Title :DARK MATTER:FLAT ROTATION CURVE OF GALAXIES-TULLY-FISHER LAW
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Abstract:

The article proposes a new model of dark matter. According to this new model, dark matter is a substance, that is a new physical element not constituted of classical particles, called *dark substance*, filling the universe and constituting what is called "emptiness". Assuming some very simple physical properties to this dark substance, we theoretically justify the flat rotation curve of galaxies and the baryonic Tully-Fisher's law.

Key words: Tully-Fisher's law, dark matter, dark halo, flat rotation curve of galaxies, dark radius.

1.INTRODUCTION

The objective of this study is to propose fundamental theoretical discoveries relative to nature of dark matter, that could be used in the basis of a general and complete theory of dark matter. In this article, we propose that a new physical element, called *dark substance*, constitutes the dark matter. According to the proposed model of dark matter, this dark substance fills all the Universe and has physical properties close to the physical properties of an ideal gas. Using those properties, we justify theoretically the flat rotation curve that is observed for some galaxies, in a new way, with density of dark substance in $1/r^2$. A simple mathematical expression of the density of dark matter (in $1/r^2$) permitting to obtain the flat rotation curve which has already been proposed, but the model of dark matter that permits to justify theoretically this mathematical expression (in $1/r^2$) has never been proposed. Moreover the study hypothesizes simple thermal properties to this dark substance which exist in our model of dark matter that permit to justify theoretically the baryonic Tully-Fisher's law. The theory called MOND [1] also proposes a theoretical justification of the flat rotation curve of some galaxies, but this theory is contrary to Newton's attraction law and moreover it is contradicted by some astronomical observations [2]. Different models of distribution of dark matter in galaxies are proposed in this study.

We remind that for many astrophysicists and physicists, the enigmas in the SCM, in particular the enigmas concerning dark matter and dark energy, make necessary a new paradigm for the SCM [3] . Our article proposes a new model of dark matter that could belong to such a new paradigm.

The model of dark matter that we propose is compatible with the Standard Cosmological Model (SCM) as it is presented in books [4][5].

2. THEORY OF DARK MATTER

2.1 Physical properties of the dark substance.

As we have seen in 1.INTRODUCTION, we stated the Postulate 1 expressing the physical properties of the dark substance:

Postulate 1:

a)A substance, called *dark substance*, fills all the Universe.

b)This substance does not interact with photons crossing it.

c)This substance owns a mass and obeys to the Boyle's law (called also Mariotte's law), to the Charles'law (called also Gay-Lussac's law), and to the following law that is their synthesis:
An element of dark substance with a mass m, a volume V, a pressure P and a temperature T verifies, k_0 being a constant:
$PV=k_0mT$

The preceding law is a valid statement for a given ideal gas G_0, replacing k_0 by a constant $k(G_0)$, and this is a consequence of the *universal gas equation*, which is also obtained using Boyle and Charles'laws. For this reason we will call it the *Boyle-Charles'law*.

We have 2 remarks consequences of this Postulate1:

-First, the dark substance is not really dark but translucent despite of its name. Indeed, because of the preceding Postulate 1b) it does not interact with photons crossing it.

-Secondly because of the Postulate 1a), what is usually called "emptiness" is not empty in reality: It is filled with dark substance.

2.2 Flat rotation curves of galaxies.

Using the fact that the dark substance behaves as an ideal gas (Postulate 1c), we are going to show that a spherical concentration of dark substance in gravitational equilibrium can constitute the dark matter in a galaxy with a flat rotation curve.

According to Postulate 1c) an element of dark substance with a mass m, a volume V, a pressure P and a temperature T verifies the law, k_0 being a constant:

$$PV=k_0mT \qquad (1)$$

Which means, setting $k_1=k_0T$:

$$PV=k_1m \qquad (2)$$

Or equivalently, ρ being the mass density of the element:

$$P=k_1\rho \qquad (3a)$$

We hypothesized that the galaxy can be modeled as a concentration of dark substance with a spherical symmetry, at an homogeneous temperature T, in gravitational equilibrium.
We considered the spherical surface S(r) (resp. the spherical surface S(r+dr)) that is the spherical surface with a radius r (resp. r+dr) and whose the centre is the center O of the galaxy. S(O,r) is the sphere filled with dark substance with a radius r and the centre O.

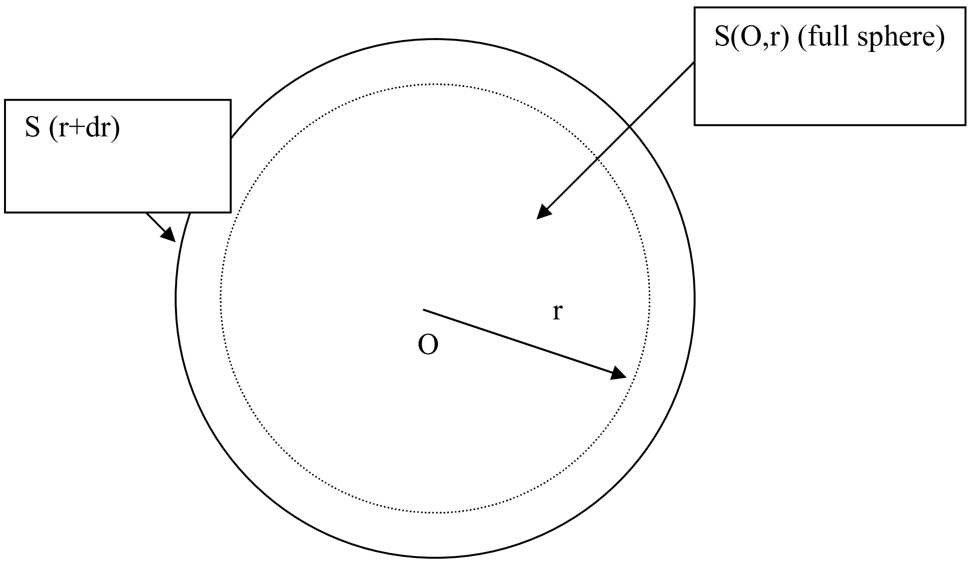


Figure 1:The spherical concentration of dark substance

The mass M(r) of the sphere S(O,r)is given by:

$$M(r) = \int_0^r \rho(x) 4\pi x^2 dx \qquad (3b)$$

Assuming a spherical symmetry for the density of dark substance, using Newton's law ($\Sigma\mathbf{F}=\mathbf{0}$ for a material element in equilibrium with a mass m, $\mathbf{F}_G(r)=m\mathbf{G}(r)$, $\mathbf{F}_G(r)$ gravitational force acting on the element, $\mathbf{G}(r)$ gravitational field defined by Newton's universal law of gravitation) and Gauss theorem in order to obtain $\mathbf{G}(r)$, we obtain the following equation (4) of equilibrium of forces on an element dark substance with a surface dS, a width dr, situated between S(O,r) and S(r+dr):

$$dSP(r+dr) + \frac{G}{r^2}(\rho(r)dSdr)(\int_0^r \rho(x)4\pi x^2 dx) - dSP(r) = 0 \qquad (4)$$

Eliminating dS, we obtain the equation:

$$\frac{dP}{dr} = -\frac{G}{r^2}(\rho(r))(\int_0^r \rho(x)4\pi x^2 dx) \qquad (5)$$

And using the equation (3) obtained using the Boyle-Charles'law assumed in the Postulate 1, we obtain the equation:

$$k_1 \frac{d\rho}{dr} = -\frac{G}{r^2}(\rho(r))(\int_0^r \rho(x) 4\pi x^2 dx) \qquad (6)$$

We then verify that the density of the dark substance ρ(r) satisfying the preceding equation of equilibrium is the evident solution:

$$\rho(r) = \frac{k_2}{4\pi r^2} \qquad (7)$$

(A density of dark matter expressed as in Equation (7) has already been proposed to explain the flat rotation curve of spiral galaxies, but it has not been proposed a model of dark matter permitting to justify theoretically this density in $1/r^2$ or to obtain the constant k_2. Here we give a theoretical justification of this density in $1/r^2$ and we are going to give the expression of the constant k_2 (Equation (8)). This is the consequence of the model of dark substance as an ideal gas, Postulate 1).

In order to obtain k_2 , we replace ρ(r) given by the expression (7) inside the equation (6), and we obtain immediately that this equation is verified for the following expression of k_2:

$$k_2 = \frac{2k_1}{G} = \frac{2k_0 T}{G} \qquad (8)$$

Using the preceding equation (7), we obtain that the mass M(r) of the sphere S(O,r) is given by the expression:

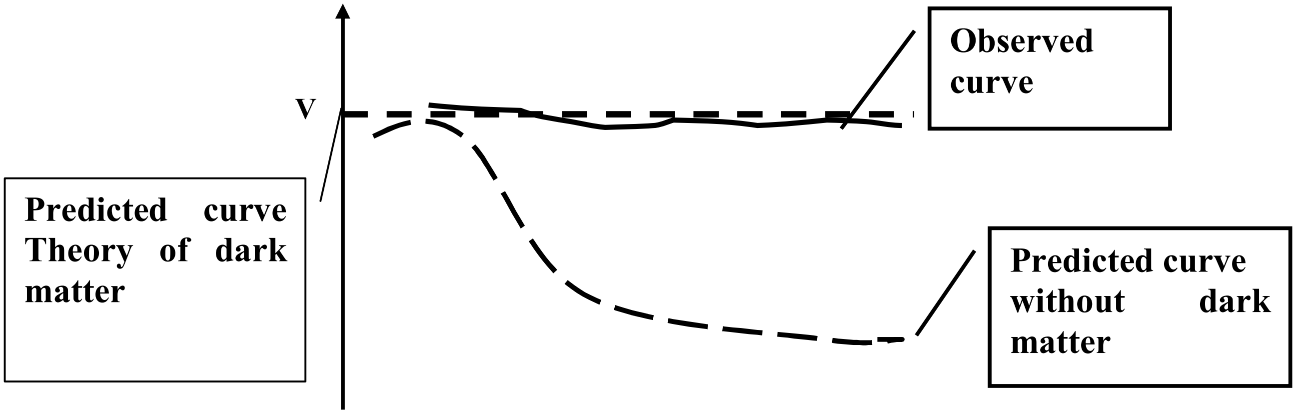
$$M(r) = \int_0^r 4\pi x^2 \rho(x) dx = k_2 r \quad (9)$$

We then obtain, neglecting the mass of stars in the galaxy, that the velocity v(r) of a star of a galaxy situated at a distance r from the center O of the galaxy is given by $v(r)^2/r = GM(r)/r^2$ and consequently :

$$v(r)^2 = Gk_2 = 2k_1 = 2k_0 T \qquad (10)$$

So we obtain in the previous equality (10) that the velocity of a star in a galaxy is independent of its distance to the centre O of the galaxy.

V(r) ORBITAL VELOCITIES OF STARS



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DISTANCE OF STARS TO THE CENTRE O OF THE GALAXY

Figure 2 :Rotation curve of galaxies

2.3 Baryonic Tully-Fisher's law.

2.3.1 Recall.

Tully and Fisher realized some observations on spiral galaxies with a flat rotation curve. They obtained that the luminosity L of such a spiral galaxy is proportional to the 4^{th} power of the velocity v of stars in this galaxy [6]. So we have the Tully-Fisher's law for spiral galaxies, K_1 being a constant:

$$L=K_1v^4 \qquad (11)$$

But in the cases studied by Tully and Fisher, the baryonic mass M of a spiral galaxy is usually proportional to its luminosity L. So we have also the law for such a spiral galaxy, K_2 being a constant:

$$M=K_2v^4 \qquad (12)$$

This 2^{nd} form of Tully-Fisher's law is known as the *baryonic Tully-Fisher's law*.

The more recent observations of Mc Gaugh [7] show that the baryonic Tully-Fisher's law (equation (12)) seems to be true for all galaxies with a flat rotation curve, including the galaxies with a luminosity not proportional to their baryonic mass.

We are going to demonstrate that using the Postulate 1 and a Postulate 2 expressing very simple thermal properties of the dark substance, (in particular its thermal interaction with baryonic particles), we can justify this baryonic law of Tully-Fisher.

2.3.2 Theory of quantified loss of calorific energy (by nuclei).

We saw in the previous equation (10) that according to our model of dark substance the square of the velocity of stars in a galaxy with a flat rotation curve is proportional to the temperature of the concentration of dark substance constituting this galaxy. So we need to determinate T:

-A first possible idea is that the temperature T refers on CMB. But this is impossible because it would imply all the stars of all galaxies with a flat rotation curve be driven with the same velocity and we know that it is not the case.

-The second possibility is that in the considered galaxy, each baryon interacts with the dark substance constituting the galaxy, transmitting to a thermal energy. We can expect that this thermal energy is very low otherwise it would already have been observed, but because of the expected very low density of the dark substance and of the considered times (we remind that the baryonic diameter of galaxies can reach 100000 light-years), it can lead to appreciable temperatures of dark substance. A priori we could expect that this loss of thermal energy for each baryon (transmitted to the dark substance) depends on the temperature of this baryon and of the temperature T of the dark substance in which the baryon is immerged, but if it was the case, the total lost thermal energy by all the baryons would be extremely difficult to calculate and moreover it should be very probable that we would then be unable to obtain the very simple baryonic Tully-Fisher's law.

The hypothesis of the study is defining the thermal transfer between dark substance and baryons, expressed in the following Postulate 2a) (Postulate 2 gives the thermal properties of the dark substance):

Postulate 2a):

-Each nucleus of atom in a galaxy is submitted to a loss of thermal energy, transmitted to the dark substance in which it is immerged.

-This thermal transfer depends only on the number n of nucleons constituting the nucleus (So it is independent of the temperature of the nucleus). So if p is the thermal power dissipated by the nucleus, it exists a constant p_0 (thermal power dissipated by nucleon) such that:

$$p=np_0 \qquad (13)$$

According to the equation (13), the total thermal power transmitted by all the atoms of a galaxy towards the spherical concentration of dark matter constituting the galaxy is proportional to the total number of nucleons of the galaxy and consequently to the baryonic mass of this galaxy. So if m_0 is the mass of one nucleon, M being the baryonic mass of the galaxy, we obtained according to the equation (13) that the total thermal power P_r received by the spherical concentration of dark substance constituting the galaxy from all the atoms is given by the following equation, K_3 being the constant p_0/m_0:

$$P_r=(M/m_0)p_0=K_3M \qquad (14)$$

Concerning the preceding Postulate 2a):

-It is possible (but not compulsory) that it be true only for atoms whose temperature is superior to the temperature T of the concentration of dark substance.

-It permits to obtain the very simple Equation (14). We will see that this equation is essential to obtain the baryonic Tully-Fisher's law.

2.3.3 Obtainment of the baryonic Tully-Fisher's law.

In agreement with the previous model of galaxy (Section 2.2), we modeled a galaxy with a flat rotation curve as a spherical concentration of dark substance, at a temperature T and surrounded itself by a medium constituted of dark substance (called "intergalactic dark substance") with a temperature T_0 and a density ρ_0.

It is natural to make the hypothesis of the continuity of $\rho(r)$: R is the radius for which the density $\rho(r)$ of the concentration of dark substance is equal to ρ_0 to obtain the radius R of the concentration of dark matter constituting the galaxy. We will call R the *dark radius* of the galaxy. So we have the equation:

$\rho(R)=\rho_0$ (15)

The equation according to (7) and (8):

$$\frac{k_2}{4\pi R^2}=\rho_0 \qquad (16)$$

$$\frac{2k_0T}{G}\times\frac{1}{4\pi R^2}=\rho_0 \qquad (17)$$

So we obtain that the radius R of the concentration of dark substance constituting the galaxy is given approximately by the equation:

$$R=(\frac{2k_0T}{4\pi G\rho_0})^{1/2}=K_4T^{1/2} \qquad (18)$$

The constant K_4 being given by :

$$K_4=(\frac{2k_0}{4\pi G\rho_0})^{1/2} \qquad (19)$$

Then we consider that the sphere with a radius R of dark substance at the temperature T is in thermal interaction with the medium constituted of intergalactic dark substance at the temperature T_0 surrounding this sphere. The simplest and most natural thermal transfer is the convective transfer. We stated this in the Postulate 2b):

Postulate 2b):

The thermal interaction between the spherical concentration of dark substance constituting the galaxy (with a density of dark substance in $1/r^2$ and a homogeneous temperature T) and the surrounding intergalactic dark substance (at the temperature T_0) can be modeled as a convective thermal transfer.

We know that if φ is the thermal flow of thermal energy on the borders of the spherical concentration of dark substance with a radius R, P_l being the total power lost by the spherical concentration of dark substance constituting the galaxy is given by the equation:

$P_l=4\pi R^2\varphi$ (20)

But we know that according to the definition a convective thermal transfer between a medium at a temperature T and a medium at a temperature T_0 and according to the previous Postulate 2b) the flow φ between the 2 media is given by the expression, h being a constant depending only on ρ_0:

$\varphi=h(T-T_0)$ (21)

The total power lost by the concentration of dark substance is:

$$P_l=4\pi R^2h(T-T_0) \qquad (22)$$

We can consider that at the equilibrium, the total thermal power P_r received by the spherical concentration of dark substance constituting the galaxy is equal to the thermal power P_l lost by this spherical concentration. According to the equations (14) and (22), (M being the baryonic mass of the galaxy), we have:

$$K_3M=4\pi R^2h(T-T_0) \qquad (23)$$

Using then the equation (18) :

$$K_3M=4\pi K_4^2hT(T-T_0) \qquad (24)$$

Making the approximation $T_0<<T$:

$$M=4\pi\frac{K_4^2}{K_3}hT^2 \qquad (25)$$

Consequently we obtain the expression of T, defining the constant K_5 :

$$T=(\frac{K_3}{4\pi K_4^2h})^{1/2}M^{1/2}=K_5M^{1/2} \qquad (26)$$

And then according to the equation (10) :

$$v^2=2k_0T=2k_0K_5M^{1/2} \qquad (27)$$

So :

$$M=(\frac{1}{2k_0K_5})^2v^4 \qquad (28)$$

So we finally obtain :

$$M=K_6v^4 \qquad (29)$$

The constant K_6 being defined by:

$$K_6=(\frac{1}{2k_0K_5})^2=\frac{4\pi K_4^2h}{4k_0^2K_3} \qquad (30)$$

$$K_6=\frac{4\pi h}{4k_0^2K_3}\times\frac{2k_0}{4\pi G\rho_0} \qquad (31)$$

$$K_6=\frac{m_0h}{2k_0G\rho_0p_0} \qquad (32)$$

We obtain the baryonic Tully-Fisher's law (12), with $K_2=K_6$. It is natural to assume that h depends on ρ_0. The simplest expression of h is $h=C_1\rho_0$, C_1 being a constant. With this relation, K_6 is independent of ρ_0, and we can use the baryonic Tully-Fisher's law to define candles used to evaluate distances in the Universe.

2.4 Superposed sphere.

We consider a spherical concentration of dark substance with a density in $1/r^2$ (that we defined in previous sections) moving in the space. Potentially we assume that its velocity or its mass be modified because of its motion, of the Archimedes's force or the absorption or the loss of dark substance by the moving concentration of dark substance. This effect could be negligible, but we have a justification that it is nil much more interesting.

Indeed according to the new proposed model of dark matter, dark substance can own 2 possible behaviors: It can behave as a substance owning a mass or as absolute emptiness. For baryonic particles immerged inside dark substance, it always behaves as absolute emptiness and consequently the velocity of baryonic particles is never modified due to an Archimedes's force generated by the motion of baryonic particles through the dark substance. According to the new theory of dark matter, the intergalactic dark substance in which the spherical concentration of dark substance (with density in $1/r^2$) is immerged also behaves as it was absolute emptiness concerning the displacement of this spherical concentration of dark substance: Neither the velocity nor the mass of the spherical concentration of dark substance are modified by its motion through the intergalactic dark substance. We will say that the spherical concentration of dark substance is a *superposed sphere* on the intergalactic dark substance surrounding it to interpret this phenomenon.

So we can define 2 kind of radius for a galaxy with a flat rotation curve: The 1st radius is the baryonic radius and the 2nd kind of radius is the dark radius, which is the radius of the superposed sphere containing the galaxy.

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3.DISCUSSION

So the proposed model of dark matter is very attractive for the following reasons:

(i)It explains the invisibility of dark matter, because dark substance constitutes what is called "emptiness".

(ii)It justifies the observations that dark matter does not interact through electromagnetic interaction for the same reason as (i).

(iii)It justifies that collision between ordinary matter and dark substance has never been observed for the same reason as (i) and also because dark substance can behave as absolute emptiness.

(iv)It permits to obtain the flat rotation curve of some galaxies very easily, using the very known law of ideal gas.

(v)It permits to obtain very easily the baryonic Tully-Fisher's Law, using very simple thermal model.

(vi)It is compatible with Newton's Laws for baryonic matter (Contrary to MOND).

(vii)It is compatible with Special and General Relativity because it does not use any inertial frame.

Dark substance being a special substance, it does not own necessarily the same physical properties as ordinary baryonic matter. So we propose 2 fundamental properties for dark substance that are different from properties of ordinary matter:

The 1st property has been exposed in the preceding section: Dark substance has the remarkable property of being able to behave sometimes as absolute emptiness and sometimes as ordinary matter with a mass. The fact that dark substance can also behave as absolute emptiness implies also the possibility that we must take $T_0=0$ in equation (21).

A 2nd fundamental property of dark substance that we will admit is that sometimes it can tend to be homogeneous, its density not obeying to Newton's Law and sometimes its density obeys to Newton's Laws. This 2nd fundamental property is important because if we admit that at the scale of a star or of a black hole the tendency to homogeneity of dark substance predominates, then there is not concentration of dark substance around stars belonging to any galaxy. Consequently it exists 2 main kinds of galaxies: Galaxies belonging to a concentration of dark matter with a density of dark substance in $1/r^2$, as for instance the Milky Way and any galaxy with a flat rotational curve, and galaxies belonging to the intergalactic dark substance with a density of dark substance that is constant, as for instance Giant Elliptical Galaxies, in which few dark matter was detected.

The previous fundamental property permits also to interpret the observation of the curve of a galaxy G with a flat rotation curve close to O_G center of G. Close to O_G the density of dark substance cannot be in $1/r^2$ because it would imply that it is infinite in O_G (which is not acceptable) and moreover because according to observation the rotation curve is not flat close to O_G ($v(O_G)=0$, then $v(r)$ increases with r). This can be explain if we admit, using the 2nd fundamental property of dark substance, that close to O_G, dark substance is homogeneous. This is true if we admit that the density of dark substance at a point P $\rho(P)$ obeys to Newton's Law if we have the necessary condition $\rho(P)<\rho_{LIMH}(T)$, $\rho_{LIMH}(T)$ being a given value that could (not necessarily) depend on the temperature $T(P)$ of the dark substance in P. Without this condition, the tendency to homogeneity predominates in P.

The proposed model of dark matter is not a complete theory of dark matter. For instance, we need to define completely the properties of dark substance, and also to interpret dynamics of galaxy clusters. A complete theory of dark matter, including study of dynamics of galaxy clusters, with a new Cosmological Model has already been published [8], but some elements of this new Cosmological model are incompatible with SCM, consequently if principles of SCM are postulated, the preceding theory of dark matter is not acceptable, contrary to the new model of dark matter exposed here.

Considering the interest of the properties of our model of dark substance exposed in this article, it is very possible, if SCM is correct, that our model of dark matter can be included in a general theory of dark matter in agreement with SCM, established by future developments.

So the new model of dark matter proposed in this article is likely to be an inevitable step in the understanding of the nature of dark matter.

4.CONCLUSION

In this article, we have modeled dark matter as a dark substance whose the physical properties, and in particular the fact that it can be modeled as an ideal gas, permitted to interpret 2 fundamental astronomical observations linked to dark matter. For instance, those physical properties permitted us to justify theoretically the flat rotation curve of galaxies and the baryonic Tully-Fisher's law. To obtain this, we interpreted galaxies with flat rotation curve as spherical concentrations of dark substance in gravitational equilibrium. We also obtained the density of dark substance close to the center O_G of a galaxy with a flat rotation curve.

We have studied according to our theory of dark matter the effects of the displacement of a concentration of dark substance on its mass and its velocity, and we have seen that those effects were nil. We saw that this theory permitted to define mathematically completely 2 kinds of radius for galaxies: The baryonic radius and the dark radius. We then exposed according to this theory the different models of distribution of dark matter in galaxies.

We have seen that the new model of dark matter was compatible with SCM.

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