actually the same thing. The introduction by the engineer of a dielectric constant and a permeability for the vacuum not equal to unity seems to the physicist to be merely an artifice, by which formulae are reduced to a form which is convenient for practical calculations." As one of the first 'modern' texts on EM it set the style for most of its successors with the first 50 pages devoted to a broad exposition and justification of vectors. In this text the approach has been based on Helmholtz's research on vortex motion in incompressible fluids, reminding the reader that these ideas had explicitly inspired Maxwell in his development of his ideas of the EM field, where continuous sources and sinks were directly introduced so that point sources could mathematically behave like Coulomb potentials. The modern view of polarizable dielectrics consisting of equal numbers of real, opposite quasi-bound charges was used here to define the "electric displacement", $\underline{D}$ in terms of the real polarization vector, $\underline{P}$ so that $\underline{D} = \underline{E} + 4\pi\,\underline{P}$. The authors are emphatic that this view of polarization was "utterly alien to the original Faraday-Maxwell conception" where in the aether view $\underline{E}/4\pi$ was regarded as the polarization of empty space.

Griffith's Text

The popular undergraduate introduction to electrodynamics by David Griffiths [98], first published in 1981 presents today's conventional approach to Maxwell's Equations by first introducing the concepts of continuous electric charge and current densities as mathematical definitions without any attempt to relate these to their grounding in the reality of the electron. Macroscopic experimental laws are then described in their continuous integral forms so that they can be mathematically converted into their differential equivalents. Finally, Maxwell's 'displacement current' ($\partial\underline{E}/\partial t$) is then just added to fix up the divergence problem with the static version of Ampère's law. Any mention of Maxwell's original derivation has been reduced to a single sentence that simply dismisses the idea of the aether. The EM potentials are likewise introduced just as mathematical 'conveniences'. Like all modern EM texts, Griffiths' discussions of electrical charge jump between those using continuous charge <u>densities</u> (e.g. Maxwell's Equations) and others using electrical <u>point</u> charges (e.g. the so-called 'Lorentz' force). One consequence of these incompatible viewpoints is seen in the standard derivations of the Liénard-Wiechert (L-W) potentials where the final form of the equations <u>requires</u> taking the limit of an extended charge distribution to zero <u>after</u> the time differences have been included so that the apparent retarded differential spatial volume (d^3x^*) includes the desired L-W factor, which introduces the 'relativistic correction' to the moving differential space volume (d^3x') that is taken to 'surround' the moving point charge. It is this subtle but never-discussed merger of two incompatible definitions of electrical charge in one single theory of EM that smuggles in the special theory of relativity.

Jackson's Text

The most widely referenced post-graduate text today on classical electrodynamics is the book by J. D. Jackson that has also gone through three major editions since it first appeared in 1965 [99]. Like Griffiths, he also just states the Lorentz force law for point charges without any discussion of the significance of the velocity term in this equation. In his introduction, Jackson does include a brief discussion of the intuitive notions of charge and current densities (ρ, $\underline{J}$), while indicating the limits of zero values for test charges, needed to define the central concepts of electrical and magnetic fields, are never ever reached in reality because of the finite charge of the electron – further noting that all charge is always an integer multiple of this value. Thus, field theories introduce force <u>densities</u> defined in terms of charge density but the link to the experimental world of reality is described in terms of forces $\underline{F}$, that require the orthogonal concept of <u>point</u> charges (Q_k), e.g. $\underline{F}_k = Q_k\,\underline{E}$.

In order to link these two concepts it is necessary for field theorists to introduce the famous Dirac 'delta' function, $\delta(x)$. Thus, since the total electrical charge is always conserved:

$$\Sigma_k\, Q_k(\underline{x}_k) = \int d^3\underline{x}\; \rho(\underline{x}) \quad \text{then} \quad \rho(\underline{x}) = \Sigma_k\, Q_k\, \delta(\underline{x} - \underline{x}_k)$$

Unfortunately Dirac's 'function' is a concept that does not comply with the basic requirements of the calculus – it is what is politely known as a mathematically 'improper' function. Jackson recognizes this dissonance, so explicitly writes that EM historically was based on macroscopic experiments and admits "The extension of these macroscopic laws, even for charges and currents in vacuum, to the *microscopic* domain was for the most part an <u>unjustified</u> extrapolation." In spite of this disclaimer, he repeats the standard (modern) approach and quickly produces the differential forms for the static laws of electricity (Coulomb: div $\underline{E} = \rho$; Ampère: curl $\underline{B} = \underline{J}$) along with the consequence of the fact that magnetism results from electric currents, i.e. there are no magnetic poles (div $\underline{B} = 0$). Once again, it is the addition of Faraday's Law of Induction (curl $\underline{E} = -\partial\underline{B}/\partial t$) that requires Ampère's static law to be fixed up by adding Maxwell's 'displacement current' to the real current <u>density</u> using the differential form of charge conservation (div $\underline{J} = -\partial\rho/\partial t$) when charge <u>density</u> varies over time. Lorenz is only mentioned through 'his' gauge condition (div $\underline{A} = -\partial\phi/\partial t$) and a footnote to his 1867 paper [17] without any indication that Lorenz had derived <u>all</u> of Maxwell's Equations using an action-at-a-distance theory entirely without the introduction of any field concepts. Again, the whole discussion of EM gauges is in terms of 'convenience' instead of being derived from fundamental principles that would demonstrate that only the Lorenz gauge is physical [100]. The absurdity of this approach to 'convenience' EM gauges is soon demonstrated by the comment: "The Coulomb gauge is often used when <u>no</u> sources are present." This describes a situation as complementary as the absence of 'test' charges at the 'field point' – in other words, Maxwell's Equations in the Coulomb gauge are ideal in **totally** <u>empty space</u>: a consistent view of the vacuum.

EM Texts ignore History

J. D. Jackson, along with one of Russia's leading experts on gauge invariance and charge conservation, L. B. Okun together wrote a review in 2001 [101] that attempted to correct the major errors in EM accreditation that 'history' (or rather, writers of textbooks) has assigned to H. A. Lorentz. In addition to re-using Heaviside's earlier 1889 magnetic force formula in 1892 without citation, now universally referred to as the "Lorentz force", Lorentz also failed to acknowledge L. V. Lorenz's retarded 4-vector EM potential (ϕ, $\underline{A}$), which resulted in the equally uncited "Lorentz gauge condition": div $A + \partial\phi/\partial t = 0$ that was introduced by Lorenz (no 't') 25 years before. Further, by avoiding the term 'vector potential' (that might have alerted readers to L. V. Lorenz or Maxwell) H. A. Lorentz also 'introduced' the key retarded and advanced integral solutions of the universal inhomogenous wave equation (now called the 'Lorentz integrals') that had been solved in 1858 by Bernhard Riemann and published posthumously in 1867 and independently by Lorenz in the same year. Since Lorentz by 1895 had studied EM intensely for over 20 years, it seems more than a little strange that he failed to acknowledge the contributions of his predecessors that were the foundation of his 'own' theory that played a major role in his award of the 1902 Nobel prize for physics. One can only repeat the authors' final statement: "It is amusing how little the authors of textbooks know about the history of physics." It can only be hoped that Jackson will include a summary of this history as an appendix in his next edition.

All contemporary authors fail to provide a metaphysical foundation for their EM field theories and only make mathematical arguments to introduce Maxwell's Equations as, in their hearts, they really believe these no longer have to be physically justified now they have been 'discovered', like the eternal Pythagoras theorem: an approach that is suitable for mathematics but not for physics. At best, they can claim that this 'micro' theory (using the artificial construct of charge density) is consistent with macroscopic EM experiments, when the electrical and magnetic properties of *bulk matter* are measured.

2.3.4 QUANTUM ELECTRODYNAMICS

The reliance on mathematics to justify the modern approach to theoretical physics reaches its apogee with the development of quantum electrodynamics that has become "the best example of quantum field theories". This is exemplified by Steven Weinberg in his magisterial text on quantum field theory who writes [102]: "The traditional approach has been to take the existence of fields for granted, relying for justification on our experience with EM, and just 'quantize' them." His own approach was to show that: "Quantum field theory is the way it is because it is the only way to reconcile the principles of quantum mechanics with those of special relativity." With great pride, Weinberg writes in his preface that his first chapter " will present the history of QFT from earliest times (1926) to 1949, when it finally assumed its modern form. In the remainder of this book I will try to keep history from intruding on physics." Obviously, this is not the approach used here.

2.4 EM ALTERNATIVES TO MAXWELL

In reviewing modern texts on EM, a reader would never guess that there were ever any alternatives to Maxwell's theory. This serious omission not only reflects the conscious rejection of the value of history in the education of today's physics students that not only exaggerates Maxwell's contributions at the expense of his Continental rivals but also (perhaps, subconsciously) reflects the historical experience of the central controllers of all religious orthodoxy to any possible challenges ("if we don't talk about them, they will go away"). In fact, there have been several useful alternatives that offer valuable insights to the conventional field theory of EM, many of them involving particulate models of electricity and/or asynchronous action-at-a-distance (AAAD). Some of these alternative (Continental) approaches are reviewed here.

2.4.1 WEBER'S SINGLE-TIME PARTICLE THEORY

Weber's Emphasis on Interactions

The great German physicist Wilhelm E. Weber (1804-1891) realized that the dynamic inter-connectedness of nature must be grounded in the interaction between pairs of elementary point-particles and not in the isolated 'atoms' of materialism "When two particles, which are spatially and temporally separated, interact then the basis of this interaction lies in the essence of both as a whole and not as single points in space and time." [103]. Weber's 1848 theory [104] interpreted electrical current as the streaming of equal and opposite microscopic electric charges. His formula for the instantaneous force between two point charges depended only on the instantaneous relative separation, velocity and acceleration and introduced a universal speed constant, c for dimensionless factors. Along with F. Kohlrausch in 1856, Weber measured this constant, which he had also shown to relate electrostatic and electromagnetic units, to be 3.1 x 10^{10} cm/sec., very close to the speed of light.

Weber's Relative EM Force Law

In 1835 Gauss noted that 'two elements of electricity in relative motion repel or attract one another differently when in motion and when in relative rest.' Although this note was not published until 1867, its contents were shared with his colleague, Wilhelm Weber, who was stimulated as a result to develop the velocity form of his force law [105].

Weber's action-at-a-distance theory of electro-dynamic interactions between two remote charges published in 1846 [106] was a purely relational theory in terms of the relative separation R_{12}, velocity V_{12} and acceleration A_{12} between the two charges Q_1 and Q_2, such that the force on charge Q_1 at any time t due to the charge Q_2 is F_{21}, where:

$$F_{21}(t) \;=\; Q_1\, Q_2 / R_{12}(t)^2\ \{1 - V_{12}(t)^2 / 2c^2\ + R_{12}(t)\, A_{12}(t) / c^2\} \quad \text{where} \quad B_{21}(t) = B_1(t) - B_2(t)$$

Weber, like all his predecessors (and almost all his followers) adopted a single-time, or God-like viewpoint. Although this has the advantage of mathematical simplicity it fails to reflect the finite time between the action and reaction at the two points of interaction, as was also suggested to Weber by Gauss.

Weber's Absolute Time

In other words, Weber's electrodynamics successfully eliminated absolute space it its approach to relational mechanics but this theory still retains the idea of absolute time since the force equations are still single-time relationships.

Franz Neumann's Vector Potential

Franz Ernst Neumann (1798-1893) was the first to introduce the concept of the vector potential in 1845 [105]. Neumann defined the (magnetic) vector potential $\underline{A}$ at any point $\underline{x}$ in space, in terms of a remote, closed static source current $\underline{I}(\underline{x}')$.

$$\underline{A}(\underline{x}) \;=\; \int d\underline{x}'\ \underline{I}(\underline{x}') / |\underline{x} - \underline{x}'|$$

Neumann was able to express Faraday's Law of Induction in terms of only this vector potential by relating the time variation of total magnetic flux Φ_m through a circuit to the induced electromotive force Ξ_m in this circuit.

$$\Xi_m \;=\; -\,1/c\ d/dt\ \Phi_m \;=\; -\,1/c\ d/dt \int d\underline{x} \bullet \underline{A}(t;\, \underline{x})$$

Kirchhoff's Potentials

In 1857, Kirchhoff published a generalization of Neumann's approach [106] that also accounted for the phenomenon of self-induction. This theory included the modern definition of the electric field in terms of the scalar and vector potentials (ϕ, $\underline{A}$). In this same paper, he also introduced the important equation describing continuity of electric charge.

$$\underline{E} = -\underline{\nabla}\phi - 1/c\, \partial\underline{A}/\partial t \quad \& \quad \underline{\nabla} \bullet (\rho\, \underline{v}) + \partial\rho/\partial t = 0$$

Maxwell uses Kirchhoff's Velocity

Assis has emphasized [107] that when Maxwell compiled his equations in 1861, where he first added the term for the displacement current (his unique contribution to EM theory $c^{-2}\partial\underline{E}/\partial t$) he deliberately used Weber's speed constant 'c' that Kirchhoff had used earlier in 1856 to derive the propagation of electrical disturbances along conductors at light-speed.

Carl Neumann's Electrical Conductors

It was Carl G. Neumann (1832-1925) who first proposed the modern theory of metallic conductors in 1871. In this theory the ponderable (heavy) positive atoms remained fixed in space in a crystalline structure while minute, negative electrical particles flowed between them [108].

Weber's FinalEM Theory

By 1880, Weber had constructed a nearly complete electrical model of nature using only positive and negative electrical particles, the laws of motion, his fundamental law of electrical interaction and the energy minimization principle. Although Rudolf Clausius (1822-1888) challenged Weber's assumption that the force between moving electric particles acts only along their line-of-centers, Bernhard Riemann removed this constraint while still assuming these continuous forces still obey Newton's law of action and reaction. Clausius was promoting his own EM theory involving absolute velocities, defined relative to the aether, which also transferred energy and momentum.

Helmholtz's Critique

Herman von Helmholtz was a life-long rival of Wilhelm Weber and never missed an opportunity to criticize Weber's work. His most serious critique was published in 1846 [109] when he proved that the sum of the kinetic and potential energies in a system must be conserved and Weber's force-model failed to comply with his criteria. Maxwell relied on this result both in 1855 and 1864 to reject Weber's model of electrodynamics. Helmholtz explicitly included four requirements involving the forces between the interacting bodies in developing his proof; these were that the forces are: 1) purely central 2) independent of time 3) independent of velocity 4) independent of acceleration. Helmholtz also failed to make explicit three additional criteria, which he had also assumed concerning the interaction between the bodies, namely that they were: a) instantaneous b) continuous over time i.e. constant forces c) could be described by a continuous (space differentiable) potential function.

C. Neumann defends Weber

In 1871, C. Neumann (frustrated by a widespread misunderstanding that is still repeated occasionally today) wrote an article that would 'once and for all, refute the objection that Weber's theory contradicts the principle of conservation of energy.' As a result, Maxwell, with good grace and admirable intellectual honesty, publicly retracted his long-held objections in the first edition of his *Treatise* [110] and admitted that Weber's EM theory did not suffer from this fundamental flaw [111].

Maxwell mystified by Weber's EM Theory

In his address to the British Association for the Advancement of Science in 1870, Maxwell admitted that the action-at-a-distance EM theory of Weber, Gauss, Riemann, Lorenz and Neumann explains <u>every</u> kind of EM phenomena but confessed that he preferred a continuous medium theory. He admitted to being mystified that 'two fundamentally opposed theories' should equally describe EM [112].

2.4.2 RETARDED ACTION-AT-A-DISTANCE

DesCartes' Contact Model

Ernst Mach (1838-1916), in his 1872 monograph *History of the Conservation of Energy* summarized DesCartes' hypothesis of contact interaction [113]: "A body can only act where it is." This view reduces the world to contact forces for continuous matter or to impact situations for discrete, particulate matter. The problem with discrete instantaneous action-at-a-distance is to provide an explanation for how when one body moves another remote body instantly can know about this change, even when they are separated by vast distances; so change in such a universe would operate like one single, integrated organism. The history of philosophy indicates that the solution to such problems was to propose a universal continuum or 'plenum'.

Medieval Resistance to AAD

The ancient antipathy to action-at-a-distance theories is traceable to the direct human experience of our sense of touch and intellectually to the medieval Schoolmen's dictum: "matter cannot act where it is not". This attitude resulted directly from the rediscovery of Aristotle's view of motion, where he simply asserted as obvious that "the moved body must be in contact with the body that causes the movement". The academicians have usually found it difficult to challenge the orthodox view.

Newton's views AAD as 'absurd'

Westfall has described how Newton began to think about mechanics as an undergraduate in 1666. Newton began, as a follower of DesCartes, with a study of the impact between two bodies while rejecting (at this time) all action-at-a-distance. It has to be recognized that Newton throughout his life hated controversy and was well aware of the trouble Galileo brought upon himself by claiming that he was describing reality. Accordingly, when he later committed in the *Principia* to action-at-a-distance as the key to understanding gravitation he obscured his convictions about its reality by claiming he was only proposing 'mathematical principles'. Even these explicit disclaimers failed to convince Huygens or Leibniz of his central concept of remote attraction when they received their personal copies from Newton. Newton consistently tried to avoid discussing their philosophical differences but Leibniz continued to object to the concept of remote action, as it implied the need for the widely hated idea of 'empty space' or 'the void'. Leibniz always represented the majority continental position, preferring DesCartes' aether, swirling everywhere in the form of vortices, to communicate action across remote distances. Even Newton himself is now (erroneously) believed to have had difficulty accepting this key feature of gravity, with critics quoting his well-known remark made in a private letter in 1693 to Richard Bentley, a leading cleric and one of Newton's first acolytes [114]: "That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it." Although this research programme is very Newtonian it its approach, it deliberately chooses (just like Newton) to fall, after an appropriate delay, into this 'absurdity'.

Riemann follows up on Gauss's Suggestion

Gauss made his suggestion of finite time delays in electrical interactions in a private letter to Weber, who had earlier been a colleague at Göttingen University. Weber never took up this idea but their most famous student and collaborator, Bernhard Riemann (1826-1866) was the first to explicitly introduce the concept of 'retarded time' in 1858 but this was not published until 1867 [115], ironically in the same issue of *The Philosophical Magazine* as L. V. Lorenz, who had independently developed the same idea [17].

Gauss's Speed "c" Suggestion

It is ironic that one of the world's greatest mathematicians, Carl Gauss (1777-1855) should add the critical *physical* idea of finite delay in 1845 to the revolutionary idea of action-at-a-distance proposed by the world's greatest natural philosopher, Sir Isaac Newton. Maxwell was fully aware of the significance of Gauss's work, as he was the first scientist to be named in his *Treatise* (even before his personal hero, Michael Faraday), where he wrote: "Gauss has brought his powerful intellect to bear on the theory of magnetism." [116] If Maxwell had fully appreciated the significance of this new, combined concept he might never have developed a theory of independent energy flowing through all of empty space. This new programme continues to build on this now almost forgotten idea that an interaction involves only the exchange of activity from one point particle to another without any need for an intervening medium.

Assis compromises Weber

Even Assis, a major supporter of Weber's EM theory, had a major difficulty accepting Weber's action-at-a-distance views; he was forced to speculate that instead of a complete vacuum there must exist a photon 'gas' between charged particles so that this gas could propagate transverse EM waves at finite speed [117], like sound waves but these are only longitudinal.

Lorenz 1867 Paper on EM

The almost forgotten Danish physicist Ludwig Valentin Lorenz (1829-1891) proposed in 1867 an alternative theory [17] that extended Kirchhoff 's current-induction model [118] introduced ten years earlier. Lorenz had rejected the then popular concept of the luminiferous aether as the physical basis of light as "it is scarcely possible to imagine a medium in which a wave motion could travel without a trace of longitudinal vibrations". He was strongly motivated by "the idea of the unity of force" and so he rejected all physical hypotheses as being too premature but based on these analytical results he confidently identified light with transverse variations in electrical current. Lorenz assumed that Kirchhoff's empirical micro-equation (describing the mutual current inductance in conductors via reciprocal, remote current densities) was just the first term in a series expansion of a new, simpler equation. This paper was the first to introduce the idea that local EM effects were due to remote source effects occurring earlier in time and not simultaneously. Most importantly, he introduced the retarded scalar and vector potentials based on his own earlier published solution to the inhomogenous wave equation. He also realized that the phase velocity of the propagating current fluctuations in a poorly conducting medium approaches that of light; in other words, he could view the vacuum as just a very poor conductor. If Lorenz had defined the "magnetic" field, like Maxwell, as the curl of the vector potential then Lorenz would have derived ALL of Maxwell's aether-based equations of EM.

Whitaker on Lorenz's 1867 Paper

In his famous history of EM and the aether, Edmund Whittaker (1873-1956) briefly reviewed Lorenz's contributions. While noting that Lorenz's equations are mathematically equivalent to the EM equation of Maxwell, Whittaker emphasizes that the retarded potentials are not identical with Maxwell's instantaneous scalar and vector potentials "for Lorenz's vector potential, $\underline{A}$ is not a circuital vector (like Maxwell's $\underline{A}$) while Lorenz's retarded scalar potential ϕ is not the same as Maxwell's instant electrostatic potential but depends on the positions occupied by the source charges at certain previous instants" [119].

Lorenz & Retardation

Lorenz was the first scientist to explicitly introduce the concept of retarded potentials into EM theory in 1867, this was ten years before Lord Rayleigh (1842-1919) used them in his own *Theory of Sound* [120]. FitzGerald was unaware of Lorenz's work but did reference Rayleigh when he (re)introduced retarded potentials into Maxwell's theory in 1883.

Jefimenko's causeless E&H

Oleg Jefimenko is firmly committed to the conventional view that only the past effects the present, he calls this view "the Principle of Causality" whereby all present phenomena are exclusively determined by past events. In Jefimenko's text on causality [121] he emphasizes that the commonly-held belief that "time-variable electric and magnetic fields cause each other" is wrong. He points out the non-causal character of Maxwell's Equations, which are only relationships between the electric field intensity, $\underline{E}$ and the magnetic intensity, $\underline{H}$ at a single point in space and the <u>same</u> time ($\underline{x}$, t). They are correlated together because both fields are linked to the same common cause: the changing remote electric current density, $\underline{J}(\underline{x}_s,t_s)$ at the earlier electric source time $t_s < t$. The separation of remote electrical interactions into a position component ($\underline{E}$) and a change-in-position component ($\underline{H}$) is a historical accident resulting only from the <u>macroscopic</u> phenomena of magnetized materials.

Borrowing Energy from the Future

Both Maxwell and Einstein relied implicitly of the reality of the concept of 'rigid rods' – a useful approximation in certain situations but not a fundamental element of reality. In particular, Maxwell used this concept to criticize delayed action-at-a-distance [122]. It is one of the contentions of this research programme that reality does permit the <u>temporary</u> 'borrowing' of energy across time. This revolutionary 'credit' concept will be elaborated upon in subsequent papers.

3. MAXWELL-HEAVISIDE AETHER THEORY

This paper has already rejected all of Maxwell's metaphysical assumptions underlying the final version of his EM theories but it is important to realize that the remaining mathematical 'skeleton' also has its own problems. In order to focus this discussion this section begins with a summary of the final mathematical equations of Maxwell's 'dynamical' theory of 1864. This summary is presented in the modern notation introduced by Heaviside when he 'gutted' Hamilton's 4D quaternions to produce the 3D vector notation better suited for a timeless and purely geometric exposition of classical electromagnetism. Section six of this paper will reverse this process and will show that a much more powerful mathematics for representing EM has been overlooked for over 150 years. Section 3.2 here quickly recreates Maxwell's original derivation to illustrate how Maxwell finally thought about this area of physics that had obsessed him for most of his life. As primarily a theory of magnetism, Maxwell focused on the concept of electro-kinetic momentum (now called vector potential), which he defined as the impulse of the electromotive force generated by the removal of all the source magnets and currents. Maxwell's most original contribution to EM theory was the addition of the 'displacement current' into Ampere's law, so that the free-space versions of the resulting equations reduced to the standard wave equations. This re-creation illustrates the numerous errors that Maxwell made in developing his original group of eight equations. The algebraic corrections made by Heaviside are then added that resulted in the famous four 'Maxwell Equations'. Fortunately for the history of EM, Heaviside's 'cleanup' eliminated any need for the controversial displacement current along with the scalar and vector potentials that Heaviside viewed (correctly) as implying 'spooky' action-at-a-distance, exemplified by Lorenz's 1867 theory. After a brief discussion of further problems with the *dynamical* paper, especially Maxwell's lack of interest in the remote sources of EM variations, this section concludes with a summary of Poynting's EM energy flow vector that is also re-created later in the CNV section.

3.1 EM FORCE FIELD EQUATIONS

Initially inspired by MacCullagh's 1839 rotational tensions in a continuous medium [123] and Faraday's 'lines-of-force' [65], Clerk Maxwell in 1855 first attempted to create a mathematical model of EM using vortices in a perfect fluid [63]. Maxwell's second attempt in 1861 was based on his infamous "gears & wheels" mechanical model of rotation [64]. In 1865 Maxwell published his final solution [66], which was constructed around energy fluctuations in a hypothetical, continuous medium that supported transverse oscillations (the 'aether'). Elastic displacements in this aetherial medium were described mathematically at the microscopic level using spatially directed partial differential operators; in modern notation: the gradient ($\underline{\nabla}\phi$), the divergence ($\underline{\nabla}\bullet\underline{A}$) and the curl or rotation ($\underline{\nabla} \wedge \underline{A}$). The result of this analysis was a set of twenty (20) equations since Maxwell used the traditional 3D Cartesian co-ordinate notation. Subsequently, in 1885 Oliver Heaviside introduced the more compact 3D vector notation [124] that reduced these to the famous four 'Maxwell Equations' [9] :

$$\underline{\nabla} \bullet \underline{E} = 4\pi\rho \qquad \underline{\nabla} \bullet \underline{B} = 0 \qquad \underline{\nabla} \wedge \underline{E} = -1/c\ \partial/\partial t\ \underline{B} \qquad \underline{\nabla} \wedge \underline{B} = 4\pi\,\underline{J}/c + 1/c\ \partial/\partial t\ \underline{E}$$

These four vector equations specified the relationships between the local electric and magnetic force *densities* ($\underline{E}$ and $\underline{B}$) at <u>every</u> point in space and time ($\underline{x}$, t) in terms of the <u>free</u> electric *charge and current densities* (ρ , $\underline{J}$) at the same location and time; in other words, they were represented by the mathematical constructs known as 'fields'. In free space ($\rho = 0$ & $\underline{J} = 0$) these equations can be combined into two forms of the homogenous 'Wave' equations:

$$\Box\,\underline{E} = 0 \qquad \Box\,\underline{B} = 0 \qquad \text{where } \Box\,\alpha \equiv (\underline{\nabla}^2 - 1/c^2\ \partial^2/\partial t^2)\,\alpha$$

These equations have solutions 'propagating from the origin at light speed, c' of the form: $\underline{E}(x, y, z; t) = \hat{\mathbf{e}}_z E_0\, f(z - c\,t)$

3.2 MAXWELL'S DYNAMICAL THEORY

It is instructive to quickly recreate the original derivation of these famous equations (but using modern vector notation and partial derivatives, not full derivatives as were used by Maxwell). Here the approach used by Maxwell in his most famous EM paper [66] will be followed closely as this will serve to remind today's readers of how Maxwell finally thought about this subject. All local references will be to the centennial version [125] published by Torrance.

As described in section 2 here, Maxwell began by proposing that all of space was filled by an aether that pervaded gross matter and possessed both elastic (dielectric) and kinetic (magnetic) properties. He imagined an electric *displacement* $\underline{D}$ induced (as in a dielectric) by the local presence of an electric field $\underline{E}$, coupled through a (linear) co-efficient of elasticity κ:

$$\underline{E}(\underline{x}, t) = \kappa\, \underline{D}(\underline{x}, t)$$ <Eqn. E, p. 67>

The key *assumption* here is that the temporal variations of this electrical displacement (the 'displacement current') were to be added to the real electric current density $\underline{J}$ to generate an effective, 'total' (or 'dynamic') current density $\underline{J}^*$:

$$\underline{J}^*(\underline{x}, t) = \underline{J}(\underline{x}, t) + \partial/\partial t\, \underline{D}(\underline{x}, t)$$ <Eqn. A, p. 61>

Maxwell used Neumann's concept of a vector potential $\underline{A}$ but interpreted now as the density of electro-kinetic momentum (that was produced by any system of magnets and/or electrical currents) to comply with Faraday's views on the reality of the magnetic field. Maxwell defined $\underline{A}$ as the total impulse of the electromotive force $\underline{E}_m$ generated by the removal of all the source magnets and currents, so that the integral is evaluated from the present time 't' or 't + τ' until the infinite future:

$$\underline{A}(\underline{x}, t) \equiv \int dt'\, \underline{E}_m(\underline{x}, t') = \tau\, \underline{E}_m(\underline{x}, t) + \underline{A}(\underline{x}, t+\tau) = \tau\, \underline{E}_m(\underline{x}, t) + \underline{A}(\underline{x}, t) + \tau\, \partial/\partial t\, \underline{A}(\underline{x}, t)$$

This defines the magnetic part of the EMF: $\underline{E}_m(\underline{x}, t) = -\partial/\partial t\, \underline{A}(\underline{x}, t)$ <Eqn. 29, p. 62>

Maxwell next identified the total EM momentum of a ***closed*** circuit with the number of lines of magnetic force N_m passing through the circuit, using either the magnetic flux intensity $\underline{B}$ or magnetic force intensity $\underline{H}$ and permittivity constant μ:

$$N_m(t) \equiv \int d\underline{l} \bullet \underline{A}(\underline{x}, t) = \iint d\underline{s} \bullet \underline{\nabla} \wedge \underline{A}(\underline{x}, t) = \iint d\underline{s} \bullet \underline{B}(\underline{x}, t) \quad \text{(using Stokes' theorem)}$$

$$\therefore\ \underline{B}(\underline{x}, t) = \underline{\nabla} \wedge \underline{A}(\underline{x}, t) = \mu\, \underline{H}(\underline{x}, t)$$ <Eqn. B, p. 63>

Maxwell then *assumed* that Ampere's Law of Steady Currents could be generalized to include dynamical currents, i.e. $\underline{J}^*$:

$$4\pi I(t) = \int d\underline{l} \bullet \underline{H}(\underline{x}, t) = \iint d\underline{s} \bullet \underline{\nabla} \wedge \underline{H}(\underline{x}, t) = 4\pi \iint d\underline{s} \bullet \underline{J}^*(\underline{x}, t)$$

$$\therefore\ \underline{\nabla} \wedge \underline{H}(\underline{x}, t) = 4\pi\, \underline{J}^*(\underline{x}, t)$$ <Eqn. C, p. 64>

Maxwell next *assumed* that the force density $\underline{E}_c$ on a conductor moving with velocity $\underline{v}$ through a magnetic field was given by:

$$\underline{E}_c(\underline{x}, t) = \underline{v} \wedge \underline{B}(\underline{x}, t)$$

Finally, Maxwell simply *assumed* that any free static charge would contribute a Coulomb potential ϕ and static force $\underline{E}_s$; so, inspired by Kirchoff's 1857 form of the electromotive force (see section 2.4.1), Maxwell just added the magnetic force on a conductor so that the total electromotive force acting on a unit charge $\underline{E}$ would simply be the sum of these three components:

$$\underline{E} = \underline{E}_s + \underline{E}_m + \underline{E}_c$$

$$\therefore\ \underline{E}(\underline{x}, t) = -\underline{\nabla}\, \phi(\underline{x}, t) - \partial/\partial t\, \underline{A}(\underline{x}, t) + \underline{v} \wedge \underline{B}(\underline{x}, t)$$ <Eqn. D, p. 66>

Maxwell now *assumed* by analogy with material dielectrics, that the quantity of 'free' positive electricity in any unit volume (the electrical density) ρ was produced by the net electrification of the different parts of the aether displacement field not neutralizing each other:

$$Q(t) = \iint d\underline{s} \bullet \underline{D}(\underline{x}, t) = \iiint dV\, \underline{\nabla} \bullet \underline{D}(\underline{x}, t) = -\iiint dV\, \rho(\underline{x}, t)$$

$$\therefore\ \underline{\nabla} \bullet \underline{D}(\underline{x}, t) + \rho(\underline{x}, t) = 0$$ <Eqn. G, p. 67>

Finally, Maxwell also adapted Kirchhoff's 1857 Equation of Charge Continuity by *assuming* that the real current $\underline{J}$ in a conducting medium could just be defined as the charge density multiplied by the local velocity:

$$\underline{J}(\underline{x}, t) = \rho(\underline{x}, t)\, \underline{v}(\underline{x}, t)$$

$$\underline{\nabla} \bullet \underline{J}(\underline{x}, t) + \partial/\partial t\, \rho(\underline{x}, t) = 0$$ <Eqn. H, p. 68>

The last equation was Maxwell's definition of specific resistance r per unit volume in a real conductor (this was never used):

$$\underline{E}(\underline{x}, t) = r\, \underline{J}(\underline{x}, t)$$ <Eqn. F, p. 67>

This set of eight (8) equations (A through H) are the original group of Maxwell's equations that were actually presented in 1864 in the 3D Cartesian co-ordinate format that was always used until the invention of the vector notation by Heaviside. Unfortunately this set of 20 equations contains several errors, not least because Maxwell was unaware of the use of modern Dimensional Analysis. These flaws emerge immediately when these equations are combined into a set of 'field' equations.

I Using (B): $\underline{\nabla} \bullet \underline{B} = \underline{\nabla} \bullet (\underline{\nabla} \wedge \underline{A}) = 0 \qquad \therefore\ \underline{\nabla} \bullet \underline{B} = 0$ (correct)

II Using (E) & (G): $\underline{\nabla} \bullet \underline{E} = \kappa\, \underline{\nabla} \bullet \underline{D} = -\kappa \rho$ Incorrect, should be: $\underline{\nabla} \bullet \underline{E} = 4\pi \rho$

III Using (D) & (B): $\underline{\nabla} \wedge \underline{E} = -\partial/\partial t(\underline{\nabla} \wedge \underline{A}) + \underline{\nabla} \wedge (\underline{v} \wedge \underline{B}) = -\partial/\partial t\, \underline{B} - \underline{v} \bullet \underline{\nabla B} = -\partial/\partial t\, \underline{B} - (d/dt\, \underline{B} - \partial/\partial t\, \underline{B})$

$\therefore\ \underline{\nabla} \wedge \underline{E} = -d/dt\, \underline{B}$ Incorrect, should be: $\underline{\nabla} \wedge \underline{E} = -\partial/\partial t\, \underline{B}/c$

IV Using (C), (B) & (E): $\underline{\nabla} \wedge \underline{B} = \underline{\nabla} \wedge (\mu\, \underline{H}) = \mu\, 4\pi\, \underline{J}^* = 4\pi\mu\, (\underline{J} + 1/\kappa\, \partial/\partial t\, \underline{E})$ Definition: $c^2 \equiv \kappa / 4\pi\mu$

$\therefore\ \underline{\nabla} \wedge \underline{B} = 4\pi\mu\, \underline{J} + \partial/\partial t\, \underline{E} / c^2$ Incorrect, should be: $\underline{\nabla} \wedge \underline{B} = 4\pi\, \underline{J}/c + \partial/\partial t\, \underline{E}/c$

V Using (C), (A) & (G): $\underline{\nabla} \bullet (\underline{\nabla} \wedge \underline{H}) = 4\pi\, \underline{\nabla} \bullet \underline{J}^* = 4\pi\, \underline{\nabla} \bullet (\underline{J} + \partial/\partial t\, \underline{D}) = 4\pi\, (\underline{\nabla} \bullet \underline{J} + \partial/\partial t\, \underline{\nabla} \bullet \underline{D}) = 0$

$\therefore\ \underline{\nabla} \bullet \underline{J} - \partial/\partial t\, \rho = 0$ Incorrect, should be: $\underline{\nabla} \bullet \underline{J} + \partial/\partial t\, \rho = 0$

So, embarrassingly, Maxwell got four out of his five equations wrong! Fortunately, the original set of equations were later 'cleaned up' (at least, algebraically) by Maxwell's disciple, Oliver Heaviside, who made the following corrections:

(E′): $\underline{E}(\underline{x}, t) = 4\pi\kappa\, \underline{D}(\underline{x}, t)$ (G′): $\kappa\, \underline{\nabla} \bullet \underline{D}(\underline{x}, t) - \rho(\underline{x}, t) = 0$ II′ $\underline{\nabla} \bullet \underline{E} = 4\pi \rho$

(A′): $\underline{J}^*(\underline{x}, t) = \underline{J}(\underline{x}, t) + \kappa\, \partial/\partial t\, \underline{D}(\underline{x}, t)$ (C′): $\underline{\nabla} \wedge \underline{B}(\underline{x}, t) = 4\pi\, \underline{J}^*(\underline{x}, t)/c$ IV′ $\underline{\nabla} \wedge \underline{B} = (4\pi\, \underline{J} + \partial/\partial t\, \underline{E})/c$

(D′): $\underline{E}(\underline{x}, t) = -\underline{\nabla}\, \phi(\underline{x}, t) - \partial/\partial t\, \underline{A}(\underline{x}, t)/c + \underline{v} \wedge \underline{B}(\underline{x}, t)/c$ V′ $\underline{\nabla} \bullet \underline{J} + \partial/\partial t\, \rho = 0$ III′ $\underline{\nabla} \wedge \underline{E} = -\partial/\partial t\, \underline{B}/c$

This 'elegant' formulation of the famous four 'Maxwell Equations' also achieved Heaviside's private agenda of eliminating the EM potentials (ϕ, $\underline{A}$) as these introduced the concept of action-at-a-distance, as exemplified by Lorenz's 1867 results. This approach also (fortunately) dropped Maxwell's only 'physical' innovation: his aetherial displacement and its temporal derivative (the 'displacement current'). All that was left then were the local 'field' variables ($\underline{E}$ and $\underline{B}$) and the real electric charges and currents (ρ and $\underline{J}$), related through the critical (and dubious) modern definition: $\underline{J} = \rho\, \underline{v}$. As we will see, this set of equations and EM variables happened to be isomorphic with Helmholtz's vector flow model of charge-density and it is in this form that "the EM theory of Maxwell" is now taught to students of physics and engineering today.

3.3 PROBLEMS WITH MAXWELL'S DYNAMICAL THEORY

At the conclusion of this revolutionary paper, Maxwell admitted that he had only presented a mathematical model and not a theory based on any real physical model; he apologized that it was only "illustrative and not explanatory" (his section 73). There are also several other problems with this paper that are rarely, if ever, discussed: possibly in deference to 'the Master'.

The first problem was one of notation; Maxwell was not aware of the 'convective' derivative and so only used the notation for full time derivatives (d/dt) and not the partial derivatives ($\partial/\partial t$). The second problem was also notational, Maxwell omitted all of the explicit space and time arguments so it is not obvious which symbols represent constants (e.g. μ) and which represent functions (e.g. ρ). Similarly, since Maxwell was developing a local field theory he just assumed that the space and time arguments were common, e.g. he actually wrote: $\underline{E} = \kappa\ \underline{D}$ for equation A. The presentation developed here corrects all these omissions and uses both partial derivatives and explicit space and time arguments for functions.

Maxwell also introduced conceptual difficulties when he represented the ideas of 'electromotive force' (the net 'driving' force in a conductor) and 'electric field intensity' (the accelerative force in a vacuum per unit charge) by the same symbol $\underline{E}$.

It is also more than annoying that Maxwell ignored dimensional scaling factors, especially the symbol 'c' for light-speed in his equations (especially equation C), so that his objective, the speed of light, has to appear 'as if by magic' from 'physical' properties of nothing (the vacuum) such as κ and μ (but, in his view, they did reflect real characteristics of the aether itself).

Maxwell totally ignored all references to the sources of the EM phenomena, so that he provided only a local (at one single point in time and space) set of mathematical equations while the causative real sources (ρ and $\underline{J}$) are at remote locations: a failure that was properly addressed by his rival, L. V. Lorenz, who identified fluctuations in the source charges and currents producing associated fluctuations in the remote charges and currents (at Maxwell's possibly empty 'field point').

3.4 EM ENERGY FLOW

In 1884, John Poynting extended Maxwell's theory to demonstrate that in free space, energy was transferred at light-speed "***by*** these electromagnetic fields" [126]. Again, in modern notation, the EM field energy density $W_0 \equiv (E^2 + B^2)/8\pi$ with its associated EM field energy current density (or 'Poynting vector') defined as $\underline{W} \equiv (\underline{E} \wedge \underline{B})/4\pi = W_0\ \underline{n}$; with these he was able to derive the energy density conservation equation:

$$\underline{E} \bullet \underline{J} + D_c / Dt\ W_0 = 0 \quad \text{where} \quad D_c / Dt\ W_0 = \partial/\partial t\ W_0 + c\ \underline{\nabla} \bullet \underline{W}$$

In this form, use has been made of the 'light-speed total time derivative', D_c / Dt. All of these results will be derived in the next paper describing a classical two-electron model of electromagnetism.

4. MODERN CLASSICAL ELECTRODYNAMICS

4.1 HELMHOLTZ FLUID DYNAMICS EM MODEL

Classical EM in the 19th Century is usually presented as a rivalry between two major approaches: the field theory of Clerk Maxwell and the action-at-a-distance theory of the 'Continentalists', exemplified by Wilhelm Weber. In fact, there was a third school headed by Herman von Helmholtz, who was an even stronger rival of Weber than Maxwell. Helmholtz was the first to pioneer the fluid or hydrodynamic model of electricity and, as a study of modern textbooks shows, this was actually how Maxwell's aetherial field approach was transformed into the modern view of CEM, with Weber's EM theory becoming almost forgotten. It is worthwhile to present a summary of this approach as it quickly demonstrates the links to the modern view and the subtle assumptions that tie these two aspects of CEM together. The approach followed here is based on the texts by Abraham and then Becker, as described in section 2.3.3 [127]. Abraham has confused the issue from the beginning by referring to Helmholtz's hydrodynamic model inspiring Maxwell to develop a similar model for Faraday's lines-of-force. This was only true in 1855 with Maxwell's first EM paper but was incorrect with respect to Maxwell's final 1864 paper that took a 'dynamical' approach that used NO hydrodynamic analogies but was constructed around a static, elastic medium (the aether). The underlying mathematical isomorphism is that both theories are vector field theories, with Maxwell focused on the medium's elastic displacement ($d\underline{x}$) while Helmholtz emphasized the fluid's displacement over time ($d\underline{x}$ / dt or velocity).

The basis of the hydrodynamic model is the subtle interweaving of the point-particle viewpoint with the continuum view of the perfect, incompressible fluid (first presented by Euler) where every point in the regions containing the fluid is considered to have its own, unique *velocity vector*. Fluid motion through space is represented by a function that varies continuously with location $\underline{x}$ (hence a mathematical 'field'). This velocity field $\underline{v}(\underline{x}, t)$ is called *irrotational* (or vortex free) when its line integral over <u>any closed path</u> is zero: in other words, there is no increase in speed when a fluid 'point' moves around any fixed location. This type of field can always be represented by the (negative) gradient of a scalar function of position $\phi(\underline{x})$.

$$\textit{Irrotational}: \int d\underline{l} \bullet \underline{v}(\underline{x}, t) = 0 \qquad \text{or} \qquad \underline{v}(\underline{x}, t) = -\underline{\nabla}\phi(\underline{x}, t) \qquad d\phi = \underline{\nabla}\phi \bullet d\underline{x} = -\underline{v} \bullet d\underline{x}$$

It is important to note that this is really a *static or timeless view of the world*, where either the closed path is traversed all at the same time (i.e. <u>instantaneously</u>) or the velocity field does <u>not</u> depend on time; in other words: $\underline{v}(\underline{x}, t) = \underline{v}(\underline{x})$. The most important characteristic of a 'perfect' fluid is that it is <u>incompressible</u>, so that in any <u>filled</u> finite, <u>fixed</u> volume of space $\mathcal{V}$ all the fluid that enters its <u>fixed</u>, defining surface over any time period $\mathcal{T}$ must equal the volume of fluid that leaves through this <u>same</u> surface during this <u>same</u> time period. These finite constraints ($\mathcal{V},\mathcal{T}$) disappear when the infinitesimal limit definition of the field's *divergence* <u>at a point</u> is introduced as the outward flow per unit volume surrounding the specified location:

$$\textit{Divergence}: \ \text{div}\, \underline{v}(\underline{x}, t) \equiv \lim_{\mathcal{V} \to 0} \{ 1/\mathcal{V} \iint d\underline{s} \bullet \underline{v}(t) \} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z = \underline{\nabla} \bullet \underline{v}$$

If the velocity field $\underline{v}$ represents fluid motion in a region without sources or sinks of fluid then in this region: $\underline{\nabla} \bullet \underline{v}(\underline{x}, t) = 0$. But when new fluid can appear at a source point $\underline{X}_p$ then $\underline{\nabla} \bullet \underline{v}(\underline{X}_p) > 0$ while if fluid disappears through a sink-hole at $\underline{X}_q$ then $\underline{\nabla} \bullet \underline{v}(\underline{X}_q) < 0$. Now, a single point source is assumed to generate new fluid equally in all directions, <u>always</u> flowing radially away from the source point (i.e. the assumption of spherical symmetry). The emission strength of this source is characterized as unity when it generates 4π units of fluid per second so that one unit of fluid flows across each steradian with the same quantity of fluid crossing all spherical surfaces equally, independent of distance. So the quantity of fluid crossing per second through the surface of a sphere of radius r from a source at the sphere's center is equal to strength e.

$$\therefore\ e = 1/4\pi \iint d\underline{s} \bullet \underline{v}(r) = r^2\, v(r) \qquad \therefore\ v(r) = e / r^2 = -\underline{\nabla}\phi(r) \qquad \therefore\ \phi(r) = e / r$$

The next step crucially assumes that the net effect of a set of sources e_j is the <u>linear</u> addition of each source; this is simply the Principle of Superposition that has always been assumed throughout mechanics but is challenged in this programme.

$$\therefore\ \phi(r) = \textstyle\sum_{\mathbf{j}} e_j / r_j \qquad \therefore\ \underline{v}(r) = -\underline{\nabla}\phi(r) = \textstyle\sum_{\mathbf{j}} \underline{v}_j$$

It is the **resemblance** of this result to Coulomb's (electrostatic) law that is **always** taken as the starting point for CEM.

The next stage in this model is to introduce sources of energy to accelerate the flow of fluid; this is done by introducing rotational flow where the fluid can acquire rotational speed around certain points or vortices (miniature 'paddlewheels'). This is described mathematically through the limit concept of rotation or *curl* over a closed surface $\mathcal{S}$.

Rotation: $\text{curl } \underline{v}(\underline{x}, t) \equiv \hat{\mathbf{e}}_x (\partial v_z/\partial y - \partial v_y/\partial z) + \hat{\mathbf{e}}_y (\partial v_x/\partial z - \partial v_z/\partial x) + \hat{\mathbf{e}}_z (\partial v_y/\partial x - \partial v_x/\partial y) = \underline{\nabla} \wedge \underline{v}$

Let $\underline{w}(\underline{x}, t) = \text{curl } \underline{v}(\underline{x}, t)$ then: $w_z = \lim_{\mathcal{S} \to 0} \{ 1/\mathcal{S} \iint d\underline{s} \bullet \underline{v}(t) \}_z = \lim_{\mathcal{S} \to 0} \{1/\mathcal{S} \iint (v_x\, dx + v_y\, dy) \}$

Whenever a vector field, like $\underline{w}$ is definable in terms of the curl of another vector field (like $\underline{v}$) then it is called *solenoidal*. Combining the two definitions of divergence and curl, at the same point, will always give a zero result: $\underline{\nabla} \bullet \underline{\nabla} \wedge \underline{v} = 0$. So, a mathematical test for the occurrence of a solenoidal field is the identity: div $\underline{w} = 0$. Conversely, if a vector field $\underline{B}$ is found to be solenoidal, then there must exist another field, called its *vector potential* $\underline{A}$, from which it can be defined:

$$\text{If } \underline{\nabla} \bullet \underline{B} = 0 \quad \text{then} \quad \underline{B} = \underline{\nabla} \wedge \underline{A}$$

A point-vortex is characterized by its rotational source density vector field $\underline{\Gamma}$. In general, any velocity vector field can be impacted by material and rotational sources; so if a region contains a continuous distribution of point-like material sources with density ρ and point-like rotational energy sources with density $\underline{\Gamma}$:

$$\underline{\nabla} \bullet \underline{v}(\underline{x}, t) = 4\pi\, \rho(\underline{x}, t) \quad \text{and} \quad \underline{\nabla} \wedge \underline{v}(\underline{x}, t) = 4\pi\, \underline{\Gamma}(\underline{x}, t)$$

Helmholtz realized that any vector may be decomposed into the sum of two component vectors, thus: $\underline{v} = \underline{v}_1 + \underline{v}_2$

Here, $\underline{v}_1$ is defined to be irrotational and generated by the material (ρ) sources while $\underline{v}_2$ is defined as solenoidal and only influenced by the rotational ($\underline{\Gamma}$) sources; therefore:

$$\underline{\nabla} \wedge \underline{v}_1 \equiv 0 \quad \& \quad \underline{\nabla} \bullet \underline{v}_2 \equiv 0 \quad \text{and} \quad \underline{\nabla} \bullet \underline{v}_1 = 4\pi\, \rho \quad \& \quad \underline{\nabla} \wedge \underline{v}_2 = 4\pi\, \underline{\Gamma}$$

Since $\underline{v}_1$ is irrotational then it has its own scalar potential ϕ, so: $\underline{v}_1 = -\underline{\nabla}\phi$ giving: $\nabla^2 \phi = -4\pi\rho$ whereas $\underline{v}_2$ is solenoidal so it must be possible to identify its own vector potential $\underline{A}$, thus: $\underline{v}_2 = \underline{\nabla} \wedge \underline{A}$ $\therefore$ $\underline{\nabla} \wedge \underline{\nabla} \wedge \underline{A} = 4\pi\, \underline{\Gamma}$ the addition of the arbitrary ('gauge') condition: $\underline{\nabla} \bullet \underline{A} = 0$ leads to the result: $\nabla^2 \underline{A} = -4\pi\, \underline{\Gamma}$ All of these Laplacian equations have solutions:

$$\phi(\underline{x}', t) = \iiint d\mathcal{V}\, \rho(\underline{x}, t) / R \quad \text{and} \quad \underline{A}(\underline{x}', t) = \iiint d\mathcal{V}\, \underline{\Gamma}(\underline{x}, t) / R \quad \text{where: } \underline{R} = \underline{x}' - \underline{x}$$

If these fields vanish at any surface at infinity then: $\underline{v} = -\underline{\nabla}\phi + \underline{\nabla} \wedge \underline{A}$ and $\iiint d\mathcal{V}\, \underline{\nabla}\phi(\underline{x}, t) \bullet \underline{A}(\underline{x}, t) = 0$

Faraday's 1831 discovery of magnetic induction can be stated in differential form for the electromotive force $\mathcal{E}$ and field $\underline{B}$:

$$\mathcal{E} = \int d\underline{l} \bullet \underline{E}_m(\underline{x}, t) = -1/c\, d/dt \iint d\underline{s} \bullet \underline{B}(\underline{x}, t) \quad \text{where } \underline{E}_m \text{ is the induced electric field intensity.}$$

This result is assumed to be equally valid for any ***closed*** path *even when there are no conductors present*. Stoke's theorem is next applied for media at rest so the induction flux only changes in so far as the magnetic field $\underline{B}$ explicitly changes.

$$\underline{\nabla} \wedge \underline{E}_m(\underline{x}, t) = -\partial/\partial t\, \underline{B}(\underline{x}, t) / c$$

When a spatial region is moving with uniform velocity $\underline{u}$ then the surface of the region alters over time, leading to:

$$d/dt \iint d\underline{s} \bullet \underline{B}(\underline{x}, t) = \iint d\underline{s} \bullet \{ \partial/\partial t\, \underline{B} + \underline{u}\, (\underline{\nabla} \bullet \underline{B}) - \underline{\nabla}\, (\underline{u} \wedge \underline{B}) \} \text{ but } \underline{\nabla} \bullet \underline{B} = 0 \;\&\; D/Dt\, A \equiv \partial/\partial t\, \underline{A} + \underline{u}\, (\underline{\nabla} \bullet \underline{A}) - \underline{\nabla}\, (\underline{u} \wedge \underline{A})$$

$$\therefore\; \underline{\nabla} \wedge \underline{E}_m(\underline{x}, t) = -D/Dt\, \underline{B}(\underline{x}, t) / c$$

The Continuity equation is: $\underline{\nabla} \bullet \underline{J} + \partial/\partial t\, \rho = 0$ while $\underline{\nabla} \bullet \underline{D} = 4\pi\rho$ so that $\underline{\nabla} \bullet (\underline{J} + 1/4\pi\, \partial/\partial t\, \underline{D}) = 0$ Let $\underline{J}^* = \underline{J} + 1/4\pi\, \partial/\partial t \underline{D}$

So this vector $\underline{J}^*$ is solenoidal *suggesting* Ampere's law: $\underline{\nabla} \wedge \underline{B} = 4 \pi \underline{J} / c$ be *generalized* to: $\underline{\nabla} \wedge \underline{B} = 4 \pi \underline{J}^* / c$

It is very important to notice that in this model, the ***force*** field intensity vector $\underline{v}(\underline{x}', t)$ is ***not*** the local electric density (at the field point, $\underline{x}'$) or the electric current $\underline{J} = \rho\, \underline{v}$ but the influence (subsequent velocity) of the remote fluid source $\rho(\underline{x}, t)$. This fluid model still requires an extra interaction specification to define how these two fluids (local and remote) interact.

4.2 CHARGE-DENSITY ELECTROMAGNETISM

In its modern reformulation, CEM is presented as oscillations in a continuous *electric charge density*, $\rho(\underline{x}, t)$. In regions of space, where this density is non-zero and it is in motion (relative to an inertial reference frame) with velocity, $\underline{v}(\underline{x}, t)$ this flow manifests as an electric current density $\underline{J}(\underline{x}, t)$, defined by the key equation: $\underline{J} \equiv \rho\, \underline{v}$. For example, Pauli introduced electrodynamics [128] through a set of N discrete, point charges $q_j(\underline{x}, t)$ in a finite spatial volume $\mathcal{V}$, creating a total (free) electric charge Q. The rate that this charge leaves this volume defines the total electric current I(t) at any instant of time t; the amount of charge flowing through one square centimeter every second around the 'field' point ($\underline{x}$, t) and orthogonal to the velocity at this point defines the current density $\underline{J}$.

$$I(t) = \iint d\underline{s} \bullet \underline{J}(t) = -\, d/dt\, Q(t) = -\, d/dt \sum_j q_j(t)$$

At this point, Pauli introduces *the most critical assumption in the whole theory of CEM* when he *defines* the electric volume charge density at the field point as a double-limit:

$$Q(t) = \lim_{N \to \infty} \sum_j \lim_{qj \to 0} \{ q_j(t) \} = \lim_{N \to \infty} \sum_j \lim_{qj \to 0} \{ \Delta\mathcal{V}_j\, (q_j / \Delta\mathcal{V}_j) \} = \iiint d\mathcal{V}\, \rho(\underline{x}, t)$$

The application of Gauss's theorem then results in the Continuity equation for electric charge at any point:

$$I(t) = \iiint d\mathcal{V}\, \underline{\nabla} \bullet \underline{J} = \iint d\underline{s} \bullet \underline{J} = -\, d/dt\, Q(t) = -\, d/dt \iiint d\mathcal{V}\, \rho(\underline{x}, t) \quad \therefore\ \underline{\nabla} \bullet \underline{J} + d/dt\, \rho = 0$$

Here, it is ***always*** assumed that the differential limit defining the instantaneous velocity $\underline{v}(x, t)$ is compatible with the static limit of charge density, as defined by Pauli above, or in Helmholtz's approach or Maxwell's 1855 hydrodynamic analogy [63]. The electric field intensity $\underline{E}(\underline{x}, t)$ is also defined by Pauli as the limit process ($q_j \to 0$) of the force $\underline{F}_e$ on an isolated test body (q_j), ***at rest***:

$$\underline{E} \equiv \lim_{qj \to 0} \{ \underline{F}_e / q_j \} \qquad \underline{F}_e \equiv Q < \underline{E} > = \iiint d\mathcal{V}\, \underline{f}_e = \iiint d\mathcal{V} \lim_{qj \to \infty} \{ \lim_{N \to 0} \{ \textstyle\sum_j q_j\, \underline{E} \} \}$$

The magnetic field intensity, in vacuo, $\underline{B}(\underline{x}, t)$ is defined through a comparable limit process using the Lorentz force $\underline{F}_m$ on a finite, electrified body with a total charge Q ***moving*** with velocity $\underline{v}$: $\underline{F}_m = Q\, \underline{v} \wedge \underline{B} / c$ and $\underline{F} = \underline{F}_e + \underline{F}_m$

$$\underline{F} \equiv = \iiint d\mathcal{V}\, \underline{f}(\underline{x}, t) = \lim_{N \to \infty} \{ \sum_j \lim_{qj \to 0} \{ (q_j / \Delta\mathcal{V}_j)\, (\underline{E} + \underline{v} \wedge \underline{B} / c)\, \Delta\mathcal{V}_j \} \} \quad \therefore\ \underline{f} = \rho\, \underline{E} + \underline{J} \wedge \underline{B} / c$$

All of these limit definitions ***assume*** that the product of macro averages (e.g. Q, $\underline{F}$ etc) are equivalent to the average of the micro products (e.g. ρ, $\underline{E}$ etc) and, most importantly, that the velocity limit can be taken independent of the other limits.

Expositions on the charge density model never talk about its ***fundamental flaw***: the infinite mutual repulsion generated by the zero spatial separation between infinitesimal spatial cells filled with similar electrically charged 'fluid'. This was also the crucial weakness in all finite models of the electron (introduced around 1900) but at least their proposers had the honesty to acknowledge this problem and admit the need for some kind of unknown mechanical counter-force that would prevent these small spheres from instantly blowing apart. The Coulomb force law always suffers from this problem, except in the case of the truly point model of the electron, but these were not the real objects measured by Coulomb.

4.3 RECOVERING MAXWELL'S EQUATIONS

The conventional approach in CEM is to generate vector field differential equations that are consistent with the macroscopic experiments that are described as integral equations. The following 'integral' laws are described along with their equivalent differential forms that all ***assume*** the validity of the transformations implied by the integral theorems of Gauss and Stokes.

1) Coulomb's Law $\int d\underline{l} \bullet \underline{E}_s(\underline{x}, t) = 0$ $\underline{E}_s(\underline{x}, t) = -\underline{\nabla}\, \phi(\underline{x}, t)$

2) Ampere's Law $\int d\underline{l} \bullet \underline{B}(\underline{x}, t) = 4\, \pi/c\; I(t)$ $\underline{\nabla} \wedge \underline{B}(\underline{x}, t) = 4\, \pi/c\; \underline{J}(\underline{x}, t)$

3) Faraday's Law $\int d\underline{l} \bullet \underline{E}_m(\underline{x}, t) = -\, d/dt \iint d\underline{s} \bullet \underline{B}(\underline{x}, t) / c$ $\underline{\nabla} \wedge \underline{E}_m(\underline{x}, t) = -\, \partial/\partial t\; \underline{B}(\underline{x}, t) / c$

4) 'Lorentz' Law $\iiint d\mathcal{V}\, \underline{E}_c(\underline{x}, t) = \iiint d\mathcal{V}\, \underline{v}(\underline{x}, t) \wedge \underline{B}(\underline{x}, t) / c$ $\underline{E}_c(\underline{x}, t) = \underline{v}(\underline{x}, t) \wedge \underline{B}(\underline{x}, t) / c$

In order to preserve compliance with the Conservation of Charge : $\underline{\nabla} \bullet \underline{J}(\underline{x}, t) + d/dt\; \rho(\underline{x}, t) = 0$

Maxwell had to introduce his 'displacement current' term into Ampere's Law: $J^*(\underline{x}, t) = \underline{J}(\underline{x}, t) + 1/4\pi\; \partial/\partial t\; \underline{E}(\underline{x}, t)$

These equations have to be supplemented with the following 'supportive' definitions:

5) $\underline{J}(\underline{x}, t) = \rho(\underline{x}, t)\, \underline{v}(\underline{x}, t)$ 6) $\underline{B}(\underline{x}, t) = \underline{\nabla} \wedge \underline{A}(\underline{x}, t)$ 7) $\underline{E} = \underline{E}_s + \underline{E}_m + \underline{E}_c$

These extra definitions generate the last two 'field' equations: 8) $\underline{\nabla} \bullet \underline{E}(\underline{x}, t) = 4\pi\, \rho(\underline{x}, t)$ 9) $\underline{\nabla} \bullet \underline{B}(\underline{x}, t) = 0$

This set of equations may be said to be the mathematical summary of the classical field theory of electromagnetism and essentially summarize the presentation of the charge density model as it is found in almost all modern textbooks. Since the differential formulations are simply the mathematical equivalents of the four 'experimental' laws (defined by equations 1 through 4) it cannot be said that "experiment confirms this theory" only that it is mathematically consistent with experiment.

However, as these experimental 'laws' were summaries of macroscopic experiments, involving vast numbers of electrons moving through complex, many-atom structures, such as macroscopic conductors, with measurements being taken over macroscopic time averages, it is more than a little optimistic to say that physics has the final, 'micro-classical' theory of EM.

Note, it is Faraday's Law that introduces both the problems with Ampere's law and is ultimately the source of all the problems of relative motion in EM that finally results in the need for the Special Theory of Relativity, as this 'law' is the only one that introduces the effect of ***changes in time*** (through the use of the time derivative).

5. SUMMARY OF CONTINUOUS NATURAL VECTORS

This section summarizes the mathematics of *Natural Vectors* introduced in the first paper [4] in this series.

5.1 DEFINITION OF SINGLE-POINT CNVs

The definition of a Natural Vector (see I-4.4) is viewed as an *imaginary scalar quaternion*, symbolized by $\mathbf{Q}$.

Natural Vector: $\mathbf{Q} \equiv i\,\mathbf{I_0}\,q_0 + \mathbf{I_1}\,q_1 + \mathbf{I_2}\,q_2 + \mathbf{I_3} q_3 \equiv \mathbf{Q}_0 + \sum_j \mathbf{Q_j} \equiv \{i\,q_0\,;\,\underline{q}\} = i\,q_0\,\mathbf{I_0} + \sum_j q_j\,\mathbf{I_j}$

Here the $\mathbf{I}_\mu$ are defined in terms of real 4x4 matrices and $\mathbf{I_0}$ is isomorphic with the unit number, while the $\{\mathbf{I_1}, \mathbf{I_2}, \mathbf{I_3}\}$ are isomorphic with Hamilton's three linearly independent imaginary quantities $\{\mathbf{i, j, k}\}$. So these bases satisfy the group multiplication rules, using the indices j, k, l = 1, 2, 3 and μ = 0, j :

$$\mathbf{I_0}\,\mathbf{I}_\mu = \mathbf{I}_\mu\,\mathbf{I_0} = \mathbf{I}_\mu \qquad\qquad \mathbf{I_j}\,\mathbf{I_k} = -\,\delta_{\mathbf{ij}}\,\mathbf{I_0} + \varepsilon_{jkl}\,\mathbf{I_l}$$

Here, $\delta_{\mathbf{ij}}$ is the Kronecker delta symbol with value +1 when both indices are equal or zero otherwise and ε_{jkl} is the cyclic permutation tensor whose value is zero unless all three indices are different when its value is +1 if the indices are cyclic (even permutation of 1,2,3) or –1 if anti-cyclic (odd permutation). The rules for (conjugate) addition and multiplication of two Natural Vectors $\mathbf{A}^*$ and $\mathbf{B}$ become:

Addition: $\mathbf{A}^* + \mathbf{B} = -\,i\,\mathbf{I_0}\,(a_0 - b_0) + (\underline{a} + \underline{b}) \bullet \underline{\mathbf{I}}$

Multiplication: $\mathbf{A}^*\mathbf{B} = \mathbf{I_0}\,(a_0\,b_0 - \underline{a} \bullet \underline{b}) + i\,\underline{\mathbf{I}} \bullet (b_0\,\underline{a} - a_0\,\underline{b}) + (\underline{a} \wedge \underline{b}) \bullet \underline{\mathbf{I}}$

Since electrons are each considered both unique and eternal they can be 'labeled' by a unique positive integer identifier 'k'. It was shown earlier (§I-5.1.4) that electrons are fermions, so each electron will have a unique position in space ($\underline{x}$) at any time, t. It is the fundamental hypothesis of this research programme that these two parameters of every electron can be mapped into their own individual Natural Vector, $\mathbf{X}$(k) or $\mathbf{X}_k$, rather than two separate traditional algebraic variables.

Hypothesis: $\{\,\underline{x}(k\!:t_k)\,\} \cong \mathbf{X}_k \equiv i\,c\,t_k\,\mathbf{I_0} + \underline{x}_k \bullet \underline{\mathbf{I}}$

The square (or 'norm') of this 'positional' NV is: $\mathbf{X}(t)^*\,\mathbf{X}(t) = (c^2\,t^2 - x^2)\,\mathbf{I_0}$

Similarly,

Velocity: $\mathbf{V}(t) \equiv \lim_{\delta t \to 0} \{\,(\mathbf{X}(t + \delta t) - \mathbf{X}(t))\,/\,\delta t\,\} = d\mathbf{X}(t)/dt = i\,c\,\mathbf{I_0} + \underline{v} \bullet \underline{\mathbf{I}}$

The "norm" of this velocity NV is: $\mathbf{V}(t)^*\,\mathbf{V}(t) = (c^2 - v^2)\,\mathbf{I_0}$

5.2 SUMMARY OF CNV FLOW VECTORS

The theory of Natural Vectors follows Hamilton by extending his 'nabla' (or 'gradient') 3D space operator ($\underline{\nabla}$) to the 4D Natural Vector ***Gradient*** applied to any scalar function ψ that is continuous in the four space-time variables $\{t\,;\,\underline{x}\}$:

Gradient: $\nabla\psi(t\,;\underline{x}) \equiv i\,\mathbf{I_0}\,\partial_0\psi(t\,;\underline{x}) + \underline{\mathbf{I}} \bullet \underline{\nabla}\psi(t\,;\underline{x})$ & $\underline{\nabla} \equiv \hat{\mathbf{e}}_1\partial_1 + \hat{\mathbf{e}}_2\,\partial_2 + \hat{\mathbf{e}}_3\partial_3 \quad \partial_0 \equiv \partial/c\,\partial t \quad \partial_1 \equiv \partial/\partial x_1$ etc

The conjugate of the NV gradient operator can be applied to any continuous natural vector (CNV) function, $\mathbf{Q}(t\,;\underline{x})$:

$$\nabla^*\mathbf{Q}(t\,;\underline{x}) = \mathbf{I_0}\,(\partial_0 q_0 - \underline{\nabla} \bullet \underline{q}) + i\,\underline{\mathbf{I}} \bullet (\underline{\nabla} q_0 - \partial_0 \underline{q}) + \underline{\mathbf{I}} \bullet (\underline{\nabla} \wedge \underline{q})$$

If $\nabla^*\mathbf{Q} = 0$ then: 1) $\underline{\nabla} \bullet \underline{q} = \partial_0 q_0$ 2) $\underline{\nabla} q_0 = \partial_0 \underline{q}$ 3) $\underline{\nabla} \wedge \underline{q} = 0$

The "***Zero Condition***" CNV operator, $\mathbf{Z}$ is defined as: $\mathbf{Z}\psi \equiv i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla} + \underline{v}\,\partial_0)\psi - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\psi)$

The CNV equivalent of the total-time differential for a scalar function: $\mathbf{I_0}\,d\psi/dt = -\mathbf{V}^*\nabla^*\psi$ if $\mathbf{Z}\psi = 0$

The "CNV ***Flow***" equation can be derived for any continuous CNV: $d\mathbf{Q}/dt + \mathbf{V}^*\nabla^*\mathbf{Q} = 0$ if $\mathbf{Z}\mathbf{Q} = 0$

5.3 SUMMARY OF CNV VOIGT VECTORS

All CNVs satisfy several useful identities, presented here; again Q_0 and α are scalar functions and $\underline{Q}$ is a vector function.

1. $\nabla^*\mathbf{X} = -2\,\mathbf{I_0}$ 2. $\nabla^*\mathbf{V} = -i\,\underline{\mathbf{I}} \bullet \partial_0\underline{v}$ 3. $\nabla^*\alpha = -i\,\mathbf{I_0}\,\partial_0\alpha + \underline{\mathbf{I}} \bullet \underline{\nabla}\alpha$

4. $\nabla^*(\alpha\,\mathbf{Q}) = \alpha\,(\nabla^*\mathbf{Q}) + (\nabla^*\alpha)\mathbf{Q}$ 5. $\nabla^*\nabla\alpha = \nabla\,\nabla^*\alpha = \mathbf{I_0}\,(\partial_0^2 - \underline{\nabla}^2)\,\alpha \equiv -\mathbf{I_0}\,\Box\alpha$

6. $\mathbf{V}^*\nabla^*(\alpha) = -\mathbf{I_0}\,(c\,\partial_0 + \underline{v} \bullet \underline{\nabla})\alpha - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla} + \underline{v}\,\partial_0)\,\alpha + \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\alpha)$

7. $\mathbf{Q}\,\nabla^*\alpha = \mathbf{I_0}\,(Q_0\,\partial_0\alpha - \underline{Q} \bullet \underline{\nabla}\alpha) + i\,\underline{\mathbf{I}} \bullet (Q_0\,\underline{\nabla}\alpha - \underline{Q}\,\partial_0\alpha) + \underline{\mathbf{I}} \bullet (\underline{Q} \wedge \underline{\nabla}\alpha)$

8. $\nabla^*(\alpha\,\mathbf{V}) = \mathbf{I_0}\,(c\,\partial_0\alpha - \underline{v} \bullet \underline{\nabla}\alpha) + i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla}\alpha - \underline{v}\,\partial_0\alpha - \alpha\,\partial_0\underline{v}) - \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{\nabla}\alpha)$

For any continuous function, $\alpha(t; \underline{x})$ we can define generic *Voigt Vectors* as these prove central in this programme.

Voigt Vector: $\mathfrak{V} \equiv -i\,c\,\alpha(t;\underline{x})\,\mathbf{I_0} + \alpha(t;\underline{x})\,\underline{v}(t) \bullet \underline{\mathbf{I}} = \alpha(t;\underline{x})\,\mathbf{V}^* = \{\, i\,\mathfrak{V}_0; \underline{\mathfrak{V}}\,\}$

The scalar and vector components of a *Voigt Vector* satisfy the following equation, named in honor of the pioneer of asynchronous EM - Ludvig V. Lorenz; this equation will also re-appear in many forms throughout this programme.

The ***Lorenz Equation***: $c\,\underline{\mathfrak{V}} + \underline{v}\,\mathfrak{V}_0 = 0$

The most important CNV associated with a continuous Voigt Vector ($\mathfrak{V}$) is its local gradient, $\nabla^*\mathfrak{V}$; the gradients play the role of forces in the corresponding electromagnetic theory. This has the explicit form:

$$\nabla^*\mathfrak{V} = -\mathbf{I_0}\;d\alpha/dt - i\,\underline{\mathbf{I}} \bullet (c\,\underline{\nabla}\alpha + \partial_0\underline{\mathfrak{V}}) + \underline{\mathbf{I}} \bullet \underline{\nabla} \wedge \underline{\mathfrak{V}}$$

This has three components, so the vector component of the CNV ***Gradient*** $\mathfrak{G}$ will have both real and imaginary parts.

Definition: $\nabla^*\mathfrak{V} \equiv i\,\mathfrak{G} = i\,[i\,G_0\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet (\underline{G}_R - i\,\underline{G}_I)] = -G_0\,\mathbf{I_0} + i\,\underline{\mathbf{I}} \bullet G_R + \underline{\mathbf{I}} \bullet G_I$

Comparing coefficients gives: $G_0 = d\alpha/dt$; $G_R = -(c\,\underline{\nabla}\alpha + \partial_0\underline{\mathfrak{V}})$; $G_I = \underline{\nabla} \wedge \underline{\mathfrak{V}}$

Now $\mathbf{V}^*\nabla^*\mathfrak{V} = \mathbf{I_0}\,(i\,c\,G_0 - i\,\underline{v} \bullet \underline{G}_R - \underline{v} \bullet \underline{G}_I) + \underline{\mathbf{I}} \bullet (c\,\underline{G}_R + \underline{v} \wedge \underline{G}_I) - i\,\underline{\mathbf{I}} \bullet (c\,G_I - \underline{v} \wedge G_R)$

But if $\mathfrak{V}$ is a ***Flow Vector*** then: $d/dt\,(\mathfrak{V}) = -\mathbf{V}^*\nabla^*\mathfrak{V} = d/dt\,(-i\,c\,\alpha\,\mathbf{I_0} + \underline{\mathbf{I}} \bullet \underline{\mathfrak{V}}) = -\mathbf{I_0}\,i\,c\,d\alpha/dt + \underline{\mathbf{I}} \bullet d/dt\,(\underline{\mathfrak{V}})$

This gives the ***Gradient Equations***: 1) $\underline{v} \bullet \underline{G}_R = 0$ 2) $\underline{v} \bullet \underline{G}_I = 0$
3) $c\,\underline{G}_R + \underline{v} \wedge \underline{G}_I) = -\,d/dt\,(\underline{\mathfrak{V}})$ 4) $c\,G_I - \underline{v} \wedge G_R = 0$

Finally, in differential terms: $-i\,\mathbf{V}^*\,\mathfrak{G} = -\mathbf{V}^*\nabla^*\mathfrak{V} = d/dt\,(\mathfrak{V}) = -i\,(d/dt\,\mathbf{X}^*)\,\mathfrak{G}$ $\therefore$ $i\,d\mathfrak{V} = d\mathbf{X}^*\,\mathfrak{G}$

There are a class of "harmonic" functions $\psi(t;\underline{x})$ that always satisfy the *Wave Equation* : $\Box\psi(t;\underline{x}) = 0$

Each such function has its own corresponding ***Associate*** CNV, defined as (note, <u>not</u> conjugate): $\mathfrak{A} \equiv \nabla\psi(t;\underline{x})$

The addition of any Associate Vector to a Voigt Vector, $\mathfrak{V}$ defines its corresponding ***Gauge Vector***, $\mathfrak{V}'$.

Definition: ***Gauge Transform*** $\mathfrak{V}' \equiv \mathfrak{V} + \nabla\psi$ where $\Box\psi = 0$

Since $\nabla\psi = i\,\mathbf{I_0}\,\partial_0\psi + \underline{\mathbf{I}} \bullet \underline{\nabla}\psi$ then $\underline{\mathfrak{V}}' = \underline{\mathfrak{V}} + \underline{\nabla}\psi$; $\mathfrak{V}_0' = \mathfrak{V}_0 + \partial_0\psi$ $\therefore$ $c\,\alpha' = c\,\alpha - \partial_0\psi$

The conjugate gradient of this Associate CNV is zero, since: $\nabla^*\nabla\psi = -\mathbf{I_0}\,\Box\psi = 0$ $\therefore$ $\nabla^*\mathfrak{V}' = \nabla^*\mathfrak{V}$ or $\mathfrak{G}' = \mathfrak{G}$

This harmonic property of the Associate CNV ensures that the Gradient of any Voigt Vector ($\mathfrak{G}$) remains invariant under a Gauge Transform.

6. THE CNV ELECTROMAGNETIC MODEL

The purpose of this section is to demonstrate the power of Natural Vectors in representing asynchronous interactions. As described in the introduction, this is ***not*** the final version of the new theory of EM in this programme: the final version is reserved for later papers. This section will show that by assuming that the key continuous variables in the modern classical theory of EM are best represented by Continuous Natural Vectors (CNVs), rather than traditional 3D vectors that are explicit functions of time, then all the equations of EM are immediately recovered simply from using the new CNV algebra.

6.1 EM CURRENT

The first hypothesis of this simple model of Classical EM (CEM) is that the 3D continuous current density $\underline{J}$ can be represented by (isomorphic to) the spatial part of the corresponding Voigt vector, referred to here as the Current CNV $\mathbf{J}$.

Hypothesis #1: $\underline{J}(t; \underline{x}) \cong \sum_{\mathbf{k}} J_{\mathbf{k}}(t; \underline{x})\, \mathbf{I_k}$ where $J_{\mathbf{k}} = \rho\, v_{\mathbf{k}}$ for k = 1,2,3

Definition. ***Current Density*** CNV: $\mathbf{J} \equiv \rho\, \mathbf{V}^* = -i\, c\, \rho\, \mathbf{I_0} + \rho\, \underline{v}(t) \bullet \underline{\mathbf{I}} = \{ i\, J_{\mathbf{0}} ; \underline{J} \}$

$\therefore$ $J_{\mathbf{0}} = -c\,\rho$ & $\underline{J} = \rho\, \underline{v}$ Charge ***Lorenz Equation***: $c\, \underline{J} + \underline{v}\, J_{\mathbf{0}} = 0$

Thus, $\mathbf{J}(t; \underline{x})$ is a Voigt vector with its functional parameter identified with the continuous, charge density $\rho(t; \underline{x})$. Since $\mathbf{V}^*$ is itself the fundamental Voigt vector then all the results of section 5.3 can be adopted immediately (with $\alpha = 1$), including the invariance results (see §I-7.1) for the acceleration, $\underline{a}$:

$$d/dt\, \mathbf{V} = 0 \quad \text{when} \quad \underline{v} \bullet \underline{a} = 0 \;\; \& \;\; \underline{v} \wedge \underline{a} = 0$$

Following Pauli and other modern expositors of CEM, the present theory also assumes that the electric charge density ρ is incompressible (this is certainly, true for electrons, as $\rho \rightarrow e$); mathematically, this is equivalent to the "Incompressibility" condition: $d\rho/dt = 0$. When this is combined with $\underline{a} = 0$ this results in the "Constant Current" condition: $d/dt\, \mathbf{J} = 0$.

This result implies that $\mathbf{J}$ is a CNV Flow Vector (see 5.2). $\therefore$ $\Box\mathbf{J} = 0 \;\Leftrightarrow\; \nabla^*\mathbf{J} = 0$

This 'Wave' Equation implies that both the charge density ρ and current density $\underline{J}$ propagate across space at speed, c.

These 'Zero' conditions imply: 1) $\underline{\nabla} \bullet \underline{J} + \partial\rho/\partial t = 0$ 2) $c^2\, \underline{\nabla}\rho + \partial\underline{J}/\partial t = 0$ 3) $\underline{\nabla} \wedge \underline{J} = 0$

Here, the first equation is the standard 'Continuity' equation (charge conservation) and the third equation indicates that $\underline{J}$ is a 'circulatory' vector (with no sources or sinks); the second equation generates the wave equation after taking its divergence.

$$\therefore\; \underline{\nabla} \bullet (c^2\, \underline{\nabla}\rho + \partial\underline{J}/\partial t) = c^2\, \underline{\nabla} \bullet \underline{\nabla}\rho + \partial/\partial t\, (\underline{\nabla} \bullet \underline{J}) = c^2\, \underline{\nabla}^2\rho - \partial^2\rho/\partial t^2 = c^2\, \Box\rho = 0 \quad \text{similarly for } \underline{J}.$$

Writing $\mathbf{Q} = \nabla\rho$ $\therefore$ $\nabla^*\mathbf{Q} = -\Box\rho\, \mathbf{I_0}$, so when $\Box\rho = 0$ then defining: $\mathbf{J}' \equiv \mathbf{J} + \nabla\rho$ $\therefore$ $\nabla^*\mathbf{J}' = \nabla^*\mathbf{J} = 0$

So, if the charge density ρ satisfies the wave equation then the CNV gradient (∇^*) of the CNV current ($\mathbf{J}$) is invariant with respect to the "Gauge transform" $\mathbf{J}'$ or equivalently: $\rho' = \rho - c^{-2}\, \partial\rho/\partial t$ and $\underline{J}' = \underline{J} + \underline{\nabla}\rho = \underline{J} - c^{-2}\, \partial\underline{J}/\partial t$

These are the usual 'covariant' requirements for charge and current density in relativistic formulations of classical EM.

6.2 EM POTENTIALS

The key hypothesis in this CNV model of classical EM is that the scalar potential (or EM energy of position per unit charge) at any point in space and time, symbolized as $\phi(t; \underline{x})$ can be represented as the temporal part A_0 of a corresponding Voigt vector, called (in honor of Maxwell) the CNV Electro-kinetic Momentum or (colloquially) the CNV Potential $\mathbf{A}$ involving a velocity $\underline{v}$ defined relative to the local inertial frame of reference.

Hypothesis #2: $\phi(t; \underline{x}) \cong -A_{\mathbf{0}}(t; \underline{x})$ $\quad \mathbf{A} = -i\,\phi\,\mathbf{I_0} + \underline{A} \bullet \underline{\mathbf{I}}$

Definition. ***Potential*** CNV: $\mathbf{A} = \alpha\,\mathbf{V}^* = -i\,c\,\alpha\,\mathbf{I_0} + \alpha\,\underline{v}(t) \bullet \underline{\mathbf{I}} = \{\, i\,A_{\mathbf{0}}\,;\,\underline{A}\,\}$

$\therefore\ c\,\alpha = \phi \quad \therefore\ \underline{A} = \phi\,\underline{v}\,/\,c$ $\quad$ Potential ***Lorenz Equation***: $c\,\underline{A} = \phi\,\underline{v} \quad \therefore\ c\,\mathbf{A} = \phi\,\mathbf{V}^*$

Since $\mathbf{V}^*$ is a Voigt vector, then: $d/dt\,\mathbf{V}^* = 0$ or $d/dt\,\underline{v} = 0$ applying this to the Lorenz equation $c\,d/dt\,\underline{A} = \underline{v}\,d/dt\,\phi$.
Now taking the divergence of this Lorenz equation, (and remembering the general form of the total derivative) then:

$$c\,\underline{\nabla} \bullet \underline{A} = \underline{\nabla} \bullet (\phi\,\underline{v}) = \underline{v} \bullet \underline{\nabla}\phi = d\phi/dt - \partial\phi/\partial t \quad \therefore\ \underline{\nabla} \bullet \underline{A} + \partial_0\phi/\partial t = 1/c\ d\phi/dt$$

Thus, the 'Lorenz Gauge' condition: $\underline{\nabla} \bullet \underline{A} + \partial_0\phi/\partial t = 0$ is equivalent to: $d\phi/dt = 0$ or $\mathbf{A}$ is invariant: $d/dt\,\mathbf{A} = 0$

These results indicate that the CNV potential $\mathbf{A}$ is a 'Flow' vector, leading to 'potential' waves: $\Box\,\mathbf{A} = 0$

6.3 EM FORCES

Since $\underline{A}$ is a potential energy density (per unit charge) then dimensionally its gradient $\underline{G}$ is a force-density (per unit charge); this suggests the following definition for the EM Force-Density CNV:

Definition. ***EM Force-Density*** CNV: $\mathbf{F} \equiv \rho\,\mathbf{G}$ $\quad$ with $\quad i\,\mathbf{G} \equiv \nabla^*\mathbf{A}$

Since both ϕ and $\underline{A}$ are real functions, then the spatial component, $\underline{G}$ is complex, thus writing: $\underline{G} \equiv \underline{E} - i\,\underline{B}$

This immediately results in the identities: $\underline{E} = -\underline{\nabla}\phi - 1/c\,\partial\underline{A}/\partial t$ and $\underline{B} = \underline{\nabla} \wedge \underline{A}$

Section 6.2 showed that $\mathbf{A}$ is a 'Flow' vector, so that: $d/dt\,\mathbf{A} + i\,\mathbf{V}^*\mathbf{G} = 0$ leading to the result: $\mathbf{V}^*\mathbf{G} = 0$

In terms of its four components: 1) $\underline{v} \bullet \underline{E} = 0$ 2) $\underline{v} \bullet \underline{B} = 0$ 3) $c\,\underline{E} + \underline{v} \wedge \underline{B} = 0$ 4) $c\,\underline{B} = \underline{v} \wedge \underline{E}$

These can be solved immediately: $c^2\,\underline{B} = -\underline{v} \wedge (\underline{v} \wedge \underline{B}) = v^2\,\underline{B} - \underline{v}\,(\underline{v} \bullet \underline{B}) = v^2\,\underline{B}$ Since $\underline{B} \neq 0$ then: $v = c$

If the scalar potential is assumed to be *spherically symmetric*, then: $\Box\phi = -4\,\pi\,\rho$ or equivalently: $\Box\mathbf{A} = -4\pi/c\ \mathbf{J}$

But since $\nabla\,\nabla^*\,\alpha = -\mathbf{I_0}\,\Box\alpha$ then the last equation transforms into ***Maxwell's (CNV) Equations***: $i\,\nabla\mathbf{G} = 4\pi/c\ \mathbf{J}$

The four parts are: 1) $\underline{\nabla} \bullet \underline{E} = 4\,\pi\,\rho$ 2) $\underline{\nabla} \bullet \underline{B} = 0$ 3) $\underline{\nabla} \wedge \underline{B} = 4\pi/c\ \underline{J} + 1/c\ \partial\underline{E}/\partial t$ 4) $\underline{\nabla} \wedge \underline{E} = -1/c\ \partial\underline{B}/\partial t$

Using the invariance of $\mathbf{J}$ or $d/dt\mathbf{J} = 0$ then: $\Box\,\mathbf{G} = 0$ or equivalently: a) $\Box\underline{E} = 0$ b) $\Box\underline{B} = 0$

Thus, the EM electric and magnetic force density functions, $\underline{E}$ and $\underline{B}$ always satisfy the wave equations, whether there are charges or currents present or not: a result derived by Lorenz in 1867.

6.4 EM GAUGE CONDITIONS

Any scalar function $\psi(t; \underline{x})$ can be the basis for the generation of an 'Associated' CNV by taking its CNV gradient. If these scalar functions also satisfy the homogenous wave equation ($\Box\psi = 0$) they are referred to as 'harmonic' functions; this class of functions has played an important role in EM from its earliest days.

Definition. ***Gauge Vector*** CNV: $\mathbf{H} \equiv \nabla\psi = i\, \partial\psi/\partial t\ 1/c\ \mathbf{I_0} + \underline{\nabla}\psi \bullet \underline{\mathbf{I}} = \{\, i\, H_0\, ;\, \underline{H}\, \}$

The auxiliary hypothesis in this CNV model of classical EM is that the CNV Electro-kinetic Momentum (or colloquially) the CNV Potential $\mathbf{A}$ is invariant under addition of any Gauge vector.

Hypothesis #3: $\mathbf{A}' \equiv \mathbf{A} + \mathbf{H}$ $\qquad$ $\mathbf{A}' = -\,i\, \phi'\, \mathbf{I_0} + \underline{A}' \bullet \underline{\mathbf{I}}$

Since ψ is harmonic, then: $\nabla^*(\nabla\psi) = 0 \;\therefore\; \nabla^*\mathbf{H} = 0$ Thus: $i\,\mathbf{G}' = \nabla^*\mathbf{A}' = \nabla^*\mathbf{A} = i\,\mathbf{G} \;\therefore\; \underline{E}' = \underline{E}$ and $\underline{B}' = \underline{B}$

Thus, the electric and magnetic force intensities $\underline{E}$ and $\underline{B}$ remain invariant (i.e. are unchanged) under a Gauge transform.

Substituting for $\mathbf{A}$ results in the standard "***Lorenz Gauge***": $\phi' = \phi - 1/c\, \partial\psi/\partial t$ & $\underline{A}' = \underline{A} + \underline{\nabla}\psi$

It is interesting to explore the additional assumption that $\mathbf{H}$ is also a Voigt vector, like $\mathbf{A}$ with the same velocity $\underline{v}$ but with its own Voigt parameter function $\beta(t; \underline{x})$.

$\mathbf{H} = \beta\, \mathbf{V}^* = -\,i\, c\, \beta\, \mathbf{I_0} + \beta\, \underline{v}(t) \bullet \underline{\mathbf{I}} = \{\, i\, H_0\, ;\, \underline{H}\, \}$ $\qquad$ $\therefore\; H_0 = -\,c\,\beta$ & $\underline{H} = \beta\, \underline{v}$

This has its own 'Lorenz' equation: $c\, \underline{H} + H_0\, \underline{v} = 0$ Taking its divergence results in: $\Box\psi = -\,1/c\; dH_0/dt = 0$

So, H_0 is a constant with the same dimensions as ϕ, so it can be redefined in terms of another constant: $H_0 \equiv -\,\phi_0$

$\therefore\; c\,\beta = \phi_0 \;\therefore\; \mathbf{H} = \mathbf{V}^*\, \phi_0 / c \equiv \mathbf{A_0} \;\therefore\; \mathbf{A}' \equiv \mathbf{A} + \mathbf{A_0}$ or $\phi' = \phi + \phi_0$ & $\underline{A}' = \underline{A} + \underline{A}_0$

This provides an interpretation of the Lorenz gauge as simply the addition of an arbitrary constant ϕ_0 to the scalar potential.

The general 'ray-like' solution to the wave equation is a one-dimensional pulse (in say the z-direction) is:

$\psi(t; \underline{x}) = \psi(\xi) = \psi(t = t_0\, ;\, z = z_0) \pm \phi_0\, \xi$ with $\xi = z - z_0 \pm c\,(t - t_0)$

$\therefore\; \mathbf{H}(t; \underline{x}) = \phi_0\, \mathbf{C}^*/\, c$ or $\mathbf{H}(t; \underline{x}) = -\,\phi_0\, \mathbf{C}/\, c$ $\qquad$ where $\underline{v} = \pm\, c\, \underline{e}_3 = \pm\, \underline{c}$

So, the Gauge CNVs are always ray-like, positive or negative constant 'pulses', contributing to the ray-like propagating potentials between the charges. This should remind the reader of the concept of the 'photon'.

6.5 EM ENERGY

Since in this theory, charge is viewed as continuous then it is always possible to define an EM energy U_0 wherever the charge density ρ is non-zero. Using the static definition of EM potential ϕ then this energy density is given by:

Definition. Energy-Density: $U_0(t; \underline{x}) \equiv -\rho(t; \underline{x})\, \phi(t; \underline{x})$

As in section 6.2, just as ϕ was assumed to be the temporal part of a Voigt vector $\mathbf{A}$, a similar hypothesis will now be made here for the scalar part of the EM CNV energy vector $\mathbf{U}$.

Hypothesis #4: $\rho(t; \underline{x})\, \phi(t; \underline{x}) \cong -U_0(t; \underline{x})$ $\qquad \mathbf{U} = -i\, \rho\, \phi\, \mathbf{I_0} + \underline{U} \bullet \underline{\mathbf{I}}$

Definition. ***Energy-Density*** CNV: $\mathbf{U} \equiv \gamma\, \mathbf{V}^* = -i\, c\, \gamma\, \mathbf{I_0} + \gamma\, \underline{v}(t) \bullet \underline{\mathbf{I}} = \{ i\, U_0\, ; \underline{U} \}$

$\therefore\ c\, \gamma = \rho\, \phi \quad \therefore\ \underline{U} = \rho\, \phi\, \underline{v} / c$ $\qquad$ Energy ***Lorenz Equation***: $c\, \underline{U} = \rho\, \phi\, \underline{v} = \phi\, \underline{J} \quad \therefore\ c\, \mathbf{U} = \phi\, \mathbf{J}$

But $c\, \mathbf{A} = \phi\, \mathbf{V}^*$ thus: $\mathbf{U} = \rho\, \mathbf{A}$ $\quad$ Since $d\rho/dt = 0$ & $d/dt\, \mathbf{A} = 0$ then: $d/dt\, \mathbf{U} = 0$ $\quad$ So $\mathbf{U}$ is conserved.

Since $\mathbf{V} = d/dt\, \mathbf{X}$ and $i\, \mathbf{G} = \nabla^* \mathbf{A}$ then: $-i\, \mathbf{V}^* \mathbf{G} = -\mathbf{V}^* \nabla^* \mathbf{A} = d/dt\, \mathbf{A} = -i\, (d/dt\, \mathbf{X}^*)\, \mathbf{G} = 0$

Resulting in: $\quad i\, d\mathbf{A} = d\mathbf{X}^*\, \mathbf{G} = 0$ $\quad$ (Total differentials)

Similarly: $d\mathbf{U} = \rho\, d\mathbf{A} = -i\, d\mathbf{X}^*\, \rho\, \mathbf{G} = -i\, d\mathbf{X}^*\, \mathbf{F} = -i\, \mathbf{V}^*\, (\mathbf{F}\, dt)$

This suggests an analogous definition to Newton's concept of Impulse. $\qquad$ Definition. ***Impulse*** CNV: $d\mathbf{I} \equiv \mathbf{F}\, dt$

Thus: $d\mathbf{U} = -i\, \mathbf{V}^*\, d\mathbf{I}$ $\qquad \therefore\ \mathbf{V}^* \mathbf{F} = i\, d/dt\, \mathbf{U} = -i\, \mathbf{V}^*\, \nabla^* \mathbf{U}$

This gives the analogue of Classical Mechanics: $i\, \mathbf{F} = \nabla^* \mathbf{U}$

Substituting the earlier identities: $\rho\, \mathbf{G} = \mathbf{F} = -i\, \nabla^* \mathbf{U} = -i\, \nabla^*(\rho\, \mathbf{A}) = -i\, \rho\, (\nabla^* \mathbf{A})$ $\quad$ So: $\nabla^*(\rho\, \mathbf{A}) = \rho\, (\nabla^* \mathbf{A})$

This is a ***very significant result***, indicating that the charge density ρ ***commutes*** with the CNV Gradient Operator ∇^*. This is obviously true for the point electron model of electricity ($\rho \to e$). It is also valid for the incompressible electric fluid model of Helmholtz and Hertz (which is the basis for the modern development of CEM) since $d\rho/dt = 0$. Furthermore, it is also valid for the 'harmonic' charge density oscillation model, where: $\rho(t; x) = \rho_0 \exp(i\, (\underline{k}.\underline{x} - \omega\, t))$.

Since the three major CNVs of this theory ($\mathbf{J}$, $\mathbf{A}$ and $\mathbf{U}$) are all Voigt 'Flow' vectors, they all share the same common velocity $v = c$ (in any inertial reference frame). They are related through the identities:

$\mathbf{U} = \rho\, \mathbf{A}$ & $c\, \mathbf{U} = \phi\, \mathbf{J}$ $\qquad$ These satisfy: $\mathbf{J}^* \mathbf{A} = c\, \rho\, \phi\, (1 - v^2/c^2)\, \mathbf{I_0}$

This suggests the Searle '***Extended Potential***' [129] $\quad \Phi \equiv (1 - v^2/c^2)\, \phi = \phi - \underline{v} \bullet \underline{A} / c$ $\quad$ a term appearing often in CEM.

It is interesting to define a "Coulomb" Energy Density CNV. $\qquad$ Definition. ***Coulomb Energy*** CNV: $\mathbf{W} \equiv \mathbf{J}^* \mathbf{A} / c$

This satisfies the identity: $c\, \mathbf{W} = \mathbf{V}\, \mathbf{U}$ $\quad$ NB This is not $\mathbf{V}^* \mathbf{U}$ so it is not a Voigt vector as $\mathbf{W} = \{ i\, \rho\, \Phi\, ; \underline{0} \} = \{ i\, W_0; \underline{W} \}$

This signifies the powerful role of complex conjugation in this theory, demonstrating the need for i in the definition of NVs.

$$W_0 = \rho\, \Phi = \rho\, \phi - \rho\, \underline{v} \bullet \underline{A} / c = \rho\, \phi - \underline{J} \bullet \underline{A} / c = -(U_0 + U_M) \qquad \therefore\ U_M = \underline{J} \bullet \underline{A} / c$$

6.6 EM MOMENTUM

The EM force density CNV $\mathbf{F}$ was defined in sections 6.3 and 6.5 in terms of the gradient of the EM Potential CNV.

$$\mathbf{F} = \rho\, \mathbf{G} = -\,i\, \rho\, \nabla^* \mathbf{A} = -\,i\, \nabla^* \mathbf{U}$$

Dimensionally it is also possible to define a force in terms of the total time differential; we will refer to this second CNV as the Heaviside Force Density.

Definition. ***Heaviside Force-Density*** CNV: $\mathbf{R} \equiv i\, \mathbf{V}^* \mathbf{F} / c = \{ i\, R_{\mathbf{0}} ; \underline{R} \}$ and $\underline{R} = \rho\, \underline{f}$

$$c\, \mathbf{R} = i\, \mathbf{V}^* \mathbf{F} = -\,d/dt\, \mathbf{U} = i\, \mathbf{J}\, \mathbf{G} = i\, \rho\, \mathbf{V}^* \mathbf{G} = i\, \rho\, (-(\underline{v} \bullet \underline{G})\, \mathbf{I_0} - i\, c\, \underline{\mathbf{I}} \bullet \underline{G} + \underline{\mathbf{I}} \bullet (\underline{v} \wedge \underline{G}))$$

This leads to the 'traditional' form mis-attributed to Lorentz (which is not a Voigt vector): $\mathbf{R} = \{ 0;\ \rho\, (\underline{E} + 1/c\, \underline{v} \wedge \underline{B}) \}$

So, the spatial 3D component is the force-density per unit charge: $\underline{f} = \underline{E} + 1/c\, \underline{v} \wedge \underline{B} = -\,\underline{\nabla}\, \Phi - 1/c\ d/dt\, \underline{A}$

This should be compared with the electric force density per unit charge: $\underline{E} = -\,\underline{\nabla}\phi - 1/c\ \partial \underline{A} / \partial t$

When this is checked against the consequences of the Zero Conditions in section 6.3, it is surprising to find that: $\underline{f} = 0$.

Since $\underline{v} \bullet \underline{f} = 0$ then no mechanical work is done through the EM interaction but energy is transferred across space.

It is possible to interpret the Heaviside 3D vector force $\underline{R}$ as the 'reactive' force (due to the remote interacting source) so that it becomes <u>mathematically</u> feasible to define an <u>equivalent</u> EM Mass-Density $\mathcal{M}$ and, by using Newton's Second Law of Motion to define the EM Momentum Density $\underline{\mathcal{P}}$.

Definitions. $\underline{R} = \mathcal{M}\ d\underline{v} / dt = -\ d\, \underline{\mathcal{P}} / dt$

Using the Charge Incompressibility condition ($d\rho/dt = 0$) and the Lorenz condition ($d\phi/dt = 0$) $\underline{R}$ can be transformed by:

$c^2\, \underline{R} = -\,\rho\, c\, d\underline{A} / dt = -\,\rho\, \phi\, d\underline{v} / dt = U_{\mathbf{0}}\, d\underline{v} / dt$ With the result: $U_{\mathbf{0}} = \mathcal{M}\, c^2$ (a well-known equation).

Since $\mathbf{U}$ is an invariant then U_0 is constant, so $\mathcal{M}$ must also be a constant $\mathcal{M}_0$, so finally: $U_{\mathbf{0}} = \mathcal{M}_0\, c^2$

$\therefore\ c^2\, d\underline{\mathcal{P}} / dt = -\,\rho\, \phi\, d\underline{v} / dt = -\,d/dt\, (\rho\, \phi\, \underline{v}) = d/dt\, (U_{\mathbf{0}}\, \underline{v}) = d/dt\, (\mathcal{M}_0\, c^2\, \underline{v})$ Thus: $\underline{\mathcal{P}} = \mathcal{M}_0\, \underline{v}$ or $\underline{\mathcal{P}}_0 = \mathcal{M}_0\, \underline{c}$

Therefore, $1/c\, \underline{A}$ behaves like Maxwell's (1864) "electro-kinetic momentum" per unit charge (hence the same symbology) and the present research confirms that this was Maxwell's most important innovation (not the 'displacement current'); indeed, this will be the focus of several of the future papers in this programme.

It should be repeated again here (as was described in the first paper) that Silberstein and Conway had separately used Hamilton's quaternion approach in 1911 and 1912 to summarize Maxwell's Equations. Although they had explicitly added the imaginary root of minus one to their formulations (i.e. they had used 'bi-quaternions') they continued to use Hamilton's original formulation of quaternions, instead of the *Natural Vector* formulation used here. As a result, they added little insight to Maxwell's original aetherial approach while the emphasis was on explicating the link to special relativity.

6.7 TEMPORAL INVARIANTS

Natural Vectors have been found to be a powerful representation of asynchronous interactions, such as the EM interaction, between two points separated across space and time. The most useful NVs are *separable* across the co-ordinates of the two points; that is, an NV $\mathbf{Q}_{12}$ is separable whenever it satisfies the following decomposition (like the position difference):

Definition. ***Separable*** NV: $\mathbf{Q}_{12}(T; \underline{X}) = \mathbf{Q}_{12}(t_1 - t_2; \underline{x}_1 - \underline{x}_2) = \mathbf{Q}_1(t_1; \underline{x}_1) - \mathbf{Q}_2(t_2; \underline{x}_2)$

This definition reflects the intrinsic, anti-symmetric nature of the interacting pair of objects, leading to the conclusion that even 'classical' electrons are *fermions*.

Some CNVs have values that overall do not change when their temporal parameter changes, these CNVs are referred to as 'temporal invariants'; they obviously satisfy the defining condition:

Definition. ***Temporal Invariant*** CNV: $d/dt\, \mathbf{Q}_0(t; \underline{x}) = 0$ or $\mathbf{Q}_0(t'; \underline{x}) = \mathbf{Q}_0(t; \underline{x})$

For example, taking the total time derivative of the EM Energy equation (see 6.5): $U_0 = -\rho\,\phi$ gives: $dU_0/dt = 0$

This is a direct result of the 'Incompressibility' condition ($d\rho/dt = 0$) and the 'Lorenz gauge' condition ($d\phi/dt = 0$). This result can be applied to the result of taking the divergence to the Potential Lorenz equation: $c\,\underline{U} = \rho\,\phi\,\underline{v}$.

$$c\,\underline{\nabla}\bullet\underline{U} = \underline{\nabla}\bullet(\phi\,\underline{J}) = \underline{\nabla}\bullet(\rho\,\phi\,\underline{v}) = \underline{v}\bullet\underline{\nabla}(\rho\,\phi) = d/dt(\rho\,\phi) - \partial/\partial t(\rho\,\phi) = -dU_0/dt + \partial U_0/\partial t = \partial U_0/\partial t$$

This indicates that the time-rate reduction in EM potential energy (U_0) is equal to the work done in moving the electrical charge through the spatial change in electrical potential (ϕ) due to an equivalent (time) stationary charge group (ρ).

Whenever a Natural Vector $\mathbf{A}$ is both separable and temporally invariant, then it will represent two (scalar and vector) conserved physical quantities; thus:

Whenever $\mathbf{A}_0(t_1 - t_2; \underline{x}_1 - \underline{x}_2) = \mathbf{A}_1(t_1; \underline{x}_1) - \mathbf{A}_2(t_2; \underline{x}_2)$ and $d/dT\,\mathbf{A}_0(T; \underline{X}) = 0$

then: $d/dt_1\,\mathbf{A}_1(t_1) = -d/dt_2\,\mathbf{A}_2(t_2) = \mathbf{B}_0$

This equation represents the ***transfer***, across space over a finite duration of time, of a joint pair of properties (A_0 and $\underline{A}$), such as energy and momentum, between one interacting point object at (t_1; $\underline{x}_1$) and the other at (t_2; $\underline{x}_2$); one of which 'loses' these properties at rates of B_0 and $\underline{B}$, while the other point object 'gains' these properties at matching rates. This 'transfer' between the two objects is simply the asynchronous action-at-a-distance (AAAD) generated by their mutual interaction but this does **NOT** imply the physical existence of some 'third-party' object (like a 'field') that 'carries' these properties and whose existence was only proposed to preserve the conservation of these properties through the duration of the interaction.

In the present theory (see §I-7.4.4), the temporal invariants are the following CNVs:

1) Velocity $\mathbf{V^*}$ 2) Momentum $\mathbf{P} = m\mathbf{V^*}$ 3) Current Density $\mathbf{J} = \rho\mathbf{V^*}$

4) EM Potential $\mathbf{A} = \phi\mathbf{V^*}/c$ 4) EM Energy $\mathbf{U} = \phi\mathbf{J}/c = \rho\mathbf{A}$ 5) Mechanical Energy $\mathbf{K} = \mathbf{PV}$

6) Rotational Momentum $\mathbf{L} = \mathbf{X^*P} - \mathbf{P^*X}$ In all cases, $\mathbf{V^*V} = (c^2 - v^2)\,\mathbf{I_0} = 0$ when $v = c$.

The most significant conserved scalar EM quantities are: ρ, ϕ, U_0 ($\rho\phi$), $\mathcal{M}_0$ ($\rho\phi/c^2$) and $\mathcal{P}_0$ ($\mathcal{M}_0\,\underline{c}$).

7. SUMMARY & CONCLUSIONS

In this final section the results and conclusions of this paper will be briefly summarized in order to draw out the major implications from the material. The paper concludes with summaries of some of the future directions in this programme.

7.1 OBJECTIVES

It is hoped that the objectives of this paper, introduced in section 1.1.2, have been met. The principal objective of this paper was to demonstrate that a particularization of Hamilton's quaternions, referred to here as *Natural Vectors*, is a superior form of representation for describing the classical theory of electromagnetism (EM) than the traditional use of 3D vectors. This area of physics was chosen first as it is the best understood of all subsequent 'field' theories and is presented as the best justification for such types of theories. Accordingly, this paper has treated electricity here from the same perspective that is found in all standard, modern textbooks on classical electromagnetism (CEM). This is not the ultimate theory of EM, which will be presented later in this series but it does demonstrate the mathematical power of the *Natural Vector* formulation.

The second objective of this paper was to demonstrate that Maxwell's theory should not be assumed to be the final word on CEM as this theory is deeply flawed in its metaphysical foundations. This paper has reviewed the state of EM theory in the 19th Century to show that Helmholtz's failed theory of EM has actually become the hidden foundation for the modern view of classical EM, which is now presented as a theory of electricity consisting of a continuous charge density. The physical, metaphysical and mathematical problems of this model have been reviewed here and its flaws should be sufficient to justify further research in this fundamental theory (while noting that the solution is not simply the quantization of this EM theory).

This paper, along with all its successors reporting on the investigations in this research programme, will continue to show that natural philosophy remains the most productive perspective in the study of the foundations of physics. As this was the first paper in this programme to address the EM basis of matter it was thought appropriate to first invest a significant effort in justifying this metaphysical perspective, which, unfortunately, is now a contentious position in modern physics.

This series will always try to place the focus of each paper in its broader historical context so that the modern reader, who is usually almost totally ignorant of the history of his own subject, will begin to appreciate the viewpoints of the major earlier scientists who contributed to the foundations. The historical perspective is also important to expose the key assumptions that usually lie behind every theory. This was the motivation behind including a review of Maxwell's contemporary rivals, especially the EM researches of the 'Continentalists', such as Weber, Neumann and Lorenz. Their Newtonian perspective is much closer in spirit to the present research programme here than the Cartesian metaphysics exemplified by Maxwell. This programme's dynamic is driven by challenging the unspoken hypotheses that lie behind all of modern physics. The major assumption that is challenged in this programme is the view that interactions occur throughout time in a continuous manner (the 'Continuum Hypothesis'). This has been the unspoken justification for the ongoing use of continuum math in physics since Newton revealed the calculus as his 'secret weapon' for summing countless, small changes: an accounting approach that is acceptable to develop numerical predictions for comparison with macroscopic experiments but one that must not be automatically assumed reflects the nature of microscopic reality.

The Study of History & Philosophy

A recent critic of developments in modern physics has quoted a remark made privately by Einstein ten years before his death to provide an explanation for the present lack of progress in theoretical physics [31]: "A knowledge of the historical and philosophical background gives that kind of independence from the prejudices of his generation from which most (of today's) scientists are suffering." This perspective is enhanced by the same author's views (shared here) that the revolution begun in the earlier years of the 20th Century by major theorists defined modern physics [130] but "they failed to complete the revolution they started" – leaving only partially successful theories that cannot be completed by the research techniques of 'normal science' (using a term popularized by one of the leading historians of science, Thomas Kuhn [131]).

7.2 METAPHYSICS OF EM

It is one of the major contentions of this research programme that progress in understanding the foundations of physics will be made more through the introduction of suitable new concepts rather than the application of mathematical innovations.

7.2.1 METAPHYSICS OF PHYSICS

Philosophy is Linguistic

Philosophy in this programme is viewed as the linguistic analysis of general concepts and their organization into networks of larger and larger logical structures. Natural philosophy is the application of this approach to the study of the natural world. This programme views theoretical physics from its historical perspective: natural philosophy, then mathematics.

Theoretical Physics is always Metaphysics

Every theoretical model in physics is grounded, either explicitly or implicitly, in a metaphysical viewpoint. In this light, all philosophical positions are ultimately just verbal propositions describing the world. Mathematics without any interpretation is meaningless, especially when it is used in scientific descriptions of the world.

Theoretical Physics is now Pythagorean

Mathematical physicists have banished philosophy from 20th Century physics because, as Pythagoreans, they have already smuggled in their own philosophy: Plato's 'eternal forms' as timeless, mysterious symbols; ongoing competition is not wanted that might threaten this long sought-for victory.

Theoretical Physics is more than Equations

The currently popular (Pythagorean) idea, that the aim of physics is to 'discover' (or more accurately, invent) "a final set of equations describing the foundation of the world", is an intellectual dead-end when the equations are too complicated to be solved and their symbols cannot be interpreted.

Maxwell calls for Interpretation of Math Symbols

In reviewing Thomson & Tait's textbook on natural philosophy, Maxwell ended with a call to his fellow mathematicians: "Mathematicians may flatter themselves that they possess new ideas which mere human language is yet unable to express. Let them make the effort to express these ideas in appropriate words without the aid of symbols." [132]

Physics can progress from its Mistakes

As Maxwell knew from his extended study of the history of science, a successful derivation of useful results in physics can sometimes be achieved even from obviously inappropriate models; for example: Carnot's discovery of the Second Law of Thermodynamics on the basis of the (invalid) caloric-fluid theory of heat. The pattern continues, Maxwell's theory of EM relied on his (incorrect) belief in the aether, so that modern physicists have followed Hertz in viewing Maxwell's theory as now only "the system of Maxwell's Equations" [133].

Field Theory as Pure Imagination

Today, field theorists deny the reality of particle trajectories but readily admit to the reality of invisible properties of each and every infinite point throughout space, such as EM and electron fields, gravitational potential, stress-energy tensors etc; none of which has been or can be directly observed or measured – only appearing as intermediate steps in mathematical expositions. These types of logical contradictions do not bother scientists who do not respect philosophy.

The Epistemology of Science

This research programme shares Ludwig Boltzmann's basic views on the epistemology of science, which views theories as representations ('maps') of nature. Such models can only explain how natural phenomena appear to human beings. Good maps use clear definitions and simple imagery to create accurate and comprehensive information that can be communicated across a cultural community. As finite humans, we cannot know the 'final truth' but fruitful research programmes can still generate a series of better and better maps. Multiple maps can provide several useful and complementary perspectives.

Theory Refutation

As physical theories are logical attempts to explain experimental facts they can only be challenged by irrefutable facts and not by rival theories – certainly not by viewpoints that are only motivated by commitments to hidden metaphysical biases.

Metaphysical Foundations – the Entity

The deepest concepts in this new research into the foundations of natural philosophy are grounded in ontology, Aristotle's starting point for the study of philosophy. The central concept of ontology is that of an 'entity': any member of a class of natural objects whose individual existence does not depend on the existence of any other object in the Universe. A complex object is not seen as just a collection of its parts (components) but a consequence of its structure (internal interactions) while its external interactions with other objects are viewed as crucial to its identity and ongoing existence.

Electrons are Fundamental

This research programme rejects the crude Mechanism of DesCartes (involving small balls of matter colliding at singular points in space and time) as too simplistic. The present theory also rejects the Idealism of Kant (imagined as the reality of energy fields in empty space) as only the reification of abstract relationships. Today, these historical views reappear as the Positivism of elementary particle physics and the Number Mysticism of impossible imaginaries (e.g. quarks, strings, etc). This new programme is based on the universality of electrons and their interactions; i.e. real existents and real processes.

Entities, Relationships & Attributes

Both classical objects and the relationships between them have usually been defined in terms of their individual attributes or properties. In this programme, the foundational, metaphysical entity is the 'electron' that is defined here only in terms of its relationships with other electrons (a recursive definition); in other words, all of the properties of an electron are those of its interaction with another electron, leading to the fundamental physical quantities: charge (e^2), mass (m), space-time ratio (c), interaction (h) and the fundamental unit of time or chronon (τ). These will later be shown to be inter-related.

Realism as the Philosophy of Physics

The philosophy of Realism has been the *de facto* theoretical foundation of physics for most of its history as it is grounded in the 'Natural Principle' – the view that the real world evolves, at every level, whether humans exist or not. In contrast, the philosophy of Positivism is based on the 'Human Fallacy' – the world consists only of the things that humans can measure. Unfortunately for this latter view, at the micro level whenever we try to measure anything then we inevitably change it because we too are made of the same 'stuff' (electrons).

Electrons are Real

When Galileo was confronted by the overwhelming orthodoxy of the intellectuals of the Catholic Church who denied the reality of the moons of Jupiter, he is reputed to have replied "but they are there." Today, when facing the vast phalanx of sophisticated philosophers and theoreticians of physics who deny that point electrons are real, one can similarly only respond "but they are there". In other words, this research programme is grounded in the ideas of operational-realism.

Time is Universal

It would seem reasonable to assume that time is more fundamental than space since <u>every</u> electron exists at every instant of time, somewhere in space but electrons are <u>not</u> found at all points in space at one instant in time.

Time is Critical

Standard field theory is the latest scientific attempt to eliminate time from physics. Although a so-called time variable often appears at the start of all such theories it is soon eliminated by either integration over all time or by multiplying by a factor involving its complex conjugate that immediately removes time from further consideration. As a 'timeless eternalist', Plato would have been pleased with this attack on his star pupil (Aristotle), who contradicted his own teacher and saw that time was central to both life (biology) and reality. This Aristotelian position is the one adopted by this research programme, so: "Father Time will never be divorced from Mother Nature". Platonists hate time (perhaps it reminds them of their own mortality). As a result, most Platonists seem to dislike algebra and natural numbers, which (as W. R. Hamilton suggested) imply mental sequencing through time, i.e. the "Science of Time". Platonists (like Einstein) have always loved geometry, the "Science of Space", with its static, scale-less, numberless points and lines. The descendents of this ancient tradition have 'geometrized' physics with theoretical physics now divided today between 'string theorists' and 'loop geometricians'.

7.2.2 AETHERIAL EM METAPHYSICS

Maxwell hijacked Newton in 1864

When Maxwell wrote his most important paper on EM [66] he co-opted the mechanical vocabulary that was based only on Newton's own innovative concepts of mass and momentum. These concepts were developed within a metaphysical model of point particles and action-at-a-distance acting instantaneously across a space that was viewed as a passive background to all of this activity. Maxwell adopted this vocabulary wholesale to describe the mysteriously active and continuous, aetherial 'matter' that he proposed as the basis for the 'mechanism' of electromagnetism. With the eventual demise of Maxwell's own aether, field theorists have continued with hijacking of Newton's continuum mathematics of particulate motion (differential calculus) and applied it to a mathematical form of continuous matter and interactions (densities and intensities) that have no foundation in any form of real matter. In fact, this approach reifies the "space between the bodies" of DesCartes, in contrast to Newton's metaphysics. The irony of history is that the particulate view of EM was always at the heart of the 'German' EM programme that Maxwell could never accept and EM textbook authors almost always seem to ignore.

Maxwell's God fills the Void

In his article on action-at-a-distance, Maxwell raised the question of whether the real, mutual influence between two remote bodies depends on the existence of some "third thing", a medium occupying the space between the bodies or do the bodies "act on each other immediately?" After describing space itself as filled with Faraday's "lines-of-force" (the aether), he was pleased to report that "the vast interplanetary and interstellar regions will no longer be regarded as waste places, which the Creator has not seen fit to fill with the symbols of the manifold order of His kingdom." [134]

Maxwell's Modeling Failure

It is difficult not to agree with Lord Kelvin's remark that Maxwell had lapsed into 'mysticism' with his *Dynamical Theory of Electromagnetism* as Maxwell had failed to provide any physical model to judge his theory as a description of reality. This paper started the trend of substituting merely mathematical symbols for conceptual understanding that has, over the last 150 years, become the standard approach to theoretical physics. Both the theories of Relativity and Quantum Mechanics have followed this approach of starting with ill-defined mathematical symbology rather than physical concepts and both have found themselves in deep, fundamental philosophical difficulties and conceptual contradictions. Maxwell gave his concepts of field and energy-density equal ontologically significant status with the 'lines-of-force', that were apparently 'discovered' by Faraday; in other words, all of these abstractions were viewed as 'real'. As the editor of an anniversary reprinting of this famous paper [125] describes, Maxwell "was dissatisfied with merely analytical mathematics concerned only with the manipulation of symbols detached from physical structures." As Maxwell himself wrote "The advance of the exact sciences depends upon the discovery and development of appropriate and exact means by which we form a mental representation of the facts." Maxwell agreed with Faraday that the primary emphasis must be on the relationships <u>between</u> things as the defining characteristics of real entities, so that we should <u>not</u> begin with free particles and then introduce their interactions (QED) but start with an investigation of the interactions themselves. **Mathematically**, there is no difference between an asynchronous, action-at-a-distance potential theory (e.g. Lorenz's ϕ and $\underline{A}$ [17]) and a local force field theory, like Maxwell's $\underline{E}$ and $\underline{B}$ fields [66], as long as both involve relating sources and reacting electric charge densities. The real distinction between Newton and Maxwell lies in Newton's view of instantaneous gravitational interactions and the finite time differences between changes in sources and reactants that have been found experimentally in EM measurements.

Maxwell's Philosophical Errors

In his *Treatise on EM* [1], Maxwell fell into the classic 19th century error of separating EM forces from mechanical forces: "Electromotive force is always to be understood to act on electricity only, not on the bodies in which electricity resides. It is <u>never</u> to be confounded with ordinary mechanical force, which acts on bodies only and not on the electricity in them." He took this further so that partial differential equations were to be used to describe energy in the form of fields, in contrast with differential equations for action between remote particles in motion. This required a distinction to be drawn between the <u>forms</u> of energy, which he viewed as some kind of existential primitive idea (God?). "There can be no doubt that Maxwell remained ill at ease with his own work right up to the end of his life, for somehow his theories did not satisfy his standards of tenability." Einstein later viewed this dissatisfaction as just due to the field theorist's necessity to still include Newton's material points (i.e. electrons) as part of the dual representations of physical reality – a dualistic problem still present today.

Although acknowledging Weber's (instantaneous) action-at-a-distance theory of EM interactions as equally mathematically valid, Maxwell could not view it as fundamental because of the "mechanical difficulties involving velocity-dependent forces between particles" – something that is not seen today as a problem for users of the 'Lorentz' force! Maxwell viewed the EM field as real undulations in an aetherial substance of finite density that permeated all gross matter while interactions with this gross matter merely modified the motion of this aether, which was viewed as the repository of equal parts linear elastic and kinetic energy, in transit between source and excited matter.

Maxwell's Failure in 1864

Maxwell's theory of microscopic EM phenomena explicitly rejected microscopic explanations, which was legitimate before the discovery of the electron at the end of the 19th Century. Maxwell was <u>not</u> interested in speculating about the origin of EM forces on moving bodies, just as Newton refused to speculate on the <u>meaning</u> of gravitation. Even Larmor's theory of the electron derived its properties from singular (vortex) properties of the aether. Maxwell inverted the modern perspective by viewing charge as being produced by the electric field affecting the continuous, elastic medium of the aether, rather than a material substance (like electrons) producing the electric field. A current of electrical conduction was viewed by Maxwell as a continuous series of charging and discharging of displacement; i.e. conductors were viewed as 'leaky' condensers with extremely short relaxation periods. Here, magnetic fields were only associated with the increase, not the rapid decrease, in displacement. In modern EM theory, the field is assumed to transmit EM energy without itself being subject to the resulting forces (i.e. no 'photon-photon' collisions). Since we now know that only real dielectric solids consist of pairs of polarizable electric particles, it is impossible to have a modern Maxwellian theory of light. "Ultimately, Maxwell's theory was unable to distinguish mathematically between currents in conductors from currents in dielectrics. … So that these questions proved to be the undoing of Maxwellian theory." Maxwell assumed 'ab initio' that the <u>sources</u> of electric and magnetic effects (i.e. matter) are distinct from the variations in the EM <u>fields</u> (i.e. aether), which only interact at those points in space and time where matter is located. Effectively, both matter <u>and</u> the EM field can always be treated as a <u>single</u>, dynamical system [135].

The final EM theory of Helmholtz in 1872 explicitly treated the aether as a polarizable medium while emphasizing the central role of scalar and vector potentials [136]. In this theory, only electrical polarizations propagate. All fields involve <u>instantaneous</u> interactions between <u>real</u> charge densities, that are due to inhomogeneities in the real current density, i.e. Coulombic electric fluids: a model that always ignores its own infinite self-repulsion.

Maxwell's Theory same as Weber's Theory?

Maxwell intuitively believed that his EM theory was mathematically equivalent to the instantaneous action-at-a-distance theories of his contemporary continental rivals, Weber and Neumann. Several authors [137] emphasized the differences between these classes of theory, especially when energy is radiated away "into the field"; however this view fails to include the receiving elements in the total interaction [138]. A comprehensive review of Weber's EM theory has been provided in the recent monograph by Assis [95].

Lorenz bypassed EM Fields

Metaphysically, Maxwell constructed his 1864 EM theory around the concepts of force fields defined in some mysterious aether; since he also relied on Coulomb's electrostatic law for the electric component of his force fields he had to introduce his concept of the 'displacement current' to derive the wave equations for both of these fields propagating in a vacuum. In contrast, in 1867 L. V. Lorenz derived all of these wave equations using only retarded scalar and vector potentials defined between remote electric charges, with no need to interpose a continuous 'field' between the source and target charges [139].

Classical EM is only an Approximation

At best, the force fields in Classical EM theory can only be interpreted as averages taken over 'mesoscopic' dimensions, so that the electric field <u>E</u> is the average unit-force on an electrical point charge due to the averaged <u>zero</u> motion of the source charges, while the magnetic field <u>B</u> is the (scaled) average unit-force on a similar electrical point charge due to the averaged <u>non-zero</u> motion of the sources. Mesoscopic averages must be calculated over thousands of source electrons so that these averages then *appear* (at least at macroscopic distances) as a continuous medium of electric charge density.

7.2.3 CONTEMPORARY VIEWS OF ELECTRODYNAMICS

Electrostatic and Magnetic Forces

The spherically symmetric law of electrostatic force has been central to the study of EM since it was first proposed in 1785 by **Coulomb**. It is usually the starting point for modern expositions of CEM. An equally important and separate hypothesis is the one attributed to Lorentz defining the effect of a local magnetic field on a moving point charge. In modern treatments of CEM it is never pointed out that charge is concentrated at a point in the moving ('magnetic') part of the EM interaction but is distributed (as charge density) in the static (or 'electric') part at the measured 'field' point.

Modern Electrodynamic Texts

The review here of three of the most influential CEM texts has shown how Maxwell's theory of EM has disappeared and has been absorbed into a modified version of Helmholtz's fluid model of EM. This has allowed the mathematical 'skeleton' of Maxwell's theory to survive while dropping the aetherial concepts that first brought it into existence. Maxwell's key idea of the 'displacement current' is now just introduced as a mathematical step to preserve algebraic consistency. Again Maxwell's original focus on the scalar and vector potentials has been converted now into just mathematical conveniences. The concept of electrical fluid (a two century's old concept) has evolved into the mathematically oriented definition of charge density. Modern EM texts jump between sections using continuous charge densities (e.g. Maxwell's Equations) and other sections using electrical point charges (the so-called 'Lorentz' force). It is this subtle but never-discussed merger of two incompatible definitions of electrical charge into one single theory of EM that now smuggles in the special theory of relativity. The major omission in all these texts is any indication that substantial alternative theories of EM were developed in the 19th Century that also accounted for all the macroscopic phenomena but this would resurrect the spectre of **action-at-a-distance** and show that 'field' theories are not the only way to develop successful microscopic theories in physics. This exposure would raise very uncomfortable philosophical questions.

7.2.4 EM ALTERNATIVES TO MAXWELL

Maxwell acknowledges his Rivals

In contrast to almost all modern authors of textbooks on EM, Maxwell was fully aware of the major research programme that had been long conducted on the Continent into the foundations of EM theory. As he wrote in the preface to the first edition of his *Treatise* [140]: "Great progress has been made in electrical science, chiefly in Germany, by cultivators of the theory of action-at-a-distance, originated by Gauss and carried on by Weber, Riemann, Neumann and Lorenz. The great *success*, which these eminent men have attained in the application of mathematics to electrical phenomena, gives additional weight to their theoretical speculations and physical hypotheses. However, these physical hypotheses are entirely alien from the way of looking at things, which I adopt. From a philosophical point of view, it is exceedingly important that the two methods should be compared as both have succeeded in explaining the principal electromagnetic phenomena; while at the same time, the fundamental conceptions of what actually takes place are radically different. … I have therefore taken the part of an *advocate* rather than that of a judge, and have rather exemplified one method than attempted to give an impartial description of both." Unfortunately, physicists have not picked up on Maxwell's **philosophical** challenge.

Weber's Contributions

After scornfully reviewing the shallow contemporary views of the development of EM theory in 19th Century Germany, ("Fashions change in science as in millinery.") the Irish theorist, Professor Alfred O'Rahilly attempted to reset the balance in his iconoclastic text on *Electromagnetics* [141] by summarizing Wilhelm Weber's rarely acknowledged contributions to modern EM theory. These major concepts included:
1) electricity having a particulate structure (not a continuous medium) 2) relating Ampere's idea of magnetism to micro-currents of these electric particles 3) attributing Ampere's forces to direct interactions between these charged particles (not between the conductors) 4) modifying Coulomb's law to allow for dynamical effects generated by their relative motion.

Continental Contributions

Section 2.4 of this paper was included to show that several key ideas in the new theory of EM interaction developed in this research programme had their origin in the contributions from the great 19th Century 'Continentalists' like Gauss, Weber, Neumann and Lorenz. These concepts converge on the idea of pairwise asynchronous interactions between point charges.

7.3 MAXWELL-HEAVISIDE AETHER THEORY

Since the modern physics student is told that he is studying Maxwell's theory of EM it was thought appropriate to include a summary here in section three to show how little the modern textbooks reflect the actual theory that Maxwell wrote in 1864. The numerous errors in this paper, as corrected by Heaviside, demonstrate that even the 'giants of physics' can still make mistakes, as Maxwell freely admitted. But their inclusion here is more to show how Heaviside fundamentally altered the philosophical basis of Maxwell's theory by dropping the potentials, especially the vital electro-kinetic ('vector') potential, that Heaviside knew drew researchers attention back to the action-at-a-distance approach of his Continental rivals, not least, the comprehensive EM theory of L. V. Lorenz [17]. Ironically, it was Heaviside's stripping of Hamilton's 4D quaternions down to 3D spatial vectors that hid both the centrality of the potentials and the 4D covariance of Maxwell's original theory. This paper reverses this process and shows that a much more powerful mathematics (based on quaternions) is far better for representing EM but, unfortunately for the history of physics, this has been overlooked for over 150 years.

7.4 MODERN CLASSICAL ELECTRODYNAMICS

Section four showed that the evolution of the modern approach to CEM is actually based on Helmholtz's failed hydro-dynamical model of EM that provided a mathematical isomorphism with parts of Maxwell's EM theory. The emphasis here was to show that the key feature of all of these 'continuum' models is their subtle inter-weaving of the point-particle model with the continuum vector field models defined in terms of calculus-like, limit definitions. It will be shown in a later paper that these limit definitions are not compatible with the limit definitions of the velocity of the actual charges. In section six it was then shown that it is the apparently simple assumption of incompressibility of charge-density (or of the velocity-field fluid) that enables these theories to be used as suitable models of EM; this is exemplified by the mathematical similarities between Coulomb's law of electrostatics and the 'emission' of incompressible fluids from a point-source.

Expositions on the charge density model never talk about its fundamental flaw: the infinite mutual repulsion generated by the zero spatial separation between infinitesimal spatial cells filled with similar electrically charged 'fluid'. This was also the crucial weakness in all finite models of the electron (introduced around 1900) but at least their proposers had the honesty to acknowledge this problem and admit the need for some kind of unknown mechanical counter-force that would prevent these small spheres from instantly blowing apart. The Coulomb force law always suffers from this problem, except in the case of the truly point model of the electron, but these were not the objects measured by Coulomb.

7.5 SUMMARY OF CONTINUOUS NATURAL VECTORS

In order to expedite the central component of this paper, the relevant details of the first paper in this series that introduced Natural Vectors [4] was summarized here in section five for the particular case of Continuous Natural Vectors. The central hypothesis in this programme is that the location and local time of each electron is better represented by a Natural Vector than by a geometric (or spatial) 3D vector representing a (globally) time dependent location.

The Power of Non-Commutative Algebra in EM

Our use of mathematical symbols can improve the efficiency of processing concepts as this allow more information to be held in human short-term memory all at the same time. Physics, since Newton's *Principia*, has been very successful in representing key physical quantities by algebraic symbols, which allows the concept of multiplication between symbols to represent new combinations of concepts beyond simple addition. Until the revolution in algebra that occurred in the middle of the 19th Century this powerful technique was limited first to real numbers and then to complex numbers; both forming only commutative algebras, so that the order of multiplication of two different concepts was irrelevant. It was the invention of quaternions by William Rowan Hamilton, matrices by Arthur Cayley and higher algebras by William Kingdon Clifford that extended algebra to non-commutative forms. This was the necessary step that was needed to represent rotations in 3D space, where the order of a series of rotations is crucial but these new algebras were too late for Maxwell. Since rotation is an intrinsic view of magnetism it was now possible to use these higher algebras to represent the phenomenon of EM in a much more compact algebraic form than was available to Maxwell, who effectively used only commutative 3D vectors. At the end of his life Maxwell intuitively recognized the importance of Hamilton's quaternions and in his great *Treatise* tried to incorporate them into his final exposition of EM. He would have had to be very prescient to merge Cayley's 1855 definition of matrices into Hamilton's 1843 revolutionary algebraic breakthrough to produce the necessary extensions referred to in this research programme as Natural Vectors or 'scalar, complex quaternions' which have been shown here to be the simplest algebra to represent the classical theory of EM.

7.6 THE CNV ELECTROMAGNETIC MODEL

The novelty in this paper is found in section six where it is shown that all the important results of CEM are quickly obtained using just the general formulas for *Voigt Vectors* and other features of the CNV formalism that were introduced in section five. The present continuum theory is based on only simple definitional hypotheses whereby the four principal EM concepts are each mapped into individual Voigt Vectors, rather than the regular mix of standard 3D vector variables. These key EM concepts included the EM current, EM potential, EM force and EM energy. Both the usual Lorenz gauge condition and the EM momentum emerge quite naturally in this approach. Unlike the conventional textbook approach (that was described in section four) that derives Maxwell's Equations from line and surface integral macroscopic laws, the present approach only needs to propose that the charge density and EM scalar potential are constant across universal time (temporal invariants).

The present approach rapidly recovers the wave equations for all the EM variables (including the traditional force densities), even in the presence of free charge and currents, showing that all these variations propagate across space at 'light-speed' c. This conclusion is shown here to be mathematically equivalent to the assumption that the scalar functions, such as the EM potential and charge-density, are temporally invariant (or, in the case of charge-density, that it is 'incompressible'). The fact that all these fundamental EM quantities form Voigt Vectors is significant. In the case of the potentials, this leads directly to the well-known "Lorenz gauge condition" [101] that is seen to be necessary for both of the potentials to propagate at light-speed across the space between the interacting charges. Section 6.4 demonstrates that this gauge condition is the immediate consequence of redefining the base-line of the EM potential energy. The standard 3D force densities (traditionally referred to as the 'electric' and 'magnetic' fields $\underline{E}$ and $\underline{B}$) simply result from the application of the CNV gradient operator ($\boldsymbol{\nabla}^*$) to the CNV potential ($\mathbf{A}$). The misnamed 'Lorentz' force [142], which history should have credited to Heaviside [83], appears here ($\mathbf{R}$) in section 6.6 simply as either the total time derivative of the CNV energy density ($\mathbf{U}$) or, equally, as the product of the current-density CNV ($\mathbf{J}$) and the force-density CNV ($\mathbf{G}$).

One of the significant differences between the present CNV approach and the standard CEM approach is the latter's need to build upon Coulomb's law of electrostatics, which imposes the requirement of spherical symmetry on the propagation of the potential functions (see section 6.3). When this extra symmetry assumption is added **then** the standard *Maxwell Equations* involving real charges and currents immediately reappear here. The default here is the appearance of point-to-point ray-like interactions between interacting 'points' of charge. This 'linear' style of interaction reflects the inherent time-symmetry that occurs between pairs of interacting point 'objects'. The CNV formalism is intrinsically covariant [143] as the 4D nature of the EM interaction manifests itself naturally in this representation (hence the *Natural Vector* appellation). Several temporal invariants arise in a natural way without the need to invoke the hypotheses of relativity or the use of the so-called 'Lorentz' transformations of space and time [144].

The most surprising new result of the present model of EM is the demonstration that the speed variable for all Voigt-like CNVs that are 'gauge invariant' always has the same value: the so-called 'light-speed' c (see section 6.4). Mathematically:

$$\mathbf{H}(t;\underline{x}) = \beta(t;\underline{x})\,\mathbf{C}^* \qquad \text{where} \qquad \mathbf{C} = (i\,\mathbf{I_0} + \mathbf{I_3})\,c \qquad\qquad \text{So, for example:} \qquad \mathbf{A} = (-i\,\mathbf{I_0} + \mathbf{I_3})\,\phi_0$$

This illustrates why the common practice of eliminating the 'instantaneous' Coulomb (or $\mathbf{I_0}$ temporal) part of the EM interaction against the longitudinal component of the EM vector potential A_3 (see Feynman [145]) has proven so useful in QED. This 'light-speed' result would seem to be quite acceptable for the potential CNV ($\mathbf{A}$) but comes as a surprise for the current CNV ($\mathbf{J}$); this raises the intriguing question of what do the traditional symbols ρ and $\underline{J}$ actually represent? It seems clear that the interpretation of these symbols here represents something different than Maxwell's ρ_m and $\underline{J}_m$; perhaps as Whitaker noted (see §2.4.2) that Lorenz's potentials were not identical with Maxwell's potentials. This 'mystery' will be resolved in the next paper in this CEM series. As noted above, Maxwell (like everyone who has followed him) assumed that his scalar potential (ϕ_m) to be spherically symmetric. In both theories $\underline{\nabla}\bullet\underline{E} = -\Box\phi$ but it was Maxwell's 1864 theory [66] (involving his 'displacement current' as corrected by Heaviside, see §3.2) that implied the equation $\Box\phi_m = -4\,\pi\,\rho_m$; also Maxwell's vector potential satisfied: $\Box\underline{A}_m = -4\,\pi\,\underline{J}_m / c$. In contrast, the CNV generates the homogenous version $\Box\mathbf{A} = 0$. This analysis reinforces the view that it is not likely that the CNV potentials (ϕ, $\underline{A}$) are identical to Maxwell's original potentials (ϕ_m, $\underline{A}_m$), even though all the other differential equations have identical forms: the CNV potentials appear to more closely resemble the Lorenz potentials.

The need for the charge density ρ to commute with the CNV gradient operator (∇*), as was shown in section 6.5, imposes severe restrictions on any possible model of electricity. It was shown here that this implied either that the point model of the electron (where $\underline{J} = e\,\underline{v}$), or the electric 'fluid' model of Helmholtz [80] or the harmonic model of continuous electric charge density would all satisfy this requirement. The use of CNV mathematics in section 6.6 also generates an equivalent mass 'quantum' ($\mathcal{M}_0$) for representing the EM interactions that exchanges both mechanical momentum and energy between the interacting charges, subject to the general relationship: $U_{\mathbf{0}} = \mathcal{M}_0\, c^2$. The classical CNV ray-like EM interaction 'traveling at light-speed' and exchanging finite quantities of momentum and of energy from point to point appears to behave here like spatially directed '**photons**' [146] – this viewpoint will be investigated in detail and reported in a later paper in this series.

7.7 CONCLUDING REMARKS

This paper has shown that the particularization of Hamilton's quaternions, referred to here [4] as *Natural Vectors*, is a better representation than 3D vectors for remote interaction theories like classical EM. The approach followed here has been in the Newtonian tradition that was extended to the study of EM by Maxwell's Continental rivals and does not use any of the field-like concepts first introduced by Maxwell [66]. The present paper has deliberately restricted itself to classical EM, where interactions are assumed to occur continuously, as this theory is regarded as the best exemplar of a classical field theory.

It has been gratifying to see how Hamilton's great invention of quaternions (and vectors!) that Maxwell recognized in his *Treatise*, fits the problem of EM, so well. The purely linear algebraic formulation of Natural Vectors generates all the scalar and vector products of 3D vectors <u>and</u> the rotational mathematics (curl) that is so characteristic of Maxwell's results. Since the present approach is purely algebraic (involving only symbolic addition and multiplication) it eliminates the need for any form of tensor analysis, which often leaves the physics very cryptic. The non-commutative nature of Natural Vectors leads naturally to the idea of anti-symmetric functions, which characterize the real objects of nature (fermions) while suggesting fruitful possibilities when quantization will be later introduced. The NV formulation is automatically covariant across space and time while its basis vectors (Hamilton's four real unit quaternions) are intrinsically more powerful and symmetric across all four dimensions of nature than the 3D unit vectors used with geometric vector calculus or even Minkowski's 4-vectors [147]. The simple structure of the Voigt Vectors (summarized in section five) that appear repeatedly throughout the various NV theories of EM guarantee the 4D covariance of each theory with respect to Galilean transformations so that they remain compatible with all aspects of classical physics right back to Newtonian mechanics. Furthermore, the failure of the so-called 'Lorentz' force vector to comply with this requirement foreshadows the problems that the theory of Special Relativity was forced to address in its attempt to make Maxwellian field theories compatible with Newtonian particle mechanics.

In this paper, the mathematical framework associated with NVs was restricted to CNVs in order to accommodate the implicit continuum hypothesis of CEM (i.e. charge-density and EM potential are continuous functions of space and time). However, it will be shown in subsequent papers in this series that the NV approach can be readily extended to discrete mathematics (using finite differences) to match more closely Newton's original approach to classical mechanics. The physical hypotheses will also be extended to richer models of electricity, which more closely reflect the known discrete reality of the micro-world. This will be the arena for the next paper in this series when the system of two continuously interacting *point* electrons will be addressed in a comprehensive (but still 'classical') electro-dynamic manner, in other words, beyond the simplistic instantaneous Coulomb approximation. The following paper after that will also show how certain implicit assumptions in all earlier EM field theories lead to the dynamical results found in Special Relativity. This later paper will remove the restriction of *continuous* interactions occurring between two point electrons: replacing the concept of '**force**' with Newton's original concept of discrete '**impulse**' but now with the addition of <u>asynchronous</u> time differences. This fourth paper in this CEM series will combine Gauss's original vision [148] with that of Newton [7] to produce a unified physics of particulate matter. A later series (QEM) will explore the consequences of quantizing the magnitude of the EM interaction between electrons, at both the atomic and nuclear levels of spatial separation.

In closing, the words of one of the major pioneers of Quantum Field Theory, Freeman Dyson, from his most important paper on the S-Matrix [149] might seem appropriate at this stage: "The elements of this paper have been essentially mathematical in character, being concerned with the consequences of a particular mathematical formalism. In attempting to assess their significance for the future one must pass from the language of mathematics to the language of physics. One must assume provisionally that the mathematical formalism corresponds to something existing in nature, and then enquire to what extent the paradoxical results of the formalism can be reconciled." The remaining papers in this research programme using Natural Vectors will demonstrate this reconciliation.

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